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Relative Performance or Team Evaluation?
Optimal Contracts for Other-Regarding Agents

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Abstract

This paper derives optimal incentive contracts for agents with other-regarding preferences. It offers a behavioral explanation for the empirically observed lack of relative performance evaluation. We analyze a principal-multi agent model and assume that agents are inequity averse or status seeking. We show that team contracts can be optimal even if the agents’ performance measures are positively correlated such that relative performance evaluation would be optimal with purely self-interested agents and even though relative performance evaluation provides additional incentives to provide effort if agents have other-regarding preferences. Furthermore, optimal incentive contracts for other-regarding agents can be low-powered as compared to contracts for purely self-interested agents.

JEL Classification: D82, D86

Keywords: Other-regarding preferences, inequity aversion, status seeking, relative performance evaluation, low-powered incentives

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1 Introduction

Incentive theory prescribes the use of relative performance evaluation (RPE) to reduce agents’ risk exposures (e.g., Lazear and Rosen 1981, Holmström 1982). Empirically, however, there is little evidence in support of this result. In a recent survey, Chiappori and Salanié (2003) conclude that “one empirical puzzle in this literature is that firms do not seem to use relative performance evaluation of managers very much” (p. 132). In this paper, we show that agents’ other-regarding preferences can be an explanation for the lack of RPE.

We derive optimal incentive contracts in a Holmström and Milgrom (1990) multi-agent moral hazard model. Our key assumption is that agents are either status seeking (Frank 1985) or inequity averse (Fehr and Schmidt 1999, Bolton and Ockenfels 2000).\(^1\) We show that the decision between RPE and team contracts then implies solving a threefold trade-off involving risk, inequality, and the incentive effect of other-regarding preferences. To see this, suppose the agents’ performance measures are positively correlated. An agent’s risk exposure can then be reduced by linking his wage negatively to other agents’ measures (RPE). While this is optimal with purely self-interested agents, with other-regarding agents there are two additional effects. First, to reduce inequality, an agent’s wage should depend positively on other agents’ measures (team contract). Second, other-regarding agents can have additional incentives, which are always positive with RPE but potentially negative with team contracts. Hence, to capitalize on these incentives, the principal should use RPE. We show that even though risk reduction and the incentive effect of other-regarding preferences drive contracts towards RPE, team contracts can nevertheless be optimal.

The standard arguments against the use of RPE are sabotage and reduced incentives to cooperate (e.g., Lazear 1989, Itoh 1991). Our paper complements these arguments and, in addition, it also applies in situations in which agents cannot affect each other’s outcome by way of sabotage or helping.

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\(^1\)Evidence for social comparisons in the workplace is provided by numerous surveys and experimental studies (e.g., Bewley 1999, Agell and Lundborg 2003, Camerer 2003). Also, many firms have a wage secrecy policy, which in itself is evidence that other employees’ pay is not a matter of indifference.
Furthermore, our paper provides a behavioral explanation why incentives are often weaker than predicted by standard moral hazard models (e.g., Williamson 1985). Holmström and Milgrom (1990) argue that “it has been somewhat of a mystery to organizational observers, why there is so much less reliance on high-powered incentives than basic agency theory would suggest” (p. 93). In our model, given incentive levels cause higher risk exposure compared to the case with purely self-interested agents, and also exposure to inequality. Optimal incentive levels are therefore lower. This result does not only complement the standard multi-tasking argument for low-powered incentives (Holmström and Milgrom 1991), but it also applies in single-tasking situations.

With regard to incentive intensities there is evidence on international differences. Conyon and Murphy (2000) provide a comprehensive comparison of CEO pay practices in the US and UK and document that American CEOs face stronger incentives than British CEOs. Within the framework of standard agency theory, this observation could be explained by differences in risk aversion, effort costs, or performance measurement. The data, however, do not suggest such transatlantic differences. The authors therefore conclude that “the traditional principal-agent model [...] does not offer promising explanations for the difference in pay levels and incentives in the two countries” (p. 664). They suggest “cultural differences” (p. 667) as an explanation: “The US, as a society, has historically been more tolerant of income inequality” (p. 667). Alesina, Di Tella, and MacCulloch (2004), for example, provide evidence that Europeans are indeed more unhappy with inequality in society than Americans. Our paper shows that cultural and agency-theoretic explanations need not be distinct. Different incentive intensities in the US and UK may both reflect equilibrium incentive contracts that differ in the ‘cultural’ dimension of agents’ other-regarding preferences. Moreover, our results imply differences in the use of RPE between the US and UK (or comparable countries), but we are not aware of any empirical work that tested this implication so far.

Our paper belongs to a strand of recent papers that analyze the effect of social comparisons in principal-multi agent models. Itoh (2004), Demougin and Fluet (2006), Demougin, Fluet, and Helm (2006), and Neilson and Stowe (2010) assume risk neutral agents. In these papers, risk must thus not be traded off against inequality or incen-
Goel and Thakor (2006) and Bartling and von Siemens (2010a) consider risk averse agents. Goel and Thakor employ the first-order approach (Holmström 1979) and show that envy can lead to low-powered team incentives. Their model is however restricted to envy, uncorrelated performance measures, and wage comparisons. We are, for example, able to analyze the role of effort costs for the sign of the incentive effect of other-regarding preferences. In their model the incentive effect is always positive, but we show that it can be negative if agents account for effort costs, especially under team contracts. Our paper identifies the conditions under which team contract are nevertheless optimal. Bartling and von Siemens analyze the impact of envy in a Grossman and Hart (1983) moral hazard model. While this model frame is very general, e.g., it is not restricted to linear schemes, it comes at the cost that closed form solutions for optimal incentive contracts cannot be derived. The focus of their paper is the interplay of risk preferences and limited liability constraints on the principal’s cost of providing incentives. They show that if and only if agents are risk averse and there are no binding limited liability constraints, then envy unambiguously increases the principal’s cost of providing incentives. As Goel and Thakor, they restrict the analysis to the case with envious agents who do not account for effort costs in their comparisons.\footnote{The impact of other-regarding preferences on incentive provision has also be analyzed in settings assuming comparisons between a principal and a single agent (Itoh 2004, Dur and Glazer 2008, Englmaier and Wambach 2010), team production (Rey Biel 2008, Bartling and von Siemens 2010b), tournaments (Grund and Sliwka 2005), and relational contracting (Kragl and Schmid 2009).}

2 The Model

Consider a principal and two identical agents, who choose effort levels $a_i$, $i \in \{1, 2\}$, at personal, quadratic cost $\psi(a_i)$, measured in monetary units. Contracts can only be written on performance indicators that take the form $q_i = a_i + \varepsilon_i$. We interpret $q_i$ as the principal’s gross profit (before wage payments) from agent $i$’s activity. Production is technologically independent such that agents affect only their own indicators. We however allow the performance indicators to be correlated. The error terms $\varepsilon_1$ and $\varepsilon_2$ are assumed to be drawn from a symmetric multivariate normal distribution with mean zero, variance $\sigma_\varepsilon^2 > 0$, and covariance $\sigma_{\varepsilon\varepsilon}$. The correlation coefficient is $\rho = \sigma_{\varepsilon\varepsilon}/\sigma_\varepsilon^2$. 


The principal is risk neutral and only interested in his expected profit. Both agents have CARA risk preferences represented by the negative exponential utility function

\[-\exp(-\eta(w_i - \psi(a_i) - L_i))\]  

(1)

where \(\eta > 0\) denotes the coefficient of absolute risk aversion and \(w_i\) agent \(i\’s\) wage. \(L_i\) denotes agent \(i\’s\) expected loss from inequality, measured in monetary units, which will be discussed in detail below.

The principal can remunerate each agent as a function of both agents’ performance indicators. We restrict attention to linear contracts. Agent \(i\’s\) wage is thus given by

\[w_i = r + vq_i + uq_j\]  

(2)

with \(i, j \in \{1, 2\}, i \neq j\), where \(r\) denotes a fixed payment, \(v\) the compensation coefficient of an agent’s own performance indicator, and \(u\) the coefficient of the respective other agent’s indicator. We consider symmetric contracts only, i.e., \(r, v,\) and \(u\) are identical for both agents.\(^3\) We speak of RPE if \(u < 0\), and of a team contract if \(u > 0\).\(^4\) We call \(u = v\) a perfect team contract, and a contract with \(u = 0\) an independent contract.

Our key assumption is that agents have other-regarding preferences. An agent’s utility does not only depend on his own payoff, i.e., wage minus effort cost, but also on the other agent’s wage and possibly effort cost. We assume that the worse-off agent incurs a loss due to unfavorable inequality. For the better-off agent we allow that agents incur a loss when they are better off (inequity aversion) or realize a gain when they are better off (status preferences). However, we assume that the loss or gain from being better off does not exceed in absolute value the loss from being worse off.

In particular, we assume that agents care for the expected loss from inequality that results from accepting the contract, given the other agent also accepts. Formally, we

\(^3\)Since contracts are symmetric ex-ante, one can argue that they are procedurally fair (e.g., Bolton, Brandts, and Ockenfels 2005). However, wages and payoffs can differ ex-post and if agents dislike this, e.g., because they are inequity averse, then they have to be compensated for the involved utility loss.

\(^4\)With negative correlation, \(u > 0\) indicates both RPE and team contracts. To avoid repetition of this case distinction, throughout the paper we implicitly assume \(\rho > 0\). The trade-offs identified in our paper equally apply with \(\rho < 0\), and no restrictions on \(\rho\) are imposed in the formal analysis.
have to distinguish the two cases \( u < v \) and \( u = v \). Consider first the case \( u < v \). Agent \( i \)'s expected loss from inequality then takes the form

\[
L_i = (v - u) \left( \alpha \int_{-\infty}^{\infty} (z - \Delta) f(z) dz + \alpha \beta \int_{-\infty}^{\Delta} (\Delta - z) f(z) dz \right)
\]

where \( z \equiv \varepsilon_j - \varepsilon_i \) and \( f(z) \) is the p.d.f. of \( z \), which is \( z \sim N(0, \sigma_z^2) \) with \( \sigma_z^2 = 2(1 - \rho) \sigma_{\varepsilon}^2 \). In the degenerate case with \( \rho = 1 \) so that \( \sigma_z^2 = 0 \), the loss from inequality is given by expression (5) below. \( \Delta \) is a measure of agent \( i \)'s inequality exposure net of \( z \),

\[
\Delta = a_i - a_j - \gamma \frac{\psi(a_i) - \psi(a_j)}{v - u}
\]

where \( \gamma \in [0, 1] \) denotes the degree to which the agents account for effort costs in their social comparisons. If \( \gamma = 0 \), effort costs are not accounted for at all, i.e., agents compare only wages. If \( \gamma = 1 \), effort costs are fully taken into account.

The agents' sensitivity to unfavorable inequality is denoted by \( \alpha \geq 0 \). Agent \( i \) is behind if \( z > \Delta \), which is captured by the first integral in (3). The agents' sensitivity to favorable inequality is denoted by \( \alpha \beta \). Agent \( i \) is ahead if \( z < \Delta \), which is captured by the second integral. If \( \beta > 0 \), agents are inequity averse. Models of inequity aversion assume that agents suffer weakly more from unfavorable than from favorable inequality, i.e., we have \( \beta \leq 1 \). If \( \beta < 0 \), agents are status seeking. Here we impose the restriction that the gain from being better off does not exceed the loss from being worse off, i.e., \( \beta \geq -1 \). Overall, we thus assume that \( |\beta| \leq 1 \) but we do not restrict the sign of \( \beta \).

Consider now the case \( u = v \). Under the perfect team contract, wages are always identical but inequality can arise with respect to effort costs. If agent \( i \) works harder than agent \( j \), agent \( i \) is behind because he bears higher effort costs. The loss from

\[5\]It cannot be optimal that \( u > v \). From the perspective of risk reduction, \( u \) is largest relative to \( v \) with perfect negative correlation; we then have \( u = v \). Moreover, expected inequality is then zero. Further increasing \( u \) relative to \( v \) could thus only increase both risk and expected inequality.

\[6\]To better understand the notion of expected loss from inequality, consider the following example. With equal probability agent 1 is either ahead of agent 2 by 10 payoff units or behind by 20. The expected loss from inequality is then \( 0.5 \cdot 20 \alpha + 0.5 \cdot 10 \alpha \beta \). The notion of expected loss from inequality does not mean that agent 1 expects to be behind by \( 0.5 \cdot 20 - 0.5 \cdot 10 = 5 \) and therefore has a loss of \( 5 \alpha \). Rather, an agent weights every possible realization of inequality with the respective sensitivity parameter \( \alpha \) or \( \alpha \beta \) and with the probability that it occurs.
inequality is then the absolute value of the effort cost difference weighted by $\alpha \gamma$. If agent $i$ works less than agent $j$, agent $i$ is ahead. The loss or gain from inequality is then the absolute value of the effort cost difference weighted by $\alpha \beta \gamma$. If both agents work equally hard, no inequality arises. Formally, we have

$$L_i = \begin{cases} 
\alpha \gamma (\psi(a_i) - \psi(a_j)) & \text{if } a_i \geq a_j \\
\alpha \beta \gamma (\psi(a_j) - \psi(a_i)) & \text{if } a_i < a_j.
\end{cases}$$

It is important to notice that the expected loss from inequality is taken ex-ante and independently from the expectation over possible wage levels. For every possible realization of the wage level $w_i$ and given effort choice $a_i$, the expected loss enters the agents’ utility function (1) as a fixed loss, equivalent to a wealth effect. This implies that the agents are risk neutral with respect to inequality but risk averse with respect to wages. One economic justification for assuming risk aversion with respect to wages is consumption smoothing. Inequality, however, does not affect consumption possibilities. Moreover, we are not aware of empirical evidence suggesting either risk neutrality or risk aversion with respect to payoff differences. Risk neutrality with respect to inequality is thus a possible modeling choice. In summary, our formulation implies that (i) agents consider ex-ante the inequality that is to be expected from accepting the contract offer, (ii) they have to be compensated for the expected inequality but not for the uncertainty over the final inequality, and (iii) for a given contract, the risk compensation for income uncertainty is not affected by the uncertainty over inequality realizations.

The property of our model that the agents’ loss from inequality does not affect risk exposure as such, i.e., given $v$ and $u$, does not mean that other-regarding preferences are irrelevant for the agents’ equilibrium risk exposure. In fact, the expected loss from inequality is determined by the magnitude and relative size of the compensation coefficients $v$ and $u$, which also determine an agent’s incentives to exert effort and his risk exposure. The resulting trade-off between incentives, risk, and inequality is at the core of the principal’s maximization problem, to which we turn next.

The principal maximizes his expected gross profit minus expected wage payments by choice of $r$, $v$, and $u$. He must thereby ensure the agents’ participation and take into
account that, given the contract, the agents maximize their utility functions by choice of effort. Maximization of an agent’s utility function is equivalent to maximization of his certainty equivalent (CE), which by standard results can be stated as

\[ CE_i = r + va_i + ua_j - \psi(a_i) - 0.5\eta(v^2 + u^2 + 2vu\rho)\sigma^2_e - L_i(a_i, a_j, v, u). \]  

(6)

An agent’s CE consists of the expected wage, effort cost, risk exposure, and the expected loss from inequality. Since production is technologically independent, the principal’s CE from contracting with an agent is given by \( CE_P = a_i - (r + va_i + ua_j) \), that is, expected revenue minus expected wage payment. In models with transferable utility, any efficient contract maximizes the joint surplus of the principal and the agents. The principal’s program can thus be stated as

\[
\max_{v,u} a_i^* - \psi(a_i^*) - 0.5\eta(v^2 + u^2 + 2vu\rho)\sigma^2_e - L_i(a_i^*, a_j^*, v, u)
\]

subject to the incentive and participation constraints

\[
a_i^* \in \arg \max_a CE_i(a_i, a_j^*, v, u)
\]

\[
CE_i(a_i^*, a_j^*, v, u) \geq 0
\]

for \( i, j \in \{1, 2\}, i \neq j \). Since the agents’ optimal effort choices are interdependent via the inequality term, the incentive constraint (8) requires that effort choices form a Nash equilibrium given \( v \) and \( u \). We restrict attention to the derivation of a symmetric equilibrium, i.e., \( a_i^* = a_j^* \). The principal’s program is finally subject to the participation constraint (9). It requires the fixed payment \( r \) to be set such that the agents receive their outside CEs, which we normalize to zero.

What is the effect of other-regarding preferences on the incentive constraint? We have to analyze the cases \( u < v \) and \( u = v \) separately. Consider first the case \( u < v \). Expected inequality is then given by (3) and incentive constraint (8) can be stated as

\[
\frac{\partial CE_i}{\partial a_i} = v - \psi'(a_i) + \alpha(v - u - \gamma\psi'(a_i)) \left( \int_{\Delta}^{\infty} f(z)d(z) - \beta \int_{-\infty}^{\Delta} f(z)d(z) \right) = 0
\]

(10)
for \( i \in \{1, 2\} \). For the derivation of (10) see Appendix A.1. Symmetry implies \( \Delta = 0 \) so that the first-order conditions (10) can be written as

\[
\psi'(a^*_i) = \frac{2v + \alpha(1 - \beta)(v - u)}{2 + \alpha(1 - \beta)\gamma}.
\] (11)

From (11) it is evident that a symmetric equilibrium can exist. It is straightforward to check that the agents’ CEs are strictly concave in effort, given \( \Delta = 0 \). The first-order conditions are thus necessary and sufficient to characterize the symmetric equilibrium.

Notice first that with \( \alpha = 0 \), equation (11) reduces to the standard incentive constraint with purely self-interested agents, \( \psi'(a^*_i) = v \). Also in the symmetric case with \( \beta = 1 \), other-regarding preferences do not affect the agents’ effort choices as neither working harder nor slacking off affects expected inequality. An agent’s effort choice is then solely determined by equating marginal cost and the own compensation coefficient \( v \). For \( \alpha > 0 \) and \( \beta \in [-1, 1) \), however, we have \( \psi'(a^*_i) \neq v \). If \( \psi''(a^*_i) > v \), there is a positive incentive effect of other-regarding preferences because any given \( v \) implements a higher effort level compared to purely self-interested agents. If \( \psi''(a^*_i) < v \), the incentive effect is negative. The sign of the incentive effect depends on the sign of \( v - u - \gamma \psi'(a^*_i) \); the magnitude is also determined by \( \alpha \) and \( \beta \). Besides the contract variables \( v \) and \( u \), the degree \( \gamma \) to which agents account for effort costs in their social comparisons has a crucial impact on the sign of the incentive effect. If \( v - u - \gamma \psi'(a^*_i) > 0 \), increasing one’s effort decreases expected inequality: \( v \) exceeds \( u \), hence the own expected wage increases by more than the other agent’s wage, and the increased own effort costs do not overcompensate for this.

We now analyze the case \( u = v \). Expected inequality is then given by (5) and the CE is not differentiable at \( a_i = a_j \). In this case, the left and right hand side derivatives

\[
\lim_{a_i \nearrow a_j} \left. \frac{\partial CE_i}{\partial a_i} \right|_{v=u} = v - (1 - \alpha \beta \gamma) \psi'(a^*_i) \geq 0 \quad (12)
\]

\[
\lim_{a_i \searrow a_j} \left. \frac{\partial CE_i}{\partial a_i} \right|_{v=u} = v - (1 + \alpha \gamma) \psi'(a^*_i) \leq 0 \quad (13)
\]

characterize the set of candidate effort choice equilibriums. However, by marginally
<table>
<thead>
<tr>
<th>( \gamma )</th>
<th>RPE ( u &lt; 0 )</th>
<th>independent ( u = 0 )</th>
<th>team ( 0 &lt; u &lt; v )</th>
<th>perfect team ( u = v )</th>
</tr>
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<tr>
<td>( \gamma = 0 )</td>
<td>+</td>
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<td>+</td>
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<td>( \gamma \in (0,1) )</td>
<td>+</td>
<td>+</td>
<td>ambiguous</td>
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<td>( \gamma = 1 )</td>
<td>+</td>
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Table 1: The incentive effect of other-regarding preferences is positive (+)/negative (−) if the agents choose higher/lower effort levels than purely self-interested agents for a given incentive intensity \( v \). It is neutral if both types choose the same effort level. The sign of the incentive effect depends on the contract type—RPE, independent, team, or perfect team contract—and on the degree \( \gamma \) to which agents account for effort costs in their social comparisons.

lowering \( u \) below \( v \) the principal can induce the effort level that is defined by the limit of (11) as \( u \to v \), \( \psi'(a^*) = 2v/(2 + \alpha(1 - \beta)\gamma) \), hence effort levels with \( \psi'(a) < 2v/(2 + \alpha(1 - \beta)\gamma) \) cannot be equilibriums. In the following we consider only this lower bound of the set of equilibrium effort choices. It is then apparent that the incentive effect is unambiguously negative, i.e., we have \( \psi'(a^*) < v \); only if \( \gamma = 0 \) it is neutral.

The overview of the incentive effect of other-regarding preferences in Table 1 shows the general directions of the influencing factors. First, the effect is always positive with RPE but, ceteris paribus, becomes smaller or even negative as \( u \) increases. Second, the less agents account for effort costs in their social comparisons, the larger their incentive to exert effort, given the contract.

We now turn to the effect of other-regarding preferences on the participation constraint. The fixed payment \( r \) must be set such that the agents are compensated not only for their effort and opportunity costs but also for their expected equilibrium loss from inequality, which we derive next. With symmetric effort choices and thus costs, the equilibrium loss from inequality is independent of \( \gamma \). It is then also independent of the level of effort such that only chance, i.e., the relative size of the error terms.

\footnote{If agents have status preferences and gain as much from being ahead as they suffer from being behind, i.e., \( \beta = -1 \), right and left hand side derivatives coincide. Then \( v = (1 + \alpha \gamma)\psi'(a_i) \) characterizes the only equilibrium. If the agents gain less from being ahead or if they suffer from it, more favorable equilibriums are possible. If \( \beta = 0 \), the incentive effect is at best neutral, i.e., the best effort choice equilibrium that is consistent with conditions (12) and (13) is given by (12) with \( \beta = 0 \). If \( \beta > 0 \), the incentive effect can be positive. The upper bound of possible equilibriums is defined by (12). Effort choice equilibriums with effort exceeding the level defined by \( v = \psi'(a) \) are possible if \( 1 - \alpha \beta \gamma < 1 \). With \( u = v \) inequality can only arise with respect to effort costs. If an agent expects the other agent to choose high effort, choosing low effort and thus low costs implies being better off. If suffering from being better off is strong, agents will match the respective other agent’s high effort choice.}
determine the relative standing (as in a symmetric tournament). If \( u < v \), the expected loss from inequality is given by (3), which given \( \Delta = 0 \) simplifies to

\[
L_i(v, u \mid a_i^* = a_j^*) = \alpha (1 + \beta) (v - u) \sqrt{\frac{1 - \rho}{\pi}} \sigma_\varepsilon.
\] (14)

The derivation of expression (14) can be found in Appendix A.2. If \( u = v \), symmetry directly implies \( L_i = 0 \). Since expression (14) equals zero if \( u = v \), it denotes an agent’s expected loss in equilibrium in both cases.

The equilibrium expected loss from inequality (14) is an intuitive expression. It increases in \( \alpha \), the agents’ sensitivity to inequitable payoffs. It also increases in \( \beta \), i.e., the less agents gain from being ahead or the more they suffer from being ahead. It is largest in the symmetric case \( \beta = 1 \), where agents suffer as much from being ahead as from being behind. It is zero in case \( \beta = -1 \), where agents gain as much from being ahead as they lose from being behind. It increases in the measurement error \( \sigma_\varepsilon \) because with higher variance more probability mass is on larger wage differences. It decreases in the correlation coefficient; it is zero if \( \rho = 1 \), i.e., if there is perfect positive correlation. Most importantly, the smaller the difference \( v - u \) of the compensation coefficients, the smaller the expected loss from inequality; it is zero if \( u = v \), i.e., if there is a perfect team contract such that the agents always receive the same wages.

In our model, since (i) equilibrium payoff differences are symmetrically distributed with zero mean and (ii) the agents’ sensitivity to unfavorable inequality (weakly) exceeds their sensitivity to favorable inequality, in expectation an agent suffers a (weakly positive) loss—even if he enjoys being better off.

### 3 Optimal Contracts

In this section, we derive the optimal incentive contracts for other-regarding agents. As benchmark, we first consider purely self-interested agents. With \( \alpha = 0 \), the principal’s program reduces to the case analyzed in Holmström and Milgrom (1990). The expected loss from inequality drops from the principal’s objective function (7), and the incentive constraint (11) reduces to \( \psi'(a_i^*) = v \). The program is then solved in two steps. First,
for each $v$ the principal minimizes the agents’ risk exposure by choice of $u$. In this case, $u$ has no impact on the principal’s program besides affecting the agents’ risk exposure. Specifically, $u$ does not affect the incentive constraint. Minimization of the agents’ risk exposure then yields $u^* = -v \rho$, where superscript $s$ indicates the case with purely self-interested agents. If the error terms are correlated, some (in case of perfect correlation all) of the risk imposed on agent $i$ can be filtered out by using the information in agent $j$’s performance measure. Hence, with purely self-interested agents, the sign of $u$, i.e., whether we have RPE, team, or independent contracts, is solely determined by the sign of the correlation coefficient $\rho$. Second, given $u^s$, the optimal $v$ is chosen to trade-off incentives and risk exposure. This yields $v^s = 1/(1 + \eta \sigma^2 \varepsilon (1 - \rho^2) \psi''(a))$. The optimal incentive intensity is thus higher if the agents are less risk averse (low $\eta$), the performance measures are less noisy (low $\sigma^2 \varepsilon$), more correlated (high absolute value of $\rho$), and if the agents react strongly to increasing incentives (low $\psi''$).

We now derive the optimal contracts for other-regarding agents. The derivation of the principal’s program (7)-(9) showed that other-regarding preferences enter the program in two ways. First, they cause an expected loss from inequality for which agents must be compensated. Second, they affect the incentive constraint. The principal should offer a (perfect) team contract to reduce the expected loss from inequality but a RPE contract to make use of the incentive effect. Therefore, in contrast to the benchmark case where $u$ can be freely set to minimize risk, now the optimal choice of $u$ involves solving a threefold trade-off between risk, inequality, and incentives.

In Section 3.1 we first analyze the symmetric case with $\beta = 1$. This simplifies the problem considerably because—as can be seen in equation (11)—symmetric inequity aversion causes no incentive effect. The optimal $u$ must thus trade-off only risk and inequality. In Section 3.2, we then analyze the general case with $\beta \in [-1,1]$.

### 3.1 The Symmetric Case

With $\beta = 1$, agents suffer as much from being ahead as from being behind. As in the benchmark case, the incentive constraint is given by $\psi'(a) = v$. The principal’s program is again solved sequentially. First, the principal determines the optimal $u$,
given \( v \). In the benchmark case, \( u \) is set to minimize the agents’ risk exposure. With \( \alpha > 0 \), however, a trade-off between risk and inequality reduction arises. Maximization of the principal’s objective function (7) with respect to \( u \) then yields

\[
\begin{align*}
    u^* &= \min \left\{ -v\rho + \frac{2\alpha \eta \sigma_v}{\pi} \sqrt{1 - \frac{\rho}{\pi}}, v \right\}.
\end{align*}
\]  

(15)

The derivation of \( u^* \) can be found in Appendix A.3. If \( \rho > 0 \), reduction of agent \( i \)'s risk exposure requires a negative weight \( u \) on agent \( j \)'s performance. Yet to reduce expected inequality, \((v - u)\) should be small, hence \( u \) be positive. If \( \rho < 0 \), both risk and inequality reduction require a positive \( u \). The risk-inequality trade-off nevertheless arises. To reduce risk, \( u \) should equal \( v \) only if there is perfect negative correlation, while \( u = v \) is always optimal with respect to inequality reduction.

Solution (15) renders the risk-inequality trade-off precise. The larger the agents’ degree \( \alpha \) of inequity aversion relative to the degree \( \eta \) of risk aversion, the larger the deviation from the risk minimizing solution. If \( \alpha \) is sufficiently large, the optimal \( u \) can be positive (team contract) even though the error terms are positively correlated such that a negative \( u \) (RPE) would be optimal with purely self-interested agents. It can even be optimal to set \( u^* = v \). In this case, the agents’ wage depends only on aggregate outcome to fully eliminate inequality. Notice, that \( u^* \) never exceeds \( v \). To reduce risk, the largest optimal value for \( u \) equals \( v \), and further increasing \( u \) not only increases risk but also expected inequality, which is zero at \( u = v \). Finally, it can be seen from (15) that there is no deviation from the risk minimizing contract if \( \rho \in \{-1, 1\} \), i.e., if the performance measures are perfectly correlated. The reason is that with perfect positive correlation, there is no inequality as the agents always produce the same output and thus always receive the same wage. With perfect negative correlation, the perfect team contract is optimal with purely self-interested agents already, and it also ensures that no inequality arises. We summarize these results in the following proposition.

**Proposition 1** If agents are symmetrically inequity averse and performance measures are not perfectly correlated, then there is a trade-off between (i) risk reduction by use of RPE and (ii) reduction of expected inequality by use of team contracts. If the degree \( \alpha \)
of regard for others is strong relative to the degree \( \eta \) of risk aversion, a (perfect) team contract can be optimal even if performance measures are positively correlated.

As argued in the introduction, standard economic theory provides a rationale for RPE based on risk reduction, whereas empirical support for RPE is scarce. Existing explanations are reduced incentives for cooperation and increased incentives for sabotage (Lazear 1989, Itoh 1991). Our paper shows that even if agents cannot affect each other’s performance measures, other-regarding preferences can explain why principals refrain from RPE. Solution (15) also shows that Holmström’s (1979) sufficient statistics result does not hold with other-regarding agents (see also Itoh (2004) and Englmaier and Wambach (2010)). According to the result, optimal contracts should condition on a measure if and only if it contains information about an agent’s performance. With other-regarding preferences the result is violated, because even if the agents’ performance measures are uncorrelated, it is optimal to condition an agent’s wage also on the other agent’s measure to trade-off inequality and risk.

In the second step of the solution of the principal’s program, the optimal incentive intensity is derived. Given \( u^* \), this yields

\[
v^* = \max \left[ \frac{1 - 2\alpha \sigma_\varepsilon \sqrt{\frac{1 - \rho}{\bar{\pi}}} (1 + \rho) \psi''}{1 + \eta \sigma_\varepsilon^2 (1 - \rho^2) \psi''}, \frac{1}{1 + 2\eta \sigma_\varepsilon^2 (1 + \rho) \psi''} \right]. \tag{16}
\]

The left term applies in case \( u^* < v^* \), the right term in case \( u^* = v^* \). The derivation of \( v^* \) can be found in Appendix A.4. The next proposition follows directly from the comparison of equation (16) and the optimal incentive intensity for purely self-interested agents in the benchmark case.

**Proposition 2** If agents are symmetrically inequity averse and if performance measures are not perfectly correlated, then the optimal incentive intensity is strictly lower for other-regarding agents than for purely self-interested agents.

**Proof:** There exists a threshold level \( \tilde{\alpha} = \eta \sigma (1 + \rho) / (2 \sqrt{(1 - \rho)} / \bar{\pi} (1 + 2\eta \sigma_\varepsilon^2 (1 + \rho) \psi'')) \) such that if \( \alpha < \tilde{\alpha} \), then \( u^* < v^* \), so \( v^* \) is given by the left entry in (16). If \( \alpha = 0 \), then \( v^* = v^s \), but \( v^* \) is strictly decreasing in \( \alpha \) while \( v^s \) is unaffected such that if \( \alpha > 0 \),
then \( v^* < v^s \). If \( \alpha \geq \bar{\alpha} \), then \( u^* = v^* \) and \( v^* \) is given by the right entry in (16). Independently of \( \alpha \) we then have \( v^* < v^s \) as \( 2(1 + \rho) > (1 - \rho^2) \).

The intuition behind the result is twofold. First, a marginal increase in \( v \) necessitates a (weakly) higher risk compensation for other-regarding than for purely self-interested agents. This follows from the trade-off between risk and expected inequality that arises only with other-regarding agents. Second, since \( u^* \leq v \), a marginal increase in \( v \) (weakly) increases expected inequality for which only other-regarding agents must be compensated. Notice finally that optimal contracts for purely self-interested and other-regarding agents coincide if the performance measures are perfectly correlated. The optimal incentive intensity is thus never higher for other-regarding than for purely self-interested agents. This provides a behavioral explanation why incentives are often weaker than predicted by the standard model. Holmström and Milgrom (1991) suggest multi-tasking as an explanation. Our paper shows that even with single-tasking, optimal incentives might be flatter than predicted by the standard moral hazard model.

### 3.2 The General Case

We now turn to the general case with \( \beta \in [-1, 1] \). The important difference to the symmetric case is that other-regarding preferences now have an incentive effect. To solve the principal’s program (7)-(9), we first determine the optimal \( u \). This yields

\[
\begin{align*}
    u^* &= \min \left[ \Phi \left( -v \rho + \frac{\alpha(1 + \beta)}{\eta \sigma \varepsilon} \sqrt{\frac{1 - \rho}{\pi}} - \frac{\lambda(1 - \chi v)}{\eta \sigma^2 \psi''} \right), v \right] \tag{17}
\end{align*}
\]

where

\[
\lambda = \frac{\alpha(1 - \beta)}{2 + \alpha(1 - \beta) \gamma}, \quad \chi = \frac{2 + \alpha(1 - \beta)}{2 + \alpha(1 - \beta) \gamma}, \quad \text{and} \quad \Phi = \frac{\eta \sigma^2 \psi''}{\eta \sigma^2 \psi'' + \lambda^2}. \tag{18}
\]

The derivation of \( u^* \) can be found in Appendix A.5. Notice first that for \( \beta = 1 \), solution (17) reduces to the solution of the symmetric case (15). But for \( \beta \in [-1, 1) \), there are now three effects that determine the sign of \( u^* \). First, to filter out common noise, \( u \) should be negative if there is positive correlation and vice versa, which is reflected by the first term in the round brackets. Second, to lower expected inequality, \( u \) should
be positive, which is reflected by the second term. Third, as summarized in Table 1, a negative \( u \) has a positive incentive effect while the effect is reduced or, depending on the size of \( \gamma \), even negative with team contracts. This is reflected by the third term.

As in the symmetric case of \( \beta = 1 \), with perfect positive correlation there is never inequality and other-regarding preferences have no impact of the principal’s program. In contrast to the symmetric case, this does not hold with perfect negative correlation. While \( u^* = v \) eliminates risk and inequality, the incentive effect of other-regarding preferences is now negative. Only if \( \gamma = 0 \), i.e., effort costs are not accounted for at all, the incentive effect is neutral. Therefore, if \( \gamma > 0 \), then the trade-off between risk, inequality, and incentive effect arises even with perfect negative correlation. The following proposition summarizes these findings.

**Proposition 3** If agents have asymmetric other-regarding preferences and there is no perfect positive correlation, then there exists a threefold trade-off between (i) risk reduction by use of RPE, (ii) reduction of expected inequality by use of team contracts, and (iii) the incentive effect of other-regarding preferences, which is always positive with RPE but can be negative with team contracts.

In the second step of the solution of the principal’s program, the optimal incentive intensity is derived. Given \( u^* \) as derived in (17), this yields

\[
v^* = \frac{\chi - \alpha (1 + \beta) \sigma_{\xi} \sqrt{1 - \rho} \left( 1 + \left( \rho - \frac{\chi}{\eta \sigma_{\xi}^2} \right) (2 - \Phi) \Phi \right) \psi'' + \lambda \left( \rho - \frac{\chi}{\eta \sigma_{\xi}^2} \psi'' \right) (1 - \Phi) \Phi}{\chi^2 + \eta \sigma_{\xi}^2 \left( 1 - \left( \frac{\chi}{\eta \sigma_{\xi}^2} \right)^2 \right)^2 (1 - \Phi) \Phi - \rho^2 (2 - \Phi) \Phi } \psi'' + \rho \lambda \chi (1 + 2(1 - \Phi)) \Phi}
\]

in case \( v^* < u^* \) and

\[
v^* = \frac{\chi - \lambda}{(\chi - \lambda)^2 + 2 \eta \sigma_{\xi}^2 (1 + \rho) \psi''}
\]

(19)

in case \( v^* = u^* \). Recall the definitions of \( \lambda \), \( \chi \), and \( \Phi \) in (18). The derivation of \( v^* \) can be found in Appendix A.5. Notice first that for \( \beta = 1 \), (19) reduces to the solution of the symmetric case (16). Furthermore, if \( u^* = v^* \) is a solution with \( \gamma = 0 \), then solutions (19) and (16) again coincide as \( \chi - \lambda = 1 \) if \( \gamma = 0 \). The reason is that the incentive effect of inequity aversion is neutral if \( \gamma = 0 \) and \( u^* = v^* \).
Figure 1 illustrates the difference between the symmetric and the general case. The panels show for different levels of $\alpha$ the optimal incentive coefficients $u^*$ and $v^*$ as functions of $\beta$. Here we consider the case with inequity averse agents, $\beta \in [0,1]$. In all panels we have set $\rho = 0.5$, i.e., there is positive correlation, and $\gamma = 0.5$, i.e., effort costs are to some extent accounted for. The parameters, $\eta$, $\sigma_\varepsilon$, and $\psi''$ are set to unity.

The left panel shows the standard case with $\alpha = 0$, which is independent of $\beta$. The negative correlation renders RPE optimal: $u^*$ is shown by the horizontal line at $-0.29$. The horizontal line at $0.57$ shows the optimal incentive intensity. With uncorrelated errors terms the optimal incentive intensity were $0.5$, but here RPE allows to reduce the agents’ risk exposure, which in turn leads to a higher optimal incentive intensity.

In the middle panel we set $\alpha = 0.3$. The lower, increasing line shows $u^*$. The symmetric case with $\beta = 1$ is represented by the right end point of the line. Despite positive correlation, here the risk-inequality trade-off renders a team contract ($u^* > 0$) optimal. However, $u^*$ decreases for lower values of $\beta$ and RPE is optimal for sufficiently low values. The reason is the incentive effect of other-regarding preferences that arises if $\beta < 1$. Given the parameters, there is a positive incentive effect: an agent decreases his expected loss from inequality by working harder. The incentive effect becomes more weight in the risk-inequality-incentive trade-off the lower $\beta$, and it is stronger, the lower $u$. Hence the optimal $u$ decreases when $\beta$ decreases. The upper, decreasing line shows the optimal incentive intensity. At $\beta = 1$, the optimal incentive intensity is $0.37$. Compared to the case with $\alpha = 0$, we have a team contract ($u^* > 0$), which increases
the agents’ risk exposure so that a lower incentive intensity is optimal. However, as $u^*$ decreases with decreasing $\beta$, two effects arise. First, $u^*$ is more in line with the risk minimizing $u$ and, second, there is a positive incentive effect that causes agents to work harder for every given $v$. Both effects cause $v^*$ to increase as $\beta$ decreases.

In the right panel we set $\alpha = 3$. Now for all of values of $\beta$, the optimal $u$ is so high that $u^* = v^*$, i.e., a perfect team contract is optimal. In contrast to the middle panel, the optimal incentive intensity now decreases as $\beta$ decreases. The reason is that the incentive effect of other-regarding preferences is negative under a perfect team contract; only with $\gamma = 0$ it is neutral. Since the incentive effect becomes more negative as $\beta$ decreases, the optimal incentive intensity decreases as well.

Finally, we would like to stress one testable implication of the general version of the model. We found that the more the agents account for effort costs in their comparisons, the less favorable the incentive effect of other-regarding preferences. Importantly, with team contracts it can become negative (see Table 1). Hence, the more the agents account for effort costs, the weaker the trend towards team contracts that we identified in this paper. When should we expect agents to account for effort costs? It appears reasonable that effort costs play a greater role if they can be easily observed, e.g., because agents work physically close to each other. If however agents cannot observe each others’ costs, as is the case, e.g., with sales forces whose members work in geographically separated areas, these costs might play a less salient role in social comparisons.

4 Conclusion

This paper proposes a behavioral explanation for the empirically observed lack of RPE. We analyze a principal-multi agent model and assume that the agents have other-regarding preferences that can take the form of inequity aversion or status preferences. We show that the principal’s decision to offer a team or a RPE contract involves solving a threefold trade-off between risk, inequality, and the incentive effect of other-regarding preferences. The model shows that team contracts can be optimal even if the agents’ performance measures are positively correlated such that RPE is optimal with purely
self-interested agents and even though the incentive effect of other-regarding preferences is always positive with RPE but can be negative with team contracts. Moreover, the model shows that optimal incentive contracts for other-regarding agents can be low-powered as compared to contracts for purely self-interested agents. This result can thus contribute to our understanding of why there is less reliance on explicit incentives than suggested by standard principal-agent models.

Appendix

A.1 Derivation of the First-Order Condition in Equation (10)

Equation (10) is derived by differentiating the CE as stated in (6).

\[
\frac{\partial CE}{\partial a_i} = v - \psi'(a_i) - \frac{\partial E[I_i]}{\partial a_i} \\
= v - \psi'(a_i) - ((v - u)(\alpha \int_{-\Delta}^{\Delta} (0 - \frac{\partial \Delta}{\partial a_i}) f(z) dz - \alpha(\Delta - \Delta) \frac{\partial \Delta}{\partial a_i}) \\
+ \alpha \beta \int_{-\infty}^{\Delta} (\frac{\partial \Delta}{\partial a_i} - 0) f(z) dz + \alpha \beta (\Delta - \Delta) \frac{\partial \Delta}{\partial a_i})) \\
= v - \psi'(a_i) - ((v - u)(\alpha \int_{-\Delta}^{\Delta} (1 + \gamma \frac{\psi'(a_i)}{v - u}) f(z) dz \\
+ \alpha \beta \int_{-\infty}^{\Delta} (1 - \gamma \frac{\psi'(a_i)}{v - u}) f(z) dz)) \\
= v - \psi'(a_i) + \alpha(v - u - \gamma \psi'(a_i)) \left( \int_{-\Delta}^{\Delta} f(z) dz - \beta \int_{-\infty}^{\Delta} f(z) dz \right)
\]

(A1)

A.2 Derivation the Expected Loss from Inequality in Equation (14)

Equation (14) is derived by integration of the expression in (3), given \( \Delta = 0 \).

\[
L_i = (v - u) \left( \alpha \int_0^\infty z f(z) dz + \alpha \beta \int_{-\infty}^0 -z f(z) dz \right) \\
= \alpha(1 + \beta)(v - u) \int_0^\infty z f(z) dz = \alpha(1 + \beta)(v - u) \frac{1}{\sqrt{2\pi\sigma_z}} \int_0^\infty z e^{-\frac{z^2}{2\sigma_z^2}} dz \\
= \lim_{y \to \infty} \frac{\alpha(1 + \beta)(v - u)}{\sqrt{2\pi\sigma_z}} \int_0^\infty z e^{-\frac{z^2}{2\sigma_z^2}} dz \\
= \alpha(1 + \beta)(v - u) \frac{1}{\sqrt{2\pi\sigma_z}} \left[ -\sigma_z e^{-\frac{y^2}{2\sigma_z^2}} \right]_0^y \\
= \alpha(1 + \beta)(v - u) \frac{1}{\sqrt{2\pi\sigma_z}} \sigma_z = \alpha(1 + \beta)(v - u) \sqrt{\frac{1 - \rho}{\pi}} \sigma_z
\]

(A2)
A.3 Derivation of $u^*$ in Equation (15)

The principal maximizes his objective function (7) with respect to $u$, where $L_i(a_i, a_j, v, u)$ is given by (14). The first-order condition is then given by

$$(1 - \psi'(a_i^*)) \frac{\partial a_i^*}{\partial u} - 0.5\eta(2u + 2v\rho)\sigma^2_\varepsilon + \alpha(1 + \beta)\sqrt{\frac{1 - \rho}{\pi}} \sigma_\varepsilon = 0. \quad (A3)$$

As $\beta = 1$, the incentive constraint is given by $\psi'(a_i^*) = v$, i.e., $u$ does not affect the agents’ effort choices and $\partial a_i^*/\partial u = 0$. Rearranging then yields the left entry in (15).

A.4 Derivation of $v^*$ in Equation (16)

Substituting $u^* = -v\rho + (2\alpha/\eta\sigma_\varepsilon)\sqrt{(1 - \rho)/\pi}$ into the objective function (7) yields

$$
2v \left(-v\rho + \frac{2\alpha}{\eta\sigma_\varepsilon}\sqrt{\frac{1 - \rho}{\pi}}\right)\rho\sigma^2_\varepsilon - \alpha(1 + \beta) \left(v + v\rho - \frac{2\alpha}{\eta\sigma_\varepsilon}\sqrt{\frac{1 - \rho}{\pi}}\right)\sqrt{\frac{1 - \rho}{\pi}}\sigma_\varepsilon
= a_i^* - \psi(a_i^*) - 0.5\eta\sigma^2_\varepsilon(1 - \rho^2)v^2 + \frac{2\alpha}{\eta} \frac{1 - \rho}{\pi} - 2\alpha\sigma_\varepsilon\sqrt{\frac{1 - \rho}{\pi}}(1 + \rho)v.
$$

Maximization with respect to $v$ yields the first-order condition

$$
(1 - \psi'(a_i^*)) \frac{\partial a_i^*}{\partial v} - \eta\sigma^2_\varepsilon(1 - \rho^2)v - 2\alpha\sigma_\varepsilon\sqrt{\frac{1 - \rho}{\pi}}(1 + \rho) = 0.
$$

Substituting the incentive constraint $\psi'(a_i^*) = v$ and using $\partial a_i^*/\partial v = 1/\psi''$, we get

$$1 - v - \eta\sigma^2_\varepsilon(1 - \rho^2)\psi''v - 2\alpha\sigma_\varepsilon\sqrt{\frac{1 - \rho}{\pi}}(1 + \rho)\psi'' = 0
\Leftrightarrow
v^* = \left(1 - \frac{2\alpha\sigma_\varepsilon}{\eta}\sqrt{\frac{1 - \rho}{\pi}}(1 + \rho)\psi''\right)/(1 + \eta\sigma^2_\varepsilon(1 - \rho^2)\psi''). \quad (A6)$$

In case $u^* = v$, the objective function (7) simplifies to

$$a_i^* - \psi(a_i^*) - 0.5\eta(v^2 + v^2 + 2v^2\rho)\sigma^2_\varepsilon. \quad (A7)$$
Maximization with respect to \( v \) then yields the first-order condition

\[
(1 - \psi'(a_i^*)) \frac{\partial a_i^*}{\partial v} - 2\eta\sigma^2_x(1 + \rho)v = 0 \quad \Leftrightarrow \quad v^* = 1 / (1 + 2\eta\sigma^2_x(1 + \rho)\psi'').
\] (A8)

The optimal \( v \) is thus given by equation (16).

### A.5 Derivation of Equations (17) and (19) in the General Case

Maximization of the principal’s objective function (7) with respect to \( u \) yields condition (A3). The incentive constraint in the general case is given by (11), which we can write as \( \psi'(a_i^*) = \chi v - \lambda u \); recall the definitions of \( \lambda \) and \( \chi \) in (18). Substituting the incentive constraint and using \( \partial a_i^*/\partial u = -\lambda/\psi'' \) yields the first-order condition

\[
-\left(1 - \chi v + \lambda u\right) \frac{\lambda}{\psi''} - 0.5\eta(2u + 2\nu\rho)\sigma^2_x + \alpha(1 + \beta)\sigma_x \sqrt{1 - \frac{\rho}{\pi}} = 0.
\] (A9)

Rearranging then results in the expression in (17). To derive \( v^* \) for the case \( v^* < u^* \), we substitute \( u^* \) as given in (17) into the objective function (7). This yields

\[
a_i^* - \psi(a_i^*) - 0.5\eta(\nu^2 + \Phi^2) \left( -\rho\nu + \frac{\lambda(1 - \chi\nu)}{\psi''} \right) \rho\sigma^2_x - (v - \Phi \left( -\rho\nu + \frac{\lambda(1 - \chi\nu)}{\psi''} \right)) \eta\sigma^2_x \Gamma = 0.
\] (A10)

where \( \Gamma \equiv \left( \frac{\alpha(1 + \beta)}{(\eta\sigma_x)} \right) \sqrt{(1 - \rho)}/\pi \). Differentiating with respect to \( v \) then yields the first-order condition

\[
(1 - \psi'(a_i^*)) \frac{\partial a_i^*}{\partial v} - 0.5\eta(2v + 2\rho^2 v\Phi^2 - \frac{2\lambda^2\chi(1 - \chi v)}{(\eta\sigma^2_x\psi'')^2} \Phi^2 - 2\rho\Gamma\Phi^2
\]
\[
+ \frac{2\rho\lambda(1 - 2\chi v)}{\eta\sigma^2_x\psi''} \Phi^2 - \frac{2\lambda(1 - 2\chi v)}{\psi''} \Phi^2 - 2\rho\Gamma\Phi^2 - \frac{2\rho\lambda(1 - 2\chi v)}{\eta\sigma^2_x\psi''} \Phi^2
\]
\[
- (1 + \rho\Phi - \frac{\lambda\chi}{\eta\sigma^2_x\psi''\Phi}) \eta\sigma^2_x \Gamma = 0.
\] (A11)
Simplifying the expression, substituting the incentive constraint \( \psi'(a_i^*) = \chi v - \lambda u^* \), where \( u^* \) is given by equation (17), and using \( \partial a_i^*/\partial v = \chi/\psi'' \) we get

\[
(1 - \chi v + \lambda \Phi) \left( -\rho v + \Gamma - \frac{\lambda(1 - \chi v)}{\eta \sigma_{\varepsilon}^2 \psi''} \right) \frac{\chi}{\psi''} - ((1 - \rho^2 (2 - \Phi) \Phi) v
- \frac{\lambda^2 \chi (1 - \chi v)}{(\eta \sigma_{\varepsilon}^2 \psi''^2)^2} \Phi^2 - \frac{\rho \lambda (1 - 2 \chi v)}{\eta \sigma_{\varepsilon}^2 \psi''} (1 - \Phi) \Phi + \frac{\lambda \chi (1 - \chi v)}{\eta \sigma_{\varepsilon}^2 \psi''^2} \Phi^2 + \rho \Gamma (1 - \Phi) \Phi \eta \sigma_{\varepsilon}^2
- (1 + \rho \Phi - \frac{\lambda \chi}{\eta \sigma_{\varepsilon}^2 \psi''} \Phi) \eta \sigma_{\varepsilon}^2 \Gamma = 0. \tag{A12}
\]

Solving for \( v \) then yields the expression in equation (19) for the case \( v^* < u^* \). We finally have to determine the optimal \( v \) for the case \( u^* = v \). With \( u^* = v \), the first-order condition is given by

\[
(1 - \psi'(a_i^*)) \frac{\partial a_i^*}{\partial v} - 2\eta(1 + \rho)v \sigma_{\varepsilon}^2 = 0 \tag{A13}
\]

Substituting the incentive constraint, which is now given by \( \psi'(a_i^*) = (\chi - \lambda)v \), and using \( \partial a_i^*/\partial v = (\chi - \lambda)/\psi'' \) yields

\[
(1 - (\chi - \lambda)v) \frac{\chi}{\psi''} - 2\eta(1 + \rho)v \sigma_{\varepsilon}^2 = 0. \tag{A14}
\]

Rearranging then results in the expression in equation (19) for the case \( v^* = u^* \).

References


