Measurement of $R = B(t \rightarrow Wb)/B(t \rightarrow Wq)$ in top-quark-pair decays using lepton+jets events and the full CDF run II dataset

CDF Collaboration; et al; Canelli, F; Kilminster, B

Abstract: We present a measurement of the ratio of the top-quark branching fractions $R = B(t \rightarrow Wb)/B(t \rightarrow Wq)$, where $q$ represents quarks of type $b$, $s$, or $d$, in the final state with a lepton and hadronic jets. The measurement uses $s=1.96$ TeV proton-antiproton collision data from $8.7$ fb$^{-1}$ of integrated luminosity collected with the Collider Detector at Fermilab during Run II of the Tevatron. We simultaneously measure $R = 0.94 \pm 0.09$ (stat+syst) and the $t\bar{t}$ production cross section $\sigma(t\bar{t}) = 7.5 \pm 1.0$ (stat+syst) pb. The magnitude of the Cabibbo-Kobayashi-Maskawa matrix element, $|V_{tb}| = 0.97 \pm 0.05$ (stat+syst) is extracted assuming three generations of quarks, and a lower limit of $|V_{tb}| > 0.89$ at 95% credibility level is set.

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Measurement of $R = B(t \to Wb) / B(t \to Wq)$ in Top–quark–pair Decays using Lepton+jets Events and the Full CDF Run II Data set
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In the standard model (SM) the top–quark decay rate into a $W$ boson and a down-type quark $q$ ($q = d, s, b$) is proportional to $|V_{tq}|^2$, the squared magnitude of the element of the Cabibbo-Kobayashi-Maskawa (CKM) matrix \[1\]. Under the assumption of a $3 \times 3$ unitary CKM matrix and using the existing constraints on $V_{ts}$ and $V_{td}$, the magnitude of the top–bottom quark coupling is $|V_{tb}| = 0.99915 \pm 0.00002 \pm 0.00005$ \[2\], and the top quark decays almost exclusively to $Wb$ final states. Any significant deviation from the expected value would imply new physics: an extra generation of quarks, non-SM top-quark production, or non-SM background to top-quark production. A direct measurement of the magnitude of the $V_{tb}$ matrix element can be obtained from the single-top-quark production cross-section \[3\], which is proportional to $|V_{tb}|^2$. The value of $|V_{tb}|$ can also be extracted from the decay rate of pair-produced top quarks. We define $R$ as the ratio of the branching fractions

\[ R = \frac{\mathcal{B}(t \to Wq)}{\mathcal{B}(t \to Wb)}. \]  

Given the unitarity of the CKM matrix and assuming three generations, $R$ is indirectly determined by the knowledge of $V_{ts}$ and $V_{td}$,

\[ R = \frac{|V_{tb}|^2}{|V_{ts}|^2 + |V_{td}|^2}, \]  

and is derived to be $0.99830 \pm 0.00004$ \[4\]. A deviation from this prediction would be an indication of non-SM physics.

This Letter reports the first CDF simultaneous measurement of $R$ and top-quark-pair-production cross section $σt\bar{t}$ performed on a data sample corresponding to an integrated luminosity of $8.7 \text{ fb}^{-1}$ collected with the CDF II detector \[5\] at the Fermilab Tevatron $p\bar{p}$ Collider at center of mass energy $\sqrt{s} = 1.96$ TeV. The analysis uses events with a lepton and multiple jets in the final state.
where one $W$ boson coming from $t\bar{t}$ production decays into a quark and an antiquark while the second $W$ boson decays into a charged lepton (electron or muon) and a neutrino.

CDF has performed several measurements of $R$ during both Run I and Run II, combining the lepton+jets final state with the dilepton final state, where both $W$ bosons decay into leptons. The most recent publication reported $R = 1.12^{+0.23}_{-0.21}$ (stat) $^{+0.13}_{-0.13}$ (syst), and $R > 0.61$ at 95% confidence level (C.L.) using 162 pb$^{-1}$ of integrated luminosity. The D0 Collaboration measured $R = 0.90 \pm 0.04$ (stat+syst) and $R > 0.79$ at 95% C.L. using data from 5.4 fb$^{-1}$ of integrated luminosity, in the lepton+jets and dilepton final states combined.

The CDF II detector consists of a charged-particle tracking system in a magnetic field of 1.4 T, segmented electromagnetic and hadronic calorimeters with a pointing geometry, and muon detectors. A silicon microstrip detector provides determination of charged-particle trajectories (tracking) over the radial range 1.5 to 28 cm, and is essential for the detection of displaced decay (secondary) vertices. A three-level, online event-selection system (trigger) is used to select events with an electron (muon) candidate in the central detector region (pseudorapidity $|\eta| < 1.1$), with $E_T$ ($p_T$) > 18 GeV (18 GeV/c), which form the data set for this analysis.

The measurement of $R$ is based on the determination of the number of $b$-quark jets in $t\bar{t}$ events reconstructed in the lepton+jets final state. The lepton+jets signature consists of a high-$p_T$ charged electron ($e$) or muon ($\mu$), large missing transverse energy $E_T$, due to the undetected neutrino from the leptonic $W$ decay, and at least three hadronic jets. Events containing muons are classified according to the coverage of the detectors used for their identification as central, when $|\eta| < 0.6$, and forward, when $0.6 < |\eta| < 1.0$. Identification of jets coming from $b$-quark fragmentation ($b$-jet tagging) is performed by the secvtx algorithm, which is based on the reconstruction of secondary vertices displaced from the primary $pp$ interaction vertex and selects a sample enriched with jets originating from $b$ quarks. The lepton+jets selection requirements are described in Ref. 10. Briefly, the analysis requires the presence of one isolated lepton ($e$ or $\mu$) with $E_T$ greater than 20 GeV, $E_T$ of at least 20 GeV, and a minimum of three jets, reconstructed using a cone algorithm with radius $\Delta R = 0.4$ in $\eta - \phi$ space, within $|\eta| < 2.0$. The jet $E_T$, after correcting for the calorimeter response, has to exceed 30, 25, and 20 GeV for the most-energetic, second-most-energetic, and any additional jet in the event, respectively. The $W$-boson transverse mass is required to be greater than 20 GeV/$c^2$. Events with one or two identified $b$-jets are selected (1 $b$-tag and 2 $b$-tag events, respectively).

The background processes include $W$-boson production in association with heavy-flavor jets ($Wb\bar{b}$, $Wc\bar{c}$, $Wc\bar{c}$), $W$-boson production in association with light-flavor jets that are incorrectly identified as $b$-jets (“mistags”), quantum chromodynamics multijet (“QCD”) events containing misreconstructed or real leptons or incorrectly-measured $E_T$, diboson events ($WW$, $WZ$, $ZZ$), single-top-quark production, and $Z$+jets events.

We divide the selected sample into subsets according to the type of lepton, number of jets in the final state, and number of identified $b$ jets (one or two). As explained in more detail below, we derive an expected event yield for each category. We then maximize the likelihood for observing the events found in each category by varying two fit parameters, $R$ and the top-quark-pair-production cross section $\sigma_{tt}$.

The $t\bar{t}$ events are modeled using the PYTHIA 12 Monte Carlo (MC) generator with top-quark mass $m_t = 172.5$ GeV/$c^2$. We estimate the backgrounds with a collection of data-driven and simulation techniques described in Ref. 10. The QCD background is modeled using data control samples 13. Mistags are estimated using a matrix (the mistag matrix) calculated in control samples and parametrized as a function of jet and event characteristics. Diboson processes are simulated using PYTHIA, single-top-quark production is simulated by POWHEG 14, while the parton shower and fragmentation is provided by PYTHIA. The ALPGEN generator, with PYTHIA supplying the parton shower and fragmentation, is used to model the $W$+jets and $Z$+jets backgrounds. A GEANT-based simulation is used to model the response of the CDF II detector 10. The cross sections used for background normalization can be found in Ref. 12. Table 4 shows the expected sample composition for all final states, after summing over lepton categories, assuming $R = 1$ and $\sigma_{tt} = 7.04 \pm 0.49$ pb 17.

The number of $b$-tagged events is the most sensitive quantity to possible values of $R$ different from one: the smaller $R$, the smaller the probability to have a $b$-jet in a top-quark-pair event. Hence, the fraction of events with one or two tags is expected to decrease with decreasing $R$. In general, the $t\bar{t}$ production cross-section measured by CDF in the lepton+jets sample assumes $R=1$. In order to avoid any bias due to this premise, we measure simultaneously $R$ and the production cross-section, since the measurement of the latter is affected by the sum of events in the different tag bins.

To perform the fit, we first divide the sample into 18 independent subsamples, organized by type of lepton, number of jets in the event (3, 4, $\geq 5$), and number of identified $b$-jets. The expected number of events in each subsample, $\mu_{exp}^{i,j}$, is given by the following expression:

$$
\mu_{exp}^{i,j} = \mu_{t\bar{t}}^{i,j} + N_B^{i,j} \epsilon_{cut}^{i,j} \epsilon_{tag}^{i,j} (R) + N_B^{i,j}, \quad (3)
$$

where $\mu_{t\bar{t}}^{i,j}$ is the expected number of $t\bar{t}$ events and $N_B^{i,j}$ the expected number of background events. The $i$ and $j$ indices indicate the $i$th jet bin with one or two identified $b$-jets (jet-tag category) and the $j$th lepton category, respectively. $\mathcal{L}$ is the integrated luminosity, $\epsilon_{cut}^{i,j}$ includes the trigger and lepton identification efficiencies, and $\epsilon_{tag}^{i,j}(R)$ is the event-tagging efficiency, i.e., the efficiency for tagging at least one jet in an event. In an
ideal case without background and assuming a b-tagging efficiency equal one for jets originating from b quarks and zero for jets originating from non-b quarks, the number of expected events with two tags is proportional to \( R^2 \), while the number of expected events with one tag is proportional to \( 2R(1-R) \). The estimates for the background processes are calculated with various values of \( R \). The differences with respect to the estimates obtained with \( R = 1 \) are found to be negligible.

The event-tagging efficiencies are calculated in \( t\bar{t} \) MC samples, using the probability to tag a jet as a b-jet according to the seckvtx algorithm, on a jet-by-jet basis. For jets originated from b and c quarks, the b-jet tagging efficiencies are corrected for differences between data and MC using a scale factor SF = 0.96 ± 0.05 [8]. For jets originated from light-flavor quarks, the probability to tag them as b-jets is obtained using the mistag matrix.

In general, \( \epsilon_{i \text{tag}} \) is calculated from the event probability to tag the \( m \)th event in MC processes with possible b-quark final states. For an event with \( n \) generic jets, the probability to have one (Eq. 4) or two (Eq. 5) tagged b-jets becomes

\[
P_{1\text{-tag}}^m = \sum_{q=1}^{n} p_{q\text{tag}} \left( \prod_{r=1, r \neq q}^n (1 - p_{r\text{tag}}) \right) \tag{4}
\]

\[
P_{2\text{-tag}}^m = \sum_{q=1}^{n-1} p_{q\text{tag}} \left( \sum_{r \geq q}^n p_{r\text{tag}} \left( \prod_{s=1, s \neq q, s \neq r}^n (1 - p_{s\text{tag}}) \right) \right) \tag{5}
\]

where \( p_{q\text{tag}} \) is either the probability to tag the \( q \)th jet, multiplied by the SF, for jets where the heavy-flavor quark is found inside the jet cone; or the mistag probability for jets matched to a light-flavor hadron, calculated using the mistag matrix.

Finally, we use the \( P_{1\text{-tag}}^m \) as an event weight to calculate the event-tagging efficiency \( \epsilon_{i \text{tag}} \) for each subsample with \( l \) tags and \( n \) jets by summing the \( P_{l\text{-tag}}^m \) weights over all of the pretagged events.

The MC sample employed for the \( t\bar{t} \) signal modeling is generated assuming \( |V_{ub}| = 1 \) so it cannot be used directly to calculate \( \epsilon_{i \text{tag}} \) as a function of \( R \) through the algorithm described above. Instead, in the MC sample we assign a random number \( F_b \) in the interval \([0, 1]\) to every jet that is matched at the parton level to a b quark from \( t\bar{t} \) decay. If \( F_b < R \) we consider this jet as genuinely originated by a b quark and use the tag probability multiplied by the SF as in Eq. 4 and 5; otherwise this jet is regarded as a light-flavor jet. This simulates a configuration in which a b quark produced in the top decay is a real \( b \) only \( R \) fraction of the time while \((1 - R) \) fraction of the time it is treated as a light-flavor quark and it is weighted by the mistag probability. This probabilistic approach allows the calculation of background and signal sample composition for any value of \( R \). This method reproduces exactly the standard calculation in the case of \( R = 1 \), simulates \( t \to Wq \) for \( R = 0 \), and allows a calculation of \( \epsilon_{i \text{tag}}(R) \) in each tag subsample and in each jet bin. Figure 4 shows the comparison of observed and expected events assuming \( R \) equal to 1, 0.5, and 0.1 in the various jet multiplicities and number of b-tagged jets in the final state.

In order to compare the prediction to the observed data in the 18 subsamples we use a likelihood function. We fit the observed event yields in each class of events to determine simultaneously \( R \) and \( \sigma_{t\bar{t}} \). The likelihood function is:

\[
\mathcal{L} = \prod_{i,j} \mathcal{P} \left( \mu_{e \exp}^{i,j}(R, \sigma_{t\bar{t}}, x_a) | N_{\text{obs}}^{i,j} \right) \prod \mathcal{G} (x_a | 0, 1), \tag{6}
\]

where \( \mathcal{P} \left( \mu_{e \exp}^{i,j}(R, \sigma_{t\bar{t}}, x_a) | N_{\text{obs}}^{i,j} \right) \) is the Poisson probability to observe \( N_{\text{obs}}^{i,j} \) events assuming the expected mean \( \mu_{e \exp}^{i,j} \) (given by Eq. 3), the index \( i \) indicates the jet-tag category, and the index \( j \) runs over the different lepton categories. The estimates of the nuisance parameters \( x_a \) are constrained to their central values and normalized to their uncertainties using Gaussian distributions \( \mathcal{G} (x_a | 0, 1) \) centered at zero with unit variance. This pro-

### Table I: Number of expected and observed events in lepton+jets data corresponding to 8.7 fb⁻¹ of integrated luminosity.

<table>
<thead>
<tr>
<th>Process</th>
<th>3 Jets</th>
<th>4 Jets</th>
<th>≥5 Jets</th>
<th>3 Jets</th>
<th>4 Jets</th>
<th>≥5 Jets</th>
</tr>
</thead>
<tbody>
<tr>
<td>( tt )</td>
<td>800 ± 67</td>
<td>777 ± 64</td>
<td>260 ± 21</td>
<td>216 ± 30</td>
<td>271 ± 36</td>
<td>97 ± 13</td>
</tr>
<tr>
<td>( W+b\bar{b} )</td>
<td>291 ± 118</td>
<td>74 ± 30</td>
<td>17 ± 7</td>
<td>48 ± 20</td>
<td>14 ± 6</td>
<td>4 ± 2</td>
</tr>
<tr>
<td>( W+c\bar{c} )</td>
<td>167 ± 68</td>
<td>47 ± 20</td>
<td>12 ± 5</td>
<td>5 ± 2</td>
<td>2 ± 1</td>
<td>0.8 ± 0.4</td>
</tr>
<tr>
<td>( W+c )</td>
<td>87 ± 35</td>
<td>17 ± 7</td>
<td>4 ± 2</td>
<td>3 ± 1</td>
<td>0.8 ± 0.4</td>
<td>0.2 ± 0.1</td>
</tr>
<tr>
<td>Single top</td>
<td>78 ± 7</td>
<td>17 ± 2</td>
<td>3.6 ± 0.3</td>
<td>18 ± 3</td>
<td>4.7 ± 0.7</td>
<td>1.1 ± 0.2</td>
</tr>
<tr>
<td>Diboson</td>
<td>45 ± 5</td>
<td>11 ± 1</td>
<td>3.1 ± 0.3</td>
<td>3.1 ± 0.5</td>
<td>0.9 ± 0.1</td>
<td>0.30 ± 0.05</td>
</tr>
<tr>
<td>Z+jets</td>
<td>32 ± 3</td>
<td>9.1 ± 0.9</td>
<td>2.4 ± 0.2</td>
<td>2.1 ± 0.2</td>
<td>0.77 ± 0.08</td>
<td>0.29 ± 0.03</td>
</tr>
<tr>
<td>Mistags</td>
<td>303 ± 42</td>
<td>74 ± 14</td>
<td>17 ± 6</td>
<td>5 ± 1</td>
<td>1.7 ± 0.4</td>
<td>0.6 ± 0.2</td>
</tr>
<tr>
<td>QCD</td>
<td>125 ± 50</td>
<td>35 ± 29</td>
<td>10 ± 9</td>
<td>6 ± 3</td>
<td>0.1 ± 1.5</td>
<td>0.1 ± 1.5</td>
</tr>
<tr>
<td>Total prediction</td>
<td>1928 ± 243</td>
<td>1061 ± 93</td>
<td>330 ± 28</td>
<td>306 ± 40</td>
<td>296 ± 38</td>
<td>104 ± 13</td>
</tr>
<tr>
<td>Observed</td>
<td>1844</td>
<td>1088</td>
<td>339</td>
<td>275</td>
<td>273</td>
<td>126</td>
</tr>
</tbody>
</table>
procedure takes into account correlations among channels by using same parameters for common sources of systematic uncertainties and allowing variations of each parameter with respect to its central value.

We perform the minimization of the negative logarithm of the likelihood \(-2 \log (\mathcal{L})\), using the MINUIT package [19]. We analytically extend \(c_{tag}(R)\) beyond \(R = 1\) during the fitting procedure, constraining each individual \(c_{tag}(R)\) to be greater than zero and their sum to be \(\leq 1\). We simultaneously fit \(R\) and \(\sigma_{t\bar{t}}\), which are the free parameters of the likelihood. We update the calculation of background yields using the value of \(\sigma_{t\bar{t}}\) determined by the fit and iterate the previous steps until the procedure converges. No dependence on the starting point was observed in the results of the iterative procedure.

The uncertainty determined by the fit comprises the statistical contribution; the systematic contribution on event-tagging efficiency, due to the systematic uncertainty on SF and the mistag matrix; the event selection efficiency, due to the lepton-identification scale-factor and the trigger efficiency; the background normalizations, including the heavy-flavor fractions; corrections for differences between MC and data heavy-flavor yields; and the luminosity [13]. We include separately the contributions due to the uncertainty on the jet-energy scale, effect of initial- and final-state radiation in the simulation (ISR/FSR), event-generator dependences, and top-quark mass. The impact of the jet-energy scale uncertainty is estimated by varying the energy of all jets in the MC samples by \(\pm 1\) standard deviation with respect to the central value for both signal and backgrounds and by repeating the iterative fits. The uncertainty arising from the choice of the MC generator is evaluated by repeating the analysis using a \(t\bar{t}\) sample generated by HERWIG [20]. The ISR/FSR uncertainty is evaluated by using \(t\bar{t}\) MC samples generated with enhanced or suppressed radiation relative to the default configuration.

Theoretical value of the top-quark-production cross section depends on top-quark mass [21]. The recursive fit of \(\sigma_{t\bar{t}}\) is expected to reduce the impact of this systematic uncertainty. In order to check this assumption, we repeat the measurement using two different MC samples for the \(t\bar{t}\) signal, simulated with \(m_t = 170\ \text{GeV}/c^2\) and \(m_t = 175\ \text{GeV}/c^2\), respectively. Central values and uncertainties on those systematic effects are included in the likelihood as nuisance parameters.

As a consistency check, the effect of each source of systematic uncertainty is estimated via simulated experiments. For each source we generate a set of simulated experiments with the same prescription but with the nuisance parameter \(x_a\), relative to the systematic effect under study, shifted by one standard deviation from its nominal value. We determine the effect of changing each source of systematic uncertainty as the change in the mean of the distributions of \(R\) and \(\sigma_{t\bar{t}}\). Table II lists the various systematic uncertainties assumed as fully uncorrelated.

The final results are summarized in Table III. Figure 2 shows the two-dimensional likelihood contour in the \((R, \sigma_{t\bar{t}})\) plane, for the fit including statistical and systematic uncertainties. The best-fit value is indicated with a “X” and can be compared to the theoretical SM prediction at the next-to-leading (NLO) order expansion in the strong-interaction coupling constant [17]. The results are in agreement with the theoretical prediction to within one standard deviation.

To determine the credibility level limit on \(R\) we follow a Bayesian statistical approach. Since \(R\) is bounded to be in the interval \([0,1]\), the prior probability density is chosen to be zero outside these \(R\) boundaries, while we consider all physical values equally probable. To obtain the posterior distribution for \(R\), we integrate over all nuisance parameters using non-negative normal distributions as priors. We also integrate over \(\sigma_{t\bar{t}}\) with the only constraint to be positive defined. The Bayesian lower limits at 68% and 95% credibility levels are shown in Fig. 3 and yield \(R > 0.785\) at 95% c.l. From Eq. (1) we ex-

![FIG. 1: Observed events for the analysis final states after summing over lepton categories, compared to expected events for different values of \(R\). For the \(t\bar{t}\) normalization the theoretical value \(\sigma_{t\bar{t}} = 7.04 \pm 0.49\ \text{pb}\) is used.](image)
From the fit results we obtain a measurement of $V_{tb}$. Assuming three generations of quarks and given the unitarity of the CKM matrix, we have $|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2 = 1$, leading to $R = |V_{tb}|^2$. From the fit results we obtain $|V_{tb}| = 0.97 \pm 0.05$ and $|V_{tb}| > 0.89$ at 95% credibility levels.

In summary, we present the simultaneous measurement of $R$ and $\sigma_t$. The X-cross corresponds to the maximum of the likelihood; the point with error bar to the NLO cross section calculation. The two-dimensional confidence regions are shown as well.

In Table III we present the results for $R$ and $\sigma_t$. The uncertainties on $R$ and $\sigma_t$ correspond to a variation of one unit of $-2 \log (L)$. The correlation parameter is $\rho = -0.434$. The magnitude of the CKM matrix element $|V_{tb}|$ is derived from $R = |V_{tb}|^2$. Lower limits at different credibility levels (c.l.) are obtained by integration of the posterior probability distribution.

<table>
<thead>
<tr>
<th>Parameter Value Lower limit Lower limit</th>
<th>(stat+syst) 68% c.l. 95% c.l.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_t$ (pb) 7.5 \pm 1.0 1.0 1.0</td>
<td>- -</td>
</tr>
<tr>
<td>$R$ 0.94 \pm 0.09 0.876 0.785</td>
<td>0.876 0.785</td>
</tr>
<tr>
<td>$</td>
<td>V_{tb}</td>
</tr>
</tbody>
</table>

In summary, we present the simultaneous measurement of $R = 0.94 \pm 0.09$ and $\sigma_t = 7.5 \pm 1.0$ pb with a correlation $\rho = -0.434$, and the determination of $|V_{tb}| = 0.97 \pm 0.05$. The results for $R$ and $|V_{tb}|$ are the most precise determination obtained by CDF and are in agreement with the standard model [2], with the previous CDF measurements [6], with the latest measurement of $R$ performed by D0 [7], and with the direct measurement of single-top-quark production cross section performed by LHC [22] and Tevatron [23] experiments.

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We use a cylindrical coordinate system where the $z$ axis is along the proton beam direction, $\phi$ is the azimuthal angle, and $\theta$ is the polar angle. Pseudorapidity is $\eta = -\ln \tan(\theta/2)$, while transverse momentum is $p_T = |p| \sin \theta$, and transverse energy is $E_T = E \sin \theta$. Missing transverse energy, $\mathcal{E}_T$, is defined as the magnitude of $-\sum_i E_T^i \hat{n}_i$, where $\hat{n}_i$ is the unit vector in the azimuthal plane that points from the beam line to the $i$th calorimeter tower. The $W$ boson transverse mass is defined as $M_W^T = \frac{1}{c^2} \sqrt{2E_T^l E_T (1 - \cos \phi_{l\nu})}$, where $E_T^l$ is the transverse energy of the lepton and $\phi_{l\nu}$ is the angle between the lepton and the $\mathcal{E}_T$. 

References:

[9] We use a cylindrical coordinate system where the $z$ axis is along the proton beam direction, $\phi$ is the azimuthal angle, and $\theta$ is the polar angle. Pseudorapidity is $\eta = -\ln \tan(\theta/2)$, while transverse momentum is $p_T = |p| \sin \theta$, and transverse energy is $E_T = E \sin \theta$. Missing transverse energy, $\mathcal{E}_T$, is defined as the magnitude of $-\sum_i E_T^i \hat{n}_i$, where $\hat{n}_i$ is the unit vector in the azimuthal plane that points from the beam line to the $i$th calorimeter tower. The $W$ boson transverse mass is defined as $M_W^T = \frac{1}{c^2} \sqrt{2E_T^l E_T (1 - \cos \phi_{l\nu})}$, where $E_T^l$ is the transverse energy of the lepton and $\phi_{l\nu}$ is the angle between the lepton and the $\mathcal{E}_T$.