A Further Look at Two-way Network Competition in Telecommunications

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ABSTRACT: This paper develops a simple reduced form model of two-way network competition with linear retail pricing. Using the techniques of supermodular games, it is demonstrated that the most important results from the existing literature do not depend on routinely invoked assumptions, such as specific functional forms or the symmetry of the network operators. In particular, it is demonstrated that (i) firms do not need to be symmetric or regulated to have incentives to collude in access pricing, and (ii) due to the effects on social welfare, enforcing colluding firms to behave noncooperatively is not necessarily desirable.

Keywords: interconnection, supermodularity, tacit collusion, telecommunications.


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1 Introduction

The problem of setting the ‘right’ interconnection rate in order to introduce competition in the telecommunications industry has been analyzed extensively. However, it seems that so far no consensus has emerged on how this body of analysis translates into regulatory policy.\(^1\) Baumol and Sidak (1994 and 1995) for instance propose the “Efficient Component Pricing Rule” (ECPR) developed by Willig (1979) and Baumol (1983) to calculate interconnection tariffs. Laffont and Tirole (1996) instead favor a “Global Price Cap” which treats access to the network as another final good provided by the incumbent and delegates pricing decisions altogether to the incumbent.\(^2\) The cost-oriented approach adopted in the European Union, Switzerland and many other OECD countries (see Jamison 1998) stands in marked contrast to both of these approaches; it is advocated by Mitchell et al. (1995, pp. 101) who hold the view that the two theory-driven proposals mentioned above are taking into account many important insights of the economic analysis of the problem, but are at the same time not very suitable for practical implementations due to strict assumptions, troublesome estimations of (super)elasticities and often complicated policy implications.

These policy rules have all been developed within the framework of “oneway” access, having in mind a situation where a powerful incumbent is holding a monopoly over inputs needed by all firms operating in the industry (the “bottleneck” problem). Under these conditions, asymmetric interconnection regulations favoring (potential) entrants seem appropriate, since there is an incentive for the incumbent to abuse its market power by charging excessive interconnection rates. As pointed out by Noam (1996, pp. 103), this rationale coexists rather uneasily with the symmetric approach called for by the existence of network externalities as described by Farrell and Saloner (1985) and Katz and Shapiro (1985). According to this rationale, the interconnection of fragmented telecommunications networks allows the realization of

\(^{1}\)Although Armstrong et al. (1996) have tried to establish a “synthesis”. See Kovacic (1996) as well.

\(^{2}\)They acknowledge, however, that additional restrictions such as an ECPR-type upper limit for interconnection fees might be necessary to prevent predatory pricing by the incumbent (see Laffont and Tirole 1996, 247)
potential gains to all participants and is therefore in the public interest. Obviously, this approach requires interconnection of all operators—indepen-
dent of market power considerations—and thereby establishes symmetry in the regula-
tory treatment of the competitors.

With the emergence of competition in mobile and even local telephony, the interest of economists and regulators has turned towards the problem of “two-way” access. The economics of network competition with two-way interconnection have recently been explored in papers by Armstrong (1998), Carter and Wright (1999), Dessein (1999a, b), Doganoglu and Tauman (1998) and Laffont et al. (1998a, b). However, these papers concentrate mainly on the analysis of competition between symmetric firms or use specific functional forms. This papers aims to add to this work by

• developing a more general reduced form model of competition in linear prices between differentiated and interconnected firms, which incorpo-
rates various approaches for discrete network choice on the consumer side and allows for asymmetric firms;

• performing comparative statics with respect to equilibrium retail prices and interconnection rates, using the techniques of supermodular games introduced by Milgrom and Roberts (1990); this approach takes advantage of the strategic complementarity in retail price competition and allows to study robust comparative statics even if profit functions are nonconcave, which is a notorious problem in two-way interconnection models;

• confirming the intuitive notion that there might well be a lack of com-
petition even if firms are asymmetric.

The focus on linear retail pricing is justified on two grounds: (i) many providers of mobile telecommunications services let their costumers opt for linear tariffs;³ (ii) the independence of a network operator’s profit from the access tariff resulting from the assumption of nonlinear retail prices (see

³In Switzerland, for example, all three operators of networks for mobile telecommunications services offer contracts featuring linear prices to potential subscribers.
Laffont and Tirole (1998b) is at odds with real world experience; assuming linear pricing helps to circumvent this problem and allows to study directly the extent to which some of the existing literature’s important results depend on specific functional forms and symmetry assumptions.

The plan of the paper is as follows. Section 2 outlines the basic set-up of a simple reduced form model of competition between differentiated network operators which accommodates the different modelling approaches found in the literature. In this model, consumers make subscription decisions and choose the number of calls to make, given the observable prices of competing suppliers of telephone services. Network operators are assumed to act as oligopolists and compete in non-regulated linear retail prices. Presumably, network operators are not able to discriminate prices for “internal” and “external” calls. Section 3 uses the techniques of supermodular games to derive the characteristics of equilibrium retail prices. Section 4 analyzes various interconnection rate equilibria of the game, alternatively studying a noncooperative, a perfectly collusive and a welfare maximizing setting of the interconnection tariffs. Section 5 illustrates the general findings by deriving an explicit analytical solution for a particular specification of the model suggested by Doganoglu and Tauman (1998). Section 6 concludes the paper.

2 The Basic Set-Up

In order to accommodate the different modelling approaches found in the literature, the model analyzed in this paper is formulated in reduced form. To be more specific, suppose the structure of the interconnection problem can be summarized as shown in Figure 1.4

Two full coverage network operators \( i, j = 1, 2, i \neq j \), compete in non-regulated linear retail prices \( p^* \). Each operator incurs constant marginal cost \( c^i \) for handling a call within its own network (“internal” call). An “external” call, i.e. a call routed from network \( i \) to network \( j \), generates an additional constant marginal cost of \( c^{ij} \); in this case, the terminating end of a call for network \( i \) is a point of interconnection where the call is taken over for

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4See Economides et al. (1996) for a related model.
completion by network $j$. Note that with the exception of the access charge, network $i$'s cost for handling outgoing and incoming calls is identical. For the completion of an external (incoming) call, the operator of network $i$ charges an interconnection fee $a^i$. In addition, suppose that there is a fixed cost $F^i$ for the operation and maintenance of the network, billing etc. which is assumed to be independent of the number of subscribers and calls. The industry configuration can thus be summarized as in Table 1.

2.1 Modeling Demand

Suppose there is a fixed number of potential subscribers for the two networks, each subscriber having preferences for one or the other network, depending

\footnote{Note that this cost structure is more general than the costs usually assumed in the literature.}
on their respective characteristics, e.g. retail price for providing services, advertising style and intensity, type of billing etc. Any consumer is assumed to subscribe to only one network. After having subscribed to one or the other network, consumers pay a linear retail price $p^i$ per call (or per appropriate unit of a call).

There are various possibilities to model the consumer’s discrete choice between differentiated networks. In the literature on network competition, discrete network choice is usually modelled employing a simple Hotelling approach (‘competition on the line’). In contrast, Doganoglu and Tauman (1998) suggest a random utility model approach which assigns nonobservable idiosyncratic taste differences to consumers. This approach captures the differentiation between telecom operators more naturally than the Hotelling approach since it takes into account that firms cannot exactly predict the potential subscribers’ behavior but face a system of expected demand. It has the additional advantage to be closely related to empirical studies of demand.

In the following, we use a framework which incorporates both of these models. We simply assume that utility maximization yields a set of well defined and differentiable demand functions $q^i(p^i, p^j)$ and $q^{ij}(p^i, p^j)$ (see section 5 for an explicit solution of a particular specification of the model).

### 2.2 Modeling Supply

As in the literature mentioned at the outset, we model supply as a differentiated oligopoly with price competition. In the two-stage game we consider, the action variables are determined in the following way.

- **stage 1:** The interconnection rates $a^i$ are simultaneously set, either by the operators themselves or by the regulator.

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6 See Armstrong (1998), Carter and Wright (1999), Dessein (1999a, b), Laffont et al. (1998a)

7 The random utility model has found widespread use in econometrics, above all in the (nested multinomial) logit form, which has been shown to be compatible with the utility maximization hypothesis by Börsch-Supan (1990) and Anderson et al. (1992); see Amemiya (1981) and McFadden (1984) for general surveys. For studies of demand in telecommunications, see Atherton et al. (1990) and Train et al. (1987).
• stage 2: Given the interconnection fees agreed upon by the operators or set by the regulator, the network operators choose their retail prices \( p_i \) noncooperatively, taking the competitor’s price \( p_j \) as given.

Note that this standard game structure implicitly assumes that interconnection tariffs cannot be altered as quickly as retail prices and therefore have a commitment value for strategic price competition on the retail market. This is a reasonable description of real world examples, since unlike retail prices, negotiated interconnection contracts are usually not unilaterally alterable and often need to be submitted to the regulatory authorities (e.g. for observation by other parties). In the following section, we shall establish the comparative statics characteristics of the equilibrium retail prices \( p_i \).

3 Retail Price Competition

Given the demand functions \( q^i(p^i, p^j) \) and \( q^{ij}(p^i, p^j) \), we can define the profit function of network operator \( i \) to be

\[
\pi^i(p^i, p^j, a^i, a^j) = (p^i - c^i)q^i(p^i, p^j) + (p^j - c^j - c^{ij} - a^{ij})q^{ij}(p^i, p^j) \\
+ (a^i - c^i - c^{ij})q^{ij}(p^i, p^j) - F^i. 
\]

Each firm sets its retail price so as to maximize (1). Suppose that there exists a unique Nash equilibrium in retail prices \( \{ p^{i*}(a^i, a^j), p^{j*}(a^i, a^j) \} \). Standard techniques suggest that the characteristics of this equilibrium can be established by deriving the first-order conditions for profit maximization and then performing comparative statics.

However, previous work has shown that under network competition, profit functions might be nonconcave in retail prices and the existence of a retail price equilibrium is thus not guaranteed in general, especially if the networks

\[\text{Armstrong (1998, 552) puts forward a more technical argument for this presumed course of events: Another ordering "does not make sense" since if both networks' retail prices are fixed, so is the number of calls going from network } i \text{ to network } j \text{ and vice versa. Then any network can set an arbitrarily high access charge to make arbitrarily high profits, and hence there can be no equilibrium.}\]

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are highly substitutable.9 Thus, an important part of the literature deals with the conditions that need to be satisfied for the existence of an equilibrium. A common feature of the different models is that firms compete in retail prices or strategic complements, respectively. Strategic complementarity, however, is a natural condition for the existence of an equilibrium. The present paper takes advantage of strategic complementarity and establishes the general characteristics of the set of possible Nash equilibria, using the techniques of supermodular games introduced by Milgrom and Roberts (1990). More specifically, the paper derives the comparative static characteristics any unique (subgame perfect) Nash equilibrium in pure strategies has to satisfy.

For this purpose, suppose the following assumptions are satisfied for \(i, j = 1, 2, i \neq j\), where subscripts denote derivatives, i.e. \(\partial \pi^i / \partial p^i \equiv \pi^i_{p^i}, \partial^2 \pi^i / \partial p^i \partial a^i \equiv \pi^i_{p^i a^i}\) (this notational convention is also used in the remainder of the paper).

(A 1) (i) \(p^i \leq \bar{p}^i\), with \(\bar{p}^i\) denoting the price for which the demand for \(i\)'s services becomes zero;
(ii) \(p^i \geq c^i + c^{ij} + a^i\);
(iii) \(a^i \geq c^i + c^{ij}\).

(A 2) \(\pi^i\) is twice continuously differentiable on \(p^i\) and \(a^i\).

(A 3) \(\pi^i_{p^i a^i} = q^{ij}_{p^i} \geq 0\) and \(\pi^i_{p^i a^i} = -q^{ij}_{p^i} \geq 0\).

(A 4) \(\pi^i_{p^i p^j} \geq 0\) (competition with strategic complements).10

(A 5) \(q^{ij}_{p^i} \leq 0, q^{ij}_{a^i} \geq 0\) and \(q^{ij}_{p^i} \geq q^{ij}_{p^i} + q^{ij}_{a^i}\).


10Writing out this condition yields
\[
\pi^i_{p^i p^j} = q^{ij}_{p^i} + (p^i - c^i)q^{ij}_{p^i p^i} + q^{ij}_{a^i} + (p^i - c^i - c^{ij} - a^i)q^{ij}_{p^i p^j} + (a^i - c^i - c^{ij})q^{ij}_{a^i} \geq 0.
\]

Assumption (A 4) thus essentially requires that the positive direct effect of a price increase on the competitor’s demand (represented by the first and the third term) dominates the possibly negative indirect effect.
Let us shortly indicate the significance of each assumption. Technically speaking, assumption (A 1) assures that the strategy set of each player is a complete lattice.\textsuperscript{11} In economic terms, the action variables $p^i$ and $a^i$ are restricted to values that allow the firms to cover costs for each service provided, i.e. weakly dominated strategies are excluded. Consequently, the retail price $p^i$ (i) cannot be higher than the price $\bar{p}^i$ leading to zero demand and (ii) must cover the cost of handling an outgoing call; in addition, (iii) the interconnection rate $a^i$ must cover $i$’s cost of an incoming call. (A 1) also implies that $a^i$ cannot be higher than $(p^j - c^j - c^i)$, otherwise it would violate the assumption of cost coverage for $j$. Note that this restriction of the action variables is without loss of generality, since it follows from the definition of a weakly dominated strategy that any Nash equilibrium of the game where strategies are restricted to satisfy (A 1) is also a Nash equilibrium of the unrestricted game. (A 2) is a technical assumption which requires that the profit functions are twice differentiable for the strategy sets defined above. By placing natural restrictions on the demand functions, assumptions (A 3), (A 4) and (A 5) jointly assure that the firms in fact play a game of strategic complements that can be analyzed using supermodularity techniques. Essentially, they require that the demand for services provided by firm $i$ ($j$, respectively) decreases (increases) with $p^i$, and that the effect on own demand is stronger than the effect on the competitor’s demand.

This set of assumptions, which will typically be satisfied in the more familiar symmetric games, assures that the game considered in this paper in fact qualifies as a (smooth) supermodular game. For such a game, there always exists a Nash equilibrium in pure strategies as well as a largest and smallest pure Nash equilibrium (Milgrom and Roberts, 1990, 1266). Given these assumptions, the characteristics of the equilibrium retail price $p^*\left(\cdot\right)$ can be summarized as follows.\textsuperscript{12}

\textsuperscript{11}A set $S$ is a lattice if for each two point set $\{x, y\} \subset S$, there is a supremum for $\{x, y\}$ and an infimum in $S$. The lattice is complete if for all nonempty subsets $T \subset S$, $\inf(T) \in S$ and $\sup(T) \in S$ (Milgrom and Roberts 1990, 1260).

\textsuperscript{12}Note that this proposition and the following ones could also be established using more traditional (implicit function) techniques for comparative statics. However, we would then need to impose the additional assumption of concave profit functions.
Proposition 1 (Nash retail prices) Suppose a unique Nash retail price equilibrium exists. In addition, suppose assumptions (A 1)-(A 5) hold. Then the equilibrium retail price $p^*_{i}(a^i, a^j, c^i, c^{ij})$ is (i) nondecreasing with the interconnection rates $a^i$ and $a^j$ (i.e. $p^*_a \geq 0, p^*_a \geq 0$) and (ii) nondecreasing with marginal costs $c^i$ and $c^{ij}$ (i.e. $p^*_c \geq 0, p^*_c \geq 0$).

Proof. (i) Theorem 6 by Milgrom and Roberts (1990, 1267) implies that the claim holds if

$$\pi^i_{p^i a^i} = q^i_{p^i} \geq 0, \pi^i_{p^i a^j} = -q^j_{p^i} \geq 0 \text{ and } \pi^i_{p^i p^j} \geq 0.$$  

This is guaranteed by (A 3) and (A 4).

(ii) The same theorem applies if the conditions

$$\pi^i_{p^i p^j} \geq 0, \pi^i_{p^i c^i} = -q^i_{p^i} - q^j_{p^j} \geq 0 \text{ and } \pi^i_{p^i c^{ij}} = -q^i_{p^i} - q^j_{p^j} \geq 0,$$

are satisfied. The first is guaranteed by (A 4); straightforward transformations of the second and the third condition lead to the simpler condition $|q^i_{p^i}| \geq q^j_{p^i}$, which is assured by (A 5).  

Intuition suggests that the equilibrium retail price $p^*_i$ is increasing with marginal costs $c^i$ and $c^{ij}$. Proposition 1 shows, however, that this is only correct if the direct (negative) effect of a price increase on own demand $q^i_{p^i}$ is stronger than the indirect (positive) effect on the competitor’s demand $q^j_{p^j}$. Perhaps more importantly, it demonstrates that the retail price increases with interconnection prices. Proposition 1 thus generalizes the literature’s earlier results found under more restrictive assumptions.\(^{13}\)

The analysis of two-way network competition supplements the traditional “bottleneck” framework analysis which does not elaborate on the link between the levels of interconnection rates and retail prices. The lack of consideration of this effect in the bottleneck framework is due to the fact that potential competitors of the vertically integrated incumbent are treated as more efficient providers of retail services with regulated prices, while strategic

\(^{13}\)See Armstrong (1998, 554), Carter and Wright (1999, 6), Doganoglu and Tauman (1998, proposition 1) and Laffont et al. (1998a, proposition 2) for a similar result.
interactions between the competitors are largely ignored. The latter holds especially for the ECPR-approach, as Brennan (1997) and Mitchell et al. (1995, p. 99) point out.

Note that proposition 1 does not rely on any symmetry assumptions—for example with respect to costs—which dominate the literature on network competition. In addition, concavity of the profit functions is not required for proposition 1 to hold. The level of the access charge is thus generally relevant (even for symmetric firms). Observe that an increase of the access tariff by network $i$ always generates two strategic effects on $j$’s retail price: it

- shifts network $j$’s profit function and thus its reaction function $R^j(\cdot)$ upward directly, and

- alters network $j$’s retail price indirectly via the ex post variation of $p^j$ as a response to the change in $a^j$, to operator $i$’s benefit (strategic complements).

The alteration of both networks’ profit functions translates into a shift of the respective reaction functions $R_i(\cdot)$ and $R_j(\cdot)$ and eventually leads to the implementation of an equilibrium with higher retail prices, which is not necessarily symmetric (see Figure 2).

At first glance, the strategic effect of increasing the interconnection rate in stage 1 looks rather similar to that appearing in many well-known two-stage games such as the Spence-Dixit model of strategic investment or related models analyzing learning by doing, advertising etc. However, there is an additional twist here: by increasing the interconnection rate $a^j$ in stage 1 of the game, player $i$ is not only able to directly shift his own reaction curve to improve his strategic position in stage 2 of the game; he also directly affects his competitor’s cost for outgoing calls, thereby initiating a desire to increase $p^i$ (see (A 3)). This effect reinforces the more common indirect effect that

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$^{14}$To see this, consider the case where network operator 2 has higher marginal costs than operator 1, i.e. $c^1 < c^2$ and $c^{12} < c^{21}$. Proposition 1 shows that operator 2 will then set a higher retail price than operator 1.

$^{15}$See Shapiro (1989, 383 ff.) for a survey of two-stage competition models.

$^{16}$That is, each action taken by any firm in stage 1 of the game has two direct effects.
emerges as a consequence of increasing \( a^j \), since firm \( i \) wants to increase the retail price \( p^j \) even if firm \( j \)'s actions are held constant, which in turn leads firm \( j \) to desire an increase in \( p^j \) because of strategic complementarity (see (A.4)). Given these mutually reinforcing effects, the retail price equilibrium must go up. Note that these effects work even if the Nash equilibrium is non-unique. Then the comparative statics summarized in proposition 1 hold for the smallest and the largest equilibrium of the set of Nash equilibria.

In sum, by setting the interconnection rate in stage 1, network operators simultaneously determine how vigorous retail competition will be in stage 2. An increase of the access charge softens retail price competition and thus serves as an instrument of tacit collusion.

4 Setting Interconnection Rates

In many OECD countries, the interconnection rate is either determined by the network operators during negotiations or, if they fail, by the regulator, in which case it is usually set at the LRIC plus a markup to recover joint and common costs. If there exists a considerable difference in the market shares of the negotiating parties, the operator’s interests in setting the interconnec-
tion fees may be conflicting. The dominant (incumbent) operator might be inclined to increase the interconnection rate and decrease the retail price in order to apply a price squeeze to his competitor and corner the market. The competing (new) operator in turn might prefer low interconnection rates in order to set low retail prices, attract subscribers, and build up his market share. In this case, the result of negotiations will critically depend on the relative power of the parties involved; the power of a party is the bigger, the less damage it expects from a breakdown of negotiations (Brock and Katz 1997, p. 106). This setting is often complicated by the fact that, if not satisfied by the result of the negotiations, the involved parties can invoke an arbitrator. Under such circumstances, the outcome of the negotiation process is very difficult to identify ex ante.\textsuperscript{17}

Rather than explicitly modeling this process, we will therefore confine ourselves to the analysis of three important benchmark cases to understand the effects of alternative mechanisms for setting interconnection tariffs:

- **noncooperative setting of access charges.** Interconnection negotiations are assumed to implement a noncooperative (subgame perfect) Nash equilibrium. While this might seem to be an uncommon outcome of negotiations, noncooperative access pricing is the common starting point in the literature which can be implemented by (i) a credible threat that will be realized if operators are not able to reach a cooperative agreement but nevertheless prefer to prevent cost-oriented regulatory intervention or (ii) a regulatory policy that prevents cooperation in the setting of access rates.

- **cooperative setting of access charges.** It is assumed that the network operators agree to maximize industry profits (without further specifying how profits are allocated).

- **welfare maximizing access charges.** Interconnection rates are assumed to be set by a social planner, maximizing the sum of the consumer and the producer surplus.

\textsuperscript{17}See Manzini and Mariotti (1997) for a game theoretic analysis of such a situation using a Rubinstein-type framework.
4.1 Noncooperative Interconnection Rates

If interconnection rates are determined noncooperatively, the network operators set \( a^{i*} \) so as to maximize (1), given the existence of a Nash retail price equilibrium in stage 2. The characteristics of the set of all noncooperative Nash equilibrium interconnection rates \( a^{i*} \) are summarized in the following proposition.

**Proposition 2 (Nash interconnection rates)** Suppose assumption (A 1)-(A 5) hold. Then both the smallest and the largest noncooperative Nash equilibrium interconnection rate \( a^{i*} \) are increasing in \( c^i \) and in \( c^{ij} \).

**Proof.** The proof is very similar to that for proposition 1. Theorem 6 by Milgrom and Roberts (1990) implies that the claim holds if

\[
\pi^i_{p^{ij}_p} \geq 0, \pi^i_{a^{i*} \epsilon} = - \left( q_{p^i}^{ij} + q_{p^j}^{ij} \right) q_{a^{i*}}^{ij} \geq 0 \text{ and } \pi^*_{a^{i*} \epsilon} = - \left( q_{p^i}^{ij} + q_{p^j}^{ij} \right) p_{a^{i*}}^{ij} \geq 0.
\]

This first condition is satisfied by (A 4). The second and third one are satisfied for \( p_{a^{i*}}^{ij} \geq 0 \) and \( \left| q_{p^i}^{ij} \right| \geq q_{p^j}^{ij} + q_{p^i}^{ij} \), which is guaranteed by proposition 1 and (A 5).

Proposition 2 shows that the set of noncooperative Nash equilibrium interconnection rates is increasing with marginal costs. For this result to hold, the effect of a retail price increase on own demand again needs to be stronger than the effect on the competitor’s demand (see proposition 1).

As indicated, proposition 2 holds for the set of Nash equilibria of the game. It does not imply, however, that the same comparative statics also apply to subgame perfect Nash equilibria of the game. That is, even if there exists a unique subgame perfect Nash equilibrium, it might not satisfy the comparative statics summarized in proposition 2. Consequently, we need to study subgame perfect equilibria separately. For that purpose, denote the profit function at the Nash retail price equilibrium as

\[
\hat{\pi}^i(a^i, a^j, c^i, c^{ij}) \equiv \pi^i(p^{ij*(a^i, a^j, c^i, c^{ij})}, p^{ij*(a^i, a^j, c^i, c^{ij})}, a^i, a^j, c^i, c^{ij}).
\]

Consider the following additional assumptions.
(A 6) \( \hat{\pi}_{a^i, a^j}^i \geq 0 \) (competition with strategic complements);
(A 7) \( \hat{\pi}_{a^i, e^j}^i \geq 0 \) and \( \hat{\pi}_{a^i, e^j}^{i,j} \geq 0 \).

Assumption (A 6) extends (A 4) in that it requires strategic complementarity for the interconnection rates explicitly (due to \( \pi_{a^i, a^j}^i = 0 \), this condition was automatically satisfied for the set of Nash equilibria). Straightforward but somewhat tedious calculations show that this condition requires

\[
\hat{\pi}_{a^i, a^j}^i = \pi_{a^i, a^j}^i + \pi_{a^i, p^j}^i \cdot p_{a^j}^i + \pi_{p^i, a^j}^i \cdot p_{a^j}^i \cdot p_{p^i}^i + [\pi_{p^i, a^j}^i \cdot p_{p^i}^i \cdot p_{p^i}^j \cdot p_{a^j}^i \cdot p_{a^j}^i] \geq 0.
\]

The indicated signs of several terms were determined above. (A 6) thus requires that the terms with undetermined sign are either positive or not too negative, which means that the strategic complementarity imposed by assumption (A 3)-(A 5) is strong enough not to be dominated by the various additional indirect effects of an access charge increase on the subgame perfection equilibrium. (A 7) is an additional comparative statics assumption similar to (A 6)\(^{18}\) which extends (A 5). The next result then follows.

**Proposition 3 (subgame perfect interconnection rates)** Suppose a unique subgame perfect Nash equilibrium exists. In addition, suppose assumptions (A 1)-(A 7) hold. Then the subgame perfect equilibrium interconnection rate \( a^i \) is increasing in \( e^i \) and \( e^{i,j} \).

**Proof.** Theorem 6 by Milgrom and Roberts (1990) implies that the claim holds if

\( \hat{\pi}_{a^i, a^j}^i \geq 0, \hat{\pi}_{a^i, e^j}^i \geq 0 \) and \( \hat{\pi}_{a^i, e^j}^{i,j} \geq 0 \).

This first condition is satisfied by (A 6). The second and third one are satisfied by (A 7). \(
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\(^{18}\)Writing out the two conditions in (A 7) yields expressions similar to that for (A 6).
Proposition 3 shows that given the additional regularity assumptions (A 6) and (A 7), the subgame perfect equilibrium interconnection rate \( a^{is} \) has the same comparative statics characteristics as the Nash equilibrium rate \( a^{*} \). We now consider how colluding network operators set their interconnection rates.

4.2 Perfectly Collusive Interconnection Rates

Suppose that the network operators are able to attain perfect collusion in stage 1, while in stage 2, they compete in retail prices for potential subscribers. Thus, they maximize total industry profits

\[
\Pi \equiv \pi^i(p^i_*(a^i, a^j), p^j_*(a^i, a^j), a^i, a^j) + \pi^j(p^i_*(a^i, a^j), p^j_*(a^i, a^j), a^i, a^j)
\]

The first-order condition for perfectly collusive interconnection rates \( a^{ic} \) then reads

\[
\frac{d\Pi}{da^i} = d\pi^i(\cdot)/da^i + d\pi^j(\cdot)/da^j \\
= d\pi^i(\cdot)/da^i + \left(\pi^j_{p^i} \cdot p^i_{a^i} + \pi^j_{p^j} \cdot p^j_{a^j} + \pi^j_{a^i}ight) = 0.
\] (2)

Recall that \( \pi^i \) is differentiable (see (A 2)). Since in equilibrium retail prices are optimally chosen, we can apply the envelope theorem, i.e. \( \pi^j_{p^i} \cdot p^i_{a^i} = 0 \), to simplify this expression. Using (1), writing out the term in brackets yields

\[
d\pi^j(\cdot)/da^j = (p^j - c^j)q^j_{p^j} \cdot p^j_{a^j} + (p^j - c^j - c^{ji} - a^j)q^{ji}_{p^j} \cdot p^j_{a^j} - q^{ji} \\
+ (a^j - c^j - c^{ji})q^{ji}_{a^j} \cdot p^j_{a^j}, \text{ for } i, j = 1, 2, i \neq j.
\] (3)

By inspection of (2) and (3), the next observation follows immediately.

**Proposition 4 (collusive interconnection rates)** Depending on the model’s specification, collusive interconnection rates \( a^{ic} \) maximizing the industry’s profits can be smaller, larger, or equal to subgame perfect Nash equilibrium rates \( a^{is} \).

Inspection of (2) reveals that the subgame perfect equilibrium interconnection rate \( a^{is} \) (which must satisfy the first-order condition \( d\pi^i(\cdot)/da^i = 0 \))
is only coincident with the access rate $a^c$ maximizing industry profits if
\[ d\pi^j(a^s)/da^i = \pi^j_{p^i} \cdot p^j_{a^i} + \pi^j_{a^i} = 0. \]
However, in general, interconnection rates maximizing industry profits will be larger or smaller than noncooperative ones, depending on the sign of this term, which is determined by

- the relative size of direct negative effect $\pi^j_{a^i} = -q^{ji}$, and
- the relative size of the indirect positive effect $\pi^j_{p^i} \cdot p^j_{a^i}$, which depends
  on the price sensitivity of demand as well as the sensitivity of the retail
  price with respect to the access charge.

The net effect of an access charge increase on the competitor’s profit is thus ambiguous. Note that this result stems from the model’s unusual feature that firm $i$’s action in stage 1 of the game has a direct effect on firm $j$’s profit. The indirect effect more familiar from many other two-stage games summarizes the strategic incentives in stage 1. Using the terminology of Fudenberg and Tirole (1984), it can be characterized as follows. An increase of the interconnection rate $a^i$ makes firm $i$ “soft” by raising $p^j$, which has a positive effect on firm $j$’s profit. Due to competition in strategic complements, operator $j$ then wants to increase his retail price $p^j$, which in turn helps firm $i$. Consequently, firm $i$ will “overinvest” or increase the interconnection rate $a^i$, respectively, and thus adopt a “fat cat” strategy. Since the strategic effect can be overcompensated by the negative direct effect, there are three different cases to consider.

**Case 1:** $d\pi^j(a^s)/da^i = 0$.

The term is equal to zero if the direct effect $\pi^j_{a^i}$ and the indirect effect $\pi^j_{p^i} \cdot p^j_{a^i}$ exactly cancel. Then network operators have congruent strategic interests. Therefore, they do not need to negotiate in order to implement the fully collusive outcome, since under these conditions, the subgame perfect equilibrium happens to be equivalent to perfect collusion. It is therefore hardly surprising that the literature on two-way interconnection identifies a potential collusion problem when studying models of symmetric firms. Note, however, that according to proposition 1, firms do not necessarily have to be symmetric to automatically attain perfect collusion. It is just their strategic interests that have to be aligned. This requirement is trivially satisfied for symmetric
firms, but might also be satisfied for specific asymmetric parametrization of the model.

**Case 2:** \( d\pi^j(a^{i*})/da^i < 0 \).

The term is negative if the indirect effect \( \pi^j_p \cdot p^*_n \) is smaller than the absolute value of the direct effect \( \pi^j_{a^i} \). Then the derivative of aggregate industry profits with respect to the interconnection rate \( a^i \) must be negative at firm \( j \)'s optimum as well, i.e. \( d\Pi(a^{i*})/da^i < 0 \). Assuming that there exists a unique interconnection rate \( a^{ic} \) maximizing industry profits \( (d\Pi(a^{ic})/da^i = 0) \), we conclude that the subgame perfect interconnection rate is higher than the collusive one. The difference to the collusive outcome stems from the fact that noncooperative operators neglect the negative externality they impose on their competitor’s profit.

**Case 3:** \( d\pi^j(a^{i*})/da^i > 0 \).

The term is positive if the indirect effect \( \pi^j_p \cdot p^*_n \) is larger than the absolute value of the direct effect \( \pi^j_{a^i} \). Then the derivative of aggregate industry profits is positive, i.e. \( d\Pi(a^{i*})/da^i > 0 \). Consequently, the subgame perfect interconnection rate is smaller than the collusive one. That is, since noncooperative operators do not take into account the positive externality they impose on their competitor’s profit, they set their interconnection lower than under collusion.

These results indicate that \( d\pi^j(a^{i*})/da^i \) can naturally be interpreted to measure the extent to which strategic incentives differ between competing operators. We have demonstrated that noncooperative firms generally adopt “fat cat” strategies and thus “overinvest” or increase the interconnection rate, respectively. Nevertheless, it is possible that they set the interconnection rate lower than under perfect collusion, since an increase of this rate might have a positive effect on the competitor’s profit \( (d\pi^j(a^{i*})/da^i > 0) \) which noncooperative operators neglect. We now consider whether noncooperative and perfectly collusive interconnection rates deviate from welfare maximizing rates.
4.3 Welfare Maximizing Interconnection Rates

By definition, welfare maximizing interconnection rates are set so as to maximize the sum of the producer surplus $\Pi$ and the consumer surplus $S$, that is

$$\max_{a^*, a^i} W = \Pi + S(p_1^*(a^i, a^j), p_2^*(a^i, a^j))$$

Welfare maximizing rates $a^{i**}$ therefore satisfy the first-order condition

$$\frac{dW}{da^i} = d\Pi(\cdot)/da^i + dS(\cdot)/da^i = 0 \quad (4)$$

with $dS(\cdot)/da^i \equiv S^{p^i} \cdot p^{i*} + S^{p^j} \cdot p^{j*}$. 

**Proposition 5 (welfare optimal interconnection rates)** Suppose the consumer surplus is decreasing in retail prices (i.e. $S^{p^i} < 0$ and $S^{p^j} < 0$), and proposition 1 holds. Then the welfare maximizing interconnection rate $a^{i**}$ is (i) always smaller than the collusive rate $a^{ic}$ and (ii) smaller than the subgame perfect equilibrium rate $a^{is}$ if $|dS(a^{is})/da^i| > d\pi^j(a^{is})/da^i$.

**Proof.** (i) From (2) we know that for collusive access charges $d\Pi(a^{ic})/da^i = 0$. Therefore, we must have $dW(a^{ic})/da^i < 0$, because $S^{p^i} < 0$, $S^{p^j} < 0$ (by assumption) and $p^{i*} > 0$, $p^{j*} > 0$ (by proposition 1). Hence, access charges must be smaller under welfare maximization than under collusion.

(ii) Using $d\pi^j(a^{is})/da^i = 0$ from the first-order condition for subgame perfect access pricing, the claim amounts to

$$\frac{dW(a^{is})}{da^i} = d\pi^j(a^{is})/da^i + dS(a^{is})/da^i < 0.$$ 

This is guaranteed by $|dS(a^{is})/da^i| > d\pi^j(a^{is})/da^i$. ■

Proposition 5 states that the welfare maximizing interconnection rate $a^{i**}$ is generally smaller than the collusive rate $a^{ic}$, and smaller than the subgame perfect rate $a^{is}$ if $|dS(a^{is})/da^i| > d\pi^j(a^{is})/da^i$, i.e. if an access charge increase has a stronger effect on the consumer surplus than on the competitor’s profit.
When are welfare maximizing interconnection rates smaller than the sub-game perfect equilibrium rates? Consider the three cases discussed above. In case 1 and 2, where \( d\pi^j(a^{i*})/da^i \) is either zero or negative, noncooperative firms generally set a subgame perfect interconnection rate \( a^{i*} \) that is higher than the welfare maximizing rate. The intuition of this result is simple, since firms either achieve the fully collusive outcome (case 1) or set even higher retail prices (case 2). In case 3, however, \( d\pi^j(a^{i*})/da^i \) is positive and the subgame perfect equilibrium rate is smaller than the collusive one. Consequently, the welfare maximizing interconnection rate may be higher than the noncooperative one.

Observe that a policy which enforces colluding firms to behave noncooperatively is not necessarily improving social welfare. To see this, suppose an access charge increase has a no (or a negative) effect on the competitor’s profit (case 1 and 2 above). We know from proposition 5 that welfare maximizing interconnection rates are then smaller than noncooperative ones. Therefore, effective two-way network competition cannot implement a social optimum. In fact, since noncooperative firms set interconnection rates at least as high as collusive ones, forcing colluding firms to behave noncooperatively generates no or even adverse welfare effects. Only if an access charge increase has a positive effect on the competitor’s profit and noncooperative firms thus set their access tariffs lower than under collusion (case 3), will forcing firms to behave noncooperatively improve social welfare relative to the collusive outcome. Hence, for a wide range of possible situations of strategic interaction, noncooperative access pricing is not preferable relative to collusion due to the adverse effects on social welfare.

5 Example of an explicit solution

Let us now illustrate the comparative statics characteristics of the reduced form model developed above, using the random utility approach proposed by Doganoglu and Tauman (1998) to model the consumer’s network choice. Suppose there are \( N \) consumers with the utility function

\[
U^i = u^i \exp(e^i),
\]

20
with $u^i(q^i) = (b - dq^i)q^i$, $b$ and $d$ being positive parameters; the term $c^i$ is i.i.d. according to the double exponential distribution (see Anderson et al. 1992). It can then be shown that the probability that a randomly chosen consumer subscribes to network $i$ is defined as

$$P^i = \frac{(b - p^i)^\eta}{(b - p^i)^\eta + (b - p^j)^\eta},$$

where $\eta$ can be interpreted as the “substitution rate”, indicating the degree of substitutability between the networks. Since the potential subscribers to the respective networks are identical and statistically independent by assumption, the number of calls initiated in network $i$ and terminated in network $j$ is proportional to the market shares $P^i(p^i, p^j)$ and $P^j(p^j, p^i) = (1 - P^i)$. The system of (expected) demand for calls therefore is

$$q^i(p^i, p^j) = \frac{N}{2d}(b - p^i)(P^i)^2,$$

$$q^{ij}(p^i, p^j) = \frac{N}{2d}(b - p^i)P^i(1 - P^i), \text{ for } i \neq j, \ i, j = 1, 2.$$

In order to derive an explicit solution for this model, we need to impose symmetry, i.e. the firms have the same costs $c^i = c^j$, $c^{ij} = c^{ji}$, charge the same access price $a^i = a^j \equiv a$ and the same retail price $p^i(a) = p^j(a) \equiv p(a)$. Firms then share the market symmetrically, i.e. $P^i = P^j = \frac{1}{2}$. Before proceeding, note that we continue to suppose that the assumptions (A 1)- (A 7) are satisfied. This requires in particular that the exogenous demand parameters $b$ and $\eta$ are chosen appropriately.

Using (1), the functional forms specified above and imposing symmetry,

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19This is so-called “balanced calling pattern” assumption made in Armstrong (1998), Carter and Wright (1999), Doganoglu and Tauman (1998) and Laffont et al. (1998a). It is a convenient approximation for fairly similar customer structures for both of the interconnecting networks. It will not apply, however, if the network operators serve systematically diverse customer segments, or if the subscribers to one network are required to pay a fee for incoming calls (as might be the case for mobile subscribers), which may induce them to keep the number of received calls as small as possible.

See Desselin (1999b) for an analysis of unbalanced calling patterns within a framework of nonlinear retail pricing.
\[
\begin{align*}
\text{noncooperation/collusion} & & \text{welfare maximization} \\
\hline
a^* = a^c &= \frac{c^i(2 - \frac{b}{2}) + c^{ij}(2 + \eta) + b\eta}{2} & a^{**} &= \frac{c^i(8 + \eta) + c^{ij}(6 + 2\eta) - 4b}{2} \\
p(a^*) = p(a^c) &= \frac{2b + 3c^i + 2c^{ij}}{4} & p(a^{**}) &= \frac{3c^i + 2c^{ij}}{2}
\end{align*}
\]

Table 2: Explicit solutions for access and retail prices

the equilibrium retail price of stage 2 can be shown to be

\[
p(a) = \frac{2b + c^i(2 + \eta) + c^{ij} + a}{4 + \eta}.
\]

As stated in proposition 1, it is increasing with \(a\) (even though the payments for call termination cancel in equilibrium) and marginal costs \(c^i\) and \(c^{ij}\). The equilibrium retail price is also increasing with the demand intensity \(b\), and decreasing with the substitution rate \(\eta\).\(^{20}\) The latter result states that a higher substitutability between the networks intensifies competition and therefore drives down the equilibrium retail price.

We have already pointed out that the strategic incentives of symmetric operators are naturally aligned (i.e. \(d\pi^j(a^{**})/da^j = 0\)). More specifically, noncooperative operators anticipate that their profits will depend on the access tariffs \(a^i\) and \(a^j\) and thus set their own tariff in stage 1 so as to maximize individual profit, given the retail price equilibrium in stage 2. Since they are symmetric, their profits are maximized at the same level \(a\) which also maximizes aggregate industry profits. Hence, noncooperative and perfectly collusive behavior generates the same interconnection rates if firms are symmetric. Table 2 summarizes the various outcomes of this game.

The results listed in Table 2 illustrate our findings stated in proposition 2 and 3. Note that under noncooperation (and collusion, since firms are symmetric), equilibrium retail and subgame perfect interconnection prices are increasing with marginal costs \(c^i\) and \(c^{ij}\) (for \(\eta < 4\)). In addition, straight-

\(^{20}\)Observe that \(p_\eta(a) = [2c^i - 2b - c^{ij} - a]/(4 + \eta)^2 < 0\). From \(a \geq c^i + c^{ij}\) and \(b > p > c^i + c^{ij} + a\) (see (A 1)) the result follows.
forward calculations show that the collusive (or noncooperative) retail price \( p(a') \) (or \( p(a^*) \), respectively) is higher than the welfare maximizing \( p(a^{**}) \) whenever the following condition is fulfilled:

\[
b > \frac{3d^j + 2d^j}{2}.
\]

Inspection of the demand functions in (6) shows that this condition is in fact the weakest possible restriction on the model’s parameter values, since it simply requires that the demand intensity \( b \) is high enough to allow for a nontrivial market equilibrium with positive demand. That is, in any symmetric equilibrium, the collusive (or noncooperative) outcome features higher retail prices—and thus higher access prices—than the welfare maximizing outcome.

6 Concluding Remarks

The purpose of this paper was to show that the most important results from the existing literature on two-way network competition with linear retail pricing do not depend on the routinely imposed assumptions of specific functional forms or symmetry of the network operators. In particular, we have confirmed the intuitive notion that network operators have an incentive to use their access tariffs as an instrument of tacit collusion and thereby soften competition on the retail market even if they are not symmetric.

Consequently, regulatory authorities essentially face the same problem, whether there is a bottleneck facility or two-way network competition: interconnection rates may be ‘too high’ relative to the welfare optimum. To be sure, the reasons for excessive interconnection rates are not the same in these two cases. In the case of a bottleneck facility, strategic entry deterrence is the main issue of concern, since the incumbent has an incentive to set an excessive interconnection rate in order to increase entry costs for potential competitors. In the case of the more mature two-way network competition considered here, however, competitors might tacitly agree to use the interconnection rates as an instrument of collusion.

In addition, we have shown that forcing colluding operators to behave
noncooperatively is not necessarily welfare improving. Noncooperative network operators adopt “fat cat” strategies when choosing the interconnection rate and generally neglect the effect that an access charge increase has on the competitor’s profit. Therefore, noncooperative interconnection rates will improve welfare relative to collusion if and only if an access charge increase has a positive effect on the competitor’s profit; otherwise, noncooperative firms will set even higher access tariffs than colluding firms.

Several important issues were not addressed in this paper. First, and most obviously, more general forms of tariffs for access and retail prices and their respective effects on network competition and social welfare should be studied. Second, the process of interconnection negotiations should be analyzed and modeled explicitly. A more thorough understanding of the negotiations’ mode of operation might prove helpful in designing restrictions to be placed on the process and allow regulatory authorities to refrain from directly regulating the level of interconnection rates. For both of these tasks, the results of this paper may serve as a useful starting point.
References


