The Shadow Costs of Repos and Bank Liability Structure[☆]

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Abstract

Making use of a structural model that allows for optimal liquidity management, we study the role that repos play in a bank's financing structure. In our model the bank's assets consist of illiquid loans and liquid reserves and are financed by a combination of repos, long–term debt, deposits and equity. Repos are a cheap source of funding, but they are subject to an exogenous rollover risk. We show that the use of repos inflicts two types of indirect ("shadow") costs on the bank's shareholders: first, it induces the bank to maintain higher liquid reserves in order to alleviate the additional default risk; second, it adds to the cost of long–term debt financing. These shadow costs limit the bank's appetite for cheap but unstable repo funding. This effect is, however, weakened under poor returns on risky assets, access to deposit funding and the depositor preference rule. We also analyze the impact of a liquidity coverage ratio, payout restrictions and a leverage ratio on the bank's financing choices and show that all these tools are able to curb the bank's reliance on repos.

Keywords: Bank financing structure; repos; liquid reserves; rollover risk; regulation.
JEL: G21; G28; G32; G35.

1. Introduction

What drives the financing decisions of modern banks? The academic literature had, until recently, paid little attention to the determinants of banks' financing structures. Nowadays, however, there is a growing interest in this issue,[1] since empirical evidence suggests that the composition of

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[1] Theoretical treatments of this topic essentially rely on the application of structural models to banking (see e.g., Hugonnier and Morellec (2015), Sundaresan and Wang (2015)).

banks' liabilities is far richer than a simple mix of equity and insured deposits.[2] In particular, in the years preceding the Global Financial Crisis, short–term wholesale debt gained popularity, as it was considered to be a relatively cheap source of funding.[3] This cost advantage was particularly relevant in the case of repos, due to their deposit–like nature, their safe harbor provisions and the investors' preferences for safe and liquid investments. However, the example of Northern Rock, brought to the edge of failure in 2008 by a run of its wholesale creditors, clearly shows that an overreliance on short–term funding may render a bank particularly fragile.

This paper develops a theoretical framework to gain insights into the main drivers behind banks' financing structures and, in particular, the maturity composition of debt. In our model, a bank can be financed by a combination of (i) short–term, secured debt that we refer to as "repos", (ii) long–term, risky debt, (iii) insured deposits, and (iv) equity. The main question we address deals with the choice between repos and long–term, risky debt. The former is relatively cheap but it is subject to an *exogenous* run of the repo creditors, whereas the latter represents a stable but costlier source of funding. The asset side of the bank's balance sheet consists of risky, illiquid assets and liquid reserves that earn no interest. The level of liquid reserves fluctuates over time and is controlled via a payout policy. The bank may draw from its liquid reserves to ensure the continuity of interest payments on debt. Moreover, liquid reserves serve as a buffer against the withdrawal losses that may be caused by a run of the repo creditors. These two features lead to an interaction between the bank's financing structure and its liquidity management policy, which turns out to be crucial for our results. The bank's liquidation policy is also related to the dynamics of its liquid reserves: in an unregulated environment, running out of liquid reserves triggers the bank's liquidation, as the bank defaults on its debt.[4] On the other hand, when the bank faces capital or liquidity requirements, a regulation–triggered liquidation might occur at strictly positive levels of the liquid reserves.

We begin by providing a formal characterization of the bank's optimal financing and payout decisions, which are jointly determined in our framework. As it is the case in many inventory models (see e.g., Jeanblanc and Shiryaev (1995) and Décamps et al. (2011)) and in recent contributions that simultaneously deal with financing and payout decisions (see e.g., Bolton et al. (2014) and Hugonnier and Morellec (2015)), the bank retains earnings below a certain level of liquid reserves and only distributes dividends whenever its liquid reserves reach this target level. The choice of the latter depends on the bank's financing structure, which affects the cost of financing and, thus,

[2]For example, according to Gropp and Heider (2010), non–deposit sources of funding constituted about 30% of the liabilities of European banks in 2004. The snapshot of the U.S. bank holding companies reported in Hanson et al. (2011) suggests that deposits amount to roughly half of a bank's liabilities, whereas the remainder represents a mix of wholesale/retail non–deposit funding and equity.

[3]For example, Chernenko and Sunderam (2014) provide evidence of the growing importance of short–term funding for European banks: in 2011, 18% of the total assets of U.S. money market funds were short–term loans provided to European banks.

[4]We assume that illiquid bank assets cannot be partially liquidated.

the dynamics of the bank's earnings. On the other hand, the choice of the financing structure depends itself on the target level of liquid reserves, since the latter affects the risk of liquidation and, therefore, the cost of long–term, risky debt.

We first solve the model numerically for a hypothetical bank that has no deposits in its financing structure and faces no regulation. Our analysis shows that including repos in the liability structure might help reduce the overall financing costs, thereby, increasing the bank's ex–ante value. However, the presence of repos in the bank's financing structure inflicts two types of indirect costs on its shareholders, which are labeled throughout the paper *shadow costs*. First, to mitigate the additional default risk caused by the possibility of a run on repos, the bank must target a higher level of liquid reserves (which earn no returns). Second, the use of repos gives rise to an additional component in the spread of the long–term risky debt, which can be interpreted as an *extra premium* rewarding the long–term creditors for the negative externalities imposed on them by the (possible) run of the repo creditors. The existence of these shadow costs plays a disciplining role by curbing the bank's incentives to excessively rely on cheap but unstable repos.

Interestingly, the magnitude of the extra premium on long–term debt resulting from the use of repos is closely related to the bank's ability to maintain high levels of liquid reserves. In particular, it is lower for the banks with higher returns on risky assets, as they are able, by increasing their target levels of liquid reserves, to better handle the additional default risk inflicted by the use of repos. For these banks, the relative cost advantage of repos is not substantial. In contrast, banks with lower returns on risky asset cannot afford to implement substantial upward adjustments of their liquid reserves and, as a consequence, must offer a higher compensation to their long–term creditors, which magnifies the initial cost advantage of repos over long–term, risky debt. This effect, which works through the interaction channel between the financing costs and the optimal management of liquidity, adds to the standard franchise–value effect that drives the shareholders' aversion to the run–triggered liquidation risk. As a result, banks with lower returns on risky assets exhibit higher proportions of repos in their financing structures.

On the second stage of our numerical analysis, we reexamine the bank's financing decisions allowing for access to (insured) deposit funding, whose volume is taken as given. Notably, the access to deposit funding induces the bank to increase its reliance on repos. The reason is that a larger volume of insured deposits in the bank's liability structure reduces the effective bank earnings. The resulting effect is, therefore, qualitatively similar to the effect of lowering the effective return on risky assets: namely, the access to deposit funding induces a bank to increase its reliance on repos. Furthermore, this effect becomes more pronounced when the insured deposits are senior to the long–term, risky debt. The seniority of the insured deposit over the long–term debt implies that long–term creditors receive a lower value in the case of a bank liquidation and, thus, demand a higher interest rate. This amplifies the cost advantage of repo financing even further.

Once we have studied the financing decisions of an unregulated bank, we consider the impact of different regulatory measures on a bank's ex–ante choice of financing structure. After the Global Financial Crisis, over–reliance on short–term debt funding in the banking sector was perceived as socially dangerous and became a serious concern for bank regulators. In this light, it is useful to gain a better understanding of how different regulatory measures affect a bank's appetite for short–term funding. We examine the effects of three regulatory tools: liquidity regulation in the spirit of the Basel III Liquidity Coverage Ratio, payout restrictions and a leverage ratio. Overall, our analysis shows that each of these tools is capable of reducing the banks' reliance on repos. Under liquidity regulation, which requires the bank to maintain a minimum level of liquid reserves as a certain proportion of its volume of repos, the bank substitutes repo funding by long–term debt. Payout restrictions induce a similar substitution effect: when forced to operate with higher target levels of liquid reserves, the bank benefits from lower costs of long–term debt financing, which downplays the cost advantage of repos. Leverage regulation, on the other hand, induces the bank to lower its volumes of both repos and long–term debt. Put into the perspective of current policy debates, this set of results suggests that developing special liquidity regulation in order to reduce a reliance of banks on repos might be redundant.

Related literature. Our paper belongs to the burgeoning body of literature that examines the interaction between the optimal capital structure and liquidity management (see e.g., Gryglewicz (2011), Bolton et al. (2014) and Hugonnier and Morellec (2015)). In these works, the capital structure affects the liquidity reserves through interest payments on debt. However, these models allow exclusively for a single type of debt (perpetual one) in the financing structure, thereby disregarding the debt–maturity–choice dimension. In our model, we introduce an additional type of debt with a short maturity (repos) and bring into focus the trade–off between financing costs and stability. Allowing for repos in the bank's financing structure creates an additional channel of interaction between liquidity management and capital structures, given that liquid reserves serve as a buffer against the withdrawal losses generated by a run of repo creditors. This feature plays an important role in our analysis, since the bank's ability to build larger liquid reserves has direct implications on its optimal debt–structure choices. At the same time, it marks a key difference between our framework and the existing structural models that deal with rollover risk such as He and Xiong (2012b), and He and Milbradt (2014, 2015), where rollover losses are absorbed by the shareholders' "deep pockets". Those models, however, focus on the impact of rollover risk on the endogenous default decisions. In our baseline setting, default occurs as soon as the liquid reserves are depleted, but the likelihood of running out of liquidity is strongly affected by both the payout and the financing decisions of the bank.

An additional key departure of our work from the papers by He and Xiong (2012b), He and Milbradt (2014, 2015) pertains to the approach to modeling the rollover risk. In their models, all debt is risky and the maturities are uniformly distributed (like in the setting of Leland and Toft (1996)). Maturing debt has to be replaced at a market price, which might be above or below its face value, thereby leading to rollover gains/losses. In the present paper we adopt an alternative approach to modeling rollover risk by considering its materialization as an *exogenous* run of repo creditors. This is motivated by the anecdotal evidence that the large actors of repo markets, such as money–market mutual funds, and even traditional banks, may face sudden liquidity needs caused by a run of their own clients/creditors, which, in turn, will push them to pull their short–term investments back.[5]

Our paper is also related to the literature on the optimal maturity composition of debt. When exploring this issue, some papers place emphasis on the role that short–term debt plays in resolving various kinds of agency problems (see e.g., Calomiris and Kahn (1991), Eisenbach (2014), Diamond and He (2014)). Alternatively, Brunnermeier and Oehmke (2013) show that short–term debt maturity may be an equilibrium outcome when a debt issuer is unable to commit to a predetermined maturity structure. We assume, instead, that the only benefit of using short–term secured debt stems from its lower cost and focus on the impact that its presence has on the long–term debt's cost. In this regard, our model is complementary to the work by Auh and Sundaresan (2014), who study a similar question within the context of a Leland–type structural model. The main feature that distinguishes our model from theirs is the fact that we allow for the joint choice of the optimal debt structure and liquidity management and, thereby, aim to capture the link between the composition of the asset and liability sides of the balance sheet. In a recent contribution, He and Milbradt (2015) study the fully dynamic choice of optimal debt maturity in a setting with a frictionless equity market. In their model, a firm can instantaneously modify its average debt maturity by substituting a fraction of the maturing long–term bonds by short–term ones (or vice versa). Increasing the fraction of debt with shorter maturity reduces the current rollover losses (that must be offset by new equity issuances), as it has a lower default risk and, thus, a higher price than debt with a longer maturity. However, this also increases the rollover frequency in the future, thereby increasing the default probability.[6] A similar trade–off between the lower cost of funding and the additional default risk inherent in short–term funding is a key driver of the results in our setting. Nevertheless, the fact that our debt maturity structure is static enables us to address the optimal choice of financing structure, whereas in He and Milbradt (2015) the latter is taken as exogenous.

The remainder of the paper is organized as follows: Section 2 presents the model. Section 3 provides a formal characterization of the bank's policies. In Section 4 we conduct a numerical analysis to identify the determinants of the bank's financing structure. In Section 5 we examine the impact of different regulatory measures on the bank's funding decisions. Section 6 concludes.

[5]A substantial body of literature dealing with rollover risk considers runs as a consequence of the coordination problem between the maturing creditors in the context of deteriorating fundamentals (see e.g., Cheng and Milbradt (2012), He and Xiong (2012a), Liang et al. (2014)). In these models, the short–term creditors run as soon as the value of the firm's fundamentals falls below a certain critical threshold. In the present paper we abstain from rationalizing runs on repos, and focus instead on the implication of runs for the financing and payout policies of banks.

[6]This mechanism is also key in Della Seta et al. (2015).

Proofs and other mathematical material are gathered in the Appendices.

2. The Model

We work in a continuous–time, infinite–horizon setting, in which all agents are risk neutral and discount the future cash flows at a rate $\rho > 0$. A group of equity investors holds a banking license and has to decide on the financing structure of a new bank. The bank's assets comprise *liquid reserves* and a portfolio of *risky assets* of a fixed size A. A fraction $(1 - \eta)A$ of the risky assets, for $\eta \in (0, 1)$ given, consists of illiquid loans that have zero value in the case of bank liquidation.[7] The remaining fraction ηA corresponds to the assets that hold their value in the case of bank liquidation and, therefore, can be used as collateral for secured borrowing transactions (e.g., sovereign bonds, agency CMOs, some approved ABS, etc.). Once the bank has been established, the risky assets generate the cash flows[8]

$$\mu dt + \sigma dW(t),$$

where μdt denotes the expected return on risky assets per unit of time, σ represents the volatility of asset returns and $W = \{W(t), t \geq 0\}$ is a standard Brownian motion, defined on the standard probability space $(\Omega, \mathcal{F}, \mathrm{P})$ that generates the filtration $\mathbb{F} = \{\mathcal{F}_t, t \geq 0\}$. The bank's revenues are taxed at a fixed rate $\theta \in (0, 1)$.

Bank balance sheet. The bank's assets may be financed using a mixture of insured deposits and the following types of securities: repos, long–term, risky debt and equity. To introduce the risk of liquidation in our model, we assume that there is no access to capital markets after the bank has been created.[9] When choosing the bank's financing structure, its shareholders take the cost and the volume of insured deposits as given and decide on how to complement the latter with non–deposit sources of funding. Throughout the paper, our analysis is centered around the choice of the initial financing structure, without allowing for any further adjustments. Although this rigid structure is assumed for technical reasons, it can be justified through the empirical evidence suggesting that the

[7] This assumption stems from the fact that the value of bank loans largely depends on the bank's private information, which cannot be easily transmitted to a new owner. It can be relaxed with minor modifications.

[8] In contrast with the Leland (1994)-type structural models, where the firm's asset cash–flow/asset value evolves according to a geometric Brownian motion, here the bank's *cash flows* follow an arithmetic Brownian motion. This is compatible with the fact that the bank's productive assets have a fixed size and it makes liquid reserves relevant. Indeed, in the presence of financial frictions, the fact that the evolution of the cash flows can oscillate vertically means that the bank may suffer operational losses that must be offset by its liquid reserves (here we assume an extreme form of financing frictions where the bank has no access to equity market after it starts operating, but the same insight would be relevant if we allowed for *costly* recapitalizations). Even though very stylized, this specification allows us to consider the link between the composition of the asset and liability sides of the bank's balance sheet. On the other hand, an underlying assumption in the Leland–type models is that shareholders bear no costs when raising new equity. As a result, there is no room for liquid reserves: all profits are distributed as dividends and all losses are offset via new equity issuances, as long as it is optimal to maintain the firm (bank) afloat.

[9] Allowing for *uncertain* recapitalization possibilities would not qualitatively alter our results.

capital structure of a bank is highly persistent and changes slowly over time (see e.g., IMF Global Financial Stability Report (2013)). The structure of the bank's balance sheet implied by our model is summarized in Table 1.

Table 1: The bank's balance sheet

Assets	**Liabilities**
Liquid reserves, $C(t)$	Deposits, P_d
Risky assets, $A \equiv (1-\theta)\mu/\rho$	Repos, P_s
	Long-term debt, P_l
	Equity, $C(t) + A - P_d - P_s - P_l$

In order to formally state the decision problem of the bank's shareholders at the time of the investment, we start by describing the menu of funding sources and their respective costs. Next, we characterize the dynamics of the liquid reserves and specify the values of the bank's securities.

The types of debt funding and their corresponding costs. Deposits in our model are insured by a deposit–insurance fund and take the form of perpetual debt with a face value of P_d. The overall cost of deposit funding to the bank amounts to $r_d P_d dt$ per unit of time, where r_d encompasses the rate of the interest payments made to depositors, as well as the deposit–insurance and management costs. Throughout the paper both P_d and r_d are taken as exogenous.

The non–deposit sources of debt financing consist of repos (instantaneously maturing, secured debt) with a volume of $P_s \leq \eta A$[10] and long–term debt with a face value of P_l. We focus on the case where the long–term debt is risky regardless of the level of repos, which requires the condition $P_l > \eta A$. Repos are instantaneously rolled over, whereas the long–term debt takes the form of a perpetual bond. The repo creditors benefit from a so–called *safe–harbor provision*, which spares them from any losses in the case of a bank liquidation (this is the case under the U.S. Bankruptcy Code). This implies that the long–term, risky debt is junior to repos. However, the long–term debt may be junior or senior to the insured deposits (we analyze the impact of the seniority rules in Section 4.2).

The bank's creditors are assumed to be subject to exogenous liquidity shocks, whose arrivals are described by the Poisson process $N = \{N(t), t \geq 0\}$ with intensity λ. When hit by a liquidity shock, the repo creditors stop rolling their debt over.[11] The long–term creditors, on the other hand,

[10]This collateral constraint states that, by liquidating its core assets, a bank is always able to repay repo creditors, even when there are no liquid reserves left. In other words, the repo creditors bear no risk. As we show in our numerical analysis, for reasonable level of parameter values (namely, when η is not too low) this constraint is never binding and thus has no direct impact on the optimal choice of financing structure.

[11]This situation may occur when the bank engages in repo transactions with a Money Market Mutual Fund

are “locked in”. We define the (stopping) time when the repo creditors run as

$$\tau^* := \inf\{t > 0 : N(t) = 1\}.$$

For simplicity, we rule out the possibility of additional repo sales once the run of the incumbent creditors has occurred.[12]

The repo funding costs the bank $r_s P_s dt$ per unit of time. Since repos are fully collateralized, the interest rate $r_s < \rho$ does not reflect the bank's liquidation risk[13] and it is exogenous (e.g., a 1–year LIBOR). The long–term debt funding costs the bank $r_l P_l dt$ per unit of time, and the interest rate $r_l \geq \rho$ is chosen *endogenously* at the time of investment so as to compensate the long–term creditors for the liquidation risk they bear.[14] It is worth mentioning that, even if the long–term debt were not subject to any liquidation risk, the long–term creditors would be compensated at a higher rate than the repo ones. The reason being that, in contrast to the repo creditors, the long–term creditors are unable to withdraw their funds in the case of a liquidity shock. As a result, their default–free rate should comprise an illiquidity premium. An alternative interpretation one may provide, based on standard liquidity arguments (see e.g., Diamond and Dybvig (1983), Gorton and Metrick (2010) or Stein (2012)), is that the repo creditors derive additional utility from holding liquid claims. Hence, they accept to be compensated at a lower rate.

As will become apparent below, the impact of repos on the overall cost of debt financing is ambiguous. On the one hand, repos may be viewed as a cheaper source of funding. On the other hand, increasing their proportion in the bank's liability structure simultaneously increases the risk of liquidation[15] and reduces the value accruing to the unsecured creditors in such a case, thereby amplifying the costs of long–term debt financing.

The dynamics of the liquid reserves. For a given cost structure of debt financing, the bank's

(MMMF). As argued by Chernenko and Sunderam (2014), the concerns of the MMMF's investors about some of its assets may induce them to pull their money back, which triggers subsequent cuts in funding of totally credit–worthy firms financed through this fund (a negative spill–over effect). In other words, a materialization of the rollover risk may not necessarily be triggered by the investors' concerns about the quality of the bank' assets, but may simply represent a negative externality of the excessive risk–taking by the provider of short–term funding. This argument underlies the assumption of an exogenous run in our model. We refer the reader to e.g. Liang et al. (2015) for a mathematically–rigorous discussion of how, in a dynamic structural model, the run intensity can be endogenously determined as a strategic decision of the banks' creditors.

[12]In fact, in the short run, the bank may be unable to replace its usual provider of short–term funding because of information asymmetries or even institutional frictions corresponding to the fact that the MMMF might be constrained by its board to lend only to a pre–approved number of counterparties (see Chernenko and Sunderam (2014)).

[13]We first study an unregulated setting, in which liquidation and default coincide. That is to say that the bank's shareholders will never default strategically at any positive level of liquidity, when the bank can still service its debt. When facing regulation, however, it might occur that the bank is subject to intervention before it becomes illiquid.

[14]Note that the use of long–term debt financing in our set–up is optimal due to the presence of tax benefits, whereas the motivation for repo financing mainly stems from their relative cost advantage.

[15]In this light, one can interpret the use of repo financing as taking on tail on the liability side of the balance sheet.

cumulative, after–tax earnings $R = \{R(t), t \geq 0\}$ evolve in the following way:

$$R(t) = (1-\theta)\Big[(\mu - r_d P_d - r_l P_l - \mathbb{1}_{\{t \leq \tau^*\}} r_s P_s)t + \sigma W(t)\Big],$$

where $\mathbb{1}_{\{\cdot\}}$ is the zero–one indicator function reflecting the fact that, if it were to withstand a run of the repo creditors, the bank would continue operating without repos on its balance sheet.

The after–tax earnings can be distributed to the bank's shareholders or they may be retained as liquid reserves. Maintaining liquid reserves, however, involves dead–weight costs that we capture via the assumption that no interest accrues on them.[16] We model the cumulative payouts made to the shareholders up to time t via a $\mathbb{F}$–adapted, non–decreasing process $L = \{L(t), t \geq 0\}$. For any initial level of liquid reserves $c_0 \geq 0$ (which in equilibrium will be optimally chosen so as to maximize the ex–ante shareholder value), the financing and payout decisions can be described by a strategy $\pi = (P_s, P_l, L)$. We define by

$$\mathcal{S} := \big\{\pi \,:\, P_s \leq \eta A, \text{ and } L \text{ is non–decreasing}\big\}$$

the set of admissible strategies that can be adopted by the bank's management. The liquid–reserves process $C^\pi = \{C^\pi(t), t \geq 0\}$ associated to a strategy $\pi \in \mathcal{S}$ satisfies

$$C^\pi(t) = c_0 + R(t) - L(t) - \mathbb{1}_{\{t \leq \tau^*\}} P_s N(t), \tag{1}$$

where the last term on the right–hand side reflects the fact that the liquid reserves may be subject to a large negative shock whose scale corresponds to the volume of the withdrawn repo funding.[17] The indicator function captures the fact that no re–issuance of repos takes place once the run has occurred, although the bank may very well remain in operation, provided that its reserves are sufficient to absorb the withdrawal loss.

We denote by $\underline{c} \geq 0$ the level of liquid reserves at which the bank is liquidated and by τ_π the corresponding liquidation time, defined for any strategy $\pi \in \mathcal{S}$ as

$$\tau_\pi := \inf\big\{t > 0 : C^\pi(t) < \underline{c}\big\}.$$

In the absence of regulation, and due to the limited liability of the bank's shareholders, it is suboptimal for them to liquidate the bank at a positive level of liquid reserves, hence $\underline{c} = 0$. This obeys the fact that the recovery value for the shareholders in the case of liquidation is always dominated by that of the bank as an ongoing concern. Technically speaking, the marginal value of liquid re-

[16]This assumption also enables us to obtain closed–form expressions for the values of the bank's liabilities.

[17]We assume that, in the case of a run, all the repo creditors pull their funds back simultaneously, as it happened, for example, in case of Lehman Brothers.

serves in the bank is always larger than or equal to one, as will be shown in the upcoming sections. Therefore, early liquidation is inefficient from the shareholders' perspective. However, as we show in Section 5, imposing liquidity or capital regulation on the bank may result in a strictly positive liquidation threshold, which would itself depend on the bank's financing structure.

It is important to notice that, in our setting, liquidation may be triggered by either the run of the repo creditors, should it occur when the level of liquid reserves $C^\pi(t) \in [\underline{c}, \underline{c} + P_s)$, or by a series of poor performances, in which case the liquid reserves get depleted gradually following adverse realizations of the Brownian risk. This shows that the role of the liquid reserves is twofold: First, they help the bank hedge against adverse profitability shocks and ensure the continuity of debt servicing. Second, they serve as a buffer against a possible large loss caused by the run of the repo creditors.

The shareholders' problem. In order to formalize the problem of the bank's shareholders, we first define the ex–ante value of equity corresponding to a strategy π and an initial level of liquid reserves c_0 :

$$U^\pi(c_0) := \mathbb{E}_0 \left[\int_0^{\tau_\pi} e^{-\rho t} dL(t) \right]. \tag{2}$$

The term inside the conditional expectation reflects the present value of the cumulated dividend payments to shareholders until the time of liquidation. In the absence of regulatory requirements, the shareholders walk away empty–handed in such a case.[18] At this point some clarification regarding τ_π and τ^* is in order. Since liquidation need not be implied by a repo run, the set $\{\tau^* < \tau_\pi\}$ is of positive measure. In other words, Expression (2) implicitly contains τ^*. This will prove to be relevant in the dynamic–programming approach that we take in the sequel. On the other hand, liquidation may be triggered by the run, in which case $\tau^* = \tau_\pi$.

The bank's shareholders choose among the set of all admissible financing and payout strategies $\mathcal{S}$, so as to maximize the value of their equity, net of their initial investment expenditure. For a given choice of c_0, the total amount of funds that must be raised to establish the bank is $A + c_0$. The difference between the total investment costs and the funding raised, namely $(A+c_0)-(P_d+P_l+P_s)$, is financed by equity. The shareholders' optimization problem can then be stated as follows:

$$\max_{c_0 \geq 0, \pi \in \mathcal{S}} \quad V^\pi(c_0) := \Big\{ U^\pi(c_0) - (A + c_0 - P_d - P_l - P_s) \Big\}. \tag{3}$$

The ex–ante market value of long–term debt under the strategy π is the expected value of the sum of the discounted, cumulated interest payments until the time of liquidation, plus the liquidation

[18] This is not necessarily the case in the presence of regulation, where the term $\mathbb{E}_0[\tilde{u}(C^\pi(\tau_\pi))]$ (the shareholders' payoff at liquidation) should be added to $U^\pi(c_0)$.

value accruing to the long–term creditors in such an event:[19]

$$D^{\pi}(c_0) := \mathbb{E}_0\left[\int_0^{\tau_\pi} e^{-\rho t} r_l P_l dt + e^{-\rho\tau_\pi}\tilde{d}(C^{\pi}(\tau_\pi))\right].$$

The value $\tilde{d}(C^{\pi}(\tau_\pi))$ accruing to the long–term creditors in the event of liquidation depends on the long–term debt's seniority relative to that of the insured deposits, as well as on whether or not the run of the repo creditors has already taken place. We assume that the long–term debt is issued in a competitive market with rational creditors, so that its interest rate is chosen so as to ensure the parity between its face and market values, i.e. $D^{\pi}(c_0) = P_l$, which implicitly defines the long–term interest rate r_l.

In order to provide some intuition regarding the solution to the shareholders' problem, we point out that the optimal payout strategy is of the so–called *barrier type*. As we formally show below, it is characterized by an optimal payout barrier (or, equivalently, an optimal target level of liquid reserves) such that all liquid reserves beyond the said barrier are distributed as dividends, while no payouts take place as long as the level of liquid reserves remains below this threshold. We will show that, in general, the optimal initial level of liquid reserves c_0 coincides with the optimal target level of liquid reserves.[20] This choice determines the ex–ante distance to liquidation and, therefore, impacts the cost of long–term debt financing via the relation $D^{\pi}(c_0) = P_l$. This is, in fact, the channel through which the payout policy affects the choice of the financing structure in our model. Simultaneously, one observes in the reserves–dynamics Equation (1) that the choice of the financing structure feeds back into the dynamics of the liquid reserves through the costs of debt servicing and, furthermore, through the scale of the rollover risk exposure.[21] As we show in the upcoming sections, this feedback mechanism plays an important role in determining the bank's optimal financing structure.

3. The Financing and Payout Decisions of an Unregulated Bank

We consider, initially, the optimal payout policy and the values of the securities of a bank assuming it has survived the run of the repo creditors. The closed–form solutions for the values of equity and debt defined in this setting will be later used as building blocks for constructing the solution to the shareholders' problem at initiation. Moreover, this setting will serve as a useful benchmark needed to assess the impact of the repo funding on the optimal financing and payout decisions of the bank's management in the numerical analysis that we perform in Section 4.

[19] Notice that the above comment regarding the implicit presence of τ^* in $U^{\pi}(c_0)$ also applies to $D^{\pi}(c_0)$.

[20] This is no longer true when payout restrictions are imposed by the regulator (see Section 5.2).

[21] The presence of an *endogenous* tail risk, related to the use of repo funding, marks the key difference between our framework and the one considered by Bolton et al. (2014).

3.1. The payout policy and the values of the bank's securities after the run

The post–run value of equity. As we have mentioned above, a run by the repo creditors does not necessarily imply the liquidation of the bank. This suggests that one may take a dynamic–programming approach so as to determine the value of the bank's liabilities. In other words, the expected present values of bank equity and long–term debt *without* repos in the bank's liability structure are to be used to determine the initial values of equity and debt. To this end, let us assume that the bank has withstood the run of the repo creditors and now operates without repos in its liabilities. In this section we take (r_l, P_l) as given and study the properties of the equity value function[22]

$$U_0(c) := \sup_L U^{(0,P_l,L)}(c), \quad c \geq \underline{c}.$$

The formal derivation of the properties of U_0 that we discuss in the sequel may be found in Appendix A.1. Since for any time horizon the probability of liquidation is decreasing in the level of liquid reserves, the marginal value of liquid reserves decreases with c, as it is the case in several inventory models with pure equity financing (see e.g., Jeanblanc and Shiryaev (1995), Milne and Robertson (1996) and Décamps et al. (2011)). This implies that the equity value function U_0, which is clearly increasing in the level of liquid reserves, is concave.[23] It is then optimal to retain earnings below a certain critical barrier b_0^* that corresponds to the level of liquid reserves at which the marginal value of an additional unit of retained earnings equals the marginal value of distributed dividends, i.e. $U_0'(b_0^*) = 1$. In the sequel we refer to the interval $(\underline{c}, b_0^*)$ as the *retention region.* Let us denote by $f_0 := r_l P_l + r_d P_d$ the instantaneous cost of debt financing after the run. The following result offers a convenient differential characterization of the equity value function U_0.

Theorem 1. *In the region $(\underline{c}, b_0^*)$, the equity value function U_0 is concave and it satisfies the ordinary differential equation*

$$\rho U_0(c) = (1-\theta)^2 \frac{\sigma^2}{2} U_0''(c) + (1-\theta)(\mu - f_0) U_0'(c), \tag{4}$$

together with the Neumann boundary conditions $U_0'(b_0^*) = 1$ and $U_0''(b_0^*) = 0$.

For each choice of r_l and P_l, Equation (4) has the general solution

$$A(r_l, P_l) e^{\beta_1 c} + B(r_l, P_l) e^{\beta_2 c},$$

where $\beta_1 = \beta_1(r_l, P_l) > 0$ and $\beta_2 = \beta_2(r_l, P_l) < 0$ are the roots of the characteristic polynomial of Equation (4).

[22]Recall that, for the unregulated bank, $\underline{c} = 0$. Nevertheless, throughout this section we define the values of the contingent claims for any $\underline{c} \geq 0$, so as to exploit the obtained results in the setting where regulation is present.

[23]The concavity of the equity value function can be interpreted as a sort of "corporate" risk aversion arising from the risk of liquidation (see e.g., Milne and Robertson (1996) for a discussion on this phenomenon).

Whenever the current level of liquid reserves c exceeds b_0^*, the shareholders' impatience outweighs their concerns regarding liquidation, and the difference $c-b_0^*$ is immediately distributed as dividends. In other words, in the region (b_0^*, ∞) the equity value function is affine:

$$U_0(c) = U_0(b_0^*) + c - b_0^*. \tag{5}$$

Our strategy to find U_0 is to take an arbitrary target level of liquid reserves $b_0 > 0$ and, using Equations (5) and (4) together with the conditions $U_0'(b_0) = 1$ and $U_0''(b_0) = 0$, to determine a candidate equity value function. Then, the optimal payout barrier can be recovered by using the boundary condition

$$U_0(\underline{c}) = \max\{\eta A + \underline{c} - P_d - P_l, 0\} =: \tilde{u}_0(\underline{c}), \tag{6}$$

where $\tilde{u}_0(\underline{c})$ is the value accruing to the shareholders in the event of liquidation. It is important to notice that Theorem 1 only provides us with necessary conditions for optimality. Therefore, if we obtain a candidate for the equity value function by means of the HJB Equation (4) we must verify that it really corresponds to the shareholder's optimal equity value. This, naturally, is equivalent to making sure that the proposed optimal strategy is indeed value maximizing. We do this in the following proposition:

Proposition 1. *For a given debt structure (P_l, P_d) and the interest rate on long–term debt r_l, the post-run value of equity is given by*

$$U_0(c; b_0^*) := \begin{cases} \frac{1}{\beta_1-\beta_2}\left[-\frac{\beta_2}{\beta_1} e^{\beta_1(c-b_0^*)} + \frac{\beta_1}{\beta_2} e^{\beta_2(c-b_0^*)}\right], & \text{for } c \in [\underline{c}, b_0^*), \\ \frac{(1-\theta)(\mu-f_0)}{\rho} + c - b_0^*, & \text{for } c \geq b_0^*. \end{cases} \tag{7}$$

The optimal, cumulative–dividend policy L^* acts on the liquid–reserves process C^π so as to keep it at or below the payout barrier $b_0^* = b_0^*(r_l, P_l)$, which is the unique solution to the equation $U_0(\underline{c}; b_0^*) = \tilde{u}_0(\underline{c})$.

In Appendix A.2 we provide a verification theorem showing that $U_0(c; b_0^*)$ is, indeed, the value function, i.e. the equation $U_0(c; b_0^*) = U_0(c)$ is satisfied for all $c \geq \underline{c}$. When $\underline{c} = 0$, the target level of liquid reserves defined via Equation (6) can be given explicitly:

$$b_0^* = \frac{1}{\beta_1 - \beta_2} \log\left(\frac{\beta_2}{\beta_1}\right)^2.$$

The long–term debt's post–run value. Let us now look at the long–term debt's market value, which we denote by D_0 in this benchmark case. This value will depend critically on the payout policy chosen by the bank's shareholders but, as long as r_l is assumed to be fixed, the long–term creditors are passive and take the payout barrier as given. Therefore, we study the properties of D_0 for a fixed, arbitrary payout barrier b_0. By standard arguments, D_0 satisfies the following differential equation:

$$\rho D_0(c) = (1-\theta)^2 \frac{\sigma^2}{2} D_0''(c) + (1-\theta)(\mu - f_0) D_0'(c) + r_l P_l \quad \text{for} \quad c \in (\underline{c}, b_0). \tag{8}$$

In order to determine D_0, we need two boundary conditions. Since the optimal payout policy implies that the level of liquid reserves never exceeds b_0 for $t > 0$, the long–term debt's value remains constant for any c that is greater than or equal to b_0. Thus, it must hold that $D_0'(b_0) = 0$. The second boundary condition is imposed at the liquidation threshold $\underline{c}$. Specifically, we have $D_0(\underline{c}) = \tilde{d}_0$, where $\tilde{d}_0 = \eta A (< P_l)$ if the long–term debt is senior to the insured deposits and $\tilde{d}_0 = \max\{\eta A - P_d, 0\}$ otherwise. Solving Equation (8) with the above–mentioned boundary conditions yields the following closed–form characterization of the market value of debt:

$$D_0(c; b_0) = \frac{r_l P_l}{\rho} - \left(\frac{r_l P_l}{\rho} - \tilde{d}_0\right) \left[\frac{\beta_1 e^{\beta_1 b_0 + \beta_2 c} - \beta_2 e^{\beta_2 b_0 + \beta_1 c}}{\beta_1 e^{\beta_1 b_0 + \beta_2 \underline{c}} - \beta_2 e^{\beta_2 b_0 + \beta_1 \underline{c}}}\right], \tag{9}$$

where the first term represents the perpetual value of the interest payments and the second term captures the impact of the liquidation and payout policies.

With the *post–run* values of equity and debt in hand, we are now in the position to define the values of the bank's securities and the optimal payout policy *before* the run.

3.2. The value of the bank's securities and the payout policy before the run

We now consider the optimal financing and payout policies when the bank has access to both long–term debt and repo financing. The main departure from the results obtained in Section 3.1 is due to the possibility of a run by the repo creditors. On the one hand, this allows for liquidation at levels of liquid reserves that are strictly higher than $\underline{c}$. On the other hand, conditional on the bank surviving a run, the dynamics of the liquid reserves experience a *regime change*, since the cost of debt servicing per unit of time will be reduced by $r_s P_s dt$. These two facts notwithstanding, as long as a run on the repos does not occur, the optimal payout policy of the bank does not deviate significantly from that pertaining to the case considered in Section 3.1. More specifically, we show in Appendix B that, given a debt structure (P_s, P_l) and a long–term interest rate r_l, there exists a target level of liquid reserves $b_1^*(r_l, P_s, P_l) \geq 0$ such that, as long as there is no run on repos, dividends are distributed so as to maintain the level of liquid reserves at or below b_1^*.[24]

The pre–run value of equity. As before, the value the of payout barrier b_1^* is closely related to

[24]For ease of reading, and as long as there are no grounds for confusion, we will refrain from writing the arguments of b_1^* in the text.

the bank's equity value function. Given the pair (P_s, P_l), the latter is defined as[25]

$$U_1(c) := \sup_L \; U^{(P_s,P_l,L)}(c).$$

We show in Appendix A.1 that U_1 is a concave function of the level of liquid reserves for the same reasons as U_0. Moreover, U_1 is also affine beyond the pre–run dividend barrier. Namely, for $c > b_1^*$, we have

$$U_1(c) = U_1(b_1^*) + c - b_1^*.$$

However, for $c \in (\underline{c}, b_1^*)$, the characterization of U_1 strongly depends on the choice of the payout barrier b_1^*. Depending on the choice of the capital structure and the underlying parameters value, the bank may end up in one of three potential scenarios: If $b_1^* \in (\underline{c}, \underline{c} + P_s]$, the bank will always be liquidated in the case of a run by the repo creditors. In the case where $b_1^* \in (\underline{c} + P_s, \underline{c} + P_s + b_0^*]$, two outcomes are possible: If the liquid reserves suffice to absorb a loss caused by a creditors' run, i.e. if $c > \underline{c} + P_s$, then the bank survives, switches regime and follows the payout policy defined in Section 3.1. In contrast, if the run occurs when $c \in [\underline{c}, \underline{c} + P_s]$, then the bank is liquidated. Finally, if $b_1^* > \underline{c} + P_s + b_0^*$ and a run occurs while $c \in (\underline{c} + P_s + b_0^*, b_1^*]$, then the bank makes a lump–sum payment of size $(c - P_s) - b_0^*$ to shareholders immediately after the run, and then follows the optimal payout strategy defined in Section 3.1.

Since it is not possible to distinguish between these scenarios from an ex–ante perspective, one would be forced to characterize candidates for the optimal equity value function in each scenario and then perform a numerical analysis to see how the model's parameters affect the choice of b_1^*. In our analysis, however, we focus solely on the case where $b_1^* \in (\underline{c} + P_s, \underline{c} + P_s + b_0^*]$. On the one hand, this turns out to be the only case that manifests itself for the range of parameters that we use in our numerical simulations. On the other hand, the first and third scenarios are somehow pathological. Namely, neither setting a target reserves level that guarantees liquidation in the case of a run by the repo creditors, nor setting a target reserves level that opens the possibility of a lump–sum payment to shareholders after the run are particularly palatable from an economic perspective.

In order to simplify the presentation, let us introduce the following operator:

$$\mathcal{L}g := (1-\theta)^2 \frac{\sigma^2}{2} g'' + (1-\theta)(\mu - f_1)g' - \rho g,$$

where $f_1 := r_d P_d + r_l P_l + r_s P_s$ and g is any twice continuously differentiable function.

[25] It should be noted that U_0 is hidden in $U_1(c)$, the reason being that the shareholders' (rational) choice of L takes into account the scenarios where a run by the repo creditors does not trigger liquidation. This is where the aforementioned dynamic–programming approach comes into play and it is manifest in Expressions (10)–(12) below. For ease of reading we abstain from presenting here the extended expression for $U_1(c)$, but provide it within the proof of Theorem 2 in Appendix A.1.

Theorem 2. *If the condition $b_1^* \in (\underline{c} + P_s, \underline{c} + P_s + b_0^*]$ holds ex–post, then the equity value function U_1 is concave and it satisfies the following system:*

$$\begin{aligned} \mathcal{L}U_1(c) - \lambda[U_1(c) - \tilde{u}_1(c)] &= 0, \quad c \in (\underline{c}, \underline{c} + P_s), & (10)\\ \mathcal{L}U_1(c) - \lambda[U_1(c) - U_0(c - P_s)] &= 0, \quad c \in (\underline{c} + P_s, b_1^*), & (11)\\ U_1(c) - U_1(b_1^*) + b_1^* - c &= 0, \quad c \geq b_1^*, & (12) \end{aligned}$$

together with the boundary conditions $U_1'(b_1^*) = 1$ and $U_1''(b_1^*) = 0$.

The *jump terms* $\lambda[U_1(c) - \tilde{u}_1(c)]$ and $\lambda[U_1(c) - U_0(c - P_s)]$ on the left–hand sides of Equations (10) and (11) reflect liquidation and a regime change after the run, respectively. Here, the value accruing to the shareholders in the case of liquidation caused by a repo creditors' run when $c \in (\underline{c}, \underline{c} + P_s)$ is given by $\tilde{u}_1(c) := \max\{\eta A + c - P_d - P_l - P_s, 0\}$, and the equity value U_0 after a run that leads to a regime change is as defined in Expression (7). As before, we require a verification result:

Proposition 2. *For a given debt structure (P_s, P_l, P_d) and the interest rate on long–term debt r_l, the pre–run value of equity $U_1(c)$ satisfies System (10)–(12) with the boundary conditions $U_1'(b_1^*) = 1$ and $U_1''(b_1^*) = 0$. The optimal, cumulative–dividend policy L^* before the run acts on the liquid–reserves process C^π so as to keep it at or below the payout barrier b_1^* which is the unique solution to the equation*

$$U_1(\underline{c}; b_1^*) = \tilde{u}_1(\underline{c}).$$

The closed–form solution to System (10) – (12) is cumbersome, so we relegate it to Appendix B.1. However, the value of equity at the optimal payout barrier b_1^* can be easily pinned down by inserting the boundary conditions $U_1'(b_1^*) = 1$ and $U_1''(b_1^*) = 0$ into Equation (11). This yields:

$$U_1(b_1^*) = \frac{(1-\theta)(\mu - f_1)}{\rho + \lambda} + \frac{\lambda}{\rho + \lambda} U_0(b_1^* - P_s),$$

where the first term reflects the value of perpetual continuation and the second one captures the possibility of a regime switch in the event of a run.[26]

The pre–run value of debt. Let us now turn our attention to the value of the long–term, risky debt. The long–term creditors anticipate the impact of the rollover risk on the bank's optimal liquidity–management policy. If a run takes place when the bank's level of liquid reserves is c and the bank survives, the long–term debt's market value changes to $D_0(c - P_s)$, where D_0 is as defined in Expression (9). Alternatively, if the run by the repo creditors pushes the bank into liquidation, the long–term creditors will collect the value $\tilde{d}_1(c)$, which depends on the relative seniority between the risky debt and the insured deposits. Under these considerations, the market value of debt

[26]Note that the effective discount rate $\rho + \lambda$ is increasing with the probability of a run.

satisfies the following system:

$$\begin{aligned} \mathcal{L}D_1(c) + r_l P_l - \lambda[D_1(c) - \tilde{d}_1(c)] &= 0, \quad c \in (\underline{c}, \underline{c} + P_s), \\ \mathcal{L}D_1(c) + r_l P_l - \lambda[D_1(c) - D_0(c - P_s)] &= 0, \quad c \in (\underline{c} + P_s, b_1^*). \end{aligned}$$

We solve the above system with the corresponding boundary conditions in Appendix B.2, allowing for the two possible scenarios of the respective seniority between the long–term debt and the insured deposits.

The pre–run payout barrier and the interest rate on long–term debt. As a last step, we determine, for a given debt structure (P_s, P_l, P_d), the equilibrium interest rate on long–term debt r_l^* and the optimal dividend barrier b_1^*. From the first–order conditions of the problem of maximizing the ex–ante value of the bank stated in Expression (3) we have the following result:

Lemma 1. *The optimal initial level of liquid reserves satisfies* $c_0 = b_1^*$.

In words, Lemma 1 establishes that the value–maximizing actions of the shareholders lead them to start the bank with exactly the target level of liquid reserves. Although simple in essence, this result is crucial, since will allow us to establish an explicit link between the optimal payout policy and the financing structure.[27] In other words, it is optimal to establish the bank with the maximum level of liquid reserves so as to reduce the ex–ante likelihood of liquidation and, thereby, the long–term debt's cost. Therefore, for a given debt structure (P_s, P_l, P_d), the equilibrium interest rate on long–term debt r_l^* and the optimal dividend barrier b_1^* are jointly determined by the system of equations[28]

$$D_1(b_1^*) = P_l \quad \text{and} \quad U_1(\underline{c}; b_1^*) = \tilde{u}_1(\underline{c}). \tag{13}$$

By establishing the aforementioned link between the optimal payout policy and the financing structure, the above system is the vehicle via which the rollover risk pertaining to the repo financing affects the optimal financing and payout decisions.

It is a priori not clear whether or not the pre–run payout barrier b_1^* exceeds the payout barrier b_0^* that the shareholders would choose after the run of the repo creditors, should the bank survive it. On the one hand, after a run the bank is no longer exposed to the rollover risk, which reduces its precautionary motives for holding liquid reserves. This may suggest that it is optimal to reduce

[27]As it is, this property holds due to the absence of the proportional equity–issuance costs. If we allowed for equity issuance with proportional deadweight costs, it would be optimal to establish the bank with an initial level of liquid reserves $c_0 < b_1^*$.

[28]The caveat that the non–linear equation $D_1(b_1^*) = P_l$ may have multiple roots is taken into account when we search for numerical solutions in the next section. Unreported numerical results show that the value of the bank evaluated at the payout barrier is decreasing over the range of admissible values of the interest rate. Thus, r_l^* is always given by the smallest root of $D_1(b_1^*) = P_l$.

the target reserves level after the run. On the other hand, the bank becomes more profitable in expectation, since it no longer has to make payments to the repo creditors. This generates a higher franchise–value effect and may induce the bank to hold more liquid reserves. Which of these two effects dominates will become apparent from the numerical analysis conducted in Section 4.

Applying the results of our analysis to the shareholders' optimization Problem (3), one can easily see that the latter reduces to the choice of the optimal financing structure:

$$\max_{(P_s,P_l)\in\mathcal{S}(b_1^*)} \quad V_1^*(P_s,P_l) = \Big\{U_1(b_1^*) - b_1^* - (A - P_l - P_s - P_d)\Big\},$$

where $\mathcal{S}(b_1^*)$ corresponds to the restriction of the strategy set $\mathcal{S}$ when the payout strategy is of barrier type at the level b_1^*.

In the sequel we solve the shareholders' optimization problem numerically, using the following parameter values: the discount rate $\rho = 5\%$, the cost of insured deposits $r_d = \{2.5\%, 4.5\%\}$, the cost of repo funding $r_s = 2.5\%$, the expected return on risky assets $\mu = \{20\%, 25\%, 30\%\}$, the volatility of the pre–tax earnings $\sigma = 18\%$, the tax rate $\theta = 35\%$, the intensity of the repo–funding withdrawal $\lambda = \{0.03, 0.05, 0.07\}$, the proportion of assets that can serve as collateral $\eta = 0.3$ and the book asset value equal to the first–best value $A \equiv (1-\theta)\mu/\rho$.[29]

4. Model Analysis

In order to better understand what drives the bank's choice between stable but costlier long–term debt and unstable but relatively cheap repo funding, we first solve the shareholders' problem for a hypothetical bank that has no access to insured deposits. Next, we introduce deposit financing and investigate the impact of the relative priority between the insured deposits and risky debt on the bank's liability structure.

4.1. No access to deposit funding

On the first stage of our numerical analysis, we disregard the possibility of deposit financing and solve the shareholders' problem for $P_d = 0$. To illustrate the impact of the repo funding, we also evaluate the optimal financing and payout policies in the setting in which the bank has no access to

[29] The choice of parameters in our numerical exercise is partially motivated by empirical evidence and partially done so as to preserve an interior solution to the problem of finding the optimal level of repo funding. In particular, the value of r_s is chosen close to the average for the 1–year LIBOR computed for the period 2000–2014. The value of η is chosen close to the estimations of Gropp and Heider (2010), reporting that a bank's collateral in average amounts to 27% of its assets. At the same time, the model turns out to be highly sensitive to the choice of the parameters λ and σ. Namely, the bank has no incentives to use any repo funding when σ is relatively low and/or when λ is relatively high.

it (we refer to this setting as *the benchmark*).[30] The optimal characteristics of the bank's financing and payout policies are reported in Table 2.

The first observation to be made in regard to the numerical results presented in Table 2 is that the presence of repo funding in the bank's liability structure impairs its ability to raise long–term debt. However, the use of repo funding increases both the overall leverage and the ex–ante value of the bank. Interestingly, banks with lower expected returns on risky assets benefit more from the value–increasing effect generated by the presence of repo funding in their financing structure. In particular, in our numerical example, the use of short–term, secured funding by the bank with $\mu = 20\%$ results in a 13% increase in the ex–ante value of the bank relative to the benchmark case, whereas for the bank with $\mu = 30\%$ the benefits of using repo funding are marginal and amount only to a 2% increase in the ex–ante bank value (this comparison is made for $\lambda = 0.03$).

An important fact to be noted is that the amount of repo funding optimally chosen by the bank in the absence of insured deposits is relatively low and the collateral constraint $P_s \leq \eta A$ is far from binding. Thus, contrary to the anecdotal and empirical evidence on banks' over–reliance on repo funding, our analysis shows that, when placed in the position of a non–financial firm, a bank would be better off abstaining from an aggressive use of repo funding. This suggests that the increased reliance on repo funding observed in practice might be partly rooted in the distortions induced by the access to (insured) deposit funding (we explore this conjecture below).

The reason why our hypothetical bank has no incentives to aggressively rely on cheap repo funding is that the use of this form of financing inflicts indirect costs on bank shareholders, which we refer to as *shadow costs*. First, the additional source of default risk inherent in repo financing increases the marginal value of liquid reserves within the bank and induces the shareholders to target a higher level of liquid reserves. This feature is illustrated in Table 2 by the positive wedge between the pre–run target level of liquid reserves b_1^* and the target level b_0^* that the bank would set if it were to survive the run. This wedge can be seen as a precautionary liquidity buffer needed to protect the bank against the rollover risk related to the use of repo funding.[31]

Yet, targeting very high levels of liquid reserves to offset the additional default risk brought about by the use of repo funding is costly (recall that liquid reserves are non-remunerated). As a result, the interest that long- term debt holders earn carries an extra premium as compensation for the negative externalities imposed on them by the bank's decision to use repos as a complementary source of funding. Thus, the use of cheap repo funding magnifies the cost of long–term debt funding.[32] This

[30]The values of equity and debt in this setting correspond to the functions U_0 and D_0 defined in Section 3.1, whereas the optimal payout barrier and the interest rate on long–term debt result from jointly solving the equations $D_0(b_0^*) = P_l$ and $U_0(b_0^*) = 0$.

[31]This finding resonates with the empirical evidence documented by Harford et al. (2014) for non–financial firms; namely, that firms tend to strengthen their liquid reserves in order to manage their refinancing risk better.

[32]A similar result is obtained by Auh and Sundaresan (2014) in a Leland–type structural model that allows for *endogenous* rollover risk.

Table 2: The optimal financing and payout policies in the absence of insured deposits

	$\lambda = 0.03$			$\lambda = 0.05$			$\lambda = 0.07$			Benchmark: $P_s = 0$		
	$\mu = 20\%$	$\mu = 25\%$	$\mu = 30\%$	$\mu = 20\%$	$\mu = 25\%$	$\mu = 30\%$	$\mu = 20\%$	$\mu = 25\%$	$\mu = 30\%$	$\mu = 20\%$	$\mu = 25\%$	$\mu = 30\%$
Bank value, $V_1^*(P_s, P_l)$	0.119	0.376	0.649	0.109	0.366	0.64	0.106	0.364	0.637	0.106	0.363	0.637
Increase in bank value due to repos	13.19%	3.59%	1.93%	3.67%	1%	0.54%	0.85%	0.23%	0.12%	-	-	-
Value of risky assets, A	2.6	3.25	3.9	2.6	3.25	3.9	2.6	3.25	3.9	2.6	3.25	3.9
Pleadgeable assets, ηA	0.78	0.98	1.17	0.78	0.98	1.17	0.78	0.98	1.17	0.78	0.98	1.17
Repos, P_s	0.159	0.145	0.135	0.087	0.08	0.074	0.042	0.039	0.036	-	-	-
Risky LT debt, P_l	1.946	2.728	3.544	1.999	2.775	3.589	2.029	2.803	3.614	2.05	2.83	3.63
Total debt, $P_s + P_l$	2.105	2.873	3.679	2.086	2.855	3.663	2.071	2.842	3.65	2.05	2.83	3.63
Repos/Total Debt	7.55%	5.05%	3.67%	4.17%	2.8%	2.02%	2.03%	1.37%	0.99%	-	-	-
Repos/Pledgeable assets	20.38%	14.87%	11.54%	11.15%	8.21%	6.32%	5.38%	4%	3.08%	-	-	-
LT interest rate, r_l^*	5.477%	5.388%	5.331%	5.467%	5.381%	5.326%	5.462%	5.379%	5.324%	5.457%	5.377%	5.32%
b_1^*	0.517	0.5113	0.5045	0.5063	0.5022	0.4962	0.4985	0.4951	0.4898	-	-	-
b_0^*	0.4892	0.485	0.4798	0.4899	0.4862	0.4813	0.4902	0.487	0.4822	0.4903	0.4875	0.4826
$(b_1^* - b_0^*)/b_0^*$	5.67%	5.43%	5.15%	3.36%	3.27%	3.08%	1.69%	1.68%	1.59%	-	-	-
Book Leverage	67.53%	76.38%	83.53%	67.15%	76.09%	83.32%	66.84%	75.88%	83.15%	66.34%	75.59%	82.83%
Repos/Total liabilities	5.1%	3.86%	3.07%	2.8%	2.13%	1.68%	1.36%	1.04%	0.82%	-	-	-
LT debt/Total liabilities	62.43%	72.53%	80.46%	64.35%	73.96%	81.64%	65.48%	74.84%	82.33%	66.34%	75.59%	82.83%
Equity/Total liabilities	32.47%	23.62%	16.47%	32.85%	23.91%	16.68%	33.16%	24.12%	16.85%	33.66%	24.41%	17.17%

Notes: This table reports the optimal financing and payout decisions of a bank that has no access to insured deposits. The last three columns characterize the bank's optimal policies in the setting in which the bank has no access to repo funding (benchmark). The optimal payout barrier and the interest rate on long–term debt in the benchmark case are obtained by jointly solving the equations $D_0(b_0^*) = P_l$ and $U_0(b_0^*) = 0$.

effect is illustrated on the left–hand side panel of Figure 1, which reports the difference between the equilibrium interest rate on long–term debt obtained in the setting in which the bank has access to repo funding and the equilibrium interest rate obtained in the benchmark case:

$$r_l^*(P_s, P_l) - r_l^*(P_l). \tag{14}$$

The above difference will be further referred to as the *extra premium* related to the presence of repos in the bank's financing structure. Overall, it is this potential increase in the cost of long–term debt funding combined with the cost of maintaining an extra cushion of liquid reserves what curbs the banks' appetite for "cheap" repo funding.

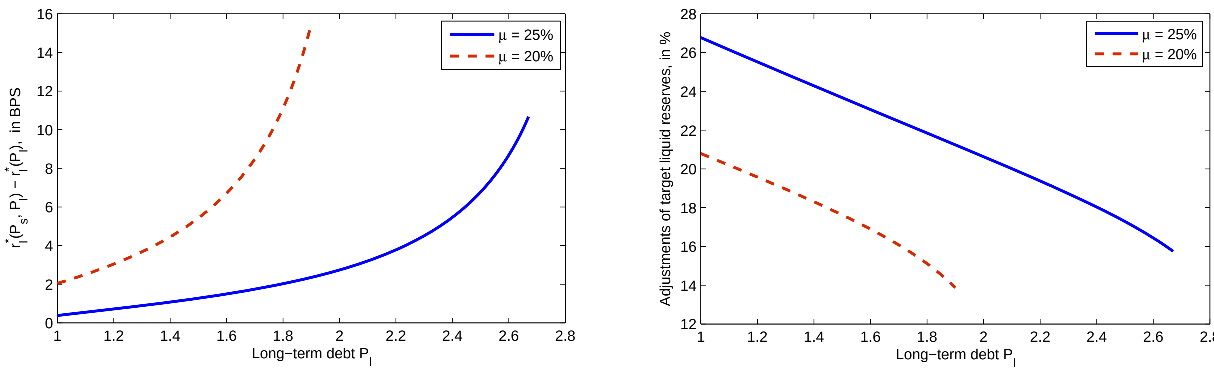

Figure 1: The impact of repos on the cost of long-term debt and payout policy

Notes: This figure illustrates the extra–premium component in the long–term debt spreads related to the use of repos (the left–hand side panel, in bps) and the scale of the upward adjustments in the target levels of liquid reserves measured relative to the benchmark levels (the right–hand side panel, in %) for two levels of the expected return μ on risky assets. For this numerical example we have taken $P_s = 0.25$ and $\lambda = 0.03$.

Note that the difference in the magnitude of the extra premium inflicted by the use of repos for banks with different levels of expected returns on risky assets μ can be easily understood by taking into account the effect of μ on the optimal payout policy. This effect is illustrated in the right–hand panel of Figure 1, which depicts the upward adjustments that a bank would make to its target level of liquid reserves relative to the levels it would set when making no use of repos. These adjustments are computed according to the following formula:

$$\frac{b_1^*(P_s, P_l) - b_0^*(P_l)}{b_0^*(P_l)} \times 100\%. \tag{15}$$

It turns out that, for the same debt structure (P_s, P_l), a bank enjoying a higher expected return on risky assets would be able to make a larger upward adjustment of its target level of liquid reserves, thereby being able to offset a larger fraction of the additional liquidation risk related to the use of repo funding. As a result, the interest rate on long–term debt issued by the banks with higher μ's exhibits a lower extra premium related to the use of repos.

Result 1. *The use of repos generates an additional component in the spreads of the long–term risky debt, which decreases with the bank expected returns on risky assets.*

The comparative statics results reported in Table 2 suggest that expected returns on risky assets are an important driver of the debt–structure choice. In particular, a bank with a lower return on assets μ exhibits a higher ratio of repo funding to total debt. There are two main effects behind this result. First, there is a direct franchise–value effect implying that the shareholders of a bank with a higher expected return on risky assets are more averse to the risk of liquidation than shareholders of a bank with a lower expected return, which induces them to use lower amounts of repos.[33] Second, there is an indirect effect of μ working via the channel of interaction between the optimal liquidity management and the financing costs. It stems from the fact that banks with lower μ face a higher wedge between the costs of repos and long–term debt financing. Indeed, lower expected returns imply higher liquidation risks (because of a lower speed of accumulation of earnings after a series of negative shocks) and, thus, higher costs of long–term debt financing. This initial cost disadvantage of the long–term debt for the banks with lower μ's is reinforced by the shadow–costs effect. As it was pointed out above, lower expected returns on risky assets undermine the bank's ability to maintain strong liquidity buffers to hedge against the rollover risk inflicted by repos. The higher extra premium that the banks with lower μ's have to pay when raising the same amount of long–term debt as the banks with higher μ's exacerbates the relative cost disadvantage of long–term debt even further, which pushes the banks with lower μ's to rely more on repo funding.

Result 2. *Repo funding plays a more prominent role in the financing structure of banks with lower returns on their risky assets.*

The final remark to be made with respect to numerical results reported in Table 2 is that both the interest rate on long–term debt and the target level of liquid reserves that the bank will fix prior to the run are *decreasing* in the intensity λ of the repo creditors' runs. This counterintuitive result is due to the fact that, when estimating the rollover risk to be low, the bank would be tempted to use a larger volume of repo funding. This would require setting a higher target level of liquid reserves in order to alleviate the consequent increase in the liquidation risk.[34] Yet, a larger volume of repo funding also reduces the expected residual value accruing to the long–term debt creditors in the case of a liquidation. This adverse effect of repo financing cannot be completely offset via an adjustment of the optimal payout policy (since maintaining high liquidity buffers is costly) and

[33] We observe this result in an unreported scenario of our numerical analysis, where we abstract from the possibility of issuing long–term risky debt and consider the optimal choice of the amount of repos for different levels of μ.

[34] The adjustment of the target level of liquid reserves, actually, explains why the quantitative effect of the run intensity on the cost of long–term debt is marginal.

will translate into higher costs of long–term debt. This positive relation between the long–term debt's cost and the relative importance of repos that emerged from our numerical analysis echoes the empirical evidence documented by Valenzuela (2013) and Gopalan et al. (2013) for non–financial firms.

4.2. Deposit funding and the depositor preference rule

Having considered the optimal financing and payout decisions of a bank without access to insured deposits, we now explore how the access to deposit funding affects the choice of the non–deposit sources of funding. We distinguish between two scenarios of priority rules. In the first scenario, deposits are junior to long–term debt. The second scenario mirrors the U.S. depositor preference law introduced on a national level in 1993, according to which domestic deposits are senior to all other debt claims.[35]

We solve the shareholders' optimization problem for the volume of insured deposits $P_d = 2$, which accounts for $50 - 75\%$ of total assets in our numerical example (conditional on the values of μ in the different scenarios), and report the results in Table 3. A comparison of the bank's optimal financing and payout decisions with those obtained in the setting with no deposit funding reveals two interesting observations.

First, it turns out that the access to deposit funding increases the relative importance of repos, which is reflected in the ratios $P_s^*/(P_s^* + P_l^*)$ and $P_s^*/(A + b_1^*)$. To gain intuition on this result, note that, when the bank (for some reason not modeled here) gives priority to deposit funding and then decides on the complementary sources of funding, the interest rate paid on deposits only affects the *effective* rate of return on risky assets. In other words, the qualitative effect of the access to deposit funding is equivalent to the effect of lowering μ, which was discussed in Section 4.1. Indeed, as shown in Table 3, banks with access to deposits funding tend to increase their reliance on repo funding.[36]

Second, making insured deposits senior to long–term debt renders the banks even more reliant on repo funding. The reason is that the depositor preference rule reduces the amount accruing to the long–term creditors in the case of liquidation, which translates into higher costs of long–term debt and, thus, exacerbates the relative cost advantage of repo funding.

Result 3. *The access to deposit funding and the depositor seniority rule exacerbate the bank's reliance on repo funding.*

Notice, however, that the quantitative impacts of deposit funding and deposit seniority rules on the optimal volume of repo funding remain relatively modest for most combinations of input

[35]The introduction of a similar law for European countries is currently being considered by the European authorities.

[36]To show that this result is not driven by the difference in the cost of repos and deposits, in Table 3 we report the bank's financing and payout policies for the particular case when the interest rates on insured deposits and repos coincide, i.e. $r_d = r_s = 2.5\%$.

parameters. In other words, claiming that the access to deposit funding is the only major factor that pushes a bank to *aggressively* substitute long–term debt by repo funding would be inappropriate. The reason being that the incentives to increase the reliance on repo funding are still mitigated by the potential shadow costs.

5. The Impact of Regulation

So far, our analysis has been focused on the management decisions of an unregulated bank. In reality, however, banks face regulatory requirements. In this section, we try to understand how various regulatory measures affect the bank's financing structure and, in particular, the relative importance of repos among its liabilities.

5.1. Liquidity regulation

We start by exploring the effect of liquidity regulation on the optimal payout and financing policies of the bank. Given that the (over)reliance of banks on repo funding is commonly recognized as a threat to financial instability, one of the objectives of the new liquidity regulation introduced in Basel III is to reduce the use of repo funding by banks (see e.g., IMF Global Financial Stability Report (2013), Chapter 3). We introduce liquidity regulation in the spirit of the Basel III Liquidity Coverage Ratio (LCR). This regulatory measure stipulates that banks must maintain a level of highly–liquid assets in a certain proportion to the volume of fund withdrawals expected in the next 30 days. In our setting, this idea is captured in simple fashion by assuming that the level of the bank's liquid reserves must always exceed a certain fraction of its repo funding. Hence, liquidity regulation in our model establishes a link between the volume of repo funding and the liquidation threshold:[37]

$$\underline{c}^*(\Lambda) = \Lambda P_s,$$

where $\Lambda \geq 0$ is a regulatory parameter reflecting the tightness of the liquidity requirements.

When solving their optimization problem under such a regulatory constraint, the bank's shareholders must take into account the feedback effect that their choice of P_s will have on the liquidation and payout policies. Intuitively, one would expect that tighter liquidity requirements should curb the bank's appetite for repo funding and should reduce its exposure to the run–triggered liquidation risk. To verify this conjecture, we turn to a numerical analysis.

The left–hand panel of Figure 2 depicts the proportions of repos and long–term debt in the bank's total financing structure as functions of Λ. Indeed, when faced with tighter liquidity requirements, the bank reduces its reliance on repo funding and can even be completely discouraged from its use. Simultaneously, the bank increases its reliance on long–term debt funding. In other words, imposing

[37] A similar approach to modeling liquidity requirements is taken by Adrian and Boyarchenko (2013).

Table 3: The impact of deposit funding and seniority rules

			P_s^*	P_l^*	$(P_s^*+P_l^*)$	$\frac{P_s^*}{P_s^*+P_l^*}$	b_1^*	b_0^*	r_l	MLev	BLev	V_1^*	$(1-\theta)(\mu-f_1)$	$\frac{P_s^*}{A+b_1^*}$
		$P_d=0$	*0.087*	*1.999*	*2.086*	*4.17%*	*0.5063*	*0.4899*	*5.47%*	*0.65*	*0.67*	*0.109*	*5.76%*	*2.80%*
$\mu=20\%$	$r_d=2.5\%$	$P_d=2$, junior	0.097	1.357	1.454	6.67%	0.5009	0.487	5.55%	0.79	1.11	1.267	4.70%	3.13%
		$P_d=2$, senior	0.123	1.066	1.189	10.34%	0.5175	0.4903	5.83%	0.75	1.02	1.145	5.51%	3.95%
	$r_d=4.5\%$	$P_d=2$, junior	0.101	0.956	1.057	9.56%	0.4608	0.4607	5.57%	0.82	1	0.666	3.52%	3.30%
		$P_d=2$, senior	0.15	0.563	0.713	21.04%	0.5159	0.487	6.29%	0.75	0.87	0.48	4.6%	4.81%
		$P_d=0$	*0.08*	*2.775*	*2.855*	*2.8%*	*0.5022*	*0.4862*	*5.38%*	*0.69*	*0.76*	*0.366*	*6.41%*	*2.13%*
$\mu=25\%$	$r_d=2.5\%$	$P_d=2$, junior	0.089	2.06	2.149	4.14%	0.5067	0.4903	5.44%	0.79	1.1	1.485	5.58%	2.37%
		$P_d=2$, senior	0.103	1.779	1.882	5.47%	0.5096	0.4863	5.59%	0.76	1.03	1.373	6.37%	2.76%
	$r_d=4.5\%$	$P_d=2$, junior	0.097	1.555	1.652	5.87%	0.501	0.4871	5.48%	0.8	0.97	0.816	4.70%	2.59%
		$P_d=2$, senior	0.12	1.2	1.32	9.09%	0.517	0.4899	5.77%	0.75	0.88	0.667	5.71%	3.19%
		$P_d=0$	*0.074*	*3.589*	*3.663*	*2.02%*	*0.4962*	*0.4813*	*5.33%*	*0.73*	*0.83*	*0.64*	*6.96%*	*1.68%*
$\mu=30\%$	$r_d=2.5\%$	$P_d=2$, junior	0.081	2.826	2.907	2.79%	0.5033	0.4873	5.36%	0.8	1.11	1.737	6.26%	1.85%
		$P_d=2$, senior	0.091	2.549	2.64	3.45%	0.5003	0.4801	5.47%	0.77	1.05	1.628	7.05%	2.08%
	$r_d=4.5\%$	$P_d=2$, junior	0.088	2.259	2.347	3.75%	0.5065	0.4903	5.40%	0.8	0.99	1.035	5.58%	2%
		$P_d=2$, senior	0.1	1.929	2.029	4.93%	0.5077	0.4851	5.56%	0.76	0.91	0.902	6.52%	2.27%

Notes: In this table we report the impact of the access to deposit funding and seniority rules on the bank's financing and payout policies for different combinations of μ and r_d. This set of results is generated for $\lambda = 0.05$. The columns *MLev* and *BLev* report the market and book leverages computed at the target level of liquid reserves b_1^*. The reported results show that: (i) the access to deposit funding and the depositor priority rule induce a higher reliance on repo funding (this feature is reflected in the ratios $P_s^*/(P_s^*+P_l^*)$ and $P_s^*/(A+b_1^*)$); (ii) the access to deposit funding exacerbates leverage, reduces the effective expected cash flows from risky assets $(1-\theta)(\mu-f_1)$ and increases the bank's value V_1^*; (iii) the implementation of the depositor seniority rule reduces leverage, increases the effective expected cash flows from risky assets $(1-\theta)(\mu-f_1)$ and reduces the bank's value V_1^*. The reduction in leverage induced by the implementation of the depositor preference rule is consistent with the numerical findings of Hugonnier and Morellec (2015) and the empirical evidence documented by Danisewicz et al. (2014).

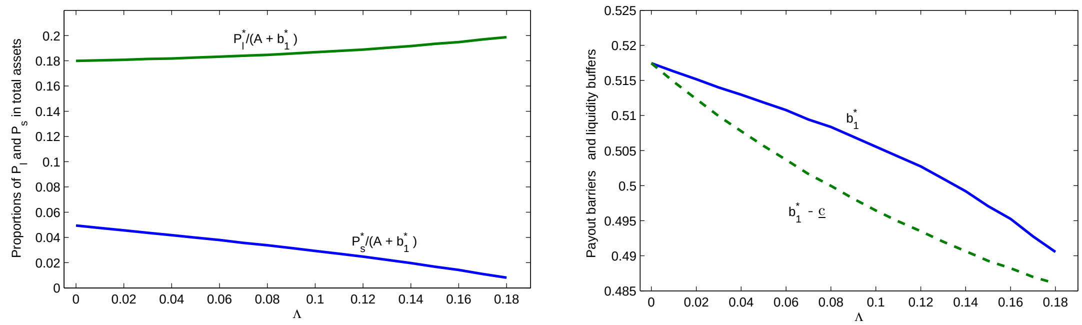

Figure 2: Liquidity requirements and bank policies

Notes: The left–hand side panel depicts the proportions of repos and long–term debt in the overall financing structure. Parameter values: $P_d = 2$, $\rho = 5\%$, $r_d = 4.5\%$, $r_s = 2.5\%$, $\mu = 20\%$, $\sigma = 18\%$, $\theta = 35\%$, $\eta = 0.3$ and $\lambda = 0.05$. Deposits are senior to long–term risky debt. In this example, the bank makes no use of repo funding for $\Lambda > 0.18$. The right–hand side panel reports the optimal payout barrier b_1^* (the solid line) and the magnitude of the target liquidity buffers $b_1^* - \underline{c}$ (the dashed line).

tighter liquidity regulation leads to substitution of repos by long–term debt funding, provided that the volume of insured deposits is fixed.

To understand the underlying mechanism at work, notice that, given any fixed P_s, the presence of liquidity requirements translates into a strictly positive liquidation threshold; hence, it increases the risk of liquidation. In principle, the bank could offset this adverse effect by making an upward adjustment of its target level of liquid reserves. Yet, due to the deadweight cost of holding liquidity, the bank would prefer to reduce its reliance on repos rather than to substantially strengthen its liquidity buffer. The reduction in the level of repo funding reduces the cost of long–term debt (via the extra–premium component), which eventually enables the bank to increase its reliance on long–term debt.

Result 4. *Tightening the liquidity requirements induces a substitution of repo funding by long–term debt funding.*

Given that tightening liquidity requirements generates a substitution effect, it is worthwhile to consider the impact of liquidity requirements on the bank's liquidation probability. In Appendix C we describe how we compute the latter, allowing for the possibility of a run by the repo creditors. We apply this methodology to evaluate the relative change in the liquidation probability caused by the implementation of liquidity regulation, which is computed according to the following formula:

$$\Delta p(T, c; \Lambda) = \frac{p(T, c; \Lambda) - p(T, c; 0)}{p(T, c; 0)},$$

where $p(T, c; \Lambda)$ is the probability of liquidation over a period of length T when the current level of liquid reserves is c and the regulatory level is Λ (see Appendix C).

We set $T = 1$ (1 year) and compute the relative changes in the liquidation probability evaluated at the target level of liquid reserves, i.e. $\Delta p(T, b_1^*(\Lambda); \Lambda)$. The numerical results reported in Figure 3 show that tighter liquidity regulation, in fact, reduces the liquidation probability despite the debt substitution effect discussed above. Moreover, this effect turns out to be more pronounced for those banks that have lower returns on risky assets.

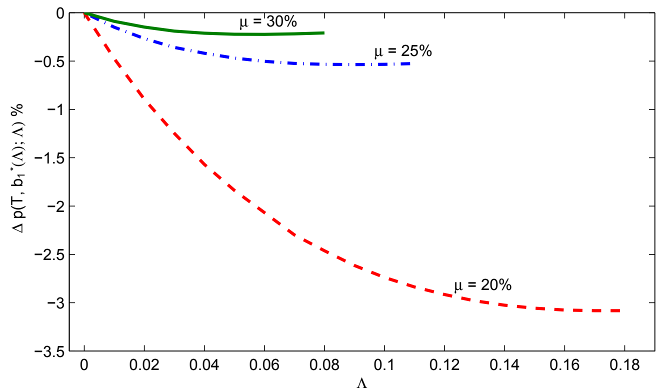

Figure 3: Relative changes in the bank's liquidation probability

Notes: This figure depicts the relative changes in the liquidation probability computed at the target level of liquid reserves for a 1–year time horizon. Parameter values: $P_d = 2$, $\rho = 5\%$, $r_d = 4.5\%$, $r_s = 2.5\%$, $\sigma = 18\%$, $\theta = 35\%$, $\eta = 0.3$ and $\lambda = 0.05$. Deposits are senior to long–term debt.

5.2. Payout restrictions

One more novel feature of the Basel III regulation was the introduction of payout restrictions on insufficiently capitalized banks.[38] A question of interest is how these payout restrictions affect the banks' ex–ante choices of financing structure and, in particular, the relative importance of repo financing. To explore this question, we study below the case where the bank's shareholders are prohibited from distributing dividends when the book value of bank equity falls below a certain critical level k_{div}, i.e. when

$$\frac{A + C^{\pi}(t) - (P_s \mathbb{1}_{\{t \leq \tau^*\}} + P_l + P_d)}{A + C^{\pi}(t)} < k_{div}. \tag{16}$$

For a given liability structure (P_s, P_l, P_d), Condition (16) can be rewritten in terms of a critical level of liquid reserves below which dividend distribution is forbidden. Specifically, the regulatory payout threshold before the run is given by

$$b_1^{reg}(P_s, P_l, P_d) = \max\Big\{\frac{P_s + P_l + P_d}{1 - k_{div}} - A, 0\Big\}, \tag{17}$$

whereas the regulatory payout threshold after the run is

[38] Specifically, payout restrictions apply to the banks that fail to meet the capital conservation buffer requirement (the requirement to maintain 2.5% of common Tier equity capital on top of the minimum capital requirements).

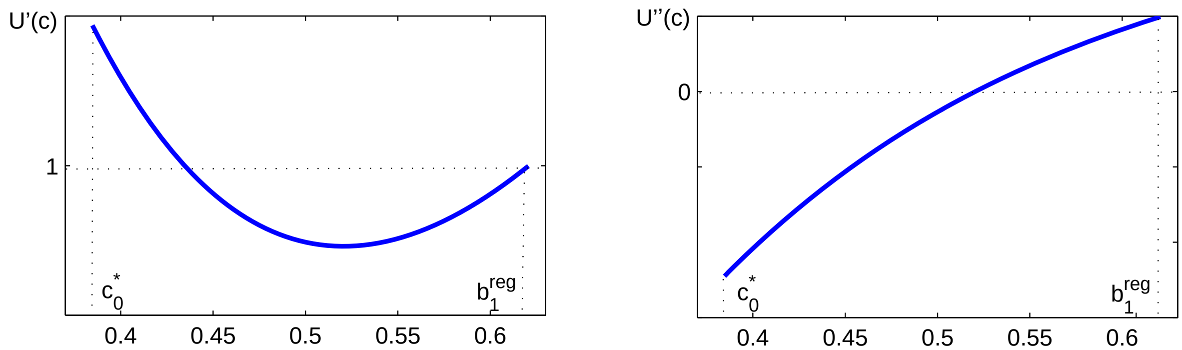

Figure 4: The impact of payout restrictions on the marginal value of c

$$b_0^{reg}(P_l, P_d) = \max\left\{\frac{P_l + P_d}{1 - k_{div}} - A, 0\right\}. \tag{18}$$

To understand how the payout restrictions affect a bank's financing policy, let us fix an arbitrary volume P_s of repos and consider the shareholders' optimization problem, which now consists of choosing the initial level of liquid reserves c_0, the volume of long–term debt P_l and the optimal payout policy characterized by the pair of payout thresholds (b_0, b_1).

The presence of the payout restrictions (b_0^{reg}, b_1^{reg})[39] has no impact on the bank's payout policy and its choice of the initial level of liquid reserves if and only if $b_1^{reg} < b_1^*$ and $b_0^{reg} < b_0^*$. In all the alternative cases the bank's payout policy will be affected by regulation. The situation when $b_0^{reg} > b_0^*$ affects the ex–ante value of the bank's securities and, thus, the choice of the pre–run payout barrier b_1^*, which will not coincide with the unconstrained, pre–run target level of liquid reserves. Regardless of whether the post–run regulation is binding or not, if $b_1^{reg} > b_1^*$, the bank is forced to abstain from distributing dividends when its liquid reserves are below b_1^{reg}. This reduces the value of the shareholders' equity and affects the marginal value of liquid reserves. Recall that, without payout restrictions, the latter was monotonically decreasing in the level of c, i.e. $U_1''(c) < 0$ over $(0, b_1)$, and $U_1''(b_1) = 0$. This is, however, no longer the case when the payout barrier is set to $b_1^{reg} > b_1^*$. In Figure 4 we plot the typical patterns of the marginal value of liquid reserves U_1' and its first derivative in the neighborhood of mandatory payout barrier $b_1^{reg} > b_1^*$. This shows that, in a neighborhood of b_1^{reg}, the equity value U_1 becomes *locally convex* and there exists a certain level of liquid reserves $c_0^* < b_1^{reg}$, such that $U_1'(c_0^*) = 1$ and $U_1''(c_0^*) < 0$.

It is easy to see from the shareholder maximization Problem (3) that, for a fixed liability structure, the value c_0^* determined via the relation $U_1'(c_0^*) = 1$ represents the optimal level of liquid reserves at which shareholders will choose to initiate the bank. As we have seen in Section 3, in the

[39]For the sake of space we omit arguments of b_1^{reg}, b_0^{reg}, b_1^*, b_0^* in the text.

absence of payout restrictions, c_0^* coincides with the target level of liquid reserves b_1^*. However, the payout restrictions reduce the expected value of dividends; hence, shareholders make lower initial equity contributions compared to the unregulated case, i.e. $c_0^* < b_1^*$. A discussion on the mechanism at work can be found in Appendix B.3.

Result 5. *When $b_1^{reg} > b_1^*$, the shareholders choose to initiate the bank at a level of liquid reserves $c_0^* < b_1^*$.*

Consider now the impact of payout restrictions on the long–term debt's cost, which is determined by the condition $D_1(c_0^*) = P_l$. The presence of the payout restrictions generates two counteracting effects. On the one hand, a lower initial capitalization (as compared to the unregulated case) increases the bank's liquidation probability for short time horizons, which in turn reduces the market value of debt. On the other hand, the probability of liquidation for long time horizons decreases, since the bank will be forced to maintain a larger liquidity buffer. Numerical simulations show that the latter effect dominates when the face value of the long–term debt is not too high. In such a case, for a given level of P_s, the bank will choose a *higher* level of long–term debt than in the unregulated case, which will in turn *increase* its ex–ante value.

Even though the value–increasing effect generated by the payout restrictions may seem counterintuitive at a first glance, it admits a very natural explanation. In fact, in the absence of the payout restrictions, shareholders do worse due to a commitment problem. Indeed, even though delaying dividend payments, i.e. setting a higher payout threshold, may be optimal from an ex–ante perspective because it reduces the cost of debt financing, thus increasing the bank's value, once the long–term debt is issued, the shareholders will have incentives to switch to the payout policy that maximizes their dividend payoffs and implies a higher liquidation risk. Rational creditors will anticipate this behavior and will demand a higher interest rate on their debt. In contrast, the payout restrictions imposed by the regulator work as a credible commitment mechanism that helps the bank reduce its cost of long–term debt financing.

The reduction in the cost of long–term debt financing generated by the commitment effect brought about by the implementation of payout restrictions in turn reduces the cost advantage of repos, thereby inducing the bank to substitute repo funding by long–term debt one. Table 4 illustrates the effects of the payout restrictions on the bank's financing and payout policies: one can observe that the implementation of payout restrictions reduces both the proportion of repos and the cost of long–term debt due to the aforementioned commitment effect.

5.3. Capital regulation

Finally, we turn to the analysis of the effect of capital regulation on the ex–ante choice of the bank's optimal strategies. Capital regulation in our model takes the form of restrictions on the (book) leverage ratio:

$$\frac{A + C^\pi(t) - (P_s \mathbb{1}_{\{t \le \tau^*\}} + P_l + P_d)}{A + C^\pi(t)} \ge k_{lev}.$$

Table 4: The impact of payout restrictions on bank policies

	k_{div}	P_s^*	P_l^*	c_0^*	b_1	b_0	r_l	MLev	BLev	V_1^*	$\frac{P_s^*}{A+b_1^*}$
$\mu = 20\%$	–	*0.229*	*0.524*	*0.5246*	*0.5246*	*0.4882*	*6.36%*	*0.76*	*0.88*	*0.50*	*7.33%*
	0.13	0.214	0.590	0.4339	0.6230	0.4872	5.94%	0.79	0.87	0.53	6.64%
	0.15	0.200	0.539	0.4396	0.6224	0.4891	5.81%	0.77	0.85	0.52	6.21%
	0.17	0.187	0.484	0.4441	0.6181	0.4901	5.71%	0.75	0.83	0.51	5.81%
	0.19	0.177	0.423	0.4498	0.6099	0.4904	5.63%	0.73	0.81	0.49	5.51%
$\mu = 25\%$	–	*0.183*	*1.167*	*0.5247*	*0.5247*	*0.4895*	*5.79%*	*0.75*	*0.89*	*0.68*	*4.85%*
	0.13	0.173	1.200	0.4435	0.6270	0.4889	5.47%	0.76	0.87	0.72	4.46%
	0.15	0.163	1.126	0.4432	0.6194	0.4870	5.41%	0.75	0.85	0.70	4.21%
	0.17	0.153	1.048	0.4441	0.6066	0.4846	5.37%	0.73	0.83	0.69	3.97%
	0.19	0.145	0.971	0.4432	0.5969	0.4817	5.33%	0.71	0.81	0.67	3.77%

Notes: This table illustrates the impact of payout restrictions on the bank's strategies. The columns $MLev$ and $BLev$ report the market and book leverages computed at the target level of liquid reserves b_1^*. The first line in each panel corresponds to the unregulated set–up. Parameter values: $\rho = 5\%$, $r_d = 4.5\%$, $r = 2.5\%$, $\lambda = 0.03$, $\sigma = 18\%$, $\theta = 35\%$, $P_d = 2$ and $\eta = 0.3$. Deposits are senior to long-term risky debt. Note that, when k_{div} increases, the bank reduces leverage so as to avoid targeting a very high level of liquidity reserves. The reduction in leverage in turn drives down the ex–ante equity value V_1^*.

The above constraint can be rewritten in terms of a level of liquid reserves at which the bank will be subject to mandatory liquidation:[40]

$$C^\pi(t) \geq \max\left\{\frac{P_s \mathbb{1}_{\{t \leq \tau^*\}} + P_l + P_d}{1 - k_{lev}} - A, 0\right\} =: \underline{c}(k_{lev}). \tag{19}$$

Again, we solve the shareholders' problem numerically by taking into account the regulatory Constraint (19). The results of our numerical analysis, reported in Table 5, show that tighter capital requirements induce the bank to target a lower leverage ratio by making cuts on *both* repos and long–term debt. This suggests that introducing special liquidity requirements to curb the bank's reliance on repos might be redundant, since the desired effect can be achieved via capital requirements alone.

It is also worthwhile to notice that, when faced with a tighter leverage ratio, a bank will adjust its financing structure so as to avoid regulatory liquidation at a strictly positive level of liquid reserves, i.e. $\underline{c}(k_{lev}) = 0$ for any level of k_{lev}. Under the assumption that hoarding liquidity entails deadweight costs, this result is very robust to changes in the parameters. In fact, any change in the level of debt that would raise the bank's liquidation threshold from zero to a strictly positive level, would induce the bank to increase its target level of liquid reserves by the same amount (see e.g., Hugonnier and Morellec (2015)). As long as the marginal cost of increasing the liquid reserves

[40]Notice that, before the run, the bank faces a higher liquidation threshold.

Table 5: The impact of leverage regulation on bank policies

	k_{lev}	P_s	P_l	$\underline{c}(k_{lev})$	b_1	b_0	r_l	MLev	BLev	V_1^*	$\frac{P_s^*}{A+b_1^*}$
$\mu = 20\%$	–	*0.229*	*0.524*	*0*	*0.5246*	*0.4882*	*6.29%*	*0.76*	*0.88*	*0.50*	*7.33%*
	0.05	0.069	0.401	0	0.5014	0.4904	5.82%	0.69	0.80	0.46	2.22%
	0.10	0.049	0.291	0	0.4968	0.4892	5.67%	0.66	0.76	0.43	1.58%
	0.15	0.035	0.175	0	0.4917	0.4865	5.56%	0.63	0.71	0.40	1.13%
	0.20	0.025	0.055	0	0.4862	0.4826	5.48%	0.60	0.67	0.37	0.81%
$\mu = 25\%$	–	*0.183*	*1.167*	*0*	*0.5247*	*0.4895*	*5.77%*	*0.75*	*0.89*	*0.68*	*4.85%*
	0.05	0.060	1.027	0	0.4957	0.4849	5.54%	0.70	0.82	0.65	1.60%
	0.10	0.041	0.884	0	0.4859	0.4792	5.44%	0.67	0.78	0.62	1.10%
	0.15	0.029	0.733	0	0.4768	0.4723	5.36%	0.64	0.74	0.59	0.78%
	0.20	0.021	0.579	0	0.4679	0.4648	5.31%	0.61	0.70	0.55	0.56%

Notes: This table illustrates the impact of leverage regulation on the bank's policies. The columns $MLev$ and $BLev$ report the market and book leverages computed at the target level of liquid reserves b_1^*. The first line in each panel corresponds to the unregulated set-up.Parameter values: $\rho = 5\%$, $r_d = 4.5\%$, $r = 2.5\%$, $\lambda = 0.03$, $\sigma = 18\%$, $\theta = 35\%$, $P_d = 2$ and $\eta = 0.3$.

exceeds the forgone tax benefits resulting from a reduction of debt, the bank prefers to reduce its levels of debt so as to avoid being liquidated at a strictly positive level of liquid reserves. This is in fact the channel through which capital regulation curbs the bank's appetite for a higher leverage in our setting. In other words, the interplay between the liquidation and the payout policies works as a transmission channel of the disciplining effect of capital regulation.

6. Conclusions

We have developed a continuous–time structural model of banking to understand what drives a bank's financing choices between repos, which carry with them rollover risk, and risky, long–term debt, which is a stable source of funding. The crucial feature of the model is that the bank's liquid reserves serve as a buffer against the losses caused by a run of the repo creditors. As a result, the bank's ability to build larger liquid reserves has direct implications on its choices of debt structure.

We have shown that the bank internalizes the rollover risk inherent in the use of repos via (i) an additional component in long–term debt spreads and (ii) the deadweight costs of maintaining an extra cushion of liquid reserves needed to mitigate the increase in the default risk. Taken together, these indirect ("shadow") costs prevent the bank from relying too heavily on cheap repo funding. Our analysis also shows that banks with higher returns on risky assets exhibit a lower proportion of repos in their financing structure. In contrast, banks with lower returns on risky assets are more reliant on repos. This model prediction is backed by two effects. First, there is a direct franchise–value effect in line with the standard risk–taking arguments, which induces banks with lower expected return on risky assets to take a higher tail risk on the liability side of its balance sheet. Second, there is a more subtle effect stemming from the fact that banks with lower returns on risky assets cannot afford to hold substantial amounts of liquid reserves and, as a consequence, face higher costs of long–term debt, which push them to substitute long–term debt financing with repos. Allowing banks to have access to insured deposits leads to the qualitatively similar implication: a large volume of insured deposit in the bank's financing structure reduces its effective earnings and weakens its capacity to maintain high levels of liquid reserves, thereby inducing the bank to increase its reliance on repos. Moreover, this effect becomes even more pronounced when insured deposits are senior to long–term debt.

We also examine the effect of regulation on a bank's ex–ante choice of financing structure, by considering three regulatory tools: liquidity regulation in the spirit of the Basel III Liquidity Coverage Ratio, payout restrictions and a capital ratio. All in all, we find that all of these tools are capable of curbing the bank's appetite for repos. Under liquidity regulation, which requires the bank to maintain a minimum level of liquid reserves as a certain proportion of its volume of repos, the bank substitutes short–term debt funding by long–term debt one. Payout restrictions induce a similar substitution effect, as the bank operates with higher target levels of liquid reserves and, thus, faces a lower liquidation risk. Capital regulation, however, induces the bank to lower the volumes of both repos and long–term debt, which suggests that developing special liquidity regulation in order to reduce the banks' reliance on repos might be redundant.

It should be acknowledged that our model's versatility, which allowed us to consider debt of different maturities and seniorities, together with insured deposits, does have as a downside: a large part of our analysis has to be done numerically. Whenever possible, we have provided the mathematical reasoning behind the model's features but, unfortunately, this could not be done in all situations. A clear avenue for future research would be the development of a fully dynamic structure of a bank's balance sheet, which is only partially the case in the current work. A related line of inquiry would be to address how bank liability and asset structures adjust following the changes in the liquidation value of risky assets caused by changes in macroeconomic conditions.

7. Appendix

Appendix A Proofs

A.1 ***Proof of Theorems 1 - 2***

In order to ease the exposition we mostly work with the generic function U_i, $i = 0, 1$, since both proofs share most of the methodology. Whenever required, we indicate where is it that the cases $i = 0$ and $i = 1$ diverge. Given that the proofs are quite long, we have chosen to label each step separately. We first show how the Hamilton–Jacobi–Bellman equations for the functions U_0 and U_1, respectively introduced in Sections 3.1 and 3.2, are obtained. We assume these functions to be twice continuously differentiable. We also prove that the mapping $c \mapsto U_i(c)$ is concave. In order to simplify the exposition we use the following notation:

$$f_0 := r_d P_d + r_l P_l, \quad f_1 := r_d P_d + r_l P_l + r_s P_s,$$

$$dC_0^L(t) := (1-\theta)(\mu - f_0)dt + (1-\theta)\sigma dW(t) - dL(t)$$

and

$$dC_1^L(t) := (1-\theta)(\mu - f_1)dt + (1-\theta)\sigma dW(t) - dL(t) - \mathbb{1}_{\{t \leq \tau^*\}} P_s dN(t).$$

Expressing $U_1(c)$ ***using the dynamic–programming principle.*** We present below the expanded representation of $U_1(c)$. To this end, recall that $\tilde{u}_1(c)$ represents the value accruing to shareholders in the case that liquidation occurs before or due to the run, whereas $\tilde{u}_0(c)$ is the shareholders' liquidation value when the default event is posterior to the repo run. We then have, following the dynamic–programming principle, that

$$\begin{aligned} U_1(c) = \sup_L \quad & \mathbb{E}_0\Bigg[\int_0^{\tau_\pi \wedge \tau^*} e^{-\rho t} dL(t) + e^{-\rho \tau_\pi} \mathbb{1}_{\{\tau_\pi < \tau^*\}} \tilde{u}_1\big(\underline{c}\big) \\ & + e^{-\rho \tau^*}\Big[\mathbb{1}_{\{\tau^* = \tau_\pi\}} \tilde{u}_1\big(C_1^L(\tau^*)\big) + \mathbb{1}_{\{\tau^* < \tau_\pi\}} U_0\big(C_1^L(\tau^*) - P_s\big)\Big]\Bigg], \end{aligned}$$

where

$$U_0(c) = \sup_L \quad \mathbb{E}_0\Bigg[\int_0^{\tau_\pi} dL(t) + e^{-\rho \tau_\pi} \tilde{u}_0(C_0^L(\tau_\pi))\Bigg]$$

is the post–run equity value function. Observe that the term $e^{-\rho \tau_\pi} \mathbb{1}_{\{\tau_\pi < \tau^*\}} \tilde{u}\big(\underline{c}\big)$ corresponds to the case where liquidation is caused by a gradual depletion of the bank's liquid reserves (via the Brownian risk) before a run by the repo creditors takes place, i.e. $C_1^L(\tau_\pi) = \underline{c}$. The term $e^{-\rho \tau^*} \mathbb{1}_{\{\tau^* = \tau_\pi\}} \tilde{u}_1\big(C_1^L(\tau^*)\big)$ is the present value of what accrues to the shareholders if the repo run results in liquidation. In this case $\underline{c} \leq C_1^L(\tau^*) \leq \underline{c} + P_s$. Finally, $e^{-\rho \tau^*} \mathbb{1}_{\{\tau^* < \tau_\pi\}} U_0\big(C_1^L(\tau^*) - P_s\big)$ is the discounted, post–run equity value should the bank survive the sudden withdrawal of funds by the repo creditors, which occurs if and only if $C_1^L(\tau^*) > \underline{c} + P_s$.

Concavity. Consider the reserves levels $c_1, c_2 > \underline{c}$ and let L_1 and L_2 be two corresponding admissible payout policies. Let $\lambda \in (0,1)$ and define

$$\tilde{c} := \lambda c_1 + (1-\lambda) c_2 \quad \text{and} \quad \tilde{L} := \lambda L_1 + (1-\lambda) L_2.$$

Clearly $\tilde{L}$ is admissible. Since $dC_i^{\tilde{L}} = \lambda dC_i^{L_1} + (1-\lambda) dC_i^{L_2}$ and the conditional–expectation operator is linear, we have

$$U_i(\tilde{c}) \geq U_i^{\tilde{L}}(\tilde{c}) = \lambda U_i^{L_1}(c_1) + (1-\lambda) U_i^{L_2}(c_2).$$

By definition, for all $\epsilon > 0$ the strategy L_1 can be chosen such that $U_i^{L_1}(c_1) \geq U_i(c_1) - \epsilon/2$, and analogously for $U_i(c_2)$. In other words, the expression

$$U_i(\tilde{c}) \geq \lambda U_i(c_1) + (1-\lambda) U_i(c_2) - \epsilon$$

holds for any positive ϵ; thus, the mapping $c \mapsto U_i(c)$ is concave.

Complementarity conditions. Next we consider the condition $1 \le U_i'$. By definition, for any $h, y > \underline{c}$ there exists a strategy L_y such that $U_i^{L_y}(y) \ge U_i(y) - h^2$. Let $\underline{c} < h < c$ and construct a strategy L by setting $L(t) = h + L^{c-h}(t)$, Then

$$U_i(c) \ge U_i^L(c) = h + U_i^{L_{c-h}}(c-h) \ge h + U_i(c-h) - h^2,$$

which is equivalent to

$$\frac{U_i(c) - U_i(c-h)}{h} \ge 1 - h.$$

By the differentiability of U_i, we may let h go to zero and conclude that $U_i'(c) \ge 1$ for all $c \ge \underline{c}$.

Differential characterizations. Below we show that for $c > \underline{c}$ it holds that

$$\mathcal{L}_0 U_0(c) := \mathcal{L} U_0(c) - (1-\theta) r_s P_s U_0'(c) \le 0$$

and

$$\mathcal{L}_1 U_1(c) := \mathcal{L} U_1(c) - \lambda\big[U_1(c) - U_0\big((c - P_s)_+\big)\big] \le 0,$$

where the operator $\mathcal{L} : \mathcal{C}^2 \to \mathcal{C}$ is defined as

$$\mathcal{L} g := (1-\theta)^2 \frac{\sigma^2}{2} g'' + (1-\theta)(\mu - f_1) g' - \rho g.$$

To this end, we fix a payout policy L with corresponding liquid–reserves process C_i^L ($C_i^L(0) = c$) and apply Itô's formula to $g_i(t,c) := e^{-\rho t} U_i(c)$:

$$\begin{aligned} e^{-\rho t} U_i\big(C_i^L(t)\big) = U_i(c) &+ \int_0^t e^{-\rho s}\Big[(1-\theta)(\mu - f_i) U_i'\big(C_i^L(s)\big) - \rho U_i\big(C_i^L(s)\big)\Big] ds \\ &+ \frac{1}{2}\int_0^t e^{-\rho s} U_i''(C_s^L) d[C_i^L, C_i^L]^c(s) \\ &+ \int_0^t e^{-\rho s}(1-\theta)\sigma U_i'(C_i^L(s)) dW(s) - \int_0^t e^{-\rho s} U_i'(C_i^L(s)) dL(s) \\ &+ \sum_{s \in \Gamma_i} e^{-\rho s}\Big[U_i(C_i^L(s_+)) - U_i(C_i^L(s)) - U_i'(C_i^L(s))\Big(C_i^L(s_+) - C_i^L(s)\Big)\Big], \end{aligned} \tag{A1}$$

where Γ_1 is the set of discontinuities of L and Γ_2 is the set of discontinuities of $\mathbb{1}_{\{t \le \tau^*\}} P_s N$. Since L and $\mathbb{1}_{\{t \le \tau^*\}} P_s N$ are of bounded variation, we have that

$$d[C_i^L, C_i^L]^c(s) = (1-\theta)^2 \sigma^2 ds.$$

Thus, Equation (A1) becomes

$$
\begin{aligned}
e^{-\rho t}U_i(C_i^L(t)) = U_i(c) + \int_0^t e^{-\rho s}\mathcal{L}U_i(C_i^L(s))ds \\
+ \int_0^t e^{-\rho s}(1-\theta)\sigma U_i'(C_i^L(s))dW(s) - \int_0^t e^{-\rho s}U_i'(C_i^L(s))dL(s) \\
+ \sum_{s\in\Gamma} e^{-\rho s}\Big[U_i(C_i^L(s_+)) - U_i(C_i^L(s)) - U_i'(C_i^L(s))\Big(C_i^L(s_+) - C_i^L(s)\Big)\Big],
\end{aligned}
\tag{A2}
$$

If we take expectations on both sides of Equation (A2), the Itô integral vanishes and, using the Dynamic Programming Principle, we obtain

$$
\mathbb{E}\Big[e^{-\rho t}U_i(C_i^L(t))\Big] \le U_i(c) - \mathbb{E}\Big[\int_0^t e^{-\rho s}U_i'(C_i^L(s))dL(s)\Big]. \tag{A3}
$$

Notice that for the Poisson jump we have

$$
\sum_{0\le s\le t}\mathbb{E}\Big[e^{-\rho s}\Big(U_1(C_1^L(s_+)) - U_1(C_1^L(s))\Big)\Big] = -\lambda\int_0^t e^{-\rho s}\Big[U_1(C_1^L(s)) - U_0(C_1^L(s) - P_s)\Big]ds. \tag{A4}
$$

Expressions (A3) and (A4) yield

$$
0 \ge \mathbb{E}\Big[\int_0^t e^{-\rho s}\mathcal{L}_i U_i(C_s^L)ds\Big] + \mathbb{E}\Big[\sum_{s\in\Gamma} e^{-\rho s}\Big(U_i(C_i^L(s_+)) - U_i(C_i^L(s)) - U_i'(C_i^L(s))(C_i^L(s_+) - C_i^L(s))\Big)\Big], \tag{A5}
$$

where Γ is the set of discontinuities of L. By the Mean Value Theorem, there exists $\hat{c} \in (C_i^L(s_+), C_i^L(s))$ such that

$$
U_i(C_i^L(s_+)) - U_i(C_i^L(s)) = U_i'(\hat{c})\Big[C_i^L(s_+) - C_i^L(s)\Big].
$$

Therefore

$$
U_i(C_i^L(s_+)) - U_i(C_i^L(s)) - U_i'(C_i^L(s))\Big[C_i^L(s_+) - C_i^L(s)\Big] = \Big[U_i'(\hat{c}) - U_i'(C_i^L(s))\Big]\Big[C_i^L(s_+) - C_i^L(s)\Big]
$$

and, by concavity of U_i, the right–hand side of the above expression, as well as the second term on the right–hand side of Expression (A5) are positive. This yields

$$
0 \ge \mathbb{E}\Big[\int_0^t e^{-\rho s}\mathcal{L}_i U_i(C_i^L(s))ds\Big]. \tag{A6}
$$

Next we multiply both sides of the equation above times $1/t$. Since

$$
\frac{1}{t}\int_0^t e^{-\rho s}\mathcal{L}_i U_i(C_s^L)ds \le \max_{s\in[0,t]} e^{-\rho s}\big|\mathcal{L}_i U_i(C_s^L)\big| < \infty,
$$

we may apply Lebesgue's Dominated Convergence Theorem and take the limit as $t \to 0$ inside the expectation operator, which yields

$$
\mathcal{L}_i U_i(c) \le 0.
$$

Variational inequalities. So far, we have shown that U_i satisfies, for $c > \underline{c}$, the set of variational inequalities

$$\mathcal{L}_i U_i(c) \leq 0 \quad \text{and} \quad 1 - U_i'(c) \leq 0.$$

Our next task is to prove that one of the inequalities is always tight. In order to do so, we resort to the Dynamic Programming Principle and write, for $t > 0$,

$$U_i(c) = \max_{L \in \mathcal{S}} \quad \mathbb{E}\Big[\int_0^t e^{-\rho s} dL_s + e^{-\rho t} U_i(C_i^L(t)) \Big].$$

Inserting Equation (A2) into the equation above we obtain

$$\begin{aligned} 0 = \sup_{L \in \mathcal{S}} \Bigg\{ & \mathbb{E}\Big[\int_0^t e^{-\rho s} \mathcal{L}_i U_i(C_i^L(s)) ds \Big] \\ & + \mathbb{E}\Big[\int_0^t e^{-\rho s} \Big(1 - U_i'(C_i^L(s)) \Big) dL(s) \Big] \\ & + \mathbb{E}\Big[\sum_{s \in \Gamma} e^{-\rho s} \Big(\Delta U_i(C_i^L(s)) - U_i'(C_i^L(s)) \Delta C_i^L(s) \Big) \Big] \Bigg\}. \end{aligned} \tag{A7}$$

If we write $\tilde{L}$ for the continuous pars of L, then Equation (A7) may be rewritten as

$$\begin{aligned} 0 = \sup_{\pi \in \Pi_p} \Bigg\{ & \mathbb{E}\Big[\int_0^t e^{-\rho s} \mathcal{L}_i U_i(C_i^L(s)) ds \Big] \\ & + \mathbb{E}\Big[\int_0^t e^{-\rho s} \Big(1 - U_i'(C_i^L(s)) \Big) d\tilde{L}(s) \Big] \\ & + \mathbb{E}\Big[\sum_{s \in \Gamma} e^{-\rho s} \Big(\Delta U_i(C_i^L(s)) + \Delta L(s) \Big) \Big] \Bigg\}. \end{aligned}$$

Notice that for all $s \in (0, t) \bigcap \Gamma$ it holds that

$$\Delta U_i(C_i^L(s)) + \Delta L(s) = \int_{C_i^L(s) - \Delta L(s)}^{C_i^L(s)} \Big(1 - U_i'(c) \Big) dc \leq 0.$$

This implies all summands on the right–hand side of Equation (A7) are non positive; thus, for $c > \underline{c}$ it holds that

$$\max \Big\{ \mathcal{L}_i U_i(c), 1 - U_i'(c) \Big\} = 0.$$

Q.E.D.

A.2 ***Proof of Propositions 1 - 2***

The functions U_0 and U_1, characterized by Equation (7) and System (10) - (12), respectively are only *candidates* for the value functions discussed in Sections 3.1 and 3.2. The proof of the propositions requires us to verify that these candidates indeed correspond to the solutions of the manager's optimizing actions. To this end, on a first step we assume there exist equilibrium r_l^*, b_0^* and b_1^* such that $U_0(0; b_0^*) = 0$, $D_1(b_1^*) = P_l$ and $U_1(0; b_1^*) = 0$ and prove the results concerning optimality of the payout policies and of $U_i(\cdot; b_i^*)$, for $i = 0, 1$. On a second step, we show that for $r_l, P_l \geq 0$ given, the equation

$$U_0(0; b_0^*(r_l, P_l)) = 0$$

has a unique positive solution and that, provided that $b_1^* \in (P_s, P_s + b_0^*]$, there exists no other $\tilde{b}_1 \in (P_s, P_s + b_0^*]$ such that equations

$$\begin{aligned} \mathcal{L}_1 U_1(c) &= 0, \quad c \in (\underline{c}, \tilde{b}_1); \\ U_1(c) - U_1(\tilde{b}_1 -) + \tilde{b}_1 - c &= 0, \quad c \geq \tilde{b}_1, \end{aligned} \tag{A8}$$

together with the corresponding boundary conditions, are satisfied.

Verification of the optimality of strategies. Let $r_l^*, P_l, P_s \geq 0$ be such that $\mu - f_i > 0$ and assume there exists b_i^* such that $U_i(0; b_i^*) = 0$. Let the processes (C_i^*, L_i^*) be a solution to the following Skorokhod problem defined on $[\underline{c}, b_i^*]$:

$$C_i^*(t) = c + \int_0^t (\mu - f_i) ds + \int_0^t \sigma dW(s) - L_i^*(t); \tag{A9}$$

$$\text{for all } 0 \leq t \leq \tau_i^*, \ \underline{c} \leq C^*(t) \leq b_i^*; \tag{A10}$$

$$\int_0^{\tau_i^*} \mathbb{1}_{\{C_i^*(t) < b_i^*\}} dL_i^*(t) = 0, \tag{A11}$$

where $\tau_i^* := \inf\{t > 0 | C_i^*(t) \leq \underline{c}\}$. The solution to the so-called Skorokhod Problem (A9)–(A11) can be found, for instance, in Karatzas and Shreve (1991). The process L_i^* is the *local time* of C_i^* at level b_i^*. Its effect on the dynamics of C_i^* is to reflect the latter downwards at level b_i^* in order to constrain it to $[\underline{c}, b_i^*]$. From Equation (A11) we see that the mass of the measure $dL_i^*(t)$ is carried by the set $\{C_i^*(t) = b_i^*\}$; thus, $L_i^*(t)$ is inactive whenever $C_i^*(t) < b_i^*$.

Now, let us show that $U_i(c; b_i^*) = \sup_L U_0^{(x_i, P_l, L)}(c)$, where $x_0 = 0$ and $x_1 = P_s$. Consider an admissible payoff strategy L_i and an initial level of liquid reserves $c_0 > \underline{c}$. Recall that the corresponding liquid–reserves process evolves according to the stochastic differential equation

$$dC_i^L(t) = (1 - \theta)\Big((\mu - f_i)dt + \sigma dW(t)\Big) - dL_i(t), \quad C_i^{L_i}(0) = c_0.$$

Proceeding as in Appendix A.1, we use the generalized Itô formula applied to $\tilde{g}_i(t, c) = e^{-\rho t} U_i(c; b_i^*)$. Using the fact that $\mathcal{L}_i U_i(C_i^{L_i}(t); b_i^*) \leq 0$, we obtain, after simplifications,

$$\begin{aligned} e^{-\rho t}\, \mathbb{E}\left[U_i(C_i^{L_i}(t); b_i^*)\right] \leq U_i(c; b_i^*) - \mathbb{E}\left[\int_0^t e^{-\rho s} U_i'(C_i^{L_i}(s); b_i^*)\big(d\tilde{L}_i(s)\big)\right] \\ + \mathbb{E}\left[\sum_{s \in \Gamma_i} e^{-\rho s}\Big(U_i(C_i^{L_i}(s_+); b_i^*) - U_i(C_i^{L_i}(s); b_i^*)\Big)\right], \end{aligned} \tag{A12}$$

where $\tilde{L}_i(s)$ is the continuous part of L_i. Let $s \in \Gamma_i$, then by the Mean Value Theorem and the fact that $U_i'(C_i^{L_i}(s); b_i^*) \geq 1$, there exists $\hat{c} \in \Big(C_i^{L_i}(s_+), C_i^{L_i}(s)\Big)$ such that

$$U_i(C_i^{L_i}(s_+); b_i^*) - U_i(C_i^{L_i}(s); b_i^*) = U_i'(\hat{c}; b_i^*)\Big(C_i^{L_i}(s_+) - C_i^{L_i}(s)\Big) \leq L_i(s_+) - L_i(s) = -\Delta L_i(s).$$

Inserting the above expression into Expression (A12) we get

$$e^{-\rho t}\,\mathbb{E}\left[U_i(C_i^{L_i}(t);b_i^*)\right] \le U_i(c;b_i^*) - \mathbb{E}\left[\int_0^t e^{-\rho s} dL_i(s)\right].$$

By continuity, $U_i(c;b_i^*)$ is bounded for $c \in [\underline{c}, b_i^*]$ and it grows linearly as c tends to infinity, therefore,

$$\lim_{t\to\infty} e^{-\rho t}\,\mathbb{E}\left[U_i(C_i^{L_i}(t);b_i^*)\right] = 0.$$

This implies that, if we set $dL_i(t) \equiv 0$ for all $t \ge \tau_i^*$,

$$U_i(c;b_i^*) \ge \mathbb{E}\left[\int_0^{\tau^*} e^{-\rho s} dL_i(s)\right]. \tag{A13}$$

Next, we consider the strategy L_i^*. Since the latter is the local time of C_i^* at level b_i^*, we may assume that $c \in [\underline{c}, b_i^*]$. Furthermore, L_i^* is a continuous processes, and on $(\underline{c}, b_i^*)$ it holds that $\mathcal{L}U_i(C_i^{L_i^*}(s);b_i^*) = 0$. Hence, for the strategy L_i^*, Itô's formula yields

$$\begin{aligned} e^{-\rho t}U_i(C_i^{L_i^*}(t);b_i^*) = U_i(c;b_i^*) &+ \int_0^t e^{-\rho s}\sigma U_i'(C_i^{L_i^*}(s);b_i^*)dW(s) \\ &- \int_0^t e^{-\rho s}U_i'(C_i^{L_i^*}(s);b_i^*)dL_i^*(s). \end{aligned} \tag{A14}$$

The measure $dL_i^*(s)$ is supported on $\{C_i^{L_i^*}(s) = b_i^*\}$ and $U_i'(b_i^*) = 1$. Therefore, taking expectations, Equation (A14) may be rewritten as

$$e^{-\rho t}\,\mathbb{E}\left[U_i(C^{L_i^*}(t)_i;b_i^*)\right] = U_i(c;b_i^*) - \mathbb{E}\left[\int_0^t e^{-\rho s} dL_i^*(s)\right].$$

Letting t tend to ∞ we have

$$U_i(c;b_i^*) = \mathbb{E}\left[\int_0^{\tau_i^*} e^{-\rho s} dL_i^*(s)\right], \tag{A15}$$

which is equivalent to $U_i(c;b_i^*) = U^{(x_i,P_l,L^*)}(c)$. Form Equation (A13), we have that for any admissible L, it holds that $U_i(c;b_i^*) \ge U_i^{(x_i,P_l,L)}(c)$. Since L_i^* is admissible, Equation (A15) yields

$$U_i(c;b_i^*) = \sup_L \quad U_0^{(x_i,P_l,L)}(c).$$

Uniqueness of b_0^* and b_1^*. Next we shown that b_0^* is the unique solution to $U_0(\underline{c};b) = \max\{\eta A - P_d - P_l, 0\}$. Recall that $\beta_2 < 0 < \beta_1$ and observe that

$$U_0(\underline{c};0) = \frac{\beta_1+\beta_2}{\beta_1\,\beta_2} = \frac{(1-\theta)(\mu - f_0)}{\rho} > 0. \tag{A16}$$

The mapping $b \mapsto U_0(\underline{c}; b)$ is decreasing, as shown by the condition

$$\frac{\partial U_0(\underline{c}; b)}{\partial b} = \frac{1}{\beta_1 - \beta_2}\left(\beta_2 e^{-\beta_1 b} - \beta_1 e^{-\beta_2 b}\right) < 0, \quad \text{for all } b > 0.$$

Furthermore, $\lim_{b\to\infty} U_0(\underline{c}; b) = -\infty$. Hence, the equation $U_0(\underline{c}; b) = 0$ has a unique solution.

In order to show there is no other $\tilde{b}_1 \in (P_s, P_s + b_0^*]$ such that System (A8), together with the corresponding boundary conditions, is satisfied, we define a parametric family of functions $\left\{U_1(\cdot : b) \middle| b \in (P_s, P_s + b_0^*]\right\}$ as the solutions to

$$\begin{aligned} \mathcal{L}_1 U_1(c) &= 0, \quad c \in (\underline{c}, b); \\ U_1(c) - U_1(b-) + b - c &= 0, \quad c \geq b, \end{aligned}$$

together with the boundary conditions $U_1'(b; b) = 1$ and $U_1''(b; b) = 0$. By definition, $U_1(\underline{c}; b_1) = 0$, so we must show that $U_1(\underline{c}; b) \neq 0$ for all $b \neq b_1$. We will do this by showing that the mapping $b \mapsto U_1(\underline{c}; b)$ is strictly decreasing. We require the following auxiliary lemma:

Lemma 2. *Let m be a solution to*

$$\mathcal{L}m(c) - \lambda m(c) := \mathcal{L}_2 m(c) = 0, \quad \textit{for} \quad c \in (\underline{c}, b)$$

such that $m'(b) < 0$ and $m(b) > 0$. Then $m'(c) < 0$ for all $c \in (\underline{c}, b)$.

Proof of Lemma 2: Let us assume that $m'(c) < 0$ does not hold for all $c \in (\underline{c}, b)$, and let $\bar{c}$ be the largest value on $(\underline{c}, b)$ that satisfies $m'(\bar{c}) = 0$. By construction, $\bar{c}$ is a positive local maximum of m. Since $m''(\bar{c}) \leq 0$, however, we would require $m(\bar{c}) \leq 0$ in order to have $\mathcal{L}_2 m(\bar{c}) = 0$, which is inconsistent with initial condition $m(b) > 0$. Therefore $m'(c) < 0$ for all $c \in (\underline{c}, b)$.

□

Next we consider two arbitrary payout thresholds b_1 and b_2 such that $b_1 < b_2$ and define

$$m(c) := U_1(c; b_1) - U_1(c; b_2), \quad c \in [c, b_1].$$

It is straightforward to show that m satisfies $\mathcal{L}_2 m(c) = 0$ subject to the boundary conditions $m'(b_1) = 1 - U_1'(b_1; b_2) < 0$ and $m''(b_1) = -U_1''(b_1; b_2) \geq 0$. Furthermore

$$\begin{aligned} m(b_1) &= U_1(b_1; b_1) - U_1(b_1; b_2) \\ &= U_1(b_1; b_1) - \left[U_1(b_2; b_2) - \int_{b_1}^{b_2} U_1'(c; b_2)\,\mathrm{d}c\right]. \end{aligned} \tag{A17}$$

It follows from the Mean Value Theorem that there exists $\hat{b} \in [b_1, b_2]$ such that

$$\int_{b_1}^{b_2} U_1'(c; b_2)\,\mathrm{d}c = U_1'(\hat{b}; b_2) \cdot (b_2 - b_1) \geq (b_2 - b_1).$$

Therefore, given that $U_1(b_1, b_1) = U_1(b_2, b_2) = (1-\theta)(\mu - f_1)/(\rho + \lambda)$, we have

$$m(b_1) \geq U_1(b_1, b_1) - U_1(b_2, b_2) + (b_2 - b_1) > 0.$$

Lemma 2 then implies that $m'(c) < 0$ holds for all $c \in (\underline{c}, b)$. Hence, $b \mapsto U_1(\underline{c}; b)$ is a strictly decreasing mapping, and, thus, there exists a unique b_1^* satisfying $U_1(\underline{c}; b_1^*) = 0$.

Q.E.D.

A.3 ***Proof of Lemma 1***

In order to prove the equality of c_0^* and b_1^* we proceed as follows: Given a debt structure (P_s, P_l, P_d) and a choice c_0 of initial reserves level, the bank's ex–ante value is given by the expression

$$V_1(c_0, P_s, P_l) = U_1(c_0) - c_0 - (I - P_l - P_s - P_d).$$

Since U_1 is concave, the first–order conditions are sufficient to determine the optimal choice of c_0; that is, c_0^* is characterized by the equation

$$\frac{\partial}{\partial c_0} V_1(c_0, P_s, P_l) = U_1'(c_0) - 1 = 0. \tag{A18}$$

We know from Appendix A.2 that the unique admissible solution to Equation (A18) is precisely b_1^*.

Q.E.D.

Appendix B Valuation of Contingent Claims

In this appendix we derive the pre–run values of bank equity and debt. Recall that:

$$\mathcal{L}_1 g(c) = (1-\theta)^2 \frac{\sigma^2}{2} g''(c) + (1-\theta)(\mu - f_1) g'(c) - \rho g(c), \tag{B1}$$

where $f_1 := r_d P_d + r_l P_l + r_s P_s$ and g is a twice continuously–differentiable function.

B.1 Equity value

For a given debt structure (P_s, P_l, P_d), an interest rate on long–term debt r_l and a liquidation threshold $\underline{c}$, consider a payout barrier b_1 that satisfies $\underline{c} + P_s < b_1 \le \underline{c} + P_s + b_0^*(P_l, r_l)$. If the run of repo creditors occurs in the region $c \in (\underline{c} + P_s, b_1]$, the bank survives and finds itself on the payout–retention region $(\underline{c}, b_0^*(P_l, r_l))$. In contrast, if the run occurs in the region $[\underline{c}, \underline{c} + P_s]$, the bank is liquidated and the shareholders receive

$$\tilde{u}_1(c) = \max\{\eta A + c - P_d - P_l - P_s, 0\}$$

upon the liquidation of the bank's assets.

In principle, one must distinguish between three possible scenarios: In the first scenario, $\tilde{u}_1(\underline{c} + P_s) = 0$, so that shareholders receive nothing should a run–triggered liquidation occur. Another possibility is $\tilde{u}_1(\underline{c} + P_s) > 0$ but $\tilde{u}_1(\underline{c}) = 0$, which means that, should a run–triggered liquidation occur, shareholders may receive a positive amount if the current level of liquid reserves is not too low. The last possibility implies $\tilde{u}_1(\underline{c}) > 0$, which means that long–term debt is riskless and shareholders will always collect a strictly positive amount in the event of liquidation. We present below the design of equity value for the case $\tilde{u}_1(\underline{c}+P_s)=0$, as it is the only case that manifests itself in our numerical analysis.[41]

Recall that U_0 denotes the post–run optimal equity value defined in Section 3.1. Before a run occurs, the equity value satisfies the system

$$\mathcal{L}_1U_1(c)-\lambda U_1(c) = 0, \quad c\in(\underline{c},\underline{c}+P_s), \tag{B2}$$
$$\mathcal{L}_1U_1(c)-\lambda[U_1(c)-U_0(c-P_s)] = 0, \quad c\in(\underline{c}+P_s,b_1), \tag{B3}$$
$$U_1(c)-U_1(b_1)+b_1-c = 0, \quad c\geq b_1, \tag{B4}$$

subject to the following conditions:

$$\begin{aligned} U_1''(b_1)&=0,\\ U_1'(b_1)&=1,\\ \lim_{c\uparrow\underline{c}+P_s}U_1(c)&=\lim_{c\downarrow\underline{c}+P_s}U_1(c),\\ \lim_{c\uparrow\underline{c}+P_s}U_1'(c)&=\lim_{c\downarrow\underline{c}+P_s}U_1'(c). \end{aligned}$$

Applying the Method of Variation of Parameters,[42] we can show that a particular solution to Equation (B3) is given by

$$H(c)=\kappa_0\Big(\kappa_1e^{\beta_1(c-P_s)}-\kappa_2e^{\beta_2(c-P_s)}\Big), \tag{B5}$$

where

$$\kappa_0=\frac{1}{(1-\theta)^2}\frac{2}{\sigma^2}\frac{\lambda}{(\beta_1-\beta_2)},\quad \kappa_1=\frac{\beta_2}{\beta_1}\frac{1}{(\beta_1-\gamma_1)(\beta_1-\gamma_2)}e^{-\beta_1b_0^*},\quad \kappa_2=\frac{\beta_1}{\beta_2}\frac{1}{(\beta_2-\gamma_1)(\beta_2-\gamma_2)}e^{-\beta_2b_0^*},$$

and $\gamma_1>0$ and $\gamma_2<0$ are the roots of the characteristic polynomial

$$(1-\theta)^2\frac{\sigma^2}{2}\gamma^2+(1-\theta)(\mu-f_1)\gamma=\rho+\lambda.$$

Then, the equity value function can be defined as follows:

$$U_1(c)=\begin{cases} A_{21}(b_1)e^{\gamma_1c}+A_{22}(b_1)e^{\gamma_2c}, & c\in(\underline{c},\underline{c}+P_s),\\ A_{11}(b_1)e^{\gamma_1c}+A_{12}(b_1)e^{\gamma_2c}+H(c), & c\in(\underline{c}+P_s,b_1),\\ \frac{(1-\theta)(\mu-f_1)}{\rho+\lambda}+\frac{\lambda}{\rho+\lambda}U_0^*(b_1-P_s)+c-b_1, & c\geq b_1, \end{cases} \tag{B6}$$

[41] Note that the formulas derived in this section do no apply to the set-up with payout restrictions. The reason is that, under payout restrictions, the super-contact condition $U_i''(b_i)=0$ does not necessary hold. The valuation of the equity claim under payout restrictions is reported in Appendix B.3.

[42] The alternative way to get this particular solution would be to make a guess about its structure, i.e. $H(c)=h_1e^{\beta_1c}+h_2e^{\beta_2c}$, and then recover the unknown constants h_1, h_2 by inserting this guess into the ODE (B3), collecting terms to get the factors of e^{β_1c}, e^{β_2c} and setting the latter to zero.

where

$$A_{11}(b_1) = -\left[\frac{\gamma_2(1-H'(b_1))+H''(b_1)}{\gamma_1(\gamma_1-\gamma_2)}\right]e^{-\gamma_1 b_1},$$
$$A_{12}(b_1) = \left[\frac{\gamma_1(1-H'(b_1))+H''(b_1)}{\gamma_2(\gamma_1-\gamma_2)}\right]e^{-\gamma_2 b_1},$$
$$A_{21}(b_1) = A_{11}(b_1) - \left[\frac{\gamma_2 H(\underline{c}+P_s)-H'(\underline{c}+P_s)}{(\gamma_1-\gamma_2)}\right]e^{-\gamma_1(\underline{c}+P_s)} \quad \text{and}$$
$$A_{22}(b_1) = A_{12}(b_1) + \left[\frac{\gamma_1 H(\underline{c}+P_s)-H'(\underline{c}+P_s)}{(\gamma_1-\gamma_2)}\right]e^{-\gamma_2(\underline{c}+P_s)}.$$

B.2 The value of long–term debt

Here we derive the market value of long–term, risky debt.[43] Under the assumption that $\underline{c}+P_s < b_1 < \underline{c}+P_s+b_0^*$, the market value of long–term debt satisfies the following system:

$$\mathcal{L}_1 D_1(c) + r_l P_l - \lambda[D_1(c) - D_0^*(c-P_s)] = 0, \quad c \in (\underline{c}+P_s, b_1), \tag{B7}$$
$$\mathcal{L}_1 D_1(c) + r_l P_l - \lambda[D_1(c) - \tilde{d}_1(c)] = 0, \quad c \in (\underline{c}, \underline{c}+P_s), \tag{B8}$$

where D_0^* denotes the post–run value of long–term debt defined in Equation (9) of Section 3.1, together with the following boundary and pasting conditions:

$$D_1'(b_1) = 0,$$
$$\lim_{c\uparrow \underline{c}+P_s} D_1(c) = \lim_{c\downarrow \underline{c}+P_s} D_1(c),$$
$$\lim_{c\uparrow \underline{c}+P_s} D_1'(c) = \lim_{c\downarrow \underline{c}+P_s} D_1'(c),$$
$$D_1(\underline{c}) = \tilde{d}_1(\underline{c}).$$

Let M_1 denote a particular solution to the non–homogeneous Equation (B7):

$$M_1(c) = \chi_0\Big(\chi_{11}e^{\beta_1(c-P_s)} - \chi_{12}e^{\beta_2(c-P_s)}\Big) + \frac{r_l P_l}{\rho},$$

where

$$\chi_0 := \frac{\lambda}{(1-\theta)^2}\frac{2}{\sigma^2}\left(\frac{\tilde{d}_0 - \frac{r_l P_l}{\rho}}{\beta_1 e^{\beta_1 b_0^* + \beta_2 \underline{c}} - \beta_2 e^{\beta_2 b_0^* + \beta_1 \underline{c}}}\right),$$
$$\chi_{11} := \frac{\beta_2}{(\beta_1-\gamma_1)(\beta_1-\gamma_2)}e^{\beta_2 b_0^*}, \quad \chi_{12} := \frac{\beta_1}{(\beta_2-\gamma_1)(\beta_2-\gamma_2)}e^{\beta_1 b_0^*}.$$

[43] This debt valuation strategy is valid for the set–up with no regulation, the set-up with liquidity regulation and the set–up with the payout restrictions. In the set–up with leverage regulation, the construction of the value of debt (not reported here) must allow for the multiple scenarios conditional on the liquidation value accruing to the long–term debt creditors, which depends on the value of the liquidation thresholds $c(k_{lev})$.

Let $M_2(c)$ denote a particular solution to the non–homogeneous Equation (B8) (as we show below, it is computed differently depending on the seniority rules). The general solution to the System (B7)-(B8) is

$$D_1(c) = \begin{cases} B_{21}(b_1)e^{\gamma_1 c} + B_{22}(b_1)e^{\gamma_2 c} + M_2(c), & c \in [\underline{c}, \underline{c} + P_s], \\ B_{11}(b_1)e^{\gamma_1 c} + B_{12}(b_1)e^{\gamma_2 c} + M_1(c), & c \in (\underline{c} + P_s, b_1]. \end{cases} \tag{B9}$$

In order to define coefficients $B_{11}(b_1), B_{12}(b_1), B_{21}(b_1)$ and $B_{22}(b_1)$, we introduce the following auxiliary functions:

$$\begin{aligned}
G_1(b_1) &= \left[\tilde{d}_1(\underline{c}) - M_2(\underline{c})\right]e^{\gamma_1 P_s} + M_2(P_s + \underline{c}) - M_1(P_s + \underline{c}) + \frac{M_1'(b_1)}{\gamma_1}e^{\gamma_1(P_s - b_1) + \gamma_1 \underline{c}}, \\
G_2(b_1) &= \gamma_1\left[\tilde{d}_1(\underline{c}) - M_2(\underline{c})\right]e^{\gamma_1 P_s} + M_2'(P_s + \underline{c}) - M_1'(P_s + \underline{c}) + M_1'(b_1)e^{\gamma_1(P_s - b_1) + \gamma_1 \underline{c}}, \\
F_1(b_1) &= e^{\gamma_2(P_s + \underline{c})} - \frac{\gamma_2}{\gamma_1}e^{\gamma_2 b_1 + \gamma_1(P_s - b_1) + \gamma_1 \underline{c}}, \\
F_2(b_1) &= \gamma_2\left[e^{\gamma_2(P_s + \underline{c})} - e^{\gamma_2 b_1 + \gamma_1(P_s - b_1) + \gamma_1 \underline{c}}\right], \\
g_1 &= e^{\gamma_1 P_s + \gamma_2 \underline{c}} - e^{\gamma_2(P_s + \underline{c})} \quad \text{and} \quad g_2 = \gamma_1 e^{\gamma_1 P_s + \gamma_2 \underline{c}} - \gamma_2 e^{\gamma_2(P_s + \underline{c})}.
\end{aligned}$$

Solving the above system of boundary, value–matching and smooth–pasting conditions yields:

$$\begin{aligned}
B_{12}(b_1) &= \frac{g_1 G_2(b_1) - g_2 G_1(b_1)}{g_1 F_2(b_1) - g_2 F_1(b_1)}, \\
B_{11}(b_1) &= -\left[\frac{M_1'(b_1) + \gamma_2 B_{12}(b_1)e^{\gamma_2 b_1}}{\gamma_1}\right]e^{-\gamma_1 b_1}, \\
B_{21}(b_1) &= [\tilde{d}_1(\underline{c}) - M_2(\underline{c}) - B_{22}(b_1)e^{\gamma_2 \underline{c}}]e^{-\gamma_1 \underline{c}} \quad \text{and} \\
B_{22}(b_1) &= \frac{G_1(b_1) - B_{12}(b_1)F_1(b_1)}{g_1}.
\end{aligned}$$

Impact of seniority rules. The value of long–term debt when it is *senior* to deposits can be computed by using the above formulas with $\tilde{d}_1(c) = c + \eta A - P_s$ and $\tilde{d}_0 = \eta A$. In this case, the particular solution to Equation (B8) is given by

$$M_2(c) = \chi_{21}c + \chi_{22},$$

where

$$\chi_{21} = \frac{\lambda}{\rho + \lambda}, \qquad \chi_{22} = \frac{r_l P_l + \lambda(\eta A - P_s)}{\rho + \lambda} + (1 - \theta)\frac{\lambda(\mu - f_1)}{(\rho + \lambda)^2}.$$

Consider now the value of long–term debt when it is *junior* to deposits. Assume that $P_d \geq \underline{c} + \eta A$, which implies that long–term creditors will receive nothing in the event of liquidation, i.e. $\tilde{d}_1(c) = 0$ and $\tilde{d}_0 = 0$. The market value of junior long–term debt satisfies the above formulas with the following modifications: $M_2(c) \equiv \frac{r_l P_l}{\rho + \lambda}$, $\tilde{d}_0 = 0$ and $\tilde{d}_1(c) \equiv 0$.

B.3 Equity value in the presence of payout restrictions

In this section we present our analysis of the effect that the regulatory constraint on the payout policy introduced in Section 5.2 has on the ex–ante value of equity. Since we are now dealing with a constrained optimization problem, the super–contact condition at the levels b_0^{reg} and b_1^{reg} no longer applies. As a consequence, the ex–ante value of equity differs significantly from that in Appendix B.1. For notational simplicity we assume that $\tilde{u}_0(0) = \tilde{u}_1(P_s) = 0$. The case with general values is analogous. We first look at the impact of payout restrictions after the run on repos and then analyze the general case on a second stage.

The impact of payout restrictions after the run. Recall that, in the unregulated case, the equity value function U_0 satisfies the ordinary differential equation

$$\rho U_0(c) = (1-\theta)^2 \frac{\sigma^2}{2} U_0''(c) + (1-\theta)(\mu - f_0) U_0'(c) \tag{B10}$$

on the region $(\underline{c}, b_0^*)$. For each choice of r_l and P_l, Equation (B10) has the general solution

$$A(r_l, P_l) e^{\beta_1 c} + B(r_l, P_l) e^{\beta_2 c}.$$

Furthermore, for $\underline{c} = 0$ and $U_0(0) = 0$ we have that $A(r_l, P_l) = -B(r_l, P_l)$. Notice that the choice of $A(r_l, P_l)$ determines the level of liquid reserves at which the condition $U_0' = 1$ is satisfied, but it bears no weight on where the second–order condition $U_0'' = 0$ is fulfilled. This occurs whenever

$$\beta_1^2 e^{\beta_1 c} - \beta_2^2 e^{\beta_2 c} = 0, \tag{B11}$$

an equation whose solution is our old acquaintance

$$b_0^* = \frac{1}{\beta_2 - \beta_1} \log \left(\frac{\beta_1}{\beta_2} \right)^2.$$

In other words, b_0^* is the unique inflection point of the family of functions that satisfy Equation (B10) and the boundary condition at zero. The fact that we impose the condition $U_0' = 1$ precisely at $c = b_0^*$ is what allows for a C^2–linear continuation of U_0 over (b_0^*, ∞).

Assume now that the payout constraint $b_0^{reg} > b_0^*$ is imposed, i.e. dividends can only be distributed at date t if the level of liquid reserves $C^\pi(t) \geq b_{reg}$. In terms of the corresponding equity value function $U_0(\cdot; b_0^{reg})$, this results in the Neumann boundary condition $U_0'(b_0^{reg}; b_0^{reg}) = 1$, which follows from the same argument as in the unregulated case and yields

$$\begin{aligned} U_0(c; b_0^{reg}) &= \frac{1}{\beta_1 e^{\beta_1 b_0^{reg}} - \beta_2 e^{\beta_2 b_0^{reg}}} \left(e^{\beta_1 c} - e^{\beta_2 c} \right) =: K(b_0^{reg}) \left(e^{\beta_1 c} - e^{\beta_2 c} \right), \quad c \leq b_0^{reg}, \\ U_0(c; b_0^{reg}) &= U_0(b_0^{reg}; b_0^{reg}) + (c - b_0^{reg}), \quad c > b_0^{reg}. \end{aligned}$$

The impact of payout restrictions before the run. The level of liquid reserves at which the shareholders will (optimally) start the firm is characterized by the FOC $U_1'(c) = 1$. In the unregulated case, as we saw in Lemma 1, this corresponds precisely to the level $c = b_1^*$. We show below, however, that in the presence of payout restrictions, if $b_1^{reg} > b_1^*$ the FOC is satisfied by another level $c_0^{reg} < b_1^{reg}$, which is then the shareholder's optimal initial choice of liquid reserves.

For $\underline{c} = 0$, $\tilde{u}_1(P_s) = 0$ and a regulatory threshold $b_1^{reg} > b_1^*$, the pre–run equity value function

$U_1(\cdot;b_1^{reg})$ satisfies the following system:

$$\begin{aligned} \mathcal{L}_1U_1(c;b_1^{reg})-\lambda U_1(c;b_1^{reg}) &= 0, \quad c\in(0,P_s), && \text{(B12)}\\ \mathcal{L}_1U_1(c;b_1^{reg})-\lambda[U_1(c;b_1^{reg})-U_0(c-P_s;b_0^{reg})] &= 0, \quad c\in(P_s,b_1^{reg}), && \text{(B13)}\\ U_1(c;b_1^{reg})-U_1(b_1^{reg};b_1^{reg})+b_1^{reg}-c &= 0, \quad c\geq b_1^{reg}, && \text{(B14)} \end{aligned}$$

together with the boundary conditions $U_1(0;b_1^{reg})=0$, $U_1'(b_1^{reg};b_1^{reg})=1$ and smooth pasting at $c=P_s$. We stress that not only is the condition $U_1''(b_1^{reg};b_1^{reg})=0$ not imposed but, in fact, it will not hold in this constrained scenario. As before, we denote by $\gamma_2<0<\gamma_1$ the roots of the characteristic polynomial

$$(1-\theta)^2\frac{\sigma^2}{2}\gamma^2+(1-\theta)(\mu-f_1)\gamma=\rho+\lambda.$$

A crucial point is the following: for $b_1^{reg}>b_1^*$ the equation $U_1''(c;b_1^{reg})=0$ has a unique solution $c(b_1^{reg})<b_1^{reg}$. This is the equivalent to Equation (B11) used above, but here the inflection point does not coincide with b_1^*, nor can it be computed in closed form. In order to find $c(b_1^{reg})$, we require the solution to Equation (B13). To this end, we use the following particular solution:

$$H_r(c)=\underbrace{\frac{\lambda K(b_0^{reg})}{(1-\theta)r_sP_s\beta_1+\lambda}}_{=:A_{1H}>0}e^{\beta_1(c-P_s)}-\underbrace{\frac{\lambda K(b_0^{reg})}{(1-\theta)r_sP_s\beta_2+\lambda}}_{=:A_{2H}}e^{\beta_2(c-P_s)}.$$

Observe that $H_r(\cdot)$ does not depend on the choice of b_1^{reg}.

Next we compute $U_h(\cdot;b_1^{reg})$, the homogeneous part of the solution to Equation (B13) (we focus on the case where $c(b_1^{reg})\in(P_s,b_1^{reg})$ since this is the only scenario that we follow in our simulations). In other words, for $c\in(P_s,b_1^{reg})$, we have

$$U_1(c;b_1^{reg})=U_h(c;b_1^{reg})+H_r(c).$$

The explicit representation of $U_h(c;b_1^{reg})$ over (P_s,b_1^{reg}) is the following:

$$U_h(c;b_1^{reg})=A_{11}e^{\gamma_1c}+A_{12}e^{\gamma_2c},$$

where

$$\begin{aligned} A_{11}&=A_{11}(b_1^{reg})=A_{21}-\frac{H_r'(P_s)-\gamma_2H_r(P_s)}{\gamma_1-\gamma_2}e^{-\gamma_1P_s},\\ A_{12}&=A_{12}(b_1^{reg})=Ge^{-\gamma_2P_s}-A_{11}(b_1^{reg}), && \text{(B15)}\\ &\text{where } G:=\frac{H_r'(P_s)-\gamma_2H_r(P_s)}{\gamma_1-\gamma_2}\Big(1-e^{(\gamma_2-\gamma_1)P_s}\Big)-H_r(P_s), \end{aligned}$$

and

$$A_{21} = A_{21}(b_1^{reg}) = \frac{\gamma_2\big[H_r'(P_s) - \gamma_1 H_r(P_s)\big]e^{\gamma_2(b_1^{reg}-P_s)} - \gamma_1\big[H_r'(P_s) - \gamma_2 H_r(P_s)\big]e^{\gamma_1(b_1^{reg}-P_s)}}{(\gamma_1-\gamma_2)(\gamma_2 e^{\gamma_2 b_1^{reg}} - \gamma_1 e^{\gamma_1 b_1^{reg}})} - \frac{1 - H_r'(b_1^{reg})}{\gamma_2 e^{\gamma_2 b_1^{reg}} - \gamma_1 e^{\gamma_1 b_1^{reg}}}. \tag{B16}$$

We then have that

$$U_1''(b_1^{reg}; b_1^{reg}) = \gamma_1^2 A_{11}(b_1^{reg})e^{\gamma_1 b_1^{reg}} + \gamma_2^2 A_{12}(b_1^{reg})e^{\gamma_2 b_1^{reg}} + H_r''(b_1^{reg}).$$

A numerical analysis of the above expression revealed that $U_1''(b_1^{reg}; b_1^{reg}) > 0$ holds for levels of $b_1^{reg} > b_1^*$ such that condition $b_1^{reg} \in (P_s, P_s + b_0^{reg}]$ holds. This, together with the facts $U_1''(P_s; b_1^{reg}) = A_{21}(\gamma_1^2 - \gamma_2^2) < 0$ and $U_1''(\cdot; b_1^{reg}) \in \mathcal{C}(0, b_1^{reg})$, allows us to apply the Intermediate Value Theorem and conclude there exists $c(b_1^{reg}) \in (P_s, b_1^{reg})$ such that $U_1''(c(b_1^{reg}); b_1^{reg}) = 0$.

We are now in a position to conclude. As mentioned above, on the domain $c < P_s$ we have that $U_1(c; b_1^{reg}) = A_{21}\big(e^{\gamma_1 c} - e^{\gamma_2 c}\big)$; therefore $U_1''(P_s; b_1^{reg}) < 0$.[44] On the other hand, as discussed above, we have that $U''(b_1^{reg}; b_1^{reg}) > 0$. Given that $U_1(\cdot; b_1^{reg})$ is of class $\mathcal{C}^2(0, b_1^{reg})$ this implies that

$$U_1''(c; b_1^{reg}) < 0 \quad \text{for} \quad c \in (P_s, c(b_1^{reg})) \quad \text{and} \quad U_1''(c; b_1^{reg}) > 0 \quad \text{for} \quad c \in (c(b_1^{reg}), b_1^{reg}).$$

In words, the graph of $U''(c; b_1^{reg})$ has a sinusoidal shape over (P_s, b_1^{reg}) and $U_1'(c(b_1^{reg}); b_1^{reg}) < 1$. This, together with the fact that $U_1'(0; b_1^{reg}) > 1$, implies there exists $c_0^*(b_1^{reg}) \in (0, c(b_1^{reg}))$ such that $U_1'(c_0(b_1^{reg}); b_1^{reg}) = 1$. By construction, the only other value of c that satisfies $U_1'(c; b_1^{reg}) = 1$ is precisely $c = b_1^{reg}$. Notice that the above discussion not only delves in the impact that payout restrictions have on the shareholders' optimal choice regarding the initial level of liquid reserves, but it also shows that the equity value function becomes concave–convex in the presence of the said regulatory constraint.

Appendix C Computing the Liquidation Probabilities

In this section we describe the numerical approximation of the liquidation probabilities discussed in Section 5.1. We follow a methodology that is in line with most of the analysis in this paper and first approximate the liquidation probabilities after a run on repos has occurred. The numerical procedure can then be easily extended to the setting in which repos are still present in the bank's balance sheet.

C.1 Computing the probability of survival after the run

Technically, it is slightly simpler to deal with the probabilities of survival, from which the liquidation probabilities follow immediately. In other words, for a given strategy $\pi = (P_s, P_l, L)$ and a time horizon T, we study the object

$$K_0(t, c, T) = \mathbb{P}\big\{\tau_\pi > T \,\big|\, C^\pi(t) = c\big\},$$

[44] This as long as the relation $c < \frac{1}{\gamma_2 - \gamma_1}\log\left(\frac{\gamma_1}{\gamma_2}\right)^2 > P_s$ holds, which is a necessary condition for $c(b_1^{reg}) \in (P_s, b_1^{reg})$ to hold.

i.e. the probability that the bank will not be liquidated before date T, given that its liquidity at date $t < T$ equals c.

In order to obtain a partial differential equation that describes K_0, we mimic the way in which we proceeded in Appendix A.1, noting that the presence of a time dimension results in a partial derivative with respect to time when making use of the Itô formula. Let $D := (\underline{c}, b_0)$, then for given time horizon T (taking advantage of the stationary nature of the liquid–reserves dynamics it is without loss of generality to consider the initial *evaluation date* to be $t = 0$), the mapping $(t, c) \mapsto K_0(t, c, T)$ solves the following boundary–value problem[45]:

$$\begin{aligned} \frac{\partial K_0(t,c)}{\partial t} + \frac{1}{2}(1-\theta)^2\sigma^2 \frac{\partial^2 K_0(t,c)}{\partial c^2} + (1-\theta)(\mu - f_0)\frac{\partial K_0(t,c)}{\partial c} &= 0, \\ K_0(T,c) &= 1 \text{ for all } c > \underline{c}, \\ K_0(t,\underline{c}) &= 0, \\ \frac{\partial K_0}{\partial c}(t, b_0) &= 0. \end{aligned} \tag{C1}$$

Observe that there is no term $\rho K_0(t, c)$ on the left–hand side of the differential equation. This obeys the fact that the probability of default is not discounted. The Dirichlet condition $K_0(t, \underline{c}) = 0$, which simply states that the probability of survival contingent on having been liquidated is zero, is precisely the reason why we work with the survival probabilities. The Neumann condition along the boundary $c = b_0$ corresponds to the reflection of C^π at the said boundary. Namely, the probability of survival cannot be increasing at $c = b_0$, since the liquid reserves are not permitted to increase beyond this level.

System (C1) is fairly standard and can be solved in numerous ways. We have chosen to use a finite element method in the spacial domain D and a $\hat{\theta}$–scheme (see e.g., Wilmott (2006) for a thorough presentation of $\hat{\theta}$–schemes) in time. A step–by–step description of how we have proceeded can be found in Barth et al. (2015).

C.2 Computing the probability of survival before the run

Two additions must be made to the method described above so as to compute the survival probabilities before the run on repos takes place, which we shall denote $K_1(t, c)$. First, the possible jump of the liquid reserves as a result of a run has to be accounted for, which results in the following boundary–value problem:

$$\begin{aligned} \frac{\partial K_1(t,c)}{\partial t} + \frac{1}{2}(1-\theta)^2\sigma^2 \frac{\partial^2 K_1(t,c)}{\partial c^2} + (1-\theta)(\mu - f_1)\frac{\partial K_1(t,c)}{\partial c} &= \lambda\Big[K_1(t,c) - K_0(t, c - P_s)\Big], \\ K_1(T,c) &= 1 \text{ for all } c > 0, \\ K_1(t,\underline{c}) &= 0, \\ \frac{\partial K_1}{\partial c}(t, b_1) &= 0. \end{aligned} \tag{C2}$$

[45]From now on we drop the argument T to simplify notation.

After rearranging we have that the partial differential equation in Expression (C2) can be written as

$$\frac{\partial K_1(t,c)}{\partial t} + \frac{1}{2}(1-\theta)^2\sigma^2\frac{\partial^2 K_1(t,c)}{\partial c^2} + (1-\theta)(\mu - f_1)\frac{\partial K_1(t,c)}{\partial c} = \lambda K_1(t,c) \tag{C3}$$

if $c \leq P_s + \underline{c}$, whereas in the case $P_s + \underline{c} < c < b_1$ we have

$$\frac{\partial K_1(t,c)}{\partial t} + \frac{1}{2}(1-\theta)^2\sigma^2\frac{\partial^2 K_1(t,c)}{\partial c^2} + (1-\theta)(\mu - f_1)\frac{\partial K_1(t,c)}{\partial c} = \lambda\Big[K_1(t,c) - K_0(t, c - P_s)\Big]. \tag{C4}$$

The discretization of the boundary–value problem in Expression (C2) can be done separately on the domains $(0,T) \times (\underline{c}, P_s + \underline{c})$ and $(0,T) \times (P_s + \underline{c}, b_1)$, respectively. Then we can do exactly as we did in Section C.1. The ex–ante probability of liquidation is simply $p(T,c) = 1 - K_1(t,c)$.

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