Parenting with Style: Altruism and Paternalism in Intergenerational Preference Transmission

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Abstract

We construct a theory of intergenerational preference transmission that rationalizes the choice between alternative parenting styles (related to Baumrind 1967). Parents maximize an objective function that combines Beckerian and paternalistic altruism towards children. They can affect their children’s choices via two channels: either by influencing their preferences or by imposing direct restrictions on their choice sets. Different parenting styles (authoritarian, authoritative, and permissive) emerge as equilibrium outcomes, and are affected both by parental preferences and by the socioeconomic environment. We consider two applications: patience and risk aversion. We argue that parenting styles may be important for explaining why different groups or societies develop different attitudes towards human capital formation, entrepreneurship, and innovation.

Keywords: Intergenerational Preference Transmission, Altruism, Paternalism, Entrepreneurship, Innovation

JEL Codes: D10, J10, O10, O40
1 Introduction

The debate about child-rearing practices has a long history. For instance, the Bible recommends strict parenting, including generous dispensing of corporal punishment.\footnote{“He who spares the rod hates his son, but he who loves him is careful to discipline him . . .” (Proverbs 13:24); or “Folly is bound up in the heart of a child, but the rod of discipline will drive it far from him.” (Proverbs 22:15)} Hard discipline and rigor are also advocated by John Locke in \textit{Some Thoughts Concerning Education}, although he argues that children should be treated progressively as reasoning beings as they grow older.\footnote{“If you would have him stand in awe of you, imprint it in his infancy; . . . For liberty and indulgence can do no good to children; their want of judgment makes them stand in need of restraint and discipline; and on the contrary, imperiousness and severity is but an ill way of treating men, who have reason of their own to guide them . . .” (Locke 1800, p. 40).} Well-being during childhood is hardly a concern to the British philosopher, who views child-rearing as a purely instrumental process aimed to take children out of the stage of immaturity as quickly as possible, and to forge a strong adult personality. Two centuries later, Montessori (1912) takes a very different stand: in her view, children have a spontaneous drive towards learning and developing from a tender age. The educators’ task is to nurture this innate drive by letting children act freely within a responsive and well-structured environment. In recent decades, the debate has raged with an unrelenting intensity, often swinging between opposite extremes. If radical anti-authoritarian parenting and schooling practices became fashionable in the 1960s and the 1970s, the “Tiger Mom” (Amy Chua) has recently become the icon of a pushy rule-oriented parenting style which is supposedly at the root of the success of many Asian children.

Developmental psychologists have categorized parenting styles and studied their effects on child development, following the seminal contributions of Baumrind (1967, 1971 and 1978). She proposed a threefold classification: \textit{authoritarian}, \textit{permissive} and \textit{authoritative}. A number of studies have since then tried to identify causal effects of child-rearing practices on children’s preferences, personalities and outcomes (see, e.g., Aunola, Stattin, and Nurmi 2000, Chan and Koo 2011, Darling and Steinberg 1993, Dornbush et al. 1987, Spera 2005, Steinberg et al. 1991).
Until recently, parenting style has remained outside the domain of mainstream economics. However, a growing body of literature has recently shown that preference heterogeneity is a driver of important social and economic phenomena such as human capital accumulation, social capital, entrepreneurship, and innovation. Moreover, there is evidence that preference traits that matter for economic outcomes can be molded by parents and educators (see, e.g., by Heckman, Stixrud, and Urzua 2006 and Algan, Cahuc, and Shleifer 2011). In spite of this, economic theories of parenting styles remain scant.

In this paper, we propose an economic theory of preference formation that casts light on the determinants and effects of parenting style. We view the choice of parenting style as the result of the interaction between parental preferences and the characteristics of the socio-economic environment. The parents’ motives are modeled as a combination of pure Beckerian altruism (i.e., the maximization of children’s well-being) and of a paternalistic element. Paternalism captures the extent to which parents disagree with their children’s natural preferences and inclinations, and try to interfere with their choices—a common experience in parenthood. The extent of paternalism is modeled as a deep (exogenous) preference parameter, heterogeneously distributed across parents. However, parents may also differ from one another in other dimensions of preferences (e.g., risk aversion) that are endogenous outcomes of their own upbringing, and that can affect their choice of parenting style.

We formalize our ideas through a dynamic model with overlapping generations, where parents take both economic and child-rearing decisions affecting their children’s current and future welfare. Parents can affect their children’s choices in two ways: either by molding their preferences or by imposing direct constraints on their choices. Echoing Baumrind’s classification, we define as permissive a parenting style that allow children to make free choices according to their natural inclinations. We define as authoritative a parenting style emphasizing parents’ intervention on children’s preferences so as to induce choices that parents regard as desirable and conducive to future success in life. In our model, the cost of such a style is that it may yield a less happy childhood. Finally, we define as authoritarian a style such that parents accept as a fact of life that adults and children
have different preferences, but disallow choices they do not approve of.

We provide a dynastic formulation of the problem. We show that paternalism introduces a time inconsistency in the dynasty’s decision problem stemming from the repeated disagreement between old and young decision makers who are simultaneously alive. The analysis, carried out via recursive methods, yields a simple intuitive condition that pins down the optimal policy rules as functions of the extent of paternalism and of the endogenous state vector (preferences, physical or human wealth, etc.).

We apply the general model to the analysis of the parental transmission of patience and risk aversion, two preference traits that have been shown to be especially important for the determination of individual and aggregate outcomes. In the case of patience, the focal point is the innate tendency of parents to care more about their children’s future (adult) felicity than do the children themselves, as witnessed, for instance, by the relentless struggle of many parents to push their reluctant children to study diligently to ensure success in later life. Likewise, risk tolerance has been argued to be an essential driver of entrepreneurship in modern societies. The tendency of young children and adolescents to accept risks that parents do not approve of is also well documented. However, instilling into children an excessive fear of risk can inhibit their willingness to seize valuable opportunities arising later in life.

Together with illustrating properties of the general model, the two applications yield a number of independent economic insights. For instance, the choice of parenting style interacts with the wealth accumulation of the dynasties. The relationship between transfers from parents to children and paternalism turns out to be U-shaped: intermediate levels of paternalism are associated with the lowest transfers. The reason is that, in either of the polar cases, parents and children end up agreeing (for opposite reasons) on the choices made by the young. This encourages parents to transfer resources to their children.

In the case of risk aversion, the choice of parenting styles hinges on the inter-

\footnote{For instance, in our previous work (Doepke and Zilibotti 2008) we argue that investment in patience has been key for the development of a spirit of capitalism at the time of the British Industrial Revolution.}
action between paternalism and the riskiness of the surrounding environment. On the one hand, parents would like their children to stay away from juvenile risks (such as gangs or street drugs). On the other hand, they would like them to be capable, later in life, to seize entrepreneurial opportunities. However, since preference traits are formed in childhood and persist throughout adult age, parents face a trade-off. The crux is the exposure to juvenile risk. If juvenile risk is pervasive (as, for instance, in the ghettoes of American cities), parents may opt to instill into their children a strong risk aversion, so that they avoid trouble. In safer environments (e.g., wealthy suburbs), parents would instead encourage risk tolerance and an entrepreneurial attitude. An important distinction is that between exogenous and endogenous risk. If juvenile risk is unavoidable (e.g., because the family leaves in a country plagued by war and terrorism), then risk tolerance is valuable, since it helps the child to cope with an uncertain life without suffering too much. On the other hand, if juvenile risk-taking is largely under the control of the child (e.g., he can choose whether or not to get involved with street gangs), then altruistic parents would emphasize the value of playing it safe. Private and public institutions affecting the return and risk of entrepreneurial activities have also an effect on the distribution of parenting styles in equilibrium.

Our paper relates to different streams of literature. There exists a limited economic literature on parenting styles, influenced by the seminal contributions of Becker and Tomes (1979) and Mulligan (1997). Weinberg (2001) focuses on parents’ influence on their children’s behavior through pecuniary incentives. He argues that, due to the scarcity of means, low-income parents have limited access to such incentives, and therefore resort to authoritarian methods such as corporal punishment. Such authoritarian methods, in turn, are at the root of the lower success of their children, and perpetuate the initial inequality. Our theory focuses on a broader set of parenting styles, and ignore, for simplicity, pecuniary costs of parenting. Lizzeri and Siniscalchi (2008) assume that altruistic parents are better informed than their children about the consequences of certain actions. They can then intervene to protect them from the consequences of ill-informed choices. However, this comes at the cost of reducing their ability to learn from experience. Their paper focuses on a different dimension (information accumulation) of parenting practices and is therefore also complementary to ours. Bhatt
and Ogaki (2012) construct a model of tough love in which parents evaluate the child’s lifetime utility with a constant high discount factor, whereas the child’s patience is assumed to be inversely related to their consumption. In this environment, parental transfers are distorted strategically to affect the child’s discount factor. Different from our paper, these authors postulate a relationship between preferences and consumption, and derive implications on parents’ behavior. In a recent empirical study, Cosconati (2009) estimates a two-period model of parenting style in which children differ in their predisposition to human capital accumulation, and argues that this affects the optimal choice of parenting style.

Our paper is also related more generally to the vast literature on cultural transmission and norms including, among others, Bisin and Verdier (2001), Hauk and Saez-Marti (2002) and Tabellini (2008 and 2010). A common assumption in this literature is imperfect empathy. Imperfectly empathic parents desire, by assumption, that their children adopt their own cultural traits (e.g., religion, culture). The intensity of their effort to shape their children’s views determines the probability of successful transmission. When transmission fails, children copy the trait of a random member of the population. Different from their approach, our model is framed in a dynamic dynastic model, where parents are both altruistic, namely, they care about the discounted utility of their children, and paternalistic, namely, they potentially disapprove of some of the choices made by their young children. In our model, even fully paternalistic parents have no exogenous drive to reproduce their own traits. Rather, preferences may be persistent across cohorts within dynasties as an equilibrium outcome. A more thorough review of the similarities and differences between the two approaches can be found in Saez-Marti and Zilibotti (2008).

In our model, authoritative parenting distorts the child’s preferences away of those that would maximize their welfare in a utilitarian sense. Such intervention can therefore be interpreted as instilling a form of “guilt” that induces the child to behave “responsibly”, and in particular to choices that adults view as inappropriate. For instance, the responsible child is induced to study diligently for an exam instead of playing joyfully with friends. This feature makes our theory closely related to the recent paper by Fernández-Villaverde, Greenwood,
and Guner (2011), where altruistic parents choose how strongly to stigmatize sex, trading off the marginal gains from instilling the taboo against its costs. The focus of their paper is how an episode of technical change, i.e., the introduction of modern contraception, has changed over time the benefits, and thus the incidence, of the taboo. However, they do not discuss alternative parenting styles.

Our paper is also related to the recent literature on time-inconsistent decision making (Laibson 1997) and temptation. In particular, Gul and Pesendorfer (2003) propose an axiomatic decision theory of a rational agent who is subject to a temptation problem. Namely, the choice set includes elements that would appeal to him, but whose choice he anticipates he would regret. The agent chooses optimally whether to succumb to temptation or resist, knowing that even resisting induces a utility loss (e.g., not ordering an appetizing dessert from the menu of a restaurant). In their environment, the decision maker may wish to restrict the choice set ex-ante. In our model, similarly, an adult may find it optimal to restrict the choice set of the next member of the dynasty. Moreover, the alternative option to mold the child’s preferences in order to prevent him from succumbing to what the parent regards as unhealthy choices is related to the cost of resisting to temptation in Gul and Pesendorfer. The reason is that manipulating the child’s preferences entails a utility loss both to the child and to the adult himself, since the latter is altruistic.

Our application to patience is related to the recent empirical literature emphasizing the importance of patience for savings and human capital investment (see, e.g., Heckman, Stixrud, and Urzua 2006, Mischel, Shoda, and Rodriguez 1992, Reyes-Garcia et al. 2007). Similarly, the application to endogenous risk aversion relates to the literature on the determinants of entrepreneurship, namely preferences, and in particular risk tolerance. The selection of risk-tolerant individuals into entrepreneurship as emphasized in the classical work of Knight (1921), and formalized by Kihlstrom and Laffont (1979) (see also, more recently, Vereshchagina and Hopenhayn 2009). The empirical literature has emphasized the existence of substantial variation in risk tolerance across different groups of people (see e.g., Guiso and Paiella 2008, Bonin et al. 2009), and a large correlation between parents’ and children’s attitude to risk (Dohmen et al. 2011).
In the following section, we develop our general framework of altruism and paternalism in a dynastic model. In Section 3, we apply the model to the transmission of time preferences across generations, whereas Section 4 analyzes the determination of risk preferences. Section 5 concludes. All proofs are contained in the mathematical appendix.

2 A Dynastic Model of Paternalism

2.1 The Decision Problems of Parents and Children

The model economy is populated by overlapping generations of two-period lived agents. Each old agent (parent) has one offspring (child), and parents are altruistic towards their children. The period utility functions depend on a preference vector, \( a \in A \), and on a non-preference vector consisting of choices in young age, \( x^y \), and in old age, \( x \). Children’s preferences can be influenced by parents. The preference vector \( a \) is acquired in young age and remains constant throughout an individual’s lifetime. However, age has also an independent effect on preferences and choice. For instance, young agents may be intrinsically less risk averse than old agents. We capture age-specific differences by assuming that, in general, \( U^y(x, a) \neq U^o(x, a) \), where \( U^y(x^y, a) \) and \( U^o(x, a) \) denote, respectively, the period utility functions of the young and of the old.

The young only make economic decisions, \( x^y \in X^y \), where \( X^y \) is the choice set of the young. Such decisions may have consequences on the old-age utility (e.g., savings or human-capital investments determine wealth in old age). When old, agents turn into parents, and make three sets of decisions. First, a second round of economic choices (e.g., inter-vivos transfers to their children), \( x \in X \), where \( X \) is the feasible choice set. Second, they mold their children’s preferences, \( a' \in A \). Third, they may impose restrictions on the choice set from which their children will be able to choose, \( X^y \in X \), where \( X \) is set of feasible choice sets from which parents can choose. We assume that parents can always choose not to impose any restriction. More formally, \( X^{FREE} = \{ \cup X^y | X^y \in X \} \in X \). As we shall see, the key feature of paternalistic preferences is to provide a motive for
parents to influence their children’s choices by either restricting their choice or by endowing them with particular preferences.

The value function of an old adult, \( v(a) \), is given by:\(^4\)

\[
v(a) = \max_{a' \in A, x \in X, X^y \in X^y} \{ U^o(x, a) + zw(X^y, a, a') \}.
\]

Here \( w(X^y, a, a') \) is the component of utility accruing to parents from their children’s experience, which is defined as:

\[
w(X^y, a, a') = (1 - \lambda) U^y(x^y(a', X^y), a') + \lambda U^o(x^y(a', X^y), a) + \beta v(a'). \tag{1}\]

The utility \( w(X^y, a, a') \) comprises both an altruistic and a paternalistic component. The altruistic component (first term) is the standard enjoyment of the child’s own utility as in Becker (1974). The paternalistic component, in contrast, evaluates the child’s actions through the lens of the parent’s utility function. The weight on paternalism is denoted by \( \lambda \), while \( z \) denotes the standard altruistic discount factor. We assume that paternalism only applies to the young, but not to the old felicity of the child. That is, the parent always agrees with the choices the child will make in old age.\(^5\)

The decision rule \( x^y(a', X^y) \) is determined by the utility maximization of the young child, given her own preferences and the choice set imposed on her by the parent:

\[
x^y(a', X^y) = \arg\max_{x^y \in X^y} \{ U^y(x^y(a'), a') + \beta v(a') \}.
\tag{2}\]

To simplify the exposition, we introduce the assumption that there exists a particular vector of preference parameters, \( a = a_x \), such that, for all feasible choices, \( x \) and \( x^y \), the period utility is maximized in a cardinal sense:

\(^4\)We abstract for simplicity from costs that parents may incur when investing in their children’s preference or restricting the children’s choice set.

\(^5\)Note that, contrary to the literature on imperfect empathy, we do not assume that parents have an intrinsic drive to reproduce their own preferences. Even a perfectly paternalistic parent could desire her child to have different preference from herself.
Assumption 1  There exists \( a \in A \) such that for all \( a \in A \) and for all feasible \( x, x^y \):

\[
\begin{align*}
U^o (x, a) & \geq U^o (x, a), \\
U^y (x^y, a) & \geq U^y (x^y, a).
\end{align*}
\]

Under this assumption, perfectly Beckerian parents would always set \( a = a \) irrespective of their own preference vector. This feature serves to sharpen the contrast between altruism and paternalism, but none of our main results hinge on the assumption.\(^6\)

### 2.2 Incentives for Preference Transmission

In this section, we analyze the optimal choice of preference transmission, denoted by \( a^{\ast} \). Given (1), \( a^{\ast} \) necessarily satisfies:

\[
\lambda U^o (x^y (a^{\ast}, X^y), a) + (1 - \lambda) U^y (x^y (a^{\ast}, X^y), a^{\ast}) + \beta v (a^{\ast}) \\
\geq \lambda U^o (x^y (a', X^y), a) + (1 - \lambda) U^y (x^y (a', X^y), a') + \beta v (a')
\]

(3)

for all \( a' \in A \).

Consider, first, the particular case in which \( x^y (a', X^y) \) is independent of \( a' \). One such example is the case in which the choice set is a singleton. An alternative example is a case in which the choice set includes different elements, but the optimal choice of the child is the same, irrespective of her preferences.

**Lemma 1** Suppose \( x^y \) is independent of \( a' \). Then, \( a^{\ast} = a \).

Intuitively, if the child’s preferences do not affect her choices in young age, the parent has no reason to deviate from the choice of preferences that maximizes the child’s happiness (\( a' = a \)).

\(^6\)In more general environments in \( a \) can be state dependent, as in our previous work (Doepke and Zilibotti 2008). For instance, \( a \) might vary with the initial vector \( s \). While such a feature could be easily incorporated, it does not lead to new insights and is therefore abstracted from in this paper.
Consider, next, the general case in which $x^y$ does depend on $a'$. Now, the parent may wish to distort the child’s preferences away of $a$ in order to manipulate her choice. To achieve this goal, a paternalistic parent is willing to inflict a utility loss on the child. In the extreme case where $\lambda = 1$ and $\beta = 0$, the parent would just impose her own preferences on the child, namely, she would choose $a'$ to maximize $U^o( x^y, a', X^y)$. In general, the parent faces a trade-off between the child’s happiness and her own desire to see the child behave in a particular way.

To cast light on such a trade-off, suppose that the child’s choice is defined over a continuous and differentiable choice set. Then, the first-order condition of the child’s problem, (2), yields:

$$U^y_{x^y, a'}(x^y, a') = 0,$$

where $x^y \in X^y$. Moving backwards to the parent’s optimal choice of preferences, the first-order condition with respect to $a'$ yields:

$$0 = \lambda U^o_{x^y, a} (x^y, a) x^y_{a'} (a', X^y) + (1 - \lambda) (U^y_{a'} (x^y (a', X^y), a') + U^y_{x^y} (a', X^y) x^y_{a'} (a', X^y)),$$

Applying the envelope theorem (from (4)), the first-order condition simplifies to:

$$0 = \lambda x^y_{a'} (a', X^y) \cdot U^o_{x^y, a} (x^y, a) + (1 - \lambda) U^y_{a'} (x^y (a', X^y), a') + \beta v_{a'} (a').$$

The first term reflects the paternalistic motive to distort preferences, which hinges, as anticipated above, on $x^y_{a'} \neq 0$. The second term yields the standard Beckerian motive to maximize the child’s utility. Thus, whenever either $\lambda = 0$ (no paternalism) or $x^y_{a'} = 0$ (preferences do not affect the child’s choice) the first term vanishes and the parent sets $a' = a$.

To complete the analysis, consider the parent’s choice of the child’s choice set:

$$X^y = \arg \max_{X^y \in A^y} w(X^y, a, a').$$

Let $\{x^y\}$ denote the singleton set consisting only of $x^y$. Moreover, let

$$x^y_* = \arg \max_{x^y} w(\{x^y\}, a, a).$$
be the parent’s wish for what the child should choose. If \( \{x^{y*}\} \in \mathcal{X}^y \), then the problem becomes trivial: the parent would restrict the child’s choice to \( X^y = \{x^{y*}\} \), and implement her bliss point by setting \( a' = a \) and imposing her most preferred economic choice. However, this option may not be available. Thus, in the general case, parents maximize their utility by choosing a combination of preference molding and of restrictions of the child’s choice set.

2.3 Parenting Styles

Depending on how the parent decides to influence the child’s preferences and choices, we define the following parenting styles (cf. Baumrind 1967):

Definition 1 We distinguish between three parenting styles:

1. A parent is said to be authoritarian if she restricts the child’s choice (\( X^y \neq X^{FREE} \)). A parent is said to be purely authoritarian if she restricts the child’s choice set to a singleton, implying that the child operates no independent choice.

2. A parent is said to be authoritative if she chooses \( a' \neq a \). A parent is said to be purely authoritative if, in addition, she allows the largest possible choice set, \( X^y = X^{FREE} \).

3. A parent is said to be permissive if she chooses \( a' = a \) and gives the child access to the largest possible choice set \( X^y = X^{FREE} \).

The analysis of the previous section implies the existence of a relationship between parenting styles and the extent of paternalism, parameterized by \( \lambda \). When \( \lambda = 0 \) (Beckerian altruism), the parent has full empathy with the child’s preferences, and adopts a permissive parenting style, by setting \( a' = a \). Conversely, when \( \lambda = 1 \), the parent entirely disregards the young child’s preferences, and looks at her choices exclusively from the adult perspective. In this case, the parent has an an incentive to manipulate preferences, adopting an authoritative style. In general, when \( \lambda > 0 \), one observes combinations of authoritative and
authoritarian elements. The strongest drive for restricting the child’s choice set arises when the parent decides not to (or is not able to) shape the child’s preferences, and yet disagrees strongly with the child’s choices.

2.4 Economic State Variables

In addition to influencing children’s preferences and choices, parents typically make other economic decisions that affect their children’s well-being. In this section, we extend the model to include such decisions, which include transfers, schooling, health, and the transmission of specific skills. The analysis of such decisions is related to the large literature—stretching back to Becker’s rotten kid theorem (Becker 1974 and 1981)—that studies the strategic relationship between parents and children when there is an incentive for the child to deviate, ex post, from the behavior prescribed by her parent. However, in the existing literature preferences are exogenous, and parents cannot affect their children’s behavior through preference manipulation.

Let \( s \) denote the state vector, excluding preferences. The choice vector \( x \) includes investment decisions (e.g., savings, human capital formation) that affect the law of motion of \( s \). Each parent is endowed with a preference, \( a \), and an economic state vector, \( s \). Formally, the difference between these two state vectors is that preferences are determined in youth (by parents’ decisions) whereas other states can be affected by decisions made within the lifetime of an individual. Each parent make three choices. First, she chooses \( x \).\(^7\) The choice \( x \) also determines the initial economic state of the child \( (s^y \in S) \) through the law of motion:

\[
s^y = g^y(s, x). \tag{5}\]

Next, the parent determines the preferences of her child \( a' \in A \). Finally, the parent also determines the choice set from which the child will be able to choose,
As above, the child only makes the economic choice \( x^y \in X^y \), where \( X^y \) is the choice set provided by the parent. Now, however, the child’s young-age decision also affects its old-age economic state \( s' \) through the law of motion:

\[
    s' = g(s^y, x^y). 
\]  

Note that parents now have access to an additional instrument to condition the children’s choice. For example, if \( s^y \) is an inter-vivos transfer, the size of the transfer will constrain the child’s consumption even without additional restrictions on the choice set, \( X^y \). On the other hand, the transfer choice interacts with the choice of preferences. As we will see below, inter-vivos transfers tend to be decreasing in the extent of disagreement between the parents and the child (which is itself endogenous) about their use.

The value function for an old adult, \( v(s, a) \), is given by:

\[
v(s, a) = \max_{a', x, X^y(s^y)} \left\{ U^o(x, a) + zw(s^y, X^y(s^y), a, a') \right\},
\]

subject to (5)-(6), where, as before,

\[
w(s^y, X^y(s^y), a, a') = \lambda U^o(x^y(s^y, a', X^y(s^y)), a) \\
+ (1 - \lambda) U^y(x^y(s^y, a', X^y(s^y)), a') + \beta v(s', a').
\]

The decision rule \( x^y(s^y, a', X^y) \) is determined by the optimizing decision on the young child, given her own preferences:

\[
x^y(s^y, a', X^y) = \arg\max_{x^y \in X^y} \{ U^y(x^y, a') + \beta v(s', a') \},
\]  

where the maximization is subject to the law of motion (6).
3 Patience

In this section, we apply the general model of the previous section to a salient dimension of individual preferences: patience. The underlying friction is that children are innately less patient than their parents would like them to be. Parents can turn children more forward-looking by instilling in them a sense of guilt about the pleasure of immediate consumption. A more patient child will be willing to undertake investments paying off in the future, such as educational effort, that parents approve of.

3.1 The Decision Problem with Endogenous Patience

We parameterize preferences by a utility function inducing a constant intertemporal elasticity of substitution, or constant relative risk aversion (CRRA). The old-age felicity is given by:

\[ U^o(x, a) = \frac{c^{1-\sigma}}{1-\sigma}, \]

where \( x = \{c, i\} \). Here \( c \) is a scalar denoting consumption, and \( i \in \{F, U\} \) (farm and urban) is a (geographic) location, where \( F \) and \( U \) are associated with different associated productivities and family organizations discussed in more detail below. We assume \( 0 < \sigma < 1 \), implying that utility is positive. The young-age felicity is given by:

\[ U^y(x^y, a) = (\psi - a)(c^y)^{1-\sigma}, \]

where \( \psi > 1 \) captures the innately high felicity from current consumption that the young enjoy, whereas \( a \in A = [0, \psi - 1] \) is the extent to which parents stifle their children’s enjoyment of young age. Note that in this application \( a \) affects only the young-age felicity. One could as well argue that patience yields a better ability to savor future consumption. This could be captured by assuming that \( U^o(x, a) = f(a) \frac{c^{1-\sigma}}{1-\sigma} \), where \( f \) is an increasing function. This specification would give similar results. Our specification implies the convenient normalization that \( a = 0 \), entailing no loss of generality.

The state \( s \) is a scalar, broadly interpreted as wealth or human capital. The in-
tergenerational law of motion of $s$ is determined in two steps. First, the parent makes an inter-vivos transfer to his child, $s^y = s - c$. Second, the child makes an investment choice:

$$s' = R_i (s^y - c^y), \quad (8)$$

where $R_i$ denotes the rate of return to investment in location $i$, and $c^y$ denotes the child’s consumption. We assume that $R_U \geq R_F$, namely, the rate of return of urban education is higher than that of investing in the farm.

Consider, next the location choice. If the child stays on the farm, the parent retains control over the family savings decisions, and the child makes no choice in young age. If the child moves to the city, instead, he receives from his parent a transfer to cover for his education and living expenses. The child can then decide how to use the transfer, sharing it between consumption (or leisure-related activities) and investment (such as education-related expenses). Due to the conflicting preferences, children living in cities divert part of the family transfer away from the intended purpose. From the parent’s standpoint, there is a trade-off between the better monitoring achieved in the farm, and the higher rate of return associated with the urban education.

To formalize the solution of this trade-off, we assume that $X^y$ comprises two subsets, $X^y = \{\hat{X}, X^{FREE}\}$, where

$$\hat{X} = \left\{ \begin{pmatrix} c \\ i \end{pmatrix} = \begin{pmatrix} c = \bar{c} \\ i = F \end{pmatrix} \right\}_{0 \leq \bar{c} \leq s^y}$$

$$X^{FREE} = \left\{ \begin{pmatrix} c \\ i \end{pmatrix} = \begin{pmatrix} 0 \leq c \leq s^y \\ i \in \{F, U\} \end{pmatrix} \right\}.$$

When $X^y \in \hat{X}$, the child can choose neither the location (the parent forces him to stay on the farm), nor the consumption level ($\bar{c}$ is chosen by the parent). When $X^y = X^{FREE}$, the child is free to choose both the location (farm vs. city) and the consumption level, subject to the budget constraint imposed by the inter-vivos transfer, $s^y = s - c$, where $s$ denotes the parent’s wealth, and $c$ denotes his consumption.
Since \( a \) does not affect the utility of old agents, we can express the value function of the old as depending only on \( s \):

\[
v(s) = \max_{c,c',X^y} \left\{ \frac{c^{1-\sigma}}{1-\sigma} + zw(s^y, X^y, a') \right\},
\]

subject to \( s^y = s - x \) and to (8).

### 3.2 Staying on the Farm

Consider, first, a parent who forces the child to stay on the farm (authoritarian parenting style). In this case, Lemma 1 implies that the parent will choose \( a' = 0 \). Moreover, since the parent determines both \( c \) and \( c_y \), then,

\[
v_F(s) = \max_{c,c^y} \left\{ \frac{c^{1-\sigma}}{1-\sigma} + z(\lambda + (1 - \lambda)\psi) \frac{(c^y)^{1-\sigma}}{1-\sigma} + z\beta v_F(s') \right\}, \tag{9}
\]

where \( s^y = s - c \) and \( s' = R_F(s^y - c^y) \). We guess that the value function is homothetic in assets, \( v_F(s) = \Omega_F s^{1-\sigma} \), where \( \Omega_F \) is a constant to be determined in equilibrium. Plugging in the guess, eliminating \( s' \) and \( s \) using the two budget constraints allows us to express the choice of \( c^y \) as follows:

\[
c^y_F = \arg \max_{c^y} \left\{ (\lambda + (1 - \lambda)\psi) \frac{(c^y)^{1-\sigma}}{1-\sigma} + z\Omega_F \frac{R_F^{1-\sigma}(s^y - c^y)^{1-\sigma}}{1-\sigma} \right\}. \tag{10}
\]

Equations (9) and (10) provide a characterization of the equilibrium conditional on an authoritarian parenting style. The complete solution (which involves determining the value of \( \Omega_F \)) can be found using standard recursive methods, and is deferred to the mathematical appendix.

### 3.3 Moving to the City

In this section, we characterize the solution under a non-authoritarian parenting style, i.e., when parents allow their children to choose their preferred location. In
In this case, since $R_U \geq R_F$, the child always chooses to move to the city. The value function of the adults can therefore be written as:

$$v_U(s) = \max_{c,a'} \left\{ \frac{c^{1-\sigma}}{1-\sigma} + zw(s^y, X^{FREE}, a') \right\},$$

where

$$w(s^y, X^{FREE}, a') = \lambda \frac{c^y(s^y, a')^{1-\sigma}}{1-\sigma} + (1 - \lambda)(\psi - a') \frac{c^y(s^y, a')^{1-\sigma}}{1-\sigma} + \beta v_U(s').$$

We guess, as above, that $v_U(s') = \Omega^U(s')^{1-\sigma}$. Using the guess, the child’s consumption choice can be written as:

$$c^yU = c^y(s^y, a') = \arg \max_{c^y} \left\{ (\psi - a') \frac{(c^y)^{1-\sigma}}{1-\sigma} + z\beta \Omega_U \frac{R_U^{1-\sigma}(s^y - c^y)^{1-\sigma}}{1-\sigma} \right\}, \tag{11}$$

where the maximization is subject to $s' = R_U(s^y - c^y)$. A complete characterization of the equilibrium is again deferred to the appendix. Here, we discuss the optimal choice of $a'$ that is the focal point of our analysis:

**Lemma 2** Conditional on a non-authoritarian parenting style, the optimal choice of the child’s preferences, $a' = (a')^U$ satisfies the following condition:

$$0 \geq -\lambda (\psi - a' - 1) c^y_a(s^y, a') - (1 - \lambda) \frac{c^y(s^y, a')^{1-\sigma}}{1-\sigma},$$

where the strict inequality holds if and only if $(a')^U = 0$.

The marginal benefit is positive as long as increasing $a'$ causes a fall of $c^y$ (i.e., if $c^y_a < 0$), which is generally true whenever the child can choose $c^y$. Intuitively, the benefit of increasing $a'$ stems from increasing the child’s drive to accumulate. The
marginal cost captures the utility loss suffered by the child when she is “brain-washed” into adhering to responsible, adult-like values. If $\lambda = 0$, the marginal benefit vanishes, and the optimal solution is to set $a' = 0$, namely, to choose a permissive parenting style. By continuity, the same solution is optimal for a range of low $\lambda$’s. In contrast, if $\lambda = 1$ the parent is not concerned with the utility loss suffered during childhood, and the second term drops out. It is then optimal to set $a' = \psi - 1$, i.e., the parent adopts a purely authoritative style, inducing the child to make the same exact choices that the parent would like him to do.

The following corollary summarizes the discussion above.

**Corollary 1** There exists $\Lambda > 0$ such that, for all $\lambda \leq \Lambda$, $(a')^U = 0$. For $\lambda = 1$, $(a')^U = \psi - 1$.

### 3.4 Equilibrium Parenting Style

In this section, we characterize the optimal parenting style. Recall that the value functions are homothetic in assets: $v_J(s) = \Omega_J s^{1 - \sigma} 1_{1-\sigma}$, where $J \in \{F, U\}$. Therefore, the crux of the analysis is to determine conditions such that $\Omega_F \geq \Omega_U$.

To this aim, consider, first, the two polar opposite cases, $\lambda = 0$ and $\lambda = 1$. In both cases, for a given rate of return, the authoritarian parent would choose the same investment level as would the child in the city. This implies that, if $R_U = R_F$, then $\Omega_U = \Omega_F$. By the same token, if $R_U = R_F$, authoritarian parenting is preferred for any intermediate level of paternalism, $\lambda \in (0, 1)$, as in this case the child—if he has a free choice—would make a saving decision that her parent dislikes. In summary, authoritarian parenting is optimal for all $\lambda$ when $R_U = R_F$, but only weakly so for $\lambda = 0$ and $\lambda = 1$.

With this observation in mind, consider the general case in which $R_U > R_F$, so that a non-trivial trade-off arises. By continuity, the argument above implies that, if $\lambda$ is either close to one or close to zero, then, respectively, a permissive and an authoritative parenting style are preferred over an authoritarian parenting style. Conversely, as long as the gap between the two rates of return is not too large,
there exists an intermediate range for $\lambda$ such that parents choose an authoritarian style. Finally, if the gap between the urban and rural return is sufficiently large, no parents would resort to authoritarian methods. The next proposition summarizes these results.

**Proposition 1** There exists $\bar{R}_F \in \{0, R_U\}$ such that:

(i) If $R_F < \bar{R}_F$, then, there exists $\lambda = \hat{\lambda}(R_F)$ such that all parents endowed with $\lambda \leq \hat{\lambda}$ adopt a permissive style, whereas all parents endowed with $\lambda > \hat{\lambda}$ adopt an authoritative style. No parent adopts an authoritarian style.

(ii) If $\bar{R}_F < R_F < R_U$, then, there exist $\lambda_1$ and $\lambda_2$, where $0 < \lambda_1 < \lambda_2 < 1$, such that

(a) all parents endowed with $\lambda \leq \lambda_1$ adopt a permissive style; (b) all parents endowed with $\lambda \in (\lambda_1, \lambda_2)$ adopt an authoritarian style; (c) if $\lambda \geq \lambda_2$, all parents adopt an authoritative style.

(iii) If $R_F = R_U$, then, all parents adopt an authoritarian style.

Intuitively, when the gap in the rates of return is too high (case (i)) no parent ever forces the child to stay on the farm. Conversely, as the gap vanishes, no parents let the child go to the city (case (iii)).

The interesting case is the intermediate range (case (ii)). Figure 1 displays a computed example in such a range that illustrates the results. The parameter values used for the figure are $\beta = z = 0.8$, $\sigma = 0.5$, $\psi = 0.3$, $R_U = 1.6$, and $R_F = 1.59$. The optimal choices for $a$, $S^y$, and $S'$ are displayed as a function of $\lambda$. As described in the proposition, the permissive parenting style is adopted for low $\lambda$, the authoritarian style prevails for intermediate $\lambda$, and parents with a $\lambda$ close to one are authoritative.

Interestingly, the authoritarian style is adopted by parents endowed with an intermediate degree of paternalism who, on the one hand, would not dare manipulate their children’s preferences, but on the other hand would disagree with their free conduct. At the extremes, parents leave liberty to their children, but follow opposite rules as far as the choice of preference is concerned: those with low $\lambda$ are permissive, while those with high $\lambda$ mold their children’s preferences.

Some of the results in Figure 1 are driven by the fact that, in our setup, parents with low $\lambda$ derive more of their total utility from their children’s well-being, be-
cause $z$ is the same for all families, whereas $\psi > 1$, so that altruistic families derive more utility from their children in young age. This is an artefact of the mathematical specification, and one might argue that it would be more realistic to assume that all parents value their children equally, and only differ in the relative evaluation of early and late utility.

To show how results would be different in this scenario, we also computed outcomes for an alternative setting where $z$ is an increasing function of $\lambda$, in such a way that all parents would give the same transfer to their children if they were able to control their children’s choices (i.e., in the farm scenario). Figure 2 shows the results. As before, parents with low $\lambda$ are permissive, parents with $\lambda$ close to one are authoritative, and parents with intermediate $\lambda$ are authoritarian. The panel for transfers shows that the transfer declines with $\lambda$ among the permissive parents. The reason is that while all families care equally about their children
(in the sense defined above), those with higher $\lambda$ disagree more with their children’s choices, which makes them less willing to provide them with transfers. For authoritarian parents, the transfer is independent of $\lambda$, because of the dependence of $z$ on $\lambda$. For authoritative parents, the transfer increase with $\lambda$, because parents with a higher $\lambda$ also choose a higher $a'$, which means that there is less disagreement between parent and child and a higher willingness to provide transfers. In terms of total accumulation of capital across generations, the fully paternalistic dynasties now do best, because they achieve high savings with the high return technology.
3.5 Additional Empirical Predictions

In this section, we consider the possibility that parents have heterogeneous abilities to shape their children preferences, depending on their education and human capital. Suppose, for instance, that there are high- and low-skill parents, and that low-skill parents can only choose their children’s preferences in the range $a \in [0, \tilde{a}]$, where $\tilde{a} < \psi - 1$. In this case, high-$\lambda$ low-skill parents would resort to authoritarian methods, whereas high-skill parents endowed with the same degree of paternalism would achieve their goals by influencing their children’s preferences. With an opportune choice of parameters, we would then obtain that high-skill parents sort themselves across the three parenting styles, whereas low-skill parents are either permissive or authoritarian, but never authoritative. This is in line with the developmental psychology literature showing that authoritative methods are more frequently in use in better educated families.

Differences in the rate of return of authoritative vs. authoritarian practices are also important. In our rural-vs.-urban location example, if the return to formal education relative to traditional activities increases over time, we will observe a progressive decline of the authoritarian parenting style. This is consistent with the observation that the rod has progressively lost its popularity in modern industrial societies.

In terms of long-run economic success, the model predicts that the dynasties that do best are either fully altruistic or fully paternalistic ones. On the one hand, fully altruistic parents agree with any propensity to save that their children have, and are prepared to make large transfers to them. On the other hand, fully paternalistic parents have indoctrinated their children into adopting the same level of savings that the parents approve of, and are also prepared to make large transfers, leading to high accumulation of assets in the dynasty. Families with intermediate paternalism are more troubled, as disagreement distorts inter-vivos transfers and tends to hamper accumulation. For instance, in Figure 1 parents with $\lambda \simeq 0.7$ disagree with their children, and yet not enough to induce them to mold their preferences. The result of the disagreement is a low transfer flow from parents to children. Authoritarian parents also incur losses, albeit for different reasons: they are willing to sacrifice a high rate of return in order to retain control on their
children’s choices. Interestingly, this choice does not stem from disregard of the children’s happiness, but, in a sense, from its opposite. These parents reject the possibility to change his child’s references in a more adult direction because of its effect on the child’s felicity (or because they are unable to convince their children, as in the case discussed at the beginning of this section).

4 Risk Aversion

In this section we apply our theory to another important dimension of preferences, namely, risk aversion. Risk aversion is known to increase with age, leading to a natural possibility of conflict between parents and children regarding risk-taking by children. The theory can explain how the transmission of risk preferences is shaped by the presence of risks that parents may like their children to avoid (such as experimenting with drugs or riding motorcycles), but also by the economic returns later on life of being risk tolerant (related, for example, to the returns to entrepreneurship).

4.1 The Decision Problem with Endogenous Risk Aversion

Preferences are parameterized by a von Neumann-Morgenstern expected utility function inducing a constant relative risk aversion (CRRA). The endogenous part of risk aversion is denoted by \( a \in [0, \bar{a}] \), where higher \( a \) implies a higher risk aversion. The old-age felicity is given by:

\[
U^o(x, a) = E \left[ \frac{c^{1-\sigma-a} - 1}{1-\sigma-a} \mid x \right],
\]

where \( c \) is a function of \( x \) to be discussed below, with the usual convention that \( U^o(x, a) = E[\log(c) \mid x] \) if \( \sigma + a = 1 \). In this example, it is natural to think of the child as an "adolescent". The adolescent felicity is given by:

\[
U^y(x, a) = E \left[ \frac{c^{1-\sigma+\psi-a} - 1}{1-\sigma+\psi-a} \mid x \right].
\]
We assume that $\psi > 0$, so that for a given underlying preference parameter $a$, adolescents are less risk averse than are adults. This captures the well-documented observation that risk aversion increases with age (see, e.g., Morin and Suarez (1983) and Pålsson (1996)). In our model, the lower risk aversion of children can lead to disagreement between parents and children about the appropriate degree of risk taking.

Note that given $c$, both $U^y$ and $U^o$ are decreasing in $a$. As a consequence, non-paternalistic parents would choose $a = 0$ for their children to maximize their happiness. However, paternalistic parents may want to increase their children’s risk aversion to influence their adolescent choices.

For simplicity, we abstract in this section from economic state variables such as saving decisions. In every period, parents and children choose from a choice set that consists of lotteries over consumption. We interpret these lotteries broadly to include juvenile risky choices such as smoking or deciding to ride motorcycles, as well as old-age decisions such as occupational choices that are associated with varying degrees of income uncertainty. Paternalistic parents may disagree with their children’s choices and hence may wish to restrict the lotteries available to the child. To focus the analysis sharply, we restrict attention to small sets of feasible lotteries. In particular, at old age there is a binary choice between a relatively risky (entrepreneurship) and a relatively safe lottery. We use $x = R$ and $x = S$ to denote risky and safe.

At young age, people can choose from another set of (juvenile) risky and safe lotteries $x^y = R^y$ and $x^y = S^y$. In addition, a third lottery $x^y = SS^y$ is also available, which is also relatively safe but first-order stochastically dominated by lottery $S^y$. As a consequence, if a parent does not restrict the choice set of the child, $X^y = X^{FREE} = \{R^y, S^y, SS^y\}$, the child will face a binary choice between $R^y$ and $S^y$ as well, because regardless of preferences it is never optimal to choose the dominated lottery $SS^y$. However, the parent may choose to restrict the child’s choice to $SS^y$ by selecting the choice set $X^y = \hat{X} = \{SS^y\}$. The parent may find this choice set attractive because it rules out their least preferred lottery, $R^y$. We assume that no choice set is available that contains $S^y$ but not $R^y$; that is, if parents want to prevent their child from making the risky choice they face a cost.

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in terms of their child choosing a safe but dominated lottery. Alternative ways of modeling the cost of restricting the child’s choice (such as a direct utility cost for either the parent or the child) would leave our results unchanged.

The specific properties of risky and safe lotteries are pinned down by the following restrictions:

**Assumption 2** (Monotonicity) The lotteries $R$, $S$, $R^y$, and $S^y$ satisfy the following restrictions:

1. $U^o(R, a) - U^o(S, a)$ is strictly decreasing in $a$ on the interval $[0, \bar{a}]$, and there exists an $a^o \in (0, \bar{a})$ such that $U^o(R, a^o) = U^o(S, a^o)$.

2. $U^y(R^y, a) - U^y(S^y, a)$ is strictly decreasing in $a$ on the interval $[0, \bar{a}]$, and there exists an $a^y \in (0, \bar{a})$ such that $U^y(R^y, a^y) = U^y(S^y, a^y)$.

3. $U^o(R^y, a) - U^o(S^y, a)$ is strictly decreasing in $a$ on the interval $[0, \bar{a}]$, and we have $U^o(R^y, a) - U^o(S^y, a) < 0$ for all $a \in [0, \bar{a}]$.

Conditions 1 and 2 state that for both adolescents and adults the relative utility derived from the risky choice is lower for individuals with higher risk aversion; thus, these conditions define the precise meaning of risky and safe lotteries in our model. The conditions also state that there are interior levels of risk aversion at both ages $a^o$ and $a^y$ that make individuals indifferent between the two choices. The third condition is concerned with how adults feel about their children’s risky and safe choices, and imposes the parallel condition that more risk averse individuals more strongly oppose the risky choice for their children. The final condition states that even the most risk-tolerant individuals would prefer that their children avoid the juvenile risk. This last condition is imposed to focus on the interesting case where there is a conflict of interest between parents and children.

As in the analysis of patience above, it will be useful to analyze the cases of a restricted choice set $\hat{X}$ and the unrestricted choice set $X^\text{FREE}$ separately. We define $v(a) = \max \{v_U(a), v_{BS}(a)\}$, where $v_U$ denotes the value function conditional on choosing $X^\text{FREE}$, while $v_{BS}$ denotes the value function conditional on choosing $\hat{X}$.
4.2 The Boarding School

We first consider the case in which the parent decides to restrict the choice set of the child by choosing the choice set $\hat{X}$, which contains only a single possible adolescent choice of $SS^y$. Interpretations of this choice set would be sending the child to a strict boarding school or moving to a safe suburb where there is no street violence and no supply of illicit drugs. These options come with a cost for the child (being separated from the parents, being disciplined by the school), hence we assume that $SS^y$ is dominated by the safe choice $S^y$ in the full choice set $X^{FREE}$. Alternatively, we can think that the cost is borne by the parent, e.g., in the form of school fees or the cost of moving into a safe neighborhood. Although we do not emphasize this alternative interpretation in the formal analysis, the results would be the same.

The restricted choice set $\hat{X}$ limits the child to the relatively safe lottery $SS^y$; no decision is taken by the child. The attraction for the parent is to prevent the child from taking the risky choice $R^y$, without having to turn the child overly risk averse. In particular, when the child is in the boarding school, the decision problem of the parent is:

$$v_{BS}(a) = \max_{a'} \{ \max \{U^o(S,a), U^o(R,a)\}$$

$$+ z [\lambda U^o(SS^y,a) + (1 - \lambda)U^y(SS^y,a') + \beta v(a')] \}.$$ (12)

Given that the child’s choice is fixed, the optimal solution for $a'$ is to maximize cardinal utility by setting $a' = 0$. If the dynasty always chooses the boarding school, all generations will have preferences $a = 0$, and lifetime utility will be given by:

$$v_{BS}(0) = \frac{1}{1 - z\beta} [U^o(R,0) + z [\lambda U^o(SS^y,0) + (1 - \lambda)U^y(SS^y,0)]].$$ 

4.3 The City

Consider, next, the choice in the city. As above, the city is a location where parents are not able to directly control their children’s choice set, so that the possible
parenting styles are authoritative and permissive. Hence, we characterize the equilibrium conditional on the dynasty always choosing the unrestricted choice set $X^{FREE}$ for the child. The value function for an old adult, $v_U(a)$, is given by:

$$v_U(a) = \max_{a'} \left\{ \max \{ U^o(S, a), U^o(R, a) \} + zw \left( X^{FREE}, a, a' \right) \right\}.$$  \hfill (13)

The utility the parent derives from the child is:

$$w \left( X^{FREE}, a, a' \right) = \lambda \left( (1 - I^Y(a')) U^o(S^y, a) + I^Y(a') U^o(R^y, a) \right) + (1 - \lambda) \left( (1 - I^Y(a')) U^y(S^y, a') + I^Y(a') U^y(R^y, a') \right) + \beta v(a').$$

Here $I^Y(a') \in \{0, 1\}$ is the decision of the young (with no loss of generality, we assume no randomization), where $I^Y = 1$ denotes taking the risky choice $R^y$ (recall that the dominated choice $S^yS^y$ is never chosen and is thus omitted from the notation).

The decision rule $I^Y(a')$ is determined by the optimizing decision on the adolescent, given her own preferences:

$$I^Y(a') = \arg\max_{I} \left\{ (1 - I) U^y(S^y, a') + IU^y(S^y, a') + \beta v(a') \right\}.$$ 

Given that the adolescent choice does not affect the continuation utility $v(a')$, the maximization problem has a simple solution:

$$I^Y(a') = \begin{cases} 
1 & \text{if } U^y(R^y, a') > U^y(S^y, a') \\
0 & \text{if } U^y(R^y, a') \leq U^y(S^y, a')
\end{cases}$$

We now move towards characterizing the solution of the choice problem. A special role is played by the level of risk tolerance $a^y$ that makes adolescents just indifferent between taking the safe and risky actions (which exists given Assumption 2). Let $W(a, a')$ denote the function that is maximized on the right-hand side.
of (13):

\[
W(a, a') \equiv \max \{U^o(S, a), U^o(R, a)\}
\]

\[
+ \lambda \left( (1 - I^Y(a')) U^o(S^y, a) + I^Y(a') U^o(R^y, a) \right)
\]

\[
+ (1 - \lambda) \left( (1 - I^Y(a')) U^u(S^y, a') + I^Y(a') U^u(R^y, a') \right) + \beta v(a').
\]

We can establish the following result.

**Lemma 3** Parents choose either \(a' = 0\) or \(a' = a^y\).

Intuitively, all else equal parents would like to give their children the lowest possible \(a'\) (namely \(a' = 0\)) since utility is decreasing in risk aversion. Strictly in terms of utility, it is good for the child to be unafraid and to be able to tolerate risk. However, because parents and children evaluate choices during adolescence differently, parents also fear that children will take too much risk (i.e., choose the lottery \(R^y\) over \(S^y\)). Depending on which of these motives dominates, they may choose either the \(a' = 0\) or the minimum risk aversion that guarantees that children will not get into “trouble” by choosing \(R^y\).

Lemma 3 implies that from the second generation onwards the distribution of preferences has positive mass at only two points: \(a = 0\) and \(a = a^y\).

The next question that arises is which parents assign which risk tolerance to their children. The following lemma can be established.

**Lemma 4** (i) If parents with \(a = 0\) choose \(a' = a^y\), then all parents with \(a > 0\) will do the same. (ii) If parents with \(a = \hat{a} > 0\) choose \(a' = 0\), then all parents with \(a < \hat{a}\) will do the same.

The lemma establishes the intuitive result that more risk-averse parents are more likely to endow their children with a high degree of risk aversion. Notice that the we do not assume that it is cheaper for risk-averse parents to invest in risk aversion (the cost of investment is zero for everyone). Rather, the result obtains because risk-averse parents are more afraid of their children taking risky actions
during adolescence. Assuming that risk-averse parents have a lower cost of investing in risk aversion or assuming some direct preference transmission (say, because of a genetic component of risk aversion) would reinforce this result.

As a final step in the characterization of the choice problem, we discuss how the nature of risk in the economic environment affects the transmission of risk tolerance. As far as risk during adulthood is concerned, this mapping is simple, as parents are altruistic towards children in old age. Then, two cases are possible. In the first (less interesting) case, \( a^y < a^o \), and children will take the risky choice in old age, irrespective of whether their parents choose \( a' = 0 \) or \( a' = a^y \). In the rest of the analysis, we ignore this case. In the second case, \( a^y < a^o \), and the parents’ preference choice determines whether the child will take risky opportunities in old age. In this case, an increase in the return to the adult risky lottery relative to the safe choice, increases the parents’ drive to endow their children with the low risk aversion \( 0 \) instead of \( a^y \).

The situation is more complicated for the juvenile choices. A key distinction here is between endogenous and exogenous risk, i.e., the extent to which juvenile risk depends on the choice of the child. To clarify this relationship, we parameterize the juvenile lottery as follows:

\[
\begin{align*}
c(S^y) &= \begin{cases} 
  c_{S,L} & \text{with probability } p_L \\
  c_{S,H} & \text{with probability } 1 - p_L
\end{cases} \\
c(R^y) &= \begin{cases} 
  c_{R,L} & \text{with probability } p_R \cdot p_L \\
  c_{R,H} & \text{with probability } p_R (1 - p_L) \\
  c_{S,L} & \text{with probability } (1 - p_R) p_L \\
  c_{S,H} & \text{with probability } (1 - p_R) (1 - p_L)
\end{cases}
\end{align*}
\]

Here \( L \) denotes a low and \( H \) a high consumption realization, so that \( c_{S,L} \leq c_{S,H} \) and \( c_{R,L} < c_{R,H} \), \( R \) denotes a relatively risky and \( S \) a relatively safe lottery, which means that we assume \( c_{R,L} < c_{S,L} \) with \( p_R > 0 \), and we also assume that \( p_L c_{R,L} + (1 - p_L) c_{R,H} > p_L c_{S,L} + (1 - p_L) c_{S,H} \), meaning the risky lottery has a higher expected...
return and is therefore not dominated. Hence, the safe choice is simply a lottery between a low and a high consumption value. The risky lottery reverts to the safe lottery with probability $1 - p_R$, and with probability $p_R$ another, more risky binary lottery is reached. Here $p_R$ can be interpreted as the arrival rate of dangerous juvenile opportunities. In other words, even an adolescent that would like to take risky choices in principle may not have the opportunity to do so. For example, to experiment with smoking or other drugs one usually first has to come into contact with people who provide access to such opportunities, and that may or may not happen for a given individual.

We can now interpret the risk inherent in lottery $S$ as exogenous, or unavoidable, juvenile risk. In contrast, the parameter $p_R$ measures the exposure to endogenous juvenile risk, i.e., risk that can be avoided if the adolescent chooses the safe option.

It can now be shown that in sufficiently paternalistic families, endogenous and exogenous juvenile risk have opposite effects on preference transmission. An increase in exogenous risk induces parents to transmit lower risk aversion to their children (i.e., transmitting $0$ instead of $\alpha_y$ becomes more attractive). Given that this type of risk cannot be avoided, paternalistic concern about the juveniles’ choices does not play a role, so that parents would like their children to be able to tolerate this risk i.e., they endow them with low risk aversion. In contrast, an increase in endogenous risk, i.e., the parameter $p_R$, increases parents’ incentive to transmit high risk aversion to their children. Parents disagree with the adolescent’s choice of $R$ over $S$, and when the risky choice becomes riskier compared to the safer alternative, this paternalistic motive gains in strength. The following proposition summarizes this result.

**Proposition 2** Holding constant the adult lotteries, $p_L$, and $E(c_{SY})$, an increase in exogenous juvenile risk ($c_{SH} - c_{SL}$) lowers parents’ incentives for transmitting high risk aversion regardless of $\lambda$. Regarding an increase in endogenous risk $p_R$, there is a cutoff $\tilde{\lambda}$ such that for parents with $\lambda < \tilde{\lambda}$ an increase in $p_R$ lowers parents’ incentive for transmitting high risk aversion, whereas parents with $\lambda > \tilde{\lambda}$ for a higher incentive for transmitting high risk aversion.
Figure 3 illustrates these results. The solid line displays the utility of the parent as a function of the the chosen risk aversion of the child for a baseline parameterization. In this example, the parent’s preferences are described by $\lambda = 0.5$ and $a = 0.66$. The utility of the parent is piecewise decreasing in the risk aversion of the child, with a one-time upward jump at $a^y$. This is a general pattern that provides the basis for Lemma 3: The optimal choice for parents is either to maximize the child’s utility by setting $a' = 0$ or to prevent the child from taking juvenile risk by setting $a^y$. In the baseline scenario (solid line), this parent is just indifferent between these choices. The dashed line shows how the utility of the parent changes when exogenous risk (the risk in lottery $S$) increases. This is a risk that the child cannot avoid; hence, affecting the child’s choice becomes less important, and making the child risk-tolerant becomes more important. As a result, the new optimal choice is $a' = 0$. The dotted line shows an alternative scenario when endogenous risk (the parameter $p_R$) goes up. The parent can induce the child to avoid this risk by setting $a^y$, and hence this is now the only optimal choice. In a steady state, the support of the distribution of preferences is $a \in \{0, a^y\}$. The following proposition characterizes the equilibrium distribution of parenting styles.

**Proposition 3** There exist two thresholds, $0 < \lambda_1 \leq \lambda_2 \leq 1$ such that, conditional on $X^y = X^{FREE}$: (i) all parents with $\lambda \leq \lambda_1$ set $a' = 0$ (permissive parenting); (ii) for $\lambda \in (\lambda_1, \lambda_2]$, all parents with $a = 0$ set $a' = 0$ (permissive parenting), whereas all parents with $a = a^y$ set $a' = a^y$ (authoritative parenting); (iii) all parents with $\lambda > \lambda_2$ set $a' = a^y$ (authoritative parenting).

The proposition establishes that there are (at most) three ranges depending on $\lambda$. Parents with low $\lambda$ choose $a' = 0$ inducing their children to take all risks, irrespective of their own $a$. In an intermediate range of paternalism, preferences are path dependent: risk-tolerant parents induce risk tolerance in their children, whereas highly risk-averse parents induce high risk aversion in their children. The parameter values are $z = \beta = 0.8$, $\sigma = \psi = 1$, $\bar{a} = 2$. These values imply that for $a = 0$ adolescents are risk-neutral, whereas adults with $a = 0$ have log utility.
Finally, highly paternalistic parents induce high risk aversion in their children, irrespective of their own risk aversion. Figures 4 to 6 illustrate these results.

The thresholds depend on the nature of both exogenous and endogenous risk. When children are exposed to a large exogenous risk, even relatively paternalistic parents prefer to make their children risk tolerant, since risk aversion only increases their "fear" in a society full of unavoidable perils. In contrast, a high exposure to endogenous risk induces even mildly paternalistic parents to adopt an authoritative parenting style inducing their children to avoid juvenile risk. This comes at the cost of a low propensity to take risky opportunities in adult age. Likewise, an increase in the return to old age risk fosters risk taking and permissive parenting style.

The analysis above has also implications for the effect of redistributive policies on risk taking and transmission of risk preferences. First, if old age "entrepreneurs" (i.e., those accepting risky lotteries) were given access to better co-insurance schemes, then entrepreneurship would become more attractive. The effect on parenting
Figure 4: Parental Utility as a Function of Child Risk Aversion for low $\lambda$

Figure 5: Parental Utility as a Function of Child Risk Aversion for Intermediate $\lambda$
style is ambiguous. If the inequality $a^y > a^o$ continued to hold, more parents would choose $a' = 0$ for their children to profit from the more attractive adult lotteries. However, if the policy change reversed the sign of the inequality, parents could have the cake and eat it, too: they could induce children to avoid juvenile risk by setting $a^y$, and yet their children would not turn down entrepreneurial opportunities in old age. In this case, the population would be both risk averse and entrepreneurial. However, such a population may also be less able to benefit from unexpected future economic opportunities (e.g., related to innovative activities) in case these were not easily insurable.

Second, redistribution schemes involving both entrepreneurial agents and agents who decline to take (adult) risks would have the opposite effects. Fewer agents would take risk, and the range of permissive parents would shrink. This would result in a society which has low juvenile risk taking (e.g., crime and drugs), but also low entrepreneurial activity.
4.4 Choosing between the Boarding School and the City

In the previous section, we compared the permissive vs. authoritative parenting style. In this section, we will compare the payoff of authoritarian parenting, pairwise, first with that of permissive parenting, and then with that of authoritative parenting. After that, we discuss the global optimum.

The utility difference between placing the child in a boarding school and permissive parenting in the city is given by:

\[ v_{BS}(a) - W(a, 0) = \lambda (U^o(SS^y, a) - U^o(R^y, a)) + (1 - \lambda) (U^y(SS^y, 0) - U^y(R^y, 0)), \]

where the first term (paternalistic component) is positive, and the second (utilitarian component) is negative.

**Proposition 4** There exist two thresholds, \(0 < \hat{\lambda}_1 \leq \hat{\lambda}_2 \leq 1\) such that, conditional on the parent choosing \(a' = 0\): (i) all parents with \(\lambda \leq \hat{\lambda}_1\) set \(X^y = X^{FREE}\) (permissive parenting); (ii) for \(\lambda \in (\hat{\lambda}_1, \hat{\lambda}_2]\), all parents with \(a = 0\) set \(X^y = \hat{X}\) (authoritarian parenting); (iii) all parents with \(\lambda > \hat{\lambda}_2\) set \(X^y = \hat{X}\) (authoritarian parenting).

As expected, utilitarian parents tend to be more permissive than paternalistic parents. In addition, given \(\lambda\), the boarding school is more likely to be chosen by highly risk averse parents. The result has a similar flavor to the comparison between authoritative and permissive parenting.

Next, the utility difference between placing the child in a boarding school and authoritative parenting in the city yields:

\[ v_{BS}(a) - W(a, a^y) = \lambda (U^o(SS^y, a) - U^o(S^y, a)) + (1 - \lambda) (U^y(SS^y, 0) - U^y(S^y, a^y)) + \beta (v(0) - v(a^y)) \]

The first term is negative for all \(a\), since \(SS^y\) is a dominated lottery. The last term is unambiguously non-negative. We have imposed so far no restriction that
would allow us to sign the second term. It seems reasonable to assume that the child would rank the strict boarding school as his least preferred option in terms of his adolescent felicity. If so, then $U^y(SS^y, 0) < U^y(S^y, a^y)$. However, this may not be true in general. The comparative statics of $\lambda$ are generally ambiguous.

The following general lessons can be learned. Authoritarian will tend to dominate over authoritative parenting if either $SS^y$ is not much worse than $S^y$ (e.g., a luxury boarding school which is strict, but fares high on other dimension of the child’s welfare), or if risk taking is especially important for individual success, which would make children exposed to authoritative parenting unfit to seize opportunities later in life. These predictions accord with the casual observation that authoritarian parenting is more prevalent in competitive and entrepreneurial societies such as China, whereas authoritative parenting is more common in Europe (where, for instance, corporal parental punishment is forbidden in many countries).

In terms of global comparisons, we have the clear-cut (and unsurprising) implication that parents with low $\lambda$ tend to be permissive. As far as more paternalistic parents are concerned, we can observe either authoritative or authoritarian parenting style, depending mainly on the economic environment, and in particular the return to risk tolerance in old age (which may, in turn, depend on financial markets and social policy), and the utility cost of authoritative vs. authoritarian parenting.

5 Conclusions

The recent economic literature has increasingly turned its attention to preference heterogeneity in order to explain both micro- and macroeconomic puzzles. The persistence of low economic development, for instance, has been linked to the prevalence of cultural traits that are not conducive to entrepreneurship and innovation (see, e.g., Roland and Gorodnichenko 2011). In turn, the developmental psychology literature has long argued that parenting style can affect individual values, preferences, and beliefs. There is, however, little understanding of the
determinants of parenting styles. In this paper, we have provided a formal economic theory of child-rearing that rationalizes the emergence of different parenting styles as an equilibrium outcome. A cornerstone of our theory is the notion of paternalism: parents do not always accept their children’s preferences and inclinations, and typically regard it as part of their parental duties to influence or impose constraints on the children’s behavior. Our theory predicts that different parenting styles are the rational outcome of the interaction between paternalism and the economic environment. After presenting a general model, we have discussed two applications to the cultural transmission of patience and risk aversion.

Although in this paper we have restricted our analysis to a decision theory problem, the theory could be extended by letting paternalism evolve as the result of an evolutionary process. Our analysis suggests that there is no golden rule about the fitness of paternalistic preferences. In one example (patience), both very high and very low paternalism bring about economic success. In another example (risk), parental paternalism reduces risk-taking and protects children from juvenile risk, but can also stifle entrepreneurship. Therefore, the success of paternalistic families depends on the preference trait, the economic environment, and on stage of economic development.

We also abstracted in this paper from self-reinforcing mechanisms operating through general equilibrium effects. In a companion paper, we study the interaction between preference formation, innovation, and growth in a model where risk tolerance is endogenous like in the second example of this paper (see Doepke and Zilibotti 2013), and the distribution of preferences has a general equilibrium effect via an endogenous occupational choice between high- and low-risk professions.

We believe that the theory proposed in this paper can provide the basic framework to study such questions and to guide the empirical analysis of the determinants and effects of alternative child-rearing practices.
A Mathematical Appendix

A.1 Proofs for Lemmas and Propositions

**Proof of Lemma 1:** The Lemma is proven by contradiction. Suppose \( a^* = \hat{a}' \neq a \). Then,

\[
(1 - \lambda) U^y (x^y, \hat{a}') + \beta U^o (x^o, \hat{a}') + \beta z \lambda U^o (\hat{a}''', \hat{a}') + \beta z (1 - \lambda) U^y (\hat{a}'''', \hat{a}') + \beta^2 z v(\hat{a}') \\
\geq (1 - \lambda) U^y (x^y, a) + \beta v (a) \\
\geq (1 - \lambda) U^y (x^y, \hat{a}) + \beta U^o (\hat{a}', a) + \beta z \lambda U^o (\hat{a}'''', a) + \beta z (1 - \lambda) U^y (\hat{a}'''', \hat{a}') + \beta^2 z v(\hat{a}'''),
\]

where \( \hat{x}'', \hat{x}''', \hat{a}''' \) denote optimal future choices given that preference parameter \( \hat{a}' \) is chosen today. Note that these choices differ from the optimal future choices conditional which would obtain if \( a' = a \). Thus, imposing these choices in the continuation after \( a' = a \) can only decrease future utility on the right-hand side of the inequality. This explains the second inequality.\(^9\) Canceling terms, the first and third line of the expression above imply:

\[
(1 - \lambda) U^y (x^y, \hat{a}') + \beta U^o (x^o, \hat{a}') + \beta z \lambda U^o (\hat{a}''', \hat{a}') \\
\geq (1 - \lambda) U^y (x^y, a) + \beta U^o (x^o, a) + \beta z \lambda U^o (\hat{a}'''', a).
\]

However, this cannot be true, since Assumption 1 implies that \( U^y (x^y, a) \geq U^y (x^y, \hat{a}') \), \( U^o (\hat{x}', a) \geq U^o (\hat{x}', \hat{a}') \), and \( U^o (\hat{x}''', a) \geq U^o (\hat{x}''', \hat{a}') \). A contradiction. Thus, \( a^* = a \). \( \square \)

**Proof of Lemma 3:** \( W \) is a piece-wise decreasing function of \( a' \) with an upward discontinuity at \( a' = a^y \), since at this point the young starts to choose the safe lottery (which increases parental utility because of Assumption 2). Thus, \( W \) is maximized either at \( a' = 0 \) or \( a' = a^y \).

**Proof of Lemma 4:** Parents choose \( a' = a^y \) for their children if the inequality:

\[
W(a, 0) - W(a, a^y) \leq 0
\]

\(^9\)This is because the parent is fully altruistic towards the old-age choices of the child, meaning that a version of the envelope theorem applies.
holds. In the expression on the left-hand side, the only term that depends on $a$ is given by:

$$\lambda \left( U^o \left( R^y, a \right) - U^o \left( S^y, a \right) \right).$$

This term is strictly decreasing in $a$ due to 2, which establishes the result. \qed

**Proof of Proposition 2:** Parents choose high risk aversion $a' = a^y$ for their children if the inequality:

$$W \left( a, 0 \right) - W \left( a, a^y \right) \leq 0$$

holds. Writing out the inequality gives:

$$W \left( a, 0 \right) - W \left( a, a^y \right) = \lambda \left( U^o \left( R^y, a \right) - U^o \left( S^y, a \right) \right) + (1 - \lambda) \left( U^y \left( R^y, 0 \right) - U^y \left( S^y, a^y \right) \right) + \beta \left( v \left( 0 \right) - v \left( a^y \right) \right).$$

When exogenous juvenile risk increases, the first term $U^o \left( R^y, a \right) - U^o \left( S^y, a \right)$ unambiguously increases, because the increase in risk lowers utility (given that that parents are risk averse) and the increase in risk enters the lottery $S^y$ with higher weight (one) than the lottery $R^y \left( 1 - p_R \right)$. The second term $U^y \left( R^y, 0 \right) - U^y \left( S^y, a^y \right)$ can be written as:

$$U^y \left( R^y, 0 \right) - U^y \left( S^y, a^y \right) = (U^y \left( R^y, 0 \right) - U^y \left( S^y, 0 \right)) + (U^y \left( S^y, 0 \right) - U^y \left( S^y, a^y \right))$$

The first term unambiguously increases by the same argument as above. To sign the term, we would like to show that the second term

$$U^y \left( S^y, 0 \right) - U^y \left( S^y, a^y \right)$$

also increases when $S^y$ becomes more risky. Consider first the case where the lottery $S^y$ satisfies:

$$c_{S,L} < 1 < c_{S,H}$$

we would like to show that for an alternative lottery $\hat{S}^y$ with $E \left( \hat{S}^y \right) = E \left( S^y \right)$ and $\hat{c}_{S,L} < c_{S,L}$ and $\hat{c}_{H,L} < \hat{c}_{H,L}$, we have:

$$U^y \left( \hat{S}^y, 0 \right) - U^y \left( \hat{S}^y, a^y \right) > U^y \left( S^y, 0 \right) - U^y \left( S^y, a^y \right).$$
or:

\[
U^y (S^y, a^y) - U^y \left( \hat{S}^y, a^y \right) > U^y (S^y, 0) - U^y \left( \hat{S}^y, 0 \right).
\]

Writing this out in detail gives:

\[
\frac{p_L (c_{S,L})^{1-\sigma+\psi-a^y} - (\hat{c}_{S,L})^{1-\sigma+\psi-a^y}}{1 - \sigma + \psi - a^y} + (1 - p_L) \frac{(c_{S,H})^{1-\sigma+\psi-a^y} - (\hat{c}_{S,H})^{1-\sigma+\psi-a^y}}{1 - \sigma + \psi} > p_L \frac{(c_{S,L})^{1-\sigma+\psi} - (\hat{c}_{S,L})^{1-\sigma+\psi}}{1 - \sigma + \psi} + (1 - p_L) \frac{(c_{S,H})^{1-\sigma+\psi} - (\hat{c}_{S,H})^{1-\sigma+\psi}}{1 - \sigma + \psi}.
\]

or:

\[
\frac{p_L (c_{S,L})^{1-\sigma+\psi-a^y} - (\hat{c}_{S,L})^{1-\sigma+\psi-a^y}}{1 - \sigma + \psi - a^y} > (1 - p_L) \left( \frac{(c_{S,H})^{1-\sigma+\psi} - (\hat{c}_{S,L})^{1-\sigma+\psi}}{1 - \sigma + \psi} \right).
\]

(14)

Consider the left-hand side. The term consists of the difference in the utility difference between consumption values \(c_{S,L}\) and \(\hat{c}_{S,L}\), evaluated at \(a = a^y\) and \(a = 0\), respectively. The derivative of the utility function with respect to \(c\) evaluated at \(c_{S,L}\) is given by:

\[
\frac{-\sigma+\psi-a}{c_{S,L}}.
\]

Now deriving this derivative further with respect to \(a\) yields:

\[
-c_{S,L}^{\sigma+\psi-a} \log (c_{S,L}).
\]

Given that \(c_{S,L} < 1\), this term is positive. This implies that the derivative of utility with respect to consumption increases with \(a\), and is thus higher at \(a = a^y\) compared to \(a = 0\). The left-hand side of (14) is thus positive. By the same argument, evaluating the same cross derivative at \(c_{S,H}\) gives:

\[
-c_{S,L}^{\sigma+\psi-a} \log (c_{S,H}),
\]

which is negative since \(c_{S,H} > 1\). The derivative of utility with respect to con-
assumption thus decreases with \( a \) in this region, and is therefore lower at \( a = a^y \) compared to \( a = 0 \). The right-hand side of (14) is therefore negative, which establishes that (14) holds. For the case where the condition

\[
c_{S,L} < 1 < c_{S,H}
\]

is not satisfied, notice that it is always possible to multiply all consumption values with a positive constant such that the condition is satisfied. Rescaling consumption in this way does not affect decisions, because utility is homothetic. The previous argument therefore also applies to the general case.

Regarding the second part of the proposition, when \( p_R \) increases the first term \( U^o (R^y, a) - U^o (S^y, a) \) unambiguously decreases, because by Assumption\( 2 \) the parents prefer \( S^y \) over \( R^y \) and hence they prefer \( S^y \) over the risky part of \( R^y \), and increasing \( p_R \) puts more weight on the risky part. In contrast, the second term \( U^y (R^y, 0) - U^y (S^y, a^y) \) unambiguously increases, because at \( a = 0 \) children prefer \( R^y \) over \( S^y \), hence they prefer the risky part of \( R^y \) to the safe part and an increase in \( p_R \) has to increase their utility. The total effect thus depends on \( \lambda \).

**Proof of Proposition 3:** A parent will choose \( a' = 0 \) if \( W (a, 0) - W (a, a^y) > 0 \), where

\[
W (a, 0) - W (a, a^y) = \lambda (U^o (R^y, a) - U^o (S^y, a)) + (1 - \lambda) (U^y (R^y, 0) - U^y (S^y, a^y)) + \beta (v (0) - v (a^y)).
\]

The term \( U^o (R^y, a) - U^o (S^y, a) \) is negative for all \( a \), while the two remaining terms are positive. When \( \lambda = 0 \), \( W (a, 0) - W (a, a^y) < 0 \). By the continuity of the value functions, the same must be true for a range of low \( \lambda \)'s.

Next, note that

\[
U^o (R^y, 0) - U^o (S^y, 0) > U^o (R^y, a^y) = U^o (S^y, a^y).
\]

Thus, \( \lambda_1 \) (if it is strictly smaller than unity) is such that

\[
\lambda_1 (U^o (R^y, a^y) - U^o (S^y, a^y)) = (1 - \lambda_1) (U^y (R^y, 0) - U^y (S^y, a^y)) + \beta (v (0) - v (a^y)).
\]
In turn, this implies that, in a right-hand neighborhood of $\lambda_1$,

$$
\lambda \left( U^o (R^y, a^y) - U^o (S^y, a^y) \right) < (1 - \lambda) \left( U^y (R^y, 0) - U^y (S^y, a^y) \right) + \beta \left( v(0) - v(a^y) \right),
$$

$$
\lambda \left( U^o (R^y, 0) - U^o (S^y, 0) \right) > (1 - \lambda) \left( U^y (R^y, 0) - U^y (S^y, a^y) \right) + \beta \left( v(0) - v(a^y) \right).
$$

Therefore, in such a right-hand neighborhood of $\lambda_1$ $W (a^y, 0) - W (a^y, a^y) < 0$ (implying $a' = a^y$), whereas $W (0, 0) - W (0, a^y) > 0$ (implying $a' = 0$).

Finally, $\lambda_2$ (if it is strictly smaller than unity) is such that

$$
\lambda_2 \left( U^o (R^y, 0) - U^o (S^y, 0) \right) = (1 - \lambda_2) \left( U^y (R^y, 0) - U^y (S^y, a^y) \right) + \beta \left( v(0) - v(a^y) \right).
$$

In turn, this implies that, in a right-hand neighborhood of $\lambda_2$,

$$
\lambda \left( U^o (R^y, 0) - U^o (S^y, 0) \right) < (1 - \lambda) \left( U^y (R^y, 0) - U^y (S^y, a^y) \right) + \beta \left( v(0) - v(a^y) \right),
$$

$$
\lambda \left( U^o (R^y, a^y) - U^o (S^y, a^y) \right) < (1 - \lambda) \left( U^y (R^y, 0) - U^y (S^y, a^y) \right) + \beta \left( v(0) - v(a^y) \right).
$$

Therefore, in such a right-hand neighborhood of $\lambda_2$ $W (a, 0) - W (a, a^y) < 0$ (implying $a' = a^y$), for $a \in \{0, a^y\}$.

\[ \square \]

**Proof of Proposition 4:** Same as previous proposition (to be written) \[ \square \]

### A.2 Characterization of the Patience Problem

#### A.2.1 Staying on the Farm

\[ c^{-\sigma} = z \beta \Omega_F R_F^{1-\sigma} (s - c - c^y)^{-\sigma} \]

\[ z (\lambda + (1 - \lambda) \psi) (c^y)^{-\sigma} = z \beta \Omega_F R_F^{1-\sigma} (s - c - c^y)^{-\sigma} \]

\[ c = (z (\lambda + (1 - \lambda) \psi))^{-\frac{1}{\sigma}} (c^y) \]

\[ c^y = (z (\lambda + (1 - \lambda) \psi))^\frac{1}{\sigma} c \]
\[
c^{-\sigma} = z\beta\Omega_FR_{F}\left(s - c\left(1 + (z (\lambda + (1 - \lambda)\psi))^{\frac{1}{2}}\right)\right)^{-\sigma}
\]
\[
c = \frac{\left(z\beta\Omega_FR_{F}\right)^{-\frac{1}{\sigma}} \left(1 + (z (\lambda + (1 - \lambda)\psi))^{\frac{1}{2}}\right)}{1 + \left(z\beta\Omega_FR_{F}\right)^{-\frac{1}{\sigma}} + \left(z (\lambda + (1 - \lambda)\psi)\right)^{-\frac{1}{2}}}
\]

A.2.2 Moving to the City

The following general characterization can be established.

**Lemma 5** Conditional on the choice set \(X^\mathcal{U} = X^{\mathcal{FREE}}\), the equilibrium is characterized by the following conditions. Let:

\[
B(\Omega_U, a') = \frac{(\lambda + (1 - \lambda)(\psi - a'))(\psi - a')^{\frac{1 - \sigma}{\sigma}} + (\beta\Omega_U)^{\frac{1}{\sigma}}R_{U}^{\frac{1 - \sigma}{\sigma}}}{(\psi - a')^{\frac{1 - \sigma}{\sigma}} + (\beta\Omega_U)^{\frac{1}{\sigma}}R_{U}^{\frac{1 - \sigma}{\sigma}}}
\]

The optimal choices then satisfy the following conditions:

\[
c^\mathcal{U} = \frac{s}{1 + (zB)^{\frac{1}{\sigma}}},
\]
\[
s^{y\mathcal{U}} = s - c^\mathcal{U},
\]
\[
c^{y\mathcal{U}} = \frac{s^{y\mathcal{U}}}{1 + \left(\frac{\beta R_{U}^{\frac{1 - \sigma}{\sigma}}\Omega_U}{\psi - (a')^{\sigma}}\right)^{\frac{1}{\sigma}}},
\]

\[
\lambda \left(\psi - (a')^{U} - 1\right) \left(\frac{1}{\sigma}\psi - (a')^{U} \frac{1}{1 + \left(\frac{\beta R_{U}^{\frac{1 - \sigma}{\sigma}}\Omega_U}{\psi - (a')^{\sigma}}\right)^{-\frac{1}{\sigma}}}\right) - \frac{1 - \lambda}{1 - \sigma} \leq 0,
\]

where the strict inequality holds if and only if \((a')^{U} = 0\). The coefficient \(\Omega_U\) is implicitly
defined by the equation:

$$\Omega_U = \frac{1 + (1 - \sigma) \left( zB(\Omega_U, a') \right)^{\frac{1}{2}}}{1 + (zB(\Omega_U, a'))^{\frac{1}{2}} \right)^{1-\sigma}}.$$  

**Proof of Lemma 5:** Given our functional form assumptions, the value function is concave and differentiable, and can be characterized using first-order conditions:

$$(\psi - a') (c^y(s^y, a'))^{-\sigma} = \beta R_U v_{U s} (s').$$

We now invoke the guess that the value function is homothetic in assets:

$$v_U(s) = \Omega_U \frac{s^{1-\sigma}}{1-\sigma},$$

where $\Omega_U$ is a constant to be determined. The derivative of the value function with respect to assets is then $v_{U s} (s) = \Omega_U s^{-\sigma}$. Replacing $v_{U s} (s')$ by $\Omega_U s^{-\sigma}$ in the first-order condition, and eliminating $s'$ using the law of motion (8) yields:

$$(\psi - c') (c^y(s^y, a'))^{-\sigma} = \beta R_U^{1-\sigma} \Omega_U (s^y - c^y(s^y, a'))^{-\sigma}.$$  \hspace{1cm} (16)

Solving for $c^y(s^y, a')$ yields

$$c^y(s^y, a') = \frac{(\psi - a')^{\frac{1}{2}} s^y}{(\psi - a')^{\frac{1}{2}} + (\beta R_U^{1-\sigma} \Omega_U (a'))^{\frac{1}{2}}}$$  \hspace{1cm} (17)

with partial derivatives:

$$c^y_{s^y} (s^y, a') = \frac{1}{1 + \left( \frac{\beta R_U^{1-\sigma} \Omega_U}{\psi - a'} \right)^{\frac{1}{\sigma}}},$$

$$c^y_{a'} (s^y, a') = \frac{1}{1 + \left( \frac{\beta R_U^{1-\sigma} \Omega_U}{\psi - a'} \right)^{\frac{1}{\sigma}}} \frac{s^y}{\psi - a'}$$

$$= \frac{1}{1 + \left( \frac{\beta R_U^{1-\sigma} \Omega_U}{\psi - a'} \right)^{\frac{1}{\sigma}}},$$

$$\frac{1}{1 + \left( \frac{\beta R_U^{1-\sigma} \Omega_U}{\psi - a'} \right)^{\frac{1}{\sigma}}}. $$

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Moving back to the parent’s problem, the first-order condition with respect to $a'$ yields:

\[ 0 = (\lambda + (1 - \lambda)(\psi - a')) c^y(s^y, a')^{1-\sigma} c_{a'}^y(s^y, a') - (1 - \lambda) \frac{c^y(s^y, a')^{1-\sigma}}{1-\sigma} \]

\[ + \beta v_{Us'}(s') \frac{\partial s'}{\partial a'} . \]

Using the fact that $v_{Us'}(S') = \Omega_U(s')^{-\sigma} = \Omega_U R_Y^{1-\sigma} (s^y - c^y(s^y, a'))^{-\sigma}$, the first order condition with respect to $a'$ yields:

\[ 0 = (\lambda + (1 - \lambda)(\psi - a')) c^y(s^y, a')^{1-\sigma} c_{a'}^y(s^y, a') \]

\[ - (1 - \lambda) \frac{c^y(s^y, a')^{1-\sigma}}{1-\sigma} \]

\[ - \beta R_Y^{1-\sigma} \Omega_U (s^y - c^y(s^y, a'))^{-\sigma} c_{a'}^y(s^y, a') . \]

Using (16) allows us to rewrite the first order condition above as:

\[ 0 = (\lambda + (1 - \lambda)(\psi - a')) c^y(s^y, a')^{1-\sigma} c_{a'}^y(s^y, a') \]

\[ - (1 - \lambda) \frac{c^y(s^y, a')^{1-\sigma}}{1-\sigma} \]

\[ - (\psi - a') c^y(s^y, a')^{-\sigma} c_{a'}^y(s^y, a') , \]

which simplifies to:

\[ 0 = -\lambda (\psi - a' - 1) c^y(s^y, a') - (1 - \lambda) \frac{c^y(s^y, a')}{1-\sigma} \]

\[ 0 = \lambda (\psi - a' - 1) \left( \frac{1}{\sigma \psi - a'} + \frac{1}{1 + \left( \frac{\beta R_Y^{1-\sigma} \Omega_U}{\psi - a'} \right)^{\frac{1}{\sigma}}} \right) - \frac{1 - \lambda}{1 - \sigma} \]
The maximization with respect to $c$ can be written as:

$$\max_c \left\{ \frac{c^{1-\sigma}}{1 - \sigma} + z \left( \lambda \frac{c^y (s^y, a')^{1-\sigma}}{1 - \sigma} \right) + (1 - \lambda) \left( \psi - a' \right) \left( \frac{c^y (s^y, a')^{1-\sigma}}{1 - \sigma} \right) + \beta \Omega_U \left( \frac{c^y (s^y - c^y)^{1-\sigma}}{1 - \sigma} \right) \right\}$$

$$= \max_c \left\{ \frac{c^{1-\sigma}}{1 - \sigma} + z \left( \frac{s - c}{1 - \sigma} \right)^{1-\sigma} \left( 1 + \left( \frac{c^y (s^y, a')^{1-\sigma}}{1 - \sigma} \right) \right) \right\}$$

$$= \left( \lambda + (1 - \lambda) \left( \psi - a' \right) \right) + \beta R_U^{1-\sigma} \Omega_U \left( \frac{\beta R_U^{1-\sigma} \Omega_U}{\psi - (a')^{\sigma}} \right)^{1-\sigma}$$

The first-order condition yields:

$$c = \frac{z^{-\frac{1}{\sigma}} \left( \frac{1}{1 + \left( \beta R_U^{1-\sigma} \Omega_U \right)^{1/\sigma}} \right)^{-\frac{1}{\sigma}} \left( \lambda + (1 - \lambda) \left( \psi - a' \right) \right) + \beta R_U^{1-\sigma} \Omega_U \left( \frac{\beta R_U^{1-\sigma} \Omega_U}{\psi - (a')^{\sigma}} \right)^{1-\sigma}}{\left( \lambda + (1 - \lambda) \left( \psi - a' \right) \right) + \beta R_U^{1-\sigma} \Omega_U \left( \frac{\beta R_U^{1-\sigma} \Omega_U}{\psi - (a')^{\sigma}} \right)^{1-\sigma}}$$

Finally, consider $\Omega$.

Using the solution for $c^y (s^y, a')$ given by (17), we obtain:

$$s' = R_U (s^y - c^y (s^y, a')) = R_U \left( \frac{\beta R_U^{1-\sigma} \Omega_U}{\psi - (a')^{\sigma}} \right)^{1/\sigma} \left( \lambda + (1 - \lambda) \left( \psi - a' \right) \right) + \beta R_U^{1-\sigma} \Omega_U \left( \frac{\beta R_U^{1-\sigma} \Omega_U}{\psi - (a')^{\sigma}} \right)^{1-\sigma} S^y.$$
Hence:

\[
\begin{align*}
  w \left( s^y, X^{\text{FREE}}, a' \right) &= \left( \lambda + (1 - \lambda)(\psi - a') \right) \frac{c^y(s^y, a')^{1-\sigma}}{1-\sigma} + \beta v(s') \\
  &= \left( \lambda + (1 - \lambda)(\psi - a') \right) \frac{\left[ \frac{(\psi-a')^{1-\sigma}}{c^y} + (\beta R_U^{1-\sigma} \Omega)^{\frac{1}{\sigma}} \right]^{1-\sigma} \Omega_U}{1-\sigma} \\
  &= \left[ \left( \lambda + (1 - \lambda)(\psi - a') \right) \left( \frac{1}{1-(zB(\Omega, a'))^{\frac{1}{\sigma}}} \right) \right]^{1-\sigma} \\
  &= \frac{\Omega_U}{1-\sigma} (s^y)^{1-\sigma}.
\end{align*}
\]

where

\[
B(\Omega_U, a') \equiv \frac{\lambda + (1 - \lambda)(\psi - a')}{\lambda + (1 - \lambda)(\psi - a')} \left( \frac{1}{1-(zB(\Omega, a'))^{\frac{1}{\sigma}}} \right) \frac{R_U^{1-\sigma} \Omega}{\Omega_U^{\frac{1}{\sigma}}}.
\]

Next, the saving problem for the parent can be written as:

\[
\max_c \left\{ c^{1-\sigma} - zB(\Omega_U, a') \left( s - x \right)^{1-\sigma} \right\},
\]

whose solution yields:

\[
c = \frac{s}{1 + (zB(\Omega_U, a'))^{\frac{1}{\sigma}}},
\]

The maximized utility of the parent is, therefore:

\[
v(s) = \frac{1}{1-(zB(\Omega, a'))^{\frac{1}{\sigma}}} \left( \frac{1}{1-(zB(\Omega, a'))^{\frac{1}{\sigma}}} \right)^{1-\sigma} \left( \frac{(zB(\Omega, a'))^{\frac{1}{\sigma}}}{1-(zB(\Omega, a'))^{\frac{1}{\sigma}}} \right) \left( s^{1-\sigma} \right).\]
We therefore conclude that:

$$\Omega_U = \frac{1 + (1 - \sigma) (zB (\Omega_U, a'))^{\frac{1}{2}}}{\left(1 + (zB (\Omega_U, a'))^{\frac{1}{2}}\right)^{(1-\sigma)}}$$
References


