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A developmental model of number representation

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Abstract: We delineate a developmental model of number representations. Notably, developmental dyscalculia (DD) is rarely associated with an all-or-none deficit in numerosity processing as would be expected if assuming abstract number representations. Finally, we suggest that the “generalist genes” view might be a plausible – though thus far speculative – explanatory framework for our model of how number representations develop.

Challenging “dogmas” in cognitive neuroscience is important for the advancement of our professional development. Therefore, we highly appreciate the article of Cohen Kadosh & Walsh (CK&W), which attempts to challenge the widely held belief that the neural representation of numerosity may be abstract rather than nonabstract. According to “neural
constructivism,” the representational features of the human neocortex are strongly modulated by a dynamic interaction between neural growth mechanisms and environmentally derived neural activity (Quartz & Sejnowski 1997; see also Fisher 2006 for discussion of the gene–environment interdependency). Hence, a closer look at the development of the brain and, in particular, at the development of neural representations of numerosity, will shed further light on the question of whether numbers are represented in an abstract or nonabstract manner.

The primary aim of this commentary is to provide more recent developmental data revealing that number representations in children are not stable, but rather undergo a developmental shift from distinct (nonabstract) to shared (abstract) representations. Beyond behavioral data, we attempt to apply a functional geneticist view as an explanatory framework for the complex data reported thus far. As outlined rather briefly by CK&W, the findings of the few developmental studies that were dedicated to examining notation-dependent effects on number processing in children largely support the authors’ notion that children’s number representations are likely to be nonabstract (Holloway & Ansari, 2009). Maybe even more interesting is the finding that 3-year-olds’ abilities to compare nonsymbolic number sets seem to rely on perceptual cues if the ambiguity between (discrete) numerical and (continuous) nonnumerical stimulus properties is overwhelming (Rousselle et al., 2004). The latter results clearly speak against the notion of an abstract number representation. A similar conclusion has been drawn by Butterworth (2005), who argues that “if put into conflict...continuous quantity seems a more powerful cue.” (p. 5; see also Mix et al. 2002). Five-year-old children are able to compare and add large sets of elements presented in different nonsymbolic modalities (dot arrays, tone sequences) (Barth et al. 2005). Importantly, the authors’ report a
significant interaction between the presentation format and the ratio of the two sets to be compared, being characterized by a steeper ratio-dependent decline in cross-modal performance. If one assumes an abstract representation of numbers, no performance differences within and across modalities should have been observed. Likewise, children’s mapping between nonsymbolic and symbolic number representations becomes more refined with age, which is contrary to expectation if one assumes abstract number representations (Mundy & Gilmore, 2009). Findings of dissociations between different notations are not restricted to typically developing children, also, dyscalculic children are reported to exhibit impaired performance when comparing symbolic Arabic digits, but not when comparing nonsymbolic object sets (Rousselle & Noel 2007).

With respect to DD, the distinction between automatic and intentional performance is an important one. Empirical evidence, supporting the authors’ claim that automatic number processing might be a more powerful tool to assess differential number processing skills, comes from a single-case study of DD (Kaufmann et al. 2004). Results are incompatible with the notion of abstract number representation, as they revealed that number processing deficiencies predominantly emerged upon automatic, but much less upon intentional, number processing. A further interesting issue in this case study was the finding of operation-specific effects in fact retrieval (addition and subtraction facts being relatively preserved, whereas multiplication facts were severely impaired; Kaufmann 2002). In our view, operation-specific effects (which have been frequently reported in the patient literature, e.g., Pesenti & van der Linden 1994) – like effects of notation – are not in line with an abstract view, but rather are strongly suggestive of the existence of distinct number representations.
Finally, we suggest that the “generalist genes” view may provide a plausible – though thus far speculative – explanatory framework for the view that number representations undergo a gradual developmental change (Kovas & Plomin 2006). In particular, concepts of polygenicity (many genes affect one trait/one cognitive domain) and pleiotropy (one gene affects many traits/cognitive domains) are not only apt to explain the frequently observed comorbidity between DD and other learning disabilities such as dyslexia or attention disorders, but may also provide a useful theoretical framework for the assumption that number representations become shared (abstract) with more experience/practice, which is inevitably accompanied by a more fine-tuned gene–environment interdependency (Fisher 2006; Kovas & Plomin 2006).

In sum, developmental findings challenge the existence of an abstract number representation. However, in our view, notation-specific effects are not necessarily indicative for the existence of nonabstract number representations, especially when it comes to developmental studies. The observed interactions between different input modalities could also be a result of deficient mapping between symbolic and nonsymbolic representations in children with and without DD. We assume that developmental progress goes along with higher overlap of brain activation between, as well as across, different numerical input modalities. Previously, we found no activation differences between approximate and exact calculation in school children (Kucian et al. 2008). These results, rather, point to a mutual neuronal network for both tasks. However, one has to keep in mind that both tasks have been presented symbolically, differing in demands solely.
Taken together, we suggest that brain activation patterns for different numerical tasks are partly overlapping and that some brain regions are dependent on notation/input modality. Moreover, activation patterns get influenced by task demands, such as automatic or intentional number processing. If a core region for number processing exists, it is plausible that this region consists of highly interconnected neurons for different numerical inputs and that activation of one neuronal population quickly spreads to other populations, leading to cross-notational activation, as proposed by CK&W. With development and higher numerical proficiency, these cross-notational activations might increase, reflecting automatization processes. There is probably a gradual difference in definition between nonabstract and abstract representation of numerosity with respect to both the strength of connections and to coactivations of different neuronal populations. If, for example, the connection is extremely tight and both neuronal populations get activated simultaneously independent of inputs, one could label it as an abstract numerical representation, although this representation is built up by distinct neuronal populations. On summarizing the above-mentioned findings, we propose the following developmental model of how number representations might be formed:
Figure 1: Our developmental model of number representation proposes an increased overlap of representations across different number notations with advanced expertise and implies a gradual difference in definition between nonabstract and abstract representation of numerosity.

References

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