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Abstract

A new theoretical model is developed which describes the structure of competition for attention and characterizes equilibria. The exogenous fundamentals of an attention economy are the space of receiving subjects with their attention capacity, and the potential set of competing firms (senders) with their radiation technology. The endogenous variables explained by the theory are equilibrium audiences (the clients belonging to a sender), their signal exposure and attention, and the diversity of senders surviving the contest for attention. Application of the equilibrium analysis suggests that international integration or progress in information technologies tend to decrease global diversity. Local diversity, perceived by the individual receivers, may increase nonetheless.

ATTENTION ECONOMIES*

Josef Falkinger[†]

October 27, 2004

Abstract

Attracting attention is a basic feature of economic life but no standard economic problem. A new theoretical model is developed which describes the general structure of competition for attention and characterizes equilibria. The exogenous fundamentals of an attention economy are the space of receiving subjects with their attention capacity, and the potential set of competing companies (senders) with their radiation technology. The endogenous variables explained by the theory are equilibrium audiences (the clients belonging to a company), their signal exposure and attention, and the diversity of senders surviving the contest for attention. Application of the equilibrium analysis to changes in information technologies and globalization suggests that international integration or range-increasing technical progress may decrease global diversity. Local diversity, perceived by the individual receivers, may increase nonetheless.

Keywords: Scarcity of attention, attention grabbing, local and global diversity, attention capacity, allocation of audiences.

JEL classification: D50, D80, L10

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1 Introduction

What determines the set of items we perceive and why does the diversity of perceived objects and agents change? – Suppose you are a reader of economic articles. Apart from your interests, you have individual characteristics like brain capacity or distractions by non-economic stuff. They put a limit to your processing of economic literature. Thus, you focus attention on important sites and outlets of publications. And important is who produces many powerful papers and gets the attention of the scientific community or, more generally, of the ensemble of producers and consumers of scientific work. Range and diversity of the important agents and places on which this ensemble focuses have changed over time. In particular, some local heroes have become global heroes and other local heroes have disappeared at all. Obviously, the circle that producers try to attract attention and consumers pay attention to producers who send powerful signals is not bound to scientific research but applies to the (economic) world in general. An account of the many current trends of attention management and the practical implications for doing business was recently presented by *Davenport and Beck* [2001] under the title "The Attention Economy".¹ The title indicates that the issue of attracting attention really adds a new dimension to an economy.

Psychologists and behavioral economics have long been aware of the problem of limited attention. As *Herbert A. Simon* pointed out, in an information-rich world a new scarcity problem arises, namely, "a scarcity of whatever it is that information consumes". What information consumes is obvious: "it consumes the attention of its recipients. Hence a wealth of information creates a poverty of attention" (*Simon* [1971], p. 40). Nonetheless, *Camerer* [2003] lists the concept of limited attention among the topics neglected until

¹A first draft of this paper was written 2001 before I have got knowledge of their book. That I independently chose almost the same title underlines that the subject is topical. My interest in it was raised by the art journal *Kunstforum*, whose December 1999 issue focused on "Ressource Aufmerksamkeit" (see also *Goldhaber* [1997] and *Frank* [1998]). *Shapiro* and *Varian* [1999] discuss business strategies for attracting attention by customizing information.

recently. *Gabaix, Laibson and Moloche* [2003] presented first experimental evidence of individual allocation of attention. Like *Simon* they focus on the question how subjects allocate their attention capacity (a given time budget) on a given set of information sources when they have to solve one or more choice problems. In such an approach, focussing on the users of information, the agenda that subjects have to deal with is given. In contrast to this, my focus is on the sender side of the problem. The reason for the crowded agenda is that many agents attempt to attract attention by sending information. Wealth of information means more senders are grabbing for attention or the given set of attention grabbing agents is trying harder by sending stronger signals. Under this focus the central question is which and how many information sources succeed in getting enough attention to be viable. Put differently: Given that the agenda of people is crowded, who and what is on the agenda if an overabundant set of potential items compete for being there.² For answering this question, an economy is considered as a system of agents (senders) who try to attract the attention of subjects (receivers) by producing and distributing information packages (signals). The senders may be firms, news agencies, scientific networks, political parties etc., promoting products, persons or ideas. They are addressed also as companies or firms in this paper, since they typically consist of more than one individual. (However, the interior structure of companies is not considered.) The central question to be answered is then: What is an equilibrium outcome if each company tries to attract the attention of subjects by exposing them to the signal "Look at me" or "Read my message"?

Several authors began to study the consequences of limited attention for economic equilibrium – in single markets or at the macroeconomic level. *Sims* [2003] pointed out that macroeconomic analysis should not rely on some exogenous notion of noise when dealing with the question how rational agents process economic data. Optimal allocation of attention endogenously determines which part of data is to be tracked as important

²Limited attention capacity has not only implications for individual behavior or business strategies but also consequences for the (equilibrium) outcome in markets or in the whole economy. To stress this aspect I use the label "attention economies" rather than "economics of attention" for addressing the subject of this paper.

and to which part no attention should be paid. *Gabaix and Laibson* [2002] provide an explanation of the equity-premium puzzle by arguing that consumers – due to attention allocation costs – can only respond to a fraction of realizations of equity returns. *Hirshleifer, Lim and Teoh* [2002] and *Hirshleifer and Teoh* [2003] analyze the implications of limited attention for firms’ information policy and the financial market equilibrium. Like in behavioral research on the allocation of attention, also in these contributions the set of information sources is exogenously given – published macroeconomic time series, equity returns, earnings of firms. The purpose of this paper is to determine endogenously the number of information sources and the volume of signals to which subjects are exposed when many information suppliers are grabbing for their attention. For this purpose I propose a receiver model, based on psychological evidence, but simply enough to be used in general equilibrium analysis. I abstract from decision processes within receivers and treat them as black boxes reacting to signal exposure in possibly heterogeneous, but exogenously given ways. Fortunately, rich empirical research on the psychology of attention (see e.g. *Kahneman* [1973] and *Pashler* [1998]) provides some robust features of how subjects react to stimulus exposures. The details of these features and their translation into a formal receiver model are presented in Section 2.3. Formulated loosely, subjects turn attention to those information sources, which are ”loud” i.e., send with relatively strong signal strength. Such unsophisticated receiver behavior is not only motivated by pragmatic reasons. Before subjects can value and choose items they must get aware of them. This involves an interaction between producers and consumers beyond the exchange of specific contents.³ The essential dimension in this ”meta-interaction” is how many units of a receiver’s attention capacity, e.g. time, are absorbed by signals targeted on her or him.

³Also trash absorbs attention capacity before you can scrap it. As is well known, the purchase of information can never be based on perfect information. Even recognized noise, destroying the possibility to focus on other things, cannot always be turned off. This is an important difference between the problem of attention economies and *Rosen’s* [1981] ”Economics of Superstars”, where consumers have a choice between services of different quality and their willingness to pay for a unit of service is higher if the service comes from a more talented producer.

In spite of analogies to conventional economic competition, reflected in formulations like "paying" and "earning" attention, attention is not just another economic good. We do not observe markets at which the good "attention" is exchanged among traders at some given price. Attentiveness is a characteristic which can hardly be fully protected by property rights. Neither is it possible for the sender of signals to acquire full control of a person's attentiveness, nor can a person who wants to stay connected with the outside world fully control the exposure to signals trying to get her attention. Moreover, attention is a general mental resource. Who and whatever exposes you to signals can absorb some of your attention. The signals may refer to products, politics, science or entertainment. Unlike with advertising single products or signalling private information in single markets, it is not the processing of specific information content which matters for an individual's attention level but the aggregate volume and strength of signals to which she or he is exposed. Thus, the interaction of subjects sending signals for attracting the attention of other subjects has something in common with congestion or emissions in a public space. But again, there are essential differences. Whereas polluters simply want to get rid of their emissions somewhere, senders produce and target signals to achieve impact. In this paper, an attention economy is modelled as a family of *senders*, which employ costly *signals* to attract the attention of *receivers* characterized by their *attention capacity*. Only those who succeed in attracting enough attention can participate in *subsequent economic interaction* which generates *earnings*. The paper doesn't consider any details of this economic interaction but assumes that earnings are positively related to attracted attention (*impact*). The focus is on competitive attention grabbing. There is no strategic interaction. Single senders have zero measure and take attention levels as given when making their choices. The senders' simultaneous pursue of impact together with the individuals' attention capacity determine equilibrium signal exposure, equilibrium attention levels and equilibrium allocations of audiences to senders. Of course, audiences can overlap and an individual is typically a member of several audiences. This gives us a measure for the diversity of senders experienced by an individual. I call it *local diversity*. By contrast, *aggregate diversity* is given by the total measure of senders which are active

in equilibrium.⁴

The comparative-static analysis shows how changes in methods of impact generation or radiation capacities affect equilibrium signal strength, attention levels and the measure of viable companies. As a main result, it is shown that an increase in the range of radiation, allowing to companies a wider diffusion of their signals, may diminish the equilibrium set of companies although each receiving subject has access to more varieties of senders than before. In other words, aggregate diversity may decrease while higher local diversity is experienced. The reason is that different local audiences turn into more homogeneous global audiences. Natural applications of this result on the impact of extended radiation range are technical innovations like the Internet. According to evidence reported by *Graham* [2001], the number of sites that attract large shares of the time spent online substantially dropped in recent years. This doesn't mean a reduction of information received by the surfers, to the contrary. But more surfers focus on the same set of information sources. Apart from technological possibilities, institutional restrictions can prevent senders from expanding their range, for instance, to foreign audiences. Deregulation, in particular international integration, is thus another example to which the comparative-static result on the impact of an increase in the range of radiation can be applied.

The paper is organized as follows: In Section 2 the general structure of attention economies is formalized. Moreover, psychological evidence for the modelling of attention and impact is provided. In Section 3 basic insights are illustrated by a simple example. Section 4 determines equilibrium allocations of audiences to given families of senders. Sec-

⁴Usual economic analysis concentrates on the specific choices out of the varieties that are available. Under that focus, diversity is closely related to interpersonal differences in tastes and talents as emphasized in *Rosen* [2002]. The following analysis points to another important aspect of diversity. Items that are not promoted powerfully enough are not part of the choice set. Equilibrium diversity is determined by attention capacities and radiation power. Even ideas or products that would be appreciated by all consumers – if they were aware of them – may not be viable since consumer attention is distracted to more powerful signals. In personal relationships you can possibly force others to listen, but in an anonymous market an audience of positive mass must be reached.

tion 5 analyses the choice of signal strength in the contest for attention and characterizes equilibrium levels of signal exposure and attention. In Section 6 the equilibrium family of senders surviving the competition for attention is derived and comparative-static results about equilibrium diversity are presented. Section 7 summarizes.

2 The structure of an attention economy

An attention economy consists of two types of agents: The companies sending signals to earn attention, and the receivers exposed to the signals. The exogenous fundamentals of the economy are the space of receivers together with their attention capacity and the space of potential senders together with their technology. Senders can choose signal strength and the audience on which the signals are targeted.

2.1 Senders and audiences

Let receivers be given by a set of subjects S . The potential audiences for a sender are given by subsets⁵ $A \subset S$, for instance, the set of Internet users with certain surfing characteristics or the subjects on the mailing list of a marketing firm. There is no natural spatial structure like in location models so that audiences could be represented as neighborhoods around senders. The reason is that modern distribution of information typically doesn't involve distance-dependent transportation costs like the distribution of commodities. Visitors of web sites are scattered all over the world and mails can be flexibly targeted. Therefore, length or radius of some neighborhood around a sender cannot characterize the size of an audience. A natural way to deal with this problem is to represent the *space of receivers* by a measure space (S, \mathcal{A}, μ) , where \mathcal{A} is a σ -field of measurable subsets of S and μ is a finite measure on \mathcal{A} . Every element of \mathcal{A} is a subset $A \subset S$ representing a potential audience. The size of the audience is given by the measure $\mu(A)$. The following analysis focuses on contests for attention in competitive environments. One requirement for competition is

⁵ \subset denotes weak inclusion.

that there are many potential audiences for whose attention the companies can compete. In particular, it should be possible to pick also small sets of receivers. Formally, the following divisibility assumption is made: For any $A \in \mathcal{A}$ with $\mu(A) > 0$ and any constant $0 < c < \mu(A)$ there exist $B \in \mathcal{A}$ so that $\mu(B) = c$. (Take, for instance, the σ -field of *Borel* sets in a real interval S and the *Lebesgue* measure.)

Senders are the economic agents, for instance firms, who want to attract the attention of audiences. Let $L = (0, l]$ be the index set of potential senders in the economy. Thus, $t \in L$ is the name ("logo") under which a company, a scientific network or any other attention-seeking agent conveys information (signals) to receivers. In general, not all potential senders will be active. Actually, it is the purpose of the following analysis to determine the subset $T \subset L$ of companies surviving the competition for attention. Perfect competition for attention requires that single companies have zero measure. It is assumed that the potential sets of active senders are *Lebesgue*-measurable. That is, the *space of senders* is given by $(L, \mathcal{B}, \lambda)$ where \mathcal{B}, λ are the σ -field of Borel sets in L and the Lebesgue measure, respectively. Every element of \mathcal{B} is a subset $T \subset L$ representing a possible family of active senders. $\lambda(T)$ measures its size.

Each potential sender $t \in L$ is endowed with a *radiation capacity*. The radiation capacity is characterized by a real integrable function $\rho : L \rightarrow \mathbb{R}_{++}$.⁶ $\rho(t)$ describes the maximal audience size (range) that can be addressed by company t . Senders are ordered according to their size, that is, ρ is non-increasing. In reality, the maximal size of audiences that a company can handle depends on two factors: the available media and the resources attributed to the management of receiver relationship. Obviously, new media have dramatically changed the range of attention-seeking senders. *Ceteris paribus*, a larger audience can be addressed through the Internet than by phone calls. In general, such technological changes are exogenous to the single company. In the short run, also the means for entertaining communication channels to receivers, for instance the organiza-

⁶ $\mathbb{R}, \mathbb{R}_+, \mathbb{R}_{++}$ denote the sets of real numbers, non-negative real numbers, positive real numbers, respectively.

tional infrastructure for maintaining mailing lists or evaluating the impact of distributed information on the addressed audience, are fixed. The following analysis assumes that capacity $\rho(t)$ is an exogenously given characteristic of sender t . The comparative-static analysis of the impact of changes in ρ on receiver attention and the diversity of companies will be an important part of the equilibrium analysis in Section 6.

Whereas range ρ is exogenous, senders can choose their *signal strength*. This is the means by which a company attracts attention and which absorbs attention capacity of receivers. For any given set $T \in \mathcal{B}$ of senders, signal strength is represented by a real integrable function $\sigma_T : T \rightarrow \mathbb{R}_+$. For every $t \in T$, $\sigma_T(t)$ describes volume and intensity of the information sent to the members of the audience of t , for instance, the number of mails to clients per period, or the loudness or conspicuousness of the transmitted signals. Production of signal strength is costly. Also the dissemination to an audience has costs. They are independent of the size of the addressed audience, provided that the audience can be handled with t 's radiation capacity $\rho(t)$. This reflects the fact that variable transportation costs are unimportant for distributing information with modern technologies. Formally, for a company t the cost of exposing an audience A of measure $\mu(A) \leq \rho(t)$ to signal strength $\sigma \geq 0$ are given by a function

$$C_t(\sigma, \mu(A)) = c_t(\sigma), \quad (2.1)$$

where c_t is differentiable with $c'_t > 0$, $c''_t \geq 0$, starting at $c_t(0) \geq 0$. This allows for fixed costs and falling average costs. They may arise from setting up signal production or distribution capacity, i.e. the capacity to address audiences of size $\rho(t)$. Note that signal costs are allowed to be t -specific.

2.2 Audience allocations

The interaction of senders and receivers depends on which subjects are attracted by which companies. For characterizing the possible outcomes in an attention economy we must describe the assignment of receivers to companies. From the perspective of a signal

sending company the question is which audience is reached by its signals. From the side of the receiving subjects the question is to which set of companies they pay attention. I call the assignment of audiences to senders an *audience allocation*. Formally, an audience allocation for a set of senders, $T \in \mathcal{B}$, is given by a relation $\mathbf{a}_T \subset T \times S$, where \mathbf{a}_T is measurable $\mathcal{B} \times \mathcal{A}$. ($\mathcal{B} \times \mathcal{A}$ is the product space with measure $\lambda \times \mu$.) If a pair (t, s) is element of \mathbf{a}_T , then sender t addresses subject s under audience allocation \mathbf{a}_T . Thus, for any $t \in T$, the section $\mathbf{a}_T(t) := \{s \in S \mid (t, s) \in \mathbf{a}_T\}$ defines the *audience* of t . From the perspective of a receiver $s \in S$, the section $M(s, \mathbf{a}_T) := \{t \in T \mid (t, s) \in \mathbf{a}_T\}$ describes the set of perceived companies. I call $M(s, \mathbf{a}_T)$ the membership of s under \mathbf{a}_T . Note that $\mathbf{a}_T \in \mathcal{B} \times \mathcal{A}$ implies that $M(s, \mathbf{a}_T)$ and $\mathbf{a}_T(t)$ are measurable \mathcal{B} and \mathcal{A} , respectively. Not all audience allocations are feasible. The audience assigned to an active company must have positive measure and cannot be larger than the company's radiation capacity $\rho(t)$. Nor can it be larger than $\mu(S)$, the measure of all subjects in the economy. Since modern technologies have a very large range I don't want to exclude the case that senders have the capacity to cover the whole economy (i.e. $\rho(t) \geq \mu(S)$). In sum, an audience allocation \mathbf{a}_T for $T \in \mathcal{B}$ is *feasible* under radiation capacity ρ if for all $t \in T : 0 < \mu(\mathbf{a}_T(t)) \leq r_t$, where $r_t := \min\{\rho(t), \mu(S)\}$. The fact that only the size of audiences is limited by the radiation technology has far-reaching implications. It reflects that audiences of companies need not be connected areas or neighborhoods in a geographical space. This would be the case if, for instance, a sender would have a fixed focus like a cone of light. In contrast to this, according to the given notion of feasibility, companies can target their signals fully flexibly, provided the targeted audiences are not larger than what they can handle by their given capacity. I think this is the adequate modelling of the media channels through which modern companies disseminate and promote information to anonymous audiences.

Membership $M(s, \mathbf{a}_T)$ gives us a measure for *local diversity* under \mathbf{a}_T . In an economy in which audience allocation \mathbf{a}_T is realized, $\lambda(M(s, \mathbf{a}_T))$ measures the variety of companies perceived by subject s . In contrast, T is the total variety of companies perceived

somewhere in the economy. I call $\lambda(T)$ *aggregate* (or *global*) *diversity* under \mathbf{a}_T .⁷ It puts an upper limit to local diversity. If for many subjects $\lambda(M(s, \mathbf{a}_T))$ is small compared to $\lambda(T)$, then the variety of companies perceived by different subjects is relatively heterogeneous across subjects. If for all s local diversity coincides with aggregate diversity, every subject experiences the same set of companies. For instance, if T is the set of active scientific networks in the world and each subject of the scientific community is a member of all $t \in T$, then local diversity is equal to global diversity. There is a uniform international scientific community paying attention to the same set of paper series etc. If many networks $t \in T$ have only local membership, then local diversity perceived by a member of a local scientific community is small compared to the global diversity experienced by a subject moving around through all local communities.

2.3 Attention, signal exposure and impact

The basic characteristic of subjects in their role as receivers in an attention economy is their *attention capacity*. An agent may be in a more or less attentive state. If (s)he is more attentive (s)he processes received information more carefully. On the one side, this capacity depends on individual psychological factors, which can vary from individual to individual since people are heterogeneous with respect to their abilities to concentrate. (This ability may be the result of some self-management regarding a person's mental resources or time.) On the other side, the attentiveness of an individual with given psychological characteristics is also influenced by the strength and volume of signals to which she or he is exposed. Any given audience allocation \mathbf{a}_T with signal strength σ_T implies for a subject $s \in S$ a certain *signal exposure*, namely:

$$\tau_s = \int_{M(s, \mathbf{a}_T)} \sigma_T d\lambda. \quad (2.2)$$

⁷By extending the notion to sets of subjects $A \in \mathcal{A}$, we can describe the variety of companies perceived in A by $M(A, \mathbf{a}_T) := \{t \in T \mid (t, s) \in \mathbf{a}_T \text{ for some } s \in A\}$. Then, $M(S, \mathbf{a}_T) = T$ for any feasible \mathbf{a}_T . Note that for all A for which $M(A, \mathbf{a}_T)$ is measurable, $\lambda(M(A, \mathbf{a}_T))$ can be seen as diversity function in the sense of *Nehring and Puppe* [2002], since $M(A, \mathbf{a}_T) = \{t \mid \mathbf{a}_T(t) \cap A \neq \emptyset\}$ and $\lambda(M(A, \mathbf{a}_T)) = \int_M d\lambda$.

From the perspective of a sender t , grabbing for attention with strength $\sigma_T(t)$, the question is how much attention a subject exposed to τ_s pays to the signals coming from t ?

The psychological literature on attention has extensively studied the limitations in perceptual systems (see *Pashler* [1998] for a review) or more generally in mental capacity (*Kahneman* [1973]). For the purpose of this paper it is necessary to translate the findings of this literature into a formal model which is simple enough to serve as a building block for general equilibrium analysis. Three general features can be identified:

First, stimulus-processing is subject to capacity limitations in the sense that "the speed or efficiency of the processing is reduced when other stimuli are processed at the same time" (*Pashler* [1998], p. 101). *Kahneman* [1973] speaks of the "spare capacity" available for processing a signal. This suggests that the extent to which σ sent by t is processed by a receiver s depends on how much of the receiver's capacity is absorbed by all other signals τ_s to which s is exposed.⁸ We can represent this by a measurable bounded function $\nu : S \times \mathbb{R}_+ \rightarrow \mathbb{R}_{++}$, where $\nu(s, \tau_s)$ is the spare capacity left when signal exposure of s is τ_s . For each $s \in S$, the function $\nu(s, \cdot)$ models the *attention capacity* of subject s . It assigns to each level τ_s the *attention level* $v = \nu(s, \tau_s)$ available for further signal σ when s is already exposed to τ_s .⁹ It is important to keep in mind that attention *capacities* $\nu(s, \cdot)$ are exogenous individual characteristics, whereas attention *levels* v depend on the economic environment, τ_s , which is exogenous to the single agent but endogenous in the economic equilibrium.

Second, attention is not necessarily a scarce resource. *Pashler* [1998] : "... capacity limits do exist beyond a certain point; with stimulus load below this level, processing appears to be ... free of capacity limits" (p. 162). In particular, as is emphasized by *Kahneman*, increasing signal exposure mobilizes mental effort. Thus, spare capacity $\nu(s, \tau_s)$

⁸Note that a single sender t has zero measure and therefore no influence on τ_s . Formally, $\tau_s = \int_{M(s, \mathfrak{a}_T)} \sigma_T d\lambda = \int_{M(s, \mathfrak{a}_{T'})} \sigma_{T'} d\lambda$ for $T' = T - \{t\}$ and $\mathfrak{a}_T(t') = \mathfrak{a}_{T'}(t')$ for all $t' \in T'$.

⁹In *Kahneman's* dual-task framework, τ_s corresponds to the primary task, and σ to the probe signal.

may even rise with τ_s – subjects wake up. But at high levels of stimulation, there is crowding of signals. Additional processing requirements cannot fully be met by additionally mobilized efforts.¹⁰ As a consequence, spare capacity $\nu(s, \tau_s)$ declines when τ_s rises. I summarize this psychological fact by the following assumption.

Assumption SA (Scarce Attention). Attention capacities represented by ν satisfy SA if for all $s \in S$ there is a threshold $\tau_s^+ \geq 0$ so that $\nu(s, \tau'_s) < \nu(s, \tau_s)$ if $\tau'_s > \tau_s \geq \tau_s^+$.

The assumption states in a formal way that attention is scarce in an information-rich economy.¹¹ Information-rich means that signal exposure has reached a certain threshold. Scarce attention means that there is crowding of signals so that higher signal exposure τ_s implies less attention left for additional signals σ . Note that the threshold τ_s^+ above which increasing signal exposure is harmful for a subject’s level of attention can be different for different subjects.

There remains one element to be specified. By definition, in an attention economy companies send signals to earn the attention of receivers. The reason is that it is profitable to have impact, i.e., to be on the subjects’ mind. A third psychological fact helps us to relate attention to impact. ”Attended events are more likely to be perceived consciously, and more likely to be perceived in detail. They have a higher probability of eliciting and controlling responses, and they are more likely to be stored in permanent memory” (*Kahneman* [1973], p. 68). Thus, if we ask to which extent a sender’s signal σ get through and influence a receiver, the answer is that this depends on the attention level v devoted to σ by the receiver. This suggests the following model for the sender-receiver relation: The (economic value of the) *impact* of a sender t on a receiver with attention level $v (> 0)$ is a non-negative real function $z_t(\sigma, v)$ with $z_t(0, v) = 0$, which increases in σ and, for

¹⁰See *Kahneman* [1973], in particular pp. 13-17, 33-37 and 199-202.

¹¹SA puts no restriction on the specific shape of ν . For instance, ν may decline very sharply if changes in α_T, σ_T lead to an increase in signal exposure. Subjects possibly concentrate on a certain amount of information and neglect the rest more or less.

$\sigma > 0$, also in v .¹² It is assumed that z_t is measurable and bounded. (Later, for some results, additional concavity and differentiability properties will be imposed.) z_t describes how signal strength and attention generate impact. Attention level v is the only receiver characteristic which matters. There is no personalized interaction and apart from their attentiveness receivers are exchangeable. Note however that impact function z_t can vary across companies. The same signal strength may generate different impact for different types of senders.¹³

2.4 Behavior and equilibrium

Receivers react to signal exposure according to their exogenously given attention capacity ν . In the contest for attention, the active role is played by the senders. Their objective is to achieve maximal impact – net of the cost of attention grabbing.

Since the signals of company t reach all members of audience $\mathbf{a}_T(t)$, the value of total impact of $t \in T$ under \mathbf{a}_T, σ_T is given by¹⁴

$$V_t(\mathbf{a}_T, \sigma_T) = \int_{\mathbf{a}_T(t)} z_t(\sigma_T(t), \nu(s, \tau_s)) d\mu(s). \quad (2.3)$$

Combining (2.1) and (2.3) we get for the net value of impact achieved by an active sender $t \in T$ in an economy with audience allocation \mathbf{a}_T and signal strength σ_T :

$$V_t^n(\mathbf{a}_T, \sigma_T) = V_t(\mathbf{a}_T, \sigma_T) - c_t(\sigma_T(t)). \quad (2.4)$$

Sender t can choose two kinds of actions to maximize V_t^n . First, t can choose signal strength $\sigma_T(t)$. This is a standard cost-benefit calculation analyzed in Section 5.

¹²Since $v = \nu(s, \tau_s)$ is the spare capacity left for σ under signal exposure τ_s , we have $z_t(\sigma, v) = z_t(\sigma, \nu(s, \tau_s))$. Thus, Assumption SA is equivalent to saying that in an information-rich economy an increase in τ_s reduces impact z_t by crowding out t 's signals.

¹³ σ comprises only that dimension of transmitted information which absorbs attention capacity. Any other aspects, which may be relevant for impact, for instance the used medium or the clarity of t 's presentation, are captured by z_t .

¹⁴Since ν and z_t are non-negative, measurable and bounded, the integral exists (note $\mu(S) < \infty$).

Secondly, t can select its audience $\mathbf{a}_T(t)$. The selection of the audience is based on the following reasoning: As long as there is a choice between more or less attentive receivers, the impact of information can be increased by targeting it on subjects with higher attention.¹⁵ This chasing for attentive audiences is analyzed in Section 4. For a given set of active companies, an equilibrium is reached if a feasible audience allocation is realized, all companies choose their optimal signal strength and no company has an incentive to retarget its signals to more attentive audiences. The equilibrium set of active firms is determined by the zero-profit condition (Section 6). Only those companies survive which achieve a non-negative value of impact (net of the cost for generating the impact).

The general analysis of audience choice and its consequences for equilibrium requires some technical apparatus. Before turning to the general results, I will illustrate basic insights in a simple representative agent framework neglecting the problem of audience selection.

3 A simple example

Suppose that attention capacities ν are identical for all $s \in S$ and given by the function $\nu(s, \tau_s) = \tau_s^{-\beta}, \beta > 0$. Moreover, assume that signal exposure is identical across receivers, i.e. $\tau_s = \tau$ for all s . Then, all subjects are equally attentive with attention level

$$\nu = \tau^{-\beta}. \tag{3.1}$$

For instance, τ can be the number of proposals brought to our attention and ν is the attention (time, processing quality) we spend on each of them.

¹⁵This requires that companies are able to observe the attention level of subjects. In a strict sense this is only possible with direct attention-monitoring methods evaluating brain waves or eye-ball movements. Although such methods exist and are further developed – *Davenport and Beck* [2001, p. 49] speak of the "Wire'em up principle" – observable proxies for the attention of audiences are more realistic. Media watch, download statistics, speed and frequency of response, self-reported data on attentiveness are examples.

Next, suppose that all potential senders have identical radiation range $\rho(t) = \rho$ and are identical also in all other respects. Let their impact function be given by $z(\sigma, v) = \sigma v$. That means, a sender which sends σ proposals to a subject that spends attention v on each proposal attracts σv of the subject's resources. Since each sender can reach $r = \min\{\rho, \mu(S)\}$ subjects, total impact of a sender is $V = r\sigma v$. Finally, let costs be $c(\sigma) = \frac{1}{2}\sigma^2 + c_0$. Then, each active sender chooses optimal signal strength $\sigma^* = rv$ and the zero-profit condition reads:

$$V^* = \frac{1}{2}r^2v^2 - c_0 = 0. \quad (3.2)$$

If $\lambda(T)$ senders are active, the aggregate level of signal emission is $\lambda(T)r\sigma^* = \lambda(T)r^2v$. In equilibrium, this must be equal to the aggregate volume of signals received by all receivers together. Since each receiver is exposed to τ signals and $\mu(S)$ is the measure of receivers in the economy, this equilibrium condition gives us

$$\tau = \frac{\lambda(T)r^2v}{\mu(S)}. \quad (3.3)$$

System (3.1) – (3.3) defines the equilibrium values of attention v , of signal exposure τ and aggregate diversity $\lambda(T)$ of active senders. Moreover, local diversity is given by the fact that (2.2) reduces to $\tau_s = \lambda(M(s, \mathbf{a}_T))rv$, since $\sigma_T(t) = \sigma^* = rv$ in the considered example. Combining this with (3.3), we have

$$\lambda(M(s, \mathbf{a}_T)) = \frac{\lambda(T)r}{\mu(S)}. \quad (3.4)$$

The equilibrium values depend, apart from the subjective characteristic β , on the size of the economy $\mu(S)$ and on the information technology to which senders have access. This technology is reflected in cost $c(\sigma)$ as well as in radiation range ρ . The latter only matters if $\rho < \mu(S)$. In this case, we get from (3.1) – (3.4): $v = \sqrt{2c_0}/\rho$, $\sigma^* = \sqrt{2c_0}$, $\tau = \rho^{1/\beta} (2c_0)^{-\frac{1}{2\beta}}$ and

$$\lambda(T) = \frac{\mu(S) \rho^{(1-\beta)/\beta}}{\bar{c}} \quad (3.5)$$

$$\lambda(M(s, \mathbf{a}_T)) = \frac{\rho^{1/\beta}}{\bar{c}} \quad (3.6)$$

with $\bar{c} \equiv (2c_0)^{\frac{1+\beta}{2\beta}}$.

Thus, if technical progress allows to grab for attention in a wider range ρ , aggregate diversity of information suppliers $\lambda(T)$ decreases (increases) if $\beta > 1$ ($\beta < 1$, respectively). In contrast, local diversity, perceived by single subjects, increases unambiguously. Obviously, this simple example serves only illustrative purposes. With other functional forms also aggregate diversity may rise. Nor can a decline in local diversity be excluded. However, as (3.4) shows, for $r = \rho < \mu(S)$, declining local diversity is only possible if $\lambda(T)$ drops highly elastically when senders are getting access to a technology with wider range ρ . I will come back on this point at the end of Section 6, where further comparative-static results are addressed including the effects of international integration.

The presented example not only assumes very simple and specific functional forms. More importantly, it disregards the senders' audience choice and the question how an equilibrium audience allocation \mathbf{a}_T is reached. Identical signal exposure $\tau_s = \tau$ was imposed by assumption. This and the assumption of identical receiver and sender characteristics allowed us to determine the measure of T and $M(s, \mathbf{a}_T)$ without any knowledge about \mathbf{a}_T . In fact, however, as shown by (2.2), signal exposure τ_s depends on \mathbf{a}_T , where \mathbf{a}_T is the result of the senders' attempt to direct their messages to the most attentive receivers. So we have to analyze the audience choice before we can say something definite.

The further analysis proceeds as follows: In a first step, the properties of equilibrium audience allocations are determined for a given set of companies T with signal strength σ_T . It is not meaningful to look for a unique sender-receiver assignment \mathbf{a}_T , since any permutation of \mathbf{a}_T that leaves signal exposure, attention and impact unchanged is also an equilibrium. The question is whether we can establish nonetheless unique equilibria in the sense that attention levels, signal exposure, impact and diversity are uniquely determined by the fundamentals of the economy. The next section shows that, under quite general conditions, in any equilibrium allocation \mathbf{a}_T for a given set T with strength

σ_T , the attention level of subjects is uniquely determined by aggregate signal emission $X : = \int_T r_t \sigma_T(t) dt$. In a further step, I analyze, for a given set of senders T with aggregate signal level X , the optimal choice of signal strength σ_t^* of a single sender $t \in T$. Then I determine the equilibrium values of X, σ_T , which are consistent with this choice. In the third step, I determine the size of equilibrium set T^* of companies which achieve sufficient impact to survive in an attention economy and describe the characteristics of companies in T^* . The results are used to resume the comparative-static analysis of the effects of technical change and international integration on sender diversity and receiver attention.

4 Equilibrium allocation of audiences

Given a set of senders, $T \in \mathcal{B}$, with signal strength σ_T . Each company $t \in T$ can reach an audience of measure $\mu(\mathbf{a}_T(t)) \leq r_t$. Depending on which feasible audience allocation is actually realized, total impact of t is $V_t(\mathbf{a}_T, \sigma_T)$. Suppose that there is another feasible audience allocation \mathbf{a}'_T under which company t would achieve a higher impact. Then t clearly would prefer \mathbf{a}'_T to \mathbf{a}_T . However, a single company hasn't the power to decide about which audience allocation is realized in the economy. It can only chose its own audience, i.e. the set of receivers $A \in \mathcal{A}$ on which the produced volume of signals, $\sigma_T(t)$, is targeted. Thus, for $t \in T$, a deviation from an audience allocation \mathbf{a}_T to another audience allocation \mathbf{a}'_T is feasible if \mathbf{a}'_T results from \mathbf{a}_T by exchanging $\mathbf{a}_T(t)$ through $\mathbf{a}'_T(t)$ and leaving all other audience assignments unchanged. This leads to the following definition of an equilibrium allocation of audiences.

Definition 1 (*Equilibrium audience allocation*). For $T \in \mathcal{B}$ with σ_T , let \mathbf{a}_T be a feasible audience allocation. (i) An audience allocation \mathbf{a}'_T is a feasible deviation from \mathbf{a}_T for $t \in T$, if \mathbf{a}'_T is a feasible audience allocation and $\mathbf{a}'_T(t') = \mathbf{a}_T(t')$ for all $t' \in T - \{t\}$. (ii) \mathbf{a}_T is an equilibrium audience allocation if for no $t \in T$ there is a feasible deviation \mathbf{a}'_T from \mathbf{a}_T with $V_t(\mathbf{a}'_T, \sigma_T) > V_t(\mathbf{a}_T, \sigma_T)$.

Whenever a company has an opportunity to increase its impact by retargeting its signals, the company will use the opportunity. It will skip less attentive receivers and use the radiation capacity to address subjects on which the produced signals have a higher impact. If audiences are allocated to senders in a way that offers no such opportunities, no company is interested in changing the audience allocation. If subjects have very heterogeneous attention capacities, it cannot be excluded that some of them are not covered at all, that is, no company targets its signals on them. Others may be fully covered by all companies. Let $U := \{s \mid M(s, \mathbf{a}_T) = \emptyset\}$, and $F := \{s \mid M(s, \mathbf{a}_T) = T\}$ be the (possibly empty) sets of uncovered and fully covered subjects, respectively. The following proposition characterizes the attention levels of covered and uncovered subjects in an equilibrium audience allocation.

Proposition 1 . For $T \in B$ with $\sigma_T > 0$ (i.e. $\sigma_T(t) > 0$ for all $t \in T$), let \mathbf{a}_T be a feasible audience allocation.

If \mathbf{a}_T is an equilibrium audience allocation, the following conditions hold: (a) For all $t \in T : \nu(s', \tau_{s'}) \leq \nu(s, \tau_s)$ for almost all $s' \in U, s \in \mathbf{a}_T(t)$. (b) $\nu(s, \tau_s) \leq \nu(s', \tau_{s'})$, for almost all $s \in S - F, s' \in F$. (c) For $v \in \mathbb{R}_{++}$, let $S_v^- := \{s \mid \nu(s, \tau_s) < v\}$, $S_v^+ := \{s \mid \nu(s, \tau_s) \geq v\}$. For all $t \in T$: If $\mu(\mathbf{a}_T(t) \cap S_v^-) \neq 0$, then $\mu(S_v^+ - \mathbf{a}_T(t)) = 0$. (d) For all $t \in T : \mu(\mathbf{a}_T(t)) = r_t$.

If \mathbf{a}_T satisfies conditions (c) and (d) then \mathbf{a}_T is an equilibrium audience allocation.

Proof. Appendix ■

The proposition provides necessary and sufficient conditions for an equilibrium. An immediate consequence of the sufficient conditions is that any audience allocation with equalized attention levels and fully utilized radiation range is an equilibrium.

Corollary 1 . \mathbf{a}_T is an equilibrium audience allocation if $\nu(s, \tau_s) = \nu(s', \tau_{s'})$, for almost all $s, s' \in S$, and $\mu(\mathbf{a}_T(t)) = r_t$ for all $t \in T$.

Proof. Equal attention levels imply condition (c), and (d) holds by assumption. ■

The necessary conditions imply that, apart from extreme cases, attention levels *must* be equal, as the following discussion of conditions (a) - (d) shows.

Condition (a) says that only the least attentive subjects are possibly not addressed by any company, that is, are uncovered by T . Since they would pay less attention than other audiences, no company would wish to retarget its signals on them. In contrast, condition (b) deals with subjects which are in the focus of all companies. Such receivers, if there are any, must have a very high attention capacity. Despite full exposure (to all $t \in T$), their attention level is at least as high as the attention level of subjects which are not fully covered by T (i.e. for which $M(s, \mathbf{a}_T)$ is a proper subset of T). Condition (c) is the most interesting one. Any company which has people with relatively low attention level among its receivers (i.e. for which $\mu(\mathbf{a}_T(t) \cap S_v^-) \neq 0$) almost surely has also all receivers with higher attention in its audience ($\mu(S_v^+ - \mathbf{a}_T(t)) = 0$). Only a company that has exhausted all subjects with relatively high attention is willing to target its signals also on less attentive receivers. The alternative would be to leave part of the radiation capacity unused. But this would be no equilibrium, as stated by (d), since additional receivers, if they are feasible, always increase the total impact of a sender. If T contains enough small companies (i.e. $\rho(t)$ is small relative to the measure $\mu(S)$ of receivers), their chase after attentive audiences tends to equalize attention levels. To see this consider an audience allocation with two subject pools $A, A' \in \mathcal{A}$. Some companies target their signals on A , others on A' or on both sets. Small companies with $\rho(t) < \min\{\mu(A), \mu(A')\}$ can fully utilize their capacity by concentrating on one set. As long as attention levels are high in one set and low in the other, small companies can always switch to the more attentive subjects. If there are many such companies, signal exposure and attention levels change. The retargeting process only stops when attention levels in the two sets are equal or if all companies have switched to the more attentive audience. But the latter implies that this audience consists of an elite whose attention capacity is so high that they process any volume of signals better than others. The following corollary proves this intuitive argument. (A set $\tilde{U} \in \mathcal{A}$ is said to be almost uncovered if $\mu(\tilde{U} \cap \mathbf{a}_T(t)) = 0$ for all

$t \in T$. And $\tilde{F} \in \mathcal{A}$ is almost fully covered if $\mu\left(\tilde{F} - \mathbf{a}_T(t)\right) = 0$ for any $t \in T$ ¹⁶.)

Corollary 2 *If heterogeneity of radiation capacities is limited, in particular if $\rho(t) = \rho(t')$ for all $t \in T$, then in any equilibrium allocation \mathbf{a}_T one of the following three conditions must be satisfied: (a) $\nu(s, \tau_s) = \nu(s', \tau_{s'})$ almost everywhere or (b) there exists an almost fully covered set of receivers with positive measure or (c) there is a set of almost uncovered subjects with positive measure.*

Proof. Appendix. ■

The restriction on $\rho(t)$ used in the proof is that either all senders have a radiation range exceeding a certain size or all of them are smaller than this size. This is a sufficient condition for the result, not a necessary one. The corollary makes no restriction concerning heterogeneity of attention capacities. In sum, under quite general circumstances, in an attention economy in which all subjects are covered by some but not by all senders, attention levels are equal in an equilibrium allocation of audiences. Obviously, attention levels can be equal in other circumstances as well, for example, if all subjects have identical attention capacities and are fully covered. The further analysis concentrates on audience allocations with equalized attention levels. The next corollary summarizes the conditions which guarantee the existence of an equilibrium with equal attention levels.

Corollary 3 . *For $T \in \mathcal{B}$ with σ_T , an equilibrium audience allocation with equal attention levels exists if the following conditions hold: There is an integrable function $\tau(s) : S \rightarrow \mathbb{R}_+$ which solves (i) $\nu(s, \tau(s)) = \nu(s', \tau(s'))$ almost everywhere. There are decompositions $\{T_s | s \in S\}$, $\{S_t | t \in T\}$ of T and S , respectively, satisfying (ii) $\int_{T_s} \sigma_T(t) dt = \tau(s)$, (iii) $\mu(S_t) = r_t$ and (iv) $T_s = \{t | (t, s) \in \mathbf{a}\}$, $S_t = \{s | (t, s) \in \mathbf{a}\}$ for some $\mathbf{a} \in \mathcal{B} \times \mathcal{A}$.*

¹⁶Note that set U of uncovered subjects is also almost uncovered. But there may be almost uncovered sets $\tilde{U} (\supset U)$ with $\mu(\tilde{U}) > 0$ for $\mu(U) = 0$. The reason is that the union $\bigcup_{t \in T} (\tilde{U} \cap \mathbf{a}_T(t))$ over an uncountable T may have positive measure even if $\mu(\tilde{U} \cap \mathbf{a}_T(t)) = 0$ for any t . In an analogous way, $\mu(F) > 0$ implies $\mu(\tilde{F}) > 0$ but not vice versa.

Proof. Apply Corollary 1 to \mathfrak{a} . ■

The corollary states two types of requirements – economic restrictions regarding heterogeneity and mathematical requirements concerning divisibility and measurability. They interact in a difficult way. Heterogeneity of receivers is no problem as long as there is a sufficiently rich sender structure so that senders can be assigned to receivers in such a way that signal load $\tau(s)$ is distributed across subjects according to their capacities. For instance, if senders are identical, condition (ii) reduces to $\lambda(T_s)\sigma = \tau(s)$. Then we have only the divisibility and measurability problem: Are there decompositions of T and S with $\lambda(T_s) = \tau(s)/\sigma$ and $\lambda(S_t) = r$ which are sections of a measurable \mathfrak{a} ? On the other hand, if receivers are homogeneous, condition (i) is trivially fulfilled for $\tau(s) = \tau$. Thus, the question that remains is: Is there a decomposition satisfying (iii) and (iv) so that $\int_{T_s} \sigma_T(t) = \tau$ for some $\tau > 0$? If also senders are identical, any $\mathfrak{a}_T \in \mathcal{B} \times \mathcal{A}$ with $\mu(\mathfrak{a}_T(t)) = r$ and $\lambda(M(s, \mathfrak{a}_T)) = \lambda(M(s', \mathfrak{a}_T))$ is an equilibrium audience allocation. The following example describes a large class of economies with heterogeneous senders, in which divisibility is no problem and an equilibrium audience allocation with equal attention levels can be constructed explicitly.

Example 1 . For $T, \sigma_T > 0$, let heterogeneity of senders be restricted by the assumption that there is a finite partition $T_k \in \mathcal{B}$, $k = 1, \dots, K$, $0 < \lambda(T_k) < \lambda(T)$ with $\rho(t) = \rho_k$ and $\sigma_T(t) = \sigma_k$ for $t \in T_k$. Sender capacities satisfy the following divisibility condition: Senders in class k either cover all subjects (i.e. $\rho_k \geq \mu(S)$) or the subject space can be properly divided among them (i.e. $\mu(S)/\rho_k$ is an integer). Moreover, suppose that receivers' attention capacities are given by a function $f(\tau_s) = \nu(s, \tau_s)$ almost everywhere.

Fact 1. For attention economies satisfying the properties of Example 1, an equilibrium audience allocation exists with $\lambda(M(s, \mathfrak{a}_T)) = \sum_k \rho_k \lambda(T_k) / \mu(S)$, $\tau_s = \bar{\tau}$ and $\nu(s, \tau_s) = f(\bar{\tau})$, almost everywhere, where $\bar{\tau} = \sum_k \sigma_k r_k \lambda(T_k) / \mu(S)$, $r_k = \min\{\rho_k, \mu(S)\}$.

Proof. Appendix. ■

In the proof of Fact 1 a concrete audience allocation is constructed satisfying the properties of equilibrium. Other audience allocations can fulfil these properties as well. However, the construction of the equilibrium audience allocation suggests that in any equilibrium audience allocation with equal attention levels, the equilibrium attention level is uniquely determined by average signal emission $\sum_k \sigma_k r_k \lambda(T_k) / \mu(S)$. (Note that $\sigma_k r_k$ is signal emission by sender k who spreads σ_k over range r_k . And $\sum_k \sigma_k r_k \lambda(T_k)$ is aggregate signal emission.) The following proposition shows that under SA this is generally the case for equilibrium audience allocations with equalized attention levels.

Proposition 2 . For $T \in \mathcal{B}$, $\sigma_T > 0$ define $X := \int_T r_t \sigma_T(t) dt$. Under SA, there exists a decreasing real function $\bar{v}(\cdot)$ so that for any equilibrium audience allocation \mathbf{a}_T with $\tau_s \geq \tau_s^+$ for almost all s the following holds: If attention levels are equal under \mathbf{a}_T , then they are given by $\bar{v}(X)$ (i.e. $\nu(s, \tau_s) = \bar{v}(X)$ almost everywhere)¹⁷.

Proof. Appendix. ■

Assumption SA and $\tau_s \geq \tau_s^+$ mean that signal exposure in the economy is sufficiently strong to make scarcity of attention relevant. Condition $\tau_s \geq \tau_s^+$ is the formal expression of what *Simon* called "information-rich world". SA states that in an information-rich economy there is indeed a new scarcity problem, namely scarcity of attention. Thus, whereas Proposition 1 and its corollaries apply to any economy, Proposition 2 characterizes attention levels in an information-rich economy with scarcity of attention. In such an economy, in any equilibrium audience allocation with equalized attention levels the equilibrium level of attention is uniquely determined. That means, we need not be concerned about multiple equilibria. The equilibrium attention level of receivers is given by $\bar{v}(X)$

¹⁷Like in all other statements $\mu(S)$ is fixed throughout the analysis. If $\mu(S)$ changes, a different function $\bar{v}(\cdot)$ is relevant. In general, the comparative-static effects of changes in $\mu(S)$ are ambiguous. But if attention capacities are identical as in Example 1, equal attention levels imply $\tau_s = \tau = X/\mu(S)$. Then the equilibrium attention level is a function of $X/\mu(S)$. At the end of this paper this fact is used to illustrate possible consequences of international integration in an attention economy (see discussion of Fact 3).

which is independent of the signal strength $\sigma_T(t)$ of a single sender ($\lambda(\{t\}) = 0$). Hence, each sender $t \in T$ can take the attention level of its receivers as given when deciding about signal strength. This decision and the implied equilibrium signal strength are analyzed in the next section.

5 Competition for attention and equilibrium signal exposure

The question solved so far was: Given a set of companies T with signal strength σ_T , what are the properties of equilibrium audience allocations under T, σ_T . This leaves open the questions: (i) Which sets of companies are equilibrium sets, and (ii) which signal strength do companies choose to have an optimal impact on their audience? In this section the second question is answered. Given an audience allocation \mathbf{a}_T for $T \in \mathcal{B}$, what are equilibrium choices $\sigma_T(t), t \in T$?

In an attention economy, the essential performance measure is impact. According to (2.4), the net value of impact of t sending with strength $\sigma \in \mathbb{R}_+$ is

$$V_t^n = \int_{\mathbf{a}_T(t)} z_t(\sigma, \nu(s, \tau_s)) d\mu(s) - c_t(\sigma). \quad (5.1)$$

Signal strength σ is *optimal* for t under \mathbf{a}_T, σ_T if V_t^n reaches a maximum at σ . A single company, having zero measure, has no influence on τ_s . Thus, any $t \in T$ takes $\nu(s, \tau_s)$ as given when deciding about its signal strength.

Definition 2 (*Equilibrium signal strength*). For $T \in \mathcal{B}$, let \mathbf{a}_T be a feasible audience allocation. (i) σ_T is an equilibrium under \mathbf{a}_T if, for all $t \in T, \sigma_T(t)$ is optimal under \mathbf{a}_T, σ_T . (ii) \mathbf{a}_T, σ_T is an equilibrium for T , if \mathbf{a}_T is an equilibrium audience allocation for T, σ_T , and σ_T is an equilibrium under \mathbf{a}_T .

As shown in Proposition 2 under quite general assumptions, in an equilibrium audience allocation \mathbf{a}_T with signal strength σ_T attention levels of subjects are given by a decreasing

function $\nu(s, \tau_s) = \bar{v}(X)$ of aggregate signal emission X . Moreover, $\mu(\mathbf{a}_T(t)) = r_t$ in an equilibrium audience allocation. Thus, the net value of total impact which is achieved by $t \in T$ is given by

$$V_t^n = r_t z_t(\sigma_T(t), \bar{v}(X)) - c_t(\sigma_T(t)). \quad (5.2)$$

According to (5.2), expanding signal strength $\sigma_T(t)$ has a positive effect on company t 's reception for any given X . But if a positive mass of companies increases signal strength, then X increases, which has an external effect on other companies. Under the assumptions of Proposition 2, $\bar{v}(X)$ decreases in X so that the external effect is negative.¹⁸ t ignores this effect when deciding about the optimal $\sigma_T(t)$. Determining equilibrium signal strength therefore requires two steps. First, one has to clarify which $\sigma_T(t)$ is optimal for individual companies $t \in T$ if aggregate signal emission is X . Second, one has to look for an equilibrium which satisfies $X = \int_T r_t \sigma_T(t) dt$.

The choice of optimal strength σ is a standard optimization problem. Since the innovation of this paper lies elsewhere, I want to keep this problem simple enough to apply differential calculus for deriving comparative-static results about the choice of signal strength. So far only measurability and boundedness of impact function $z_t(\sigma, v)$ was required. Moreover, $z_t(0, v) = 0$, z_t increasing in σ and, for $\sigma > 0$, increasing in v . From now on it is assumed that z_t is twice continuously differentiable and satisfies the following concavity property:¹⁹

$$\frac{\partial^2 z_t}{\partial \sigma^2} < 0, \frac{\partial^2 z_t}{\partial v \partial \sigma} \geq 0. \quad (5.3)$$

Moreover, $\lim_{\sigma \rightarrow 0} \frac{\partial z_t}{\partial \sigma} = \infty$ and $\lim_{\sigma \rightarrow \infty} \frac{\partial z_t}{\partial \sigma} = 0$.

¹⁸The idea to internalize such negative external effects is probably the reason why people have proposed to impose a price on sending electronic mails. See *Kraut, Sunder, Morris, Telang, Filer and Cronin* [2002] for laboratory experiments on such proposals.

¹⁹These are sufficient conditions guaranteeing generally the existence of a positive optimal signal strength with comparative-static properties as required for an equilibrium. They need not hold in concrete examples, in which the solution can be calculated explicitly.

With this specification the outcome of the optimization of signal strength is a monotonous function of audience size and attention level.

Proposition 3 . For T, σ_T let \mathbf{a}_T be a feasible audience allocation with $\nu(s, \tau_s) = v$ almost everywhere. If cost and impact functions satisfy (2.1) and (5.3), respectively, then: (a) A company's optimal signal strength is given by a differentiable function $\sigma_t^*(\mu(\mathbf{a}_T(t)), v)$ with $\partial\sigma_t^*/\partial\mu > 0$ and $\partial\sigma_t^*/\partial v \geq 0$. (b) For all $t \in T$, the maximal net-value of impact is given by a differentiable increasing function of $\mu(\mathbf{a}_T(t))$ and v .

Proof. Appendix. ■

According to Part (a), larger companies send more strongly. According to (b), they are also better off. Moreover, any company is better off when attention is high. But no company is inclined to reduce signal strength when v is high. To the contrary, $\partial\sigma^*/\partial v \geq 0$, i.e. senders tend to increase their attention grabbing effort if facing more attentive receivers..

The next task is to characterize equilibrium signal strength. For this purpose two further purely technical restrictions have to be imposed on the functions representing attention capacities, signal impacts and costs. First, it is assumed that the space of receivers (S, \mathcal{A}, μ) is given by a real interval S with \mathcal{A} the σ -field of *Borel* sets in S and μ the *Lebesgue* measure. Moreover, it is assumed that for all s the function representing attention capacities, $\nu(s, \tau)$, is differentiable with respect to τ . Secondly, it is assumed that impact and cost functions vary across t in a measurable way. Formally, for all σ, v , the derivatives $\partial z_t(\sigma, v)/\partial\sigma$ and $c'_t(\sigma)$ are measurable functions of t . (Note that this is trivially fulfilled if z_t, c_t are identical for all t .) These properties guarantee the following facts which are useful for the derivation of equilibrium signal strength.

Lemma 1 . Function $\bar{v}(\cdot)$ in Proposition 2 is differentiable ($d\bar{v}/dX < 0$). Function $\sigma_T(t) := \sigma_t^*(\mu(\mathbf{a}_T(t)), v)$ of optimal signal strength, derived in Proposition 3, is measurable.

Proof. Appendix. ■

With this preparation, equilibrium signal strength can be determined as follows: In an equilibrium audience allocation $\mu(\mathbf{a}_T(t)) = r_t$. Thus, according to Proposition 3, optimal strength of $t \in T$ is given by $\sigma_t^*(r_t, v)$. Since r_t is an exogenous characteristic of t , argument r_t can be omitted without loss of information. Thus, in the further exposition I write $\sigma_t^*(v)$ instead of $\sigma_t^*(r_t, v)$. Equilibrium signal strength σ_T must satisfy two conditions: It must be consistent with the optimal choice of individual companies, i.e.:

$$\sigma_T(t) = \sigma_t^*(v) \quad (5.4)$$

for all $t \in T$. And σ_T must be consistent with attention level v . If the assumptions of Proposition 2 are fulfilled, this attention level is given by a decreasing function $\bar{v}(X)$, i.e.

$$v = \bar{v}(X) \quad \text{with } X = \int_T r_t \sigma_T(t) dt. \quad (5.5)$$

(Lemma 1 guarantees that σ_t^* is measurable so that the integral defining X exists.) Combining the two conditions (5.4) and (5.5), we obtain the equation

$$X = Z(X, T, r_T) \quad (5.6)$$

with $Z(X, T, r_T) := \int_T r_t \sigma_t^*(\bar{v}(X)) dt \geq 0$ and r_T denoting the function on T assigning to each $t \in T$ range r_t . According to Lemma 1, $d\bar{v}/dX < 0$. Since σ_t^* is differentiable and non-decreasing in v , $\partial Z/\partial X \leq 0$ so that equation (5.6) defines for each T, r_T a unique equilibrium level $X^*[T, r_T]$ of aggregate signal emission. Together with (5.4) and (5.5), this defines also a unique attention level $v_T^* := \bar{v}(X^*[T, r_T])$ and a unique equilibrium signal strength $\sigma_T^*(t) := \sigma_t^*(v_T^*)$. The following theorem summarizes this important result.

Theorem 1 . *Under SA, in any equilibrium \mathbf{a}_T, σ_T for $T \in \mathcal{B}$, in which attention levels are equalized and $\tau_s \geq \tau_s^+$ for almost all s , equilibrium signal strength σ_T^* and equilibrium attention level v_T^* are uniquely determined by T and r_T .*

Proof. Main text. ■

Assumption SA and $\tau_s \geq \tau_s^+$ mean that receivers are strained by the prevailing signal exposure, so that their attention capacity is decreasing in the relevant range. In other words, signal exposure is strong enough to make the economy information rich and attention scarce. Uniqueness of σ_T^* and v_T^* allows us to derive comparative-static properties of equilibria in an attention economy. Regarding the attention level induced by a given set of active senders we have the following result.

Theorem 2 . *Under the assumptions of Theorem 1: If $\rho(t) < \mu(S)$ and $\rho(t)$ increases to $\tilde{\rho}(t) > \rho(t)$ on a measurable subset $T_0 \subset T, 0 < \lambda(T_0)$, then equilibrium attention v_T^* declines.*

Proof. Appendix. ■

The theorem means that in an information-rich economy expansion of ρ aggravates the problem of scarce attention. Two channels are effective. An increase in the range of radiation, allowing a wider diffusion of signals, induces companies to increase their signal strength in competing for attention. The reason is that diffusion of signals within the feasible range has zero marginal cost so that the production of impact by addressing signals on audiences is subject to economies of scale. At the same time, there is a second effect leading to higher signal exposure of subjects even without such economies of scale at the company level. Any given signal strength reaches more subjects when the radiation range of a company is extended. As long as the set of companies doesn't change, this necessarily means more overlap among audiences. For instance, if a national scientific network extends its range to the international level it has to address subjects belonging to other national or international communities. As a consequence, these subjects will be exposed to more publications. If signal exposure was high before, relative to threshold τ_s^+ defined by SA, then the increased volume of signals will be perceived with less attention. Of course, the described phenomenon isn't specific to scientific communities. Info-stress

and attention deficit are not uncommon. According to Theorem 2, a responsible economic factor is the possibility to address larger audiences.

The extension of the range of one company may wipe out other companies so that T , which up to now has been taken as given, changes. This brings us to the question of how equilibrium diversity is determined in an attention economy. This question is answered in Section 6. Before turning to this section, I want to illustrate Theorem 1 and Theorem 2 by the following example.

Example 2 . Suppose that T can be partitioned in K measurable sets T_1, \dots, T_K with $\rho(t) = \rho_k$ for $t \in T$, where the divisibility condition of Example 1 is satisfied. Like in Example 1 let $\nu(s, \tau_s)$ be given by a non-increasing function $f(\tau_s)$. Moreover, for $t \in T_k$, $c_t(\sigma) = c_1^k \sigma + c_0^k$ for some constants $c_0^k \geq 0$, $0 < c_1^k < r_k$ and, for $\sigma > 1$, $z_t(\sigma, v) = g_k(v) \ln \sigma + h_k(v)$, where $g_k(v) \geq 1$, $h_k(v) \geq 0$ for $v > 0$, and $g'_k \geq 0$, $h'_k \geq 0$ with one inequality holding strictly. (For $\sigma \leq 1$, $z_t(\sigma, v) = 0$ is assumed.)

Fact 2. For any T satisfying the properties of Example 2, (a) an equilibrium \mathbf{a}_T, σ_T with equalized attention levels exists, and (b) in any such equilibrium $\sigma_k^* = r_k g_k(v^*) / c_1^k$, $v^* = f(\tau^*)$ where signal exposure of subjects is given by a function $\tau^*(\lambda(T_1), \dots, \lambda(T_K), \rho_1, \dots, \rho_K)$ with $\partial \tau^* / \partial \lambda(T_k) > 0$ and $\partial \tau^* / \partial \rho_k > 0$ if $\rho_k < \mu(S)$, $k = 1, \dots, K$.

Proof. Appendix. ■

6 Viability and equilibrium diversity in an attention economy

The notion of a competitive environment requires: Agents must be viable, and free entry is allowed. In an attention economy, in which it is vital to attract attention, a natural notion of viability is that companies achieve a non-negative net-value $V_t^n \geq 0$ from sending costly

signals for having impact on audiences. Free entry means that companies can participate in the contest for attention if they want.

For any single company the situation looks as follows: Given a set $T \in \mathcal{B}$ of active companies, a feasible audience allocation \mathbf{a}_T and some signal strength $\sigma_T > 0$, the net-value of impact achieved by $t \in T$ under \mathbf{a}_T, σ_T is $V_t^n(\mathbf{a}_T, \sigma_T)$ given by (2.4). Company $t \in T$ is viable if $V_t^n(\mathbf{a}_T, \sigma_T) \geq 0$. Companies $t' \in L - T$ face the following entrance problem. If $t' \notin T$ participates by targeting $\sigma > 0$ on an audience $A \in \mathcal{A}$, the set of active companies becomes $T' = T \cup \{t'\}$. Since $\lambda(\{t'\}) = 0$, this doesn't change attention levels of subjects. Thus, t' achieves maximal impact when picking an audience A^* with $\mu(A^*) = \min\{\rho(t'), \mu(S)\}$, in such a way that $\nu(s, \tau_s) \geq \nu(s', \tau_{s'})$ for all $s \in A^*$, $s' \in S - A^*$, and maximal net-value of impact when choosing optimal signal strength $\sigma_{t'}^* = \arg \max_{A^*} \int z_{t'}(\sigma, \nu(s, \tau_s)) d\mu(s) - c_{t'}(\sigma)$. Denote by $V_{t'}^{n*}(\mathbf{a}_T, \sigma_T)$ the maximal net-value of impact achieved by t' when participating in the contest for attention in an optimal way. Company t' is viable if $V_{t'}^{n*}(\mathbf{a}_T, \sigma_T) \geq 0$. It definitely has an interest to enter if the achieved value is strictly positive. If $V_{t'}^{n*}(\mathbf{a}_T, \sigma_T) = 0$, it is indifferent with respect to entry.

Definition 3 (*Free entry equilibrium*). $T, \mathbf{a}_T, \sigma_T$ is a free entry equilibrium if \mathbf{a}_T, σ_T is an equilibrium for T (according to Definition 2) and the following property is satisfied: For all $t \in T$, $V_t^n(\mathbf{a}_T, \sigma_T) \geq 0$, and $V_{t'}^{n*}(\mathbf{a}_T, \sigma_T) \leq 0$ for any $t' \in L - T$.

The further analysis is restricted to economies in which attention levels are equalized in equilibrium. In such an economy, equilibrium signal strength and equilibrium attention level are uniquely determined for a given set T of active companies (see Theorem 1). They are given by

$$\sigma_T^*(t) = \sigma_t^*(v_T^*) \text{ with } d\sigma_t^*/dv \geq 0 \quad (6.1)$$

$$v_T^* = \bar{v}(X^*) \text{ with } d\bar{v}/dX^* < 0 \quad (6.2)$$

respectively, where X^* is implicitly defined by the condition (see (5.6)):

$$X^* = \int_T r_t \sigma_T^*(t) dt. \quad (6.3)$$

This implies that also the maximal net-value of impact achieved by a company $t \in T$ is unique in an equilibrium for T .

For any $t \in T$ facing equalized attention levels v , the net-value which t achieves when addressing a maximally feasible audience with signal strength σ is given by²⁰ $V_t^n(\sigma, v) := r_t z_t(\sigma, v) - c_t(\sigma)$, where c_t, z_t satisfy (2.1), (5.3). The maximal net-value under v is $V_t^n(\sigma_t^*(v), v)$. Thus, in an equilibrium with equalized attention levels v_T^* , for every $t \in L$, the maximal net-value that can be achieved by t is

$$V_t^{n*} \equiv V_t^n(\sigma_t^*(v_T^*), v_T^*) = r_t z_t(\sigma_t^*(v_T^*), v_T^*) - c_t(\sigma_t^*(v_T^*)). \quad (6.4)$$

(Note that no single company has an impact on v and any company, $t \in T$ as well as $t' \in L - T$, is confronted with the same attention level v).²¹

Theorem 1 has shown for a given set of active companies T , that in an equilibrium with equalized attention levels the equilibrium level of attention is unique. The following theorem shows that in any free-entry equilibrium with equalized attention levels the equilibrium level of attention is unique.

Theorem 3 . *For given fundamentals $(\rho(t), z_t, c_t, t \in L)$, and attention capacities $\nu(s, \cdot)$, $s \in S$) there exists a unique $v^* \in \mathbb{R}_{++}$ so that in any free-entry equilibrium $T^*, \mathbf{a}_{T^*}, \sigma_{T^*}^*$ full-filling the assumptions of Theorem 1 (and thus (6.1) - (6.4)) the equilibrium level of attention is equal to v^* . Also aggregate signal emission X^* is uniquely determined.*

²⁰So far function symbol V_t^n was used for $V_t^n(\mathbf{a}_T, \sigma_T)$ in a given context \mathbf{a}_T, σ_T . Now, the relevant context is captured by a single variable v . For saving notation I use the same symbol to denote net-value as a function of σ and v .

²¹The notational distinction between $V_t^n(\mathbf{a}_T, \sigma_T)$, for $t \in T$, and $V_t^{n*}(\mathbf{a}_T, \sigma_T)$ for $t \in L - T$, is no longer necessary. Both values are given by $V_t^n(\sigma_t^*(v), v)$, where v is the attention level implied by \mathbf{a}_T, σ_T .

Proof. Appendix. ■

The theorem says that in an information-rich economy with scarcity of attention, competitive grabbing for attention leads to unique levels of signal exposure and attention, determined by sender technologies and receiver capacities. However, the theorem leaves open how many senders are active in equilibrium.

According to Definition 3, for identifying a possible equilibrium set T^* of active companies we must check for which $t \in L$ the values given by (6.4) are positive, zero, or negative. Since companies may differ in capacity $\rho(t)$, impact function z_t or cost c_t , it is not possible to compare them without further restrictions. In the following an ordering on the set of potential companies L is assumed. I call t *more powerful* (or *stronger*) than t' under v if $r_t \sigma_t^*(v) > r_{t'} \sigma_{t'}^*(v)$, i.e. if total signal emission coming from t is higher than from t' when both companies send with optimal strength. The following monotonicity property says that stronger companies are also more valuable.

Assumption M (Monotonicity). Heterogeneity of $\rho(t), z_t, c_t, t \in L$, is restricted in such a way that, for all $v \in \mathbb{R}_{++}$, $V_t^n(\sigma_t^*, v) > V_{t'}^n(\sigma_{t'}^*, v)$ if t is more powerful than t' under v (i.e. if $r_t \sigma_t^*(v) > r_{t'} \sigma_{t'}^*(v)$).

Power is represented by a high $r_t \sigma_t^*$, that means, by sending widely and loudly. To assume that this is valuable in terms of impact seems to be natural in an attention economy. Being small and decent doesn't pay. Property M trivially holds if all potential companies have identical fundamentals so that no company is stronger than another. Also if they differ only in range $\rho(t)$ while having identical impact and cost functions, Property M follows immediately from (6.4) and the envelope theorem. But M is generally satisfied when the fundamentals of companies can be ranked in a clear way. I call t *non-inferior* to t' if $r_t \geq r_{t'}$, $\partial z_t / \partial \sigma \geq \partial z_{t'} / \partial \sigma$, $c_t \leq c_{t'}$.²² If at least one inequality holds strictly, t is

²²Since z_t, c_t are functions of v and σ , the respective inequalities can hold locally or globally. In the following analysis only the local properties at $\sigma_t^*(v)$ are relevant. However, we make comparative-static analysis with respect to v .

superior to t' . Thus, t is superior if it has a larger range or if it is better in the sense of having higher marginal impact or lower marginal cost of signal production. The following lemma shows how this ranking of companies in terms of fundamentals implies a ranking in terms of power and value.

Lemma 2 . *Suppose that L can be ordered so that t is non-inferior to t' if $t \leq t'$. Then: (a) t is more powerful than t' if and only if t is superior to t' . (b) If for all v, t and t' , $\partial z_t / \partial \sigma > (=) \partial z_{t'} / \partial \sigma$ implies $z_t > (=, \text{resp.}) z_{t'}$, and $c'_t < (=) c'_{t'}$ implies $c_t < (=, \text{resp.}) c_{t'}$, then Property M is fulfilled.*

Proof. Appendix. ■

Part (a) shows that companies can be ranked according to their emission power if they can be ranked according to radiation capacity, marginal impact and marginal cost of signal provision. Part (b) shows that this ranking implies a ranking in terms of the net-values of impact which can be achieved by the respective companies. For instance, if companies have different radiation capacities $\rho(t)$ but have access to the some technology for signal and impact productions so that c_t and z_t are identical, then companies with larger range send more powerfully and achieve a higher net-value of impact. In an analogous way, a ranking according to power and net-value is possible if companies have access to the same radiation technology so that $\rho(t) = \rho(t')$ for all $t, t' \in L$, but some of them have a cost advantage in signal production or an impact advantage in the sense that they achieve higher impact with the same signal strength. According to (b), a sufficient condition for M is that an advantage at the margin means also an advantage overall.

Theorem 4 . *Let $T^*, \alpha_{T^*}, \sigma_{T^*}^*$ be a free-entry equilibrium fulfilling the assumptions of Theorem 1 (and thus (6.1) – (6.4)). Under M: (a) $\lambda(T^*)$ is uniquely determined by the fundamentals of the attention economy. (b) If L can be ordered so that t is non-inferior to t' if $t \leq t'$, then: t superior to t' and $t' \in T^*$ imply $t \in T^*$ and $V_t^{n*} > V_{t'}^{n*}$. (V_t^{n*} evaluated at the (unique) equilibrium attention level v^* .)*

Proof. Appendix. ■

Part (a) of the theorem answers the question of how many senders are viable in an economy with scarce attention. Even though in general there is no unique equilibrium set of active companies T^* and no unique equilibrium audience allocation, equilibrium measure $\lambda(T^*)$ is uniquely determined. Hence, comparative-static analysis about equilibrium diversity is possible. If senders are identical it doesn't matter which subset $T \subset L$ of equilibrium measure is active. If there is heterogeneity of potential senders, in an equilibrium those companies dominate which are large or have a better technology for impact production (Part (b) of the theorem). They send more powerfully (see Lemma 2) and survive the contest of attention. An immediate consequence of (b) is that $t' \in L - T^*$ cannot be superior to $t \in T^*$.

In the following comparative-static analysis $\mu(S)$ and attention capacities are kept unchanged while the companies' fundamentals change from $b_t \equiv (r_t, z_t, c_t)$ to $\tilde{b}_t = (\tilde{r}_t, \tilde{z}_t, \tilde{c}_t)$, $t \in L$. I say that \tilde{b} is a *progress* over b on $E \in \mathcal{B}$ if \tilde{b} and b coincide on $L - E$ and, for every $t \in E$, \tilde{b}_t is advantageous compared to b_t in the following sense: For $t \in E$, $\tilde{r}_t \geq r_t$, $\tilde{z}_t \geq z_t$, $\partial \tilde{z}_t / \partial \sigma \geq \partial z_t / \partial \sigma$, $\tilde{c}_t \leq c_t$, $\tilde{c}'_t \leq c'_t$ and at least one of the following properties hold: $\tilde{r}_t > r_t$, or $\tilde{z}_t > z_t$ and $\partial \tilde{z}_t / \partial \sigma > \partial z_t / \partial \sigma$, or $\tilde{c}_t < c_t$ and $\tilde{c}'_t < c'_t$. In words, companies in E experience progress in radiation capacity or in impact or signal production.

Theorem 5 . *Consider two (otherwise identical) attention economies with sender fundamentals b and \tilde{b} , respectively. Assume that under b as well as under \tilde{b} Assumption M is satisfied. Let $T^*, \alpha_{T^*}, \sigma_{T^*}^*$ and $\tilde{T}, \alpha_{\tilde{T}}, \tilde{\sigma}_{\tilde{T}}$ be free-entry equilibria for the respective economies satisfying the assumptions of Theorem 1. Let $T_0 := \{t \in T^* \mid V_t^{n*} = 0\}$ be the (possibly empty) set of marginal companies under b . If \tilde{b} is a progress over b on some $E \subset T^*$, $\lambda(E) > 0$, then: (a) $\tilde{v} \leq v^*$. (b) $\lambda(\tilde{T}) < \lambda(T^*)$, if $\tilde{v} = v^*$ or if $\lambda(T_0 - E) > 0$. (\tilde{v}, v^* denote the respective equilibrium attention levels. Their uniqueness is guaranteed by Theorem 3.)*

Proof. Appendix. ■

The theorem answers the question, how in an information-rich economy with scarcity of attention, equilibrium attention and equilibrium diversity depend on the fundamentals of the economy. If there is progress in the means of grabbing for attention, equilibrium attention certainly does not increase (Part (a)) and necessarily declines if no senders vanish (first if-clause in Part (b)). Regardless of whether or not the attention level declines, a sufficient condition for a reduction of the measure of active senders is that not all marginal senders are subject to the considered progress (second if-clause in Part (b)). However, as Example 3 will show, also a uniform increase in radiation possibilities for all potential senders, or international integration may lead to declining attention and diversity. The economic mechanism behind this result is as follows: If companies get access to a radiation technology allowing a wider range of receivers, to a more powerful impact technology, or to less costly signal production, they produce and distribute stronger signals. Since the increased signal exposure leads to a decline in the receivers' attention level, other senders with weaker signals are no longer sufficiently perceived to achieve viable impact. From the point of view of receivers, this means that the diversity of senders changes. However, the impacts on aggregate and local diversity must be carefully distinguished.

Aggregate diversity addressed in Theorem 4 and 5 is important from the perspective of an outside observer of the world. How many different senders – i.e. producers and emitters of signals pointing to ideas, products, issues – survive in a given attention economy? Why may small networks, local journals or national scientific communities vanish? By contrast to this global perspective, for every single receiver the experienced variety of senders is given by the measure $\lambda(M(s, \mathbf{a}_{T^*}))$ of the senders of which s is a member. I called it local diversity. By definition, for all $s \in S$, $M(s, \mathbf{a}_{T^*}) \subset T^*$. Local diversity is limited by aggregate diversity. Nonetheless, $M(s, \mathbf{a}_{T^*})$ may increase for some or even all s even if $\lambda(T^*)$ is reduced. More formally, let $m^* \equiv (1/\mu(S)) \int_S \lambda(M(s, \mathbf{a}_{T^*})) d\mu(s)$ denote average local equilibrium diversity and $\bar{r} \equiv (1/\lambda(T^*)) \int_{T^*} r_t dt$ denote average range of active senders. Since the aggregate measure of memberships is equal to the aggregate

measure of audiences,²³ local diversity experienced on average is given by the equation

$$m^* = \bar{r}\lambda^*/\mu(S) \text{ with } \lambda^* \equiv \lambda(T^*). \quad (6.5)$$

Obviously, the effect of r_t -changes on m^* is ambiguous if the effect on λ^* is negative. In the simple example considered in Section 3, an increase of the radiation capacity of senders can lead to a decline in aggregate diversity, while diversity experienced from the local perspective of every single receiver definitely rises. The further analysis looks at the robustness of this outcome. As a first step, I examine the comparative-static properties of attention economies satisfying the features of Example 2. Moreover, I consider the effects of international integration.

Example 3 . *Suppose that potential senders are identical with $\rho(t) = \rho > 1$, $c_t(\sigma) = \sigma + c_0$ and $z_t(\sigma, v) = g(v) (\ln \sigma + \gamma)$ if $\sigma > 1$, where $c_0 > 0$, $0 \leq \gamma \leq 1$ and $g(v) = g_0 v^\alpha$, $g_0 \in \mathbb{R}_{++}$, $\alpha > 0$. (Note that these are instances of the cost- and impact functions discussed in Example 2.) Moreover, for all $s \in S$, $\nu(s, \tau_s) = \tau_s^{-\beta}$, $\beta > 0$.*

We know already that for such an economy an equilibrium with equalized attention levels exists for a given set $T \subset L$ of active companies. (It is assumed that L is sufficiently large so that not all potential senders are active in an equilibrium.) Moreover, since all potential senders are identical, M is satisfied and thus, according to Theorem 4, the measure $\lambda(T^*)$ of active companies is unique in a free entry equilibrium. Both aggregate and local diversity are characterized by the following fact:

Fact 3. In an attention economy as described in Example 3: (a) Aggregate diversity is given by $\lambda^* = \lambda_0 \mu(S) r^{(1-\alpha\beta)/(\alpha\beta)}$, where $r = \min \{\rho, \mu(S)\}$ and λ_0 is a positive constant which is positively related to γ, g_0 and negatively related to c_0 . (b) Local diversity is given by $m^* = \lambda_0 r^{1/(\alpha\beta)}$.

Proof. Appendix. ■

²³Set $\sigma_T = 1$, $\mu(a_T(t)) = r_t$ in Lemma A1, to get $\int_S \lambda(M(s, a_T)) d\mu(s) = \int_T r_t dt$.

For the evaluation of comparative-static effects on equilibrium diversity two cases must be distinguished. If $\mu(S) \leq \rho$, λ^* reduces to $\lambda_0 \mu(S)^{1/(\alpha\beta)}$ and $m^* = \lambda^*$. If $\mu(S) > \rho$, then $r = \rho$ and λ^* decreases (increases) with ρ if $\alpha\beta > 1$ ($\alpha\beta < 1$, respectively), whereas local diversity m^* unambiguously increases. Note that α is the elasticity of economic impact with respect to attention level v . And β is the elasticity with which a receiver's spare capacity for processing further signals, $\nu(s, \tau_s)$, declines when signal exposure τ_s rises. Thus $\alpha\beta > 1$, if economic impact sensitively depends on attention and/or attention declines with signal exposure relatively rapidly.

From an economic point of view the comparative-static analysis leads to important insights concerning effects of "globalization" on diversity. Globalization in an attention economy means that senders get access to a larger space of receivers. This can have two reasons: International integration or progress in the radiation technology.

International integration is relevant if the radiation technology is sufficiently advanced so that audiences larger than the population of an isolated economy can be addressed. Integration allows to senders to exploit these possibilities. To see the effects on diversity, consider two identical economies with receiver set S^i , $\mu(S^i) = \mu$, $i = H, F$. Senders have access to a radiation technology which allows them to cover more than S^i , but not the whole world $S^W = S^H \cup S^F$, $\mu(S^W) = 2\mu$. This means, they have identical radiation capacities with $\mu < \rho < 2\mu$. Thus, $r_a = \min\{\rho, \mu\} = \mu$ in the "closed economy" and $r_W = \min\{\rho, 2\mu\} = \rho$ in the integrated world. Using this in Fact 3, we obtain for equilibrium diversity in the closed economy: $\lambda_a^* = m_a^* = \lambda_0 \mu^{1/(\alpha\beta)}$. This gives for total diversity in the world: $\lambda_{tot}^* = 2\lambda_a^*$. In the integrated world with cross-border radiation, equilibrium diversity is given by $\lambda_W^* = \lambda_0 2\mu \rho^{(1-\alpha\beta)/(\alpha\beta)}$ and $m_W^* = \lambda_0 \rho^{1/(\alpha\beta)}$. Comparing autarky diversity with diversity in the integrated world, we get: $m_W^* > m_a^*$ since $\rho > \mu$. Thus, local diversity experienced by an average receiver is larger under international integration than in a closed economy. However, with respect to global diversity the effect of integration is ambiguous. We have $\lambda_W^* < \lambda_{tot}^*$ if $1 > (\mu/\rho)^{1-1/(\alpha\beta)}$. Since $\mu < \rho$ was assumed, this implies $\lambda_W^* < \lambda_{tot}^*$ if $\alpha\beta > 1$. For $\alpha\beta < 1$, $\lambda_W^* > \lambda_{tot}^*$. Although each subject

is exposed to more different senders, fewer senders are active after integration. The reason is that senders have a radiation technology under which international radiation is feasible, and international integration allows full use of the radiation range. As a consequence, more subjects are exposed to the same senders and, if $\alpha\beta > 1$, i.e. if the reaction of attention to signal exposure and the relation of economic impact to attention are relatively elastic, some senders are driven out of the market.

The second source of globalization in an attention economy is technical progress leading to an increase in the feasible range of radiation. To study the effect on diversity, suppose there is an integrated world with $\mu(S^W) = \mu_W$ and identical senders whose range increases from ρ_I to $\rho_{II} > \rho_I$ where $1 < \rho_I < \mu_W$ is assumed. Then, according to Fact 3, diversity under range ρ_I is given by $\lambda_I^* = \lambda_0 \mu_W \rho_I^{(1-\alpha\beta)/(\alpha\beta)}$, $m_I^* = \lambda_0 \rho_I^{1/(\alpha\beta)}$ whereas under range ρ_{II} diversity is $\lambda_{II}^* = \lambda_0 \mu_W r_{II}^{(1-\alpha\beta)/(\alpha\beta)}$, $m_{II}^* = \lambda_0 r_{II}^{1/(\alpha\beta)}$ where $r_{II} = \min\{\mu_W, \rho_{II}\} > \rho_I$. Thus, again $m_{II}^* > m_I^*$, and $\lambda_{II}^* < \lambda_I^*$ ($\lambda_{II}^* = \lambda_I^*$, $\lambda_{II}^* > \lambda_I^*$) if $\alpha\beta > 1$ ($\alpha\beta = 1$, $\alpha\beta < 1$, respectively).

In the above example local diversity rises with r even when global diversity falls. Although this is not necessarily so in general, it is not just an incidental possibility. According to (6.5), for $\bar{r} < \mu(S)$, local diversity can only fall if aggregate diversity drops highly elastically in reaction to wider radiation. To get a sufficient condition for rising local diversity consider symmetric companies and receivers with $\nu(s, \tau) = f(\tau)$, $f' < 0$. Then, $V^n = r z(\sigma, v) - c(\sigma)$ and the first-order condition for optimal signal strength $\sigma^*(r, v)$ is:

$$r z_\sigma(\sigma, v) - c'(\sigma) = 0. \quad (6.6)$$

Combining this with zero-profit condition

$$r z(\sigma^*(r, v), v) - c(\sigma^*(r, v)) = 0, \quad (6.7)$$

we get $v(r)$ with $v'(r) = -\frac{z}{r z_v}$. (Subscripts of z denote partial derivatives.)

On the other hand, $v = f(\tau)$, where $\tau = m \sigma^*(r, v)$. In sum, we have the equation

$$f(m \sigma^*(r, v(r))) - v(r) = 0, \quad (6.8)$$

characterizing equilibrium diversity m^* as a function of range r . Implicit differentiation gives us:²⁴

$$\frac{dm^*}{dr} > 0 \text{ iff } \frac{z|D|}{rm} > |f'| (z_\sigma z_v - z z_{\sigma v}), \quad (6.9)$$

where $D \equiv r z_{\sigma\sigma} - c'' < 0$ is the second-order condition for (6.6). According to (6.9), a positive impact of r on local diversity is more likely if attention drops little under increased signal exposure (i.e. $|f'|$ is small). Moreover, a sufficient condition for $dm^*/dr > 0$ is:

$$\frac{z_v}{z} \leq \frac{z_{\sigma v}}{z_\sigma}. \quad (6.10)$$

Hence, local diversity certainly rises with r if the (marginal) effect of signal strength on impact is at least as sensitive to receiver attention as the level of impact, i.e. if the interaction of signal strength and attention level is important for generating impact.²⁵

7 Conclusion

The presented theory explains the basic mechanisms at work in an economy in which earning attention and achieving impact are prerequisites of economic viability.

The exogenous fundamentals are on the one side the space of receivers and their attention capacities, and on the other side the potential set of senders and their radiation and impact technologies.

The endogenous variables explained by the theory are equilibrium audiences (the clients belonging to a company), equilibrium signal exposure and attention, and the measure of active senders in a free-entry equilibrium. Although there are multiple equilibrium allocations of audiences to senders, equilibrium signal strength, equilibrium attention level and equilibrium measure of active senders can be uniquely characterized.

²⁴From (6.8) we get $\frac{dm}{dr} >, =, < 0$ if $|v'| >, =, < |f'| m \left(\frac{\partial \sigma^*}{\partial r} + \frac{\partial \sigma^*}{\partial v} v' \right)$, respectively. Using $v'(r) = -\frac{z}{r z_v}$ from (6.7) and $\frac{\partial \sigma^*}{\partial r} = \frac{z_\sigma}{|D|}$, $\frac{\partial \sigma^*}{\partial v} = \frac{r z_{\sigma v}}{|D|}$ from (6.6) we can rewrite this condition as (6.9).

²⁵For $z(\sigma, v) = g(\sigma) h(v)$, $g' > 0$, $h' > 0$, condition (6.10) trivially holds.

For an information-rich economy with scarcity of attention, the theory predicts that changes allowing to senders more powerful signal emission – for instance, an extension of feasible radiation ranges, cheaper signal production, more effective methods of impact generation, but also international integration – tend to decrease global diversity of senders and attention levels of subjects. Declining local diversity, measured by the variety of senders experienced by the individual receivers, is less likely.

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Appendix

We prove first two lemmas which are important in several proofs.

Lemma A1. For $T, \sigma_T, \mathfrak{a}_T \subset T \times S$, let $b(t, s) := \begin{cases} \sigma_T(t), & \text{if } (t, s) \in \mathfrak{a}_T \\ 0 & , \text{ otherwise} \end{cases}$

be the signal emission on $s \in S$ from $t \in T$ under \mathfrak{a}_T . For any $\mathfrak{a}_T \in \mathcal{B} \times \mathcal{A}$: (a) $b(t, s)$ is measurable $\mathcal{B} \times \mathcal{A}$. (b) $\int_{M(s, \mathfrak{a}_T)} \sigma_T(t) dt$ is a measurable function on S . (c) $\int_S \int_{M(s, \mathfrak{a}_T)} \sigma_T(t) dt d\mu(s) = \int_T \mu(\mathfrak{a}_T(t)) \sigma_T(t) dt$.

Proof. (a) Define $\tilde{\sigma}_T(t, s) = \sigma_T(t)$. For any $y \in \mathbb{R}_+$, the set $\{(t, s) | \tilde{\sigma}_T(t, s) > y\} = \{t | \sigma_T(t) > y\} \times S$ is a measurable rectangle, since σ_T is measurable \mathcal{B} . Thus, $\tilde{\sigma}_T(t, s)$ is measurable $\mathcal{B} \times \mathcal{A}$. Since $b(t, s) = I_{\mathfrak{a}_T} \tilde{\sigma}_T$, where $I_{\mathfrak{a}_T}$ is the indicator function on \mathfrak{a}_T , also $b(t, s)$ is measurable $\mathcal{B} \times \mathcal{A}$. Properties (b) and (c) follow from *Fubini's theorem* (see e.g. *Billingsley [1995]*, p. 234). Note that $b(t, s)$ is non-negative. Moreover, $\int_T b(t, s) dt = \int_{M(s, \mathfrak{a}_T)} \sigma_T(t) dt$ and $\int_S b(t, s) d\mu(s) = \int_{\mathfrak{a}_T(t)} \sigma_T(t) d\mu(s) = \mu(\mathfrak{a}_T(t)) \sigma_T(t)$, since $b(t, s) = 0$ for $t \notin M(s, \mathfrak{a}_T)$ or $s \notin \mathfrak{a}_T(t)$. QED.

Lemma A2. Suppose that \mathfrak{a}_T is an equilibrium audience allocation for $T, \sigma_T > 0$. If there exist sets $A, A' \in \mathcal{A}$ of positive measure so that $\nu(s, \tau_s) < \nu(s', \tau_{s'})$ for all $s \in A, s' \in A'$, then: $\mu(A' - \mathfrak{a}_T(t)) = 0$ or $\mu(\mathfrak{a}_T(t) \cap A) = 0$, for all $t \in T$.

Proof. Suppose that $\mu(A' - \mathfrak{a}_T(t)) > 0$ and $\mu(\mathfrak{a}_T(t) \cap A) > 0$ for some $t \in T$. The divisibility assumption imposed on (S, \mathcal{A}, μ) implies that $A' - \mathfrak{a}_T(t)$ and $\mathfrak{a}_T(t) \cap A$ contain measurable subsets B' and B , respectively, with $\mu(B) = \mu(B') > 0$. Let \mathfrak{a}'_T be the audience allocation resulting from \mathfrak{a}_T when $\mathfrak{a}_T(t)$ is replaced by $\mathfrak{a}'_T(t) := (\mathfrak{a}_T(t) - B) \cup B'$. Since $\mu(\mathfrak{a}'_T(t)) = \mu(\mathfrak{a}_T(t)) \leq \rho(t)$, \mathfrak{a}'_T is a feasible deviation for t . We show that it is attractive for t to deviate from \mathfrak{a}_T to \mathfrak{a}'_T . If t retargets its signals from $\mathfrak{a}_T(t)$ to $\mathfrak{a}'_T(t)$, then for $s \in B$ membership changes to $M(s, \mathfrak{a}'_T) = M(s, \mathfrak{a}_T) - \{t\}$, whereas

for $s \in B'$ membership changes to $M(s, \mathbf{a}'_T) = M(s, \mathbf{a}_T) \cup \{t\}$. Since singletons have zero measure, $\int_{M(s, \mathbf{a}_T)} \sigma_T(t) dt = \int_{M(s, \mathbf{a}'_T)} \sigma_T(t) dt$ for all $s \in S$. Thus, signal exposure τ_s is not affected by the deviation from \mathbf{a}_T to \mathbf{a}'_T . Since $B \subset A$ and $B' \subset A'$, we have $\nu(s, \tau_s) < \nu(s', \tau_{s'})$ for $s \in B$ and $s' \in B'$. Moreover, since z_t is increasing in v we obtain $V_t(\mathbf{a}'_T, \sigma_T) = \int_{\mathbf{a}'_T(t)} z_t(\sigma_T(t), \nu(s, \tau_s)) d\mu(s) - \int_B z_t(\sigma_T(t), \nu(s, \tau_s)) d\mu(s) + \int_{B'} z_t(\sigma_T(t), \nu(s, \tau_s)) d\mu(s) > V_t(\mathbf{a}_T, \sigma_T)$. This is a contradiction to the assumption that \mathbf{a}_T is an equilibrium audience allocation. QED.

The rest of the Appendix contains the proofs of the claims in the text.

Proof of Proposition 1. Necessity of (a) – (d): (a) Suppose there are $A, A' \in \mathcal{A}$ with positive measure so that for all $s \in A, s' \in A', \nu(s, \tau_s) < \nu(s', \tau_{s'}), M(s', \mathbf{a}_T) = \emptyset$ and $s \in \mathbf{a}_T(t)$ for some t . According to Lemma A2, $\mu(A' - \mathbf{a}_T(t)) = 0$ or $\mu(A \cap \mathbf{a}_T(t)) = 0$. The first clause implies that almost all $s' \in A'$ belong to $\mathbf{a}_T(t)$ which contradicts $M(s', \mathbf{a}_T) = \emptyset$. The second clause contradicts the assumption that all $s \in A$ belong to $\mathbf{a}_T(t)$. (b) Suppose there are $A, A' \in \mathcal{A}$ with positive measure so that $\nu(s', \tau_{s'}) < \nu(s, \tau_s), M(s', \mathbf{a}_T) = T$ and $M(s, \mathbf{a}_T) \neq T$ for all $s \in A, s' \in A'$. According to Lemma A1, $\mu(A - \mathbf{a}_T(t)) = 0$ or $\mu(A' \cap \mathbf{a}_T(t)) = 0$ for all $t \in T$. However, the first clause cannot hold for $t \in T - M(s, \mathbf{a}_T)$, and the second clause contradicts $M(s', \mathbf{a}_T) = T$ for $s' \in A'$. (c) Follows immediately from Lemma A1. (d) Suppose that $\mu(\mathbf{a}_T(t)) < r_t$. The divisibility property imposed on (S, \mathcal{A}, μ) implies that a measurable set $A \subset S - \mathbf{a}_T(t)$ exists with $\mu(A) = r_t - \mu(\mathbf{a}_T(t))$. Let \mathbf{a}'_T be the audience allocation resulting from \mathbf{a}_T when $\mathbf{a}_T(t)$ is replaced by $\mathbf{a}'_T(t) := \mathbf{a}_T(t) \cup A$. Since $\mu(\mathbf{a}'_T(t)) = r_t$, \mathbf{a}'_T is a feasible deviation for t . Moreover, by targeting the unused radiation capacity $\rho(t) - \mu(\mathbf{a}_T(t))$ on A , company t attracts additional attention implying $V_t(\mathbf{a}'_T, \sigma_T) > V_t(\mathbf{a}_T, \sigma_T)$. (Note that $\mu(A) > 0$ and $z_t(\sigma_T(t), v) > 0$ for $\sigma_T(t) > 0$.) This is a contradiction to the assumption that \mathbf{a}_T is an equilibrium allocation.

Sufficiency of (c) and (d): Because of (d), no company can increase the measure of its audience. Thus, $V_t(\mathbf{a}_T, \sigma_T)$ can only be increased if t retargets its signals $\sigma_T(t)$ from

a measurable subset $A \subset \mathfrak{a}_T(t)$ to a measurable subset $A' \subset S - \mathfrak{a}_T(t)$, $\mu(A') \leq \mu(A)$, so that $\int_A z_t(\sigma_T(t), \nu(s, \tau_s)) d\mu(s) < \int_{A'} z_t(\sigma_T(t), \nu(s, \tau_s)) d\mu(s)$. This is only possible if there exist measurable $B \subset A$, $B' \subset A'$ with $\mu(B) = \mu(B') > 0$ so that $\nu(s, \tau_s) < \nu(s', \tau_{s'})$ for almost all $s \in B$, $s' \in B'$. In sum, there must be B, B' with $\mu(B \cap \mathfrak{a}_T(t)) = \mu(B) > 0$, $\mu(B' \cap \mathfrak{a}_T(t)) = 0$ and higher attention levels in B' than B . This is in contradiction to Condition (c). Thus no such deviation exists. QED.

Proof of Corollary 2. Take S_v^-, S_v^+ as defined in Proposition 1.

Step 1: Since $\mu(\mathfrak{a}_T(t)) = \min\{\rho(t), \mu(S)\}$, $\rho(t) \geq \mu(S_v^+)$ implies $\mu(\mathfrak{a}_T(t) \cap S_v^-) \neq 0$ or $\mu(\mathfrak{a}_T(t)) = \mu(S_v^+)$. This implies $\mu(S_v^+ - \mathfrak{a}_T(t)) = 0$, according to Proposition 1 (c). An analogous argument leads to $\mu(\mathfrak{a}_T(t) \cap S_v^-) = 0$ if $\rho(t) < \mu(S_v^+)$.

Step 2: By definition, $\mu(S_v^+)$ is a non-increasing function of v starting at $\mu(S_0^+) = \mu(S)$ and eventually reaching zero since ν is bounded. $\mu(S_v^+) = \mu(S)$ implies that almost all subjects have at least attention level v , whereas $\mu(S_v^+) = 0$ means that almost all subjects have lower attention level than v . Hence, $\nu(s, \tau_s) = v^+$ for almost all s if and only if $\mu(S_v^+) = \mu(S)$ for all $v \leq v^+$ and $\mu(S_v^+) = 0$ for all $v > v^+$. (The if part is obvious. For the only if part suppose that $0 < \mu(S_v^+) < \mu(S)$ for some $v > 0$. Then also $\mu(S_v^-) > 0$ and $\nu(s, \tau_s) \geq v > \nu(s', \tau_{s'})$ for $s \in S_v^+, s' \in S_v^-$.)

Step 3: Suppose that Property (a) in Corollary 2 does not hold. Then, according to Step 2, $0 < \mu(S_v^+) < \mu(S)$ for some v . Suppose that heterogeneity of radiation capacities is restricted so that either $\rho(t) \geq \mu(S_v^+)$ for all t or $\rho(t) < \mu(S_v^+)$ for all t . In the first case, Step 1 implies $\mu(S_v^+ - \mathfrak{a}_T(t)) = 0$ for all t and thus Part (b) of the corollary holds for $\tilde{F} = S_v^+$. In the second case, Step 1 implies (c) with $\tilde{U} = S_v^-$. QED.

Proof of Fact 1. For $k = 1, \dots, K$, define $n_k := \max\{1, \mu(S)/\rho_k\}$ and $m_k := \lambda(T_k)r_k/\mu(S)$. By assumption, n_k is a natural number. Decompose S into n_k subsets $S_k^i, i = 1, \dots, n_k$, of equal size $\mu(S_k^i) = r_k$ so that $\bigcup_i S_k^i = S$ and $S_k^i \cap S_k^j = \emptyset$ for $i \neq j$. Moreover, decompose T_k into n_k subsets $T_k^i, i = 1, \dots, n_k$, of equal size $\lambda(T_k^i) = m_k$ so

that $\bigcup_i T_k^i = T_k$ and $T_k^i \cap T_k^j = \emptyset$ if $i \neq j$. Then, the audience allocation \mathbf{a}_T defined by $\mathbf{a}_T(t) = S_k^i$ if $t \in T_k^i$ satisfies the following properties: (i) $\mu(\mathbf{a}_T(t)) = r_k$ for all $t \in T$. (ii) For any $s \in S$ and $k \in \{1, \dots, K\}$ there is $i(k, s) \in \{1, \dots, n_k\}$, so that $s \in S_k^{i(k, s)}$. Moreover, $M(s, \mathbf{a}_T) \cap T_k = T_k^{i(k, s)}$ and $\tau_s = \int_{M(s, \mathbf{a}_T)} \sigma_T(t) dt = \sum_k \sigma_k \lambda(T_k^{i(k, s)}) = \sum_k \sigma_k m_k$. Thus, for all s , $\nu(s, \tau_s) = f\left(\sum_k \sigma_k m_k\right)$. Property (i) guarantees that no company can increase its impact by increasing the size of its audience and Property (ii) implies equal attention levels. Thus, according to Corollary 1, \mathbf{a}_T is an equilibrium. QED.

Proof of Proposition 2. Suppose that there are two audience allocations $\mathbf{a}_T^1, \mathbf{a}_T^2$ with the required properties, i.e. $\nu(s, \tau_s^i) = \tilde{v}_i$ for some constant \tilde{v}_i and $\tau_s^i \geq \tau_s^+$, where $\tau_s^+ = \int_{M(s, \mathbf{a}_T^i)} \sigma_T(t) dt, i = 1, 2$. Suppose $\tilde{v}_1 \neq \tilde{v}_2$, say $\tilde{v}_2 < \tilde{v}_1$. Then, because of SA, $\tau_s^2 > \tau_s^1$ for almost all s . This implies $\int_S \tau_s^2 d\mu(s) > \int_S \tau_s^1 d\mu(s)$, in contradiction to Lemma A1 (c), according to which $\int_S \tau_s d\mu(s) = X$ for any allocation \mathbf{a}_T with $\mu(\mathbf{a}_T(t)) = r_t, t \in T$. Hence, for any given $X, \tilde{v}_1 = \tilde{v}_2 \equiv \tilde{v}$. Define $\bar{v}(X) = \tilde{v}$. Suppose next that $\rho(t), t \in T$, or σ_T change, so that X increases to X' . Let $\mathbf{a}_T, \mathbf{a}'_T$ be audience allocations with the required properties (i.e. $\mu(\mathbf{a}_T(t)) = r_t, \mu(\mathbf{a}'_T(t)) = r'_t, \tau_s \geq \tau_s^+, \tau'_s \geq \tau_s^+, \nu(s, \tau_s) = \tilde{v}, \nu(s, \tau'_s) = \tilde{v}'$, where notation is analogous to before). $X' > X$ implies $\int_S \tau'_s d\mu(s) > \int_S \tau_s d\mu(s)$ and thus $\tau'_s > \tau_s$ on a set $A \subset S$ with positive measure. Because of SA, $\nu(s, \tau'_s) < \nu(s, \tau_s)$ for $s \in A$ and thus $\tilde{v}' < \tilde{v}$. QED.

Proof of Proposition 3. (a) By definition, $V_t^n = \mu(\mathbf{a}_T(t)) z_t(\sigma, v) - c_t(\sigma)$, if $\nu(s, \tau_s) = v$ for almost all s . Thus, the first-order condition for $\max_{\sigma} V_t^n(\sigma)$ is given by

$$F \equiv \mu(\mathbf{a}_T(t)) \frac{\partial z_t(\sigma, v)}{\partial \sigma} - c'_t(\sigma) = 0.$$

The assumptions in (2.1) and (5.3) guarantee that the equation has a solution and implicitly defines $\sigma_t^*(\mu(\mathbf{a}_T(t)), v) > 0$ with $\partial \sigma_t^* / \partial \mu > 0$ and $\partial \sigma_t^* / \partial v \geq 0$. The second-order condition $\partial F / \partial \sigma < 0$ holds because of $\partial^2 z_t / \partial \sigma^2 < 0$ and $c''_t \geq 0$. (b) According to (a), at σ_t^* we have $V_t^n = \mu(\mathbf{a}_T(t)) z_t(\sigma_t^*(\cdot), v) - c_t(\sigma_t^*(\cdot))$. Applying the envelope theorem, we

obtain $\partial V_t^n / \partial \mu = z_t > 0$ and $\partial V_t^n / \partial v = \mu(\mathbf{a}_T(t)) \partial z_t / \partial v > 0$. QED.

Proof of Lemma 1. For $v \in (0, \nu(s, \tau_s^+))$, the equation $\nu(s, \tau) = v$ defines for all s signal exposure $\tau(s, v) > \tau_s^+$ where $\tau(s, v)$ is differentiable in v with $\partial \tau / \partial v < 0$. Thus, $\int_S \tau(s, v) ds$ is a differentiable and decreasing function of v . Since $\int_S \tau(s, v) ds = X$, according to Lemma A1 (c), v is a differentiable decreasing function of X .

According to the proof of Proposition 3, for given $\mu(\mathbf{a}_T(t)), v$, optimal signal strength $\sigma_t^*(\mu_T(\mathbf{a}_T(t)), v)$ is determined by the first-order condition $F(t, \sigma) = \mu(\mathbf{a}_T(t)) \frac{\partial z_t(\sigma, v)}{\partial \sigma} - c_t'(\sigma) = 0$. Since $\partial F / \partial \sigma < 0$, $\sigma_t^* \leq y$ if and only if $F(t, y) \leq 0$ for any $y \in \mathbb{R}_+$. Thus, $\{t | \sigma_T(t) \leq y\} = \{t | F(t, y) \leq 0\}$ which is a measurable set, if the functions $a(t) := \partial z_t(\sigma, v) / \partial \sigma$ and $b(t) := c_t'(\sigma)$ are measurable. QED.

Proof of Theorem 2. Only the impact on $X^*[T, r_T]$ has to be proved. The effect on v_T^* follows from SA. According to Proposition 3, $\sigma_t^*(\tilde{r}_t, \bar{v}(X)) > \sigma_t^*(\rho(t), \bar{v}(X))$ for all $t \in T_0$. (Note $\tilde{r}_t = \min\{\mu(S), \tilde{\rho}(t)\} > \rho(t)$.) This implies $Z(X, T, \tilde{r}_T) > Z(X, T, r_T)$, where Z is defined in (5.6). Since $\partial Z / \partial X \leq 0$, the equilibrium level \tilde{X} defined by the equation $Z(\tilde{X}, T, \tilde{r}_T) = \tilde{X}$ is higher than the level defined by $Z(X, T, r_T) = X$. QED.

Proof of Fact 2. Let \mathbf{a}_T be the audience allocation constructed in the proof of Fact

1. Chose v and τ such that $v = f(\tau)$ and

$$\tau = \frac{1}{\mu(S)} \sum_k \lambda(T_k) r_k^2 g_k(v) / c_k^1. \quad (\text{A.1})$$

Then for $t \in T_k$:

$$V_t^n = r_k [g_k(v) \ln \sigma + h_k(v)] - c_1^k \sigma - c_0^k \quad (\text{A.2})$$

and $\arg \max V_t^n = \sigma_k^*$ where

$$\sigma_k^* = r_k g_k(v) / c_k^1. \quad (\text{A.3})$$

For $\tau_s = \tau$, Lemma A1, Part (c) implies $\mu(S) \tau = \int_T r_t \sigma_T(t) dt = \sum_k r_k \sigma_k \lambda(T_k)$ and

thus:

$$\tau = \frac{X}{\mu(S)} \text{ with } X = \sum_k r_k \sigma_k \lambda(T_k). \quad (\text{A.4})$$

Substituting σ_k^* for σ_k we get (A.1) with $v = f(\tau)$. Thus, $\sigma_T(t) = \sigma_k^*$, $t \in T$, is an equilibrium under \mathbf{a}_T . Obviously, \mathbf{a}_T is an equilibrium for T, σ_T since $\nu(s, \mathbf{a}_T) = v$ for all s and $\mu(\mathbf{a}_T(t)) = r_t$ for all $t \in T$.

For proving (b), note that in any equilibrium with equalized attention levels we must have $\tau_s = \tau$, $f(\tau) = v$ and $\mu(\mathbf{a}_T(t)) = r_k$, $t \in T_k$. Thus, (A.2) – (A.4) hold. They imply equation (A.1). Since $v = f(\tau)$, $f' \leq 0$, $g'_k \geq 0$, this equation implicitly defines τ as a function $\tau^*(\lambda(T_1), \dots, \lambda(T_K), \rho_1, \dots, \rho_K)$ with $\partial\tau^*/\partial\lambda(T_k) > 0$ and $\partial\tau^*/\partial\rho_k > 0 (= 0)$ if $\rho_k < \mu(S)$ ($\geq \mu(S)$, respectively). (Substitute $r_k = \min\{\rho_k, \mu(S)\}$ into (A.1).) (Moreover, $\tau^*(\cdot)$ is lower (higher) in a large economy of size $\mu(S')$ than in a small economy $\mu(S) < \mu(S')$ if $\rho_k < \mu(S)$ ($\rho_k \geq \mu(S)$, respectively) for all $k = 1, \dots, K$.) QED.

Proof of Theorem 3. Suppose that $T_i^*, \mathbf{a}_{T_i^*}, \sigma_{T_i^*}^*$, $i = 1, 2$, are free-entry equilibria with equilibrium attention levels $v_{T_i^*}^* := \bar{v}_i(X_i^*)$, respectively. For simplifying notation, set $v_i := v_{T_i^*}^*$ and $\sigma_t^i := \sigma_t^*(v_i)$, for any $t \in L$. Moreover, denote by τ_s^i signal exposure of s in equilibrium i and define $B_i := T_i^* - (T_1^* \cap T_2^*)$. Then,

$$X_1^* = \int_{T_1^* \cap T_2^*} r_t \sigma_t^1 dt + \int_{B_1} r_t \sigma_t^1 dt, \quad X_2^* = \int_{T_1^* \cap T_2^*} r_t \sigma_t^2 dt + \int_{B_2} r_t \sigma_t^2 dt. \quad (\text{A.5})$$

Assume that $v_1 = \bar{v}_1(X_1^*) < v_2 = \bar{v}_2(X_2^*)$. (Since Indices 1 and 2 can be exchanged, the following contradiction also applies to $v_2 < v_1$, which establishes $v_1 = v_2$.) Note first that $v_1 < v_2$ implies $V_t^2 \equiv V_t^n(\sigma_t^2, v_2) \geq V_t^n(\sigma_t^1, v_2) > V_t^n(\sigma_t^1, v_1) \equiv V_t^1$, for any $t \in L$. (The first inequality follows from the definition of σ_t^i as the optimal choice under v_i . The second inequality follows from $\partial V_t^n / \partial v = r_t \partial z_t / \partial v > 0$.) I show that $v_1 < v_2$ and $V_t^2 > V_t^1, t \in T_1^*$, lead to a contradiction: (i) On the one hand, since SA with $\tau_s^i \geq \tau_s^+$ is assumed, $v_1 < v_2$ implies $\tau_s^1 > \tau_s^2$ and $\int_S \tau_s^1 d\mu(s) > \int_S \tau_s^2 d\mu(s)$. Thus, according to Lemma A1, $X_1^* > X_2^*$. (ii) On the other hand, viability of $t \in T_1^*$ requires $V_t^1 \geq 0$ so that $V_t^2 > V_t^1$ implies $V_t^2 > 0$. Hence, $T_1^* \subset T_2^*$, $B_1 = \emptyset$, $B_2 = T_2^* - T_1^*$. Moreover, for all

$t \in T_1^* = T_1^* \cap T_2^*$, $\sigma_t^1 \leq \sigma_t^2$ because of $\partial\sigma_t^i/\partial v \geq 0$ (see Proposition 3). Combining this with (A.5), we conclude $X_1^* \leq X_2^*$. This contradicts (i). Finally, $v_1 = v_2$ for all s implies $\tau_s^1 = \tau_s^2$ and thus $X_1^* = \int_S \tau_s^1 d\mu(s) = \int_S \tau_s^2 d\mu(s) = X_2^*$. QED.

Proof of Lemma 2. (a) The first-order condition defining $\sigma_t^*(v)$ is $F(\sigma, v) \equiv r_t \partial z_t(\sigma, v) / \partial \sigma - c_t'(\sigma) = 0$. Since $\partial F / \partial \sigma < 0$, $\sigma_t^* \geq \sigma_{t'}$, if t is non-inferior to t' . $\sigma_t^* > \sigma_{t'}$, if t superior to t' . Thus, t superior to t' is sufficient for $r_t \sigma_t^* > r_{t'} \sigma_{t'}^*$. It is also necessary: Suppose t is not superior to t' . Then t is identical to t' or t' is superior to t , which would imply $r_t \sigma_t^* \leq r_{t'} \sigma_{t'}^*$. (b) By definition, $V_t^n(\sigma_t^*, v) = r_t z_t(\sigma_t^*, v) - c_t(\sigma_t^*)$. According to (a), $r_t \sigma_t^* > r_{t'} \sigma_{t'}^*$ implies t is non-inferior to t' and (i) $r_t > r_{t'}$, or (ii) $\partial z_t / \partial \sigma > \partial z_{t'} / \partial \sigma$ or (iii) $c_t' < c_{t'}'$. In case (i) $V_t^n > V_{t'}^n$ (evaluated at the respective arguments) follows from the assumption that non-inferiority implies $z_t \geq z_{t'}$ and $c_t \leq c_{t'}$. In the cases (ii), (iii), it follows from non-inferiority of t and the assumption that (ii) implies $z_t > z_{t'}$ and (iii) implies $c_t < c_{t'}$. QED.

Proof of Theorem 4. (a) Suppose that there are two equilibria T_1^*, T_2^* with $\lambda(T_1^*) < \lambda(T_2^*)$ and thus $0 \leq \lambda(B_1) < \lambda(B_2)$, where $B_i := T_i^* - (T_1^* \cap T_2^*)$. According to Theorem 2, $v_1 = v_2 = v^*$, and thus $X_1^* = X_2^*$, $\sigma_t^1 = \sigma_t^2 \equiv \sigma_t$, $V_t^n(\sigma_t^i, v_i) = V_t^n(\sigma_t, v^*)$ for any $t \in L$. (Notation as in proof of Theorem 3.) This implies

$$V_t^* \equiv V_t^n(\sigma_t, v^*) = 0 \text{ for } t \in B_1 \cup B_2. \quad (\text{A.6})$$

(Note that $B_1 \subset T_1^*$, $B_2 \subset T_2^*$ imply $V_t^n(\sigma_t, v^*) \geq 0$ for $t \in B_1 \cup B_2$. But since $B_j \cap T_i^* = \emptyset$, $i \neq j$, $V_t^n(\sigma_t, v^*) = V_t^n(\sigma_t^i, v_i) \leq 0$ if $t \in B_j$.)

Moreover, according to (A.5), $X_1^* = X_2^*$, $\sigma_t^1 = \sigma_t^2$ imply

$$\int_{B_1} r_t \sigma_t dt = \int_{B_2} r_t \sigma_t dt. \quad (\text{A.7})$$

Since $\lambda(B_1) < \lambda(B_2)$, equation (A.7) can only hold if $\bar{B}_i \subset B_i$, $\lambda(\bar{B}_i) > 0$, exist so that $r_t \sigma_t > r_{t'} \sigma_{t'}$ for $t \in \bar{B}_1, t' \in \bar{B}_2$. Thus, according to M , $V_t^n(\sigma_t, v^*) > V_{t'}^n(\sigma_{t'}, v^*) \geq 0$ for $t \in \bar{B}_1 \subset B_1, t' \in \bar{B}_2 \subset B_2$, in contradiction to (A.6).

(b) $t' \in T^*$ implies $V_{t'}^* \geq 0$. According to Lemma 2 (a), t superior to t' implies t is more powerful than t' and thus, according to $M, V_t^* > V_{t'}^* \geq 0$. Hence, $t \in T^*$. QED.

Proof of Theorem 5. Theorem 3 and 4 guarantee that unique values for equilibrium attention v^*, \tilde{v} , aggregate signal emission X^*, \tilde{X} , and measures $\lambda(T^*), \lambda(\tilde{T})$ of the set of active companies exist in the two equilibria.

(a) Assume that $v^* < \tilde{v}$: Denote $\tilde{V}_t^n(\sigma, v) := \tilde{r}_t \tilde{z}_t(\sigma, v) - \tilde{c}_t(\sigma)$ and $\tilde{\sigma}_t(v) := \arg \max \tilde{V}_t^n(\sigma, v)$. Then, $\tilde{V}_t \equiv \tilde{V}_t^n(\tilde{\sigma}_t(\tilde{v}), \tilde{v}) > \tilde{V}_t^n(\tilde{\sigma}_t(v^*), v^*)$, since $\partial \tilde{V}_t^n / \partial v = \tilde{r}_t \partial \tilde{z}_t / \partial v > 0$, due to the envelope theorem. Moreover, for all $t \in L$, $\tilde{V}_t^n(\tilde{\sigma}_t(v^*), v^*) \geq \tilde{V}_t^n(\sigma_t^*(v^*), v^*) \geq V_t^n(\sigma_t^*(v^*), v^*) \equiv V_t^{n*}$. (The first inequality follows from the definition of $\tilde{\sigma}_t$ as the optimal choice under \tilde{V}_t^n . The second inequality follows from $\tilde{V}_t^n(\sigma, v) \geq V_t^n(\sigma, v)$ by construction (\tilde{b} was assumed to be a progress over b). In sum, $\tilde{V}_t^{n*} > V_t^{n*}$ for all $t \in L$. According to the proof of Theorem 3, $\tilde{V}_t^{n*} > V_t^{n*}, t \in T^*$ and $v^* < \tilde{v}$ lead to a contradiction. (Note that the argument did not rely on the underlying fundamentals.) Thus, $v^* \geq \tilde{v}$. This proves Part (a).

(b) (i) By construction, for all $t \in E, \tilde{\sigma}_t(v) > \sigma_t^*(v)$ and $\tilde{r}_t \tilde{\sigma}_t(v) > r_t \sigma_t^*(v)$ for any v . Moreover, for $t \in E, \tilde{V}_t^n(\sigma, v) = \tilde{r}_t \tilde{z}_t(\sigma, v) - \tilde{c}_t(\sigma) > V_t^n(\sigma, v) = r_t z(\sigma, v) - c_t(\sigma)$ for any σ, v . For $t \in L - E, \tilde{\sigma}_t(v) = \sigma_t^*(v), \tilde{r}_t \tilde{\sigma}_t(v) = r_t \sigma_t^*(v)$ and $\tilde{V}_t^n(\sigma, v) = V_t^n(\sigma, v)$.

(ii) Assume $v^* = \tilde{v}$: Then, $X^* = \tilde{X}$ (employ argument at the end of proof of Theorem 3). According to (i), $\tilde{V}_t^n(\tilde{\sigma}_t(v), v) \geq \tilde{V}_t^n(\sigma_t^*(v), v) > V_t^n(\sigma_t^*(v), v)$ for all $t \in E$. Thus, $E \subset \tilde{T} \cap T^*$ since $E \subset T^*$ by assumption. Moreover, for all $t, \tilde{\sigma}_t \geq \sigma_t^*$ (I omit the argument $\tilde{v} = v^*$) with strict inequality for $t \in E$. This implies $\int_{\tilde{T} \cap T^*} \tilde{r}_t \tilde{\sigma}_t dt > \int_{\tilde{T} \cap T^*} r_t \sigma_t^* dt$. Using this, $X^* = \tilde{X}$ and an analogous decomposition to (A.5) for X^*, \tilde{X} , we obtain:

$$\int_{B^*} r_t \sigma_t^* dt > \int_{\tilde{B}} \tilde{r}_t \tilde{\sigma}_t dt, \quad (\text{A.8})$$

where $B^* := T^* - (\tilde{T} \cap T^*)$ and $\tilde{B} := \tilde{T} - (\tilde{T} \cap T^*)$. For $t \in B^*, t' \in \tilde{B}$, we have:

$$\tilde{r}_{t'} \tilde{\sigma}_{t'} \geq r_t \sigma_t^*. \quad (\text{A.9})$$

(Suppose that $\tilde{r}_{t'} \tilde{\sigma}_{t'} < r_t \sigma_t^*$. Then, $\tilde{r}_{t'} \tilde{\sigma}_{t'} < \tilde{r}_t \tilde{\sigma}_t$, because of (i), and $\tilde{V}_t^n(\tilde{\sigma}_t, v) > \tilde{V}_{t'}^n(\tilde{\sigma}_{t'}, v)$

due to M . Thus $t \in \tilde{T}$, since $t' \in \tilde{B} \subset \tilde{T}$. This contradicts $B^* \cap \tilde{T} = \emptyset$.) Using (A.9) in (A.8), we obtain $\lambda(B^*) > \lambda(\tilde{B})$ and thus $\lambda(T^*) > \lambda(\tilde{T})$. (iii) Assume $\tilde{v} < v^*$: Then, $\tilde{V}_t^n(\tilde{\sigma}_t(\tilde{v}), \tilde{v}) = V_t^n(\sigma_t^*(\tilde{v}), \tilde{v}) < V_t^n(\sigma_t^*(\tilde{v}), v^*) \leq V_t^n(\sigma_t^*(v^*), v^*) \leq 0$ for every $t \in (T_0 - E) \cup (L - T^*)$. (The first equality follows from $t \notin E$. The next two inequalities follow for any $t \in L$ from $\partial V_t^n / \partial v > 0$ and the optimality of $\sigma_t^*(v^*)$. The last inequality holds for any $t \in T_0 \cup (L - T^*)$.) Thus, $(T_0 - E) \cup (L - T^*) \subset L - \tilde{T}$, $\lambda(L - \tilde{T}) \geq \lambda(T_0 - E) + \lambda(L - T^*) > \lambda(L - T^*)$ and $\lambda(\tilde{T}) < \lambda(T^*)$, since $\lambda(T_0 - E) > 0$ by assumption. QED.

Proof of Fact 3. (a) Suppose that the set of active senders is T with $\lambda(T)$. Applying (A.2) and (A.3), we get $\sigma_t^* = rg(v)$ and $V_t^{n*} = y \ln y + (\gamma - 1)y - c_0$ for $y \equiv rg(v)$. Thus $V_t^{n*} >, =, < 0$ if $rg(v) >, =, < \bar{y}(c_0, \gamma)$ where $\bar{y}(c_0, \gamma)$ is defined by the condition $\ln \bar{y} = 1 - \gamma + c_0/\bar{y}$. (Note that $\bar{y} > 1$ and \bar{y} increases with c_0 and decreases with γ .) According to (A.1), $\tau = r^2 g(v) \lambda / \mu$ where λ denotes $\lambda(T)$ and μ denotes $\mu(S)$. Using this and $g(v) = g_0 v^\alpha$ in $\nu(s, \tau_s) = \tau_s^{-\beta}$ we get $\tau^* = (g_0 r^2 \lambda / \mu)^{1/(1+\alpha\beta)}$, $v^* = \tau^{*-\beta}$ and $g(v^*) = \tilde{g}_0 (\mu / (r^2 \lambda))^{\alpha\beta/(1+\alpha\beta)}$ with $\tilde{g}_0 \equiv g_0^{1/(1+\alpha\beta)}$. Substitution of $g(v^*)$ into the condition $V_t^{n*} >, =, < 0$ gives us $\lambda^* = \lambda_0 \mu(S) r^{(1-\alpha\beta)/(\alpha\beta)}$ where $\lambda_0 \equiv (\tilde{g}_0 / \bar{y})^{1+1/(\alpha\beta)}$. (b) Local diversity depends on the realized audience allocation. For the equilibrium audience allocation constructed in the proof of Fact 1, the measure of membership of every $s \in S$ is given by $m^* = r / \mu(S)$. Since all active senders are identical, attention levels must be equalized in any equilibrium audience allocation. Moreover, equal attention levels imply an equal measure of membership for all $s \in S$. Hence, $m^* = \lambda^* r / \mu(S)$ in any equilibrium. Substituting $\lambda^* = \lambda_0 \mu(S) r^{(1-\alpha\beta)/(\alpha\beta)}$ into $m^* = \lambda^* r / \mu(S)$ we get $m^* = \lambda_0 r^{1/(\alpha\beta)}$. QED.

Table of notations

(S, \mathcal{A}, μ)	measure space of receiver subjects s
$A \in \mathcal{A}$	audience
$\mu(A)$	measure of audience A
$(L, \mathcal{B}, \lambda)$	measure space of potential senders t
$T \in \mathcal{B}$	set of active senders
$\mathbf{a}_T \in \mathcal{B} \times \mathcal{A}$	audience allocation
$\mathbf{a}_T(t) := \{s \mid (t, s) \in \mathbf{a}_T\}$	audience of t under allocation \mathbf{a}_T
$M(s, \mathbf{a}_T) := \{t \mid (t, s) \in \mathbf{a}_T\}$	membership of s
$\lambda(T)$	aggregate diversity
$\lambda(M(s, \mathbf{a}_T))$	local diversity
$\rho(t)$	radiation capacity of t
$r_t = \min \{\rho(t), \mu(S)\}$	range of t
$\sigma_T : T \rightarrow \mathbb{R}_+$	signal strength
τ_s	signal exposure of s
$\nu(s, \cdot) : \mathbb{R}_+ \rightarrow \mathbb{R}_{++}$	attention capacity of s
$v = \nu(s, \tau_s)$	attention level of s under exposure τ_s
$z_t(\sigma, v)$	impact of sender t when sending σ to receiver with attention v
$C_t(\sigma, \mu(A)) = c_t(\sigma)$	cost of reaching audience of measure $\mu(A) \leq r_t$ with strength σ
V_t	total value of impact achieved by t
$V_t^n = V_t - c_t$	value of t net of cost c_t
$X := \int_T r_t \sigma_T(t) dt$	aggregate signal emission