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Abstract

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KEYWORDS: successive oligopolies, vertical integration, efficiency, foreclosure

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1 Introduction

In many industries, vertically integrated and separated firms coexist. And, many times, it is the larger firms that are integrated. In the U.S. petroleum refining industry, for instance, many large refiners are integrated backward into the exploration and extraction of crude oil or forward into marketing and distribution. Yet, other firms choose a non-integrated structure, separated from both oil production and retailing. Interestingly, these non-integrated firms tend to be smaller, doing less refining.¹

Another well-known example for such asymmetric vertical integration is the package holidays industry, where tour operators assemble holiday packages by contracting with suppliers of transport and accommodation services. In its investigation of the U.K. market, the European Commission (1999a, no. 73) noted that the “polarization of the market into large integrated companies and smaller non-integrated companies is a widely recognized trend in the industry.”²

Asymmetric vertical structures are also documented, e.g., for the beer industry in the U.K. (Slade 1998a), the retail gasoline market in Vancouver (Slade 1998b), the U.S. cable television industry (Waterman and Weiss 1996, Chipty 2001), and the Mexican footwear industry (Woodruff 2002). The integration patterns observed in these industries are broadly consistent with the notion that large firms tend to be integrated, whereas smaller firms are often non-integrated.

In this paper, we examine how asymmetric vertical structures come about and why integrated firms tend to be large in many industries. However, we also explore countervailing forces, which might explain why the vertical structure is not always asymmetric and why, even for asymmetric structure, there are also cases where the small firms are integrated. We adopt a reduced-form approach towards analyzing vertical-integration decisions in successive oligopolies, avoiding the notorious difficulties with interpreting intricate closed-form solutions of specific models. We focus on the case of two downstream firms who may differ with respect to the efficiency at which they transform an intermediate input into a final good. These firms simultaneously decide about taking over one of at least two upstream firms at fixed acquisition cost, thereby getting access to the input good at marginal cost. Firms that decide to remain separated continue to

¹See Bindemann (1999) and Aydemir and Buehler (2002) for details on vertical integration in the oil industry.
²Also, Damien Neven was quoted as saying that there are essentially two ways of doing business in this industry: Either “stay small and buy inputs or produce large volumes and integrate vertically.” (European Commission 1999a, no. 73)
buy the input from the upstream market at the relevant input price. Our line of argument features four key ingredients.

(1) Efficiency and Foreclosure Effects. We note that previous literature has given conditions under which vertical integration reduces the integrating firm’s marginal costs (efficiency effect) and raises rivals’ marginal costs (foreclosure effect). The efficiency effect can result from pure technological economies of scope or the elimination of a mark-up from an imperfectly competitive upstream market. The foreclosure effect stems from the supply reduction on the upstream market associated with the elimination of an independent upstream supplier.³

(2) Positive Own and Adverse Cross Effects. We argue that when vertical integration has both an efficiency effect and a foreclosure effect, it is likely to increase the integrating firm’s downstream equilibrium output and mark-up (positive own effect), and reduce the rival’s output and mark-up (adverse cross effect). The positive own effect results because lower own costs and higher competitor costs both tend to increase own output and mark-up in standard oligopoly models. The adverse cross effect emerges because both the increased efficiency of the integrating firm and the reduced efficiency of the rival tend to reduce the rival’s output and mark-up.

Thus motivated, we view the existence of a combined positive own effect and adverse cross effect as the essential property of vertical integration. In doing so, we acknowledge that, in the real world as well as in theoretical models, neither effect must necessarily arise.⁴ However, we shall show in Section 6 that both properties hold in many models of successive oligopolies.

(3) Integration Incentives. We show that—together with some additional properties—the simultaneous existence of a positive own effect and an adverse cross effect implies the following statements:

(a) At least when firms are symmetric initially, integration decisions are strategic substitutes, i.e. a firm’s returns to integration are (weakly) lower when its rival is integrated rather than separated;

(b) When a firm has higher transformation efficiency than its rival, that firm’s net gain from integration is higher.

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³Since vertical integration will often reduce demand on the wholesale market as well, it does not necessarily increase the upstream price. As a result, the foreclosure effect is a less robust feature of standard successive-oligopoly models than the efficiency effect.

⁴For instance, in-house production may be more costly when specialized suppliers are more efficient at providing the input (as suggested by Stigler 1951), and the own effect might therefore be negative. More details are discussed in Section 4.3.3.
To explain these statements, we relate back to step (2). Consider statement (a): If integration affects a firm’s own mark-up positively and the competitor’s output negatively, then the mark-up increase resulting from integration is more valuable if the competitor is separated, as it applies to a greater output. That is, there is a demand/mark-up complementarity in the product market supporting the strategic substitutes property of integration decisions. The demand/mark-up complementarity also supports statement (b): Low-cost firms have high demand and mark-up and thus benefit more from an increase of demand and mark-up brought about by vertical integration.

(4) Characterizing Equilibria. We use standard game-theoretic techniques to show that (a) and (b) together imply our following main results:

(i) There may be asymmetric equilibria where only one of the firms integrates, even if firms are perfectly symmetric initially.

(ii) Low-cost firms are more likely to integrate than high-cost firms. As low costs coincide with high market shares in standard oligopoly models, our analysis suggests that large firms are more likely to integrate.

Result (i) is in line with earlier studies examining asymmetric vertical market structures in more specific settings. However, we believe that our reduced-form approach exposes the basic underlying economic forces that drive the result more clearly. Our analysis indicates that asymmetric vertical market structures may be more plausible than previous literature suggests: In particular, they may occur for various types of product-market competition, for different functional forms of demand, and when there are many upstream suppliers.

To our knowledge, result (ii) has not been addressed in the previous theoretical literature, which has largely ignored the effects of efficiency differences on the incentives to integrate. It fits nicely, however, with the

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5These papers include Ordover et al. (1990), Abiru et al. (1998), Elberfeld (2002), and Jansen (2003); see Section 6 for details.

6Though our paper is general in these respects, it is not a strict generalization of all papers addressing endogenous vertical integration. For instance, in Ordover et al. (1990), a firm that has not integrated obtains a second chance to do so after input prices are determined.

7However, in Dufeu (2004) some firms are more efficient at producing upstream goods, whereas others are more efficient at producing downstream goods.

8Clearly, other reasons might be put forward to explain why large firms are more likely to integrate. For instance, small firms might simply face more severe financing constraints than large firms and thus have difficulties acquiring upstream firms.
impression that firm size and vertical integration are correlated (see, e.g., Adelman 1955, and Chandler 1977).

Even though we adopt a fairly general reduced-form approach, results (i) and (ii) should not be expected to hold universally. At least the underlying sufficient conditions (a) and (b) might be violated. For instance, the strategic substitutes property may break down when vertical integration generates a strong foreclosure effect, that is, a substantial increase of upstream prices. In this case, the potential cost reduction from own integration might be higher when the competitor is integrated rather than separated, so that firms have an incentive to follow suit to escape foreclosure.

Our paper is potentially relevant for policy analysis. For instance, result (i) relates to the familiar Chicago school argument that strategic vertical integration cannot generate competitive harm, because non-integrated firms can always counter integration by vertically integrating themselves so as to assure input supply at competitive prices (“bandwagoning”, see e.g. Bork 1978). Our result indicates that bandwagoning may be unprofitable even when the conditions are favorable, i.e., when firms are symmetric initially and face the same costs of acquiring an upstream firm. Result (ii), in turn, cautions against the notion that antitrust authorities should frown on asymmetric market structures: If low-cost firms are more likely to integrate than high-cost firms, asymmetric vertical integration tends to improve the industry’s productive efficiency by shifting output from less efficient separated firms to more efficient integrated firms.10

The remainder of the paper is organized as follows. Section 2 introduces the analytical framework, taking ingredients (1) and (2) above for granted. Section 3 provides a linear Cournot example fitting into our framework. Section 4 analyzes vertical-integration decisions in reduced form and provides our main results, which amount to ingredients (3) and (4) above. Section 5 outlines generalizations of our analysis to (i) more than two firms, (ii) upstream sales of integrated firms, and (iii) endogenous acquisition costs. Section 6 discusses the relation to the literature. In particular, this section supplies ingredient (1). Section 7 concludes. The exact derivation of ingredient (2) is left to an Appendix.

9Of course, as conditions (a) and (b) are sufficient but not necessary for (i) and (ii) to hold, it is still possible that our results go through for non-global violations of (a) and (b).

10Riordan (1998) discusses a similar effect in a model where the dominant firm has an exogenous cost advantage relative to a competitive fringe.
2 Analytical Framework

Consider an industry where, initially, there are two separated downstream firms $i = 1, 2$ and at least two separated upstream firms. The upstream firms produce an intermediate good that the downstream firms transform into a final product. Importantly, downstream firms may differ with respect to their efficiency in this activity.

In stage 1, the downstream firms decide whether to integrate backwards by acquiring one of the upstream firms ($V_i = 1$) or not ($V_i = 0$). In stage 2, the remaining separated upstream firms set wholesale prices or quantities for the downstream market, unless both downstream firms are integrated. In stage 3, downstream firms compete in the product market.

As will become clear below, stages 1 and 2 determine the marginal downstream costs $c_i, i = 1, 2$. For given values of $c_i$, downstream product-market competition in stage 3 is given in reduced form as follows.

Assumption 1 For every $c \equiv (c_1, c_2)$, there exists a unique product-market equilibrium resulting in downstream outputs $q_i(c)$, mark-ups $m_i(c)$ and profits $\pi_i(c) = q_i(c) \cdot m_i(c), i = 1, 2$, such that $q_i(c), m_i(c)$ and thus $\pi_i(c)$ are all non-increasing in $c_i$ and non-decreasing in $c_j, j \neq i$.

Assumption 1 holds for many standard oligopoly models.

Next, we discuss how the $c_i$'s in stage 3 are determined in the preceding stages. We assume that to produce one unit of the final product, downstream firms require one unit of the intermediate product, which is produced at constant marginal cost. An integrated firm produces the input in-house, whereas a vertically separated firm buys the intermediate product from the imperfectly competitive upstream market at the equilibrium price.\(^\text{11}\) We let $w_i$ denote firm $i$'s marginal cost of obtaining the intermediate product, which is the cost of producing the product in-house for an integrated firm ($V_i = 1$), and the cost of obtaining it on the upstream market for a separated firm ($V_i = 0$). Transforming the intermediate product into the final good adds marginal transformation costs $t_i$. Let $\bar{t} \equiv \max(t_1, t_2)$ and define firm $i$'s exogenous efficiency level as $Y_i \equiv \bar{t} - t_i$.

We write $w_i(V, Y)$ for the upstream price as a function of the integration decisions $V = (V_1, V_2)$ and the exogenous efficiency levels $Y = (Y_1, Y_2)$. Firm $i$'s marginal costs are thus

$$c_i (V, Y) = w_i (V, Y) + \bar{t} - Y_i. \quad (1)$$

\(^\text{11}\)This equilibrium price is either set directly by a single upstream firm (if $V = (1, 0)$ or $V = (0, 1)$) or results from upstream competition in stage 2 (if $V = (0, 0)$).
In stage 1, acquisition costs are assumed to be given by a constant $F > 0$. Thus, downstream firms choose $V_i \in \{0, 1\}$ so as to maximize

$$\pi_i (c_1(V, Y), c_2(V, Y)) - V_i F.$$ 

Using Assumption 1, equilibrium product-market profits, mark-ups and outputs are functions of the firms’ vertical structures and efficiency levels:

**Notation 1 (equilibrium quantities)** For $i = 1, 2$, denote downstream profits, mark-ups and outputs, respectively, as

$$\Pi_i (V, Y) = \pi_i (c_1(V, Y), c_2(V, Y));$$

$$M_i (V, Y) = m_i(c_1(V, Y), c_2(V, Y));$$

$$Q_i (V, Y) = q_i(c_1(V, Y), c_2(V, Y)).$$

We require that these quantities satisfy the following condition:

**Assumption 2** Product-market profits are exchangeable, i.e. for all $V', V'' \in \{0, 1\}$ and $Y', Y'' \in [0, \infty)$,

$$\Pi_1(V', V''; Y', Y'') = \Pi_2(V'', V', Y', Y'').$$

Analogous properties hold for $Q_i$ and $M_i$.

This symmetry assumption requires that the firms’ profits depend only on their actions and efficiency levels, but not on their identity.

The properties of $M_i (V, Y)$ and $Q_i (V, Y)$ reflect all the information from stages 2 and 3 that we require for our analysis. We shall therefore state the remaining assumptions in terms of these functions. Below, we shall show that well-known specific models satisfy our assumptions. We require that integration (weakly) increases own output and mark-up.

**Assumption 3** For $i, j = 1, 2$, equilibrium output $Q_i$ and mark-up $M_i$ are non-decreasing in $V_i$.

In the Appendix, we show how Assumption 3 can be derived from more primitive assumptions. Intuitively, consider (3) and (4): $V_i$ affects $Q_i$ and $M_i$ via $c_i$ and $c_j$. The motivation for assuming a positive effect of $V_i$ on $Q_i$ (and $M_i$) is that $V_i$ reduces $c_i$ either by eliminating the mark-up from imperfect upstream competition or by exploiting purely technological economies of scope—this is the efficiency effect of integration. The effect of $V_i$ on
c_j is less clear: At least if integrated firms are not active on the wholesale market, backward integration of a downstream firm means that supply and demand on the whole-sale market typically both fall. If the supply effect dominates, an increase in V_i increases the wholesale price and thus c_j, j ≠ i—this is the foreclosure effect of integration, which reinforces Assumption 3, as higher competitor costs tend to increase own output and mark-up by Assumption 1. If the demand effect dominates, c_j falls. In this case, Assumption 3 is less plausible as decreasing competitor costs decrease own output and mark-up. The effects of efficiency levels on outputs and mark-ups are captured as follows.

**Assumption 4** For i, j = 1, 2, i ≠ j, equilibrium outputs Q_i and mark-ups M_i are (i) non-decreasing in Y_i and (ii) non-increasing in Y_j.

In the Appendix, we show how Assumption 4 can be derived from more primitive assumptions. Intuitively, as to (i), higher own efficiency reduces own costs by (1) if the direct effect of efficiency is not compensated by a strong increase in the input price, i.e., if ∂w_i/∂Y_i < 1. Indirect effects on competitor costs should reinforce this effect: Higher transformation efficiency means that firm i demands more intermediate input, thereby tending to increase the wholesale price for the competitor. As to (ii), if the competitor j becomes more efficient, this should adversely affect Q_i and M_i by decreasing c_j. In addition, higher Y_j should also increase the wholesale price by increasing input demand, thereby driving up c_i for separated firms, thus reinforcing the negative effect on Q_i and M_i.\(^{12}\)

### 3 A Simple Cournot Example

To provide a simple example where all four assumptions are satisfied simultaneously, we consider the following 2×2 firms model.\(^{13}\) Suppose downstream firms compete à la Cournot, with linear demand given by P(Q) = a − Q, where Q = Q_1 + Q_2 and a > 0. For simplicity, assume that the marginal cost of producing the input good is constant and normalized to zero. For an integrated firm, therefore, w_i = 0. If both firms are integrated (V = (1, 1)), the model corresponds to standard Cournot competition. If both firms are separated (V = (0, 0)), we suppose Cournot competition

\(^{12}\)If firm i is integrated, the second effect is absent, as an integrated firm does not rely on the wholesale market.

\(^{13}\)The model is adapted from Salinger (1988) who considers arbitrary numbers of homogeneous firms.
upstream precedes downstream competition. If only one firm is separated \((V = (0, 1)\) or \(V = (1, 0)\)), the remaining upstream firm sets a linear monopoly price for the separated downstream firm.\(^{14}\)

Table 1 summarizes the results, with \(\alpha \equiv a - \bar{f}\). Assumption 1 is a standard property of the linear Cournot model. Inspection of Table 1 further shows that Assumptions 2, 3 and 4 hold for all admissible \(Y\).\(^{15}\)

<table>
<thead>
<tr>
<th>(V = (0, 0))</th>
<th>(V = (1, 0))</th>
<th>(V = (0, 1))</th>
<th>(V = (1, 1))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Q_1 = M_1)</td>
<td>(\frac{4\alpha + 11Y_1 - 11Y_2}{12})</td>
<td>(\frac{5\alpha + 7Y_1 - 2Y_2}{12})</td>
<td>(\frac{2\alpha + 1Y_1 - 2Y_2}{12})</td>
</tr>
<tr>
<td>(Q_2 = M_2)</td>
<td>(\frac{4\alpha - 7Y_1 + 11Y_2}{12})</td>
<td>(\frac{2\alpha - 2Y_1 + 4Y_2}{12})</td>
<td>(\frac{5\alpha - 2Y_1 + 7Y_2}{12})</td>
</tr>
<tr>
<td>(\Pi_1)</td>
<td>(\left(\frac{4\alpha + 11Y_1 - 11Y_2}{12}\right)^2)</td>
<td>(\left(\frac{5\alpha + 7Y_1 - 2Y_2}{12}\right)^2)</td>
<td>(\left(\frac{2\alpha + 1Y_1 - 2Y_2}{12}\right)^2)</td>
</tr>
<tr>
<td>(\Pi_2)</td>
<td>(\left(\frac{4\alpha - 7Y_1 + 11Y_2}{12}\right)^2)</td>
<td>(\left(\frac{2\alpha - 2Y_1 + 4Y_2}{12}\right)^2)</td>
<td>(\left(\frac{5\alpha - 2Y_1 + 7Y_2}{12}\right)^2)</td>
</tr>
</tbody>
</table>

4 Analyzing Integration Decisions

Next, we analyze integration decisions. We use the following notation:

**Notation 2 (integration effects)** For \(i = 1, 2, j \neq i, f = \Pi, Q, M, \) let

\[
\Delta f_1 (V_2; Y) \equiv f_1 (1, V_2; Y) - f_1 (0, V_2; Y);
\]

\[
\Delta f_2 (V_1; Y) \equiv f_2 (V_1, 1; Y) - f_2 (V_1, 0; Y).
\]

Thus, \(\Delta \Pi_i (V_j; Y)\) is firm \(i\)'s integration incentive, i.e., the effect of its integration on own profits. \(\Delta Q_i\) and \(\Delta M_i\) denote the output and mark-up effects of own integration. We shall use the following terminology:

**Definition 1**

(i) Vertical integration decisions are strategic substitutes if \(\Delta \Pi_i (V_j; Y)\) is non-increasing in \(V_j\) for \(i, j = 1, 2, j \neq i\).

(ii) Low-cost firms have higher integration incentives than high-cost firms if, for \(H > L\) and \(\bar{V} \in \{0, 1\}\),

\[
\Delta \Pi_1 (\bar{V}; H, L) \geq \Delta \Pi_2 (\bar{V}; H, L).
\]  \(^{(5)}\)

\(^{14}\)We assume in this example that integrated firms do not supply the intermediate good on the wholesale market. Our general approach, however, also makes sense without this restriction (see section 5.2).

\(^{15}\)We assume that efficiency differences are small enough that each firm produces a positive output.
4.1 Symmetric Firms

We first give conditions under which asymmetric equilibria emerge in a symmetric setting \((Y_1 = Y_2 = 0)\). These conditions hold in our example of Section 3. We require the following result.

**Lemma 1 (strategic substitutes)** Suppose that, for \(Y = 0\) and \(i, j = 1, 2, i \neq j\), the following statements hold:

(i) Vertical integration has an **adverse cross effect**, i.e
\[
Q_i \text{ and } M_i \text{ are non-increasing in } V_j.
\]  

(ii) Vertical integration does not raise the competitor’s output and mark-up increase from vertical integration, i.e.
\[
\Delta Q_i (V_j; Y) \text{ and } \Delta M_i (V_j; Y) \text{ are non-increasing in } V_j.
\]  

Then vertical-integration decisions are strategic substitutes.

**Proof.** By exchangeability, it suffices to consider firm 1’s incentive to integrate, \(\Delta \Pi_1 (V_2; Y)\). Rewriting this profit differential yields
\[
\Delta \Pi_1 (V_2; Y) = Q_1 (1, V_2; Y) \cdot \Delta M_1 (V_2; Y) + M_1 (0, V_2; Y) \cdot \Delta Q_1 (V_2; Y).
\]  

By Assumption 3, both \(\Delta M_1 (V_2; Y)\) and \(\Delta Q_1 (V_2; Y)\) are non-negative. By (6), \(Q_1\) and \(M_1\) are both non-increasing in \(V_2\). By (7), \(\Delta M_1 (V_2; Y)\) and \(\Delta Q_1 (V_2; Y)\) are non-increasing in \(V_2\). Thus (8) is non-increasing in \(V_2\). \(\blacksquare\)

The adverse cross effect condition (6) makes sense for two reasons. First, if firm \(j\)’s integration reduces its marginal costs, firm \(j\) becomes a stronger downstream competitor (**cross-efficiency effect**). Second, after integration, the integrated upstream firm may have an incentive to curtail supply to the input market, thereby raising the cost of a non-integrated rival (**foreclosure effect**).\(^{16}\)

The intuition for Lemma 1 is as follows: Vertical integration by firm \(i\) (weakly) increases firm \(i\)’s demand and mark-up by Assumption 3, whereas vertical integration by firm \(j\) decreases these quantities by (6). Now, as

\(^{16}\)Whether vertical foreclosure emerges in equilibrium depends on subtle details of the specific model under consideration: For instance, foreclosure will typically occur for low numbers of upstream suppliers (Salinger 1988) or high costs of switching suppliers (Chen 2001).
firm $j$’s integration reduces the mark-up $M_i$, the positive effect of a given output increase $\Delta Q_i$ on firm $i$’s profits is smaller when firm $j$ is integrated. Similarly, the positive effect of a given mark-up increase $\Delta M_i$ on firm $i$’s profits is smaller when firm $j$ is integrated, because firm $j$’s integration reduces firm $i$’s demand $Q_i$. Thus, if $\Delta Q_i$ and $\Delta M_i$ are independent of the competitor’s vertical structure, integration decisions are strategic substitutes. A fortiori, when the integration of firm $j$ reduces $\Delta Q_i$ and $\Delta M_i$, as required by (7), the strategic substitutes property holds.

Thus, demand/mark-up complementarities in the product market are likely to make integration decisions strategic substitutes. However, condition (7) may be violated when firm $j$’s integration gives rise to a strong foreclosure effect, as firm $i$ will then have relatively high costs when it is separated. Thus, firm $i$’s cost reduction from own integration is likely to be higher when firm $j$ is integrated. In this case, other things being equal, output and mark-up increases $\Delta Q_i$ and $\Delta M_i$ will be higher when firm $j$ is integrated. As a result, in spite of demand/mark-up complementarities in the product market, the strategic substitutes property of vertical-integration decisions may be violated when the foreclosure effect is strong.\footnote{A similar effect occurs in Ordover et al. (1990): If the competitor’s integration raises the upstream price excessively, the separated firm has an incentive to integrate.}

Using Lemma 1, we now characterize the equilibria of the game.

**Proposition 1 (symmetric costs)** Suppose $Y = 0$, and conditions (6) and (7) hold. Then, for suitable values of $F$, there exist equilibria of the integration game where exactly one firm integrates.

The proof is straightforward. Lemma 1 shows that integration decisions are strategic substitutes. Therefore, for suitable values of $F$, we have

$$\Delta \Pi_i (0; Y) \geq F \geq \Delta \Pi_i (1; Y). \tag{9}$$

Thus, if only one firm integrates, it is not profitable for either firm to deviate.

There are two caveats to the conclusion of Proposition 1. First, recall that the strategic substitutes condition may be violated if there is a strong foreclosure effect. In the opposite polar case where integration decisions are strategic complements rather than substitutes, vertical integration by firm $j$ renders vertical integration more profitable for firm $i$ (i.e. $\Delta \Pi_i (1; Y) > \Delta \Pi_i (0; Y)$).\footnote{Obviously, intermediate cases can arise where integration decisions are neither globally substitutes nor complements. In such cases, asymmetric equilibria are still possible.} As a result, only symmetric equilibria can exist. Second,
even if integration decisions are strategic substitutes, symmetric equilibria will still arise when \( F \) is so high that the l.h.s. of (9) is violated: Then, no firm will integrate. Similarly, when \( F \) is so low that the r.h.s. of (9) is violated, all firms integrate.

### 4.2 Efficient Firms Are More Likely to Integrate

Next, we consider the notion that low-cost firms are more likely to integrate than high-cost firms. A strong version of this statement would be that \( Y_1 > Y_2 \) implies \( V_1 \geq V_2 \) in any equilibrium of the integration game. Yet, Proposition 1 suggests that such a result cannot hold: If \( Y_1 = Y_2 = 0 \), there may well be an equilibrium with only firm 2 integrating, because \( \Delta \Pi_i(0; Y) \geq \Delta \Pi_i(1; Y) \). If \( \Delta \Pi_i(0; Y) > \Delta \Pi_i(1; Y) \) for \( Y = 0 \) and \( \Pi_i(V; Y) \) is a continuous function of \( Y \) in a neighborhood of \( Y = 0 \), such an equilibrium still exists when firm 2 is slightly less efficient than firm 1.

However, efficient firms may be more likely to integrate in the sense that the following two statements hold.

(i) If there is an equilibrium where only the inefficient firm integrates, there is also an equilibrium where only the efficient firm does.

(ii) If there is an equilibrium where only the efficient firm integrates, there is not necessarily an equilibrium where only the inefficient firm does.

To support these claims, we first give conditions guaranteeing that low-cost firms face higher integration incentives than high-cost firms.

**Lemma 2 (efficiency/integration complementarity)** Suppose the following conditions hold, with \( H > L \):\(^{19}\)

\[
\Delta Q_1(V_2; H, L) \geq \Delta Q_1(V_2; L, H); \\
\Delta M_1(V_2; H, L) \geq \Delta M_1(V_2; L, H).
\]

Then (5) holds, i.e., low-cost firms have higher integration incentives than high-cost firms.

**Proof.** See Appendix.

To grasp the intuition of Lemma 2, note that by exchangeability, we have \( \Delta \Pi_2(V; H, L) = \Delta \Pi_1(\tilde{V}; L, H) \), and we can thus write (5) as

\[
\Delta \Pi_1(\tilde{V}; H, L) \geq \Delta \Pi_1(\tilde{V}; L, H).
\]

\(^{19}\)By exchangeability, analogous conditions must then hold for \( \Delta Q_2 \) and \( \Delta M_2 \).
By Assumption 3, firm 1’s integration incentive comes from the increase in output $\Delta Q_1$ and mark-up $\Delta M_1$. A relatively efficient firm 1 with high state $H$ has higher output and mark-up than a relatively inefficient firm with state $L < H$. Thus, for the relatively efficient firm 1, the value of a given output increase $\Delta Q$ from integration is higher than for the inefficient firm 2, as the output increase applies to a greater mark-up. Similarly, the value of a given mark-up increase $\Delta M$ is higher for the efficient firm, as its output is higher. Therefore, demand/mark-up complementarities support condition (12). A countervailing force could arise if the sizes of the output and mark-up increases were higher for relatively inefficient firms. Conditions (10) and (11) prevent this. We now use Lemma 2 to show that conditions (10) and (11) together imply statements (i) and (ii).

**Proposition 2 (asymmetric costs)** Suppose conditions (10) and (11) hold. Further assume that firm 1 is more efficient than firm 2, i.e. $Y_1 > Y_2$.

(i) If there is a pure strategy equilibrium $(V_1^*, V_2^*)$ with $V_1^* < V_2^*$, there also is a pure strategy equilibrium with $V_1^* > V_2^*$.

(ii) If there is a pure strategy equilibrium $(V_1^*, V_2^*)$ with $V_1^* > V_2^*$, there is not necessarily a pure strategy equilibrium with $V_1^* < V_2^*$.

**Proof.** (i) An equilibrium of the integration game where only the inefficient firm integrates requires that, for $H > L$,

$$\Delta \Pi_2 (0; H, L) \geq F \geq \Delta \Pi_1 (1; H, L).$$

(13)

The equilibrium where only the efficient firm integrates requires

$$\Delta \Pi_1 (0; H, L) \geq F \geq \Delta \Pi_2 (1; H, L).$$

(14)

If (5) holds, (13) implies (14). Thus, Lemma 1 implies the result.

(ii) To find a counterexample, suppose (10) and (11) hold with inequality, and output and mark-up are strictly increasing in $V_i$. Then, (5) also holds with inequality. If (6) and (7) also hold with inequality, then

---

20Conditions (10) and (11) hold in the Cournot example of Section 3.

21Note that our result does not assume that integration decisions are strategic substitutes. With this assumption, the game would be supermodular after reordering the strategy space of one of the players, and we could apply standard methods of monotone comparative statics (Milgrom and Roberts 1990, Th. 5) to obtain the conclusion of Proposition 2.
\( \Delta \Pi_i (V_j; Y) \) is decreasing in \( V_j \). If \( \Pi_i (V, Y) \) is continuous in \( Y \) near zero, then for \( H > L \), but sufficiently close to \( L \), Lemmas 1 and 2 imply:

\[
\Delta \Pi_2 (1; H, L) < \Delta \Pi_1 (1; H, L) < \Delta \Pi_2 (0; H, L) < \Delta \Pi_1 (0; H, L).
\]

Thus, for suitable levels of \( F \), (14) holds, but (13) does not. ■

Proposition 2 states that efficient firms are more likely to integrate. However, it does not rule out the possibility that only symmetric equilibria exist.

4.3 Applications and Limitations

We now discuss in which kind of industries the assumptions underlying Propositions 1 and 2 are plausible.

4.3.1 Example 1: Pure Efficiency Effects

Both propositions apply to a setting with efficiency effects, but without foreclosure effects. These conditions are likely to hold when (i) upstream competition is imperfect, (ii) there are economies of scope between upstream and downstream operations, and (iii) final products are weak substitutes.\(^{22}\)

When there are efficiency effects, but no interactions between integration decisions and competitor costs, Assumption 3 is naturally satisfied.\(^{23}\) Also, when there are only efficiency effects, the caveat that integration decisions will not be strategic substitutes when there is a strong foreclosure effect (i.e., condition (7) is violated) is no longer valid. Therefore, Propositions 1 and 2 appear particularly plausible when vertical integration generates efficiency effects only.

4.3.2 Example 2: Efficiency and Foreclosure Effects

We now modify condition (iii) from Example 1 by assuming that final products are close substitutes, so that foreclosure may be an issue. As argued earlier, if foreclosure effects are strong, the strategic substitutes

\(^{22}\)For instance, in the U.S. motion picture entertainment industry a few large integrated majors compete against smaller independent distributors (Litman 2001). Competition is imperfect both at the production and the distribution stage, and there are important integration efficiencies, including the ability to produce and distribute motion pictures using all types of media. Furthermore, there is substantial product differentiation.

\(^{23}\)In particular, condition (16) of Lemma 3 in the Appendix, which is crucial for deriving Assumption 3, holds.
property of vertical-integration decisions might be violated, so that equi-
libria with asymmetric vertical structures might not arise in a setting where
firms are symmetric initially. Yet, we have also emphasized that the efficiency/integration complementarity does not rely on strategic substitutes.
Thus, Proposition 2, which states that large efficient firms are more likely
to integrate, is still likely to hold in such a setting.

For instance, the vertical integration of gasoline refiners and retailers
in the U.S. has been suspected to generate sizeable foreclosure effects. In
this industry, which is dominated by a number of large integrated firms,
competition is clearly imperfect both at the refining and retailing stage,
there are considerable integration efficiencies (see Vita 2000), and final
products are close substitutes. Another well-documented example that
is consistent with the set-up described here and with the prediction of
Proposition 2 is the above-mentioned market for foreign package holidays
in the U.K. (see European Commission 1999a).

4.3.3 Example 3: Low Integration Incentives for Efficient Firms

There are at least two types of industries where the assumptions underlying
Proposition 2 may be violated, so that it is more likely to see large low-cost
separated firms and small high-cost integrated firms.

First, suppose large separated downstream firms obtain the interme-
diate product at more favorable terms than their smaller counterparts,
i.e., large buyers pay lower prices. This may occur, for instance, because
suppliers compete for large buyers (Snyder 1996, 1998), or large buyers
have higher bargaining power (Scherer and Ross 1990). In the food retail
distribution sector of several European countries, buyer power plays an im-
portant role (European Commission 1999b). The integration patterns in
this sector should thus not necessarily be expected to correspond to the
predictions of Proposition 2.

Second, consider industries where vertical integration serves as a quality-
enhancing device. For instance, in the hotel industry small establish-
ments sometimes adopt a “boutique strategy” and produce all vertically-related
services (e.g. construction, marketing, food processing) in-house to become
a premium hotel, whereas larger chains buy many of these services from
contractors. Such industries differ from our setting in that integration has a
quality effect and almost certainly increases rather than decreases marginal
costs. Nevertheless, such industries can be understood in terms of our as-
sumptions. It seems unreasonable to expect Assumption 3 to hold: While
the quality premium from integration suggests a high mark-up, output is
typically lower for an integrated "boutique firm". Therefore, we cannot appeal to the strategic substitutes property or the efficiency/integration-complementarity to justify our results. Consequently, the predictions of our analysis are likely to be inadequate for such industries.

5 Generalizations

We now discuss the plausibility of our results without the restrictions that (i) there are two downstream firms, (ii) integrated firms do not engage in wholesale activity and (iii) acquisition costs are constant.

5.1 Large Numbers of Downstream Firms

We sketch without proofs how our analysis generalizes to \( I > 2 \) downstream firms. We replace the requirement \( "i, j = 1, 2, j \neq i" \) in Assumptions 1, 3 and 4 and in Lemma 1 by \( "i, j = 1, ..., I, j \neq i'" \) and generalize Assumption 2 (exchangeability) to \( I > 2 \) firms as in Athey and Schmutzler (2001). Further, for \( f = M, Q, \Pi \), we generalize the notation \( \Delta f_i (V_j; \mathbf{Y}) \) to \( \Delta f_i (\mathbf{V}_{-i}; \mathbf{Y}) \) in an obvious way. We replace \( \Delta f_i (V_j; \mathbf{Y}) \) with \( \Delta f_i (\mathbf{V}_{-i}; \mathbf{Y}) \) in (7), (10) and (11). We call the modified conditions \( (6)'\), \( (7)'\), \( (10)'\) and \( (11)'\). Propositions 1 and 2 can then be restated as follows:

**Proposition 3 (symmetric costs)** For \( I \geq 2 \) firms, suppose \( (6)' \) and \( (7)' \) hold. Then, for suitable values of \( F \) and arbitrary \( k \in \{1, ..., I - 1\} \), there exist equilibria of the integration game with \( \mathbf{Y} = \mathbf{0} \) where exactly \( k \) firms integrate.

**Proposition 4 (asymmetric costs)** Suppose \( (10)' \) and \( (11)' \) hold. Suppose for some \( k, l \in \{1, ..., I\} \), \( Y_k > Y_l \).

(i) If there is a pure strategy equilibrium \( \mathbf{V}^* \) with \( V_k^* < V_l^* \), there also is a pure strategy equilibrium with \( V_k^* > V_l^* \).

(ii) If there is a pure strategy equilibrium \( \mathbf{V}^* \) with \( V_k^* > V_l^* \), there is not necessarily a pure strategy equilibrium with \( V_k^* < V_l^* \).

5.2 Upstream Sales

We return to the case of two downstream firms. To model the possibility of upstream sales, we continue to work with the reduced-form profit function \( \Pi_i (\mathbf{V}, \mathbf{Y}) \). However, we now suppose that \( \Pi_i \) may also contain profits
from upstream sales, $\Pi^U_i (V, Y)$. Similarly, $Q^U_i (V, Y)$ and $M^U_i (V, Y)$ denote equilibrium upstream outputs and mark-ups, whereas $Q^D_i (V, Y)$ and $M^D_i (V, Y)$ are the corresponding downstream quantities. The objective function of firm $i$ is thus

$$\Pi_i (V, Y) = Q^D_i (V, Y) \cdot M^D_i (V, Y) + Q^U_i (V, Y) \cdot M^U_i (V, Y).$$

The following properties are plausible:

(i) $Q^U_i$ and $\Pi^U_i$ are non-decreasing in $V_i$.\(25\)

(ii) $Q^U_i, M^U_i$ and thus $\Pi^U_i$ are non-increasing in $Y_i$.

Property (i) must hold because separated downstream firms do not sell anything on the upstream market by definition, whereas integrated firms sell non-negative quantities. Thus, the potential gains from upstream sales are another motivation for vertical integration. As to property (ii), note that if an integrated firm reduces its costs, its downstream output increases, whereas its competitor’s output decreases. Thus, the competitor will require less inputs on the wholesale market, which decreases firm $j$’s wholesale demand. As a result, both $Q^U_i$ and $M^U_i$ should decrease if $Y_i$ increases. Thus, the prospect to sell upstream goods on the wholesale market is less attractive for relatively efficient firms, which reduces the incentive for efficient firms to integrate.

Now consider the cross effects of changes in $V_j$ on $Q^U_i$ and $M^U_i$. To avoid double marginalization, the downstream unit of firm $j$ will demand less inputs from firm $i$ after integration. Therefore, we should expect a negative effect of competitor integration on own upstream sales, which reinforces the strategic substitutes property.\(26\)

### 5.3 Endogenous Acquisition Costs

Now suppose that for downstream firm $i$, the cost of acquiring an independent upstream firm is given by a function $A(V_j, Y)$ rather than by a constant. That is, acquisition costs depend both on vertical structure and

\(24\)Whether it is reasonable for integrated firms to supply competitors depends on the conjectures of integrated firms about the competitor’s behavior in the upstream market (see, e.g., Salinger 1988, and Schrader and Martin 1998).

\(25\)For separated firms, the upstream mark-up $M^U_i$ is not well-defined, as $Q^U_i = 0$.

\(26\)However, with more than two downstream firms, there might be a competing effect: If integration leads competitor $j$ to reduce its sales to the wholesale market, this should have positive effects on the upstream sales of the integrated firm $i$.  

http://www.bepress.com/bejte/advances/vol5/iss1/art1
efficiency levels. Assuming that acquisition costs reflect the opportunity costs of the firms that are being taken over, it is likely that $A$ is non-decreasing in $Y_i$ and $Y_j$. Intuitively, equilibrium output is higher when downstream firms are more efficient. Thus, input demand and the profits of upstream firms should be higher. Since acquisition costs reflect the opportunity costs of takeover targets, they should approximately amount to the profits of that firm in the absence of a merger. Thus, acquisition costs should be expected to increase in downstream efficiency, at least if the downstream competitor is integrated.

The effect of a change in $V_j$ on $A$ is again ambiguous. Intuitively, firm $j$’s vertical integration will have a negative effect on both demand and supply for the remaining separated upstream firm. If the demand effect dominates, outputs, mark-ups and thus profits of the upstream firm to be integrated by downstream firm $i$ should fall. Acquisition costs should therefore decrease with $V_j$. Conversely, if the supply effect dominates, the profit of the upstream firm and thus acquisition costs should increase with $V_j$. As a result, there may be circumstances where the conclusion of Proposition 1 does not hold when acquisition costs are endogenous: If the demand effect dominates, firm $i$’s acquisition becomes less expensive, and $V_i$ and $V_j$ are less likely to be strategic substitutes.

6 Related Literature

In this section, we relate our paper to existing literature. First, we discuss how the four key ingredients of our analysis have been addressed elsewhere.\footnote{We note in passing that Assumptions 1 and 2 are satisfied in most models in the literature for wide parameter values; however, in Ordover et al. (1990), multiple equilibria may arise for some specifications of demand.} Second, we sketch the main results on the endogenous emergence of asymmetric vertical integration.

6.1 The Key Ingredients

(1) Efficiency and Foreclosure Effects. Salinger (1988) considers a fixed-proportion linear Cournot model with arbitrary numbers of homogeneous firms, where integration always causes an efficiency effect as it eliminates successive mark-ups. Whether integration also generates a foreclosure effect depends on parameter values. More specifically, vertical integration—which amounts to an exogenous change in the number of integrated firms—
increases the wholesale price if more than half of the firms producing the intermediate product are vertically integrated.\textsuperscript{28}

Ordover et al. (1990) examine a fixed-proportion model with endogenous integration decisions. Two upstream firms produce a homogenous input good and compete in prices. Two downstream firms produce differentiated products and compete in prices. Their model thus rules out an efficiency effect unless there is only one upstream firm, but gives rise to a foreclosure effect in equilibrium.

Hart and Tirole (1990) analyze different variants\textsuperscript{29} of a model where two upstream firms produce a homogenous input good and compete in prices, whereas two downstream firms engage in Cournot competition. Non-linear contracts between upstream and downstream firms are allowed. In the so-called “ex post monopolization” variant of the model, the more efficient upstream firm integrates with one of the downstream firms and slightly undercuts the less efficient upstream firm to supply the other downstream firm. This results in an efficiency effect of integration.\textsuperscript{30} Integration has no foreclosure effect, as it does not raise the downstream rival’s costs.

Chen (2001) considers a fixed-proportion model where two or more upstream firms produce a homogenous input good and may have different marginal costs.\textsuperscript{31} Two downstream firms produce differentiated final products and compete in prices. Vertical integration always generates an efficiency effect. There may also be a foreclosure effect, though the intuition differs from previous literature: Integration changes a downstream firm’s pricing incentive, as it becomes a supplier to its rival. This multimarket interaction softens competition in the downstream market and changes the rival’s incentive in selecting input suppliers.

Summing up, previous literature suggests that vertical integration helps gaining competitive advantage by cutting own costs or by raising rivals’ costs.

\textbf{(2) Positive Own Effects and Negative Cross Effects.} As we

\textsuperscript{28}Gaudet and van Long (1996) obtain a similar result even when integrated firms purchase from the upstream market to increase rivals costs, and integration decisions are endogenous.

\textsuperscript{29}These variants differ with respect to the bargaining power pertaining to upstream and downstream firms.

\textsuperscript{30}With non-linear upstream prices and homogeneous suppliers, integration may have no effect whatsoever on downstream marginal costs (Rey and Tirole, forthcoming). Yet, even in this case, there is the standard textbook argument (e.g. Besanko et al. 2000, 173) that post-integration costs might be lower because of economies of scope between upstream and downstream production.

\textsuperscript{31}To our knowledge, Linnemer (2003) is the only related paper allowing for asymmetric cost structures at the downstream level.
noted, the efficiency and foreclosure effects of vertical integration motivate our analysis. Yet, what we really need is less restrictive: Integration should have positive own effects (Assumption 3) and adverse cross effects (Condition (6)). This does not necessarily require both efficiency effects and foreclosure effects to be present. For instance, in Salinger (1988), both Assumption 3 and Condition (6) are satisfied universally, even though foreclosure will only arise for a subset of parameter values.

(3) Integration Incentives. The key intuition of our paper is that the combined positive own effect and adverse cross effect of integration tend to imply that (a) vertical-integration decisions are strategic substitutes, and (b) efficient firms face higher integration incentives. The driving force are demand/mark-up complementarities in the product market. While this observation is new in the context of vertical-integration decisions, similar mechanisms have been exploited in other fields. For instance, Bagwell and Staiger (1994) and Athey and Schmutzler (2001) use the related idea that cost-reducing investments are strategic substitutes in the context of many oligopoly models. Complementarities between demand-enhancing and mark-up-increasing activities are crucial for this result.32

(4) Characterizing Equilibria. Finally, we analyzed the implications of properties (a) and (b) for the equilibria of the integration game. In Proposition 2, we have examined how the complementarity between cost efficiency and vertical integration affects equilibrium market structure. Proposition 2 bears some similarity to Theorem 1 in Athey and Schmutzler (2001), which considers the relation between a state variable (which could, for instance, be interpreted as the efficiency level of a firm) and an investment variable (which could, for instance, be interpreted as the integration decision) in games with strategic substitutes. However, note that we do not use the strategic substitutes property in the derivation of Proposition 2.33

6.2 Endogenous Asymmetric Equilibria

Earlier literature has dealt with the conditions supporting asymmetric integration equilibria. Both symmetric and asymmetric equilibria have been

32Demand/mark-up complementarities also drive the complementarity between product and process innovations highlighted by Athey and Schmutzler (1995); Aydemir and Schmutzler (forthcoming) analyze the role of demand/mark-up complementarities in the context of horizontal acquisitions.

33More generally, although we do not explicitly refer to particular results from the monotone comparative statics literature (e.g. Milgrom and Roberts 1990), the techniques used to prove the result are similar.
observed in Cournot as well as Bertrand models.

Employing a linear Cournot model, Gaudet and van Long (1996) show that, depending on the number of firms in the upstream and downstream market, asymmetric equilibria may arise where some but not all firms integrate, even if firms are perfectly symmetric in all other aspects. Abiru et al. (1998) confirm that, for unequal numbers of Cournot oligopolists at each stage, asymmetric integration is possible for certain parameter values. Employing yet another variant of the linear Cournot model, Elberfeld (2002) also obtains asymmetric integration equilibria in a symmetric setting, and he derives the conditions under which there is a negative relation between market size and the extent of vertical integration. Jansen (2003) provides conditions under which some (otherwise symmetric) upstream firms choose vertical integration, whereas others choose vertical separation, using a variant of a model proposed by Gal-Or (1990), where downstream demand is linear and firms compete à la Cournot.

Ordover et al. (1990) study a 2×2 firms model with price competition and differentiated downstream goods. They show that asymmetric vertical integration does emerge in equilibrium, provided the downstream firms’ revenues are increasing in the input price (and the latter does not increase too much). However, these authors also show that if downstream firms compete in quantities, the only equilibrium entails no vertical integration.

7 Conclusions

This paper provides two main results on successive oligopolies. First, even in a symmetric setting it is possible that some firms integrate vertically, and others do not. Second, when downstream firms differ with respect to their initial efficiency levels, efficient firms are more likely to integrate.

The first result cautions against the notion that bandwagoning prevents asymmetric vertical integration. The second result is consistent with the observation that, in many vertically-related industries, the large firms tend to be the integrated ones: Efficiency works towards a high market share, and, as our analysis shows, it also works towards vertical integration.

Both results are generated by demand/mark-up complementarities in the product market, which are present in many models of successive oligopolies. However, as we have pointed out, there are countervailing forces that may, in principle, upset these findings. For instance, the strategic substitutes property, which is necessary for asymmetric equilibria to emerge in a symmetric setting, may be violated when strong foreclosure effects are present. It might also fail to hold if the integration of a competitor drives
down acquisition costs, thus making further integration more attractive. Further, there are reasons why more efficient firms are not necessarily more likely to integrate: For instance, because their competitors have relatively low demand, they do not expect to sell much on the upstream market.

We conjecture that our approach can be generalized further. For instance, while we defined our cost function \( c_i(V, Y) \) using a one-to-one technology, our main results can be derived for non-linear technologies as long as the relevant quantities \( M_i(V; Y), Q_i(V; Y), \) etc. are well-defined and Assumptions 2-4 and the supplementary conditions used in the propositions hold.

8 Appendix

8.1 Justifying Monotonicity Assumptions

In this section, we show how our Assumptions 3 and 4 as well as Condition (6) can be derived from more primitive conditions.

**Lemma 3** Suppose Assumption 1 holds. Further assume that, for \( i, j = 1, 2, j \neq i \),

\[
\begin{align*}
\text{\( c_i(V; Y) \) is non-increasing in \( V_i \),} \\
\text{\( c_i(V; Y) \) is non-decreasing in \( V_j \).}
\end{align*}
\]

Then Assumption 3 and Condition (6) hold.

**Proof.** By (15), integration weakly reduces own costs. By (16), it weakly reduces competitor costs. Thus, \( Q_i \) and \( M_i \) are non-decreasing in \( V_i \) by Assumption 1, and Assumption 3 follows. Similarly, firm \( j \)'s integration weakly reduces its costs and increases firm \( i \)'s costs; by Assumption 1, it therefore weakly decreases firm \( i \)'s output and mark-up. ■

The result is still true when the effect of \( V_j \) on \( c_i \) is negative, but sufficiently small. The statement about condition (6) requires the additional condition that \( c_i(V; Y) \) is decreasing in \( V_i \) rather than merely non-increasing. Next, we provide a similar result for the effects of changes in efficiency.

**Lemma 4** Suppose Assumptions 1 and 2 hold. Further assume that, for \( i, j = 1, 2, j \neq i \),

\[
\begin{align*}
\text{\( c_i(V; Y) \) is non-increasing in \( Y_i \),} \\
\text{\( c_i(V; Y) \) is non-decreasing in \( Y_j \).}
\end{align*}
\]

Then, for \( i = 1, 2, j \neq i \), Assumption 4 holds.
Proof. From $Q_i(V, Y) = q_i(c_1(V, Y), c_2(V, Y))$,

$$\frac{\partial Q_i}{\partial Y_i} = \frac{\partial q_i}{\partial c_i} \frac{\partial c_i}{\partial Y_i} + \frac{\partial q_i}{\partial c_j} \frac{\partial c_j}{\partial Y_i}.$$ 

Using (17) and Assumption 1, $\frac{\partial Q_i}{\partial Y_i} > 0$ if (18) holds.

The argument for $M_i$ and for the effects of $Y_j$ is similar. ■

The conclusion of Lemma 4 still holds when $c_i(V; Y)$ is decreasing in $Y_j$, as long as this effect is small relative to the efficiency effects.

8.2 Proof of Lemma 2

Using exchangeability, (6) holds if, for $V_2 = 0$ and $V_2 = 1$,

$$\Pi_1(1, V_2; L, H) - \Pi_1(0, V_2; L, H) \leq \Pi_1(1, V_2; H, L) - \Pi_1(0, V_2; H, L).$$

This condition would clearly hold if

$$Q_1(1, V_2; L, H) \cdot \Delta M_1(V_2; L, H) + M_1(0, V_2; L, H) \cdot \Delta Q_1(V_2; L, H) \leq Q_1(1, V_2; H, L) \cdot \Delta M_1(V_2; H, L) + M_1(0, V_2; H, L) \cdot \Delta Q_1(V_2; H, L).$$

For the first inequality, note that, by Assumption 3, $\Delta M_1(V_2; L, H) \geq 0$ and $\Delta Q_1(V_2; L, H) \geq 0$; by Assumption 4,

$$M_1(0, V_2; L, H) \leq M_1(0, V_2; H, L)$$

and $Q_1(1, V_2; L, H) \leq Q_1(1, V_2; H, L)$.

The second inequality follows from (10) and (11).

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