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Jung, R C; Winkelmann, R
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Abstract

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Voluntary and Involuntary Labor Mobility:  
A Bivariate Poisson Regression Approach*

Robert C. Jung  
University of Konstanz

Rainer Winkelmann  
SELAPO, University of Munich

SUMMARY  
The study introduces a distinction between two types of labor mobility: Direct job to job changes (which are assumed to be voluntary) and job changes after experiencing an unemployment spell (assumed to be involuntary). Exploiting the close relationship between those two phenomena we adopt a bivariate regression framework for our empirical analysis of data on male individuals in the German labor market. To account for the non-negative and discrete nature of the two counts of job changes in a ten year interval a new econometric model is proposed: the bivariate Poisson regression proves to be superior to the univariate specification. Further, the empirical content of distinguishing between two types of mobility is subject to a test, and, in fact, supported by the data: The hypothesis that both measures are observationally equivalent can be rejected.

JEL codes: C25, J24, J60

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1 Introduction

Labor mobility is a pervasive feature of market economies. Individuals typically hold several jobs during their working career. In a recent paper, Topel and Ward (1992) report an average of 9 job changes during lifetime for males in the US. Moreover, most of these changes occur at early stages of the career. In the US, an average of two out of three lifetime job changes occur during the first ten years after entering the job market (Topel and Ward, 1992). Similar own calculation for males in the German labor market, based on the German socio–economic panel, indicate a lower overall mobility of typically 3 lifetime job changes. Again, the majority of these job changes occur at early stages of the career (45% in the first ten years). This paper aims at providing some insights into two important conceptual questions that have to be addressed by any empirical study on labor mobility. First, what is the appropriate econometric technique for modeling labor mobility and second, is it admissable to treat all job changes alike, or is it preferable to distinguish between job changes with and without an intervening spell of unemployment?

Both theoretical and empirical work on labor mobility has traditionally identified mobility with either the durations of job tenure or the propensity to change jobs within a given interval, leading to the use of the corresponding econometric techniques, either duration analysis, or Probit and Logit models. We argue here in favor of a less common approach, the use of count data models for studying mobility. The arguments are set out in some detail in the next section. While the use of count data models for modeling labor mobility has been previously proposed by Gilbert (1979) and Börsch-Supan (1990), the methodology is extended by allowing for bivariate modeling.

The necessity of a bivariate econometric model arises, since a distinction is made between direct changes, i.e. job changes without an intervening spell of unemployment, and job changes via unemployment. This is in the spirit of Jovanovic (1984) who models movements in and out of unemployment, as well as changes of employers. Here, the view is taken that direct job changes reflect ‘voluntary mobility’, while job changes via unemployment constitutes the part of mobility which is ‘involuntary’. The distinction between the two types of mobility is thus not based on the type of separation: Separations initiated by the employer (and labeled ‘layoff’) versus separations initiated by the employee (and labeled ‘quit’) or some intermediate cases (‘separation in mutual agreement’) (See McLaughlin 1991). Here, the distinctive feature is the intervening spell of
unemployment. This distinction is based on the assumption that individuals ‘dislike’ unemployment, whether it is initiated by a quit or by a layoff, and would always prefer direct job changes to job changes via unemployment if the option existed, leading to the labels of ‘voluntary’ and ‘involuntary’ mobility.

Given this framework, individual working histories can be characterized by two counts: first, the number of voluntary job changes, and second, the number of involuntary job changes. One could proceed modeling both aspects separately. However, they can be expected to be closely related, representing a competing risk during individual work histories. Therefore, a new econometric model is implemented, based on the bivariate Poisson distribution. It allows for the simultaneous estimation of the correlation structure between the dependent variables and the regression coefficients.

SECTION 2 provides arguments for the use of count data models, and discusses some limitations. The bivariate Poisson regression model is presented in SECTION 3. Data and sampling issues are discussed in SECTION 3, while SECTION 4 contains the regression results. The results indicate that the correlation between voluntary and involuntary job changes is significant. A comparison of the regression coefficients with the coefficients of univariate estimations displays a remarkable stability. However, a gain in efficiency can be noted. Finally, the results allow to test whether the two types of mobility have possibly been generated by the same regression model, indicating that they are observationally equivalent and their distinction is artificial. This hypothesis is clearly rejected.

2 Labor Mobility, Counts, and Human Capital

There are several reasons suggesting the use of count data models instead of the alternatives provided by duration models or panel-probit models. First, it is the only feasible approach if no information on the timing of the events, but just the number of occurrences is available. This is, for instance, the case for the retrospective question in the German socio-economic panel:

‘How many jobs have you held between 1974 and 1984?’.

Second, it might still be a good choice, even if information on the timing is available. The argument rests on the fact that mobility, especially for the German case, is a rare event.
Own calculations based on the German socio-economic panel show that the average male individual in Germany changes jobs only 3 times during his working career of typically around 40 years. Considering the ten year period 1974-84, the average number of job changes is given by 0.76 (This number was calculated considering only male individuals which were in the labor force both at the beginning and at the end of the period.) If one disregards job changes with an intervening spell of unemployment, this number reduces to 0.52. Also, 61% of the individuals have never changed employer during the ten year period (their employment spells are right censored), and 69% had no ‘direct job change’. This implies that using Probit models with annual intervals, on average in nine out of ten years no event is observed. If the individuals with right censored employment spells were employed in 1974, an information that is not available, these spells are both left and right censored. For these individuals, both duration and count data models (reporting the outcome ”0”) use the same amount of information.

The low mobility implies little information in the data. We therefore argue that it is justified to aggregate to a longer period and ignore the timing of the events. This is what a count data model does, leading to models which are relatively simple to formulate and to estimate. The objection that a probit model could then be used as well neglects the presence of repeated mobility. For instance, despite the low mean number of job changes, 18% of the individuals did experience repeated mobility. This information about the endogeneous variable is used in a count data model, but not in a Probit model (where repeated mobility can only be incorporated as a regressor).

There are also some important limitations. Analyzing labor mobility with a count data model is essentially a reduced form approach. In particular, labor mobility certainly depends on perceived wage gains from changing job, which in turn are a function of past mobility. The interactions between wage dynamics and labor mobility cannot be observed using count data. Here, just the result of the interaction is modeled using economic theory for suggesting the relevant regressor variables.

Among the most prominent determinants of mobility are current job tenure and overall labor market experience. These have been stressed by recent microeconomic approaches explaining individual labor mobility in the framework provided by the theory of human capital (Mincer and Jovanovic 1981, McLaughlin 1991). One main prediction of the human capital approach to labor mobility is that, as long as human capital investments have
partly a firm specific component, labor mobility is negatively correlated with current job
tenure. The negative correlation arises since the firm specific human capital accumulated
with tenure is lost when changing the employer. The amount of firm specific human
capital at a specific point in time depends both on the duration, as well as on the speed,
of accumulation. The latter, in turn, may be affected by education and other individual
characteristics.

A similar pattern exist for experience-mobility profiles. Again, a negative relationship
is predicted. However, a distinction has to be made between ‘true’ experience effects, and
indirect effects via job tenure (Mincer and Jovanovic, 1981): Experience-mobility profiles
pick up tenure effects if the latter are not (or cannot) be controlled for. Let the propensity
to change the job \( m \) be a function of both tenure \( T \) and experience \( X \). Then

\[
\frac{dm}{dX} = \frac{\partial m}{\partial T} \cdot \frac{dT}{dX} + \frac{\partial m}{\partial X}.
\]

Only \( \frac{\partial m}{\partial X} \) is a ‘true’ experience effect. It is joined by an indirect tenure effect since
tenure grows with experience. Clearly, \( 0 < \frac{dT}{dX} < 1 \), and mobility declines with
experience also if there is no true experience effect, solely due to the increase of firm specific
human capital over time. Since the definition of ‘current’ job tenure has no meaning
when observing repeated mobility, tenure and experience effects cannot be separated in a
count data approach. However, labor market experience (at the beginning of the period)
certainly is an important determinant of the number of job changes in a given period.

3 Bivariate Poisson distribution

Statistical theory has derived quite different bivariate Poisson distributions. Kocherlakota
and Kocherlakota (1992, pp.87-90) provide a comprehensive discussion. The formulation
chosen here is a natural extension of the univariate Poisson distribution while allowing
for correlation among the random variables under inspection. We therefore follow several
other authors and denote it simply as the bivariate Poisson. It arises in several ways.
Among those, the so called trivariate reduction method provides useful insights and there-
fore will be presented here. Let the random variables \( V_1, V_2 \) and \( U \) be independently
Poisson distributed with \( V_j \sim Po(\theta_j), j = 1, 2, \) and \( U \sim Po(\gamma) \). New random variables \( Y_j \)
can be constructed by
\[ Y_j = V_j + U \quad j = 1, 2 \tag{2} \]

\( Y_j \) is a convolution of two Poisson random variables (Feller, 1968) and \( Y_j \sim \text{Po}(\theta_j + \gamma) \) with probability generating function \( G_{Y_j}(s_j) = \exp \left[(\theta_j + \gamma)(s_j - 1)\right] \). The joint probability generating function \( G(s_1, s_2) \) takes the following form:

\[ G(s_1, s_2) = E(s_1^{Y_1} s_2^{Y_2}) = E(s_1^{V_1} V_2^{Y_2}(s_1 s_2)^U) = \exp \left[ \theta_1(s_1 - 1) + \theta_2(s_2 - 1) + \gamma(s_1 s_2 - 1) \right]. \tag{3} \]

This gives rise to the probability function

\[ P(Y_1 = y_1, Y_2 = y_2) = \exp \left[ -(\theta_1 + \theta_2 + \gamma) \sum_{j=0}^{s} \frac{\gamma^j}{j!} \frac{\theta_1^{y_1-j}}{(y_1-j)!} \frac{\theta_2^{y_2-j}}{(y_2-j)!} \right], \tag{4} \]

with \( s = \min(y_1, y_2) \). The covariance between \( Y_1 \) and \( Y_2 \) can be derived as follows:

\[ \text{Cov}(Y_1, Y_2) = \text{Cov}(V_1 + U, V_2 + U) \]
\[ \overset{\star}{=} \text{var}(U) \]
\[ = \gamma \tag{5} \]

where \( \star \) directly follows from the independence of \( U \) and \( V_j \). Normalization by the standard errors of the two random variables yields the typical correlation form:

\[ \text{Corr}(Y_1, Y_2) = \frac{\gamma}{\sqrt{(\theta_1 + \gamma)(\theta_2 + \gamma)}}. \tag{6} \]

The correlation is non-negative. As for the bivariate normal, zero correlation is a necessary and sufficient condition for the independence of the random variables \( Y_1 \) and \( Y_2 \).

Following the standard approach in univariate Poisson regression we model the marginal expectation of \( Y_1 \) and \( Y_2 \), respectively, as a loglinear function of exogenous variables.

\[ \theta_j + \gamma = \exp(x_j' \beta_j) \quad j = 1, 2 \tag{7} \]
The $k$-dimensional vector of covariates $x_j$ may include a constant term. The formulation (7) is quite flexible allowing for different regression functions for the two dependent variables as well as for identical ones (The two sets of regressors $x_j$, $j = 1, 2$ will be identical in the empirical application). Substitution of $\theta_1$ and $\theta_2$ using $\exp(x'_j \beta_j) - \gamma$ in the probability function (4) and multiplication over all observations yields the likelihood function, which is given in logarithmic form here:

$$\log L(\beta_1, \beta_2, \gamma \mid y_{11}, \ldots, y_{1n}; y_{21}, \ldots, y_{2n}; x_{11}, \ldots, x_{1n}; x_{21}, \ldots, x_{2n}) =$$

$$n \gamma - \sum_{i=1}^{n} \exp(x'_{1i} \beta_1) - \sum_{i=1}^{n} \exp(x'_{2i} \beta_2) + \sum_{i=1}^{n} \log B_i \quad (8)$$

with

$$B_i = \sum_{k=0}^{s_i} \frac{\gamma^k}{k!} \frac{[\exp(x'_{1i} \beta_1) - \gamma]^{y_{1i} - k}}{(y_{1i} - k)!} \frac{[\exp(x'_{2i} \beta_2) - \gamma]^{y_{2i} - k}}{(y_{2i} - k)!},$$

where $s_i = \min(y_{ji})$, $j = 1, 2$ and $y_j$ is the $n$-dimensional vector of the observed dependent variables. For $\gamma = 0$, the log-likelihood can be factored into two independent parts

$$\log L(\beta_1, \beta_2, \gamma \mid .) = \log L(\beta_1 \mid y_{11}, \ldots, y_{1n}; x_{11}, \ldots, x_{1n}) + \log L(\beta_2 \mid y_{21}, \ldots, y_{2n}; x_{21}, \ldots, x_{2n}) \quad (9)$$

each of which is the log-likelihood of an univariate Poisson regression. A similar specification of the model (8) has been presented by King (1989), denoted as seemingly unrelated Poisson regression model (SUPREME). Also, Gourieroux, Monfort and Trognon (1984b) provide a model closely related to ours. They suggest the use of robust Poisson regression methods in order to correct for a misspecified mean function.

We were not able to show global concavity of the (log-)likelihood function (8) yet. But given the fact that we combine two random variables whose marginal distributions are Poisson - where it is relatively easy to show global concavity of the corresponding likelihood function - and our experience grown out of numerous regression runs from different starting values we have the impression of a well behaved likelihood function. The first and second partial derivatives of the loglikelihood function (8) are given in the appendix. A standard Newton–Raphson procedure has been implemented in GAUSS using analytical gradient and Hessian.

The limitations of the univariate Poisson regression model - especially the equality of mean and variance - equally apply for the bivariate case. The assumptions for the
The consequences of over- and underdispersion are identical to the univariate case. The estimation of the parameter vector $\beta_j$, though consistent, is not efficient any more. Further, for the case of overdispersion the estimated standard errors are biased downwards leading to wrong inference. Since we do not correct for unobserved individual heterogeneity we expect to observe exactly this phenomenon in our estimates. Basically, the literature offers two possible solutions to the problem. The parametric approach is to explicitly allow for the variation of the marginal expectation over all individuals by introducing an additional error term $\varepsilon$ in (7):

$$\theta_{ji} + \gamma = \exp(x'_{ji} \beta_j + \varepsilon_i) \quad j = 1, 2, \ i = 1, \ldots, n.$$  \hspace{1cm} (10)

To arrive at a closed form solution for the resultant distribution often the Gamma distribution for $\varepsilon$ is used (See Cameron and Trivedi (1986) for the univariate case or Gourieroux (1989) for the bivariate case.) In the bivariate case this leads to severe computational problems which have not been solved so far to our knowledge. The semiparametric approach avoids the specification of the error distribution and therefore provides robust results. In this paper we adopt the following strategy: since the Poisson estimate for the parameter vector $\beta$ is consistent no matter what kind of dispersion is prevailing, as long as the mean function is correctly specified, we choose the bivariate Poisson model and additionally calculated robust standard errors by pre- and postmultiplying the matrix formed by the sum of the first derivative cross products with the inverted Hessian (Gourieroux, Monfort and Trognon, 1984a).
4 Data

In SECTION 2, an argument was made for the use of count data models in the econometric analysis of labor mobility. The first wave of the German socio-economic panel, collected in 1984, provides such (retrospective) information on both the number of jobs and the number of unemployment spells for individual labor market histories during the ten year period 1974-84. This period is certainly characterized by varying labor market conditions, in particular by a significant increase in unemployment, which in turn affects the overall pattern of voluntary and involuntary labor mobility. Still, the period is so long that macro effects can be expected to play a minor role on average, so that the observed individual mobility patterns may be indicative also for different time periods with other macroeconomic conditions.

Using the information on the number of employers and the number of unemployment spells, two measures of labor mobility can be developed. First, to derive a measure of INVOLUNTARY JOB CHANGES assume that

i) spells of unemployment are always involuntary,
ii) people do not return to the same job after a spell of unemployment,
iii) changes without a spell of unemployment are always voluntary, and
iv) that the individuals have been employed at the beginning of the period.

Assumption ii) is less unrealistic for the German labor market than it might be for labor markets in other countries, since the phenomenon of temporary layoffs is of negligible size. Given these assumptions, the number of involuntary job changes is given by the number of unemployment spells. Second, the number of VOLUNTARY JOB CHANGES is given by the total number of jobs, reduced by one (i.e. the number of total job changes), minus the INVOLUNTARY JOB CHANGES. Under the above assumptions, this difference is always non-negative.

To avoid additional complexities in the discussion of labor mobility, the choice of employer is treated separately from the labor force participation decision. To exclude individuals that potentially moved in and out of the labor force during the considered period women are excluded from the sample. Further, the sample contains only males that started their working career before or in 1974, i.e. were either full time employed, regular part time employed, or unemployed, and did not retire before 1984. The resulting
sample includes 2124 observations.

A cross tabulation of the dependent variables is given in TABLE 1. The mode for both VOLUNTARY and INVOLUNTARY JOB CHANGES is at zero. The means are 0.52 and 0.36, respectively. The proportion of frequent movers is close to the one a (marginal) homogeneous Poisson distribution would predict: 11% empirical (7%), as opposed to 10% (5%) for a Poisson distribution with mean 0.52 (0.36). The variance mean relation is 2.22 for VOLUNTARY and 3.32 for INVOLUNTARY JOB CHANGES, indicating a tendency for overdispersion at the marginal level. This appears to provide a first check for the validity of the bivariate Poisson model, since the latter postulates Poisson marginals, and overdispersion violates the Poisson assumption. However, also the marginals of the bivariate Poisson distribution are conditioned on the covariates in the regression model, and overdispersion at the marginal level is (theoretically) compatible with mean-variance equality conditional on covariates. Practical experience, however, indicates that overdispersion of this magnitude never vanishes when conditioning on covariates. Finally, the empirical correlation of 0.06 is positive, though relatively small.

The choice of explanatory variables corresponds to standard variables in previous empirical work (See Mincer and Jovanovic, 1981, for instance). It is assumed that the same explanatory variables potentially affect both types of mobility, whereas differences in the estimated regression vectors are allowed for. The issue, whether INVOLUNTARY and VOLUNTARY JOB CHANGES in fact display distinct regression parameters, due to different underlying structural processes, or whether they cannot be distinguished empirically (i.e. the hypothesis of an identical vector cannot be rejected), is subject to a test in the next section.

Since the process generating the counts is kept in a black box, time varying covariates cannot be incorporated in the analysis. Further, all variables measured after the period carry the risk of being inflicted by endogeneity. The time varying explanatory variables considered here relate either to the beginning of the period, or to the beginning of the overall work history. A second set of variables, free of the above problems, are demographic variables. Both type of variables are used to explain a propensity towards mobility prior to the considered period.

A further classification of the explanatory variables distinguishes between one group affecting the human capital, and another affecting transaction cost. The variables of
greatest interest for the interpretation of the mobility process in terms of human capital considerations are the education level, as well as experience in the labor market. The education level is given by the YEARS OF SCHOOLING. This variable is not given in the Socio-economic panel and it is constructed by attributing to the highest schooling degree the corresponding number of years it usually requires to take that degree, and adding years of professional education. The YEARS OF SCHOOLING vary between 8 and 22 years. Most of the education is general training, increasing the stock of general human capital. Another variable crucial for the stock of human capital is professional EXPERIENCE. This variable is measured at the beginning of the period, i.e. in 1974. EXPERIENCE is not constructed via the usual formula AGE–YEARS OF SCHOOLING–6, but instead uses retrospective information on age at entrance into the labor market, to provide a measure independent of potential measurement errors in the YEARS OF SCHOOLING. Actually, calculations show that both ways of calculating EXPERIENCE provide a variable with almost identical mean and variance. This increases the confidence into the retrospective data source.

As mentioned before, the reduced form approach provides no proper way to distinguish between tenure and ‘true’ experience effects. Also, the extent to which tenure and experience increase during the ten year period is determined by the endogeneous dynamics of the process, which are not modeled explicitly. However, the convex experience-mobility profile reported for instance in Mincer and Jovanovic (1981) should also prevail here. A term of squared experience is included to allow for this.

A major variable influencing transaction costs is the marital status. SINGLE takes the value 1 if the person is and always was a single and 0 if he is or was married, i.e. including divorced persons and widows/ers. Being single lowers transaction costs and facilitates job changes. Less clear a priori is the effect of nationality. GERMAN is 1 if the person has German nationality and 0 otherwise.

The rest of the variables describe the professional situation. According to insider outsider theories, membership in a union or in a comparable professional organization (UNION) should reduce both voluntary and involuntary job changes. Unions may raise firm’s labor turnover costs and may increase the wages of insiders. We control for the occupational status in the first job. The categories are ORDINARY and QUALIFIED BLUE and WHITE COLLAR workers, respectively. The reference group are civil servants.
with an expected lower mobility due to lifetime tenure.

A last remark applies to the concept of period-at-risk. Clearly, neither VOLUNTARY nor INVOLUNTARY JOB CHANGES can occur while being unemployed. The “risk” for both events is zero during an unemployment spell. This violates a basic Poisson assumption and can be accounted for by using an adjustment for the length of the period at risk which in general varies over individuals. Under the Poisson model, the expected number of counts in a given period is proportional to the length of the period at risk. This proportionality can be either imposed by taking the logarithm of the period at risk with a unit coefficient into the regression. Alternatively, the coefficient may also be estimated. (McCullagh and Nelder (1989) call this adjustment a logarithmic offset.)

In the socio-economic panel, the necessary information for calculating the offset is provided by the cumulative months of unemployment during the ten year period. The maximum in the data is 72 of 120 possible months. However, the offset is only possible if the period at risk is not endogeneous, as is the case for INVOLUNTARY JOB CHANGES: Individuals with more unemployment spells also have a lower period at risk. We did use the offset for the VOLUNTARY JOB CHANGES. This did not change the results and in the next section, the results without offset are reported.

5 Results and Conclusions

The regression results are given in TABLES 2 and 3. TABLE 2 contains the results of the bivariate Poisson regression. Two types of asymptotic $t$-values are calculated. The first assumes a correct specification of the model and is based on standard errors calculated from the Hessian, while the second uses robust standard errors correcting for overdispersion. In fact, the robust $t$-values are smaller throughout, indicating the presence of overdispersion in our sample.

A significant positive covariance of $\gamma = 0.022$ between the two counts is estimated. Also, most of the parameters estimates are significant in both regression equations. The joint test of the model with intercepts and covariance only, against the unrestricted alternative is clearly rejected by a likelihood-ratio test (483.08 > $\chi^2_{20,0.95} = 31.41$). The experience-mobility profiles are convex in both cases corresponding to the findings in previous studies. Interestingly, YEARS OF SCHOOLING has a significant negative impact
only on involuntary changes, and not on voluntary ones. This is compatible with the idea that YEARS OF SCHOOLING measures general human capital. As mentioned in the introduction, only firm specific capital reduces the probability of voluntary job changes, while an increase in general human capital reflects higher opportunities in both incumbent and outside firms.

Single individuals are less costly to lay off, and thus have a higher risk of involuntary job changes, while the expected positive effect of being a single on voluntary changes is not reflected in the data. Interestingly, union membership has a strong negative impact on both voluntary and involuntary mobility. Voluntary mobility is highest for qualified white collar workers, while involuntary mobility is highest for ordinary blue collar workers.

Is there a gain in using the bivariate as opposed to the separate estimation of univariate Poisson regressions? The results of the latter are given in TABLE 3. Separate estimation amounts to setting $\gamma = 0$ and thus assuming independence. Both models are nested, and a likelihood-ratio test, comparing the bivariate value to the sum of the log-likelihood values of the separate estimations again provides evidence for the bivariate Poisson regression model ($25.64 > \chi^2_{1,0.95} = 3.84$).

Another issue is, whether the gain in explanatory power is joined by a change in the parameter estimates. A comparison of the parameter estimates displays a remarkable robustness across the specifications. The estimated robust standard errors, however, are smaller in the bivariate approach compared to the univariate for almost all parameter estimates. The main advantage of using the bivariate Poisson is clearly an increase in efficiency.

In a final step, a further restriction is imposed, the equality of the regression parameters across equations. The results of the joint estimation are given in the third column of TABLE 3. Using again likelihood-ratio tests, the restriction is clearly rejected both against the separate estimation ($139.2 > \chi^2_{11,0.95} = 19.68$) and against the bivariate Poisson model ($164.84 > \chi^2_{12,0.95} = 21.03$). The result of a significant difference in the parameter estimates confirms and justifies ex-post a working assumption underlying this paper: that the distinction between INVOLUNTARY and VOLUNTARY JOB CHANGES is a meaningful concept. The empirical evidence found in this paper suggests that both types of mobility have a different underlying behavioral structure, and that this differences should also be accounted for in the theoretical modeling of labor mobility.
### TABLE 1: Cross Tabulation of Voluntary and Involuntary Job Changes

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### TABLE 2: Results of Bivariate Poisson Regression

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<td>2.972</td>
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References


King, G. 1989 “A seemingly unrelated Poisson regression model” *Sociological Methods and Research*.


Appendix: First and Second Derivatives of the Log-likelihood $\log L$

Using

$$B_{1i} = \frac{\gamma^k}{k!} \frac{[\exp(x'_i\beta_1) - \gamma]^{y_{1i} - k}}{(y_{1i} - k)!} \quad \text{and} \quad B_{2i} = \frac{[\exp(x'_i\beta_2) - \gamma]^{y_{2i} - k}}{(y_{2i} - k)!},$$

we can write $B_i$ in (8) as follows: $B_i = \sum_{k=0}^{s_i} B_{1i}B_{2i}$ (with $s_i = \min(y_{1i}, y_{2i})$) which is helpful for the following derivation. The first partial derivatives of (8) are:

$$\frac{\partial \log L}{\partial \gamma} = n - \sum_{i=1}^{n} \left( \frac{1}{B_i} \frac{\partial B_i}{\partial \gamma} \right)$$

(12)

$$\frac{\partial \log L}{\partial \beta_j} = -\sum_{i=1}^{n} \exp(x'_{ji}\beta_j)x_{ji} + \left( \frac{1}{B_i} \frac{\partial B_i}{\partial \beta_j} \right)$$

(13)

where $j = 1, 2$ and

$$\frac{\partial B_i}{\partial \gamma} = \sum_{k=0}^{s_i} \left( \frac{\partial B_{1i}}{\partial \gamma} B_{2i} + \frac{\partial B_{2i}}{\partial \gamma} B_{1i} \right)$$

(14)

with

$$\frac{\partial B_{1i}}{\partial \gamma} = \frac{\gamma^{k-1}}{k!} \frac{[\exp(x'_{1i}\beta_1) - \gamma]^{y_{1i} - k - 1}}{(y_{1i} - k)!} \left\{ k \left[ \exp(x'_{1i}\beta_1) - \gamma(y_{1i} - k) \right] \right\}$$

(15)

$$\frac{\partial B_{2i}}{\partial \gamma} = \frac{(y_{2i} - k) \left[ \exp(x'_{2i}\beta_2) - \gamma \right]^{y_{2i} - k - 1}}{(y_{2i} - k)!}$$

(16)

$$\frac{\partial B_i}{\partial \beta_1} = \sum_{k=0}^{s_i} \left( \frac{\gamma^{k}}{k!} \frac{(y_{1i} - 1) \left[ \exp(x'_{1i}\beta_1) - \gamma \right]^{y_{1i} - k - 1}}{(y_{1i} - k)!} \exp(x'_{1i}\beta_1)x_{1i} \frac{[\exp(x'_{2i}\beta_2) - \gamma]^{y_{2i} - k}}{(y_{2i} - k)!} \right)$$

(17)

$$\frac{\partial B_i}{\partial \beta_2} = \sum_{k=0}^{s_i} \left( \frac{\gamma^{k}}{k!} \frac{(y_{1i} - 1) \left[ \exp(x'_{1i}\beta_1) - \gamma \right]^{y_{1i} - k}}{(y_{1i} - k)!} \times \left( \frac{(y_{2i} - j) \left[ \exp(x'_{2i}\beta_2) - \gamma \right]^{y_{2i} - k - 1}}{(y_{2i} - k)!} \exp(x'_{2i}\beta_2)x_{2i} \right) \right)$$

(18)

has been used in (13). The second partial derivatives of (8) are:

$$\frac{\partial^2 \log L}{\partial \gamma^2} = \sum_{i=1}^{n} \left[ \left( \frac{1}{B_i} \frac{\partial^2 B_i}{\partial \gamma^2} \right) - \left( \frac{1}{B_i} \frac{\partial B_i}{\partial \gamma} \right)^2 \right]$$

(19)

$$\frac{\partial^2 \log L}{\partial \beta_j \partial \beta'_j} = -\sum_{i=1}^{n} \exp(x'_{ji}\beta_j)x_{ji} \times \sum_{i=1}^{n} \left[ \left( \frac{1}{B_i} \frac{\partial^2 B_i}{\partial \beta_j \partial \beta'_j} \right) - \left( \frac{1}{B_i} \frac{\partial B_i}{\partial \beta_j} \frac{\partial B_i}{\partial \beta'_j} \right) \right]$$

(20)

$$\frac{\partial \log L}{\partial \beta_j} = \sum_{i=1}^{n} \left[ \left( \frac{1}{B_i} \frac{\partial B_i}{\partial \beta_j} \right) - \left( \frac{1}{B_i} \frac{\partial B_i}{\partial \beta_j} \frac{\partial B_i}{\partial \beta_j} \right) \right]$$

(21)

$$\frac{\partial^2 \log L}{\partial \beta_j \partial \beta'_j} = -\sum_{i=1}^{n} \left[ \left( \frac{1}{B_i} \frac{\partial^2 B_i}{\partial \beta_j \partial \beta'_j} \right) - \left( \frac{1}{B_i} \frac{\partial B_i}{\partial \beta_j} \frac{\partial B_i}{\partial \beta'_j} \right) \right],$$

(22)

where again $j = 1, 2$ holds. We refrain from giving all the formulas necessary in order to evaluate the complete Hessian matrix due to space limitations.