Essays on consumption risk in international asset markets

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The Faculty of Business, Economics and Informatics of the University of Zurich hereby authorizes the printing of this dissertation, without indicating an opinion of the views expressed in the work.

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Chairman of the Doctoral Board: Prof. Dr. Steven Ongena
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Chapter 1

Introduction

This thesis presents three essays which focus on the empirical linkages between the behaviour of asset prices and consumption. Given the influential role of financial markets for the macroeconomy, understanding risks associated with the financial assets is of importance not only for professional investors. As documented in the asset pricing literature, the behaviour of asset prices displays some pervasive empirical regularities. Asset returns are predictable over time and across assets. Predictability over time is strongly associated with business cycles, such that prices are low and expected returns high in times of low macroeconomic performance. Predictability across assets refers to systematic differences in average returns across assets sorted on some fundamental characteristics. An economic explanation for these variations in expected returns is variations in risk. Thus, the predictability pattern is not necessarily a sign of market malfunctioning, but reflects risk compensation in equilibrium. In a basic consumption-based asset pricing model, the risk of an asset is determined by its consumption risk. The main economic intuition of the model is simple. As the investor only cares about his consumption volatility, a higher covariance of the asset’s payoff with consumption means higher risk and leads to higher expected return in equilibrium. Therefore, consumption risk is measured as the contemporaneous covariance of the asset return with consumption growth. The poor empirical performance of this measure led to the emergence of several model extensions along the dimensions of long-run consumption risk and time-varying volatility, among others, to tie together the patterns in asset prices and consumption. This thesis studies these linkages between consumption and asset prices on international markets.
from an empirical perspective.

Chapter two focuses on the linkages between consumption and aggregate wealth in Switzerland. It constructs a proxy for the Swiss consumption to wealth ratio and examines the predictive power of this ratio for the fluctuations in the stock and housing market. It further analyses the effect of fluctuations in these asset markets on aggregate consumption. Chapter three studies whether and how variations in uncertainties about market returns and about consumption growth affect the cross section and the time series of stock returns. Chapter four focuses on the importance of short- and long-run consumption risks in international stock markets. I use a cointegration framework and focus on the relationship between dividends, exchange rates and consumption to explore how consumption risk in international stock returns change over investment horizons.

Each chapter represents a self-contained paper. In the remainder of this introduction, I summarize the main results of each paper, embed the findings in a broader economic context and highlight how the papers contribute to the existing literature.

1.1 Chapter two

Campbell and Mankiw (1989) show that the ratio of consumption to wealth today is a function of future expected returns on the total wealth. This relation, which is derived from a simple intertemporal budget constraint with minimal theoretical restrictions, provides an important link between the macroeconomy and the financial markets. In a seminal paper, Lettau and Ludvigson (2001) makes this framework empirically applicable by proposing the cointegrating residual between log consumption, log asset wealth and log labor income as a proxy for the consumption-wealth ratio. Following this influential paper, a large literature has emerged focusing on diverse aspects of the link between consumption and aggregate wealth. Lettau and Ludvigson (2001) show for the U.S. that their proxy can predict stock market returns at business cycle frequencies. This constitutes an important finding, as expected excess returns seem to vary countercyclically (Fama and French (1989)), but financial predictor variables only have forecasting power over longer horizons. Lettau and Ludvigson (2004) quantitatively assess the nature of shocks driving the macroeconomic and financial variables. Concerns about the stability of the cointegrating
relationship and the accuracy of the empirical proxy have been pointed out in Brennan and Xia (2005) and Rudd and Whelan (2006). Further, Hamburg, Hoffmann and Keller (2007), Aono and Iwaisako (2013) and Della Corte, Sarno and Valente (2010) construct empirical proxies of the consumption-wealth ratio for countries other than the U.S.

The contribution of this chapter is to construct a proxy for the Swiss consumption-wealth ratio and to empirically examine its link to the Swiss stock and housing market. The ratio is constructed as the residual from a cointegrating relationship between consumption, housing wealth, financial wealth and labor income. I examine how successful the Swiss consumption-wealth ratio is in predicting returns on the Swiss stock and housing market. Further, I provide evidence on how fluctuations in financial markets affect consumption in Switzerland and distinguish between the different nature of the shocks driving the macroeconomic and financial variables. I find that the temporary deviations from the long-run equilibrium predict changes in house and stock prices, where changes in house prices are predictable at shorter horizons. Further, a dynamic analysis of the cointegrated system reveals that movements in consumption and labor income are of permanent nature, whereas house and stock prices are mainly driven by temporary shocks. This finding shows that, consistent with the findings in the U.S., most of the booms and busts in stock and housing markets do not affect consumption. I estimate a financial wealth effect of two percent for Switzerland, which is internationally relatively low. This relatively low effect could be driven by the fact that the Swiss working population holds stocks through pension funds which it cannot use for consumption until retirement. An estimate of the marginal propensity to consume out of housing wealth is around three percent which highlights the relative importance of housing.

1.2 Chapter three

This chapter studies empirically the relation between time-varying aggregate uncertainty in the economy and stock returns. The questions it addresses are as follows: can fluctuations in aggregate uncertainty rationalize the time variation of expected returns on the aggregate U.S. stock market? Is the conditional volatility a source of risk in pricing the cross section of stocks?

The recent literature in consumption-based asset pricing has emphasized the role of so-called
long-run risks to explain asset pricing anomalies. The long-run risk models capture the intuition that investors care not only about contemporaneous consumption but also about long-run growth perspectives. Fluctuations in the long-run growth perspectives are perceived as very risky and thus demand a high risk compensation in equilibrium. The model specification of Bansal and Yaron (2004) characterizes long-run risk as a small and highly persistent component in consumption growth. Although the shocks to the long-run risk component are small, they command a large risk premium which helps to solve the equity premium puzzle. This result is due to their employed recursive utility specification of Epstein and Zin (1989) which delivers a preference for early resolution of uncertainty under standard parameterization. In combination with stochastic volatility, their model also generates time-varying risk premia.

An empirical identification of the shocks to the long-run risk components is difficult. Nevertheless, a large literature following their work shows that long-run risk can be important in explaining several asset pricing puzzles (Bansal, Dittmar and Kiku (2009); Bansal, Kiku and Yaron (2009); Hansen, Heaton and Li (2008); Bansal, Dittmar and Lundblad (2005)). Parker and Julliard (2005) have shown for the U.S. that focusing on the lower frequency provides a better measure of the true consumption risk. They find that a large fraction of the variation across the twenty-five Fama-French portfolio returns can be explained by the covariance of their one-period asset returns with realized consumption growth over three years. Thus, there is considerable evidence that the low frequency component in consumption growth is able to rationalize the cross-sectional variation in stock returns. But there are very few evidence for the conditional volatility channel (Bansal et al. (2012); Campbell et al. (2012)). This paper contributes to this literature by studying how stochastic volatility affects both the cross section and the time series of stock returns.

I build on the work of Campbell (1993) to explore the effects of aggregate uncertainty on asset prices. Aggregate uncertainty in this context refers to conditional volatility of future stock market returns and of future consumption growth. To evaluate whether changing volatility helps in explaining the cross section and the time series of stock returns, I derive a long-run consumption function which makes the investor’s consumption decision dependent on expected volatility.

Colacito and Croce show in a series of papers that long-run risk can also resolve puzzles in international finance such as the uncovered interest rate parity puzzle and the Brandt, Santa Clara and Cochrane (2006) puzzle.
Since expected volatilities of market returns and consumption growth are latent variables, I use a vector autoregressive model (VAR) in combination with a multivariate generalized autoregressive conditional heteroscedasticity (GARCH) model to extract the volatility shocks.

Equilibrium models with efficient markets are so far not able to generate enough time-variation in expected returns. I find that the consumption-wealth ratio has significant predictive power for stock market volatilities, a relation that becomes apparent once some theoretically motivated structures are imposed. I then evaluate whether volatility risk is able to rationalize time series predictability and find that predictability of excess stock returns cannot be fully explained by changes in stock market volatility. The estimated conditional Sharpe ratio (mean excess return per unit of volatility risk) for the aggregate U.S. stock market displays significant and countercyclical fluctuations.

I find that fluctuations in the stock market volatility are a priced source of risk and help in explaining the cross section of stock returns. In particular, growth stocks have lower expected returns because they perform better when there is news about higher aggregate uncertainty and lower market returns in the future. Thus, they are better intertemporal hedges.

1.3 Chapter four

Chapter four focuses on the importance of short- and long-run consumption risks in international stock markets. Do short- and long-run investors face different risks when investing in international stock markets? To answer this question, I decompose the stock returns into a cash flow component, price components and exchange rate changes and estimate the covariance risk of these payoff components with consumption at different investment horizons. I find that a single consumption-based risk factor possesses significant explanatory power for the cross section of equity returns in the long run. This long-run consumption risk is solely reflected in the cash flow component. Transitory risks in the prices die out in the long run and matter more in the short run. The long-run risk in cash flow is also able to rationalize the international value premium. Higher returns on value stocks relative to growth stocks are explained by higher long-run consumption risk exposures in value stocks’ cash flows. Further, the estimated price of risk is higher in the long run than at short horizons, which is in line with the long-run risk model implication.
This paper contributes to the international asset pricing literature along several dimensions. By employing a cointegration framework, I empirically explore the long-run risk explanation for international equity returns. Further, the return decomposition allows a more versatile analysis of the risk-return relationship on international asset markets. I also find that my dividend-consumption ratio predicts dividend growth in all the countries in my sample, which represents a novel contribution to the literature on dividend growth predictability.

International asset pricing models are typically based on the assumption that financial markets are perfect and therefore, with no investment barriers, the investment opportunity set is the same across countries. If the investors also face the same consumption opportunities across countries, the expected returns are solely determined by the assets’ exposures to global risk factors. Then, the domestic capital asset pricing model (CAPM) can be extended in an international setting to the World CAPM by using the excess return on the world market portfolio as the risk factor. Solnik (1974), Adler and Dumas (1983) and Stulz (1981) allow for differences in consumption opportunity sets and derive models where exchange rate risk is priced. Solnik (1974) develops a model in which investors have different consumption baskets and zero local inflation. Then, changes in the exchange rate mirror deviations from purchasing power parity (PPP). Since investors care about returns in domestic currency, PPP deviations imply that the real returns of the same asset differ across countries. Thus, the perception of risk and the returns expected by the investors vary across countries. Adler and Dumas (1983) introduce stochastic inflation and derive a multifactor model with exchange rate risks. Stulz (1981) shows that expected returns are linearly related to a measure of world consumption risk.

The empirical literature on international consumption-based asset pricing is limited due to measurement errors in international consumption data. Wheatley (1988) uses a consumption CAPM to test for international equity market integration and does not find conclusive evidence for the hypothesis that the consumption-based model holds internationally. Cumby (1990) tests the consumption-based model for four major equity markets and finds that the observed returns are consistent with the model predictions only if the tests are conditioned on the period of higher financial integration during the 1980s. Li and Zhong (2005) analyse the explanatory power of a habit-formation model and show that it outperforms various other models. They also document
that the unconditional international consumption-CAPM has some explanatory power for the cross section of equity returns, but fails to explain the risk-free rate.

The empirical literature focused extensively on the ability of the world CAPM and the international CAPM with exchange rate risk to explain the cross section of average equity returns. Dahlquist and Sallstrom (2002) evaluate both the unconditional and conditional versions of these models and find that national market returns are well captured by all of the models. Returns on portfolios sorted by value and size characteristics cannot be explained by the unconditional world CAPM, whereas the conditional version of the model with foreign exchange risk explains a large part of the cross-sectional variations. Other studies also provide evidence that currency risk is priced and that the price of risk is varying over time (Dumas and Solnik (1995), Santis and Gerard (1997)). More recently, Brusa, Ramadorai and Verdelhan (2015) show that the dollar and the carry factor two currency risk factors which explain the cross section of returns on interest rate-sorted currency portfolios, help to account for the variations in average international equity returns. The evidence suggests that, despite increased globalization, international asset prices continue to depend on both global and local risk factors, although local factors such as exchange rate risk are consistent with fully integrated capital markets (see Karolyi and Stulz (2003); Lewis (2011) for a survey). This chapter contributes to the empirical literature by applying a different methodology and by focusing on the long run.

Chapter four also relates exchange rate fluctuations to fundamentals over different horizons. Early attempts to link exchange rates to macroeconomic fundamentals find that the fundamentals do not help to predict exchange rates (Meese and Rogoff (1983)). As a result, people perceived exchange rate fluctuations as random. More recently, studies focusing on risk-based explanations for exchange rate fluctuations reveal that exchange rate movements are not random. Exchange rates display systematic variations which can be captured by the dollar and carry risk factor. Verdelhan (2012) finds that the share of systematic variations in bilateral exchange rates uncovered by these factors are large among developed countries. A higher share indicates a higher importance of global versus local shocks due to the global nature of the common factors and therefore also indicates higher market integration. The large systematic variations uncovered by

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2The dollar factor reflects the cross-sectional average of changes in all exchange rates relative to the U.S. dollar and the carry factor reflects average changes in exchange rates between the high and low interest rate currencies.
these factors suggest that exchange rate fluctuations are significantly determined by exposures to
the global shocks. A natural candidate for the common global factor in the long-run risk models
is the low frequency component in consumption. Colacito and Croce (2011, 2013) show that
long-run risk models can reproduce several international finance anomalies, once the long-run
consumption risk components are allowed to be highly correlated across countries. They pro-
vide a link between real exchange rate fluctuations and long-run growth perspectives. Colacito
and Croce (2011) address the anomaly exposed by Brandt, Cochrane and Santa-Clara (2006)
that consumption growth do not covary enough to explain the observed exchange rate volatility.
Colacito and Croce (2013) simultaneously generate the lack of correlation between exchange
rate changes and consumption growth differentials (Backus and Smith (1993) puzzle) and the
tendency of high interest rate currencies to appreciate (uncovered interest rate parity puzzle). A
potential determinant of the highly cross-correlated low frequency component is the degree of fi-
nancial integration. I test whether a country’s long-run risk exposure in dividends to the common
consumption risk factor is related to its financial openness. The results from a panel cointegrating
regression are mixed. A country’s exposure does increase with financial openness, but the effect
vanishes when the separate effect of the financial openness measure is included. The analysis of
the exchange rate channel suggests that exchange rate risk does not matter in the long run. This
finding is consistent with the view that purchasing power parity tends to hold in the long run and
currency risk dies out.
Chapter 2

A Swiss consumption to wealth ratio
2.1 Introduction

In a seminal paper, Lettau and Ludvigson (2001) propose an empirical proxy \((c_{ay})\) for the consumption-wealth ratio and define it as the cointegrating residual between consumption, asset wealth and labor income. They find that this proxy is a useful state variable that summarizes consumers’ expectations and contains significant predictive power for stock returns over business cycle frequencies. Further, Lettau and Ludvigson (2004) use the cointegrating relationship between consumption and the wealth components to study the nature of the shocks driving the financial and the real side of the economy. They find that consumption is mainly driven by permanent shocks, whereas movements in asset wealth are of temporary nature. Thus, only a small part of the variation in asset wealth affects consumption. These two studies, which are based on U.S. data, highlight that the consumption-wealth ratio provides an important link between the macroeconomy and the financial markets.

I follow their framework and construct a proxy for the consumption-wealth ratio in Switzerland. Total wealth is disentangled into housing wealth, financial wealth and labor income. Since data on Swiss housing and financial wealth is only available after the year 2000, I approximate these wealth components by a house price index and a stock market price index over the whole sample period. The price indices seem to appropriately describe the dynamic properties of the wealth components, as the correlations between the indices and the wealth components are high. I find evidence for at least one cointegrating relationship among the four variables. Imposing one cointegrating relationship, I provide evidence that deviations from the long-run equilibrium predict changes in house and stock prices. Changes in house prices are predictable at short horizons up to 5 years, whereas stock market returns are predictable at business cycle frequencies of 3 to 6 years. Consistent with the findings from the U.S., the residual does not predict changes in consumption and labor income.

Most of the studies focusing on the Anglo-Saxon countries find that the consumption-wealth ratio predicts changes in asset wealth. Hamburg, Hoffmann and Keller (2007) find for Germany that the equilibrium errors signal changes in labor income rather than asset prices. They interpret this finding as a result of the structural differences in the financial system. Anglo-Saxon countries have, due to their market-based financial system, a higher rate of stock market participation of the
households than the continental European countries with their bank-based system. A study by Birchler et al. (2011) finds that 17 percent of the population in Switzerland directly owned stocks in year 2010. Although the population share of direct stock-ownership has decreased from 30 percent in the year 2000, it is internationally comparatively high.[3] In addition, the Swiss working population holds stocks and bonds indirectly through pension funds, which it cannot access until its retirement. Housing wealth, with an average share of 50 percent in household’s portfolio over the last decade, is the largest component in household’s net wealth. Therefore, it is important to study the role of housing and financial wealth separately.

I show that, in Switzerland, the real effects of fluctuations in housing wealth is stronger than of financial wealth. Estimates of wealth effects on consumption based on my cointegration framework establish an effect of 3 percent for housing wealth and 1.95 percent for financial wealth. A number of empirical studies have applied the cointegration framework to study wealth effects on consumption in different countries. Lettau and Ludvigson (2004), Ludvigson and Steindel (1999) and Davis and Palumbo (2001) find estimates of wealth effects around 3 to 5 percent for the United States. Tan and Voss (2003) yield similar results for Australia. My findings for Switzerland are of similar magnitude. A potential reason of the relatively low financial wealth effect in Switzerland could be its pension system, since stock holdings through pension funds cannot be accessed until retirement. Further, the analysis of the cointegrated system based on a vector error correction model asserts that fluctuations in consumption and labor income are of permanent nature, while house and stock prices are mainly driven by temporary shocks.

I also find that the cointegrating relationship is not stable over different sample periods. The cointegrating vector and the adjustment coefficients change significantly over time. The stability of the cointegrating relationship is also a matter of concern in the U.S. data. Brennan and Xia (2005) argue that the predictive power of cay is mainly due to a look-ahead bias as the cointegrating parameters are estimated in-sample. They show that if the cointegrating parameters are re-estimated each period using only data available, cay loses its out-of-sample forecasting power. Lettau and Ludvigson (2005b) respond that in order to detect long-run equilibrium with some confidence, one needs long sample data. Long samples are however subject to structural breaks.

[3] A similar survey for Germany finds a population share of direct and indirect stockownership of 12.6% in 2010. (DAF-Kurzstudie 1/2011)
and regime shifts. Hoffmann (2006) finds evidence for a second cointegrating relationship after controlling for deterministic trends and a structural break. He argues that this evidence can help to circumvent the problem with look-ahead bias as with observable great ratios, one does not need to estimate the coefficients of \( c_{ay} \) from a long sample ex-post to identify transitory components in asset prices.

This paper is structured as follows. Section two presents the framework linking consumption to wealth and discusses the implications of multiple cointegrating vectors. Section three describes the econometric framework, the data and the results. Section four concludes.

### 2.2 The consumption-wealth framework

#### 2.2.1 The main framework

I extend the framework proposed by Campbell and Mankiw (1989) and Lettau and Ludvigson (2001) to include housing wealth. They provide a general framework which links consumption and wealth to expected returns. The framework starts with the intertemporal budget constraint of a representative household

\[
W_{t+1} = (1 + R_{wt+1})(W_t - C_t) \tag{2.1}
\]

where \( W_t \) is the aggregate wealth, \( C_t \) is consumption and \( R_{wt+1} \) is the net return on aggregate wealth. If the aggregate consumption-wealth ratio is assumed to be a stationary variable, the budget constraint can be log-linearized around the steady state value of the consumption-wealth ratio. The resulting approximation leads to our key equation which holds ex post as well as ex ante:

\[
c_t - w_t = E_t \sum_{j=1}^{\infty} \rho_w^j (r_{wt+j} - \Delta c_{t+j}) + \frac{\rho_w}{1 - \rho_w} \kappa \tag{2.2}
\]

where \( \rho_w \equiv \frac{W - C}{W} < 1 \) is the steady-state investment rate and \( \kappa \) is a constant arising from the log-linearization. Small letters denote variables in log. Equation (2.2) shows that fluctuations in log consumption-wealth ratio must forecast changes in either future returns or consumption.

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4 The derivation is provided in the appendix of chapter 3.
5 The linearization constant is not relevant for the analysis and is therefore left out from now on.
growth or both. In order to make equation (2.2) empirically applicable, I follow Lettau and Ludvigson (2001). Unobservable aggregate wealth is decomposed into asset wealth and human capital $W_t = A_t + H_t$. Assuming the share of asset wealth $\gamma = \frac{A}{W}$ and human capital $1 - \gamma = \frac{H}{W}$ to be stationary, an approximate expression for log aggregate wealth can be written as

$$w_t \approx \gamma a_t + (1 - \gamma)h_t.$$  

Lettau and Ludvigson assume that the nonstationary component of human capital $H_t$ can be described by aggregate labor income $Y_t$, such that

$$h_t = \kappa + y_t + z_t$$

where $z_t$ is a zero-mean stationary random variable. This assumption can be rationalized by different specifications.\(^6\)

Further, the return on aggregate wealth is decomposed as shown in Campbell (1996)

$$r_{w,t+1} \approx \gamma r_{a,t+1} + (1 - \gamma)r_{h,t+1}.$$  

From the equations above, a proxy for the consumption-wealth ratio, referred to as the cay residual, can be derived

$$c_t - \gamma a_t - (1 - \gamma)y_t = E_t \sum_{j=1}^\infty \rho_j^t \left[ \gamma r_{a,t+j} + (1 - \gamma)r_{h,t+j} - \Delta c_{t+j} \right] - (1 - \gamma)z_t \quad (2.3)$$

where $[1, -\gamma, -(1 - \gamma)]$ is the cointegrating vector. I further decompose asset wealth into wealth due to financial assets and real estates

\(^6\)Labor income can be seen as the dividend on human capital as in Campbell (1996)

$$1 + R_{h,t+1} = \frac{H_{t+1} + Y_{t+1}}{H_t}$$

where log-linearization implies

$$z_t = E_t \sum_{j=0}^\infty \rho^t_j \left[ \Delta Y_{t+1+j} - r_{h,t+1+j} \right].$$
\[ A_t = FA_t + RE_t. \]

Taking logs of the equation above and linearizing around the long-run mean of \(\frac{RE_t}{FA_t}\), i.e. the average housing wealth to financial wealth ratio, yields

\[ a_t \approx \delta f a_t + (1 - \delta)re_t \]

where \(\delta = \frac{FA}{A}\) is the share of financial asset wealth in total asset wealth. Thus, our key equation (2.2) expands to

\[ c_t - \gamma(1 - \delta)re_t - \gamma \delta f a_t - (1 - \gamma)y_t = E_t \sum_{j=1}^\infty \rho_j^c [\gamma \delta f a_{t+j} + \gamma (1 - \delta) r_{re,t+j}] + \ldots \quad (2.4) \]

\[ \ldots + E_t \sum_{j=1}^\infty \rho_j^w [(1 - \gamma)r_{ht,t+j} - \Delta c_{t+j}] - (1 - \gamma)z_t \]

where a linear combination of the logs of consumption \(c\), real estate \(re\), financial assets \(fa\) and labor income \(y\) represents another proxy for the aggregate consumption-wealth ratio and is denoted as \(cre fay_t\). Cointegration among the four variables implies that movements in the residual must forecast at least one of the variables on the right-hand side of the equation (2.4).

### 2.2.2 More than one cointegrating relationship

Equation (2.4) implies that the residual \(cre fay_t\) is stationary and therefore, the four variables are cointegrated. However, that does not mean that all the variables are connected by a single long-run equilibrium relationship. There may be more than one linearly independent cointegrating vector. To have a more formal analysis of this issue, I apply the idea used in [Hoffmann (2006)] that the full cointegrated system can be represented as a linear combination of pairwise cointegrated systems which follow different equilibrium paths. The framework of the linearized budget constraint in the previous subsection is based on the assumption that the shares of consumption and the wealth components in total wealth are stationary, i.e. \(c_t - w_t, re_t - w_t, fa_t - w_t\) and \(y_t - w_t\) are all stationary. Thus, a linear combination of these ratios must be stationary too. This suggests pairwise cointegration of consumption with the wealth components, such that \(c_t - re_t, c_t - fa_t\)
and $c_t - y_t$ are all stationary. These pairs can be connected by different equilibrium relations, but the following linear combination of the ratios leads to the full cointegrated system

$$
cref_{ay,t} = c_t - \gamma(1-\delta)re_t - \gamma\delta fa_t - (1-\gamma)y_t
$$

$$
= c - \psi re_t - \phi fa_t - (1 - \psi - \phi)y_t
$$

$$
= (1 - \psi - \phi)(c_t - y_t) + \phi(c_t - fa_t) + \psi(c_t - re_t)
$$

(2.5)

where $\psi \equiv \gamma(1-\delta)$ and $\phi \equiv \gamma\delta$. The equations above show that the stationarity assumptions made to derive the proxy for the aggregate consumption-wealth ratio ($cref_{ay}$) imply the presence of three cointegrating relationships.

If there exists more than one cointegrating vector, there are several issues which need to be addressed. A second or third cointegrating relationship complicates the framework. The proxy for consumption-wealth ratio cannot be identified without imposing additional restrictions. Since the cointegrating space is more than one-dimensional, the share of each wealth component in total wealth cannot be estimated in a single regression equation. Hoffmann (2006) suggests a solution to that problem. Agents determine their optimal portfolio by minimizing the temporary fluctuations in their portfolio. Those optimal portfolio shares are used to create a proxy for the aggregate consumption-wealth ratio. The existence of three cointegrating vectors could resolve the concern about the look-ahead bias. This concern arises when the cointegrating coefficients are estimated over the full sample which might induce a bias into the forecasting regressions. If the ratios are (trend-) stationary with cointegrating vector $[1, -1]$, then there is no need to estimate the cointegrating coefficients. The residual $cref_{ay}$ is just a linear combination of the three directly observable ratios.

### 2.3 Empirical implementation

This section introduces the econometric framework to analyse the cointegrated system described in the previous section. The joint dynamics of the system are characterized by a vector error correction model (VECM). It is shown that the estimates of the VECM parameters can be used to back out the permanent and transitory shocks. A brief summary of the methodology used
to obtain a permanent-transitory decomposition is presented. Once the structural shocks are identified, one can apply variance decompositions and impulse response analysis to examine the dynamic properties of the cointegrated system.

2.3.1 Econometric framework

Cointegration implies the existence of a vector error correction model as shown by Engle and Granger (1987). The error correction model links the long-run equilibrium relationship implied by cointegration to the short run adjustment coefficients. The adjustment coefficients describe through which variables the error correction mechanism takes place. The model has the following representation

\[ \Gamma(L) \Delta x_t = \text{const.} + \alpha \left[ \beta^\prime, \delta_{\text{trend}} \right] \begin{bmatrix} x_{t-1} \\ t \end{bmatrix} + \varepsilon_t \]  

(2.6)

where \( \Gamma(L) \) is a lag polynomial, \( \alpha \) is the vector of adjustment coefficients, \( \beta \) is the cointegrating vector and \( \delta_{\text{trend}} \) captures a linear trend if it is restricted to lie in the cointegrating space. \( x_t = [c_t, r_t, f_a_t, y_t] \) contains consumption \( c_t \), real estate \( r_t \), financial assets \( f_a_t \) and labor income \( y_t \).

Lettau and Ludvigson (2004) show that it is important to distinguish between the permanent and transitory elements of consumption and wealth. “Perhaps the more relevant finding of this paper is ... that consumption responds differently to temporary changes in wealth than to permanent changes.” Therefore, the analysis based on the permanent and transitory decomposition of the cointegrated system is done in two steps. First, the decomposition is applied to the level of the variables in \( x_t \). Second, cointegration is used to identify the permanent and transitory shocks and to analyse the dynamics of the system. Next, I formally identify the permanent and transitory elements. Following Proietti (1997), who provides a generalization of the Gonzalo and Granger (1995) decomposition, the permanent and transitory elements of the variables are

\[ x_t = x_t^p + x_t^\tau \]  

(2.7)

\[ x_t^p = C(1)\Gamma(1)x_t \]  

(2.8)

\[ x_t^\tau = (I - C(1)\Gamma(1))x_t \]  

(2.9)
where $C(1) = \beta_\perp(\alpha_\perp' \Gamma(1) \beta_\perp)^{-1} \alpha_\perp$ is the long-run impact matrix expressed in terms of the VECM parameter estimates as shown by [Johansen (1995)]. $\alpha_\perp, \beta_\perp$ are the orthogonal complements of $\alpha$ and $\beta$ such that $\alpha_\perp' \alpha = 0$. To identify permanent and transitory shocks I follow [Hoffmann (2001)]. He suggests a solution procedure based on a long-run recursive identification scheme and QR decomposition of the long-run impact matrix. The permanent shocks $\pi_t$ are identified by premultiplying the equation (2.6) with $\alpha_\perp'$ to get rid of $\alpha \beta' x_{t-1}$ which captures the transitory dynamics of the system. Transitory shocks $\tau_t$ are identified by making them orthogonal to the permanent ones. Thus,

$$
\eta_t = \begin{bmatrix} \pi_t \\ \tau_t \end{bmatrix} = \begin{bmatrix} \alpha_\perp' \\ \alpha_\perp \Omega^{-1} \end{bmatrix} \epsilon_t = S^{-1} \epsilon_t \tag{2.10}
$$

where $\Omega$ is the covariance matrix of $\epsilon$. This identification of shocks is sufficient to conduct variance decompositions. But in general, it is not enough to compute the impulse responses to each shock. I need to further identify the permanent and transitory shocks among themselves. The relation of the transformed shocks $\eta_t$ to $x_t$ is given by the Wold representation

$$
\Delta x_t = C(L) SS^{-1} \epsilon_t. \tag{2.11}
$$

The structural shocks $\eta_t$ are linked to the reduced form shocks $\epsilon_t$ as follows

$$
S^{-1} \epsilon_t = \eta_t \tag{2.12}
$$

where $S = \left[ \Omega \alpha_\perp \left( \alpha_\perp' \Omega \alpha_\perp \right)^{-1} \alpha \left( \alpha_\perp' \Omega^{-1} \alpha \right)^{-1} \right]$.

### 2.3.2 Data

Building a proxy for the consumption-wealth ratio for Switzerland is a challenging task due to the lack of data availability. The state of Swiss wealth-related data is poor. Official quarterly estimates of labor income only goes back to the year 1990. Further, estimates of household’s net worth are available only on an annual basis starting from the year 2000. Therefore, the wealth-related quarterly series used in my analysis covering the period 1973 Q1 - 2010 Q4 are
constructed from different data sets and using interpolation methods for some missing values.

Quarterly estimates of consumption and labor income are obtained from the government agency SECO (state secretariat for economic affairs). As the quarterly series of labor income starts only in 1990, annual estimates from the SNB for the period from 1970 to 1990 are interpolated by using a cubic spline and added to the official estimates. Data on household wealth and its subcategories are provided by the Swiss national bank (SNB) on an annual basis from 2000 to 2009. The two main components of household net wealth are net financial assets and real estate. I use the MSCI (Morgan Stanley Capital International) equity index and the house price index to proxy the two main components of the household wealth over the whole sample period. I argue that the dynamic properties of the two wealth components are well approximated by the stock market price index and a house price index, since the indices are highly correlated with the annual estimates of the wealth components over the period 2000-2009. As a robustness check I also use the market value of the country’s stock market index as a more direct measure of stock market wealth.

2.3.3 A first look at the data

Table (2.1) summarizes some descriptive statistics of $x_t = [c_t, r_t, f_a_t, y_t]$. The use of price indices as proxies for financial and housing wealth raises some concerns about the accuracy of the approximations. However, the proxies are highly correlated with the annual wealth estimates from the SNB over the time period of 2000-2009. The correlation coefficient of stock market index with net financial assets is 0.98 and the correlation of house price index with housing wealth estimates is 0.8. Both price indices together can explain up to 80 percent of the variation in household’s net wealth. Figure (2.1) shows how well the price indices fit the observed net wealth.

I start my analysis with some tests on the time series properties of $x_t$. For a correct specification of the econometric framework, I assume the time series in $x_t$ to be $I(1)$. Unit root tests confirm this assumption for all the variables in $x_t$. Next, I test whether there is a long-run equilibrium relationship among the variables.

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8See data appendix for a more detailed description.
2.3.4 Cointegration analysis

I argue in the theory section that there can be more than one cointegrating relationship among the four variables and that the full cointegrated system can consist of a linear combination of pairwise cointegrated systems. Each pair can follow different equilibrium paths and the ratio of each pair should be stationary. Equation (2.5) characterizes the full system as a linear combination of the ratios of consumption to each wealth component. Figure (2.2) depicts our proxy of the consumption-wealth ratio \( cr/f ay_t \) along with the ratios of consumption to the proxy of each wealth component \( c_t - y_t, c_t - re_t, c_t - fa_t \). The cointegrating residual \( cr/f ay_t \) is obtained by imposing one cointegrating relationship among the four variables. But the plots of the pairwise ratios show that they are in fact trending. There seems to be a slow downward drift in \( c_t - y_t \), whereas there is large downward trend in \( c_t - fa_t \) between the early 1980s and 2000.

Thus, I test for trend-stationarity of the ratios. Table (2.2) uses the Johansen cointegration tests and a residual based test to see whether the ratios can be characterized as trend-stationary. Panel A and B of table (2.2) show that the null of no cointegration cannot be rejected, once a deterministic trend is included in the cointegrating space. Panel C also provides evidence for trend-stationarity of the ratios. The deterministic trends are all significant and a unit root test on the regression residuals rejects the null at the 10 percent significance level. These results suggest to impose a second or even a third cointegrating relationship in the VECM framework of equation (2.6). However, Panel A and B also show the estimated cointegrating vector for each pair of variables and they deviate significantly from their theoretical value of \([1, -1]\).

In order to test for the presence and the number of cointegrating relationships, the test procedure proposed by Johansen (1991) is applied. Table (2.3) shows the test results. Panel A and B differ in terms of the specification of the deterministic components. Panel A includes only a constant term in the VECM representation, but no deterministic term is restricted to lie in the cointegrating space. The test results provide evidence that the system is cointegrated and there are two linearly independent cointegrating vectors. The second panel shows the test results when there is a linear trend in the cointegrating space. This specification captures the case when a linear combination of the variables removes the stochastic trend but not the deterministic trend. Thus, it tests whether the system contains trend-stationary relations. This specification also suggests two
cointegrating relationships among the four variables at the 10 percent significance level. The two cointegrating relations can be theoretically motivated in a heterogenous agent framework, where the economy consists of a rich and a poor agent. The rich owns stocks and houses and the poor consumes only out of labor income. Both budget constraints need to hold, thus they are individually cointegrated. But they also need to be cointegrated on aggregate. Thus, a linear combination of both budget constraints needs to be stationary. The different trends observed in the data can be motivated by different slopes on the consumption of the rich and the poor. Although this is a potentially interesting extension of the framework, I adopt a narrower focus and confine my attention to a more statistical analysis.

Even though the test results suggest that I should impose two cointegrating relationships, there are two reasons which speak for imposing only one cointegrating relationship. Under the specification of panel A in table (2.3), the test statistic for the second cointegrating relationship is only marginally significant. In addition, the estimates of the cointegrating vector deviate substantially from their theoretical value if a second or a third cointegrating vector is imposed. Thus, I cannot restrict the cointegrating vector of each ratio to be $[1, -1]$. I therefore proceed by imposing one cointegrating relationship which makes the interpretation of our results more simple and intuitive.

It should be noted that the cointegrating relationship is not stable over different sample periods. The cointegrating vector and the adjustment coefficients change significantly over time. More importantly, the coefficients in the cointegrating vector on house and stock prices change their signs.

2.3.5 VECM results

The existence of a cointegrating relation implies that there exists an error-correction representation, such that at least one of the variables must restore the long-run equilibrium. Table (2.4) presents the VECM estimates. The adjustment coefficients $\alpha$ show through which variables the residual is pulled back towards its mean. Only the coefficients in the house price and stock price equations are significant and positive. Thus, the proxy $crefay$ has forecasting power for changes in asset wealth. A positive deviation from the equilibrium is restored by positive subsequent
changes in house and stock prices. The adjustment coefficients in the consumption and labor
equation are insignificant, but there are some significant short run coefficients. That means, con-
sumption and labor income do not contribute to the error correction mechanism, but their growth
rates are predictable by their own lags. The VECM results suggest that consumption hardly
contains transitory component consistent with the permanent income hypothesis. Whereas, fluc-
tuations in the proxies for asset wealth seem to be driven mainly by temporary shocks. These
findings are comparable to what Lettau and Ludvigson (2004) find for the U.S.

Table (2.4) also provides the point estimates of the cointegrating vector \([1, \beta_{re}, \beta_{fa}, \beta_y]\). The
cointegrating relation is shown below

\[
c_t = 2.631 + 0.132r_{et} + 0.077f_{at} + 0.318y_t .
\] (2.13)

The values in parenthesis are the standard errors. The coefficients are significantly different
from zero and can be interpreted as the share of each wealth component in total wealth. The
approximation of real estate and financial wealth by price indices could explain why the co-
efficients do not sum up to one. Based on these cointegration coefficients, I can calculate the
so-called “wealth effects”, i.e. the marginal propensity to consume out of each wealth compo-
nent. I find that the marginal effect of real estate wealth on consumption is 3 percent and the
financial wealth effect is 1.95 percent in Switzerland. These estimates are obtained by multi-
plying \(\beta_{re}\) and \(\beta_{fa}\) with the most recent values of \(C_{t RE}\) and \(C_{t FA}\) as suggested by Ludvigson and
Steindel (1999). The magnitude of the estimates are close to traditional estimates of the wealth
effect. More interestingly, my findings emphasize that housing wealth is more influential for
consumer spending than financial wealth.

As described in the econometric framework section, I identify permanent and transitory
shocks and components using the estimates from the VECM. First, I apply the permanent and
transitory decomposition to the variables in level. Figures (2.4) and (2.5) plot the transitory and
permanent components of the variables. The graphs show that consumption and labor income
follow a permanent path, whereas house and stock prices display large temporary fluctuations.
It is also noteworthy that the temporary fluctuations in the price indices are highly correlated.
These findings confirm the intuition that most of the fluctuations in asset wealth are transitory
and consumption and income are mainly driven by permanent shocks.

Next, I am interested in the dynamic properties of the cointegrated system. How do consumption and the wealth components react to permanent and transitory shocks? I conduct a forecast error variance decomposition and an impulse response analysis for transitory shocks. Since I impose one cointegrating relationship, there are three permanent and one transitory shock. The permanent shocks are not uniquely identified, but the impulse response to a transitory shock can be obtained.

Figure (2.6) depicts the impulse responses of consumption, labor income and the price indices to a transitory shock. The findings are in line with the results above. Only stock and house prices respond to a transitory shock. The response is quite large and persistent. The shock affects consumption only marginally. Table (2.5) reports the relative contribution of transitory shocks to the forecast error variance in each variable. These results further confirm that transitory shocks do not matter much for labor income and consumption, whereas house and stock prices are strongly affected. Transitory shocks account for a higher share of error variance in house prices than stock prices. At one quarter forecast horizon, 40 percent of the forecast error in house prices are due to transitory shocks relative to 18 percent for stock prices. Their variance shares decrease slowly over time and at the four year horizon, transitory shocks still account for 8.8 percent, respectively 5.1 percent, of the variability.

The dynamic analysis of the cointegrated system provides evidence that movements in house and stock prices are of transitory nature and they adjust to restore the long-run equilibrium. This finding implies that the cointegrating residual should have predictive power for the changes in price indices.

2.3.6 Predictive regressions

In this section, I assess the predictive power of the cointegrating residual. Therefore, the following long-run forecast regressions are estimated

\[
\frac{1}{K} \sum_{k=1}^{K} \Delta x_{t+k} = \alpha_K + \beta_K \Delta r + \eta_{t+k}.
\]  (2.14)

Thus, growth rates of \( x_t = [c_t, re_t, f a_t, y_t] \) over different horizons are regressed on the cointe-
grating residual. Table 2.6 reports for each regression the slope coefficients, the Newey-West corrected t-statistics and the adjusted $R^2$. Panel A and D show that $c ref a y_i$ has no forecasting power for future consumption and labor income growth. Panel B provides evidence that the cointegrating residual is able to predict changes in house price over shorter horizons. The predictive power peaks at four quarters with an adjusted $R^2$ of 0.19. Stock prices seem to be predictable at longer horizons as shown in panel C. The slope coefficients are significant after three years and the predictive power rises over the forecast horizon up to 23 percent at six year horizon.

The results from the long horizon forecasting regressions are consistent with the findings from the permanent and transitory decompositions. Consumption and labor income mainly follow their permanent level and are thus unaffected by short term deviations from the equilibrium. In fact, those deviations are mainly generated by temporary fluctuations in stock and house prices. The mean reversion property of house prices is evident at shorter horizon than for stock prices.

### 2.4 Conclusion

This paper constructs a proxy for the log consumption-wealth ratio for Switzerland. Cointegration tests provide evidence for the existence of at least one cointegrating relationship. Imposing one cointegrating vector, I examine the dynamic properties of the cointegrated system. I find that the relative contribution of transitory shocks to the variability in house and stock prices is high, while consumption and labor income are mainly driven by permanent shocks. The finding that consumption and wealth are to a large extent driven by different forces should be taken into account when assessing the effect of booms and busts in house and stock prices on consumption. I also find that the proxy for the consumption-wealth ratio possesses significant predictive power for returns on the housing and stock market over business cycle frequencies. These results are in line with the findings from studies conducted on U.S. data.
2.5 Appendix A: Data sources

*Consumption* is measured as total personal consumption expenditure. The quarterly series are in millions of Swiss francs, at current prices and seasonally adjusted. My source is the State Secretariat for Economic Affairs (SECO).

*Labor income* is obtained from the Swiss national accounts and defined as compensation of employees, i.e. the total amount of gross wages paid to an employee. The quarterly series are in millions of Swiss francs, at current prices and seasonally adjusted. My source is the SECO. The quarterly series only cover the period from 1990 Q1 to 2010 Q4. For the years 1970 to 1990, annual estimates of labor income from the Swiss National Bank (SNB) are added. Those annual data are made quarterly by using the cubic spline interpolation method.

*Household wealth* data is available only on an annual basis from the year 2000 to 2009 from the SNB. The two main components of household net wealth are net financial assets and real estate. Real estate wealth is proxied by the single-family house price index from the SNB. The two proxies used for net financial assets are the MSCI (Morgan Stanley Capital International) equity price index and the market value of the country’s stock market index from Datastream. The market value is in millions of Swiss francs. The stock market proxies cover the sample period from 1973 Q1 to 2010 Q4.

*Population* is from the IMF IFS. Annual data are made quarterly by using the cubic spline. Population measure is used to express all the variables except the price indices in per capita terms.

*Price deflator.* Consumption, labor income and the proxies for total wealth are deflated by the personal consumption expenditure deflator. The deflator is a chain price index with the reference year 2000. Data is obtained from the SECO.
### 2.6 Appendix B: Tables and Figures

Table 2.1: Descriptive statistics of the annualized growth rates

<table>
<thead>
<tr>
<th></th>
<th>consumption</th>
<th>house price index</th>
<th>stock market index</th>
<th>labor income</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>mean</strong></td>
<td>0.009</td>
<td>-0.001</td>
<td>0.031</td>
<td>0.013</td>
</tr>
<tr>
<td><strong>std</strong></td>
<td>0.037</td>
<td>0.084</td>
<td>0.393</td>
<td>0.035</td>
</tr>
<tr>
<td><strong>min</strong></td>
<td>-0.152</td>
<td>-0.312</td>
<td>-1.686</td>
<td>-0.088</td>
</tr>
<tr>
<td><strong>max</strong></td>
<td>0.121</td>
<td>0.287</td>
<td>1.070</td>
<td>0.107</td>
</tr>
</tbody>
</table>

Notes: Sample period 1973 Q1 - 2010 Q4. The variables are expressed in real terms using a consumption deflator.
Table 2.2: Trend stationarity of the ratios

<table>
<thead>
<tr>
<th></th>
<th>c − y</th>
<th>c − hpi</th>
<th>c − stock</th>
<th>Critical Value 90%</th>
<th>CV 95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A:</td>
<td>No deterministic terms in cointegrating space</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>test statistic (trace test: r = 0)</td>
<td>15.431</td>
<td>6.675</td>
<td>6.189</td>
<td>15.58</td>
<td>17.84</td>
</tr>
<tr>
<td>β₂</td>
<td>-0.760**</td>
<td>-1.129</td>
<td>-0.147**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>std error</td>
<td>0.032</td>
<td>0.663</td>
<td>0.030</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Panel B:</td>
<td>With deterministic trend in cointegrating space</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>test statistic (trace test: r = 0)</td>
<td>26.762**</td>
<td>23.186*</td>
<td>23.299*</td>
<td>22.76</td>
<td>25.32</td>
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<td>Cointegrating-vector estimate [1, β₂]</td>
<td></td>
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<tr>
<td>β₂</td>
<td>-0.495**</td>
<td>-0.128**</td>
<td>0.090**</td>
<td></td>
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<tr>
<td>std error</td>
<td>0.067</td>
<td>0.024</td>
<td>0.021</td>
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<td>δ_{trend}</td>
<td>-0.001**</td>
<td>-0.002**</td>
<td>-0.004**</td>
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<td></td>
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<tr>
<td>std error</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td></td>
<td></td>
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<tr>
<td>Panel C:</td>
<td>Regression ratio = const + δt + u_t : ADF test on u_t</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>δ</td>
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<td>0.002**</td>
<td>-0.013**</td>
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<td>tstat</td>
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<td>const</td>
<td>0.032**</td>
<td>4.073**</td>
<td>6.391**</td>
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<td></td>
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<tr>
<td>tstat</td>
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<td>227.492</td>
<td>126.224</td>
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<tr>
<td>ADF t-test</td>
<td>-2.087**</td>
<td>-1.901*</td>
<td>-2.569**</td>
<td>-1.625</td>
<td>-1.945</td>
</tr>
</tbody>
</table>

Notes: Panel A and B show the Johansen cointegration test statistics and their critical values for the null hypothesis of r = 0 cointegrating relationship. Panel C runs a unit root test (augmented Dickey-Fuller test) on the residual u_t. *, ** indicate that the null hypothesis of no cointegration is rejected at the 10%, respectively at the 5% significance level.
Table 2.3: Johansen cointegration tests of the full system

<table>
<thead>
<tr>
<th>Panel A: Trace test - without a trend in the cointegration space</th>
</tr>
</thead>
<tbody>
<tr>
<td>rank</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Trace test - with a trend in the cointegration space</th>
</tr>
</thead>
<tbody>
<tr>
<td>rank</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
</tbody>
</table>

Notes: Table shows the trace test statistics and their critical values. It tests for the number of cointegrating relationships between $c_t$, $r_t$, $f_a$, and $y_t$. *, ** indicate that the null of $r$ cointegrating vector is rejected at the 10%, respectively at the 5% significance level. In Panel A, there is no deterministic trend included. In panel B, the deterministic trend is restricted to lie in the cointegrating space. The maximum eigenvalue test suggests similar results (not shown here).

Table 2.4: Estimates of a vector error correction model: No deterministic terms in cointegrating space

<table>
<thead>
<tr>
<th></th>
<th>$Δc_t$</th>
<th>$Δr_t$</th>
<th>$Δf_a$</th>
<th>$Δy_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Δc_{t−1}$</td>
<td>-0.354**</td>
<td>0.076</td>
<td>0.581</td>
<td>-0.184**</td>
</tr>
<tr>
<td>tstat</td>
<td>-4.025</td>
<td>0.386</td>
<td>0.598</td>
<td>-2.125</td>
</tr>
<tr>
<td>$Δr_{t−1}$</td>
<td>0.074</td>
<td>0.046</td>
<td>-0.257</td>
<td>-0.008</td>
</tr>
<tr>
<td>tstat</td>
<td>1.920</td>
<td>0.536</td>
<td>-0.603</td>
<td>-0.220</td>
</tr>
<tr>
<td>$Δf_{a,t−1}$</td>
<td>0.009</td>
<td>0.021</td>
<td>0.052</td>
<td>-0.005</td>
</tr>
<tr>
<td>tstat</td>
<td>1.166</td>
<td>1.262</td>
<td>0.631</td>
<td>-0.674</td>
</tr>
<tr>
<td>$Δy_{t−1}$</td>
<td>0.140</td>
<td>0.581**</td>
<td>-1.698</td>
<td>0.258**</td>
</tr>
<tr>
<td>tstat</td>
<td>1.465</td>
<td>2.715</td>
<td>-1.605</td>
<td>2.739</td>
</tr>
<tr>
<td>$α$</td>
<td>-0.050</td>
<td>0.244**</td>
<td>0.807**</td>
<td>0.015</td>
</tr>
<tr>
<td>tstat</td>
<td>-1.509</td>
<td>3.311</td>
<td>2.215</td>
<td>0.451</td>
</tr>
<tr>
<td>constant</td>
<td>0.003**</td>
<td>-0.002</td>
<td>0.012</td>
<td>0.003**</td>
</tr>
<tr>
<td>tstat</td>
<td>3.233</td>
<td>-1.291</td>
<td>1.341</td>
<td>3.559</td>
</tr>
<tr>
<td>R2bar</td>
<td>0.108</td>
<td>0.132</td>
<td>0.028</td>
<td>0.033</td>
</tr>
</tbody>
</table>

Notes: Table shows the coefficient estimates for the following vector error correction model $ΔX_{t+1} = Γ_0 + Γ_1 ΔX_t + αβ'X_t + ε_t$, where $X_t = [c_t, r_t, f_a, y_t]$. ** indicates that the coefficient is significantly different from zero at the 5% significance level.
Table 2.5: Variance decomposition

<table>
<thead>
<tr>
<th>Forecast horizon k</th>
<th>Variance share of transitory shocks</th>
<th>c_{t+k} - E_t(c_{t+k})</th>
<th>re_{t+k} - E_t(re_{t+k})</th>
<th>fa_{t+k} - E_t(fa_{t+k})</th>
<th>y_{t+k} - E_t(y_{t+k})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.083</td>
<td>0.399</td>
<td>0.179</td>
<td>0.007</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.069</td>
<td>0.243</td>
<td>0.104</td>
<td>0.005</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.059</td>
<td>0.156</td>
<td>0.076</td>
<td>0.003</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>0.054</td>
<td>0.112</td>
<td>0.061</td>
<td>0.002</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>0.052</td>
<td>0.088</td>
<td>0.051</td>
<td>0.002</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>0.050</td>
<td>0.074</td>
<td>0.044</td>
<td>0.002</td>
<td></td>
</tr>
</tbody>
</table>

Notes: The variance decomposition shows the share of forecast error variance in the levels due to transitory shocks. K is in quarters.

Table 2.6: Long horizon forecasts

<table>
<thead>
<tr>
<th>Forecast horizon K</th>
<th>Panel A</th>
<th>Panel B</th>
<th>Panel C</th>
<th>Panel D</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>∑<em>{k=1}^{K} Δc</em>{t+k} = α_K + β_K re_{t+k} + u_{t+k}</td>
<td>∑<em>{k=1}^{K} Δre</em>{t+k} = α_K + β_K fa_{t+k} + u_{t+k}</td>
<td>∑<em>{k=1}^{K} Δfa</em>{t+k} = α_K + β_K cre_{t+k} + u_{t+k}</td>
<td>∑<em>{k=1}^{K} Δy</em>{t+k} = α_K + β_K fa_{t+k} + u_{t+k}</td>
</tr>
<tr>
<td>1</td>
<td>β_K = 0.004</td>
<td>tstat = 3.284</td>
<td>tstat = 0.256</td>
<td>tstat = 0.007</td>
</tr>
<tr>
<td>4</td>
<td>tstat = 0.137</td>
<td>R2bar = 0.122</td>
<td>tstat = 0.256</td>
<td>tstat = -0.006</td>
</tr>
<tr>
<td>8</td>
<td>tstat = -0.007</td>
<td>R2bar = 0.191</td>
<td>tstat = 0.137</td>
<td>tstat = -0.006</td>
</tr>
<tr>
<td>12</td>
<td>R2bar = 0.229</td>
<td>R2bar = 0.051</td>
<td>R2bar = 0.400</td>
<td>R2bar = -0.006</td>
</tr>
<tr>
<td>16</td>
<td>R2bar = -0.007</td>
<td>R2bar = -0.007</td>
<td>R2bar = 0.031</td>
<td>R2bar = -0.006</td>
</tr>
<tr>
<td>20</td>
<td>R2bar = 0.074</td>
<td>R2bar = 0.074</td>
<td>R2bar = 0.074</td>
<td>R2bar = 0.002</td>
</tr>
<tr>
<td>24</td>
<td>R2bar = 0.044</td>
<td>R2bar = 0.044</td>
<td>R2bar = 0.044</td>
<td>R2bar = 0.002</td>
</tr>
</tbody>
</table>

Notes: Standard errors are adjusted for autocorrelation and heteroscedasticity using the Newey and West (1987) estimator. Forecast horizon K is in quarters. cre_{t+k} is the residual from the cointegrating regression of log consumption c_{t+k} on log housing wealth re_{t+k}, log financial wealth fa_{t+k} and log labor income y_{t+k}. R2bar is the adjusted R^2 statistic. *, ** indicates that the coefficient is significantly different from zero at the 10%, respectively at the 5% significance level.
Figure 2.1: Plot of the observed and fitted net wealth

Notes: Figure plots the time series of the observed and a fitted net household wealth. The blue line depicts the net wealth data from the Swiss national bank. The green line shows the net wealth approximated by the price indices on stock and housing market over the period 2000 - 2009.
Notes: Figure plots the great ratios. $crefay$ denotes the cointegrating residual among the four variables. $c - y$ is the log consumption to labor income ratio. $c - re$ is the ratio of log consumption to house price index and $c - fa$ denotes the ratio of log consumption to stock market price index.
Figure 2.3: Equilibrium error based on one cointegrating relationship

Note: Figure plots the estimated cointegrating residual $c_{refay}$, based on one cointegrating relationship among log consumption, log house price index, log stock market index and log labor income.
Figure 2.4: Transitory components

The transitory components

Note: Figure plots the transitory components of log consumption $c$, log house price index $hpindx$, log stock market index $stock$ and log labor income $y$. 
Figure 2.5: Actual series and its permanent components

Note: Figure plots the actual series and the permanent components of log consumption $c$, log house price index $hpindx$, log stock market index $stock$ and log labor income $y$. 
Note: Figure shows the cumulative impulse responses of log consumption $c$, log house price index $hpindx$, log stock market index $stock$ and log labor income $y$ to a transitory shock.
Chapter 3

The consumption-wealth ratio, aggregate uncertainty and stock returns
3.1 Introduction

This paper studies empirically the relation between time-varying aggregate uncertainty in the economy and stock returns. The questions it addresses are as follows: Can fluctuations in aggregate uncertainty rationalize the time variation of expected returns on the aggregate U.S. stock market? Is conditional volatility a source of risk in pricing the cross section of stocks?

There are several theoretical models with efficient markets which motivate the idea of a volatility risk factor. The long-run risk model by Bansal and Yaron (2004) focuses on the low frequency fluctuations in consumption growth and its variance. The main idea of their model is that investors perceive fluctuations in long-run consumption growth and in aggregate economic uncertainty as very risky and thus demand a high compensation for those risks. In a subsequent work, Bansal, Kiku and Yaron (2007) emphasize the role of stochastic volatility in generating empirically plausible implications. There is evidence that long-run consumption growth is able to rationalize the cross-sectional variation in stock returns. Parker and Julliard (2005) suggest to measure long-run consumption risk as the covariance of one-period asset returns with realized consumption growth over three years. Their risk measure explains a large fraction of the variation in returns across the 25 Fama-French portfolios. Another theoretical model to motivate a volatility risk factor is the Intertemporal CAPM by Merton (1973). It delivers the insight that the conditional distribution of returns, which the investor faces in the future, describes the investment opportunity set. Therefore, shocks to state variables which reflect changes in the investment opportunity set affect the pricing kernel and exposure to those shocks should explain the variation in risk premia. Campbell and Vuolteenaho (2004) decompose the investment opportunity set into a cash flow and discount rate component. They find that different exposures to news about future cash flows and discount rate explain the size and value anomaly. But there are only few empirical studies which focus on the role of conditional volatility for both the cross section and the time series of stock returns. Bansal et al. (2012) and Campbell et al. (2012) allow for stochastic volatility in their asset pricing models and they find that their volatility risk measure helps to explain the value premium.

A large number of studies document predictable variations in stock market volatility. Schwert (1989) finds that stock market volatility is higher during recessions than at other times. Paye
(2012) looks at a large set of potential predictor variables. The strong contemporaneous relation between volatility and business conditions implies that lagged volatility provides an efficient indicator of the economic state. He finds that only few variables like default spread and commercial paper to treasury spread contain useful information beyond that already contained in lagged volatility to predict volatility. Lettau and Ludvigson (2010) review the evidence on the risk-return trade-off. They find that market volatility is forecastable not only at high frequency but also at longer horizons ranging from one quarter to several years. They also find a positive and significant conditional risk-return relationship. Empirical researchers have found little evidence that periods of high stock market volatility are periods of high expected returns. Bollerslev, Engle and Wooldridge (1988) report weak evidence for a positive risk-return relationship.

I build on the work of Campbell (1993) to explore the effects of conditional volatility on asset prices. I derive a long-run consumption function which makes the investor’s consumption decision dependent on expected returns and expected volatility. Those conditional first and second moments reflect expectations about future investment opportunities, which I use as risk factors. Since conditional mean and conditional volatility are latent variables, they have to be estimated from the data. A natural candidate to model conditional volatility is a generalized autoregressive conditional heteroscedasticity (GARCH) model. Thus, I estimate a vector autoregressive model (VAR) in combination with a multivariate GARCH model to extract shocks to the conditional volatility of returns and of consumption growth. This empirical implementation allows to evaluate whether changing volatility helps in explaining the cross section and time series of stock returns.

I find that fluctuations in the stock market uncertainty are a priced source of risk and help in explaining the cross section of stock returns. In particular, growth stocks have lower expected returns because they perform better when there is news about higher uncertainty. Thus, they are better intertemporal hedges with respect to stock market volatility risk. Predictability of excess stock returns cannot be fully explained by changes in stock market volatility. I find that conditional Sharpe ratio (mean excess return per unit of volatility risk) for the aggregate U.S. stock market fluctuates considerably and is countercyclical.

The paper is structured as follows. Section two presents an intertemporal asset pricing model.
to motivate the idea that movements in volatility are an additional source of risk. It also has implications for the predictability properties of the consumption-wealth ratio. Section three describes the econometric framework which makes the present value relation empirically applicable. In section four, a linear factor model is used to test the pricing implications of the conditional long-run volatility. Section five concludes.

3.2 Theoretical motivation

As a theoretical foundation for my empirical analysis, I use an intertemporal asset pricing model in which time-varying volatility is introduced. The model is based on [Campbell (1993)], and the basic assumptions and the derivations are in the appendix.

3.2.1 An alternative representation of the consumption-wealth ratio

A log linearized version of an aggregate budget constraint implies that variations in consumption-wealth ratio indicate variations in future market returns and/or future consumption growth.

\[ c_t - w_t = E_t \sum_{j=1}^{\infty} \rho_j (r_{m,t+j} - \Delta c_{t+j}) + \frac{\rho}{1 - \rho} \kappa. \]  

(3.1)

A more theoretically restricted decomposition of consumption-wealth ratio can be derived by substituting out consumption growth from the budget constraint using the log Euler equation under Epstein-Zin preferences

\[ E_t \Delta c_{t+1} = \psi \log \delta + \psi E_t r_{m,t+1} + \frac{1}{2} \theta \psi Var_t (\Delta c_{t+1} - \psi r_{m,t+1}). \]  

(3.2)

For the empirical implementation, the following representation of the consumption-wealth ratio is of interest

\[ c_t - w_t = (1 - \psi) E_t \sum_{j=1}^{\infty} \rho_j r_{m,t+j} - \frac{\theta}{2 \psi} E_t \sum_{j=0}^{\infty} \rho_j Var_{t+j} (\Delta c_{t+j+1} - \psi r_{m,t+j+1}) + \mu \]  

(3.3)

where \( c_t - w_t \) is the log consumption-wealth ratio, \( \rho \equiv \frac{W-C}{W} \) is the mean reinvestment rate, \( \theta \equiv \)
\( \frac{1 - \gamma}{1 - \psi} \) with risk aversion parameter \( \gamma \) and the intertemporal elasticity of substitution \( \psi \). \( r_m \) is the log total wealth return and \( \mu \) is a constant. Now, this equation has two interesting implications. First, it implies that fluctuations in consumption-wealth ratio could be due to movements in the volatility terms. Second, movements in the uncertainty measures should affect asset prices and can be used to explain the cross section of average asset returns. Thus, the model highlights the importance of volatility risk as an additional risk factor.

### 3.2.2 Predictability properties

We know that a proxy for the consumption-wealth ratio, \( cay \), forecasts future stock market returns. But it does not forecast future consumption growth. In the empirical implementation, I show that consumption-wealth ratio has some predictive power for the variance term. But this must be consistent with the finding that consumption growth is not predictable by \( cay \). Now, an explanation to make the two findings consistent would be if part of the predictability of some components in equation (3.3) cancels each other out. In the empirical part, I find no predictability in the consumption volatility. The main predictable variation comes from the stock market volatility.

### 3.2.3 Pricing implications

Equation (3.3) also shows that the optimal consumption and investment decision of the representative investor depends on his expectations about future market returns and the variance of future consumption growth relative to market returns. I interpret the variance term as a measure of aggregate uncertainty. The right- and left-hand-side components are linked by preference parameters. The preference parameter \( \psi \) determines whether an increase in expected returns has a positive or negative impact on consumption today. \( \psi < 1 \) means that the income effect will dominate over the substitution effect and higher expected returns are reflected in higher consumption today relative to wealth. An increase in the variance term affects consumption relative to wealth negatively if \( \theta > 0 \), for example if \( \gamma > 1 \) and \( \psi < 1 \). Changes in expectations about future market returns and aggregate uncertainty reflect changes in the set of investment opportunities. Fol-

---

\(^{9}\) Lettau and Ludvigson (2001)
lowing this logic, those news variables should be able to explain the cross-sectional variation in average returns. Thus, they can be used as risk factors.

This intuition can be seen more formally. As derived in equation (3.43) in the appendix, the risk premium for any asset $i$ can be expressed as

$$E_t \left( r_{i,t+1}^f - r_{i,t+1}^f + \frac{1}{2} \text{Var}_t \left( r_{i,t+1}^f \right) \right) = - \text{Cov}_t \left( m_{t+1}, r_{i,t+1}^f \right).$$  \hspace{1cm} (3.4)

The log stochastic discount factor $m$ under Epstein-Zin-Weil preferences is

$$m_{t+1} = \theta \ln \delta - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1) r_{m,t+1}.$$  \hspace{1cm} (3.5)

To substitute out consumption growth in the pricing kernel, consider the log-linearized budget constraint from equation (3.33) in the appendix

$$\Delta c_{t+1} + (c_t - w_t) - (c_{t+1} - w_{t+1}) = r_{m,t+1} + \kappa + \left( 1 - \frac{1}{\rho} \right) (c_t - w_t).$$  \hspace{1cm} (3.6)

Since only the shocks to the pricing kernel are priced, take the $(E_{t+1} - E_t)$ operator on both sides of the equation. This eliminates all the variables known at time $t$

$$(E_{t+1} - E_t) \Delta c_{t+1} = (E_{t+1} - E_t) [r_{m,t+1} + (c_{t+1} - w_{t+1})].$$  \hspace{1cm} (3.7)

In the pricing kernel equation (3.5), take the $(E_{t+1} - E_t)$ operator on both sides and substitute out consumption growth using equations (3.7) and (3.3)

$$(E_{t+1} - E_t) m_{t+1} = \left( - \frac{\theta}{\psi} + \theta - 1 \right) (E_{t+1} - E_t) r_{m,t+1} + \frac{\theta \psi}{\theta - 1} (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{m,t+1+j} \ldots \hspace{1cm} (3.8)$$

Using $\theta \equiv \frac{1-\gamma}{1-\psi}$, the innovation to pricing kernel can be simplified and the risk premium on
an asset $i$ is

\[ E_i (r_{i+1}^f) - r_{i+1}^f + \frac{1}{2} \text{Var}_t (r_{i+1}^f) = \gamma \text{Cov}_t (N_{rt+1}, r_{i+1}^f) \]

\[ \ldots + (\gamma - 1) \text{Cov}_t (N_{DR,t+1}, r_{i+1}^f) \]

\[ \ldots - \frac{1}{2} \theta^2 \text{Cov}_t (N_{Vr,t+1}, r_{i+1}^f) \]

\[ \ldots - \frac{1}{2} \left( \frac{\theta}{\psi} \right)^2 \text{Cov}_t (N_{Vc,t+1}, r_{i+1}^f) \]

\[ \ldots + \frac{1}{2} \theta^2 \text{Cov}_t (N_{Vrc,t+1}, r_{i+1}^f) \]

\[ \text{(3.9)} \]

where $N_{rt+1} \equiv (E_{t+1} - E_t) r_{m,t+1}$ denotes news about current market returns. News about future market returns are defined as $N_{DR,t+1} \equiv (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{m,t+j+1}$. News about aggregate uncertainty consist of the following three terms: news on future discounted stock market variance $N_{Vr,t+1} \equiv (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \sigma_{t+j}^2 (r_{m,j+1})$, news on future discounted consumption growth variance $N_{Vc,t+1} \equiv (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \sigma_{t+j}^2 (\Delta c_{t+j})$, and news on the covariance between the two $N_{Vrc,t+1} \equiv (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \text{Cov}_{t+j} (\Delta c_{t+j}, r_{m,t+j+1})$. The first term in equation (3.9) is the compensation for the market risk, the other terms are compensations for risks related to changes in future investment opportunity set. The risk factors describing changes in the investment opportunity set are the news related to future market returns and the news related to future uncertainties about market returns and consumption growth.

### 3.3 Empirical implementation

The log-linear representation of the theory is very useful for empirical purposes, since the theoretical implications can be tested using linear models. To test those implications, I need to estimate the latent variables in equation (3.3) and (3.9). I use a vector autoregressive model to generate forward projections of future market returns and volatilities. The VAR framework also allows to extract the risk factors in the pricing equation in a simple and elegant way. The information set available for the investor is captured by a minimal amount of state variables.
3.3.1 Data

I use U.S. data in quarterly frequency and per quarter rates. The variables used in the VAR and for the cross-sectional test are very standard in the empirical asset pricing literature. Therefore, I only provide a brief description of the data.

As a proxy for returns on the wealth portfolio, I use the value-weighted returns on the broad CRSP stock market index. Returns are deflated using the price index for personal consumption expenditure. The source for the stock market return data is the Center for Research in Security Prices (CRSP). Log excess real returns is the difference between log real stock market returns and log real three month t-bill rates. In the empirical analysis, I only use log real excess returns.

As for consumption, I use consumption of nondurables plus services minus clothing and shoes. The reason for using that concept is that utility is maximized over the flow of consumption. Consumption of durable goods happens over several periods. Therefore, this component is excluded from the true total consumption. When estimating the long-run relation between consumption and wealth, it is implicitly assumed that the log of true total consumption is constantly proportional to log consumption of nondurables.

I use two state variables which help to forecast future returns and future volatility, the \( cay \) variable proposed in Lettau and Ludvigson (2001) and the yield spread between Moody’s Baa and Aaa corporate bond yields. \( cay \) is a proxy of the log consumption-wealth ratio which depicts the deviation from a common trend among consumption, asset wealth and labor income. It has significant forecasting power for future excess stock market returns over the business cycle frequency. The corporate bond spread is used as an indicator for default risk in the aggregate economy.

For the cross-sectional analysis, I use the twenty-five Fama-French portfolios sorted by size (market capitalization) and value (book-to-market ratio) as test assets. All the data are quarterly.

\(^{10}\) Of course, there are important components in the wealth portfolio like human capital and housing which are not traded on the stock market. The underlying assumption is that returns on the equity market portfolio are perfectly correlated with returns on aggregate wealth.

\(^{11}\) I use returns and excess returns interchangeably in the text.

\(^{12}\) i.e. \( c_t = \lambda c_{t-1} \)

\(^{13}\) The definition of the variables and the sources of the data are described in the appendix. The Moody’s corporate bond yields can be obtained from the federal reserve bank of St. Louis.

\(^{14}\) Data is on Kenneth French’s website at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.
and the sample period is 1952 Q1 - 2011 Q4. Table (3.1) shows the summary statistics of the variables.

### 3.3.2 Econometric framework

I assume that the economy follows a vector autoregressive process (VAR) of order one. The VAR collects all the information available at time $t$ and generates the best possible forecasts. In the VAR, I include the conditional variances of stock market returns and consumption growth and the conditional covariance between the two as additional state variables. Thus, expectations about future aggregate uncertainty are allowed to affect investor’s expectations about future returns.

#### 3.3.2.1 Vector autoregressive model

In a first stage, I fit the following VAR(1) model

$$Y_t = \nu + AY_{t-1} + \eta_t$$  \hspace{1cm} (3.10)

where $Y_t = [rx_t, \Delta c_t, cay_t, def_t]'$, $\nu$ is a $(Kx1)$ vector of intercepts and $\eta_t$ is a $(Kx1)$ vector of residuals. The system of variables in the VAR consists of log real excess stock market returns, real per capita consumption growth and two forecasting variables for future returns and volatilities. $cay$, a proxy for log consumption-wealth ratio and the spread between Baa and Aaa corporate bond yields are used as predictors. The variances and covariances of the fitted residuals are allowed to change over time and the volatility dynamics are captured by a multivariate GARCH(1,1) process.

#### 3.3.2.2 BEKK GARCH model

The vector of innovations $\eta_t$ in the first stage regression is assumed to be normally distributed with mean zero and time-varying variances and covariances, such that

$$\eta_t = H_t^{\frac{1}{2}} \varepsilon_t$$  \hspace{1cm} (3.11)
where \( \varepsilon_t \sim i.i.d. N(0, I_K) \). The conditional distribution of \( \eta_t \) is also Gaussian \( \eta_t \sim N(0, H_t) \). \( H_t \) is the conditional variance-covariance matrix of \( \eta_t \)

\[
H_t = E_{t-1} \left( \eta_t \eta_t' \right) = \text{Var}_{t-1} (\eta_t).
\]

(3.12)

I extract the residuals of excess returns and consumption growth from the first stage regression. The dynamics of their conditional variance-covariance matrix is captured by the BEKK-GARCH(1,1) model proposed in Engle and Kroner (1995)

\[
H_t = PP' + BH_{t-1}B' + C_{t-1} \eta_t^{t-1}C'.
\]

(3.13)

The advantage of this specification is that \( H_t \) is guaranteed to be positive definite due to the quadratic nature of the underlying process. \( P \) is a lower triangular matrix. \( B \) and \( C \) are \((2x2)\) matrices of parameters. The conditional variances and covariance of market return and consumption growth depend on the past variances and innovations of market returns as well as of consumption growth. The parameters \( \theta = \text{vec}(P, B, C) \) are obtained by maximizing the Gaussian log-likelihood function of the GARCH model

\[
\ln L(\theta) = \sum_{t=1}^{T} \left\{ \frac{1}{2} \left[ \ln 2\pi - \ln |H_t| - \eta_t H_t^{-1} \eta_t' \right] \right\}.
\]

(3.14)

Based on the residuals of the first stage regression, the following BEKK model is estimated
\[
\begin{bmatrix}
H_r & H_{cr} \\
H_{rc} & H_c
\end{bmatrix}_t =
\begin{bmatrix}
0.034 & 0.000 \\
(0.021) & (0.0006)
0.000 & 0.001 \\
(0.0006) & (0.0003)
\end{bmatrix}
+ \ldots
+ \begin{bmatrix}
0.871 & 0 \\
(0.174) & (0.014)
0 & 0.871 \\
(0.005) & (0.060)
\end{bmatrix}
\begin{bmatrix}
H_r & H_{cr} \\
H_{rc} & H_c
\end{bmatrix}_{t-1}
+ \ldots
+ \begin{bmatrix}
0.284 & 0 \\
(0.194) & (0.009)
0 & 0.284 \\
(0.006) & (0.173)
\end{bmatrix}
\begin{bmatrix}
\eta_t^2 & \eta_{cr} \\
\eta_{rc} & \eta_t^2
\end{bmatrix}_{t-1}
(3.15)
\]

where standard errors are in the parenthesis. The coefficients indicate that there are not much interactions between the volatility terms. The conditional variances and covariance estimated from this model are used in a next step as additional state variables in a VAR(1) model

\[
Y_t = \nu + AY_{t-1} + \eta_t
(3.16)
\]

with \( Y_t = [r_x, H_r, H_{rc}, H_c, cay, def]^\intercal_t \). From that VAR, I generate the long-run forecasts and the news series as follows

\[
N_{r,t+1} = e_r \eta_{t+1}
(3.17)
\]
\[
N_{DR,t+1} = e_r \rho A(I - \rho A)^{-1} \eta_{t+1}
(3.18)
\]
\[
N_{V,r,t+1} = e_r \rho (I - \rho A)^{-1} \eta_{t+1}
(3.19)
\]
\[
N_{V,rc,t+1} = e_r \rho (I - \rho A)^{-1} \eta_{t+1}
(3.20)
\]
\[
N_{V,c,t+1} = e_r \rho (I - \rho A)^{-1} \eta_{t+1}
(3.21)
\]
\[
N_{CF,t+1} = (e_r + e_r \rho A(I - \rho A)^{-1}) \eta_{t+1}.
(3.22)
\]
The long-run forecasts excess returns and volatilities are generated by replacing \( \eta_{t+1} \) with \( Y_t \) in the formulas above.

### 3.3.3 Results

Table 3.6 provides the estimates of the VAR parameters and their t-statistics. It shows that the conditional variance of stock returns has a significant effect on expected returns. The main predictor of excess returns is \( cay \). The corporate bond yield spread \( def \) considerably helps to forecast future expected stock market variance. Current excess return has a significant negative impact on future stock market variance. Further, the \( R^2 \)s in the volatility equations are quite high.

Table 3.5 shows the correlation coefficients between the news series. News about current market excess returns are strongly negatively correlated with both \( N_{DR} \) and \( N_{VR} \). News about future excess returns and excess returns volatility are slightly positively correlated. The signs of the correlations are perfectly in line with the theoretical implications. One would expect that higher risk states are associated with a positive shock on the expected return volatility and expected risk premium. The correlation coefficients are computed over the whole sample. The plots of the news series in figure 3.2 and figure 3.3 provide additional insights. In times of distress, the discount rate and stock market volatility news are highly correlated. The vertical lines in the figures mark four major events of distress. The marked events are: the oil price shock in 1973 and the following recession until March 1975, the stock market crash in October 1987, September 11, 2001, and the last recession from December 2007 until June 2009. In those time periods, there are large positive shocks in \( N_{DR} \) and \( N_{VR} \) and large negative shocks in \( N_{CF} \). During the late ‘90s and before 1973, discount rate news and market volatility news display a negative comovement. The news series can be interpreted as the proximate causes of fluctuations in market prices. Thus, the last and the 1973 recession were particularly bad due to the more permanent nature of their shocks, the negative cash flow news were much stronger during those periods.
3.3.3.1 Estimating the deep parameters

Equation (3.3) restricts the relation between the consumption-wealth ratio and the conditional first and second moments through the preference parameters and the constant $\rho$ which serves here as a discount factor. Thus, I estimate the deep parameters by fitting the left- and the right-hand side of the following equation using a gridsearch procedure

$$c_t - \omega_t = (1 - \psi)E_t \sum_{j=1}^{\infty} \rho^j r_{m,t+j} - \frac{1}{2} \psi E_t \sum_{j=1}^{\infty} \rho^j \text{Var}_t (\Delta c_{t+j}) - \ldots$$

$$\ldots - \frac{1}{2} \psi \theta E_t \sum_{j=1}^{\infty} \rho^j \text{Var}_t (r_{m,t+j}) + \ldots$$

$$\ldots + \theta E_t \sum_{j=1}^{\infty} \rho^j \text{Cov}_t (r_{m,t+j}, \Delta c_{t+j}) + \mu. \quad (3.23)$$

The left-hand side is proxied by the $cay$ variable. The right-hand-side variables are the long-run forecasts generated from the VAR model in equation (3.16). The gridsearch procedure minimizes the squared residuals between the observed and the predicted $cay$, i.e. the right- and the left-hand side. Hence, the estimation of a forward-looking long-run consumption function delivers estimates of the preference parameters. Figure (3.4) shows the weighted average of the variances and covariance and compares it to other measures of uncertainty. The upper right graph shows the time series of my long-run uncertainty measure and an index of political uncertainty measured by Baker, Bloom and Davis (2012). This index reflects the number of news related to political uncertainty in 10 major U.S. newspapers, the size of disagreement between economic forecasters and the number of temporary tax provisions. Thus, the index quantifies how uncertain people are about future economic policies. The correlation between the two uncertainty measures in the upper-right graph is 0.54. The high correlation with the VIX index (0.63) is not surprising since my aggregate uncertainty measure is mainly driven by the stock market volatility. The correlation between the VIX and the political uncertainty measure is 0.49.

Table (3.3) presents the estimates of the preference parameters. The constant $\rho$ is difficult to estimate, since it is defined as the mean reinvestment rate out of total wealth. I follow the

---

\[15\] The weighted average of the variances and covariance are formed according to equation (3.23) using estimated preference parameters.
literature and exogenously specify that $\rho$ equals 0.99 which means that 4 percent of total wealth is consumed annually on average.\textsuperscript{16} Given $\rho = 0.99$, the parameter estimate for the intertemporal elasticity of substitution is below one and the risk aversion parameter is slightly above one. These parameter specifications establish a positive relation between future expected returns and $cay$ and a negative one between future variances and $cay$. The positive relation between $cay$ and future returns is also found in the literature.\textsuperscript{17} Further, Lettau and Ludvigson (2010) find a negative relation between $cay$ and future stock market volatilities.

3.3.3.2 Predictability properties

Table (3.2) shows the evidence for the negative relation between $cay$ and future stock market volatility. It runs long-horizon forecast regressions of realized stock market volatility over subsequent $h$ periods on $cay$.\textsuperscript{18} The predictability results survive the inclusion of the lagged realized volatility and show forecasting power at the short horizon.

A somewhat harder test of the predictability properties of $cay$ is to look at the relative variance decomposition of the theoretically decomposed consumption-wealth ratio in equation (3.3). More formally,

$$\text{Var}(cay_t) = \text{Cov} \left( \frac{1}{1-\psi} E_{t} r^{f_{t}}, cay_t \right) + \text{Cov} \left( -\frac{\theta}{2\psi} E_{t} V_{t}^{f_{t}}, cay_t \right). \quad (3.24)$$

Table (3.4) shows how much variations in expected cumulated discounted returns and volatilities contribute to variations in $cay$. The variance contributions are estimated from the direct regressions

$$\hat{E}_r = \alpha_r + \beta_r cay_t + u_{r,t} \quad (3.25)$$

$$\hat{V} = \alpha_V + \beta_V cay_t + u_{V,t}. \quad (3.26)$$

\textsuperscript{16}This value is not totally random, but rather based on data. If $\rho$ is computed as $\rho = 1 - \beta_a \exp{(c-a)}$ where $\beta_a$ is the share of asset wealth in total wealth which is obtained from the cointegrating regression $c_t = \beta_a a_t + \beta_y y_t + \ldots + u_t$, then we get $\rho \approx 0.99$ on quarterly basis.

\textsuperscript{17}see Lettau and Ludvigson (2001)

\textsuperscript{18}Realized volatility is computed as volatility of daily stock market returns within a quarter.
91.9 percent of variations in $cay$ are due to variations in excess returns. But the volatility channel also plays a significant role with a relative variance contribution of 18.7 percent. Decomposing the variance term further as in equation (3.23), I find that the variance contribution of stock market volatility is 19.5 percent and that of the covariance term is -0.8 percent. Consumption growth volatility does not play any role. That means, $cay$ has significant predictive power for stock market volatilities. $cay$ negatively predicts future volatility and positively predicts future stock market returns. The conditional long-run returns and volatilities are negatively related. These results are qualitatively consistent with the results in Lettau and Ludvigson (2010).

Figure (3.1) plots the left- and the right-hand side of equation (3.3). The left-hand side is a proxy of the consumption-wealth ratio made observable through the $cay$ variable. The right-hand side is a theoretically motivated proxy of the consumption-wealth ratio and reflects the proximate causes of asset market fluctuations. The estimation of the long-run consumption function delivers a close fit between the two proxies as well as reasonable estimates of the preference parameters. The sum of future discounted returns is highly correlated with the observed $cay$, which reflects the insight from the empirical literature that $cay$ is a good predictor of future market returns. The aggregate volatility does comove significantly with expected excess returns.

### 3.3.3.3 Excess Sharpe ratio variability

With the estimates of the conditional excess returns and conditional standard deviation of excess returns from the VAR in equation (3.16), I am now able to provide an estimate of the conditional Sharpe ratio. There is little evidence on how the Sharpe ratio varies over time. Hansen and Jagannathan show that the maximum value of Sharpe ratio poses a restriction on the volatility of the set of stochastic discount factors used to price assets. Similarly, the time series behavior of the Sharpe ratio should also restrict the set of discount factors. Figure (3.5) shows that the time variation of this ratio can be quite large. Thus, predictability of stock market returns cannot be explained by changes in stock market volatility. I find that the Sharpe ratio for the aggregate stock market is countercyclical. The price of risk falls in times of expansion and rises in times of distress (e.g. during the great recession and the 1973 crisis).
3.4 A cross-sectional test

The asset pricing equation (3.9) implies that the risk premium on any asset \( i \) is determined by market risk, by discount rate news and by news about uncertainty. The intuition is that, after controlling for current market risk, changes in expectations about future market returns and future aggregate uncertainty reflect improvement or deterioration of the investment opportunity set. Thus, investors demand compensation for those risks.

3.4.1 Methodology

To test this intuition, the pricing equation is written in the beta representation.

\[
E(R_{ei}^t) = \beta_i^r \lambda_r + \beta_i^{DR} \lambda_{DR} + \beta_i^V \lambda_V + \beta_i^{V r} \lambda_{VR} + \beta_i^{V c} \lambda_{VC} \tag{3.27}
\]

where the underlying risk factors \( f_t = [N_r, N_{DR}, N_{V r}, N_{V rc}, N_{V c}] \) reflect news about current excess stock market returns, future excess returns and future volatilities. I test whether the risk factors can explain the size and value premium. Test assets are the 25 Fama-French portfolios sorted by size and book-to-market ratio. To estimate the risk prices \( \lambda \) and the exposures of the portfolios towards the risk factors, I use the Fama and MacBeth procedure. I take the simple real excess returns on the test assets to avoid the Jensen’s inequality term in the pricing equation. In a first stage, time series regressions of portfolio excess returns on the factors generate the factor loadings \( \beta^i \)

\[
R_{e}^t = a^i + \beta^i f_t + \epsilon^i. \tag{3.28}
\]

In a second stage, cross-sectional regressions of excess returns on betas at each time period \( t \) gives the risk prices. The second stage regression does not include a constant

\[
R_{\epsilon_i} = \beta^i \lambda + \alpha. \tag{3.29}
\]

Estimates of \( \lambda \) and \( \alpha \) are the averages over time: \( \hat{\lambda} = \frac{1}{T} \sum_{t=1}^{T} \hat{\lambda}_t \) and \( \hat{\alpha} = \frac{1}{T} \sum_{t=1}^{T} \hat{\alpha}_{i,t} \).

Fama-MacBeth standard errors do not account for the fact that the \( \beta \)s are generated regressors. As an alternative, I estimate the GMM standard errors which do not require the assumption
of i.i.d. (independent and identically distributed) errors and correct for the effect of generated regressors. I map the whole pricing equation into GMM. The moment condition contains both time series regression and the cross-sectional regression

$$g_T(b) = \begin{bmatrix} E \left( R^{e_i} - a + \beta f_t \right) \\ E \left( R^{e_i} - a + \beta f_t \right) f_t \\ E \left( R^{e_i} - \beta^i \lambda \right) \end{bmatrix} = 0.$$  

(3.30)

I use a prespecified weighting matrix $a_T = \begin{bmatrix} I_N & 0 \\ 0 & \beta' \end{bmatrix}$ which sets the first two moment conditions exactly to zero in estimation. The third moment condition is overidentified and therefore weighted by $\beta$ which gives us the OLS cross-sectional estimate of $\lambda$. Standard errors can be derived using the standard GMM formulas. A detailed discussion of this methodology is in Cochrane (2005).

### 3.4.2 Results

Table (3.6) compares the five-factor model with the CAPM. The risk price on the stock market volatility risk is negative and much lower in absolute terms than the price for market risk and discount rate risk. The Fama-MacBeth standard errors suggest that all risk prices are significantly different from zero. The t-statistics based on the GMM standard errors lead to insignificant risk price on discount rate news. Figure (3.6) plots the fitted versus the observed excess returns. It shows that augmenting the CAPM by volatility risk and discount rate news generates a better fit and those additional risk factors are able to explain a part of the cross-sectional variations in average returns. Table (3.8) shows the exposure of the 25 portfolios to market risk, discount rate risk and stock market volatility risk. Table (3.9) provides the t-statistics based on GMM standard errors. Almost all of the $\beta$ coefficients on the discount rate and volatility risks are insignificant. Exposures to market risk are significant. Value stocks are negatively exposed to the volatility risk which together with the negative risk price imply a positive risk premium. But the growth portfolios have a positive exposure which leads to negative risk premium for volatility risk. This result is counterintuitive. The exposures to volatility risk $\beta_{t,V_t}$ do explain the value
spread. Growth stocks perform better in riskier time than value stocks which leads to lower risk 
premium in equilibrium. The exposures to the discount rate news $\beta_{i,DR}$ are mostly positive. Thus, 
a positive discount rate news leads to a higher risk premium which is in line with the insight ob- 
tained from figure (3.2). It shows that in times of high distress, both volatility and discount rate 
news increase. The correlation coefficient over the whole sample period is also slightly positive. 
Therefore, riskier states are on average associated with positive discount rate and volatility news. 
The exposures on discount rate news do not explain the value spread. Exposure of value stocks 
are on average slightly higher than those of growth stocks, which implies that value stocks per- 
form better in riskier states than growth stocks. But the levels of and the variation among the 
beta coefficients are quite small. Exposures of growth and value stock to market risk $\beta_{i,r}$ also do 
not help to explain the value spread. Growth stocks comove higher with the market than value 
stocks. Thus, growth stocks are better intertemporal hedges than value stocks with respect to 
volatility risks.

3.5 Conclusion

A theoretically motivated decomposition of the consumption-wealth ratio implies that fluctu- 
ations in that ratio can also indicate movements in aggregate uncertainty about future market 
returns and consumption growth. I find that the volatility channel plays a significant role. Move- 
ments in the uncertainty measure also have pricing implications. Volatility risk is a separate 
source of risk which is priced in the market. In particular, growth stocks are better intertemporal 
hedges than value stocks, since they have higher covariance with news about volatility risk.
### 3.6 Appendix A: Tables and Figures

Table 3.1: Summary statistics of the variables used in the VAR

<table>
<thead>
<tr>
<th></th>
<th>mean</th>
<th>std deviation</th>
<th>autocorrelation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$rx$</td>
<td>0.012</td>
<td>0.086</td>
<td>0.083</td>
</tr>
<tr>
<td>$V_r$</td>
<td>0.007</td>
<td>0.002</td>
<td>0.811</td>
</tr>
<tr>
<td>$V_{rc}$</td>
<td>0.000</td>
<td>0.000</td>
<td>0.734</td>
</tr>
<tr>
<td>$V_c$</td>
<td>0.000</td>
<td>0.000</td>
<td>0.806</td>
</tr>
<tr>
<td>$cay$</td>
<td>-0.002</td>
<td>0.017</td>
<td>0.920</td>
</tr>
<tr>
<td>$def$</td>
<td>0.002</td>
<td>0.001</td>
<td>0.904</td>
</tr>
<tr>
<td>$\Delta c$</td>
<td>0.005</td>
<td>0.005</td>
<td>0.452</td>
</tr>
</tbody>
</table>

Notes: All the variables are in quarterly frequency and per quarter rates. $rx$ is the log real excess returns, $V_r$ denotes the conditional variance of excess returns, $V_c$ the conditional variance of consumption growth and the $V_{rc}$ is the conditional covariance between the two. $cay$ is a proxy for the log consumption-wealth ratio, $def$ denotes the the default risk variable, which is the difference between log BAA and log AAA corporate bond yields, $\Delta c$ denotes real consumption growth. Sample period is 1952 Q3 - 2011 Q4.
Table 3.2: Predictive regressions

<table>
<thead>
<tr>
<th>Forecast horizon ( h )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_1 )</td>
<td>-0.028**</td>
<td>-0.070**</td>
<td>-0.114**</td>
<td>-0.159**</td>
<td>-0.207</td>
<td>-0.264</td>
<td>-0.319</td>
<td>-0.377</td>
<td>-0.428</td>
<td>-0.478</td>
</tr>
<tr>
<td>( tstat )</td>
<td>-2.342</td>
<td>-2.216</td>
<td>-2.185</td>
<td>-1.964</td>
<td>-1.768</td>
<td>-1.684</td>
<td>-1.598</td>
<td>-1.552</td>
<td>-1.509</td>
<td>-1.477</td>
</tr>
<tr>
<td>( \beta_2 )</td>
<td>0.438**</td>
<td>0.785**</td>
<td>1.080**</td>
<td>1.389**</td>
<td>1.644**</td>
<td>1.817**</td>
<td>1.903**</td>
<td>2.006**</td>
<td>2.117**</td>
<td>2.299**</td>
</tr>
<tr>
<td>( R^2_{adj} )</td>
<td>0.197</td>
<td>0.211</td>
<td>0.218</td>
<td>0.233</td>
<td>0.234</td>
<td>0.221</td>
<td>0.202</td>
<td>0.194</td>
<td>0.189</td>
<td>0.189</td>
</tr>
</tbody>
</table>

Notes: Table shows the results from the long-run forecast regression: \( \sum_{j=1}^{h} \delta_{y+j} = \beta_0 + \beta_1 cay + \beta_2 rv + \epsilon_{t+h} \) where \( rv \) is the realized volatility of daily stock market returns within a quarter. Forecast horizon is in quarter. ** indicates that the coefficient is significantly different from zero at the 5% significance level. Sample 1952 Q1 - 2011 Q4. T-statistics are computed using Newey-West standard errors with \( h-1 \) lags.
Table 3.3: Estimates of the deep parameters

\[ \rho = \frac{W - C}{W} = 0.99 \]

\[ \gamma = 1.1 \]

\[ \psi = 0.9 \]

\[ \theta = 0.9 \]

\[ \mu = 0 \]

Notes: The parameters are estimated by fitting the following equation using a gridsearch procedure

\[ cay_t = (1 - \psi)E_t \sum_{j=1}^{\infty} \rho^j r_{m,t+j} - \frac{1}{2} \theta E_t \sum_{j=1}^{\infty} \rho^j \text{Var}_t (\Delta c_{t+j}) - \ldots \]

\[ -\frac{1}{2} \psi \theta E_t \sum_{j=1}^{\infty} \rho^j \text{Var}_t (r_{m,t+j}) + \theta E_t \sum_{j=1}^{\infty} \rho^j \text{Cov}_t (r_{m,t+j}, \Delta c_{t+j}) + \mu. \]

The steady state reinvestment rate \( \rho \) is fixed at 0.99 at quarterly frequency. \( \gamma \) is the risk aversion parameter. \( \psi \) is the intertemporal elasticity of substitution, \( \theta = \frac{1}{1-\psi} \), and \( \mu \) is a constant. Sample period 1952 Q4 - 2011 Q4.

Table 3.4: Relative variance contribution of expected returns and volatilities

<table>
<thead>
<tr>
<th>( y_t )</th>
<th>( (1 - \psi)E_t r_{t}^{lr} )</th>
<th>( \frac{\theta}{2\psi}E_t V_{lr}^{lr} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_{cay} )</td>
<td>0.919**</td>
<td>0.187**</td>
</tr>
<tr>
<td>tstat</td>
<td>42.976</td>
<td>9.614</td>
</tr>
<tr>
<td>const.</td>
<td>0.002**</td>
<td>0.000</td>
</tr>
<tr>
<td>tstat</td>
<td>4.985</td>
<td>1.115</td>
</tr>
</tbody>
</table>

Notes: Table shows the estimates of the following regression: \( y_t = \beta_0 + \beta_{cay} cay_t + \epsilon_t \). ** indicates that the coefficient is significantly different from zero at the 5% significance level. Sample 1952 Q4 - 2011 Q4.

Table 3.5: Correlation between the news series

<table>
<thead>
<tr>
<th>( N_r )</th>
<th>( N_{DR} )</th>
<th>( N_{Vr} )</th>
<th>( N_{Vc} )</th>
<th>( N_{CF} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_r )</td>
<td>0.083</td>
<td>-0.449</td>
<td>0.069</td>
<td>0.066</td>
</tr>
<tr>
<td>( N_{DR} )</td>
<td>-0.419</td>
<td>0.060</td>
<td>0.006</td>
<td>0.000</td>
</tr>
<tr>
<td>( N_{Vr} )</td>
<td>-0.169</td>
<td>-0.452</td>
<td>0.546</td>
<td>0.000</td>
</tr>
<tr>
<td>( N_{Vc} )</td>
<td>0.128</td>
<td>-0.239</td>
<td>-0.067</td>
<td>0.387</td>
</tr>
<tr>
<td>( N_{CF} )</td>
<td>0.643</td>
<td>0.395</td>
<td>-0.379</td>
<td>-0.560</td>
</tr>
</tbody>
</table>

Notes: Off-diagonal elements show the correlation coefficients. Standard deviations are on the diagonal. Sample 1952 Q4 - 2011 Q4.
Table 3.6: VAR parameters

<table>
<thead>
<tr>
<th></th>
<th>$\beta$</th>
<th>$r_{x,t}$</th>
<th>$V_{t+1,r}$</th>
<th>$V_{t+1,rc}$</th>
<th>$V_{t+1,c}$</th>
<th>$cay_{t+1}$</th>
<th>$def_{t+1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$r_{x,t}$</td>
<td>0.053</td>
<td>-0.006**</td>
<td>0.000*</td>
<td>0.000</td>
<td>0.001</td>
<td>-0.002**</td>
</tr>
<tr>
<td></td>
<td>$V_{t,r}$</td>
<td>7.660*</td>
<td>0.825**</td>
<td>0.000</td>
<td>0.000*</td>
<td>-0.422</td>
<td>0.004</td>
</tr>
<tr>
<td></td>
<td>$V_{t,rc}$</td>
<td>-133.573</td>
<td>-1.316</td>
<td>0.672**</td>
<td>0.006</td>
<td>-14.988</td>
<td>-2.883**</td>
</tr>
<tr>
<td></td>
<td>$V_{t,c}$</td>
<td>-1390.165</td>
<td>8.697</td>
<td>0.509</td>
<td>0.776**</td>
<td>120.469</td>
<td>5.401</td>
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<tr>
<td></td>
<td>$cay_{t}$</td>
<td>0.897**</td>
<td>0.002</td>
<td>0.000</td>
<td>0.000</td>
<td>0.910**</td>
<td>-0.001</td>
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<tr>
<td></td>
<td>$def_{t}$</td>
<td>3.611</td>
<td>0.193**</td>
<td>0.003*</td>
<td>0.000</td>
<td>-0.047</td>
<td>0.925**</td>
</tr>
<tr>
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<td>$const$</td>
<td>-0.018</td>
<td>0.001**</td>
<td>0.000</td>
<td>0.000**</td>
<td>0.002</td>
<td>0.000</td>
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<tr>
<td></td>
<td>$R^{2}_{adj}$</td>
<td>0.041</td>
<td>0.754</td>
<td>0.549</td>
<td>0.651</td>
<td>0.847</td>
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<tr>
<th></th>
<th>$t-stat(ols)$</th>
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<td></td>
<td>$r_{x,t}$</td>
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<td>-8.984</td>
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<td>1.092</td>
<td>0.286</td>
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<td>$V_{t,r}$</td>
<td>2.052</td>
<td>20.746</td>
<td>0.171</td>
<td>-2.175</td>
<td>-1.426</td>
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<td>$V_{t,rc}$</td>
<td>-0.778</td>
<td>-0.720</td>
<td>12.873</td>
<td>1.431</td>
<td>-1.102</td>
<td>-3.329</td>
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<tr>
<td></td>
<td>$V_{t,c}$</td>
<td>-0.857</td>
<td>0.503</td>
<td>1.032</td>
<td>18.462</td>
<td>0.937</td>
<td>0.660</td>
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<td>$cay_{t}$</td>
<td>2.704</td>
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<td>34.635</td>
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<td>0.634</td>
<td>3.173</td>
<td>1.911</td>
<td>1.477</td>
<td>-0.105</td>
<td>32.195</td>
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<tr>
<td></td>
<td>$const$</td>
<td>-0.509</td>
<td>2.019</td>
<td>0.053</td>
<td>4.695</td>
<td>0.555</td>
<td>1.428</td>
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<td></td>
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<td>0.818</td>
<td>-3.921</td>
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<td>1.232</td>
<td>0.302</td>
<td>-5.514</td>
</tr>
<tr>
<td></td>
<td>$V_{t,r}$</td>
<td>1.839</td>
<td>19.681</td>
<td>0.132</td>
<td>-1.697</td>
<td>-1.445</td>
<td>0.162</td>
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<td>$V_{t,rc}$</td>
<td>-0.738</td>
<td>-0.842</td>
<td>13.806</td>
<td>1.347</td>
<td>-1.041</td>
<td>-2.400</td>
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<tr>
<td></td>
<td>$V_{t,c}$</td>
<td>-0.780</td>
<td>0.516</td>
<td>1.170</td>
<td>22.191</td>
<td>1.029</td>
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<tr>
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<td>$cay_{t}$</td>
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<td>$def_{t}$</td>
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<td>1.026</td>
<td>-0.099</td>
<td>20.082</td>
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<tr>
<td></td>
<td>$const$</td>
<td>-0.498</td>
<td>2.264</td>
<td>0.057</td>
<td>4.792</td>
<td>0.589</td>
<td>1.096</td>
</tr>
</tbody>
</table>

Notes: Newey-West standard errors are with one lag. *, ** indicates that the coefficient is significantly different from zero at the 10%, respectively at 5% significance level. Sample 1952 Q4 - 2011 Q4.
Table 3.7: Fama-MacBeth regressions with 25 Fama-French portfolios

<table>
<thead>
<tr>
<th>Model</th>
<th>factor</th>
<th>$\lambda_r$</th>
<th>$\lambda_{DR}$</th>
<th>$\lambda_{V_r}$</th>
<th>$\lambda_{V_{rc}}$</th>
<th>$\lambda_{V_c}$</th>
<th>$R^2_{adj}$</th>
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<tbody>
<tr>
<td>CAPM</td>
<td>$\lambda_f$</td>
<td>0.019**</td>
<td></td>
<td></td>
<td></td>
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<td>-0.424</td>
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<tr>
<td></td>
<td>tstat(fm)</td>
<td>3.396</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>tstat(gmm)</td>
<td>3.427</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5-factor model</td>
<td>$\lambda_f$</td>
<td>0.018**</td>
<td>0.041**</td>
<td>-0.007**</td>
<td>0.000**</td>
<td>0.000</td>
<td>0.599</td>
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<tr>
<td></td>
<td>tstat(fm)</td>
<td>3.135</td>
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<td>-4.568</td>
<td>-2.229</td>
<td>-0.431</td>
<td></td>
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<td></td>
<td>tstat(gmm)</td>
<td>2.643</td>
<td>1.145</td>
<td>-2.351</td>
<td>-1.089</td>
<td>-0.284</td>
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</tbody>
</table>

Notes: Table shows the t-statistics based on the Fama-MacBeth as well as the GMM standard errors. Fama-MacBeth regressions $E(R^e_i) = \beta \lambda + \alpha_i$ with 25 Fama-French portfolios. ** indicates that the coefficient is significantly different from zero at the 5% significance level. Sample period: 1952 Q4 - 2011 Q4.
Table 3.8: Exposures of the 25 Fama-French portfolios: Betas

<table>
<thead>
<tr>
<th></th>
<th>$\beta_i$</th>
<th>growth</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>value</th>
<th>value - growth</th>
</tr>
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<tbody>
<tr>
<td>small</td>
<td>1.536**</td>
<td>1.318**</td>
<td>1.124**</td>
<td>1.088**</td>
<td>1.133**</td>
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</tr>
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<td>1.469**</td>
<td>1.234**</td>
<td>1.082**</td>
<td>1.051**</td>
<td>1.099**</td>
<td>-0.370</td>
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</tr>
<tr>
<td>3</td>
<td>1.378**</td>
<td>1.131**</td>
<td>1.022**</td>
<td>0.983**</td>
<td>1.001**</td>
<td>-0.377</td>
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</tr>
<tr>
<td>4</td>
<td>1.246**</td>
<td>1.110**</td>
<td>1.002**</td>
<td>0.969**</td>
<td>0.989**</td>
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</tr>
<tr>
<td>big</td>
<td>0.980**</td>
<td>0.905**</td>
<td>0.803**</td>
<td>0.786**</td>
<td>0.802**</td>
<td>-0.178</td>
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</tr>
<tr>
<td>big - small</td>
<td>-0.556</td>
<td>-0.413</td>
<td>-0.321</td>
<td>-0.303</td>
<td>-0.331</td>
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<tr>
<td>average</td>
<td>1.322</td>
<td>1.140</td>
<td>1.007</td>
<td>0.975</td>
<td>1.005</td>
<td>-0.317</td>
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</table>

<table>
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<tr>
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<th>$\beta_i$</th>
<th>growth</th>
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<th>3</th>
<th>4</th>
<th>value</th>
<th>value - growth</th>
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<td>0.045</td>
<td>0.049</td>
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<tr>
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<td>0.105</td>
<td>0.056</td>
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<td>0.114</td>
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<tr>
<td>3</td>
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<td>0.044</td>
<td>0.067</td>
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<td>0.105</td>
<td>0.027</td>
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</tr>
<tr>
<td>4</td>
<td>0.017</td>
<td>0.088**</td>
<td>0.035</td>
<td>0.030</td>
<td>0.000</td>
<td>0.017</td>
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<td>-0.135**</td>
<td>-0.111</td>
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</tr>
<tr>
<td>big - small</td>
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<td>-0.014</td>
<td>-0.033</td>
<td>-0.130</td>
<td>-0.166</td>
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<td>0.060</td>
<td>0.023</td>
<td>-0.001</td>
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<table>
<thead>
<tr>
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<th>$\beta_i$</th>
<th>growth</th>
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<th>3</th>
<th>4</th>
<th>value</th>
<th>value - growth</th>
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<td>-2.908</td>
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<tr>
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<tr>
<td>3</td>
<td>1.039</td>
<td>0.108</td>
<td>-0.549</td>
<td>-0.674</td>
<td>-1.390</td>
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<tr>
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<td>0.381</td>
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<td>-0.913</td>
<td>-1.986*</td>
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<td>0.625</td>
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<td>-0.761</td>
<td>-0.841</td>
<td>-1.486</td>
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<tr>
<td>big - small</td>
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<td>0.249</td>
<td>-0.044</td>
<td>-0.295</td>
<td>0.111</td>
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<tr>
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<td>-0.256</td>
<td>-0.862</td>
<td>-1.428</td>
<td>-2.546</td>
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</table>

Notes: Table shows the point estimates for the beta coefficients. Sample: 1952 Q4 - 2011 Q4. Averages are computed over the size categories. *, ** indicates that the coefficient is significantly different from zero at the 10%, respectively at 5% significance level.
Table 3.9: Exposures of the 25 Fama-French portfolios: T-statistics

<table>
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<th>value</th>
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<td>1.307</td>
<td>0.413</td>
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<tr>
<td>4</td>
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<td>0.768</td>
<td>0.696</td>
<td>0.007</td>
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<table>
<thead>
<tr>
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<th>3</th>
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<th>value</th>
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<td>-1.799</td>
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<tr>
<td><strong>big</strong></td>
<td>1.302</td>
<td>1.117</td>
<td>-0.195</td>
<td>-1.225</td>
<td>-1.315</td>
</tr>
</tbody>
</table>

Notes: Table shows the t-statistics to the beta estimates from table 3.8. Sample: 1952 Q4 - 2011 Q4. The t-statistics are calculated using GMM standard errors.
Figure 3.1: Fit of the theoretically decomposed $cay$

Notes: Graph shows the estimation of the equation (3.3): $cay_t = \mu + (1 - \psi)E_t r^{lr} - \frac{1}{2} \frac{\theta}{\psi} E_t V^{lr}$. It depicts the observed variable $cay_t$ together with the estimates of the theoretically decomposed consumption-wealth ratio.
Notes: Figure plots the smoothed news series. Series are smoothed by an exponential weighted moving average: $MA_{i,t} = \alpha \text{News}_{i,t-1} + (1 - \alpha)MA_{i,t-1}$ where $\alpha = 0.8$ implies a half life of 24 quarters. The news series are normalized by their own standard deviation. So, the scale is in units of standard deviations.
Figure 3.3: News series 2

Notes: Figure plots the smoothed news series. Series are smoothed by an exponential weighted moving average: $MA_{x,t} = \alpha \text{News}_{x,t-1} + (1 - \alpha)MA_{x,t-1}$ where $\alpha = 0.8$ implies a half life of 24 quarters. The news series are normalized by their own standard deviation. So, the scale is in units of standard deviations.
Notes: $EV^{lr}$ depicts the long-run forecast of aggregate volatility. Pol. uncertainty is the Baker, Bloom and Davis [2013] measure of political uncertainty.
Notes: Figure above shows the one period ahead forecast of the first and second moment of the log real excess stock market returns based on the VAR model. The figure below plots the conditional Sharpe ratio
\[
\frac{E[r_{t+1}]}{\sqrt{V_{t+1}}},
\]
Figure 3.6: CAPM and the 5-factor model

Notes: Figure shows the fit of the traditional CAPM on the left-hand side and the fit of the 5-factor model on the right-hand side. The five factors are the news on current excess market returns, on the future discount rates and on the variances and covariance of excess market returns and consumption growth. Sample 1952 Q4 - 2011 Q4.
3.7 Appendix B

3.7.1 An intertemporal CAPM with time-varying volatility

The representative household faces the following intertemporal budget constraint

\[ W_{t+1} = R_{m,t+1} (W_t - C_t) \]  \hspace{1cm} (3.31)

where the total wealth \( W \) includes human capital and financial assets. All of the household’s total wealth is assumed to be tradable. \( R_m \) is the gross return on the total invested wealth which I denote here as the return on the market portfolio. \( C \) denotes consumption.

Divide by \( W_t \) and take the log

\[ \Delta w_{t+1} = r_{m,t+1} + \log (1 - \exp (c_t - w_t)). \]  \hspace{1cm} (3.32)

If the \( x_t \equiv c_t - w_t \) is assumed stationary, one may log-linearize the aggregate budget constraint around the long-run mean \( x_0 \equiv c - w \). Define \( \rho \equiv 1 - \exp(x_0) \)

\[ \log (1 - \exp(x_t)) \approx \log(\rho) - \left(1 - \frac{1}{\rho}\right)x_0 + \left(1 - \frac{1}{\rho}\right)x_t \]
and insert into equation (3.32)

\[ \Delta w_{t+1} = r_{m,t+1} + \kappa + \left(1 - \frac{1}{\rho}\right)(c_t - w_t) \]

\[ \Delta c_{t+1} + (c_t - w_t) - (c_{t+1} - w_{t+1}) = r_{m,t+1} + \kappa + \left(1 - \frac{1}{\rho}\right)(c_t - w_t) \] \hspace{1cm} (3.33)

\[ c_t - w_t = \rho (r_{m,t+1} - \Delta c_{t+1}) + \rho (c_{t+1} - w_{t+1}) + \rho \kappa \]

which gives us the log-linearized budget constraint. Forward iteration yields

\[ c_t - w_t = \rho (r_{m,t+1} - \Delta c_{t+1}) + \ldots + \rho^n (r_{m,t+n} - \Delta c_{t+n}) + \rho^n (c_{t+n} - w_{t+n}) + \sum_{j=1}^{n} \rho^j \kappa. \]
Under the transversality condition: \( \lim_{n \to \infty} \rho^n (c_{t+n} - w_{t+n}) = 0 \), since \( \rho < 1 \)

\[ c_t - w_t = \sum_{j=1}^{\infty} \rho^j (r_{m,t+j} - \Delta c_{t+j}) + \frac{\rho_w}{1 - \rho_w} \kappa. \]

The log-linearized budget constraint takes a present value form which holds ex post as well as ex ante

\[ c_t - w_t = E_t \sum_{j=1}^{\infty} \rho^j (r_{m,t+j} - \Delta c_{t+j}) + \frac{\rho}{1 - \rho} \kappa \] (3.34)

where \( \rho = \frac{w - c}{w} \) is the steady-state investment rate, \( \kappa \) is a constant arising from the log-linearization and small letters denote variables in logarithm. Equation (3.34) shows that fluctuations in log consumption-wealth ratio must forecast changes in either future returns or consumption growth or both.

The representative agent has recursive utility introduced in Epstein and Zin (1989)

\[ U_t = \left\{ (1 - \delta) C_t^{-1/\gamma} + \delta \left( E_t U_{t+1}^{1-\gamma} \right)^{\frac{\theta}{\gamma}} \right\}^{1-\theta} \] (3.35)

where \( \theta = \frac{1 - \gamma}{1 - \psi} \), \( \gamma \) is the risk aversion parameter, \( \psi \) the intertemporal elasticity of substitution and \( \delta \) is the subjective discount factor. Under this form of budget constraint and Epstein-Zin preferences, the Euler equation for any asset \( i \) can be derived using dynamic programming arguments

\[ 1 = E_t \left\{ \delta \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\gamma}} \right\}^\theta \left\{ \frac{1}{R_{m,t+1}} \right\}^{1-\theta} R_{i,t+1}. \] (3.36)

I assume that the joint conditional distribution of asset returns and consumption is lognormal and heteroscedastic. Thus, I allow the variances and covariances of consumption growth and returns to change over time. The Euler equation in logarithm for the wealth portfolio under these assumptions is

\[ E_t \Delta c_{t+1} = \psi \log \delta + \psi E_t r_{m,t+1} + \frac{1}{2} \frac{\theta}{\psi} \text{Var}_t (\Delta c_{t+1} - \psi r_{m,t+1}). \] (3.37)

Combining this Euler equation (3.37) with the budget constraint in the present value form
(equation 3.34) gives a consumption function which makes the relation between consumption, future returns on the market portfolio and volatilities dependent on structural parameters

\[ c_t - w_t = E_t \sum_{j=1}^{\infty} \rho^j \left[ (1 - \psi) r_{m,t+j} - \frac{\theta}{2\psi} Var_t (\Delta c_{t+j} - \psi r_{m,t+j}) \right] + \mu \]  

(3.38)

where \( \mu \equiv \frac{\rho}{1-\rho} (\kappa - \psi \log \delta) \).

### 3.7.2 A general pricing equation

The basic pricing equation

\[ 1 = E_t [M_{t+1} R_{t+1}^f] \]  

(3.39)

\[ E_t [R_{t+1}^f] - R_{f,t} = -R_{f,t} \text{cov}_t (M_{t+1}, R_{t+1}^f). \]  

(3.40)

For \( x \sim N(\mu_x, \sigma_x) \), use the log-normal facts

\[ E_t [\exp(x)] = \exp \left[ E_t (x) + \frac{1}{2} \text{Var}_t (x) \right]. \]  

(3.41)

Thus, with \( m = \ln M \) and \( r = \ln R \) and under the assumption that \( m \) and \( r \) are jointly normally distributed

\[ E_t [\exp (m_{t+1} + r_{t+1}^f)] = 1 \]

\[ \ln \exp \left[ E_t (m_{t+1} + r_{t+1}^f) + \frac{1}{2} \text{Var}_t (m_{t+1} + r_{t+1}^f) \right] = 0 \]

\[ E_t (m_{t+1}) + E_t (r_{t+1}^f) + \frac{1}{2} \text{Var}_t (m_{t+1}) + \frac{1}{2} \text{Var}_t (r_{t+1}^f) + \text{Cov}_t (m_{t+1}, r_{t+1}^f) = 0. \]

For a riskless asset: \( \text{Var}_t (r_{t+1}^f) = 0 \) and \( \text{Cov}_t (m_{t+1}, r_{t+1}^f) = 0 \)

\[ r_{t+1}^f + E_t (m_{t+1}) + \frac{1}{2} \text{Var}_t (m_{t+1}) = 0. \]  

(3.42)
The risk premium for asset $i$ is then

$$E_t (r_{i,t+1}^f) - r_{i,t+1}^f + \frac{1}{2} \text{Var}_t (r_{i,t+1}^f) = -\text{Cov}_t (m_{t+1}, r_{i,t+1}^f).$$

(3.43)
3.8 Appendix C: Data source and definitions

Data on consumption, wealth and income are in real and per capita terms. The definition of the variables are the same as in Lettau and Ludvigson (2004).

Consumption is measured as consumption expenditure on nondurables and services minus clothing and shoes. Data is taken from the National Income and Product Accounts (NIPA) tables released by the Bureau of Economic Analysis (BEA). The quarterly series are seasonally adjusted and at annual rates. The real measures are chain-weighted. Thus real consumption of nondurables and services minus clothing and shoes is constructed using the “Fisher of Fishers” aggregation method and scaled up to match the mean of the total real personal consumption expenditures.

Labor income consists of wages and salaries + transfer payments + employer contributions for employee pensions and insurance - employee contributions for social insurance - taxes. My source is the NIPA Table 2.1 from the BEA.

Wealth. Data on household net worth is obtained from the Flow of Funds Account of the Federal Reserve System. Household net worth includes consumer durable goods.

Population is constructed by dividing total real disposable income by total real disposable income per capita. The source is NIPA table 2.1.

Price deflator. Consumption and labor income are deflated by the total personal consumption expenditure deflator. The deflator is a chain-type price index with the reference year 2005. Data is taken from the BEA.
Chapter 4

Horizon-dependent consumption risk in international equity returns
4.1 Motivation

I analyse how consumption risk is related to international stock returns over different investment horizons. Do short- and long-run investors face different risks? For that purpose, international equity returns are decomposed into a cash flow component, price components and exchange rate changes. The covariance risk of these payoff components with consumption is then tracked over different investment horizons. This approach allows us to study the structure of risk incorporated in the asset’s payoff and the risk-return relationship beyond one period.

I find that consumption risk in the cash flow component dominates in the long run and fails to explain the risk premia across assets in the short run. Conversely, price changes matter more over shorter horizons. This result suggests that short- and long-run investors do face different risks on international equity markets.

International asset pricing models typically relate differences in expected returns across countries to differences in the assets’ exposures to global risk factors. These models are valid in a world with perfect financial markets where the investment and consumption opportunity sets are the same for all investors. [Campbell and Hamao (1992) and Ferson and Harvey (1993) emphasize the view that in perfectly integrated financial markets, global macroeconomic risks would drive the time- and cross-sectional variation of expected returns. Nitschka (2010), following this chain of logic, shows that if markets are sufficiently integrated, national risk factors should be able to capture the cross-sectional variation in international risk premia and finds evidence for this argument by focusing on the European stock markets.

Based on this literature, the analysis is restricted to a sample of highly developed countries. I argue that their capital markets are highly integrated and therefore strongly driven by common risk factors. As a result, I employ an asset pricing model which prices international assets with a single consumption-based risk factor. I hereby take the perspective of a U.S. investor and focus on U.S. consumption risk. Although studies find that local factors such as exchange rate risk are important in driving cross-sectional dispersion of international returns, I argue that risk compensations at lower frequency are more affected by global factor. Lewis and Liu (2012) argue that the potential welfare gains from better international risk sharing is low, since their model implied correlation of persistent consumption risk is close to one. This suggests that
persistent consumption shocks are of global nature. Their finding supports the view that the financial market integration among the developed economies is high and long-run consumption risk is an important common factor driving the risk premia in international equity markets.

I build on Bansal, Dittmar and Kiku (2009) and extend their framework to an international setting. Motivated by the theoretical literature on the long-run risk explanation for variations in the risk premia, they focus on the cointegrating relation between dividends and consumption and argue that this relationship measures the amount of long-run consumption risk embodied in the asset’s cash flow component. They find that this measure of risk successfully accounts for cross-sectional variation in the U.S. equity risk premia at both short and long horizons. Shocks to the low frequency component of consumption are hard to identify empirically. The identification of the long-run risk in this paper is obtained by using cointegration. Bansal, Dittmar and Kiku (2009) present an economic theory which implies that dividends and consumption share a common trend. If this long-run equilibrium relationship exists, cointegration can be a useful device to uncover that economic structure and to identify the forces driving the cointegrated system in the short and the long run. Thus, cointegration is not only used to uncover the long-run risk exposure, but the identification of the long- and short-run dynamics also improves the predictions of the model.

Using a cointegration approach also allows me to contribute to the literature on dividend growth predictability by providing evidence that dividend growth is predictable by the dividend-consumption ratio for all the countries in the sample. A large empirical literature documents that the U.S. dividend growth is not predictable by the U.S. dividend-price ratio. Rangvid, Schmeling and Schrimpf (2012) show that this result is not robust in an international setting. Dividend growth rates of small and medium countries are predictable by their dividend yield.

I also contribute to the empirical literature on international asset pricing by focusing on short- and long-run consumption risks. The empirical literature focused extensively on the ability of the world CAPM and the international CAPM with exchange rate risk to explain the cross section of average equity returns. Dahlquist and Sallstrom (2002) evaluate both the unconditional and conditional versions of the models and find that national market returns are well captured by all of the models. Returns on portfolios sorted by value and size characteristics cannot be
explained by the unconditional world CAPM, whereas the conditional version of the model with foreign exchange risk explains a large part of the cross-sectional variations. Other studies also provide evidence that currency risk is priced and that the price of risk is varying over time (Dumas and Solnik (1995), Santis and Gerard (1997)). More recently, Brusa, Ramadorai and Verdelhan (2015) show that the dollar and the carry factor help to account for the variations in average international equity returns. The evidence suggests that, despite increased globalization, international asset prices continue to depend on both global and local risk factors. The empirical literature on the international consumption-based pricing models is limited due to measurement errors in international consumption data. Wheatley (1988) uses a consumption CAPM to test for international equity market integration and does not find conclusive evidence for the hypothesis that the consumption-based model holds internationally. Li and Zhong (2005) document that the unconditional international consumption-CAPM has some explanatory power for the cross section of equity returns, but fails to explain the risk-free rate. Consistent with this literature, I find that the model is rejected at the short horizon. But the model contains significant explanatory power in the limit, as the observed and the model-predicted risk premia are strongly and positively correlated. The difference in the performance of the model over the short and the long run suggests that local factors matter less in the long run. Local factors reflect differences in national capital markets such as the openness of the capital markets or currency risk due to own monetary and fiscal policy. My findings suggest that exchange rate risk is not priced in the long run and therefore does not contribute to explain the variations in risk premia. This implies that country-specific effects due to real exchange rate risk diminish over longer holding periods, since purchasing power parity (PPP) tends to hold in the long run. If PPP holds, there is no currency risk.

Since the common consumption-based risk factor in the model is proxied by the U.S. consumption, I provide an alternative statistical way to derive a common global factor. I apply the method proposed in Kasa (1992) to test for the number of cointegrating relations among five national stock markets and extract one common stochastic trend. I show that the common stochastic trend in international stock prices is reflected in the trend in their dividends. This indicates the

19 see Lewis (2011)
importance of cash flow risk in the long run. A comparison of the common trend components with U.S. consumption shows that both time series follow the same trend over the long horizon.

The paper is structured as follows. First, I present the theoretical and the econometric framework. Then, I briefly review the data and discuss the results on predictability, the pricing implications and the role of exchange rates. The last section concludes.

4.2 The theoretical framework

I build on the framework proposed by Bansal, Dittmar and Kiku (2009) to examine the riskiness of returns on international equity markets. The returns in local currency are expressed in U.S. dollar

\[
R_{USD,t+1}^{US} = R_{local}^{USD} E_{t+1}^{\text{USD}} E_t
\]

\[
= P_{t+1}^{local} + D_{t+1}^{local} E_{t+1}^{\text{USD}} E_t
\]

where \( E_t \) is the nominal exchange rate and reflects the price of the local currency in terms of U.S. dollar such that an increase in \( E_t \) constitutes a depreciation of the U.S. dollar relative to the local currency. The superscript local indicates that the variables are denominated in the currency of the respective country other than the U.S. Log-linearizing the returns on the international equity portfolios according to Campbell and Shiller (1988) yields the equation for the log return approximation\(^{20}\)

\[
r_{t+1}^{\text{USD}} - \pi_{t+1}^{\text{US}} = \kappa_0 + (\kappa_1 - 1) z_{t+1}^{\text{nom,local}} + \Delta z_{t+1}^{\text{nom,local}} + \Delta d_{t+1}^{\text{nom,local}} + \Delta e_{t+1}^{\text{USD}} - \pi_{t+1}^{\text{USD}}
\]

\[
= \kappa_0 + (\kappa_1 - 1) z_{t+1}^{\text{nom,local}} + \Delta z_{t+1}^{\text{nom,local}} + \Delta d_{t+1}^{\text{nom,local}} + \Delta d_{t+1}^{\text{real,USD}}.
\]

The returns are in real terms and denominated in a common currency, namely the U.S. dollar. The log real return can be approximated by the log price-dividend ratio \( z = p_t - d_t \), the change in it and the log real dividend growth. \( \pi_t^{\text{US}} \) is the U.S. inflation and the \( \kappa \)s are the log-linearization

\(^{20}\)The derivation of the Campbell and Shiller log return approximation is shown in the appendix. Further, small letters denote variables in log.
constants. *nom* indicates that the variable is in nominal terms. The advantage of using the log return approximation is that I can now easily decompose the return into a cash flow component and price components.

Since I am interested in the riskiness of returns and its components over different investment horizons, I compound the returns over $s$ horizons denoted by $r_{t+1 \rightarrow t+s} \equiv \sum_{j=1}^{s} r_{t+j}$. The compounded returns are characterized by the return components summed up over $s$ horizons $x_{t+1 \rightarrow t+s} \equiv \sum_{j=1}^{s} x_{t+j}$ where $x_{t+1} = \{z_{t+1}, \Delta z_{t+1}, \Delta d_{t+1}\}$. Putting the return components into an asset-pricing framework, one can determine what kind of risks matter at which horizon.

The basic pricing equation under the assumption of a representative agent which prices all the assets at any horizon $s$ can be written as

$$E_t [\exp (m_{t+1 \rightarrow t+s} + r_{t+1 \rightarrow t+s})] = 1.$$ (4.3)

The pricing equation states that the expected return on any asset is equal to one when discounted with the stochastic discount factor $m_t$. In case of perfectly integrated asset markets, this equation should also hold for international asset returns. Thus, if capital markets are sufficiently integrated, international equity returns should also be driven by national risk factors. I impose the perspective of a U.S.-investor and focus on U.S. consumption growth as the risk factor. The standard consumption-based log discount factor under power utility is given by $m_t = \log(\rho) - \gamma \Delta c_t$, where $\rho$ is the time preference parameter and $\gamma$ denotes the risk aversion parameter.

For empirical purpose, let me reformulate the pricing condition as an expected return-beta representation where the asset’s risk premium is determined by the asset’s exposure to the risk factor and the market price of risk. Since only shocks to the stochastic discount factor are priced, I define $\eta_{c,t+1} \equiv \Delta c_{t+1} - E_t (\Delta c_{t+1})$ to be the innovation in consumption growth over one period. Then, $\eta_{c,t+1 \rightarrow t+s}$ is defined as the shock to the $s$-period cumulative consumption growth and $\sigma_{c,s}^2$ denotes the variance of this $s$-period innovation. Similarly, $\eta_{r,t+1 \rightarrow t+s}$ refers to the innovation in the returns summed up over $s$ periods, $r_{t+1 \rightarrow t+s}$ and $\sigma_{r,s}^2$ denotes its conditional variance. Under the assumption that asset returns and consumption growth are conditionally jointly lognormal.
distributed and homoscedastic, the conditional risk premium at any horizon \(s\) is given by\(^{21}\)

\[
\frac{1}{s} E_t \left[ r_{t+1→t+s} + 0.5 \sigma_{r,t,s}^2 - sr_{t,s}^f \right] = \frac{1}{s} \gamma \sigma_{c,s}^2 \frac{\text{cov}(\eta_{c,t+1→t+s}, \eta_{r,t+1→t+s})}{\sigma_{c,s}^2} \tag{4.4}
\]

\[
E \left( \frac{1}{s} E_t \left[ r_{t+1→t+s} + 0.5 \sigma_{r,t,s}^2 \right] \right) = E \left( r_{f,t}^f \right) + \lambda_s \beta_s. \tag{4.5}
\]

The equation is divided by the investment horizon \(s\). Thus, the conditional risk premium per unit of time is determined by the horizon-dependent exposure to consumption risk and the market price of risk over the \(s\)-horizon. The price of risk itself is given by the risk aversion parameter and the variance of the \(s\)-period consumption growth.

Note that the conditional risk premium is also determined by the conditional variance of the \(s\)-horizon return \(\sigma_{r,s}^2\). Return predictability implies that this conditional variance per unit of time is not the same at all horizon \(s\) and therefore varies across investment horizon. But in the long run, predictable variations should vanish due to their temporary nature and as \(s\) goes to infinite, the return \(r_t\) follows a random walk process. Then, we know that the conditional variance of a random walk process is linearly increasing in time, such that \(\sigma_{r,s}^2 = s \sigma_r^2\), which indicates a linear time trend in the \(s\)-period risk premium in the long run.

I am now interested in the underlying structure of the overall risk compensation. That is, I want to determine how much of the consumption risk is reflected in the asset’s cash flow and price components. Take the log return approximation in equation (4.2), then the innovation in \(s\)-horizon return \(\eta_{r,t+1→t+s}\) is the sum of the three components \(\eta_{\Delta d,t+1→t+s}\), \(\eta_{\Delta z,t+1→t+s}\) and \(\eta_{c,t+1→t+s}\). Substituting \(\eta_{r,t+1→t+s}\) in equation (4.4) gives us

\[
\beta_s = \frac{\text{cov}(\eta_{c,t+1→t+s}, \eta_{\Delta d,t+1→t+s})}{\sigma_{c,s}^2} + \frac{\text{cov}(\eta_{c,t+1→t+s}, \eta_{\Delta z,t+1→t+s})}{\sigma_{c,s}^2} + \ldots \\
+ (\kappa_1 - 1) \frac{\text{cov}(\eta_{c,t+1→t+s}, \eta_{z,t+1→t+s})}{\sigma_{c,s}^2}. \tag{4.6}
\]

The asset’s overall exposure \(\beta_s\) is composed of the exposure of the cash flow component and the capital gain components to consumption risk such that

\[
\beta_s = \beta_{\Delta d,s} + \beta_{\Delta z,s} + (\kappa_1 - 1) \beta_z,s. \tag{4.7}
\]

\(^{21}\)The derivation of the beta representation from a general pricing equation is given in the appendix of chapter three.
The asset’s exposure to consumption risk is dependent on the investment horizon $s$. Therefore, the risk exposure and the risk compensation should be different for short- and long-run investors. Although the cash flow beta $\beta_{\Delta d,s}$ and the price level beta $\beta_{z,s}$ are relevant for both the short- and the long-run investors, the risk compensation for short-run investors may additionally be affected by shocks to changes in the price-dividend ratio. The underlying intuition for that channel is simple. At shorter horizons, investors care more about transitory price fluctuation. As the horizon $s$ increases, the exposure of transitory movements in prices vanishes and the exposure of the cash flow component to consumption risk becomes more dominant. In the limit, $\beta_{\Delta z,s}$ goes to zero as the variance of the cumulative consumption growth $\sigma_{c,s}^2$ dominates the covariance between consumption growth and changes in the stationary variable $z_t$. The asset beta for a long run investor should mainly be determined by the risk in the cash flow component. The effect of the price level beta should be negligible as $(\kappa_1 - 1)$ is close to zero.\footnote{$\kappa_1$ is between 0.96 and 0.98 in the data.}

$$
\lim_{s \to \infty} \beta_{s} = \beta_{d} + (\kappa_1 - 1)\beta_{z,s}.
$$

(4.8)

The cash flow beta in the limit $\beta_{d}$ is nothing else than the cointegration parameter between consumption and dividends. Thus, the theory implies a cointegration restriction, which motivates a cointegration framework to identify the structure underlying the overall risk compensation. More specifically, I can use cointegration as an identification device to uncover long run consumption risk in dividends.

### 4.3 Econometric framework

To estimate the horizon-dependent risk exposures, I model the joint dynamics of the variables involved as a vector autoregressive process of order one. The recursive structure of the VAR allows to compute the covariance matrix and the betas over different horizons.

The theory derived in the previous section predicts that log dividends and log consumption are cointegrated. Therefore, I start by estimating the cointegration parameter $\beta_{d}$ using the following...
cointegration specification

\[ d_t = \tau_0 + \beta_d c_t + \varepsilon_{d,t}. \]  

(4.9)

If dividends and consumption are cointegrated, the cointegrating residual \( \varepsilon_{d,t} \) has significant predictive power for future consumption or dividend. I include this error term in the first-order VAR

\[ X_t = BX_{t-1} + u_t \]  

(4.10)

with \( X_t' = [\Delta c_t, \varepsilon_{d,t}, \Delta d_t, \Delta z_t] \) and \( u_t' = [u_{c,t}, u_{d,t}, u_{\Delta d,t}, u_{\Delta z,t}] \). All variables are demeaned. From the VAR, I can compute the variances and covariances of the shocks to the cumulative consumption growth and shocks to the sum of the return components over \( s \) periods. The \( s \)-period variance-covariance matrix \( \Sigma_s \) is constructed according to the following formula which is derived in the appendix

\[ \Sigma_s = \frac{1}{s} \left( C_s \Sigma_u C_s' \right) + \left( \frac{s-1}{s} \right) \Sigma_{s-1}. \]  

(4.11)

\( \Sigma_u \) is the variance-covariance matrix of the error term \( u_t \) and \( C_k = C_{k-1} + B^{k-1} \) for \( k = 1, \ldots, s \) and \( C_0 = 0 \). In the limit, the long-run covariance matrix is given by

\[ \Sigma_{lr} = (I - B)^{-1} \Sigma_u (I - B)^{-1}'. \]  

(4.12)

Now, it is straightforward to compute the horizon-dependent betas. The decomposition of the asset’s overall risk exposure at any horizon \( s \) as in equation (4.7) is constructed as

\[ \beta_s = \frac{\Sigma_s(1,4)}{\Sigma_s(1,1)} + \frac{\Sigma_s(1,5)}{\Sigma_s(1,1)} + (\kappa_s - 1) \frac{\Sigma_s(1,3)}{\Sigma_s(1,1)} \]  

(4.13)

where \( \Sigma(i, j) \) is the covariance between variable \( i \) and \( j \) in the \( X \) vector.
4.4 Data

I use stock market data and macroeconomic data from 17 developed countries. The countries are: Australia, Austria, Belgium, Canada, Denmark, France, Germany, Italy, Japan, Netherlands, Norway, Singapore, Spain, Sweden, Switzerland, United Kingdom and United States. I employ an annual frequency. The country selection is based on data availability, i.e. only developed economies as defined by the Morgan Stanley Capital International (MSCI) for which stock market data over the full sample period are available, are chosen. I use the MSCI price indices \( P_l \) and the MSCI total return indices \( R_l \) for each country expressed in local currency and in U.S. dollar. \( P_l \) measures the market price performance of the portfolio and \( R_l \) measures the price performance and income from regular cash distributions which are reinvested. The dividend series and the price-dividend ratios can then be extracted from the price indices and total return indices. I follow Campbell (2003) to construct these series. From the observed price index \( P_l \) and the total return index \( R_l \), the dividend yield for each portfolio is computed as

\[
\frac{D_t}{P_t} = \frac{R_{l_t}}{R_{l_{t-1}}} \frac{P_{l_{t-1}}}{P_{l_t}} - 1 = \left( \frac{P_t + D_t}{P_{t-1}} \right) \frac{P_{t-1}}{P_t} - 1
\]  

(4.14)

and the dividend series \( D_t \) is

\[
D_t = \frac{D_t}{P_t} P_t
\]  

(4.15)

For each country, I take either the stock market portfolio or the MSCI value and MSCI growth portfolio as test assets. The sample period for the market indices covers 1971 to 2014 and for the value and growth indices, it runs from 1976 to 2014. I compare the characteristics of the constructed dividend series with those reported in Rangvid, Schmeling and Schrimpf (2012). The close fit of the characteristics makes me comfortable in using the constructed data on dividends and dividend yields. The portfolio returns and dividend series are deflated using the U.S. consumer price index taken from the Bureau of Economic Analysis (BEA). U.S. consumption of

\[\text{Cash payments to shareholders by repurchasing shares are not included in the traditional per share dividends. But they do affect future dividend per share over the lower share base.}\]

\[\text{The value and growth indices are constructed by MSCI using eight historical and forward-looking variables for every security. Each security is placed into either the value or the growth index which targets 50% of the underlying market index. The variables are book-to-market ratio, 12-month forward earnings-to-price ratio, dividend yield, long-term and short-term forward earnings per share ratio, current internal growth rate, long-term historical earnings per share and sales per share growth trend.}\]
nondurables and services per capita is used as a risk measure and is also taken from the BEA.

Table (4.1) shows the summary statistics for the log real returns and log real dividend growth rates of each portfolio denominated in U.S. dollar. The average market return across countries is 6.5 percent and the average value premium is 2.5 percent. This relatively low average premium is due to the fact that the growth and the value portfolio each target 50 percent of the market. Further, the cross-sectional variation of the average returns are quite high. The difference between the minimum and maximum average returns is 7.7 percent for the market portfolios, 6.8 percent for value portfolios and 8.6 percent for growth portfolios. The average log real dividend growth is around 2 percent. Figure (4.2) plots the mean log real excess returns on the value and growth portfolios against each other and thus visualizes the value premium in each country. The excess returns are all denominated in U.S. dollar and in excess of the U.S. risk-free rate. All the countries except for Denmark have a positive value premium. As for the risk-free rate, I take the 3-month treasury bill rates. Data on gross foreign assets relative to GDP as a measure of financial openness are taken from Lane and Milesi-Ferretti (2006).

4.5 Results

4.5.1 Cointegration results

Tables (4.2) and (4.3) present the results from two different cointegration tests. The trace test based on Johansen (1991) is a likelihood ratio test to determine the number of cointegrating vectors. If the null hypothesis of \( r = 0 \) cointegrating vector is rejected, there is at least one cointegrating vector. The null of \( r = 1 \) should not be rejected for a system with two variables to be cointegrated. I consider here a specification with an intercept in the cointegrating vector. The Engle and Granger (1987) test runs a unit root test on the residuals of the regression in equation (4.9). If the null of a unit root in the residuals is rejected, one has evidence for the presence of cointegration. I show the test statistics along with the 90-percent critical values. The results from the Johansen test suggest that U.S. consumption is cointegrated with dividends on the market-, value- and growth-portfolios of most countries. The Engle and Granger unit root test provides evidence for cointegration for only one-third of the portfolios. Due to the low power of this test,
I interpret my findings in favor of cointegration for most portfolios.

### 4.5.2 Predictability results

There is a large literature which documents that variations in the U.S. dividend yield is due to variations in expected returns and cannot be explained by expected dividend growth. The lack of dividend growth predictability by the dividend yield in the U.S. does not mean that dividend growth rates are not predictable at all. Lettau and Ludvigson (2005a) find that U.S. dividends are predictable by a proxy for consumption-dividend-income ratio. Rangvid, Schmeling and Schrimpf (2012) show in an international setting that dividend growth rates are highly predictable by the dividend-price ratio in small and medium-sized countries. I contribute to this literature by providing evidence that dividend growth is predictable by the dividend-consumption ratio for all the countries in the sample.

If dividends and the U.S. consumption are cointegrated, then deviations from the common stochastic trend must be restored by subsequent changes in dividends, consumption or both. I estimate a vector error correction model to find out which variable drives the adjustment process. The model takes the following form

\[
\Delta X_{t+1} = \Gamma_0 + \Gamma_1 \Delta X_t + \alpha \beta' X_t + \varepsilon_t
\]  

(4.16)

where \(X_t = [d_{it}, c_{US,t}]\) and \(i\) represents the test asset. The cointegration error \(\beta' X_t\) gives the current deviation from the joint trend and the adjustment vector \(\alpha\) shows the speed of convergence towards the equilibrium. Table (4.6) and table (4.8) show the point estimates of the adjustment coefficients \(\alpha\) and their t-statistics. The coefficients in the consumption growth equation are all close to zero with only few estimates significantly different from zero. In contrast, the coefficients in the dividend growth equation are all negative, much higher in size and mostly significant. The results clearly show that dividends bring the system back to equilibrium, whereas consumption does not contribute to the error correction mechanism. Thus, the cointegrating residual has predictive power for future dividend growth.

To test the predictive power of the error correction term over different horizons, I run long-horizon predictive regressions of future changes in log dividend and log U.S. consumption on the
current cointegrating residual $\varepsilon_{i,d,t}$ from equation (4.9)

$$
\frac{1}{K} \sum_{j=1}^{K=10} \Delta x_{i,t+j} = \alpha_K + \beta_K \varepsilon_{i,d,t} + \eta_{i,t+K}
$$

(4.17)

where $\Delta x_{i,t} = \{\Delta d_{i,t}, \Delta c_{US,t}\}$ and $i$ indicates the market portfolio of country $i$. Table (4.7) presents the results for dividend growth predictability in five major countries. The regression coefficients are negative and significant at longer forecast horizons. The error correction term predicts dividend growth over longer horizons. U.S. consumption growth is not predictable from the error correction term. Although not reported, the results hold for all the countries in the sample. These results are in line with the evidence from the error correction model.

4.5.3 Cross-sectional analysis

I now explore the cross-sectional pricing implications. I analyse how much of the cross-sectional variations in average excess returns is explained by the exposures of return components to consumption risk at various investment horizons. In figure (4.2) and figure (4.3), I plot the cointegration parameter $\beta_d$ of each portfolio against its mean excess returns. The exposure of the cash flow component to consumption risk in the limit is positively related to the mean excess returns. The fitted line shows that one additional unit of long-run risk exposure in dividends translates into 1.3 percent excess returns on average. This relationship is observable among country portfolios as well as among value and growth portfolios. This finding emphasizes the importance of the cash flow component for long-run investors.

Do short- and long-run investors face different risks? To answer this question, I plot the observed risk premia versus the consumption risk premia reflected in the different return components over the short and the long run. Figure (4.4) plots the observed mean excess returns against the returns fitted by the cash flow beta. The first panel shows the plot over the horizon of one year, whereas the second plot considers investments over an infinite horizon. I find that at a short horizon, risk in the cash flow component does not account for the cross-sectional variation. But as the horizon increases, cash flow considerations become more important in determining the risk compensations for the investors. In the limit, the fitted and the observed returns are scattered close along the forty-five degree line.
The figures display variations in mean returns not only across countries but also across the different styles of portfolios. Considering the divergence between the value and growth stocks, table (4.9) offers an explanation. On average, value portfolios have higher long-run cash flow betas than growth portfolios which means that the cash distributions of value firms are more closely and positively related in the long run to consumption. The value premium can be explained by this higher long-run consumption risk exposure in the cash flow component of value stocks. Thus, consumption risk in cash flow determines the risk compensations for long-run investors.

How much does covariance risk in capital gains help in explaining the cross-sectional pattern? Figure (4.5) and figure (4.6) suggest that in the long run, risks in price changes are not relevant at all. There is no comovement between price changes and consumption over a long horizon. The long-run capital-gains betas are in line with the theoretical prediction and concentrated around zero. However, over the investment horizon of one year, there is a large variation in the price change betas across countries. Although those betas do not help much to explain the overall cross-sectional variation, they do have some explanatory power among the growth portfolios. Similarly, as illustrated in figure (4.7), the contribution of fluctuations in the level of price-dividend ratio $z_t$ to the risk compensations is slightly higher at shorter horizons. Thus, transitory risks in price changes matter more at short horizons and die out in the long run.

I add up all the components and determine the assets’ overall exposures. The cross-sectional fit of the overall asset betas are depicted in figure (4.8). At longer horizons, the model is able to explain a considerable amount of the variations across countries as well as across portfolios sorted on value characteristics. The results suggest that risk compensations in international equity market are dominated by consumption risk in cash flows in the long run. Whereas transitory risks reflected in price changes are more important in the short run. Table (4.10) reports the estimates from the cross-sectional pricing regressions. It shows that the model does not perform well in the short run. But in the long run, equity returns are positively related to risk premia associated with consumption. The price of long-run consumption risk is positive ($\lambda_1 = 1.01$ percent) and significantly different from zero (t-statistic = 2.81). The adjusted $R^2$ of the model is 21 percent. The average zero-beta rate $\lambda_0$ is 5.44 percent. The model implied risk-free rate is therefore much higher than e.g. the U.S. risk-free rate of 1.4 percent. The inability of the model to account for
the risk-free rate in the data is consistent with the findings in the literature.

The price of risk in figure (4.10) is increasing over investment horizon. That means people are more averse to risks related to the low frequency movements due to their persistent nature. This finding is consistent with the long-run risk models, which imply that the price of long-run consumption risk is much higher than that of contemporaneous consumption risk.

Figure (4.9) further shows that my risk factor can jointly account for cross-sectional variations in returns on international value and growth portfolios as well as on the twenty-five national Fama and French portfolios sorted by size and value characteristics. Again, the explanatory power in the long run is driven by the cash flow component.

4.5.4 The role of exchange rates

Within the limits of my framework, I can separate the real exchange rate channel and model it as an additional risk factor or as a separate payoff component.\(^{25}\) First, consider exchange rate changes as a separate payoff component from investment in foreign assets. Then, my theoretical framework motivates a cointegration procedure to uncover long-run consumption risk in exchange rates, which provides a direct link between exchange rate movements and a fundamental macroeconomic variable. This approach can be motivated by the literature on the risk-based explanations for currency returns. Early studies following Meese and Rogoff (1983) find that macroeconomic fundamentals do not help to predict exchange rates. As a result, people perceived exchange rate fluctuations as random. More recent attempts to link exchange rates to economically motivated risk factors reveal that exchange rate movements are not random, but contain a systematic component. Verdelhan (2012) shows that the systematic variations in bilateral exchange rates can be accounted for by two common currency risk factors, the dollar and the carry factor.\(^{26}\) These risk factors, which can be related to downside or volatility risk, are priced in the currency and the equity market (Lustig, Roussanov and Verdelhan (2011), Brusa, Ramadorai and Verdelhan (2015), Menkhoff et al. (2012)). Verdelhan (2012) finds that the share of systematic variations in bilateral exchange rates uncovered by these factors are large among

\(^{25}\)Focusing on nominal exchange rates instead of real exchange rates does not significantly change the results.

\(^{26}\)The dollar factor reflects the cross-sectional average of changes in all exchange rates relative to the U.S. dollar and the carry factor reflects average changes in exchange rates between the high and low interest rate currencies.
developed countries, which suggests that exchange rate fluctuations can be significantly determined by exposures to the global shocks. He argues that a higher share of systematic variations indicates higher importance of global versus local shocks due to the global nature of the common factors and therefore also indicates higher market integration. In the long-run risk models, the low frequency component in consumption is a natural candidate for the common global factor. Colacito and Croce (2011, 2013) show that long-run risk models can reproduce several international finance anomalies, once the long-run consumption risk components are allowed to be highly correlated across countries.\footnote{Colacito and Croce (2011) address the anomaly exposed by Brandt, Cochrane and Santa-Clara (2006) that consumption growth do not covary enough to explain the observed exchange rate volatility. Colacito and Croce (2013) simultaneously generate the lack of correlation between exchange rate changes and consumption growth differentials (Backus and Smith (1993) puzzle) and the tendency of high interest rate currencies to appreciate (UIP puzzle).}

Their model provides a link between exchange rates and long-run growth perspectives.

Next, I study whether the cointegration framework is appropriate to uncover a link between exchange rates and a long-run component. To formalize the intuition, let us rewrite the return equation (4.2) and introduce a separate real exchange rate component \( \Delta q_{t+1} = \Delta e_{t+1} + \pi_{t+1}^{local} - \pi_{t+1}^{US} \), such that

\[
\Delta q_{t+1} = \Delta e_{t+1} + \pi_{t+1}^{local} - \pi_{t+1}^{US},
\]

(4.18)

This specification allows us to disentangle the real exchange rate channel from the cash flow component which is now denominated in local currency. Following the procedure outlined in the theory section, the asset’s overall exposure is determined by

\[
\beta_s = \beta_{d,s} + \beta_{z,s} + (\kappa_1 - 1) \beta_{z,s} + \beta_{q,s}.
\]

(4.19)

In the limit, the asset betas are given by

\[
\lim_{s \to m} \beta_s = \beta_d + (\kappa_1 - 1) \beta_{z,s} + \beta_q.
\]

(4.20)

\( \beta_q \) provides an estimate of the exposure in real exchange rate to the long-run consumption
risks. Figure 4.11 depicts how $\beta_q$ differs across countries and whether it contributes to the cross-sectional variations in equity returns. The negative relation is due to few outliers. Among the rest of the test assets, there is no systematic relation between the exposures in exchange rate and the risk premia. A similar explanation as for $\Delta z$ could apply here. Risks associated with real exchange rates are more of temporary nature and die out in the long run.

Second, consider the case where the exchange rate component in the model takes the role of a risk factor. If exchange rate risk matters for the pricing of equities, international investors are compensated in equilibrium for bearing the risk of lower income on foreign investments due to adverse movements in exchange rates. I introduce this additional risk factor in the model. From a pricing perspective, log changes in real exchange rates reflect differences in log stochastic discount factors of the home and foreign country under the assumption of complete markets

$$m_{t+1}^{local} = m_{t+1}^{US} + \Delta q_{t+1}$$ (4.21)

where I take the U.S. as the home country with $m_{t+1}^{US}$ denominated in U.S. dollar and $m_{t+1}^{local}$ is the foreign pricing kernel in terms of local currency. Then, any risky asset is priced according to the following pricing equation

$$E_t \left[ \exp \left( m_{t+1}^{US} + \Delta q_{t+1} + r_{t+1}^{local} \right) \right] = 1$$ (4.22)

and the risk premium on the asset is determined by its covariance risk with the home pricing kernel and the exchange rate

$$E_t \left[ r_{t+1}^{local} + 0.5 \sigma_r^2 - r_t \right] = -\text{cov}_t \left( m_{t+1}^{US}, r_{t+1}^{local} \right) - \text{cov}_t \left( \Delta q_{t+1}, r_{t+1}^{local} \right)$$ (4.23)

$$= \gamma \text{cov}_t \left( \Delta c_{t+1}^{US}, r_{t+1}^{local} \right) - \text{cov}_t \left( \Delta q_{t+1}, r_{t+1}^{local} \right).$$ (4.24)

Summing up over $s$ horizon, the risk premium can be characterized as follows

$$E \left( \frac{1}{s} E_t \left[ r_{t+1-s}^{local} + 0.5 \sigma_r^2 - r_t \right] \right) = \frac{1}{s} \left[ \gamma \text{cov} \left( \eta_{t+1-s}, \eta_{t+1-s} \right) - \text{cov} \left( \eta_{t+1-s}, \eta_{t+1-s} \right) \right].$$ (4.25)
The risk compensation for a long-run investor is potentially determined by the cointegration parameter between dividends, consumption and exchange rates. For more than half of the test assets, these three variables seem to be cointegrated with at least one cointegrating vector as shown in table (4.5). I interpret the beta estimates from the cointegrating regression as long-run risk exposures and plot the long-run exposure in dividends to exchange rate risk against the mean excess returns in figure (4.12). Although the negative relation in the picture is consistent with the model implication in equation (4.25), the relation is weak and imprecisely measured due to the lack of cointegration for some test assets. I therefore conclude that exchange rate risk is not priced in the long run, which is consistent with the view that purchasing power parity tends to hold in the long run. The country-specific effects matter less over longer holding periods.

4.5.5 Financial integration and risk exposure

Since the long-run risk exposures in dividends are crucial in explaining the cross section of international risk premia, a natural question to ask is whether the exposures to the risk factor are related to some country-specific characteristics. A potential determinant of the exposure is the extent of financial integration. Higher financial integration will likely increase the country’s exposure to a common risk factor. I test this intuition by estimating a panel cointegrating regression of the following form

\[ d_{i,j,t} = \tau_0 + \delta c_t + \gamma_j + \theta_t + \epsilon_{i,j,t} \]  

(4.26)

with country-fixed effects \( \gamma_j \) and time-fixed effects \( \theta_t \). \( c_t \) is U.S. consumption and \( d_{i,j,t} \) is dividend on portfolio \( i \) in country \( j \) at time \( t \). \( i \) indicates a country’s value or a growth portfolio. As a measure of financial integration, I take gross foreign assets \( GFA_{j,t} \) relative to \( GDP_{j,t} \) and interact it with the risk factor. I introduce the interaction term by imposing the following structure on the coefficient \( \delta \)

\[ \delta = \delta_0 + \delta_1 X_{j,t} \]  

(4.27)

where \( X_{j,t} = \log \left( \frac{GFA_{j,t}}{GDP_{j,t}} \right) - \log \left( \frac{GFA_{t}}{GDP_{t}} \right) \). \( \frac{GFA}{GDP} \) is the cross-country average at time \( t \). \( \delta_1 \) tells us whether the exposure to the global factor increases with financial integration. Table (4.11) provides the results for different regression specifications. The first model specification shows
the cointegrating parameter between dividends and consumption from the pooled regression. Including the interaction variable and the country-fixed effects do not alter the cointegration coefficient. The coefficient on the interaction term is slightly positive and significant. Therefore, more financial integration does increase the country’s exposure to the common factor. This effect remains if I additionally control for country- and time-fixed effects, but it is not robust to the inclusion of the separate effects of the openness measure on dividends.

4.6 An alternative risk factor: Common stochastic trends in international stock markets

My results suggest that risk premia on country’s equity portfolios are in the long run driven by the exposures of the cash flow component to a common risk factor. In order to check the validation of this finding, I apply an alternative approach and build on an idea formulated in an early study by Kasa (1992). He was interested in the question whether the random walk components in national stock market indices are different or whether they respond to a single global factor. He therefore tests for common stochastic trends among national stock markets. Based on Kasa (1992), I test for the number of common stochastic trends among the price indices of five national stock markets and extract the common trends in their price indices and dividends. If mainly risks in the cash flow component matter in the long run, then common stochastic trends in international stock markets should be reflected in the common trends in their dividend payments. The five stock markets are: Canada, Germany, Japan, United Kingdom and United States.

Using the estimated cointegrating vector $\hat{\beta}$, the common stochastic trend in the five national stock price indices, given by $X_t$, can be extracted as:

$$
X_t = \beta_\perp (\beta_\perp \beta_\perp)^{-1} \beta_\perp X_t + \beta (\beta' \beta)^{-1} \beta' X_t \quad (4.28)
$$

where the first component on the right-hand side of the equation is the common stochastic trend and the second part is the stationary component. The intuition for identifying the trend component is simple. Since a linear combination of variables in $X_t$ is stationary in the space spanned by $X_t$, I also run a Johansen-type decomposition to extract common trends. The results are very similar.
\( \beta \), the trend component of \( X_t \) moves along the space spanned by \( \beta_\perp \) which is the orthogonal complement. The common trend is the weighted average of the five markets given by \( (\beta_\perp' \beta_\perp)^{-1} \beta_\perp' X_t \). The term \( (\beta_\perp' \beta_\perp)^{-1} \beta_\perp' \) gives relative market weights and \( \beta_\perp \) gives the factor loadings of each market to the common trend.

In table (4.12), I test for the number of cointegrating relationships among the five equity markets. If I impose a lag of five years in the test specification, the results indicate four cointegrating relationships which translates into one common stochastic trend. I motivate the choice of the lag length economically. Evidence from the literature on return predictability suggest that there are mean reversion in stock prices typically over the business cycle frequency or longer. I therefore impose a VAR of higher order to capture that mean reversion property.

Figure (4.14) plots the common stochastic trends in prices and in dividends along with the U.S. consumption. It shows that U.S. consumption and the common trend in the cash flow component move closely together in the long run. I interpret this result as a justification for using U.S. consumption as a global risk factor.

4.7 Conclusion

I analyse the riskiness of international equity returns over different investment horizons. I use a consumption-based risk factor to rationalize the risk compensations observed on international equity markets. My findings imply that the risk premia in the long run are mostly determined by consumption risk in the cash flow component. Whereas, transitory risks in the price component and exchange rate do not contribute to risk compensations in the long run but matter more at shorter horizons.

I study whether the cross-country differences in the long-run consumption risk exposures in dividends are related to a country-specific characteristic such as the financial market openness. The results indicate a positive relation between financial integration and risk exposure. But the results are not robust when accounted for the separate effects of financial integration.

I assume that asset markets are sufficiently integrated. Then, national risk factors such as U.S. consumption have explanatory power on international market and can be treated as a common risk factor. As a way to check the plausibility of this line of argument, I use a statistical approach
to extract a common factor. I estimate a common stochastic trend in international stock prices and show that the trend in prices is reflected in their dividends. A comparison of the stochastic trends with U.S. consumption shows that there is a strong comovement over the long run. So, the use of consumption as a global risk factor can be justified at least over long horizons.
4.8 Appendix A: Tables and Figures

Table 4.1: Summary statistics

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Notes: Table shows the mean and the standard deviation of the log real returns and the log real dividend growth rates of the market, value and growth portfolios for each country. The returns are denominated in U.S. dollar, deflated by U.S. CPI and at annual frequency. Sample period covers 1971 – 2014 for the market portfolios and 1976 – 2014 for the value and growth portfolios.
Table 4.2: Cointegration tests based on countries’ stock market indices

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</table>

Notes: Table shows the test statistics from the Johansen and Engle-Granger cointegration tests. The critical values at the 10% significance level are also given. For each test asset, I test the following cointegration specification: $d_t = \tau_0 + \beta d_{US,t} + \epsilon_t$. Test assets are the countries’ market portfolios. A star * under the trace test indicates that the null of $r$ cointegrating vectors can be rejected at 10% significance level. The starred values for the Engle-Granger test indicate that the null of a unit root in the residual $\epsilon_t$ can be rejected at 10% significance level suggesting the presence of a cointegrating relationship. Sample period: 1971 - 2014.
Table 4.3: Cointegration tests based on countries’ value and growth portfolios

<table>
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<th>Growth portfolios</th>
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<td>Trace test</td>
<td>Engle-Granger</td>
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<td>$r = 0$</td>
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<td>17.79</td>
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<td>20.16*</td>
</tr>
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<td>5.43</td>
<td>-3.17*</td>
<td>14.03</td>
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</table>

Notes: Table shows the test statistics from the Johansen and Engle-Granger Cointegration tests. The critical values at the 10% significance level are also given. For each test asset, I test the following cointegration specification: $d_t = \tau_0 + \beta_{DCUS, t} + \epsilon_t^d$. Test assets are the countries’ value and growth portfolios. The stars * under the trace test indicate that the null of $r$ cointegrating vectors can be rejected at 10% significance level. The starred values for the Engle-Granger test indicate that the null of a unit root in the residual $\epsilon_t^d$ can be rejected at 10% significance level suggesting the presence of a cointegrating relationship. Sample period: 1976 - 2014.
Table 4.4: Test for cointegration between $q_t$ and $c_t$

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<td>9.21*</td>
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<td>8.66*</td>
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</table>

Notes: Table shows the test statistics from the Johansen cointegration test. The critical values at the 10% significance level are also given. For each country, I test the following cointegration specification: $q_t = \tau_0 + \beta_d c_{U,S} + \epsilon_t$. $q_t$ is the log bilateral real exchange rate of a country relative to the U.S. A star * indicates that the null of $r$ cointegrating vectors can be rejected at 10% significance level. Sample period: 1976 - 2014.
Table 4.5: Test for cointegration between $d_t$, $q_t$ and $c_t$

<table>
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<th>Value portfolios</th>
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<th></th>
<th>Growth portfolios</th>
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<td>$r = 0$</td>
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<td>17.79</td>
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<td>37.68*</td>
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<td>7.78*</td>
<td>35.11*</td>
<td>18.74*</td>
<td>7.21</td>
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</tbody>
</table>

Notes: Cointegration specification is $d_t = \tau_0 + \beta_q q_t + \beta_d c_{US,t} + \epsilon_t$. Log dividends $d_t$ is in real terms and denominated in local currency. $q_t$ denotes log real bilateral exchange rates relative to the U.S. where an increase in $q_t$ indicates a real depreciation of the U.S. dollar. Test assets are the countries’ value and growth portfolios. * indicates that the null of $r$ cointegrating vectors can be rejected at 10% significance level. Sample period: 1976 - 2014.
Table 4.6: Predictability results for stock market portfolios

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<th>alphas</th>
<th>t-stats</th>
<th>t-stats</th>
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<td>Δc</td>
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<td>-0.01**</td>
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<td>-0.01**</td>
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Notes: Table shows the adjustment coefficients $\alpha$ from the VECM-Model $\Delta X_{t+1} = \Gamma_0 + \Gamma_1 \Delta X_t + \alpha \beta' X_t + \varepsilon$, where $X_t = [d_t, c_{t,S_t}]$. The model is estimated for each country’s market portfolio. t-stats indicate the t-statistics. The stars *, ** indicate that the coefficients are significantly different from zero at the 10%, respectively at the 5% significance level.
Table 4.7: Long-horizon forecast regressions

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<th>9</th>
<th>10</th>
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<td>-0.15</td>
<td>-0.11</td>
<td>-0.08</td>
<td>-0.12</td>
<td>-0.10</td>
<td>-0.14</td>
<td>-0.12</td>
</tr>
<tr>
<td></td>
<td>tstat</td>
<td>-3.49</td>
<td>-1.82</td>
<td>-1.68</td>
<td>-1.62</td>
<td>-1.48</td>
<td>-2.52</td>
<td>-2.73</td>
<td>-5.19</td>
<td>-6.62</td>
</tr>
<tr>
<td></td>
<td>$R^2_{adj}$</td>
<td>0.17</td>
<td>0.06</td>
<td>0.04</td>
<td>0.04</td>
<td>0.03</td>
<td>0.12</td>
<td>0.13</td>
<td>0.31</td>
<td>0.40</td>
</tr>
</tbody>
</table>

Notes: Table shows the results from long-horizon forecast regressions for five major countries where $\varepsilon_{d,t}^i$ denotes the residual from the cointegrating regression of market portfolios $i$’s dividend on U.S. consumption: $d_{i,t} = \tau_0 + \beta_i d_{c,t}^US + \varepsilon_{d,t}^i$. Forecast horizon K is in years. Sample 1971 - 2014. T-statistics are computed using Newey-West standard errors with $K - 1$ lags. The stars *, ** indicate that the coefficients are significantly different from zero at the 10%, respectively at the 5% significance level.
<table>
<thead>
<tr>
<th>Country</th>
<th>α (value)</th>
<th>t-value (value)</th>
<th>α (growth)</th>
<th>t-value (growth)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUS</td>
<td>-0.42**</td>
<td>-2.99</td>
<td>-0.25</td>
<td>-1.43</td>
</tr>
<tr>
<td>AUT</td>
<td>-0.38**</td>
<td>-2.32</td>
<td>-0.28</td>
<td>-1.50</td>
</tr>
<tr>
<td>BEL</td>
<td>-0.30**</td>
<td>-2.30</td>
<td>-0.35</td>
<td>-2.44</td>
</tr>
<tr>
<td>CAN</td>
<td>-0.09</td>
<td>-0.77</td>
<td>-0.38</td>
<td>0.00</td>
</tr>
<tr>
<td>DEN</td>
<td>-0.32**</td>
<td>-2.62</td>
<td>-0.29</td>
<td>-1.89</td>
</tr>
<tr>
<td>FRA</td>
<td>-0.26*</td>
<td>-1.97</td>
<td>-0.53</td>
<td>-3.09</td>
</tr>
<tr>
<td>GER</td>
<td>-0.46**</td>
<td>-3.17</td>
<td>-0.47</td>
<td>-2.38</td>
</tr>
<tr>
<td>ITA</td>
<td>-0.24**</td>
<td>-2.27</td>
<td>-0.07</td>
<td>0.00</td>
</tr>
<tr>
<td>JPN</td>
<td>-0.39**</td>
<td>-3.01</td>
<td>-0.27</td>
<td>-1.75</td>
</tr>
<tr>
<td>NED</td>
<td>-0.37**</td>
<td>-2.41</td>
<td>-0.46</td>
<td>-2.76</td>
</tr>
<tr>
<td>NOR</td>
<td>-0.29**</td>
<td>-2.28</td>
<td>-0.37</td>
<td>-2.36</td>
</tr>
<tr>
<td>SGP</td>
<td>-0.17</td>
<td>-1.63</td>
<td>-0.25</td>
<td>-1.51</td>
</tr>
<tr>
<td>ESP</td>
<td>-0.12</td>
<td>-0.91</td>
<td>-0.39</td>
<td>-3.28</td>
</tr>
<tr>
<td>SWE</td>
<td>-0.53**</td>
<td>-3.61</td>
<td>-0.51</td>
<td>-2.87</td>
</tr>
<tr>
<td>CHE</td>
<td>-0.55**</td>
<td>-3.34</td>
<td>-0.16</td>
<td>-1.59</td>
</tr>
<tr>
<td>UKD</td>
<td>-0.63**</td>
<td>-4.11</td>
<td>-0.72</td>
<td>-3.37</td>
</tr>
<tr>
<td>USA</td>
<td>-0.45**</td>
<td>-2.63</td>
<td>-0.33</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Notes: Table shows the adjustment coefficients $\alpha$ from the VECM-Model $\Delta X_{t+1} = \Gamma_0 + \Gamma_1 \Delta X_t + \alpha \beta' X_t + \epsilon_t$, where $X_t = [d_t, c_{US}]$. The model is estimated for each country’s value and growth portfolio. t-stats indicate the t-statistics. The stars *, ** indicate that the coefficients are significantly different from zero at the 10%, respectively at the 5% significance level.
Table 4.9: Long-run exposure of cash flow component to consumption risk ($\beta_d$)

<table>
<thead>
<tr>
<th>Country</th>
<th>Value</th>
<th>Growth</th>
<th>Value - Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUS</td>
<td>2.05</td>
<td>0.61</td>
<td>1.44</td>
</tr>
<tr>
<td>AUT</td>
<td>1.84</td>
<td>0.36</td>
<td>1.48</td>
</tr>
<tr>
<td>BEL</td>
<td>1.89</td>
<td>-0.18</td>
<td>2.08</td>
</tr>
<tr>
<td>CAN</td>
<td>0.51</td>
<td>-0.61</td>
<td>1.12</td>
</tr>
<tr>
<td>DEN</td>
<td>1.77</td>
<td>2.36</td>
<td>-0.58</td>
</tr>
<tr>
<td>FRA</td>
<td>2.54</td>
<td>1.64</td>
<td>0.89</td>
</tr>
<tr>
<td>GER</td>
<td>2.37</td>
<td>1.28</td>
<td>1.09</td>
</tr>
<tr>
<td>ITA</td>
<td>2.11</td>
<td>3.12</td>
<td>-1.01</td>
</tr>
<tr>
<td>JPN</td>
<td>1.71</td>
<td>-0.33</td>
<td>2.04</td>
</tr>
<tr>
<td>NED</td>
<td>2.94</td>
<td>0.66</td>
<td>2.28</td>
</tr>
<tr>
<td>NOR</td>
<td>2.95</td>
<td>0.92</td>
<td>2.04</td>
</tr>
<tr>
<td>SGP</td>
<td>2.52</td>
<td>0.57</td>
<td>1.95</td>
</tr>
<tr>
<td>ESP</td>
<td>0.76</td>
<td>1.24</td>
<td>-0.47</td>
</tr>
<tr>
<td>SWE</td>
<td>3.36</td>
<td>2.85</td>
<td>0.51</td>
</tr>
<tr>
<td>CHE</td>
<td>2.82</td>
<td>2.32</td>
<td>0.50</td>
</tr>
<tr>
<td>UKD</td>
<td>1.50</td>
<td>1.15</td>
<td>0.35</td>
</tr>
<tr>
<td>USA</td>
<td>0.41</td>
<td>0.80</td>
<td>-0.40</td>
</tr>
</tbody>
</table>

Notes: Table shows the exposures of the cash flow component to consumption risk over an infinite horizon for each country’s value and growth portfolio. I.e. it shows $\beta_d$ from the cointegrating regression $d_t = \tau_0 + \beta_d cUS_t + \epsilon_d$ for each test asset.

Table 4.10: Cross-sectional fit by horizon

<table>
<thead>
<tr>
<th></th>
<th>$s = 1$</th>
<th>$s = \infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_{0,s}$</td>
<td>6.24**</td>
<td>5.44**</td>
</tr>
<tr>
<td>t-statistics</td>
<td>16.79</td>
<td>10.31</td>
</tr>
<tr>
<td>$\lambda_{1,s}$</td>
<td>0.10</td>
<td>1.01**</td>
</tr>
<tr>
<td>t-statistics</td>
<td>1.11</td>
<td>2.81</td>
</tr>
<tr>
<td>adj. $R^2$</td>
<td>-0.01</td>
<td>0.21</td>
</tr>
</tbody>
</table>

Notes: Table shows the estimates from the cross-sectional regression $E(R_{i,t}) = \lambda_{0,s} + \lambda_{1,s} \beta_{i,s}$. The asset $i$'s overall exposure over investment horizon $s$ is given by $\beta_{i,s} = \beta_{d,i,s} + \beta_{z,i,s} + (\kappa_1 - 1) \beta_{z,i,s}$ where $i$ = country’s value or growth portfolio. The estimates for $\lambda$ are in terms of annual percentage points. The stars ** indicate that the coefficients are significantly different from zero at 5% significance level.
Table 4.11: Panel regressions: Long-run consumption risk exposure in dividends and financial openness

<table>
<thead>
<tr>
<th></th>
<th>1)</th>
<th>2)</th>
<th>3)</th>
<th>4)</th>
<th>5)</th>
<th>6)</th>
<th>7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>-15.48**</td>
<td>-15.48**</td>
<td>-15.77**</td>
<td>1.16</td>
<td>-15.48**</td>
<td>-15.71**</td>
<td>1.21</td>
</tr>
<tr>
<td>t-statistics</td>
<td>-18.48</td>
<td>-18.86</td>
<td>-24.54</td>
<td>0.00</td>
<td>-18.86</td>
<td>-24.35</td>
<td>0.00</td>
</tr>
<tr>
<td>cUS</td>
<td>1.59**</td>
<td>1.59**</td>
<td>1.59**</td>
<td>-0.05</td>
<td>1.59**</td>
<td>1.59**</td>
<td>-0.05</td>
</tr>
<tr>
<td>t-statistics</td>
<td>18.88</td>
<td>19.27</td>
<td>25.20</td>
<td>0.00</td>
<td>19.27</td>
<td>25.20</td>
<td>0.00</td>
</tr>
<tr>
<td>Xj × cUS</td>
<td>0.01**</td>
<td>0.02**</td>
<td>0.02**</td>
<td>-0.04</td>
<td>-0.05</td>
<td>-0.05</td>
<td>-0.05</td>
</tr>
<tr>
<td>t-statistics</td>
<td>7.23</td>
<td>2.71</td>
<td>2.97</td>
<td>-0.56</td>
<td>-0.72</td>
<td>-0.79</td>
<td></td>
</tr>
<tr>
<td>Xj</td>
<td></td>
<td></td>
<td></td>
<td>0.54</td>
<td>0.61</td>
<td>0.61</td>
<td></td>
</tr>
<tr>
<td>t-statistics</td>
<td>0.73</td>
<td>0.98</td>
<td>1.07</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>adj. R²</td>
<td>0.23</td>
<td>0.26</td>
<td>0.57</td>
<td>0.64</td>
<td>0.26</td>
<td>0.57</td>
<td>0.64</td>
</tr>
</tbody>
</table>

Notes: Regression: \( d_{it} = \tau_0 + \delta_0 c_{us,t} + \delta_1 X_{j,t} c_{us,t} + \delta_2 X_{j,t} + \gamma_f + \theta_i + \epsilon_{i,j,t} \), where \( X_{j,t} = \log \left( \frac{GFA_j}{GDP_j} \right) - \log \left( \frac{GFA}{GDP} \right) \). \( i \) indicates a country's value or growth portfolio. \( GFA_j \) denotes the gross financial assets of country \( j \) at time \( t \) taken from the Lane and Milesi-Ferretti data set. The stars ** indicate that the coefficients are significantly different from zero at 5% significance level. Sample period: 1976 – 2011.

Table 4.12: Test for cointegration among five equity markets

<table>
<thead>
<tr>
<th># of cointegrating relations</th>
<th>Trace test</th>
<th>90% - CV Trace</th>
<th>( \lambda_{\text{max}} ) test</th>
<th>90% - CV ( \lambda_{\text{max}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r = 0 )</td>
<td>289.39*</td>
<td>65.95</td>
<td>157.45*</td>
<td>30.82</td>
</tr>
<tr>
<td>( r = 1 )</td>
<td>131.94*</td>
<td>45.25</td>
<td>80.54*</td>
<td>24.92</td>
</tr>
<tr>
<td>( r = 2 )</td>
<td>51.39*</td>
<td>28.43</td>
<td>34.45*</td>
<td>18.96</td>
</tr>
<tr>
<td>( r = 3 )</td>
<td>16.94*</td>
<td>15.58</td>
<td>16.91*</td>
<td>12.78</td>
</tr>
<tr>
<td>( r = 4 )</td>
<td>0.03</td>
<td>6.69</td>
<td>0.03</td>
<td>6.69</td>
</tr>
</tbody>
</table>

Notes: I perform the Johansen’s trace test and the maximum eigenvalue test to estimate the number of common stochastic trends among the price indices of five national stock markets (Canada, Germany, Japan, U.K. and the U.S.). Table shows the test statistics along with their critical values at 10% significance level. A star * indicates that the null of \( r \) cointegrating vectors can be rejected at 10% significance level. Number of lags is \( L = 5 \) years in the vector error correction model: \( \Delta X_t = \mu + \alpha \beta' X_{t-1} + \sum_{j=1}^{L} \Delta X_{t-j} + \epsilon_t \). Results imply four cointegrating relations and therefore one common stochastic trend among the five equity markets.
Figure 4.1: The value premium

Real excess returns, in log (1976–2014)

Notes: Figure shows the observed mean log excess returns of value and growth portfolios for each country. The log excess returns are dollar denominated log real portfolio returns minus the log real U.S. risk-free rate.
Figure 4.2: Risk premia vs. long-run risk exposure in dividends (market portfolios)

Notes: Figure shows the observed mean log excess returns and long-run consumption risk exposure in dividends. Sample: market portfolios, 1971 - 2014.
Figure 4.3: Risk premia vs. long-run risk exposure in dividends (value and growth portfolios)

Notes: Figure shows the observed mean log excess returns and long-run consumption risk exposure in dividends. Sample: value and growth portfolios, 1976 - 2014.
Figure 4.4: Mean excess returns: Observed vs. Fitted by $\Delta d$

Notes: Figure plots the observed risk premia against the risk premia predicted by the cash flow exposure over the short ($s = 1$ year) and long horizon ($s = \infty$): $E(R_{x_i,t}) = \lambda_{\Delta d,i} + \beta_{\Delta d,i} \lambda$, where $i =$ country’s value and growth portfolios.
Figure 4.5: Risk premia vs. risk exposure in price changes $\Delta z$

Notes: Figure shows mean log excess returns and consumption risk exposure in price changes ($\Delta z$) over $s = 1$ year and over infinite horizon. Sample: value and growth portfolios, 1976 - 2014.
Figure 4.6: Mean excess returns: Observed vs. Fitted by $\Delta z$

Notes: Figure plots the observed risk premia against the risk premia predicted by the risk exposure in price changes over the short ($s = 1$ year) and long horizon ($s = \infty$): $E(R_{x,i}) = \lambda_{\Delta z,i,s} \beta_{x,i,s}$ where $i =$ country’s value and growth portfolios.
Figure 4.7: Mean excess returns: Observed vs. Fitted by $z$

Notes: Figure plots the observed risk premia against the risk premia predicted by the risk exposure in price levels over the short ($s = 1$ year) and long horizon ($s = \infty$): $E(R_{x,t}) = \lambda_{x,z}^i \beta_{x,i,t}$ where $i$ = country’s value and growth portfolios.
Figure 4.8: Mean excess returns: Observed vs. Fitted by all components

Notes: Figure plots the observed risk premia against the risk premia predicted by the asset $i$’s overall exposure to consumption risk over the short ($s = 1$ year) and long horizon ($s = \infty$): $E(R_{xi,t}) = \lambda_s \beta_{i,s}$ where $\beta_{i,s} = \beta_{d,i,s} + \beta_{c,i,s} + (\kappa - 1) \beta_{g,i,s}$ where $i =$ country’s value and growth portfolios.
Figure 4.9: Pricing jointly international and U.S. portfolios

Notes: Figure plots the observed risk premia against the risk premia predicted by the asset $i$’s overall exposure to consumption risk over the short ($s = 1$ year) and long horizon ($s = \infty$): $E(R_{i,t}) = \lambda_s \beta_{i,s}$ where $\beta_{i,s} = \beta_{d,t,i} + \beta_{z,t,i} + (\kappa - 1) \beta_{c,t,i}$. Sample: countries’ value and growth portfolios + the 25 Fama-French portfolios (U.S.) sorted by size and value.
Notes: Figure shows $\lambda_s$ from $E(R_{X_t}) = \lambda_s \beta_s$ where $\beta_s = \beta_{\Delta d, i, s} + \beta_{\Delta z, i, s} + (\kappa_1 - 1) \beta_{\varepsilon, i, s}$. Sample: value and growth portfolios.
Figure 4.11: Risk premia vs. long-run consumption risk in real exchange rates

Notes: Figure plots observed risk premia against the long-run consumption risk exposure in real exchange rates obtained from $q_t = \alpha + \beta_q c_t^{1/5} + \epsilon_t$. Returns are expressed in local currency. Sample: value and growth portfolios, 1976 - 2014.
Figure 4.12: Risk premia vs. long-run exchange rate risk exposure in dividends

Notes: Figure shows mean log excess returns plotted against long-run real exchange rate risk exposure in dividends obtained from $d_t = \alpha + \beta_q q_t + \beta_c c_t + \epsilon_t$. Returns are expressed in local currency. Sample: value and growth portfolios, 1976 - 2014.
Figure 4.13: National stock market price indices and the common trend

Notes: Figure plots the estimated common stochastic trend among the five national stock markets together with their market price indices. The price indices are expressed in U.S. dollar and in real terms.
Figure 4.14: Common trends in price and dividends, U.S. consumption, real

Note: Figure plots the time series of the common stochastic trends in prices and dividends together with log real U.S. consumption.
4.9 Appendix B:

4.9.1 Campbell and Shiller log-linear return approximation

Let us start with the equity return denominated in U.S. dollar

\[ R_{USD}^{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} E_{t+1} \]  

(4.29)

where \( E_t = \frac{USD_{loc}}{loc} \) and \( loc_t \) denotes local currency at time \( t \). So, if \( E_t \) increases, the U.S. dollar depreciates relative to the local currency. Note that \( P_t \) and \( D_t \) are in local currency. Take the log of equation (4.29) and rearrange to get

\[
\log (R_{USD}^{t+1}) = \log (P_{t+1} + D_{t+1}) - \log (D_{t+1}) + \Delta \log (E_{t+1}).
\]

Define \( Z_{t+1} \equiv \frac{P_{t+1}}{D_{t+1}} \) and let variables in small letters denote the log of the variable. Then take the first order Taylor expansion of \( \log (1 + \exp (z_{t+1})) \) around the long-run mean \( E (z_{t+1}) = \bar{z} = (p - d) \)

\[
\log (1 + \exp (z_{t+1})) \approx \log (1 + \exp (\bar{z})) + \frac{1}{1 + \exp (\bar{z})} \exp (\bar{z}) (z_{t+1} - \bar{z})
\]

\[
\approx \log (1 + \exp (\bar{z})) + \frac{\exp (\bar{z})}{1 + \exp (\bar{z})} \bar{z} + \frac{\exp (\bar{z})}{1 + \exp (\bar{z})} z_{t+1}
\]

\[
\approx \kappa_0 + \kappa_1 z_{t+1}
\]

and thus,

\[
r_{USD}^{t+1} \approx \kappa_0 + (\kappa_1 - 1) z_{t+1} + \Delta z_{t+1} + \Delta d_{t+1} + \Delta e_{t+1}.
\]

(4.30)

To get the real returns, I deflate the returns by U.S. inflation \( \pi_{t+1}^{US} \)

\[
r_{t+1}^{USD} - \pi_{t+1}^{US} \approx \kappa_0 + (\kappa_1 - 1) z_{t+1} + \Delta z_{t+1} + \Delta d_{t+1} + \Delta e_{t+1} - \pi_{t+1}^{US}.
\]

(4.31)

Recall that the log price-dividend ratio \( z_{t+1} \) and the log dividend growth \( \Delta d_{t+1} \) are measured
in local currency. So, real returns expressed in U.S. dollar can be approximated by price-dividend ratio in local currency and real dividend growth in U.S. dollar.

4.9.2 Alternative formulation of the VAR

In order to reduce the number of parameters I need to estimate, I restrict the joint dynamics of the variables. I estimate a vector autoregressive model in three variables given by

\[ Y_t = AY_{t-1} + \eta_t \] (4.32)

where \( Y_t' = [\Delta c_t, \varepsilon_d, t, z_t] \) and \( \eta_t' = [u_{c,t}, u_{E,t}, u_{z,t}] \). I can express the VAR in equation (4.10) in terms of the VAR coefficients in equation (4.32) and the cointegration parameter. Thus, I estimate nine parameters instead of twenty-five. Having more degrees of freedom should increase the precision of the estimates. The equation for dividend growth \( \Delta d_t \) is derived from the cointegration restriction

\[ \Delta d_t = \beta_d \Delta c_t + \Delta \varepsilon_d, \] (4.33)

and \( \Delta z_t = z_t - z_{t-1} \). The relation between the residual vectors \( u_t \) and \( \eta_t \) is given by

\[
\begin{pmatrix}
u_{c,t} \\
u_{E,t} \\
u_{z,t} \\
u_{\Delta d,t} \\
\Delta u_{z,t}
\end{pmatrix} =
\begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\beta_d & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
u_{c,t} \\
u_{E,t} \\
u_{z,t} \\
\Delta u_{z,t}
\end{pmatrix}.
\] (4.34)

The VAR in equation (4.10) has then the following structure
\[
\begin{pmatrix}
\Delta c_t \\
\varepsilon_{d,t} \\
\z_t \\
\Delta d_t \\
\Delta z_t
\end{pmatrix}
= \begin{pmatrix}
a_{11} & a_{12} & 0 & 0 \\
a_{21} & a_{22} & a_{23} & 0 \\
a_{31} & a_{32} & a_{33} & 0 \\
\beta_d a_{11} + a_{21} & \beta_d a_{12} + a_{22} - 1 & a_{23} & 0 \\
a_{31} & a_{32} & a_{33} - 1 & 0
\end{pmatrix}
\begin{pmatrix}
\Delta c_{t-1} \\
\varepsilon_{d,t-1} \\
\z_{t-1} \\
\Delta d_{t-1} \\
\Delta z_{t-1}
\end{pmatrix}
+ \begin{pmatrix}
u_{c,t} \\
\varepsilon_{d,t} \\
\z_{t} \\
\beta_d u_{c,t} + \varepsilon_{d,t} \\
u_t
\end{pmatrix},
\]

Note that I set \(a_{13} = 0\) since country-specific price-dividend ratios should not predict U.S. consumption growth. From the VAR, I can extract the shocks to the variables cumulated over \(s\) periods and compute the horizon-dependent covariance matrices and betas. Let us define the prediction error over \(k\) periods as

\[
PE(k) = X_{t+k} - E_t X_{t+k} = \sum_{j=0}^{k-1} B^j u_{t+k-j}.
\]

Given that the error term \(u_t\) is i.i.d., the variance of the prediction error is

\[
Var(PE(k)) = \sum_{j=0}^{k-1} B^j \Sigma u B^j.
\]

The innovation in the variables cumulated over \(s\) periods is then

\[
\sum_{k=1}^{s} X_{t+k} - E_t \left( \sum_{k=1}^{s} X_{t+k} \right) = \sum_{k=1}^{s} C_k u_{t+1+k-s-k}
\]

where \(C_k = C_{k-1} + B^{k-1}\) and \(C_0 = 0\). The variance-covariance matrix of the cumulated prediction errors for the period \(s\) can be computed recursively, such that

\[
\tilde{\Sigma}_s = \left( C_s \Sigma u C_s' + \tilde{\Sigma}_{s-1} \right)
\]

where \(\tilde{\Sigma}_0 = 0\). The covariance matrix is divided by horizon \(s\) and denoted as \(\Sigma_s = \tilde{\Sigma}_s / s\). Its
recursive structure is given by

\[
\Sigma_s = \frac{1}{s} \left( C_s \Sigma u C_s' \right) + \left( \frac{s-1}{s} \right) \Sigma_{s-1}.
\]  \hspace{1cm} (4.40)

As \( s \) goes to infinity, the long-run covariance matrix can be written as

\[
\Sigma_{lr} = (I - B)^{-1} \Sigma u (I - B)^{-1}'.
\]  \hspace{1cm} (4.41)
Bibliography


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Research Assistant 2007 - 2008
Chair of International Trade and Finance, University of Zurich

TEACHING EXPERIENCE
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