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Higgs and Z-boson production with a jet veto

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We derive first next-to-next-to-leading logarithmic resummations for jet-veto efficiencies in Higgs and Z-boson production at hadron colliders. Matching with next-to-next-to-leading order results allows us to provide a range of phenomenological predictions for the LHC, including cross-section results, detailed uncertainty estimates and comparisons to current widely-used tools.

In searches for new physics at hadron colliders such as the Tevatron and CERN’s Large Hadron Collider (LHC), in order to select signal events and reduce backgrounds, events are often classified according to the number of hadronic jets — collimated bunches of energetic hadrons — in the final state. A classic example is the search for Higgs production via gluon fusion with a subsequent decay to $W^+W^-$ [1, 2]. A severe background comes from $t\bar{t}$ production, whose decay products also include a $W^+W^-$ pair. However, this background can be separated from the signal because its $W^+W^-$ pair usually comes together with hard jets, since in each top decay the $W$ is accompanied by an energetic ($b$) quark.

Relative to classifications based on objects such as leptons (used e.g. to identify the $W$ decays), one of the difficulties of hadronic jets is that they may originate not just from the decay of a heavy particle, but also as Quantum Chromodynamic (QCD) radiation. This is the case in our example, where the incoming gluons that fuse to produce the Higgs quite often radiate additional partons. Consequently, while vetoing the presence of jets eliminates much of the $t\bar{t}$ background, it also removes some fraction of signal events. To fully interpret the search results, including measuring Higgs couplings, it is crucial to be able to predict the fraction of the signal that survives the jet veto, which depends for example on the transverse momentum threshold $p_{t,veto}$ used to identify vetoed jets.

One way to evaluate jet-veto efficiencies is to use a fixed-order perturbative expansion in the strong coupling $\alpha_s$, notably to next-next-to-leading order (NNLO), as in the Higgs-boson production calculations of [3–5]. Such calculations however become unreliable for $p_{t,veto} \ll M$, with $M$ the boson mass, since large terms $\alpha_s^2 L^{2n}$ appear ($L = \ln(M/p_{t,veto})$) in the cross section at all orders in the coupling constant. These enhanced classes of terms can, however, be resummed to all orders in the coupling, often involving a functional form $\exp(L g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L)/\pi + \ldots)$. There exist next-to-next-to-leading logarithmic (NNLL) resummations, involving the $g_n(\alpha_s L)$ functions up to and including $g_3$, for a number of quantities that are more inclusive than a jet veto: e.g. a Higgs or vector-boson transverse momentum [6–9], the beam thrust [10], and related observables [11, 12]. To obtain estimates for jet vetoes, some of these calculations have been compared to or used to reweight [10, 13, 16] parton-shower predictions [17, 18] matched to NLO results [19, 20]. However, with reweighting, neither the NNLO nor NNLL accuracy of the original calculation carry through to the jet veto prediction.

Recently there has been progress towards NNLL calculations of the jet veto efficiency itself. Full NLL results and some NNLL ingredients for Higgs and vector-boson were provided in [21]. Ref. [22] used these and other ingredients in the soft-collinear effective theory framework to consider resummation for the Higgs-boson case beyond NLL accuracy. In this letter we show how to use the results of [21] together with those from boson $p_t$ resummations [6–9] to obtain full NNLL accuracy. We also examine the phenomenological impact of our results, including a matching to NNLO predictions. Given the ubiquity of jet cuts in hadron-collider analyses, the understanding gained from our analysis has a potentially wide range of applications.

The core of boson transverse-momentum ($p_t^B$) resummations lies in the fact that soft, collinear emissions at disparate rapidities are effectively emitted independently. Summing over all independent emissions, one obtains the differential boson $p_t$ cross section

$$\frac{d\Sigma(B)}{dp_t^B} = \sigma_0 \int \frac{d^2b}{4\pi^2} e^{-ib\cdot p_t^B} \prod_{i=1}^n \int [dk_i|M^2(k_i)(e^{ib\cdot k_i} - 1)],$$

(1)

where $\sigma_0$ is the leading-order total cross section, $[dk_i|M^2(k_i)$ is the phase-space and matrix-element for emitting a soft, collinear gluon of momentum $k_i$, while the exponential factors and $b$ integral encode in a factorised form the constraint relating the boson $p_t$ and those of individual emissions $\delta^2(p_t^B - \sum_{i=1}^{n} k_{is})$ [23]. The $-1$ term in the round brackets arises because, by unitarity, virtual corrections come with a weight opposite to that of real emissions, but don’t contribute to the $p_t^B$ sum.

To relate Eq. (1) to a cross section with a jet-veto, let us first make two simplifying assumptions: that the
independent-emission picture is exact and that a jet algorithm clusters each emission into a separate jet. The resummation for the cross section for the highest jet $p_t$ to be below some threshold $p_{t0}$, considering jets at all rapidities, is then equivalent to requiring all emissions to be below that threshold:

$$\Sigma (k_0) = \sigma_0 \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=1}^{n} \int [dk_i] M^2(k_i) (\Theta (p_i - k_i) - 1)$$

$$= \sigma_0 \exp \left[ - \int [dk_i] M^2(k_i) \Theta (k_i - p_i) \right], \quad (2)$$

with the same universal matrix element $M^2(k_i)$ entering Eqs. (1) and (2).

Eq. (2) is clearly an oversimplification. Firstly, even within the independent emission picture, two emissions close in rapidity $y$ and azimuth $\phi$ can be clustered together into a single jet. Let us introduce a function $J(k_1, k_2)$ that is 1 if $k_1$ and $k_2$ are clustered together and 0 otherwise. Concentrating on the 2-emission contribution to Eq. (2), one sees that clustering leads to a correction given by the difference between the veto with and without clustering:

$$\mathcal{F}^{\text{clust}} \sigma_0 = \frac{\sigma_0}{2!} \int [dk_1][dk_2] M^2(k_1) M^2(k_2) \times J(k_1, k_2)$$

$$\left( \Theta (p_i - k_i, 12) - \Theta (p_i - k_i, 11) \right) \Theta (p_i - k_i, 21). \quad (3)$$

where $k_12 = k_1 + k_2$ (throughout, we assume standard E-scheme recombination, which adds 4-vectors). This contribution has a logarithmic structure $\alpha_s^2 L$, i.e. NNLL, with each emission leading to a power of $\alpha_s$, while the $L$ factor comes from the integral over allowed rapidities ($|y| \lesssim \ln (M/p_{t, \text{veto}})$).

For more than two emissions, two situations are possible: (1) three or more emissions are close in rapidity, giving extra powers of $\alpha_s$ without extra log-enhancements (N^3LL and beyond); (2) any number of extra emissions are far in rapidity, each giving a factor $\alpha_s L$, i.e. also NNLL. The latter contribution is simple because, independently of whether the two nearby emissions clustered, those that are far away must still have $p_{t1} < p_{t, \text{veto}}$. Thus the full “clustering” correction to the independent-emission picture is a multiplicative factor $1 + \mathcal{F}^{\text{clust}}$, as first derived in detail in the appendix of [21] using results from [24].

For the generalised-kt jet-algorithm family [25, 29], with a jet radius parameter $R$, we have $J(k_1, k_2) = \Theta (R^2 - (y_1 - y_2)^2 - (\phi_1 - \phi_2)^2)$. At NNLL accuracy Eq. (3) evaluates to $\mathcal{F}^{\text{clust}} = 4\alpha_s^2 (p_{t, \text{veto}}) C_L f^{\text{clust}}(R)/\pi^2$ with [21]

$$f^{\text{clust}}(R) = \left( -\frac{\pi^2 R^2}{12} + \frac{R^4}{16} \right) C,$$  

for $R < \pi$; $C = C_F = \frac{4}{3}$ or $C_A = 3$ respectively for incoming quarks (e.g. $q\bar{q} \to Z$) or incoming gluons (e.g. $gg \to H$).

Next, we address the issue that gluons are not all emitted independently. This is accounted for in Eq. (1) because, to order $\alpha_s^2$, the $M^2(k)$ quantity that appears there should be understood as an effective matrix element

$$[dk] M^2(k) = [dk] \left( M^2(k) + M_{\text{loop}}^2(k) \right)$$

$$+ \int d^2k_1 [dk_b] M_{\text{correl}}^2(k_a, k_b) \delta^2(k_{a, ab} - k_t), \quad (5)$$

where $M^2(k)$ is the pure $O(\alpha_s)$ matrix element, $M_{\text{correl}}^2(k_a, k_b)$ the correlated part of the matrix element for emission of two soft-collinear gluons or a quark-antiquark pair, including relevant symmetry factors, and $M_{\text{loop}}^2$ the corresponding part of the $\alpha_s^2$ 1-loop matrix element. The separation into correlated and independent emissions is well defined because of the different colour factors that accompany them in the generic case [30–32]. The 6-function in Eq. (5) extracts two-parton configurations with the same total $p_t$ as the 1-gluon configurations.

For a jet veto, part of the result from the effective matrix element carries through: when two correlated emissions are clustered into a single jet, it is their sum, $k_{t, ab}$, that determines the jet transverse momentum. Therefore the same effective matrix element can be used in Eq. (2), as long as one includes an additional correction to account for configurations where the two emissions are clustered in separate jets:

$$\mathcal{F}_{\text{correl}} \sigma_0 = \sigma_0 \int [dk_a][dk_b] M_{\text{correl}}^2(k_a, k_b) \times (1 - J(k_a, k_b))$$

$$\left( \Theta (p_i - k_{ta}) \Theta (p_i - k_{tb}) - \Theta (p_i - k_{ta, \text{ab}}) \right) \quad (6)$$

At NNLL, $\mathcal{F}_{\text{correl}} = 4\alpha_s^2 (p_{t, \text{veto}}) C_L f_{\text{correl}}(R)/\pi^2$ with

$$f_{\text{correl}}(R) = \left( \frac{(-131 + 12\pi^2 + 132 \ln 2) C_A}{72} \right) \ln \frac{R}{0.61 C_A - 0.015 n_f + \mathcal{O}(R^2)} \quad (7)$$

for generalised-kt algorithms, in the limit of small $R$. Ref. [21] includes a numerical result for all $R < 3.5$ and analytical terms up to $R^6$, used in the rest of this article. It did not, however, make the relation with the boson $p_t$ resummation.

All remaining contributions to a NNLL resummation, such as the 3-loop cusp anomalous dimension or a multiplicative $C_1 \alpha_s$ term are either purely virtual, so independent of the precise observable, or involve at most a single real emission, so can be taken from the boson $p_t$ resummations [6, 9]. Thus the full NNLL resummed cross

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1 For generic processes, subtleties can arise with spin-correlation effects [34]. These are simpler for jet vetoes, which don’t correlate distinct collinear regions.
section for the jet-veto is given by:

\[ \Sigma_{\text{NNLL}}^{(j)}(p_t, \text{veto}) = \sum_{i,j} \int d\alpha d\beta |M^\alpha|^2 \delta(\alpha, \beta - M^2) \]

\[ \times \left[ f_i(x_1, e^{-L} \mu_F) f_j(x_2, e^{-L} \mu_F) \left( 1 + \frac{\alpha_s}{2\pi} H^{(1)} \right) + \frac{\alpha_s}{2\pi} 1 - 2\alpha_s\beta_0 L \sum_k \int dx_i \frac{dz}{z} \left( C_{ki}(x_1) f_i(x_1, e^{-L} \mu_F) \times f_j(x_2, e^{-L} \mu_F) + \{(x_1, i) \leftrightarrow (x_2, j)\} \right) \right] \]

\[ (1 + F^{\text{clus}} + F^{\text{corr}}) \times e^{L g_1(\alpha_s L)} + g_2(\alpha_s L) + g_3(\alpha_s L), \]

(8)

where the coefficient functions \( H^{(1)} \) and \( C_{ki}^{(1)} \) and resummation functions \( g_1, g_2 \) and \( g_3 \) are as derived for the boson \( p_T^B \) resummation (reproduced for completeness in the supplemental material to this letter 33, together with further discussion on the connection to boson \( p_T^R \) resummation). The results are expressed in terms of \( L = \ln(Q/p_T, \text{veto}) \), \( \alpha_s = \alpha_s(\mu_R) \); the resummation, renormalisation and factorisation scales \( Q, \mu_R \) and \( \mu_F \) are to be chosen of order of \( M \).

A form similar to Eq. (5) was derived independently in 22 for Higgs production, also using ingredients from 21. It differs however at NNLL in that the combination of \( f^{\text{clus}} + f^{\text{corr}} \) is accompanied by an extra \( -\zeta_3 C_A \).

Ref. 22 had used a NNLL analysis of the \( R \to \infty \) limit to relate jet and boson-\( p_T \) resummations. A subtlety of this limit is that one must then account for a \( N^3LL \) \( \alpha_s^2 R \) term, which for \( R \gtrsim \ln M/p_T \) is promoted to an additional NNLL \( \alpha_s^2 \ln M/p_T \) contribution 33.

One check of Eq. (8) is to expand it in powers of \( \alpha_s \), \( \Sigma_{\text{NNLL}}^{(j)}(p_T) = \alpha_s^2 \sum\infty_{n=0} \Sigma_{\text{NNLL,n}}^{(j)}(p_T) \), and compare \( d\Sigma_{\text{NNLL},2}^{(j)}(p_T)/d\ln p_T \) to the NLO Higgs+1 jet prediction 36 38 from MCFM 39. \( d\Sigma_{\text{NNLL}}^{(j)}(p_T)/d\ln p_T \). NNLL resummation implies control of terms \( \alpha_s^2 L^3 \ldots \alpha_s^2 \) (constant terms in this quantity and so the difference between MCFM and the 2nd order expansion of the resummation should vanish for large \( L \). This is what we find within reasonable precision. The precision of the test can be increased if one considers the \( O(\alpha_s^2) \) difference between the jet and boson-\( p_T \) resummations, which has fewer logarithms and so is numerically easier to determine in MCFM. It is predicted to be

\[ \frac{d\Sigma_{\text{NNLL},2}^{(j)}(p_T)}{d\ln p_T} - \frac{d\Sigma_{\text{NNLL},2}^{(B)}(p_T)}{d\ln p_T} = -\frac{4\alpha_s^2 \sigma_0}{\pi^2} \left( f^{\text{clus}}(R) + f^{\text{corr}}(R) \right) + \zeta_3 C. \]

(9)

This is compared to MCFM’s LO H+2-jet result in the upper panel of Fig. 1. There is excellent agreement at small \( p_T \), for each of three \( R \) values. The result of 22 (BN, only for \( R = 0.5 \)) is also shown for comparison.

The above test can be extended one order further by examining the order \( \alpha_s^2 \sigma_0 \) difference between the jet and boson \( p_T \) differential distributions. The comparison between our predictions and MCFM H+2-jet NNLO results 42 43 is given in Fig. 1 (lower panel), for each of three \( R \) values. To facilitate visual interpretation of the results, the expected \( \alpha_s^2 \sigma_0 L^2 \) term has been subtracted. The residual \( \alpha_s^2 \sigma_0 L \) term is clearly visible in the MCFM results and, within the fluctuations, coincides well with our predictions, providing a good degree of corroborating evidence for the correctness of our results beyond order \( \alpha_s^2 \sigma_0 \).

To illustrate the phenomenological implications of our work, we examine the jet veto efficiency \( \epsilon(p_T) \equiv \Sigma^{(j)}(p_T)/\sigma_{\text{tot}} \), where \( \sigma_{\text{tot}} \) is the total cross section, known up to NNLO 44 48. We combine (“match”) the resummation with fixed-order predictions, available from fully differential NNLO boson-production calculations 4 5 50 51 or NLO boson+jet calculations 52 53 implemented in MCFM 53. We use three matching schemes, denoted \( a, b \) and \( c \), straightforward extensions 35 of those used at NLL in 21.
Our central predictions have $\mu_R = \mu_F = Q = M/2$ and scheme $a$ matching, with MSTW2008NNLO PDFs \cite{54}. We use the anti-$k_t$ \cite{23} jet-algorithm with $R = 0.5$, as implemented in FastJet \cite{55}. For the Higgs case we use the large $m_{\text{top}}$ approximation and ignore $b\bar{b}$ fusion and $b's$ in the $gg \rightarrow H$ loops (corrections beyond this approximation have a relevant impact \cite{16,54}). To determine uncertainties we vary $\mu_R$ and $\mu_F$ by a factor of two in either direction, requiring $1/2 \leq \mu_R/\mu_F \leq 2$. Maintaining central $\mu_{R,F}$ values, we also vary $Q$ by a factor of two and change to matching schemes $b$ and $c$. Our final uncertainty band is the envelope of these variations. In the fixed-order results, the band is just the envelope of $\mu_{R,F}$ variations.

The results for the jet-veto efficiency in Higgs and Z-boson production are shown in Fig. 2 for 8 TeV LHC collisions. Compared to pure NNLO results, the central value is slightly higher and for Higgs production, the uncertainties reduced, especially for lower $p_{t,\text{veto}}$ values. Compared to NNLO+NLL results \cite{21}, the central values are higher, sometimes close to edge of the NNLO+NLL bands; since the NNLO+NLL results used the same approach for estimating the uncertainties, this suggests that the approach is not unduly conservative. In the Higgs case, the NNLO+NLL uncertainty band is not particularly smaller than the NNLO+NLL one. This should not be a surprise, since \cite{21} highlighted the existence of possible substantial corrections beyond NNLL and beyond NNLO. For the Higgs case, we also show a prediction from POWHEG \cite{23,41} interfaced to Pythia 6.4 \cite{17} at parton level (Perugia 2011 shower tune \cite{41}), reweighted to describe the NNLL+NLO Higgs-boson $p_t$ distribution from HqT (v2.0) \cite{7}, as used by the LHC experiments. Though reweighting fails to provide NNLO or NNLL accuracy for the jet veto, for $p_{t,\text{veto}}$ scales of practical relevance, the result agrees well with our central prediction. It is however harder to reliably estimate uncertainties in reweighting approaches than in direct calculations.

Finally, we provide central results and uncertainties for the jet-veto efficiencies and 0-jet cross sections (in pb) with cuts (in GeV) like those used by ATLAS and CMS, and also for a larger $R$ value:  

<table>
<thead>
<tr>
<th>$R$</th>
<th>$p_{t,\text{veto}}$</th>
<th>$\epsilon(7 \text{ TeV})$</th>
<th>$\sigma_{\text{q-jet}}(7 \text{ TeV})$</th>
<th>$\epsilon(8 \text{ TeV})$</th>
<th>$\sigma_{\text{q-jet}}(8 \text{ TeV})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>25</td>
<td>$0.63^{+0.07}_{-0.05}$</td>
<td>$9.6^{+1.3}_{-1.1}$</td>
<td>$0.61^{+0.07}_{-0.06}$</td>
<td>$12.0^{+1.6}_{-1.4}$</td>
</tr>
<tr>
<td>0.5</td>
<td>30</td>
<td>$0.68^{+0.06}_{-0.05}$</td>
<td>$10.4^{+1.2}_{-1.1}$</td>
<td>$0.67^{+0.06}_{-0.05}$</td>
<td>$13.0^{+1.5}_{-1.5}$</td>
</tr>
<tr>
<td>1.0</td>
<td>30</td>
<td>$0.64^{+0.03}_{-0.05}$</td>
<td>$9.8^{+0.8}_{-1.1}$</td>
<td>$0.63^{+0.04}_{-0.05}$</td>
<td>$12.2^{+1.1}_{-1.4}$</td>
</tr>
</tbody>
</table>

Interestingly, the $R = 1$ results have reduced upper uncertainties, due perhaps to the smaller value of the NNLL $f(R)$ correction (a large $f(R)$ introduces significant $Q$-scale dependence). The above results are without a rapidity cut on the jets; the rapidity cuts used by ATLAS and CMS lead only to small, < 1%, differences \cite{21}.

For the 0-jet cross sections above, we used total cross sections at 7 TeV and 8 TeV of 15.3$^{+1.1}_{-1.2}$ pb and 19.5$^{+1.4}_{-1.5}$ pb respectively \cite{57,58} (based on results including \cite{45,49}) and took their scale uncertainties to be uncorrelated with those of the efficiencies. Symmetrising uncertainties, we find correlation coefficients between

![Figure 2](https://example.com/image2.png)
the 0-jet and $\geq 1$-jet cross sections of $-0.43 \ ( -0.50)$ for $R = 0.4 \ (R = 0.5)$, using the covariance matrix in [33].

Code to perform the resummations and matchings will be made available shortly.

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Note added: as our manuscript was being finalised, Ref. [67] appeared. It claims issues in NLL resummations of jet vetoes, however does not address the all-order derivation of the NLL $R$-dependent terms in [21]. Its claim is further challenged by the $\alpha_s^3$ numerical check in Fig. 1.


SUPPLEMENTAL MATERIAL

We here provide material that completes the discussion of the letter, including more explicit formulae, some derivations and supplementary figures.

1. Explicit resummation formulae

In the present section we report the explicit expressions for the resummation functions \( g_1 \), \( g_2 \) and \( g_3 \) computed in [3, 7], as functions of \( \lambda = \alpha \beta_0 L \), with \( L = \ln(Q/p_t) \). \( \alpha_s \) denotes \( \alpha_s(\mu_R) \) unless otherwise stated, and \( Q \) is the resummation scale (see main text)

\[
g_1(\lambda) = \frac{A^{(1)} 2\lambda + \ln(1-2\lambda)}{2\lambda},
\]

\[
g_2(\lambda) = \frac{1}{2\pi\beta_0} \ln(1-2\lambda) \left( A^{(1)} \ln \frac{M^2}{Q^2} + B^{(1)} \right) - \frac{A^{(2)} 2\lambda + (1-2\lambda) \ln(1-2\lambda)}{4\pi^2\beta_0^2 \left( 1-2\lambda \right)},
\]

\[
+ A^{(1)} \left( -\frac{\beta_1 \ln(1-2\lambda)((2\lambda-1) \ln(1-2\lambda)-2)-4\lambda}{2\pi\beta_0} \ln \frac{\mu_R^2}{Q^2} \right),
\]

\[
g_3(\lambda) = \left( A^{(1)} \ln \frac{M^2}{Q^2} + B^{(1)} \right) \left( -\frac{\lambda}{1-2\lambda} \ln \frac{\mu_R^2}{Q^2} + \frac{\beta_1 2\lambda + \ln(1-2\lambda)}{2\beta_0^2 \left( 1-2\lambda \right)} \right) - \frac{1}{2\pi\beta_0} \ln \frac{\mu_R^2}{Q^2}
\]

\[
+ A^{(1)} \left( \frac{\lambda \beta_0 \beta_2 (1-3\lambda) + \beta_2^2 \lambda}{\beta_0^2 (1-2\lambda)^2} + (1-2\lambda) \ln(1-2\lambda) \left( \beta_0 \beta_2 (1-2\lambda) + 2\beta_2^2 \lambda \right) + \frac{\beta_2^2 (1-4\lambda) \ln^2(1-2\lambda)}{4\beta_0^4 (1-2\lambda)^2} \right)
\]

\[
- \frac{\lambda^2 \ln^2 \frac{\mu_R^2}{Q^2}}{\beta_0^2 (1-2\lambda)^2} \right) + \frac{\beta_1 2\lambda + \ln(1-2\lambda)}{2\beta_0^2 (1-2\lambda)^2} \ln \frac{\mu_R^2}{Q^2},
\]

where, for Higgs, \( A^{(1)} = 2C_A \) and \( B^{(1)} = -4\pi\beta_0 \), while for Drell-Yan, \( A^{(1)} = 2C_F \) and \( B^{(1)} = -3C_F \). The remaining coefficients can be expressed in a unique way as \( [3, 53, 60] \):

\[
A^{(2)} = A^{(1)} K_{CMW}^{(1)}, \quad A^{(3)} = A^{(1)} K_{CMW}^{(2)} + \pi \beta_0 C d^{(2)}, \quad B^{(2)} = -2\gamma^{(2)} + 2\pi\beta_0 C\zeta_2,
\]

\[
\beta_0 = \frac{11C_A - 2n_f}{12\pi}, \quad \beta_1 = \frac{17C_A^2 - 5C_A n_f - 3C_F n_f}{24\pi^2}, \quad \beta_2 = \frac{2857C_A^2 + (54C_F^2 - 615C_F C_A - 1415C_A^2) n_f + (66C_F + 79C_A) n_f^2}{3456\pi^3},
\]

in terms of the Casimir \( C = C_A \) for Higgs and \( C = C_F \) for Drell-Yan, and of the well known constants

\[
K_{CMW}^{(1)} = C_A \left( \frac{67}{18} - \frac{\pi^2}{6} \right) - \frac{5}{9} n_f, \quad d^{(2)} = C_A \left( \frac{808}{27} - 28\zeta_3 \right) - \frac{224}{54} n_f,
\]

\[
K_{CMW}^{(2)} = C_A^2 \left( \frac{245}{24} - \frac{67}{9} \zeta_2 + \frac{11}{6} \zeta_3 + \frac{11}{5} \zeta_2^2 \right) + C_F n_f \left( \frac{-55}{24} + 2\zeta_3 \right) + C_A n_f \left( \frac{-209}{108} + \frac{10}{9} \zeta_2 - \frac{7}{3} \zeta_3 \right) - \frac{1}{27} n_f^2.
\]

Here \( \gamma^{(2)} \) [61, 62] are the coefficients of the \( \delta(1-z) \) term in the NLO splitting functions \( P^{(1)} \). For Higgs production we have

\[
\gamma^{(2)} = C_A \left( \frac{8}{3} + 3\zeta_3 \right) - \frac{1}{2} C_F n_f - \frac{3}{2} C_A n_f,
\]

whilst for the Drell-Yan process

\[
\gamma^{(2)} = C_F \left( \frac{3}{8} - \frac{\pi^2}{2} + 6\zeta_3 \right) + C_F C_A \left( \frac{17}{24} + \frac{11}{18} \pi^2 - 3\zeta_3 \right) - C_F n_f \left( \frac{1}{12} + \frac{\pi^2}{9} \right).
\]
We finally report the expressions for the collinear coefficient function \( C_{ij}^{(1)}(z) \) and the hard virtual term \( H^{(1)} \) in eq. (17)

\[
C_{ij}^{(1)}(z) = -P_{ij}^{(0),\epsilon}(z) - \delta_{ij} \delta(1 - z) C_F \frac{Q^2}{\mu_F^2} + P_{ij}^{(0)}(z) \ln \frac{Q^2}{\mu_F^2},
\]

\[
H^{(1)} = H^{(1)} - \left( B^{(1)} + \frac{A^{(1)}}{2} \ln \frac{M^2}{Q^2} \right) \ln \frac{M^2}{Q^2} + q \frac{2\pi\beta_0}{\beta_0} \ln \frac{\mu_F^2}{M^2},
\]

where \( q \) is the \( \alpha_s \) power of the LO cross section (\( q = 2 \) for Higgs production and \( q = 0 \) for Drell-Yan). The coefficient \( H^{(1)} \) encodes the pure hard virtual correction to the leading order process, it is given by

\[
\text{Higgs :} \quad H^{(1)} = C_A \left( 5 + \frac{7}{6} \pi^2 \right) - 3C_F,
\]

\[
\text{Drell – Yan :} \quad H^{(1)} = C_F \left( -8 + \frac{7}{6} \pi^2 \right).
\]

Finally, \( P_{ij}^{(0),\epsilon}(z) \) is the \( \mathcal{O}(\epsilon) \) term of the LO splitting function \( P_{ij}^{(0)}(z) \):

\[
P_{qq}^{(0),\epsilon}(z) = -C_F(1 - z),
\]

\[
P_{qg}^{(0),\epsilon}(z) = -C_F z,
\]

\[
P_{gq}^{(0),\epsilon}(z) = -z(1 - z),
\]

\[
P_{gg}^{(0),\epsilon}(z) = 0.
\]

2. Full matching formulae

We start by recalling the three prescriptions discussed in [21] for defining the jet-veto efficiency at NNLO accuracy. In this section, in order to simplify the notation, we will refer to the integrated jet-veto distribution \( \Sigma^{(j)} \) as \( \Sigma \).

\[
\epsilon^{(a)}(p_{t,\text{veto}}) = \frac{\Sigma_0(p_{t,\text{veto}}) + \Sigma_1(p_{t,\text{veto}}) + \Sigma_2(p_{t,\text{veto}})}{\sigma_0 + \sigma_1 + \sigma_2},
\]

\[
\epsilon^{(b)}(p_{t,\text{veto}}) = \frac{\Sigma_0(p_{t,\text{veto}}) + \Sigma_1(\sigma_{t,\text{veto}}) + \Sigma_2(\sigma_{t,\text{veto}})}{\sigma_0 + \sigma_1},
\]

\[
\epsilon^{(c)}(p_{t,\text{veto}}) = 1 + \frac{\Sigma_1(\sigma_{t,\text{veto}})}{\sigma_0} - \frac{\Sigma_1(\sigma_{t,\text{veto}})}{\sigma_0} \frac{\Sigma_1(\sigma_{t,\text{veto}})}{\sigma_0} + \frac{\Sigma_2(\sigma_{t,\text{veto}})}{\sigma_0}
\]

with

\[
\Sigma_i(\sigma_{t,\text{veto}}) = \sigma_i + \tilde{\Sigma}_i(\sigma_{t,\text{veto}})
\]

being the \( \mathcal{O}(\alpha_s^i) \)–th correction relative to the Born cross section where

\[
\tilde{\Sigma}_i(\sigma_{t,\text{veto}}) = - \int_{p_{t,\text{veto}}}^{\infty} dp_t \frac{d\sigma_i(p_t)}{dp_t},
\]

can be determined from MCFM, while \( \sigma_i \) is the \( i \)th order contribution to the total cross section (cf. [44–49]). The above three prescriptions differ by terms \( \mathcal{O}(\alpha_s^2) \), which are beyond the control of current fixed-order calculations.

The jet-veto matched efficiency should tend to one and the differential distribution should vanish at the maximum allowed transverse momentum \( p_{t,\text{veto}}^{\text{max}} \)

\[
\epsilon(p_{t,\text{veto}}^{\text{max}}) = 1, \quad \frac{d\epsilon}{dp_{t,\text{veto}}}(p_{t,\text{veto}}^{\text{max}}) = 0.
\]

2 Often in the literature, the hard coefficient \( \mathcal{H}^{(1)} \) is considered as part of the \( \delta(1 - z) \) term in the coefficient function \( C_{ij}^{(1)}(z) \), so it comes with a factor \( 1/(1 - \alpha_s\beta_0 \mathcal{L}) \) in eq. (8). This results in a different convention for the resummation coefficient \( B^{(2)} \) which will differ by an amount \( 2\pi\beta_0\mathcal{H}^{(1)} \) from what reported here.
To fulfil such requirements, we modify the resummed logarithms as follows

\[
L \rightarrow \tilde{L} = \left(1 - \frac{p_{t,\text{veto}}}{p_{t,\text{max}}} \right)^{1/p} \ln \left( \frac{Q}{p_{t,\text{veto}}} \right)^{p} - \left( \frac{Q}{p_{t,\text{max}}} \right)^{p} + 1, \tag{24}
\]

where \( p \) is some integer power. By default we choose \( p = 5 \) \cite{21}. The factor \( 1 - \frac{p_{t,\text{veto}}}{p_{t,\text{max}}} \) is necessary to fulfil eq. \( (23) \) but it is largely irrelevant in practice since \( p_{t,\text{max}} \) is much larger than the typical values of the jet transverse momentum veto (in practice, we set \( p_{t,\text{max}} = \infty \)). We introduce three multiplicative matching schemes \( \hat{\Sigma} \), each of them corresponding to one of the three efficiency definitions \( \epsilon^{(a)}, \epsilon^{(b)}, \epsilon^{(c)} \). To simplify the notation, we split the luminosity factor in the square brackets of Eq. \( (8) \) into two terms \( \mathcal{L}^{(0)}(\tilde{L}) \) and \( \mathcal{L}^{(1)}(\tilde{L}) \), which start at order \( \alpha_s^{(0)} \) and \( \alpha_s^{(1)} \) respectively,

\[
\mathcal{L}^{(0)}(\tilde{L}) = \sum_{i,j} \int dx_1 dx_2 \delta(x_1 x_2 s - M^2) f_i(x_1, e^{-L} L_F) f_j(x_2, e^{-L} L_F),
\]

\[
\mathcal{L}^{(1)}(\tilde{L}) = \frac{\alpha_s}{2\pi} \sum_{i,j} \int dx_1 dx_2 \delta(x_1 x_2 s - M^2) \left[ f_i(x_1, e^{-L} L_F) f_j(x_2, e^{-L} L_F) \mathcal{H}^{(1)} \right. \\
\left. + \frac{1}{1 - 2\alpha_s \beta_0} \sum_k \int_{x_1}^{x_2} \frac{dz}{z} C_{ki}(z) f_i \left( \frac{x_1}{z}, e^{-\tilde{L}} L_F \right) f_j \left( x_2, e^{-\tilde{L}} L_F \right) + \{ (x_1, i) \leftrightarrow (x_2, j) \} \right]. \tag{26}
\]

The first of the three matching schemes then reads

\[
\hat{\Sigma}^{(a)}_{\text{matched}}(p_{t,\text{veto}}) = \frac{1}{\sigma_0} \sum_{\text{NNLL}}(p_{t,\text{veto}}) \left[ \sigma_0 \left( 1 - \frac{\mathcal{L}^{(1)}(\tilde{L})}{\mathcal{L}^{(0)}(\tilde{L})} \right) + \frac{\Sigma^{(1)}(p_{t,\text{veto}}) - \Sigma^{(0)}_{\text{NNLL}}(p_{t,\text{veto}})}{\sigma_0} \right], \tag{27}
\]

and the corresponding jet-veto efficiency is

\[
\epsilon^{(a)}_{\text{matched}}(p_{t,\text{veto}}) = \frac{\Sigma^{(a)}_{\text{matched}}(p_{t,\text{veto}})}{\Sigma^{(a)}_{\text{matched}}(p_{t,\text{veto}})}. \tag{28}
\]

The second scheme can be derived from the previous one by replacing \( \Sigma^{(2)}(p_{t,\text{veto}}) \) with \( \Sigma^{(2)}_{\text{matched}}(p_{t,\text{veto}}) \). For the vetoed cross section we get

\[
\hat{\Sigma}^{(b)}_{\text{matched}}(p_{t,\text{veto}}) = \frac{1}{\sigma_0} \sum_{\text{NNLL}}(p_{t,\text{veto}}) \left[ \sigma_0 \left( 1 - \frac{\mathcal{L}^{(1)}(\tilde{L})}{\mathcal{L}^{(0)}(\tilde{L})} \right) + \frac{\Sigma^{(1)}(p_{t,\text{veto}}) - \Sigma^{(0)}_{\text{NNLL}}(p_{t,\text{veto}})}{\sigma_0} \right], \tag{29}
\]

while for its efficiency

\[
\epsilon^{(b)}_{\text{matched}}(p_{t,\text{veto}}) = \frac{\Sigma^{(b)}_{\text{matched}}(p_{t,\text{veto}})}{\Sigma^{(b)}_{\text{matched}}(p_{t,\text{veto}})}. \tag{30}
\]

Finally, the third matching scheme is directly formulated for the efficiency resulting in

\[
\epsilon^{(c)}_{\text{matched}}(p_{t,\text{veto}}) = \frac{1}{\sigma_0} \sum_{\text{NNLL}}(p_{t,\text{veto}}) \left[ \sigma_0 \left( 1 - \frac{\mathcal{L}^{(1)}(\tilde{L})}{\mathcal{L}^{(0)}(\tilde{L})} \right) + \frac{\Sigma^{(1)}(p_{t,\text{veto}}) - \Sigma^{(0)}_{\text{NNLL}}(p_{t,\text{veto}})}{\sigma_0} \right], \tag{31}
\]

\[
+ \frac{\Sigma^{(2)}(p_{t,\text{veto}}) - \Sigma^{(1)}(p_{t,\text{veto}})}{\sigma_0} \sum_{\text{NNLL}}(p_{t,\text{veto}}) \left( \Sigma^{(1)}(p_{t,\text{veto}}) - \Sigma^{(0)}_{\text{NNLL}}(p_{t,\text{veto}}) \right). \]
3. Details of relation between jet and boson-$p_t$ resummations

This section collects a number of results to help relate jet and boson $p_t$ resummations. Firstly we demonstrate that the $g_n(\alpha_s L)$ from a boson $p_t$ resummation can be directly carried over to jet $p_t$ resummation for $n \leq 3$. Then we obtain a form for the boson $p_t$ resummation that is suitable for expansion and comparison with fixed-order results. Finally we consider the large $R$ limit of the jet-$p_t$ resummation, which was used in [22] to attempt to obtain a relation between jet and boson $p_t$ resummations.

a. Relating $g_n(\alpha_s L)$ between boson and jet resummations

One of the main ingredients of our results are the $g_n(\alpha_s L)$ functions that are used in boson $p_t$ resummations. These stem from the rightmost integral in Eq. (11), which involves a Fourier transformation, whereas in Eq. (2) we need related integrals but with a theta-function instead of the $(\exp(ibk_t) - 1)$ factor.

We start from the expression for the resummed $p_t$ distribution in eq. (11) and concentrate on the part of the matrix-element in the right-hand integral that is responsible for the leading logarithms. Integrating over azimuthal angles we obtain:

$$\frac{d\Sigma^{(b)}(p_t)}{p_t dp_t} = \sigma_0 \int bdb J_0(bp_t) \exp[-\mathcal{R}(b)], \quad \mathcal{R}(b) = \int [dk] M^2(k) (1 - J_0(bk_t)). \quad (32)$$

We wish to show that we can safely perform the replacement

$$ (1 - J_0(bk_t)) \rightarrow \Theta(k_t - b_0/b), \quad b_0 = 2e^{-\gamma_E}, \quad (33)$$

up to and including NNLL accuracy. Integrating over rapidity $\mathcal{R}(b)$ has the form

$$\mathcal{R}(b) = \int_0^M \frac{dk_t}{k_t} F \left( \frac{\alpha_s \ln \frac{M}{k_t}}{k_t} \right) (1 - J_0(bk_t)) \quad F \left( \frac{\alpha_s \ln \frac{M}{k_t}}{k_t} \right) = 4C_F \frac{\alpha_s}{\pi} \ln \frac{M}{k_t} \frac{1}{1 - 2\alpha_s b_0 \ln \frac{M}{k_t}}. \quad (34)$$

To evaluate separately real and virtual contributions in eq. (34), we introduce a dimensional regulator and write

$$\mathcal{R}(b) = F(\alpha_s \partial_k) \int_0^M \frac{dk_t}{k_t} \left( \frac{k_t}{M} \right)^{-\epsilon} (1 - J_0(bk_t)) \left. \right|_{\epsilon=0}, \quad (35)$$

which yields

$$\mathcal{R}(b) = R_{\text{LL}}(b_0/b) + \delta \mathcal{R}(b), \quad (36)$$

where, neglecting terms suppressed by powers of $1/(bM)$,

$$R_{\text{LL}}(b_0/b) = \int_0^M \frac{dk_t}{k_t} F \left( \frac{\alpha_s \ln \frac{M}{k_t}}{k_t} \right) \Theta(k_t - b_0/b), \quad (37)$$

and

$$\delta \mathcal{R}(b) = F(\alpha_s \partial_k) \frac{(b/b_0)^\epsilon}{\epsilon} \left[ -1 + e^{-\gamma_E} \frac{\Gamma(1 - \frac{\epsilon}{2})}{\Gamma(1 + \frac{\epsilon}{2})} \right] \left. \right|_{\epsilon=0} \quad \text{(38a)}$$

$$= F(\alpha_s \partial_k) \left( \frac{b}{b_0} \right)^\epsilon \left| \frac{\zeta}{12} + \mathcal{O}(\epsilon^4) \right| \left. \right|_{\epsilon=0}. \quad \text{(38b)}$$

This gives at most a term $\alpha_s^n \ln^{n-2}(Mb/b_0)$, i.e. a $N^3\text{LL}$ term. A similar argument can be applied to contributions to $\mathcal{R}(b)$ arising from less singular regions, giving also rise to terms that are beyond NNLL. Consequently, to NNLL accuracy, the same $g_1$, $g_2$ and $g_3$ functions can be used in both the jet and boson resummation.
b. Evaluation of the boson-\( p_t \) integrated cross section

To facilitate comparisons between the jet and boson-\( p_t \) resummations at fixed order, it is convenient to have an expression for the boson-\( p_t \) resummation whose fixed-order expansion can be straightforwardly obtained. The full expression for the cumulative \( p_t \) cross section can be found in [2, 3, 4] and reads

\[
\Sigma^{(B)}(p_t) = \int_0^\infty dy J_1(y) |\mathcal{M}_B| e^{-R(b_0/b)} \left( \mathcal{L}^{(0)}(\ln(Qb/b_0)) + \mathcal{L}^{(1)}(\ln(Qb/b_0)) \right) ,
\]

where

\[
-R(b_0/b) = \ln(Qb/b_0)g_1(\alpha_s \ln(Qb/b_0)) + g_2(\alpha_s \ln(Qb/b_0)) + \frac{\alpha_s}{\pi} g_3(\alpha_s \ln(Qb/b_0))
\]

is the full NNLL radiator. As discussed above, the resummation functions \( g_1, g_2 \) and \( g_3 \) are those used for the jet veto case. To perform the inverse Fourier transform we expand \( R(b_0/b) \) and the full luminosity factor around \( b = b_0/p_t \) and neglect subleading logarithmic terms getting, at NNLL accuracy,

\[
\Sigma^{(B)}(p_t) = \int_0^\infty dy J_1(y) |\mathcal{M}_B|^2 \left[ \mathcal{L}^{(0)}(\ln(Q/p_t)) + \mathcal{L}^{(1)}(\ln(Q/p_t)) + \partial_{\ln p_t} \mathcal{L}^{(0)}(\ln(Q/p_t)) \ln(y/b_0) \right] \times \left( \frac{y}{b_0} \right)^{-R'} e^{-R(p_t)} \left( 1 - \frac{1}{2} R'' \ln^2(y/b_0) \right) ,
\]

where we have performed the change of variable \( y = b_0 p_t \), and we have made use of \( R' \) and \( R'' \), the first and second derivatives of \( R \) with respect to \( \ln(Q/p_t) \). To order \( \alpha_s L, R' = 4\alpha_s C \ln(Q/p_t)/\pi \). Moreover, from eq. (22), we see that the variation of \( \mathcal{L}^{(0)}(L) \) reads

\[
\partial_{\ln p_t} \mathcal{L}^{(0)}(L) = \frac{\alpha_s}{\pi} \sum_{i,j,k} \int dx_1 dx_2 \delta(x_1 x_j - M^2) \left[ (P_{ki}^{(0)} \otimes f_i) (x_1, e^{-L} \mu_F) f_j (x_2, e^{-L} \mu_F) + \{(x_1, i) \leftrightarrow (x_2, j)\} \right].
\]

It is straightforward to show that eq. (11) evaluates to

\[
\Sigma^{(B)}(p_t) = |\mathcal{M}_B|^2 e^{-R(p_t)} \left[ \mathcal{L}^{(0)}(\ln(Q/p_t)) \left( 1 - \frac{1}{2} R'' \partial_{R'}^2 \right) \right. \left. + \mathcal{L}^{(1)}(\ln(Q/p_t)) - \partial_{\ln p_t} \mathcal{L}^{(0)}(\ln(Q/p_t)) \partial_{R'} \right] e^{-\gamma_E R'} \frac{\Gamma(1 - \frac{R'}{2})}{\Gamma(1 + \frac{R'}{2})} \quad \text{(43)}
\]

In this notation, the result for the jet-veto cross section is simply \( |\mathcal{M}_B|^2 e^{-R(p_t)} \mathcal{L}^{(0)}(\ln(Q/p_t)) + \mathcal{L}^{(1)}(1 + \mathcal{F}^{\text{clust}} + \mathcal{F}^{\text{correl}}) \). It is therefore immediate to evaluate the differences between the two formulae at any given fixed order and in particular to derive Eq. (10): making use of the fact that \( e^{-\gamma_E R'} \frac{\Gamma(1 - \frac{R'}{2})}{\Gamma(1 + \frac{R'}{2})} \) has an expansion of the form \( 1 + \frac{\zeta_3}{6} R^3 + \mathcal{O} \left( R^5 \right) \), one sees that the only terms in the difference that survive at order \( \alpha_s^2 L \) are the \( \mathcal{F}^{\text{clust}} \) and \( \mathcal{F}^{\text{correl}} \) contributions and the \( R'' \partial_{R'}^2 \) term of Eq. (10), with the latter giving

\[
-\sigma_0 \frac{1}{2} R'' \partial_{R'}^2 e^{-\gamma_E R'} \frac{\Gamma(1 - \frac{R'}{2})}{\Gamma(1 + \frac{R'}{2})} = \sigma_0 \left( -\frac{\alpha_s^2}{\pi} \zeta_3 C^2 \ln \frac{Q}{p_t} + \mathcal{O} \left( \alpha_s^2 L^0 \right) + \mathcal{O} \left( \alpha_s^3 L^2 \right) \right)
\]

which is the source of the \( \zeta_3 \) in Eq. (9).³

³ One point to note in evaluating the difference between the jet and boson \( p_t \) resummations at order \( \alpha_s^2 L^2 \) is that it is necessary to account also for the difference between \( C_2 \) terms for the two resummations. One of the properties of this difference of \( C_2 \) terms is that it is \( Q \) dependent that ensures that the final prediction for the difference of \( \alpha_s^3 L^2 \) terms is \( Q \)-independent.

To produce figure 1 the difference of \( 3 \)-jet calculation.
One natural way of relating jet and boson-\(p_t\) resummations is to make the observation that for an infinite jet radius, all partons will be clustered into a single jet, which will have a transverse momentum that balances exactly that of the boson. This approach was taken in Ref. [22] and here we examine it in detail.

First, let us consider the properties of \(\mathcal{F}^{\text{clust}}\) and \(\mathcal{F}^{\text{correl}}\) for large \(R\). It is straightforward to see that \(\mathcal{F}^{\text{correl}}\) vanishes for large \(R\), since in Eq. (47) the two partons will always clustered together, giving \(1 - J(k_1, k_2) = 0\). For \(\mathcal{F}^{\text{clust}}\), the NNLL component for \(R > \pi\) can be evaluated in closed form and is given by

\[
\mathcal{F}^{\text{clust}} = -4 \frac{\alpha_s^2(p_t, \text{veto}) C^2}{\pi^2} \ln(Q/p_t) \left(\left(\frac{\pi}{6} R^2 - \frac{R^4}{8\pi}\right) \arctan \frac{\pi}{\sqrt{R^2 - \pi^2}} + \left(\frac{R^2}{8} - \frac{\pi^2}{12}\right) \sqrt{R^2 - \pi^2}\right). \tag{45}
\]

This has the property that it vanishes as \(1/R\) for large \(R\). Thus it would appear that at order \(\alpha^2 L\) the difference between jet and boson-\(p_t\) resummations should be given by [22]

\[
\frac{d\Sigma^{(J)}_{\text{NNLL,2}}(p_t)}{d\ln p_t} - \frac{d\Sigma^{(B)}_{\text{NNLL,2}}(p_t)}{d\ln p_t} = (f(R) - f(\infty)) \alpha_s^2 \sigma_0 = f(R) \alpha_s^2 \sigma_0, \tag{46}
\]

which differs from the result in Eq. (19) (here \(f(R) = f^{\text{correl}}(R) + f^{\text{clust}}(R)\)).

To understand the origin of this difference, it is helpful to examine the structures that lead to \(\mathcal{F}^{\text{clust}}\) vanishing for large \(R\). A first observation is that for large \(R\), \(J(k_1, k_2)\) can be written as

\[
J(k_1, k_2) = \Theta \left(R - |\Delta y| + \frac{\Delta \phi^2}{2R} + O \left(\frac{1}{R^2}\right)\right), \quad \Delta y \equiv y_1 - y_2, \quad \Delta \phi = \phi_1 - \phi_2. \tag{47}
\]

Neglecting the term of order \(1/R\) will allow us to simplify our discussion and so we will instead examine a “rapidity-only” jet algorithm with the clustering function

\[
J_{\text{rap}}(k_1, k_2) = \Theta(R - |\Delta y|). \tag{48}
\]

Let us now evaluate Eq. (44) with \(J_{\text{rap}}\). We break the problem into rapidity, transverse momentum and azimuthal integrals. Each emission \(i\) is limited to a rapidity \(|y_i| < \ln(M/k_i)\). Assuming that we can neglect terms \(\ln(k_{1i}/k_{2i})\) from the rapidity integration, we can write the latter as

\[
\int dy_1 dy_2 \Theta \left(|y_1| - \ln\frac{M}{k_{1i}}\right) \Theta \left(|y_2| - \ln\frac{M}{k_{2i}}\right) \Theta(R - |y_1 - y_2|) = 4R \ln\frac{M}{k_{1i}} - R^2 + O(R \ln \zeta), \tag{49}
\]

where \(\zeta = k_{2i}/k_{1i}\) and we have included the constraint that \(J_{\text{rap}}(k_1, k_2)\) is non-zero. We can then write Eq. (44) as

\[
\mathcal{F}^{\text{clust}} = 4 \frac{\alpha_s^2 C^2}{\pi^2} \int_0^1 \frac{d\zeta}{\zeta} \int_{-\pi}^\pi \frac{d\phi}{2\pi} \int_{-\pi}^\pi \frac{dk_t}{2\pi} \int_{R^2/2}^R \left(4R \ln\frac{M}{k_{1i}} - R^2\right), \tag{50}
\]

where we have dropped the \(O(R \ln \zeta)\) term of Eq. (49). Performing the \(k_{1i}\) integration gives

\[
\mathcal{F}^{\text{clust}} = 4 \frac{\alpha_s^2 C^2}{\pi^2} \int_0^1 \frac{d\zeta}{\zeta} \int_{-\pi}^\pi \frac{d\phi}{2\pi} \ln\left(1 + \zeta^2 + 2\zeta \cos \phi\right) - \frac{R^2}{2} \ln^2\left(1 + \zeta^2 + 2\zeta \cos \phi\right) \tag{51}
\]

Because \(\int_0^{2\pi} d\phi \ln(1 + \zeta^2 + 2\zeta \cos \phi) = 0\), the first term in square brackets vanishes. This was the only term that had a NNLL \(\alpha_s^2 \ln M/p_t\) factor and so at NNLL accuracy \(\mathcal{F}^{\text{clust}}\) is zero at large \(R\), modulo \(1/R\) corrections associated with the \(1/R\) term in Eq. (47). The only element that survives the azimuthal integration in Eq. (51) is the second term in square brackets, resulting in

\[
\mathcal{F}^{\text{clust}} = -4 \frac{\alpha_s^2 C^2}{\pi^2} R \zeta_3. \tag{52}
\]

This is N$^3$LL, so beyond our accuracy. Note, however, that it is enhanced by a factor of \(R\). In the large \(R\) limit, the separation between partons is limited to be at most \(2 \ln M/p_t\) and thus the \(R\) factor is effectively replaced with a coefficient of order \(\ln M/p_t\). Consequently the apparently N$^3$LL term of Eq. (52) is “promoted” and becomes a NNLL \(\alpha_s^2 \ln M/p_t\) contribution. This is not accounted for in the purely NNLL \(R\)-dependent analysis that led to Eq. (46).

The exact infinite \(R\) result can be obtained at order \(\alpha_s^2 L\) by evaluating \(\mathcal{F}^{\text{clust}}\) with \(J(k_1, k_2) = 1\), giving

\[
\mathcal{F}^{\text{clust}} = 16 \frac{\alpha_s^2 C^2}{\pi^2} \int_0^1 \frac{d\zeta}{\zeta} \int_{-\pi}^\pi \frac{d\phi}{2\pi} \int_{-\pi}^\pi \frac{dk_t}{2\pi} k_{t,1} \ln\frac{M}{k_{t,1}} - \ln \zeta \ln\frac{M}{k_{t,1}} = -4 \frac{\alpha_s^2 C^2}{\pi^2} \zeta_3 \ln M/p_t + O(\alpha_s^2) \ln M/p_t. \tag{53}
\]

Note the agreement of the \(\zeta_3\) term here with that derived in Eq. (44). It is this contribution that corresponds to the \(\zeta_3\) term in Eq. (49).
FIG. 3. Comparison of NNLO, NLL+NNLO and NNLL+NNLO results for the jet-veto efficiency for Higgs (left) and Z-boson (right) production at 7 TeV. The Higgs plot also includes the result from a POWHEG (revision 1683) [20, 40] plus Pythia (6.426) [17, 41] simulation in which the Higgs-boson $p_t$ distribution has been reweighted to match the NNLL+NNLO prediction from HqT 2.0 [7] as in [21]. The lower panels show the results normalised to the central NNLL+NNLO curves.

4. Correlation matrix between 0-jet and inclusive 1-jet cross sections

As discussed in [21], the prescription that we propose for determining the uncertainties on the 0-jet cross section is to treat the uncertainties on the jet-veto efficiency and on the total cross section as uncorrelated. This gives the following covariance matrix for the uncertainties of the 0-jet ($\sigma_{0\text{-}jet}$) and inclusive 1-jet ($\sigma_{\geq 1\text{-}jet}$) cross sections:

$$
\begin{pmatrix}
\epsilon^2 \delta^2_{\sigma} + \sigma^2 \delta^2_{\epsilon} & \epsilon(1-\epsilon)\delta^2_{\sigma} - \sigma^2 \delta^2_{\epsilon} \\
\epsilon(1-\epsilon)\delta^2_{\sigma} - \sigma^2 \delta^2_{\epsilon} & (1-\epsilon)^2 \delta^2_{\sigma} + \sigma^2 \delta^2_{\epsilon}
\end{pmatrix}
$$

(54)

5. Results at 7 TeV

For completeness, we show in Fig. 3 results for 7 TeV centre of mass energy. The changes relative to the 8 TeV results are modest, with very slightly higher efficiencies at 7 TeV. This can be understood because at higher centre of mass energy, the PDFs are probed at lower $x$ values, where the scale dependence is steeper, causing the efficiencies to drop off more rapidly as one decreases $p_{t,veto}$.

6. $R$ dependence of results

Figure 4 shows the jet veto efficiency as a function of $p_{t,veto}$ for several different jet-radius ($R$) values. Increasing the jet radius, more radiation is captured and therefore a jet is more likely to pass the $p_{t,veto}$ threshold and so be vetoed. Consequence the jet-veto efficiency is expected to be lower for larger $R$ values. This is precisely as observed in Fig. 4.

Quantitatively, the differences between the $R = 0.4$ and $R = 0.5$ results (the values used respectively by ATLAS and CMS) are small compared to the uncertainties on the predictions. In contrast, for $R = 1$ the differences compared to the smaller-$R$ results are not negligible. One interesting feature, commented on briefly in the main text, is that for the Higgs-boson case, the uncertainties are somewhat smaller for $R = 1$ than for $R = 0.4$ and $R = 0.5$, especially the upper part of the uncertainty band. This can be understood with the help of the observation that the upper edge of the uncertainty band for the small $R$ values is set by the $Q = M_H$ variant of the resummation (recall that our default
FIG. 4. Jet veto efficiency at NNLL+NNLO as a function of \( p_{t,veto} \), comparing several jet-radius values; shown for \( pp \) collisions at a centre-of-mass energy of 8 TeV, for gluon-fusion Higgs production with \( M_H = 125 \text{ GeV} \) (large \( m_{\text{top}} \) limit) and for Z-boson production. Uncertainty bands are shown only for \( R = 0.4 \) and \( R = 1.0 \) in order to enhance the clarity of the figure. The \( R = 0.5 \) uncertainty band is to be found in Fig. 2. The lower panels show the predictions normalised to the central \( R = 0.5 \) results.

\( Q = M_H/2 \). Using \( Q = M_H \) increases the size of \( L \). Since the \( f_{\text{correl}}(R) + f_{\text{clust}}(R) \) function grows for small \( R \) and multiplies \( \alpha_s^2 L \), a smaller \( R \) value magnifies the impact of an increase in \( Q \).

If, experimentally, one were to consider using larger \( R \) values for performing jet vetoes in order to reduce the theoretical uncertainties, one concern might be the greater contamination of the jet’s \( p_t \) from the underlying event and pileup. To some extent this could be mitigated by methods such as subtraction \([64]\), filtering \([65]\) or trimming \([66]\). Note that with subtraction and filtering (when the latter uses two filtering subjets, or more) our jet-veto predictions remain unchanged at NNLO and at NNLL accuracy.