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The Promise and Pitfalls of Restructuring Network Industries

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Keywords: access pricing, investment, double marginalization, vertical foreclosure, product differentiation.

JEL: D43, L43.

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1 Introduction

It is widely believed that introducing competition is the key to achieving the full benefits of privatization in previously monopolized and regulated network industries, such as telecommunications, electricity or railways.\(^1\) The recent wave of “deregulation” in these industries—i.e. the introduction of competition into statutory monopolies—is certainly consistent with this view. The traditional approach towards introducing downstream competition has been to restructure the industry by breaking up the integrated dominant firm and prohibiting the upstream monopolist to reenter the downstream market. Well-known divestitures of this type include the breakup of AT&T in the United States in 1984 and the breakup of British Rail in Great Britain in 1994. A less radical approach—often adopted in the 1990’s by European countries deregulating their national telecommunications markets—allows the upstream monopolist to remain integrated and attempts to create a level playing field for the downstream competitors by regulating the access prices.\(^2\) Yet another approach was adopted in the recent deregulation of the German electric power industry, where regulations have been removed altogether and even access charges are freely determined by the industry.

Industrial organization theory suggests that irrespective of the approach adopted, the restructuring of network industries with natural monopoly characteristics upstream is subject to the following potential pitfalls:

- **double marginalization**: the introduction of imperfect downstream competition leads to successive markups which imply higher prices for the final good and lower aggregate welfare;\(^3\)

- **underinvestment**: downstream competition may reduce the monopolist’s incentive to invest in network infrastructure.

- **vertical foreclosure**: when competing with new entrants, the monopolist may have incentives to raise downstream rivals’ cost by charging

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\(^1\)See Newbery (2000) for a survey on the state of the debate.

\(^2\)See Laffont and Tirole (2000) for a recent analysis of the role of competition in telecommunications.

\(^3\)The classic reference is Spengler (1950); Tirole (1988, Chapter 4) and Perry (1989) provide surveys on market outcomes in vertically related industries.
excessive wholesale or access prices.⁴

Somewhat surprisingly, both regulators and antitrust authorities have rarely addressed these potential pitfalls when restructuring vertically integrated network industries, even though the extensive literature on interconnection is either explicitly or implicitly based on the problem of vertical foreclosure.

This paper takes the pitfalls of restructuring network industries seriously and studies both pricing and investment behavior of a network monopolist under the most common forms of market structure, i.e. (i) vertical integration without downstream competition, (ii) vertical separation, where the upstream monopolist is fully separated from the imperfectly competitive downstream market, and (iii) liberalization, where the upstream monopolist is allowed to operate in the imperfectly competitive downstream market. We consider network industries producing a given number of final products that are imperfect substitutes. More specifically, we will assume that the effect of a price increase on own demand dominates the cross-effects on competitors’ demands, i.e. the analysis is best applied to industries with strongly differentiated or branded products, such as railways or telecommunications services.

The paper adds to the literature in several respects. First, it extends earlier work by Greenhut and Ohta (1976 and 1978), Perry (1978), and Haring and Kaserman (1978) which focused on integration and separation in the Cournot model, by considering the case of liberalization and analyzing Bertrand competition with differentiated final products. The extension to the liberalization scenario is important for two reasons: (i) liberalization plays a significant role in the restructuring of European network industries; (ii) under liberalization, the well-known double marginalization effects crucial for comparing integration and separation are supplemented by strategic pricing incentives on the part of the integrated network monopolist, with ambiguous net effects on access and retail prices.

Second, the paper examines the network operator’s incentive to invest into cost reduction under the different forms of vertical market structure. The

⁴See Rey and Tirole (forthcoming) for a recent survey of the foreclosure literature.
underinvestment problem is an important aspect of restructuring network industries that has largely been ignored in the previous literature.\footnote{See Buehler \textit{et al.} (forthcoming) for an analysis of the network operator’s incentive to invest into infrastructure quality.}

Third, the paper provides the first analysis of the pros and cons of a network monopolist’s downstream participation that allows for strategic third-degree price discrimination by the network monopolist, introducing a real possibility of a vertical price squeeze.\footnote{In related papers, Mandy (2000) and Weisman and Kang (2001) analyze the incentives to “sabotage” downstream rivals using non-price strategies.} As pointed out above, this is particularly relevant for the liberalization scenario.

Throughout the analysis, the paper deliberately abstracts from access or retail price regulations, both in the benchmark case of vertical integration and the cases of separation and liberalization, thus isolating the effects generated by the restructuring of the industry. As a result, we may discuss the role that access or retail price regulations should play in moving the industry from one equilibrium to another, rather than imposing a particular equilibrium by imposing arbitrary rules of price regulation.

Under reasonable assumptions on demand, we derive the following results. \textit{First}, vertically separating an integrated network industry is likely to increase retail prices, since it tends to reduce the network monopolist’s perceived price elasticity of demand. The driving force of this result is the well-known double marginalization associated with vertical separation. The effect of liberalization on retail prices, however, is less clear, since pricing decisions are not only affected by double marginalization, but also by the integrated firm’s incentive to strategically manipulate access and retail prices so as to foreclose separated downstream rivals. \textit{Second}, as the monopolist’s investments into cost reduction are driven by aggregate demand for the intermediate good, restructuring an integrated network industry may not only lead to higher retail prices, but also to lower investment into cost reduction, thereby generating adverse welfare effects. \textit{Third}, using a simple linear demand model, we demonstrate that in the case of liberalization, practicing vertical foreclosure might not be profitable from the integrated firm’s point of view. Put differently, even if access prices are not regulated, the inte-
grated firm may find it more profitable to divert the competitor’s demand by lowering its retail price rather than raising the competitor’s access charge.

The policy implications of this exercise are instructive. The introduction of imperfect downstream competition does not necessarily generate welfare gains in the absence of downstream innovation, such as new products or more efficient firms.\footnote{This result may be overturned if there is sufficient downstream innovation. To study the conditions for this to happen, one would have to endogenize the entry and exit decisions of potential downstream competitors and analyze their effect on overall efficiency over time, which is beyond the scope of this paper.} In particular, if regulatory and antitrust authorities are aiming at lower retail prices and higher social welfare, they may find the breakup of a dominant vertically integrated firm undesirable. Similarly, in order to move a vertically separated or even liberalized industry into a more favorable equilibrium than the one that emerges under integration, carefully crafted access price regulations are necessary to limit the network monopolist’s market power. The phasing out of “residual regulation” called for by some policy makers thus appears to be problematic. More generally, it is not evident that containing the monopolist’s market power is easier or less costly under separation or liberalization than under integration. It therefore remains to be explained by models of dynamic innovation in the downstream sector why the standard practice is to replace vertically integrated monopolies with regulated retail prices by (partially) separated upstream monopolies with regulated access (and retail) prices.

The remainder of the paper is organized as follows. Section 2 gives the basic setup of the model. It discusses the main assumptions and outlines the cost structures of the various firms. Section 3 develops the case of vertical integration as a benchmark. Section 4 compares equilibrium prices and investment under vertical separation and integration. Section 5 analyzes the case of liberalization and compares its equilibrium outcome with vertical integration and separation. Section 6 concludes.
2 The Basic Setup

We model the production and selling of a differentiated final product provided over a network as an industry with a vertical structure. Suppose that in order to produce the final good (e.g. railway services), the seller needs access to an intermediate good produced by a monopolist. For simplicity, assume that to provide one unit of the final product (e.g. one passenger mile), one unit of the homogenous intermediate good (e.g. one mile of track) is required. The sellers’ variable costs are constant and normalized to zero. The differentiated final product is sold on $n$ markets with downward-sloping demand $D_i(p)$, $i = 1, ..., n$, where $p = (p_1, ..., p_n)$ is the vector of retail prices. Aggregate demand for the intermediate good is thus given by $D(p) \equiv \sum_{i=1}^{n} D_i(p)$. The constant unit cost $c(e)$ of providing the intermediate good depends on the level of effort $e$ that is exerted by the network operator to reduce this cost; implementing a positive effort is costly, which is reflected in a strictly increasing cost function $\psi(e)$. Finally, suppose that there is a fixed cost $F$ of operating the network.

In the various industry configurations, we model the provision of the final good as a two-stage game with the following course of events.

- **Stage 1**: The network monopolist chooses both the cost-reducing effort $e$ and the per unit access charge $a_i$ for each of the downstream firms $i = 1, ..., n$ (i.e. there is no fixed component in access charges).

- **Stage 2**: Given the access charges $a_i$, each downstream firm $i$ sets the retail price $p_i$ for the provision of the final good.

Observe that in the case of vertical integration, the network operator is also owner of all retail firms $i = 1, ..., n$ and thus faces a simple optimization problem. In the case of vertical separation, the network operator and the downstream competitors play a sequential game which can be solved using backward induction. In the case of liberalization, where the network operator is vertically integrated with at least one downstream firm and faces downstream competitors, a sequential game between the integrated network operator and its downstream competitors is played (see Figure 1).
Throughout the paper, we require that the following basic assumptions are satisfied:

[A1] The final products are demand substitutes and strategic complements, i.e. \( \frac{\partial D_i(p)}{\partial p_i} < 0 \), \( \frac{\partial D_i(p)}{\partial p_j} \geq 0 \) and \( \frac{\partial^2 D_i(p)}{\partial p_i \partial p_j} \geq 0 \), \( i, j = 1, \ldots, n, i \neq j \).

[A2] Demand functions are exchangeable in the sense that the following conditions hold:

(i) For any firm \( i \), a permutation of the vector \( p_{-i} \) does not change the demand \( D_i(p_i, p_{-i}) \).

(ii) If \( i \neq j \) such that \( p_i = p_j \) and \( p_{-i} \) is identical with \( p_{-j} \) up to permutation, then \( D_i(p) = D_j(p) \).

[A3] The own effect of a price change dominates the cross effects both in terms of the level and slope of demand, i.e. \( \sum_j \frac{\partial D_i(p)}{\partial p_j} < 0 \) and \( \frac{\partial^2 D_i(p)}{\partial p_i^2} + \sum_{j \neq i} \frac{\partial^2 D_i(p)}{\partial p_i \partial p_j} \leq 0 \), \( i, j = 1, \ldots, n \).

[A4] The marginal cost of the network monopolist is decreasing in effort such that \( c'(e) < 0 \) and \( c''(e) > 0 \). The cost of providing effort is positive and increasing, i.e. \( \psi'(e) > 0 \), \( \psi''(e) > 0 \).

To understand the role of assumptions [A1]-[A3], consider the parameter restrictions placed on the linear demand system given by

\[
D_i(p) = \alpha_i - \beta_i p_i + \sum_{j \neq i} \gamma_{ij} p_j.
\]

Note that if the goods are demand substitutes and prices are larger than marginal cost, it suffices that \( \frac{\partial^2 D_i(p)}{\partial p_i \partial p_j} \geq 0 \) in order for the goods to be strategic complements in the neighborhood of a price equilibrium (Tirole, 1988, p. 337).

As usual, \( p_{-i} \) denotes the vector of prices of all firms except firm \( i \).

See Vives (1999, p. 150) for a discussion of such conditions. [A3] will be crucial for the proof of Lemma 1 below, since it assures \( dp_i/da_i > 0 \) under separation (see Appendix B).
Assumption [A1] requires $D_i(p)/\partial p_i = -\beta_i < 0$ and $\partial D_i(p)/\partial p_j = \gamma_{ij} \geq 0$, which implies $\beta_i > 0$ and $\gamma_{ij} \geq 0$. The additional condition $\partial^2 D_i(p)/\partial p_i \partial p_j \geq 0$ does not restrict parameter values for the linear demand model. Part (i) of [A2] requires that permutation of $p - i$ does not affect $D_i(p_i, p- i)$, which immediately implies $\gamma_{ij} = \gamma_i$ for all $j$. Part (ii) of [A2] imposes that $D_i(p) = D_j(p)$ if $p_i = p_j$ and $p - i$ is identical to $p - j$ up to permutation. It follows that parameters must be symmetric, i.e. $\alpha_i = \alpha, \beta_i = \beta$ and $\gamma_i = \gamma$ for all $i$. [A3] requires that the own demand effect of price changes dominates the cross effects for the level and slope of demand. Using [A1] and [A2], the first condition implies $\sum_j \partial D_i/\partial p_j = -\beta + (n - 1) \gamma < 0$; the second condition does not restrict the parameters of the linear model.

Summing up, the linear demand model satisfies assumptions [A1]-[A3] if and only if

$$D_i(p) = \alpha - \beta p_i + \sum_{j \neq i} \gamma p_j, \quad \text{with } \beta > (n - 1) \gamma > 0. \quad (1)$$

To illustrate the results derived for general demand functions below, we shall often refer to the linear demand model given in (1), assuming $n = 2$ for simplicity. Appendix A provides a detailed analysis of this model.

### 3 Vertical Integration

Suppose that there is a vertically integrated monopolist whose divisions $i = 1, ..., n$ serve all markets with demand $D_i(p)$ for the final good, i.e. the integrated monopolist sets a market price $p_i^I$ for each division $i$, where the superscript $I$ denotes “integration”. Its profit maximizing problem is then given by

$$\max_{p, e} \Pi(p, e) = \sum_{i=1}^{n} [p_i - c(e)] D_i(p) - \psi(e) - F.$$ 

The first-order conditions for equilibrium prices $p_i^I$ and equilibrium effort $e^I$ are given by

$$\frac{p_i^I - c(e^I)}{p_i^I} = \frac{1}{\varepsilon_{ii}} \sum_{j \neq i} [p_j^I - c(e^I)] D_j(p^I) \varepsilon_{ji}^I / R_i^I \varepsilon_{ii}^I, \quad i = 1, ..., n, \quad (2)$$
and
\[-c'(e^I)D(p^I) = \psi'(e^I),\]
(3)

with
\[\varepsilon_{ji} \equiv -\left(\frac{\partial D_j(p)}{\partial p_i}p_i\right)_{p^I} D_j\]
denoting the price-elasticity of demand in division \(j\) with respect to the price in division \(i\). \(R_i^I \equiv p_i^I D_i(p^I)\) is the revenue of division \(i\) under vertical integration, where \(p^I\) is the vector of equilibrium retail prices under integration. (2) is the familiar Lerner index for a multiproduct monopoly with separable costs and dependent demands (see e.g. Tirole, 1988, p. 70). For later reference, let us rewrite (2) as
\[
\frac{p_i^I - c(e^I)}{p_i^I} = E_i^I + X_{-i}^I, \tag{4}
\]
with \(E_i^I \equiv 1/\varepsilon_{ii}^I\) denoting the inverse price elasticity of demand for a monopolist serving market \(i\), and
\[
X_{-i}^I \equiv -\frac{\sum_{j \neq i}[p_j^I - c(e^I)]D_j(p^I)\varepsilon_{ji}^I}{R_i^I \varepsilon_{ii}^I}
\]
denoting the pricing externalities originating in market \(i\) and affecting all other markets \(j \neq i\). It is important to note that a vertically integrated monopolist takes into account that the final products offered by its different divisions are substitutes (\(\varepsilon_{ji} < 0\) for \(i \neq j\)) and thus sets higher markups than each of its division would set individually.

\section{4 Vertical Separation}

Under vertical separation, there is an upstream network monopolist completely separated from downstream operations and a set of independent downstream oligopolists \(i = 1, \ldots, n\). Given the access charge \(a_i^S\) chosen by the upstream monopolist and the vector of retail prices \(p_{-i}^S\) set by all other
vertically separated downstream firms, firm $i$ chooses its retail price so as to

$$\max_{p_i} \Pi_i(p, a_i) = (p_i - a_i^S) D_i(p_i, p_{-i}^S).$$

The equilibrium retail price $p_i^S$ is thus given by

$$\frac{p_i^S - a_i^S}{p_i^S} = \frac{1}{\varepsilon_{ii}^S}, \quad i = 1, \ldots, n,$$

where $\varepsilon_{ii}^S$ is the price elasticity of demand for firm $i$, now evaluated at $p^S$ rather than $p^I$. Given the vector of access prices $a = (a_1, \ldots, a_n)$ from the game’s first stage, the equilibrium retail prices $p_i^S(a)$ in the second stage are functions of these access prices and characterized by the best-response functions $p_i^I(a_i, p_{-i}^S) = p_i^S$. With equilibrium retail prices denoted as $p^S(a) = (p_1^S(a), \ldots, p_n^S(a))$, the upstream firm’s problem can be written as

$$\max_{a, e} \Pi(a, e) = \sum_{i=1}^n [a_i - c(e)] D_i(p^S(a)) - \psi(e) - F.$$

The first-order condition for equilibrium access prices is then given by

$$\frac{a_i^S - c(e^S)}{a_i^S} = -\frac{D_i(p^S)}{a_i^S \sum_j \frac{\partial D_i}{\partial p_j} \frac{\partial p_j}{\partial a_i}} \sum_{j \neq i} \left[ a_j^S - c(e^S) \right] \sum_k \frac{\partial D_i}{\partial p_k} \frac{\partial p_k}{\partial a_i}.$$

(6)

Let us simplify (6) using the definition of $\varepsilon_{ji}$ introduced above, as well as the elasticities of retail prices with respect to access charge $a_i$ given by

$$m_{ji} \equiv \frac{(\partial p_j / \partial a_i) a_i}{p_j}.$$

In addition, let $R_i^S \equiv a_i^S D_i(p^S)$ denote the monopolist’s access revenue from product $i$ under vertical separation. First-order condition (6) then simplifies to

$$\frac{a_i^S - c(e^S)}{a_i^S} = E_i^S + X_{-i}^S,$$

(7)
with
\[ E^S_i \equiv \frac{1}{\sum_j \varepsilon_{ij}^S m_{ji}^S} \]
denoting the inverse price elasticity of demand for a vertically separated upstream monopolist serving market \( i \), and
\[ X^S_{-i} \equiv -\frac{\sum_{j \neq i}[a_j^S - c(e^S)]D_j(p^S) \sum_k \varepsilon_{jk}^S m_{ki}^S}{R_i^S \sum_j \varepsilon_{ij}^S m_{ji}^S} \]
denoting the pricing externalities affecting markets \( j \neq i \). While the first-order condition for equilibrium effort
\[ -d'(e^S)D(p^S) = \psi'(e^S) \] (8)
has the same form as under integration, equilibrium access prices \( a^S = (a_1^S, ..., a_n^S) \) are now given by a generalized form of the Lerner index. First-order condition (7) indicates that the monopolist must account for the fact that price changes in market \( i \) are first translated into retail price changes through \( m_{ji} \) before they affect access demand through \( \varepsilon_{ij} \). More specifically, equation (7) shows that the industry’s vertical separation affects the pricing incentives of the network monopolist in two related ways:

(i) Vertical separation changes the perceived inverse price elasticity of demand in market \( i \) from \( E^I_i \) to \( E^S_i \). Instead of directly affecting demand via \( \varepsilon_{ii} \), an increase of the monopolist’s price first affects the pricing decisions of the downstream firms via the elasticities of retail prices \( m_{ji} \). Only through the associated changes of retail prices does an increase of \( a_i \) affect the demand for access in market \( i \).

(ii) Vertical separation changes the pricing externalities between markets from \( X^I_{-i} \) to \( X^S_{-i} \). As under vertical integration, the upstream monopolist accounts for the externalities between markets \( i \) and \( j \neq i \) when setting its prices. But just as within each market, variations of access prices now only indirectly affect the demand for the final good. An increase of the access price \( a_i \) generates demand effects—if any—in markets \( j \neq i \) only after translation via \( m_{ji} \) into changes of retail prices.
To compare the equilibrium outcomes under vertical integration and separation, it is thus crucial to know how downstream firms pass on increases of access charges. The following Lemma establishes that firms only partially pass on changes in access charges.

Lemma 1 Suppose that there is a unique interior equilibrium under separation characterized by $a^S$ and $p^S$. Then

$$0 \leq m^S_{ji} < m^S_{ii} \leq 1, \quad i, j = 1, ..., n, i \neq j.$$

Proof. See Appendix. ■

Intuitively, Lemma 1 states that firm $i$ will not find it optimal to fully pass on increases of its access charge $a_i$ to its customers, i.e. $m_{ii} \leq 1$. At the same time, firms $j \neq i$ welcome the increase of $a_i$ since the associated increase of $p_i$ shifts out their demand schedules, allowing them to adjust their prices $p^S_j$ upwards. Therefore, $m_{ji} > 0, j \neq i$, and the substitution effects generated by the cost increase of firm $i$ are mitigated.

4.1 Equilibrium Retail Prices for Given Effort

We now study the conditions for which retail prices are higher under separation than integration. For general demand functions, a direct comparison of $p^S_i(a^S(e^S))$ and $p^I_i(e(e^I))$ using (2) and (5) is not instructive. Instead, we proceed in two steps. First, we fix the effort level at $e \equiv \bar{e}$ and compare the Lerner index for retail prices under integration with the Lerner index for access charges under separation, using the fact that—for a given effort level—retail prices under separation must be higher than access charges by the profit maximizing behavior of downstream firms. Second, we show that if effort is chosen endogenously, the effort level is generally smaller under separation than integration. Therefore, marginal cost must be higher under vertical separation than under integration, which reinforces higher retail prices under separation.

Comparing (4) and (7), our first main result gives a sufficient condition for the retail prices to be higher under separation than under integration:
Proposition 1 Suppose effort is fixed at $\bar{e}$. Then changing the industry’s structure from integration to separation strictly increases retail prices if

$$(E^S_i - E^I_i) + (X^S_{-i} - X^I_{-i}) \geq 0 \quad \text{for all } i.$$ (9)

Proof. See Appendix. ■

The intuition of proposition 1 is straightforward. Recall that $E^I_i = 1/\varepsilon^I_{ii}$ and $E^S_i = 1/((\varepsilon^S_{ii}m^S_{ii}) + \sum_{j \neq i} \varepsilon^S_{ij}m^S_{ji})$ denote the monopolists’ perceived inverse price elasticity of demand in market $i$ under integration and separation, respectively; similarly, $X^I_{-i}$ and $X^S_{-i}$ denote the pricing externalities generated in market $i$. Proposition 1 thus simply states that retail prices increase if separating the industry reduces the monopolist’s perceived price elasticity of demand more strongly than it reduces the pricing externalities.

According to Lemma 1, it is natural to expect vertical separation to reduce the perceived price elasticity of demand: First, $m_{ij} \geq 0$ implies $\sum_{j \neq i} \varepsilon^S_{ij}m^S_{ji} \leq 0$. Second, under vertical separation $\varepsilon^I_{ii}$ is scaled downwards by $m_{ii} \leq 1$. Both effects unambiguously support a reduction of the perceived price elasticity. However, there may be a countervailing effect, since $\varepsilon^I_{ii}$ and $\varepsilon^S_{ii}$ are evaluated at different sets of retail prices. Yet, for this countervailing effect to dominate, we must have $\varepsilon^S_{ii} \geq \varepsilon^I_{ii}$. There is no obvious reason why this should be the case.

The linear demand model with $n = 2$ illustrates that the monopolist’s perceived price elasticity is likely to decrease: Using the equilibrium prices given in (A1), (A4) and (A5) of Appendix A, we can show that due to $\beta > \gamma$ by assumption, we have

$$E^S_i = \frac{(\beta - \gamma)(\gamma + 2\beta)(\alpha - \beta c + \gamma c)}{(\alpha + \beta c - \gamma c)(2\beta^2 - \gamma^2)} > E^I_i = \frac{(\beta - \gamma)(\alpha - \beta c + \gamma c)}{\beta(\alpha + \beta c - \gamma c)},$$ (10)

i.e. the perceived price elasticity of demand decreases with the industry’s vertical separation ($E^S_i - E^I_i > 0$).

Condition (9) indicates, however, that there might be a countervailing effect associated with the pricing externalities between markets. In fact, for
the linear demand model, we find

\[ X_{S}^{i} = \frac{\gamma \beta (\alpha - \beta c + \gamma c)}{(\alpha + \beta c - \gamma c)(2\beta^2 - \gamma^2)} < X_{I}^{i} = \frac{\gamma (\alpha - \beta c + \gamma c)}{\beta (\alpha + \beta c - \gamma c)} \]  \hspace{1cm} (11) \]

for \( \beta > \gamma \), i.e. vertical separation reduces the pricing externalities between markets \( (X_{S}^{i} - X_{I}^{i} < 0) \). In the linear demand model, these two effects of vertical separation cancel, so that the retail prices under integration are just equal to the access prices under separation \( (p_{I}^{i} = a_{i}^{S}, i = 1, 2) \). Condition (9) is thus satisfied with equality, and the retail prices under separation are strictly higher than under integration (see Result 1 in Appendix A), as predicted by Proposition 1.

Proposition 1 relies on a given level of marginal cost. However, the level of marginal cost is not exogenous but depends on the endogenous choice of effort. In the next section, we shall study this choice of effort and how it affects equilibrium prices.

4.2 Choice of Effort

Consider the first-order conditions (3) and (8) for equilibrium choice of effort. Observe that in both market configurations, the equilibrium effort \( e^{*} \) chosen by the upstream monopolist satisfies a condition of the form

\[ -e'(e^{*})D(p^{*}) = \psi'(e^{*}). \]  \hspace{1cm} (12) \]

If the equilibrium retail prices were the same both under integration and separation, i.e. \( p^{*} = p_{I}^{i} = p_{S}^{i} \), the equilibrium effort \( e^{*} \) would have to be the same in both market configurations. However, if for any given level of effort, equilibrium retail prices are higher under separation and \( D(p) \) is decreasing in \( p \), the equilibrium effort \( e^{*} \) must be smaller under separation than under integration. As a consequence, the marginal cost of the upstream monopolist \( c(e^{S}) \) is higher under separation than under integration \( c(e^{I}) \). The next proposition summarizes the second result.

**Proposition 2** Suppose the assumptions of proposition 1 hold. Then, if the network monopolist chooses effort endogenously, effort is higher under
vertical integration than under separation, i.e. \( e^I > e^S \).

Intuitively, if retail prices are higher under separation than under integration, aggregate demand for the intermediate good is smaller under separation. Therefore, the incentive to invest into cost reduction is also smaller. Note that this result reinforces higher retail prices under vertical separation (Proposition 1), since the mark-ups of access prices are based on a higher level of marginal cost when effort is smaller.

5 Liberalization

In the case of liberalization the upstream monopolist also operates in the downstream market. To simplify, assume that the upstream monopolist operates only firm 1 downstream (see Figure 1). Of course, the pricing rule of the competing downstream firms remains unaltered, but now demand is evaluated at a different set of retail prices \( p^L = (p^L_1, \ldots, p^L_n) \). The equilibrium retail prices \( p^L_i \) of the competing downstream firms thus satisfy the first-order condition

\[
\frac{p^L_i - a^L_i}{p^L_i} = \frac{1}{\varepsilon^L_{ii}}, \quad i = 2, \ldots, n, \quad (13)
\]

with \( \varepsilon^L_{ii} \) denoting the price elasticity of demand for firm \( i \)'s services under liberalization.

The upstream monopolist’s problem is given by

\[
\max_{p_1, a, e} \Pi(p_1, a, e) = [p_1 - c(e)] D_1(p^L(a)) + \sum_{i \neq 1} [a_i - c(e)] D_i(p^L(a)) - \psi(e) - F.
\]

The first-order condition for the equilibrium retail price in market 1 is given by

\[
\frac{p^L_1 - c(e^L)}{p^L_1} = E^L_1 + X^L_{-1}, \quad (14)
\]
with \( E^L_i \equiv 1/\varepsilon_{1i} \) and

\[
X^L_{1i} \equiv -\frac{\sum_{j \neq i} [a^L_j - c(e^L)] D_j(p^L) \varepsilon_{1j}^L}{R^L_i \varepsilon_{1i}^L}.
\]

Equilibrium access prices for the markets \( i = 2, \ldots, n \) satisfy the first-order condition

\[
\frac{a^L_i - c(e^L)}{a^L_i} = E^L_i + X^L_{1i},
\]

with

\[
E^L_i \equiv \frac{1}{\sum_j \varepsilon_{ij}^L m_{ji}^L}
\]

denoting the inverse price elasticity of demand for a monopolist serving market \( i \), and

\[
X^L_{1i} \equiv -\frac{[p^L_i - c(e^L)] D_1(p^L) \sum_k \varepsilon_{1k}^L m_{ki}^L}{R^L_i(p^L) \sum_j \varepsilon_{ij}^L m_{ji}^L} - \frac{\sum_{j \neq i, j \neq 1} [a^L_j - c(e^L)] D_j(p^L) \sum_k \varepsilon_{jk}^L m_{ki}^L}{R^L_i(p^L) \sum_j \varepsilon_{ij}^L m_{ji}^L}
\]

denoting the pricing externalities affecting markets \( j \neq i \). Equilibrium effort is given by the standard rule

\[
-c'(e^L) D(p^L) = \psi'(e^L).
\]

Note that whereas two first-order conditions are needed to characterize the upstream monopolist’s behavior under separation, three first-order conditions are needed for liberalization. Consider the integrated downstream firm’s retail price. Equation (14) indicates that the retail price in market 1 is coordinated with the access prices set in all other markets \( j \neq 1 \) and therefore internalizes the pricing externalities. The access prices, in turn, are set according to (15) which is very similar to (7), in particular with respect to the perceived inverse price elasticity of demand in market \( i \). The difference between these two conditions thus mainly concerns the pricing externalities between markets: \( X^L_{1i} \) accounts for the fact that under liberalization, the monopolist can set the retail price rather than the access price in market.
1. Finally, (16) indicates that the monopolist’s effort is set according to the same rule as under vertical integration.\textsuperscript{11}

5.1 Liberalization v. Integration

To compare the equilibrium outcomes under liberalization and vertical integration, we proceed as above. First, we examine the conditions under which the retail prices under liberalization are higher than those under integration for a given effort level. Second, we study the incentive to exert cost reducing effort.

Comparison of (4), (14) and (15) yields the following result:

**Proposition 3** Suppose effort is fixed at \( \bar{e} \). Then changing the industry’s structure from integration to liberalization strictly increases retail prices if

\[
(E^L_1 - E^I_1) + (X^L_1 - X^I_1) > 0, \quad \text{and} \\
(E^L_i - E^I_i) + (X^L_i - X^I_i) \geq 0, \quad \text{for all } i \neq 1.
\]

**Proof.** See Appendix. \( \blacksquare \)

Inspection of Proposition 3 indicates that the intuition of Proposition 1 remains intact, i.e. changing the industry structure from integration to liberalization increases retail prices if, for each market, the monopolist’s perceived demand elasticity is reduced more strongly than the pricing externalities. However, it is not as natural as before to expect that all retail prices increase, since there are no cross-price elasticity terms unambiguously supporting a decrease of the perceived price elasticity in market 1.

In particular, under liberalization the integrated monopolist has an incentive to set a low retail price in market 1 so as to divert demand from its downstream competitors, thereby generating higher demand for the good that is produced with lower marginal cost (due to the absence of double marginalization). This incentive is clearly absent under vertical separation.

Nevertheless, the linear demand model suggests that it is still fairly natural to expect that retail prices turn out to be higher under liberalization.\textsuperscript{11} Of course, all these terms also need to be evaluated at different access and retail prices compared to vertical separation and integration.
than integration. In fact, Result 2 in Appendix A demonstrates that for linear demand, we have $p_i^L > p_i^I$, $i = 1, 2$.

Consider now the incentive to exert effort under liberalization. Since the relevant first-order condition (16) has the same form as under integration and separation, an analogous argument as above can be applied. Proposition 4 summarizes the result.

**Proposition 4** Suppose the assumptions of proposition 3 hold. Then, if the network monopolist chooses effort endogenously, effort is higher under vertical integration than under liberalization, i.e. $e_I > e_L$.

Proposition 4 implies that if changing the industry’s structure from integration to liberalization increases retail prices for a given effort $\bar{e}$, it will do so even more when effort is endogenous.

We now proceed to a more detailed comparison of liberalization with vertical separation.

### 5.2 Liberalization v. Separation

We have pointed out above that under liberalization, the upstream monopolist has an incentive to divert demand from his downstream competitors to market 1 so as to generate higher demand for the good that is produced with lower marginal cost. This can be established by setting a relatively low retail price in market 1, or by setting relatively high access prices for the other markets. Both strategies place the separated downstream rivals at a competitive disadvantage. However, the broad notion of “placing competitors at a disadvantage” is not sufficiently precise. One needs to distinguish between discrimination that is truly anticompetitive and discrimination that harms rivals precisely because it is competitive (Klass and Salinger, 1995, p. 677). The bulk of the recent literature on *vertical foreclosure*\(^{12}\) therefore argues, starting from the notion of raising rivals’ cost (Salop and Scheffman, 1983, 1987), that an integrated firm acts anticompetitively when rais-

ing rivals’ costs, but not when lowering the cost level of its own downstream subsidiaries.

When comparing liberalization and separation, we follow this distinction and say that there is vertical foreclosure if \( a^L_i > a^S_i, i = 2, ..., n \). Of course, retail prices remain important: Even if access charges are higher than under separation, retail prices may actually turn out to be lower if the integrated monopolist subjects its competitors to a price squeeze. That is, vertical foreclosure is not necessarily associated with negative welfare effects.

Unfortunately, for general demand functions little can be said about the occurrence of vertical foreclosure and the comparison of access and retail prices under liberalization and separation. We therefore confine ourselves to making the following observation:

**Observation 1** The monopolist’s pricing behavior under liberalization is generally different from that under vertical separation.

To see that the monopolist’s pricing behavior under liberalization must be different from that under separation, suppose the contrary, i.e. assume that for a given effort level \( \bar{e} \), the integrated monopolist sets all access prices such that \( a^L = a^S \) and chooses its retail price in market 1 so as to satisfy \( p^L_1 = p^S_1 \). First-order conditions (5) and (13) then imply \( p^L = p^S \). Now consider first-order conditions (15) and (7) for equilibrium access pricing. Since the elasticities \( \varepsilon_{ij} \) and \( m_{ji} \) are evaluated at the same prices both under liberalization and separation, the inverses of the perceived price elasticities in market \( i \) are equal, i.e. \( E^L_i(p^S, a^S) = E^S_i \). Now consider the pricing externalities. Since we must have \( p^S_1 > a^S_1 \), it follows that \( X^L_i(p^S, a^S) > X^S_i \). This in turn implies \( a^L_i > a^S_i \), hence a contradiction.

Let us consider the linear demand model discussed in Appendix A to further explore the relation between liberalization and separation. The comparison of Lerner indices turns out to be rather tedious, and we thus resort to a direct comparison of the explicit solutions for the various equilibrium prices (holding effort constant). Result 3 in Appendix A shows that retail prices are lower under liberalization than under separation \( (p^L_i < p^S_i, i = 1, 2) \). That is, changing the industry’s vertical structure from separation to liberalization increases social welfare, because the integrated monopolist produces at lower
marginal cost and thus sets a lower retail price, which induces its downstream competitor to reduce its retail price. This is the efficiency effect of firm 1’s vertical integration.

Yet, this is not the whole story, since there might be a foreclosure effect of firm 1’s integration—reflected in an increase of firm 2’s access charge—which may give rise to higher retail prices (relative to separation). However, Result 4 in Appendix A demonstrates that in the linear demand model, the foreclosure effect of firm 1’s integration is “reversed”: the access charge falls relative to separation \( a_L^2 < a_S^2 \). The next observation summarizes this finding.

**Observation 2** In the linear demand model, vertical foreclosure does not emerge in equilibrium.

Intuitively, the result follows from the fact that the monopolist has lower marginal costs under liberalization, which will induce it to reduce its prices both on the downstream and the upstream market. This efficiency effect dominates the strategic incentive to raise the access charge in the upstream market in the linear demand model, so that the net effect on the access charge is negative.

### 6 Conclusions

The above analysis suggests that if a network industry’s final products are differentiated, changing the vertical structure from unregulated, integrated monopoly to separation or liberalization may be detrimental to social welfare if not supplemented by adequate access or retail price regulation. The argument holds a fortiori for the restructuring of a reasonably regulated integrated monopoly. Therefore, one may wonder why many countries have recently attempted to restructure their network industries.

An obvious answer to this question would be that the costs of regulation are lower under separation or liberalization than under integration (accounting for the costs of restructuring). It remains to be seen whether the accumulating empirical evidence is consistent with this view. Another answer could probably be derived from a public choice model of the regulation of
network industries. Finally, one could argue that opening up the downstream sector to competition not only effectively constrains the pricing behavior of each active downstream firm, but also leads to a selection of more efficient firms over time, bringing about innovation that is unlikely to occur under integrated statutory monopoly. While such selection may indeed occur, it is unclear how introducing imperfect downstream competition can help to control the network monopolist’s market power.

This brings us to the scope for future research. First, we abstracted from downstream entry for simplicity. Endogenizing the entry decisions of potentially more efficient downstream competitors producing differentiated products may prove instructive. Second, we focussed on cost-reducing investment, neglecting other types of downstream or upstream investment, such as investment in network quality or advertisement for the final product. Third, we ignored the fact that an integrated firm’s incentive to foreclose its downstream competitors by price discrimination depends on the costs of this strategy relative to non-price discrimination or sabotage. Allowing for different types of vertical foreclosure should enrich our understanding of a network monopolist’s strategic behavior.
Appendix A: The Linear Demand Case

This appendix discusses the special case with linear demand and \( n = 2 \) downstream firms. As pointed out in section 2, the linear demand model satisfying assumptions [A1]-[A3] is given by

\[
D_i(p_i, p_j) = \alpha - \beta p_i + \gamma p_j, \quad i, j = 1, 2, i \neq j,
\]

with \( \beta > \gamma > 0 \) (see eq. (1)). Throughout, we assume that marginal cost is given by the constant \( c > 0 \), i.e. we abstract from effort considerations (Propositions 2 and 4 indicate that effort considerations reinforce the results derived here).

7.1 Vertical Integration

The integrated monopolist solves the problem

\[
\max_{p_i, p_j} \Pi(p_i, p_j) = \sum_i (p_i - c) (\alpha - \beta p_i + \gamma p_j) - F, \quad i, j = 1, 2, i \neq j.
\]

Straightforward calculations yield the symmetric equilibrium retail prices

\[
p_i^I = \frac{\alpha + (\beta - \gamma) c}{2(\beta - \gamma)}, \quad i = 1, 2.
\] (A1)

7.2 Vertical Separation

Consider the second stage of the game. For a given access charge \( a_i \), downstream firm \( i \) chooses its retail price so as to

\[
\max_{p_i} \Pi_i(p_i, p_j) = (p_i - a_i) (\alpha - \beta p_i + \gamma p_j).
\]

The first-order condition reads

\[
\frac{\partial \Pi_i}{\partial p_i} = (\alpha - \beta p_i + \gamma p_j) - \beta (p_i - a_i) = 0.
\] (A2)

Retail prices are thus given by

\[
p_i^S(a_i, a_j) = \frac{\alpha \gamma + 2 \beta \alpha + 2 \beta^2 a_i + \beta \gamma a_j}{4 \beta^2 - \gamma^2}, \quad i, j = 1, 2, i \neq j.
\] (A3)
In the first stage of the game, the separated upstream monopolist chooses access charges so as to

$$\max_{a_i, a_j} \Pi(a_i, a_j) = \sum_i (a_i - c) (\alpha - \beta p_i^S(a_i, a_j) + \gamma p_j^S(a_j, a_i)) - F,$$

$$i, j = 1, 2, i \neq j.$$

Equilibrium access charges are thus given by

$$a_i^S = \frac{\alpha + (\beta - \gamma) c}{2(\beta - \gamma)}, \quad i = 1, 2. \quad (A4)$$

Plugging $a_i^S$ into $p_i^S(a_i, a_j)$ yields symmetric equilibrium retail prices

$$p_i^S = \frac{\beta^2 c + 3\beta \alpha - \beta c \gamma - 2 \alpha \gamma}{2(2\beta - \gamma)(\beta - \gamma)}, \quad i = 1, 2. \quad (A5)$$

### 7.3 Liberalization

Consider the second stage of the game. For a given access charge $a_2$, downstream firm 2 chooses its retail price so as to

$$\max_{p_2} \Pi_2(p_1, p_2) = (p_2 - a_2) (\alpha - \beta p_2 + \gamma p_1).$$

The first-order condition is given by

$$\frac{\partial \Pi_2}{\partial p_2} = \alpha - 2\beta p_2 + \gamma p_1 + \beta a_2 = 0.$$

The integrated monopolist solves the problem

$$\max_{p_1} \Pi_2(p_1, p_2) = (p_1 - c) (\alpha - \beta p_1 + \gamma p_2) + (a_2 - c) (\alpha - \beta p_2 + \gamma p_1) - F.$$

The first-order condition is

$$\frac{\partial \Pi_1}{\partial p_1} = \alpha - 2\beta p_1 + \gamma p_2 + \beta c + \gamma a_2 - \gamma c = 0.$$

Solving for equilibrium retail prices, we get

$$p_i^L(a_2) = \frac{2\beta \alpha - 2\beta c \gamma + 2\beta^2 c + 3\beta \gamma a_2 + \alpha \gamma}{4\beta^2 - \gamma^2}, \quad (A6)$$
\[ p^I_2(a_2) = \frac{2\beta \alpha + \beta c \gamma + (2\beta + \gamma^2)a_2 + \alpha \gamma - \gamma^2 c}{4\beta^2 - \gamma^2}. \]  
(A7)

Now, consider the first stage of the game. Differentiating the integrated firm’s profit function with respect to \( a_2 \), it follows from the envelope theorem that

\[ \frac{\partial \Pi_i}{\partial a_2} = [(p_i - c) \gamma - (a_2 - c) \beta] \frac{\partial p_2}{\partial a_2} + (\alpha - \beta p_2 + \gamma p_1) = 0. \]  
(A8)

Solving the system of equations given by (A6), (A7) and (A8) yields the following equilibrium prices:

\[
\begin{align*}
 a^L_2 &= \frac{8\beta^3 c + 8\beta^3 \alpha - 8\beta^3 c \gamma + 2\beta^2 c \gamma^2 - 3\beta c^3 + \alpha \gamma^3 + \gamma^4 c}{2 (\beta - \gamma) (\gamma^2 + 8\beta^2) \beta}; \\
p^I_1 &= \frac{8c\beta^3 + 8\beta^2 \alpha - 10\beta^2 c \gamma + 2\beta \alpha \gamma + 5\beta \gamma^2 c - \alpha \gamma^2 - 3c \gamma^3}{2 (\beta - \gamma) (\gamma^2 + 8\beta^2) \beta}; \\
p^L_2 &= \frac{4\beta^4 c + 12\beta^3 \alpha - 4\alpha \gamma \beta^2 - 4\beta^2 c \gamma^2 + 2\beta \alpha \gamma^2 + \beta c \gamma^3 - \alpha \gamma^3 - \gamma^4 c}{2 (\beta - \gamma) (\gamma^2 + 8\beta^2) \beta}.
\end{align*}
\]  
(A9), (A10), (A11)

7.4 Comparing Equilibrium Prices

We now show a number of useful results to hold for the linear demand model.

**Result 1** Retail prices are higher under separation than under integration, i.e. \( p^S_i > p^I_i \), \( i = 1, 2 \).

By first-order condition (A2), we must have \( p^S_i > a^S_i \) in any interior equilibrium with strictly positive demand \( (\alpha - \beta p_i + \gamma p_j > 0) \). Comparison of equations (A4) and (A1), in turn, indicates that \( a^S_i = p^I_i \). It follows immediately that \( p^S_i > p^I_i \).

**Result 2** Retail prices are higher under liberalization than under integration, i.e. \( p^L_i > p^I_i \), \( i = 1, 2 \).

Consider the retail price of downstream firm 1. By (A10) and (A1), we have \( p^L_1 > p^I_1 \) if

\[ \frac{8c\beta^3 + 8\beta^2 \alpha - 10\beta^2 c \gamma + 2\beta \alpha \gamma + 5\beta \gamma^2 c - \alpha \gamma^2 - 3c \gamma^3}{2 (\beta - \gamma) (\gamma^2 + 8\beta^2) \beta} > \frac{\alpha + (\beta - \gamma) c}{2 (\beta - \gamma)}. \]

This condition is equivalent to \( 2\gamma (\beta - \gamma) (\alpha - \beta c + \gamma c) > 0 \), which is satisfied due to \( \beta > \gamma > 0 \) by assumption for strictly positive demand.
With this result in place, \( p^L_2 > p^L_1 \) is sufficient to guarantee that \( p^L_2 > p^L_1 \). To see that \( p^L_2 > p^L_1 \) actually holds, combine the first-order conditions (A6) and (A7) to get
\[
(4\beta^2 - \gamma^2) (p_2 - p_1) = (2\beta - \gamma) (\beta - \gamma) (a_2 - c) > 0,
\]
which implies \( p^L_2 > p^L_1 \).

**Result 3** Retail prices are higher under separation than under liberalization, i.e. \( p^S_i > p^L_i \), \( i = 1, 2 \).

Consider the retail price of downstream firm 1. By (A5) and (A10), we have \( p^S_1 > p^L_1 \) if
\[
\frac{\beta c + 3\beta \alpha - \beta c \gamma - 2\alpha \gamma}{2(2\beta - \gamma)(\beta - \gamma)} > \frac{8\beta^3 + 8\beta^2 \alpha - 10\beta^2 c \gamma + 2\beta \alpha \gamma + 5\beta \gamma^2 c - \alpha \gamma^2 - 3\gamma^3}{2(\beta - \gamma)(\gamma^2 + 8\beta^2)}.
\]
Straightforward calculations show that the latter condition is equivalent to
\[
(\beta - \gamma)(3\gamma^2 - 4\beta \gamma + 8\beta^2)(\alpha - \beta c + \gamma c) > 0,
\]
which is satisfied because of \( \beta > \gamma > 0 \) by assumption for strictly positive demand.

Similar arguments show that \( p^S_2 > p^L_2 \).

**Result 4** The access charge under separation is higher than under liberalization, i.e. \( a^S_i = a^S_2 > a^L_i \).

By (A4) and (A9), we have \( a^S_2 > a^L_2 \) if
\[
\frac{\alpha + (\beta - \gamma) c}{2(\beta - \gamma)} > \frac{8\beta^4 c + 8\beta^3 \alpha - 8\beta^3 c \gamma + 2\beta^2 \gamma^2 - 3\beta c \gamma^3 + \alpha \gamma^3 + \gamma^4 c}{2(\beta - \gamma)(\gamma^2 + 8\beta^2)\beta}.
\]
Calculations similar to those outlined above show that the latter condition is satisfied due to \( \beta > \gamma > 0 \) by assumption for strictly positive demand.

### 8 Appendix B

**Proof of Lemma 1.** We first show that \( 0 < m_{ii} \leq 1 \). Let us start with deriving \( \partial p_i / \partial a_i \). Suppressing the superscript \( S \), the system of first-order conditions for equilibrium retail pricing under vertical separation is given by
\[
\Pi^i_i = D_i(p) + (p_i - a_i) \frac{\partial D_i(p)}{\partial p_i} = 0, \quad i = 1, ..., n. \quad (B1)
\]
Differentiating the $\ell^{th}$ equation of this system with respect to $a_i$, accounting for the changes in all downstream prices, yields

$$
\sum_{k=1}^{n} \left[ (1 + \delta_{\ell k}) \frac{\partial D_{\ell}(p)}{\partial p_k} + (p_{\ell} - a_{\ell}) \frac{\partial^2 D_{\ell}(p)}{\partial p_{\ell}\partial p_k} \right] \frac{dp_k}{da_i} = \delta_{\ell i} \frac{\partial D_i(p)}{\partial p_i},
$$

where $\delta_{\ell k}$ denotes the Kronecker delta (equal to one if $\ell = k$ and zero otherwise). Let

$$
\Pi_{ij} \equiv (1 + \delta_{ij}) \frac{\partial D_i(p)}{\partial p_j} + (p_i - a_i) \frac{\partial^2 D_i(p)}{\partial p_i\partial p_j}, \quad i, j = 1, \ldots, n.
$$

denote the second derivative of downstream firm $i$’s profit function with respect to $p_i$ and $p_j$. In matrix form, the system of equations can then be written as

$$
\begin{bmatrix}
\Pi_{11} & \ldots & \Pi_{1n} \\
\vdots & \ddots & \vdots \\
\Pi_{n1} & \ldots & \Pi_{nn}
\end{bmatrix}
\begin{bmatrix}
\partial p_1/\partial a_i \\ \vdots \\ \partial p_n/\partial a_i
\end{bmatrix}
= \begin{bmatrix}
0 \\
\vdots \\
0
\end{bmatrix},
$$

where the nonzero entry on the right-hand side is in the $i^{th}$ row. Applying Cramer’s rule and accounting for exchangeability (by [A2]) yields

$$
\frac{dp_i}{da_i} = \frac{\partial D_i/\partial p_i}{(\Pi_{ii})^2 + (n-2)\Pi_{ii}\Pi_{ij} - (n-1)(\Pi_{ij})^2} > 0, \quad j \neq i. \quad (B2)
$$

In the unique interior equilibrium, the denominator of (B2) must be positive. The numerator, in turn, is positive by [A3], since the latter implies $\Pi_{ii} + (n-1)\Pi_{ij} < 0$. Manipulating the numerator yields

$$
\frac{dp_i}{da_i} = \frac{(\Pi_{ii})^2 + (n-2)\Pi_{ii}\Pi_{ij} - [\Pi_{ii} - \partial D_i/\partial p_i] \left[ \Pi_{ii} + (n-2)\Pi_{ij} \right]}{(\Pi_{ii})^2 + (n-2)\Pi_{ii}\Pi_{ij} - (n-1)(\Pi_{ij})^2}.
$$

Now, observe that $dp_i/da_i \leq 1$ if

$$
[\Pi_{ii} - \partial D_i/\partial p_i] \left[ \Pi_{ii} + (n-2)\Pi_{ij} \right] \geq (n-1)(\Pi_{ij})^2. \quad (B3)
$$
We know from assumption [A3] that

$$|\Pi_{ii} + (n-2)\Pi_{ij}| > \Pi_{ij}.$$  

For (B3) to hold, it thus suffices to show that

$$|\Pi_{ii} - \partial D_i/\partial p_i| \geq (n-1)\Pi_{ij}.$$  \hspace{1cm} \text{(B4)}

Again, assumption [A3] guarantees that (B4) is satisfied. $m_{ii} \leq 1$ now follows immediately from the first-order condition (B1). Thus, we have shown that $0 < m_{ii} \leq 1$.

We now show that $0 \leq m_{ji} < m_{ii}$ for $j \neq i$. Solving the system of equations for the derivative $dp_j/da_i$ yields

$$\frac{dp_j^S}{da_i} = -\frac{\partial D_i/\partial p_i\Pi_{ij}}{(\Pi_{ii})^2 + (n-2)\Pi_{ii}\Pi_{ij} - (n-1)\left(\Pi_{ij}\right)^2} \geq 0,$$

where the denominator is the same as in (B2) and the numerator is nonnegative by assumption [A1]. Clearly, $dp_i/da_i > dp_j/da_i$ if

$$|\Pi_{ii} + (n-2)\Pi_{ij}| > \Pi_{ij},$$

or if

$$|\Pi_{ii}| > (n-1)\Pi_{ij},$$

which is guaranteed by assumption [A3]. We now use the fact that the demand functions are symmetric, which implies that retail prices are also symmetric. We thus have $0 \leq m_{ji} < m_{ii}$. It now follows immediately that $0 \leq m_{ji} < m_{ii} \leq 1$. □

\textbf{Proof of Proposition 1.} For any given effort level $\bar{e}$, it is sufficient for retail prices to be higher under separation that access prices under separation are at least as high as retail prices under integration, i.e.,

$$a_i^S(\bar{e}) \geq p_i^I(c(\bar{e})) \implies p_i^S(a^S(\bar{e})) > p_i^I(c(\bar{e})), \text{ for all } i.$$

Using (4) and (7), we may write the relation $a_i^S(\bar{e}) \geq p_i^I(c(\bar{e}))$ in terms of Lerner indices as

$$E_i^S + X_{S_i}^S \geq E_i^I + X_{-i}^I, \text{ for all } i.$$
The claim follows immediately from rewriting the latter condition as
\[
(E_i^S - E_i^I) + (X_i^S - X_i^I) \geq 0, \text{ for all } i.
\]

**Proof of Proposition 3.** First, consider market 1. For any given effort \( \bar{e} \), the retail price is higher under liberalization than under integration if
\[
E_1^L + X_{-1}^L > E_1^I + X_{-1}^I.
\]

Now consider the other markets \( i \neq 1 \). Using (4), (14) and (15) the fact that retail prices must be larger than access prices by the profit maximizing behavior of downstream firms, we may write \( p_i^L > p_i^I, i \neq 1 \), in terms of Lerner indices as
\[
E_i^L + X_{-i}^L \geq E_i^I + X_{-i}^I, \quad \text{for } i \neq 1.
\]

Rewriting these conditions as
\[
(E_i^L - E_1^I) + (X_{-1}^L - X_{-1}^I) > 0, \quad \text{and} \quad (E_i^L - E_1^I) + (X_{-i}^L - X_{-i}^I) \geq 0, \quad \text{for all } i \neq 1.
\]

establishes the claim. ■

**References**


Church, J., Gandal, N. (2000), ‘Systems Competition, Vertical Merger and


Figure 1: Different types of market structure