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# Deregulating Network Industries: Dealing with Price-Quality Tradeoffs\*

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## **Abstract**

This paper examines the effects of introducing competition into monopolized network industries on prices and infrastructure quality. Analyzing a model with reduced-form demand, we first show that deregulating an integrated monopoly cannot simultaneously decrease the retail price and increase infrastructure quality. Second, we derive conditions under which reducing both retail price and infrastructure quality relative to the integrated monopoly outcome increases welfare. Third, we argue that restructuring and setting very low access charges may yield welfare losses, as infrastructure investment is undermined. We provide an extensive analysis of the linear demand model and discuss policy implications.

**Keywords:** infrastructure quality, deregulation, investment incentives, access charges, regulation

**JEL Classification:** D43, L43

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# 1 Introduction

It is commonly accepted among economists and regulators alike that deregulating network industries—i.e. introducing competition into previously monopolized industries such as electric power, water, gas, or railroad transportation—can bring about considerable welfare gains.<sup>1</sup> Yet, recent experience with various forms of deregulation is mixed: On the one hand, there are a number of success stories, typically associated with the restructuring of telecommunications. On the other hand, there are disturbing reports on ill-fated attempts to deregulate utilities such as the Californian electric power industry or the British railroad industry.<sup>2</sup> Not surprisingly, these restructuring disasters have triggered a public debate on the pros and cons of deregulating network industries.

In this paper, we focus on one particular aspect of this debate, namely the notion that deregulating an integrated monopoly is likely to bring about a *degradation of infrastructure quality*. Abstracting from vertical relations, Spence (1975; 1977) has established that the quality level provided by a profit maximizing monopolist will generally deviate from the socially optimal level. When we consider a network monopolist's provision of quality in a vertically-related industry subject to regulation, however, the picture is less clear, as institutional and regulatory details have subtle effects on investment incentives. In a recent paper, Buehler et al. (2004) demonstrate that, for the case of a bilateral monopoly, the network monopolist's incentive to invest into infrastructure quality is typically smaller under vertical separation than under vertical integration, although it is possible to construct counter-examples.

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<sup>1</sup>For instance, in his presidential address to the European Economic Association, Newbery (1997, 358) put forward the following thesis: “[...] introducing competition into previously monopolised and regulated network utilities is the key to achieving the full benefits of privatisation.”

<sup>2</sup>See, e.g., Joskow (2000), Borenstein (2002) and the special issue of the *Journal of Industry, Competition and Trade* (2002) for the restructuring of the Californian electric power industry. *The Economist* has documented the crisis in the British railroad industry in numerous articles, some with telling titles such as “How not to run a railway” (November 25, 2000) and “Britain off the rails” (March 17, 2001).

In the present paper, we go beyond the case of bilateral monopoly and extend previous research along two dimensions: First, in addition to vertical integration and separation, we consider the case of liberalization, where the network monopolist also operates in the downstream market. Both under separation and liberalization, we allow for varying degrees of (imperfect) downstream competition. Second, we provide a welfare analysis allowing us to discuss how to deal with the price-quality tradeoffs encountered when deregulating network industries.

We model an industry where an essential input—the network infrastructure—can be provided at various levels of quality. Network quality is costly to provide but increases the value of the final product to customers. Under vertical integration, a single firm chooses both the level of infrastructure quality and the retail price. In the other market configurations, the upstream monopolist determines infrastructure quality. Depending on the market configuration, the upstream monopolist's profits stem from access revenues (under vertical separation with various degrees of downstream competition) or both access and retail revenues (under liberalization with various degrees of downstream competition). Downstream firms, in turn, pay an access charge which is exogenously set by a regulator and independent of network quality. The latter assumption may be justified by noting that quality is usually difficult and costly to specify and therefore cannot be described *ex ante* in a contract and ascertained *ex post* by a court. That is, quality is observable but non-verifiable (see, e.g., Laffont and Tirole 1993).

Using a simple model with demand depending on price and quality, we derive the following main results. *First*, regulatory authorities do indeed face a price-quality tradeoff when deregulating network industries, as deregulating an integrated monopoly cannot yield both a lower retail price and higher infrastructure quality. This follows from the fact that in order to generate the same investment incentive as under integration, the access charge under separation or liberalization must be at least as large as the retail price under integration. *Second*, we identify necessary and sufficient conditions under which restructuring and setting the access charge

such that both retail price and infrastructure quality decrease relative to the integrated monopoly outcome increases welfare. *Third*, deregulation may generate welfare losses even if retail prices decrease. This may come about when the level of the access charge is low enough to severely undermine the network monopolist's incentive to invest into infrastructure quality.

The remainder of the paper is structured as follows. In section 2, we introduce the analytical framework. In section 3, we discuss the equilibrium outcomes under the various regimes. In section 4, we derive our main results based on an analysis of the feasible price-quality bundles under each of these regimes. In section 5, we provide an in-depth analysis of the linear demand model. In section 6, we discuss limitations and extensions. Section 7 concludes.

## 2 Analytical Framework

### 2.1 Assumptions

Consider a public utility which provides its final product using a network infrastructure, and suppose that a provider of the final product (e.g. electricity or transportation services) must have access to the network (e.g. the transmission grid or railroad tracks). For simplicity, suppose that one unit of network access is required to produce one unit of the final product. Let  $D(p, \theta)$  denote the demand for the final product, where  $p \geq 0$  is the retail price and  $\theta \geq 0$  reflects network quality.<sup>3</sup> In line with the literature (see, e.g., Laffont and Tirole 1993), we assume that quality  $\theta$  is observable but non-verifiable, and hence non-contractible. This implies, in particular, that access charges cannot be made contractually dependent on  $\theta$ . We think it is natural to assume that quality is non-contractible in the present context, as explicitly specifying quality standards is extremely difficult and costly. Network quality  $\theta$  can thus hardly be described *ex ante* in a regulatory contract, let alone ascertained

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<sup>3</sup>If quality encompasses multiple dimensions,  $\theta$  should be interpreted as a real-valued index summarizing the various aspects of quality.

ex post by a court. This is consistent with the observation that, in practice, access charges typically reflect network quality highly imperfectly. We therefore view quality-independent access charges as a useful approximation to the real world that allows us to expose the basic tradeoffs emanating from the deregulation of network industries.

In the various market configurations considered below, we let  $n \geq 1$  denote the number of firms operating in the downstream market. We assume that the regulator prescribes a linear access charge  $a \geq 0$ , whereas retail prices remain unregulated.<sup>4</sup> We think this is a natural setting, as the deregulation of network industries typically aims at introducing (imperfect) downstream competition, while using regulation to contain the network monopolist's remaining (upstream) market power. Finally, we let the marginal cost of output be constant and normalized to zero and abstract from fixed costs of operating the network.<sup>5</sup>

Throughout the paper, we require that the following basic assumptions are satisfied, where subscripts denote partial derivatives.

**[A1]** The demand for the final product,  $D(p, \theta)$ , is twice continuously differentiable and satisfies  $D_p(p, \theta) < 0$  and  $D_\theta(p, \theta) > 0$ . We let  $P(Q, \theta)$  denote inverse demand, with  $P_Q(Q, \theta) < 0$  and  $P_\theta(Q, \theta) > 0$ .

**[A2]** The revenue function  $R(p, \theta) \equiv pD(p, \theta)$  satisfies

$$R_{p\theta}(p, \theta) = D_\theta(p, \theta) + pD_{p\theta}(p, \theta) \geq 0 \quad \text{for all } (p, \theta).$$

**[A3]** Producing quality level  $\theta$  costs  $K(\theta)$ , with  $K'(\theta) > 0$ ,  $K''(\theta) > 0$ , and  $K(0) = 0$ .

Assumption [A1] makes demand depend positively on quality and negatively on the price. That is, network quality has a demand-enhancing effect, creating

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<sup>4</sup>As noted by Mandy and Sappington (forthcoming), it is common practice to use uniform and non-discriminatory access charges. It will become clear, though, that for reasonable values, a fixed component in access charges does not change the predictions of our analysis.

<sup>5</sup>The introduction of fixed costs does not affect the results (provided they are not too large).

an incentive for the network provider to provide quality. [A2] imposes that the marginal revenue of a quality increase is non-decreasing in the retail price. This assumption allows for both parallel shifts ( $D_{p\theta} = 0$ ) and rotations of the demand schedule ( $D_{p\theta} \neq 0$ ). All we require is that  $D_{p\theta}$  is not too strongly negative (i.e., the demand schedule must not rotate too much counter-clockwise).<sup>6</sup> [A3] requires that the cost function is increasing and convex in quality.<sup>7</sup>

We now describe the instruments available to regulatory authorities.

## 2.2 The Regulator's Instruments

We consider the following policy instruments available to regulatory authorities:

- (i) **Market structure regulation.** When deregulating public utilities, regulatory authorities may determine the preferred *market configuration*. In particular, when restructuring an integrated monopoly, regulatory authorities may decide whether (and to what extent) to introduce downstream competition and whether to break up the integrated network monopoly. That is, they may decide about the industry's future horizontal and vertical structure. We will demonstrate below that by selecting a particular market configuration—such as vertical separation or liberalization with a downstream oligopoly—regulators select a particular set or “menu” of feasible price-quality bundles.
- (ii) **Access charge regulation.** Once regulatory authorities have opted for a particular market configuration (e.g. vertical separation with  $n$  downstream

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<sup>6</sup>Many standard demand functions satisfy this assumption. For instance, for linear demand  $D(p, \theta) = \alpha - \beta p + \theta$ , with  $\alpha, \beta > 0$ , an increase in  $\theta$  gives rise to a parallel shift of the demand schedule as  $D_{p\theta}(p, \theta) = 0$ , whereas for  $D(p, \theta) = \alpha - \beta p / \theta$ , with  $\alpha, \beta > 0$  we have a clock-wise rotation due to  $D_{p\theta}(p, \theta) = \beta / \theta^2 > 0$ . We consider the linear demand example in more detail in section 5.

<sup>7</sup>More generally, the network provider's cost function may depend on the level of output  $Q$ , i.e.,  $\tilde{K}(Q, \theta)$ . We restrict attention to the case where costs are additively separable in  $Q$  and  $\theta$  and marginal costs of output are constant and normalized to zero.



firms) and thus implemented a menu of feasible price-quality bundles, a particular price-quality bundle is selected by determining the level of the *access charge*. In an ideal world, the latter would be chosen so as to maximize social welfare contingent on the market configuration. In doing so, regulatory policy has to deal with the following tradeoff: Increasing the access charge enhances the incentive to invest into infrastructure quality, but also increases the retail price. Conversely, decreasing the access charge undermines the incentive to invest into infrastructure quality, while reducing the retail price.

Below, we examine how regulatory authorities can use these instruments to deal with price-quality tradeoffs when deregulating network industries.

### 3 Alternative Market Configurations

We consider market configurations (or regimes) that differ in the number of downstream firms  $n \geq 1$ , and in whether the network monopolist is permitted to operate in the downstream market. We call a regime “liberalization” if the vertically integrated network monopolist operates in a downstream market composed of  $n$  firms and denote it by  $L(n)$ .<sup>8</sup> If the network monopolist is banned from the downstream market, we speak of “separation” and denote it by  $S(n)$ . Finally, we denote the benchmark of vertically integrated monopoly by  $I$ . For notational convenience, we let  $\rho \in \{I, S(n), L(n)\}$  represent the relevant regime under consideration.

We now proceed to characterizing the price-quality bundles in the welfare optimum and the various regimes  $\rho \in \{I, S(n), L(n)\}$ .

#### 3.1 Welfare Optimum

Let

$$W(p, \theta) = \int_p^\infty D(\tilde{p}, \theta) d\tilde{p} + pD(p, \theta) - K(\theta) \quad (1)$$

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<sup>8</sup>That is, the integrated network monopolist faces  $n - 1$  downstream competitors.

denote the social welfare function, where the first two terms represent gross consumer surplus (i.e., net consumer surplus plus consumer expenditure), and the third term refers to the cost of providing infrastructure quality  $\theta$ . The welfare-maximizing bundle  $(p^*, \theta^*)$  solves

$$W_p = pD_p \leq 0 \quad (2)$$

$$W_\theta = \int_p^\infty D_\theta d\tilde{p} + pD_\theta - K' \leq 0. \quad (3)$$

As the marginal cost of output is zero, the welfare maximizing price  $p^*$  must be equal to zero. The socially optimal level of quality  $\theta^*$  is strictly larger than zero, implying that the social cost of a marginal increase of quality, given by  $K'(\theta)$ , is just equal to the marginal increase of gross consumer surplus (i.e., (3) holds with equality).<sup>9</sup>

Since the vertical structure's profit is negative in the welfare optimum, we also consider the constrained welfare optimum assuring the vertical structure a non-negative profit. We provide this optimization problem and its solution in the Appendix. Below, we shall use the outcome under integrated monopoly as the relevant benchmark, as it is often taken to approximate the price-quality bundle provided by a monopolistic public utility prior to restructuring.

### 3.2 Vertical Integration

Under vertical integration,  $\rho = I$ , the monopolist solves the following profit maximization problem:

$$\max_{Q, \theta} \pi(Q, \theta) = QP(Q, \theta) - K(\theta).$$

Assuming an interior solution, the integrated monopolist's choices  $p^I$  and  $\theta^I$  solve

$$\pi_Q = P + QP_Q = 0 \quad (4)$$

$$\pi_\theta = QP_\theta - K' = 0. \quad (5)$$

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<sup>9</sup>Given  $p^* = 0$ ,  $\theta^* = 0$  cannot satisfy (3), as  $W_\theta = \int_0^\infty D_\theta d\tilde{p} > 0$  by [A1] and [A3].

For given  $\theta$ , the equilibrium price  $p^I$  is strictly larger than zero by (4), causing the integrated monopolist's output to fall short of the optimal output. (5) indicates that the equilibrium quality  $\theta^I$  is also strictly larger than zero. Whether the integrated monopolist undersupplies quality is less clear: As is well known from the literature, the monopolist's provision of quality depends on the output gap and the effect of quality on the marginal willingness to pay of the average and the marginal consumer (see, e.g., Tirole 1988).

### 3.3 Vertical Separation

Under vertical separation with  $n$  downstream firms competing à la Cournot,  $\rho = S(n)$ , the network monopolist and the  $n$  independent providers of the final product play a sequential game, taking the regulated access charge  $a$  and the number of competitors  $n$  as given. This is a simple two-stage game, which can be solved using backward induction. At the second stage of the game, given  $\theta$ , downstream firm  $i$  chooses its output so as to

$$\max_{q_i} \pi^D(q_i, Q_{-i}; \theta, a, n) = q_i (P(Q, \theta) - a),$$

where  $Q_{-i} = \sum_{j \neq i} q_j$  denotes the sum of the competitors' outputs and  $Q = q_i + Q_{-i}$  denotes aggregate output. Assuming an interior solution, the first-order condition is given by

$$\pi_{q_i}^D = (P - a) + q_i P_Q = 0. \quad (6)$$

Applying symmetry and letting  $q^D(\theta; a, n)$  denote each downstream firm's equilibrium output (given quality level  $\theta$ ), aggregate output is given by  $Q^D(\theta; a, n) = nq^D(\theta; a, n)$ .

In the first stage, the upstream network monopolist chooses quality  $\theta$  so as to

$$\max_{\theta} \pi^U(\theta; a, n) = anq^D(\theta; a, n) - K(\theta).$$

At an interior solution, the monopolist's choice of quality as a function of  $a$  and  $n$  solves

$$\pi_{\theta}^U = anq_{\theta}^D - K' = 0. \quad (7)$$

Let  $(p^{S(n)}, \theta^{S(n)})$  denote the equilibrium price-quality bundle attained under vertical separation. Note that these equilibrium values are functions of both the access charge  $a$  and the number of downstream competitors  $n$ ; varying  $a$  thus produces the menu of feasible price-quality bundles, given  $\rho = S(n)$ . That is, the choice of  $a$  is crucial for comparing  $(p^I, \theta^I)$  and  $(p^{S(n)}, \theta^{S(n)})$ . We shall discuss this in more detail below.

For future reference, let us briefly consider two polar cases. First, setting  $n = 1$  corresponds to a vertically separated industry with a downstream monopoly. Secondly, vertical separation with *perfect* downstream competition emerges when a large number of firms ( $n \rightarrow \infty$ ) compete à la Cournot. In this case, the retail price will be driven down to marginal cost irrespective of quality  $\theta$ , so that  $p^{S(\infty)} = a$ .

### 3.4 Liberalization

Under liberalization with  $n$  downstream firms,  $\rho = L(n)$ , there is a vertically integrated network monopolist competing with  $n - 1$  independent downstream firms à la Cournot. Without loss of generality, let downstream firm  $i = 1$  be the vertically integrated monopolist. In the second stage of the game, downstream firms choose their outputs according to (6).<sup>10</sup> In the first stage, the integrated upstream monopolist selects  $\theta$  so as to

$$\max_{\theta} \pi^U(\theta; a, n) = P(Q, \theta)q_1^D + aQ_{-1}^D(\theta; a, n) - K(\theta),$$

where  $Q_{-1}^D(\theta; a, n) = (n - 1)q_{-1}^D(\theta; a, n)$  denotes the output of the firms operating at the downstream market only. At an interior solution, the integrated network monopolist's choices of output and quality solve

$$\pi_{\theta}^U = \left( P_Q \left[ (n - 1) \frac{\partial q_{-1}^D}{\partial \theta} \right] + P_{\theta} \right) q_1^D + a(n - 1) \frac{\partial q_{-1}^D}{\partial \theta} - K' = 0. \quad (8)$$

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<sup>10</sup>Note that Cournot competition at the downstream level will now be asymmetric as the integrated firm's marginal costs of zero will generally differ from competitors' marginal costs of  $a$ .

We let  $(p^{L(n)}, \theta^{L(n)})$  denote the resulting equilibrium price-quality bundle under liberalization.

Again, perfect downstream competition will drive the retail price down to the access charge, just as under separation with perfect competition, i.e.,  $p^{L(\infty)} = a$ . As a result, the network monopolist will have the same incentive to invest into infrastructure quality as under separation with perfect downstream competition. It follows immediately that liberalization with perfect downstream competition produces the same outcome as vertical separation with perfect downstream competition for valid access charges.

## 4 Analyzing Price-Quality Tradeoffs

In this section, we focus on the question of how regulatory authorities might use the available policy instruments—market structure regulation and access charge regulation—when faced with price-quality tradeoffs. More specifically, we shall determine the set of feasible price-quality bundles under each regime  $\rho$  and ask how to best set the access charge once a particular regime is in place.

Our first result states that, in our setting, regulatory authorities do face a price-quality tradeoff when deregulating a vertically integrated network monopoly providing the price-quality bundle  $(p^I, \theta^I)$ .

**Proposition 1 (tradeoffs)** *Suppose [A1]-[A3] hold. Further assume that  $dp/dn \leq 0$  and  $d\theta/dn \geq 0$  under  $\rho \in \{S(n), L(n)\}, n \geq 1$ . Then a price-quality bundle  $(p', \theta')$ , such that  $p' \leq p^I$  and  $\theta' \geq \theta^I$ , with at least one inequality being strict, is not feasible under  $\rho \in \{S(n), L(n)\}$  for  $n \geq 1$ .*

**Proof.** With perfect downstream competition, the network provider's revenues are given by  $\tilde{R}(\theta; a) \equiv aD(\theta; a)$ . Hence, the marginal returns to quality are  $\tilde{R}_\theta(\theta; a) = aD_\theta(\theta; a)$ , and the effect of an increase in  $a$  on marginal returns to quality is given by

$$\tilde{R}_{a\theta} = D_\theta + aD_{a\theta} \geq 0,$$

which is nonnegative by [A2]. That is, perfect competition replicates the bundle  $(p^I, \theta^I)$  for  $p^I = a$ , but it cannot provide a bundle  $(p', \theta')$  such that  $p' \leq p^I$  and  $\theta' \geq \theta^I$ , with at least one inequality being strict. The claim follows from  $dp/dn \leq 0$  and  $d\theta/dn \geq 0$  under  $\rho \in \{S(n), L(n)\}$  for  $n \geq 1$ . ■

Proposition 1 demonstrates that restructuring a vertically integrated monopoly cannot simultaneously reduce the retail price and increase infrastructure quality. More specifically, it shows that any price reductions must come at the cost of sacrificing infrastructure quality (and vice versa).

To understand the intuition for this result, first consider  $S(n)$ . We want to argue that the network provider's investment incentive,  $\tilde{R}_\theta$ , is smaller than under integration for any access charge  $a \leq p^I$  due to lower returns to increasing  $\theta$ . To see this, fix the access charge at  $a = p^I$  and consider the effect of a quality increase. Holding retail prices constant, the *direct effect* of a quality increase is an increase in demand and thereby access revenues. However, there will also be an *indirect (price-mediated) effect* on demand, as retail prices will endogenously rise with increases in  $\theta$  by [A2]. Since the network owner controls the retail price under integration, the returns to increasing network quality must be higher than if prices were unchanged by a simple optimality argument.<sup>11</sup> As a result, the investment incentive must be higher under vertical integration for  $a = p^I$ . It should be clear that the result holds a fortiori for  $a < p^I$ . For  $a > p^I$ , the retail price must be higher than  $p^I$  by the profit maximization of downstream firms. Hence, higher network quality than under vertical integration can be achieved only by means of higher retail prices.

Next, consider investment incentives under  $L(n)$ . Any access charge  $a > 0$  leads to a cost asymmetry between downstream firms, as the integrated firm's marginal costs are zero, whereas all other firms' marginal costs are strictly positive. As the access charge rises, this asymmetry increases for any quality level  $\theta$ . It follows that there is some critical access charge at which the asymmetry is sufficiently large to squeeze non-integrated downstream firms out of the market. More specifically,

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<sup>11</sup>After all, the integrated network owner could simply keep prices unchanged.

for any access charge  $a \geq p^I$ , profits of the non-integrated competitors will be compressed to zero, which produces an outcome identical to integrated monopoly. Hence, for  $a \geq p^I$ , both price and quality will be the same as under integration.

Having shown that neither  $S(n)$  nor  $L(n)$  dominate the benchmark of integrated monopoly in terms of both price and quality, we proceed to a more detailed comparison of regimes. In particular, we compare the benchmark outcome  $(p^I, \theta^I)$  with the feasible outcomes under  $S(n)$  and  $L(n)$  in terms of welfare. Since these comparisons critically rely on the regulator's choice of the access charge, we first provide a general result on the welfare effects of infinitesimally changing the access charge, starting from any feasible price-quality bundle  $(p, \theta)$  under regime  $\rho$ . Since deregulation typically aims at reducing retail prices, we formulate the result in terms of infinitesimal reductions of the access charge. To avoid excessive notation, we let  $dp/d\theta|_{(p,\theta)}^\rho$  denote the slope of the menu of feasible price-quality bundles under regime  $\rho$ , evaluated at the price-quality bundle  $(p, \theta)$ . Similarly,  $dp/d\theta|_{(p,\theta)}^W$  denotes the slope of the welfare function evaluated at  $(p, \theta)$ .

**Proposition 2 (local welfare change)** *Suppose [A1]-[A3] hold. Consider regime  $\rho \in \{S(n), L(n)\}, n \geq 1$ , where the access charge  $a$  is such that the resulting price-quality bundle is  $(p, \theta)$ . Then marginally **decreasing the access charge** increases welfare if and only if*

$$\frac{dp}{d\theta}\Big|_{(p,\theta)}^\rho > \frac{dp}{d\theta}\Big|_{(p,\theta)}^W, \quad (9)$$

where

$$\frac{dp}{d\theta}\Big|_{(p,\theta)}^W \equiv -\frac{W_\theta}{W_p} = -\frac{\int_p^\infty D_\theta d\tilde{p} + pD_\theta - K'}{pD_p}.$$

**Proof.** See the Appendix. ■

Proposition 2 states that—starting from an initial access charge  $a$  resulting in  $(p, \theta)$  under regime  $\rho$ —marginally reducing the access charge increases welfare if and only if the slope of the menu of feasible price-quality bundles under regime  $\rho$  is larger than the slope of the welfare function evaluated at  $(p, \theta)$ . Figure 1 provides an

illustration of Proposition 2, where  $(p^I, \theta^I)$  is the benchmark bundle and condition (9) is satisfied.

<Figure 1 around here>

This result is useful for answering the question of whether there is scope for welfare-improving deregulation through liberalization. To see this, consider the benchmark bundle  $(p^I, \theta^I)$ , which is also feasible under  $L(n)$  for  $a \geq p^I$ . Proposition 2 shows that there is scope for welfare-improving deregulation from  $I$  to  $L(n)$ , provided that (9) holds for  $\rho = L(n)$ .<sup>12</sup>

Note that Proposition 2 can be used to evaluate the welfare effects of deregulation only for regimes capable of reproducing  $(p^I, \theta^I)$ . For instance, Proposition 2 cannot be applied to comparisons involving  $I$  and  $S(n)$ ,  $n < \infty$ , as  $(p^I, \theta^I)$  is not a feasible bundle under the latter regime. Nevertheless, the result is useful for evaluating access charge regulation under any regime  $\rho \in \{S(n), L(n)\}$ , as it allows to determine how changes in access charges will affect welfare.

Our next result considers the welfare effects of non-infinitesimal changes in the access charge. We show that even if there is scope for welfare-increasing reductions of the access charge, there is a risk of setting the access charge too low.

**Proposition 3 (global welfare change)** *Consider regime  $\rho \in \{S(n), L(n)\}$ ,  $n \geq 1$ , where the access charge  $a$  is such that the resulting price-quality bundle is  $(p, \theta)$ . Suppose [A1]-[A3] and (9) hold, i.e., marginally decreasing the access charge increases welfare. Then **decreasing the access charge** by a non-infinitesimal amount may reduce welfare.*

**Proof.** We establish the claim using a graphical argument. Suppose decreasing the access charge by a non-infinitesimal amount does not reduce welfare. Consider regime  $\rho$  in figure 1, where all reductions of the access charge resulting in price

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<sup>12</sup>A similar result holds for  $S(\infty)$ . It seems unlikely, though, that restructuring will lead to perfect downstream competition.



quality bundles between  $(p^I, \theta^I)$  and  $(p', \theta')$  increase welfare. Larger reductions, such as the one giving rise to  $(p'', \theta'')$ , reduce welfare. ■

Proposition 3 points to the risk of setting the access charge too low: Even if there is scope for improving welfare by reducing the access charge, reducing it by a large amount may yield welfare losses. Intuitively, this result follows from the fact that reducing the access charge not only reduces the retail price but also discourages investment in network quality. The latter effect may well dominate for large reductions in access charges, giving rise to a negative welfare effect.

## 5 The Linear Demand Model

In this section, we provide an analysis of the linear demand model, where we can derive closed-form solutions for prices and qualities. Let us assume that demand is given by

$$D(p, \theta) = \alpha - \beta p + \theta, \quad \alpha > 0, \beta > \frac{1}{2}, \quad (10)$$

where  $\alpha$  and  $\beta$  are exogenous parameters. Further suppose that  $K(\theta) = \theta^2$ . Note that the linear demand example satisfies [A1]-[A3].

Table 1 summarizes the equilibrium prices and quantities as functions of the model parameters, as well as the respective menu equations describing the feasible price-quality bundles in  $(p, \theta)$ -space.

<Table 1 around here>

Figure 2 further provides an illustration of the linear demand model for parameter values  $\alpha = 4$  and  $\beta = 0.9$ .<sup>13</sup> Note that price-quality bundles in the southeast corner are preferred to bundles in the northwest corner. The welfare optimum (*WO*) is in the lower-right corner at  $(p^*, \theta^*) = (0, 5)$ , whereas the constrained welfare optimum (*CWO*) is at  $(p_c^*, \theta_c^*) \approx (1.7, 3.1)$ . The price-quality bundle  $(p^I, \theta^I) \approx (3.1, 1.5)$  under vertical integration—where we both have a higher price and lower quality than

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<sup>13</sup>Qualitatively, the choice of specific parameter values does not affect figure 2.

in the (constrained) welfare optimum—will be our main point of reference below. All other regimes are represented by straight lines. Any point on any of these lines can be obtained by fixing the access charge at a specific level, i.e., changes in access charges reflect movements on the lines, whereas changes in regimes are reflected in jumps from one line to another.<sup>14</sup>

<Figure 2 around here>

Using table 1 and figure 2, we now derive a number of results that highlight the relevance of Propositions 1-3 above. First, inspection of figure 2 confirms for the linear model that there is a tradeoff when deregulating an integrated monopoly (see Proposition 1):

**Result 1** *A bundle  $(p', \theta')$  such that  $p' \geq p^I$  and  $\theta' \geq \theta^I$ , with at least one inequality being strict, is not feasible. For improvements in one dimension only, the following results hold:*

- (i) Increasing quality to any  $\theta' > \theta^I$  is feasible under  $\rho = S(n), n \geq 2$  only, and it comes at the cost of increasing the retail price to some  $p' > p^I$ .*
- (ii) Reducing the retail price to  $p' < p^I$  is feasible for  $S(n), n \geq 1$  and  $L(n), n \geq 2$ , but it comes at the cost of reducing quality to  $\theta' < \theta^I$ .*

Figure 2 further indicates that liberalization will perform better than separation when restructuring aims at reducing retail prices. Comparison of the menus  $S(1)$  and  $S(2)$  shows that any quality level  $\theta$  attainable under  $S(1)$  may be attained at a lower retail price under  $S(2)$ . Thus,  $S(2)$  in a sense dominates  $S(1)$ . Similar arguments show that  $S(\infty)$  dominates both  $S(1)$  and  $S(2)$ . That is, the more intense downstream competition, the lower the retail price at which a given quality level may be attained. The problem with  $S(\infty)$  is that, in practice, deregulation is

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<sup>14</sup>For each line, the left boundary is given by the restriction  $\theta \geq 0$ , the right by the highest valid access charge under the respective regime.

unlikely to lead to perfect downstream competition. It is thus useful to compare the various regimes to  $L(2)$ , which represents the price-quality bundles feasible under liberalization with two downstream firms.

Clearly, under  $L(2)$ , quality levels above  $\theta^I$  are not feasible for the reasons discussed above. However, for any quality level feasible under  $S(1)$ ,  $S(2)$  and  $L(2)$ , liberalization offers the lowest retail price and will thus be strictly preferred to  $S(1)$  and  $S(2)$ . More generally, for a given number of downstream firms, liberalization is preferred to vertical separation if deregulation aims at reducing retail prices.<sup>15</sup> The result still holds if the number of downstream firms approaches infinity. In this case, liberalization becomes similar to vertical separation, which is reflected in a counter-clockwise rotation of  $L(2)$  onto the lower part of  $S(\infty)$  where  $p \leq p^I$  (see figure 2).

Next, we examine whether there is scope for welfare-improving restructuring in the linear demand model, starting from the benchmark of vertically integrated monopoly. Substituting (10) into (1) and totally differentiating yields

$$\left. \frac{dp}{d\theta} \right|_{(p^I, \theta^I)}^W = \frac{1}{\beta} \approx 1.1,$$

whereas, from table 1, we have

$$\left. \frac{dp}{d\theta} \right|_{(p^I, \theta^I)}^{L(2)} = \frac{1 + 6\beta}{5\beta} \approx 1.4.$$

Since  $dp/d\theta|_{(p^I, \theta^I)}^{L(2)} > dp/d\theta|_{(p^I, \theta^I)}^W$ , it follows immediately that restructuring from  $I$  to  $L(2)$  yields welfare gains for suitable access charges (see Proposition 2). A similar result holds for restructuring from  $I$  to  $S(\infty)$ ,<sup>16</sup> with the qualification that, in practice, restructuring is unlikely to lead to perfect downstream competition. The next result summarizes these findings.

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<sup>15</sup>In fact, the predictions of the linear demand model are even stronger than this: Any quality level below  $\theta^I$  can be achieved at a lower retail price using a liberalized market with  $n$  downstream competitors than under separation with  $n + 2$  downstream competitors for any  $n \geq 2$ .

<sup>16</sup>For  $S(\infty)$ , we have  $dp/d\theta|_{(p^I, \theta^I)}^{S(\infty)} = 2$  (see table 1).

**Result 2** *In the linear demand model, there is scope for welfare-improving deregulation. In particular, for suitable access charges, restructuring from  $I$  to  $L(2)$  or  $S(\infty)$  improves welfare.*

In figure 2, iso-welfare curves contain all price-quality bundles that yield some constant level of social welfare. For instance,  $W^{L(2)}$  gives all price-quality bundles that yield the same welfare as  $L(2)$  provided the access charge is set optimally. At this optimal access charge, the line of feasible price-quality bundles  $L(2)$  is tangent to the iso-welfare curve  $W^{L(2)}$ . The notation is similar for all other market configurations. Observe that iso-welfare curves farther southeast denote higher levels of welfare.

Finally, we consider non-infinitesimal reductions of the access charge, focusing on the restructuring from  $I$  to  $L(2)$ . Figure 2 indicates that, if the access charge is set too low, welfare will actually be lower than under integration. More specifically, if the regulator chooses a very low access charge—a point on  $L(2)$  to the left of the intersection with  $W^I$ —, the resulting welfare will be smaller than  $W^I$ . This implies that there is a risk of setting the access charge too low when restructuring (see Proposition 3).<sup>17</sup> In this setting, the critical reduction of the access charge,  $\Delta a = p^I - a' > 0$ , can be calculated by equating the menu equation for  $L(2)$  with the welfare function  $W(p^I, \theta^I)$  and then solving for the access charge  $a' \approx 1.1$  associated with the intersection point. The next result summarizes these findings.

**Result 3** *In the linear demand model, setting the access charge too low may reduce welfare relative to  $W(p^I, \theta^I)$ . In particular, restructuring from  $I$  to  $L(2)$  and reducing the access charge below  $a' \approx 1.1$  reduces welfare.*

Result 3 indicates that, starting from  $a = p^I \approx 3.1$ , the access charge may be reduced by almost 65% before restructuring from  $I$  to  $L(2)$  yields welfare losses.<sup>18</sup>

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<sup>17</sup>In contrast, if the access charge is set too high, the worst possible outcome is that  $L(2)$  is equivalent to  $I$ , and welfare will thus be the same (this will happen for  $a \geq p^I$ ).

<sup>18</sup>The retail price will then fall by about 29% to roughly 2.2.

That is, at least in the linear model, regulatory authorities have a lot of leeway when regulating the access charge.

## 6 Limitations and Extensions

We now discuss the plausibility of our results when (i) the network monopolist can discriminate against downstream rivals, and (ii) downstream firms can contribute to the provision of quality.

### 6.1 Discrimination of Downstream Rivals

It is well known that the integrated monopolist may have an incentive to engage in non-price discrimination against its downstream rivals (see, e.g., Economides 1998, Mandy 2000, Beard et al. 2001, Weisman and Kang 2001). In a recent paper, Mandy and Sappington (forthcoming) show that both cost-increasing and demand-reducing sabotage may be profitable from the integrated firm's point of view under downstream Cournot competition.<sup>19</sup> That is, non-price discrimination of downstream rivals may occur under the regime  $\rho = L(n), n \geq 2$ .

To see how non-price discrimination of downstream rivals may affect our results, we consider the case of *cost-increasing* sabotage.<sup>20</sup> Following Mandy and Sappington, let  $s \geq 0$  denote the units of cost-increasing sabotage undertaken by the integrated monopolist, assuming that each unit of cost-increasing sabotage increases rivals' marginal costs by one unit. Further, let  $\kappa(s)$  denote the direct costs incurred when undertaking  $s$  units of cost-increasing sabotage, where  $\kappa(s)$  is a non-

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<sup>19</sup>However, these authors do not consider investment in infrastructure quality.

<sup>20</sup>The analysis of *demand-reducing* sabotage (e.g., quality discrimination against downstream rivals) is beyond the scope of the paper. In addition, the upstream firm could conceivably lower its own costs while raising the costs of downstream rivals by allowing the quality of the inputs supplied to them to decline. At the same time, such quality degradation could be costly if: (a) regulators penalize such behavior; or (b) the upstream monopolist cannot discriminate between the quality of inputs supplied to rivals and itself.

negative function increasing in  $s$ .<sup>21</sup> At the second stage of the game, given  $\theta$  and  $s$ , downstream firm  $i$  now chooses its output so as to

$$\max_{q_i} \pi^D(q_i, Q_{-i}; \theta, s, a, n) = q_i (P(Q, \theta) - a - s), \quad (11)$$

whereas the integrated upstream monopolist selects  $q_1, \theta$  and  $s$  so as to

$$\max_{q_1, \theta, s} \pi^U(q_1, \theta, s; a, n) = P(Q, \theta)q_1 + aQ_{-1}^D(\theta, s; a, n) - K(\theta) - \kappa(s). \quad (12)$$

Inspection of (11) indicates that, from the downstream competitors' point of view, cost-increasing sabotage is equivalent to an increase in the access charge. That is, sabotage adds to the asymmetry between the integrated network monopolist and its downstream competitors, making it more likely that the latter are squeezed out of the market. In fact, if undertaking sabotage is costless, i.e.,  $\kappa(s) = 0$  for all  $s \geq 0$  (and the regulator does not prevent sabotage), the monopolist always chooses  $s$  so that  $a + s = p^I$ , forcing its competitors to exit from the market. The resulting price-quality bundle will be  $(p^I, \theta^I)$ . However, as sabotage is typically costly, downstream competitors are unlikely to be fully forced out if the access charge is not very high. In any case, the increase of the downstream competitors' perceived marginal costs reduces the efficiency at which the industry's output is produced. As a result, the retail price  $p$  at which a given quality level  $\theta$  may be attained should be expected to be higher than in the absence of sabotage.

We now want to argue that Propositions 1-3 are not affected by the introduction of cost-increasing sabotage. To see this, first note that assumptions [A1]-[A3] still hold under sabotage. Next, consider Proposition 1. Let us first focus on the case where the downstream market is perfectly competitive and the access charge is  $a \geq p^I$ . In this case, our above discussion suggests that downstream competitors are squeezed out of the market. The integrated firm's profit maximization then yields  $(p^I, \theta^I)$ . Now, for  $a < p^I$ ,  $s$  will be chosen such that the retail price is no

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<sup>21</sup>We borrow this assumption from Mandy and Sappington (forthcoming). Note that with quality discrimination,  $\kappa(s)$  might turn out to be negative (see Reiffen 1998).

larger than  $p^I$ , implying that the associated quality level is at best  $\theta^I$ . That is, with perfect competition, an outcome that dominates the price-quality bundle under  $I$  in both dimensions is not feasible. Assuming that  $\pi_{s\theta}^U(\theta, s; a, n) < 0$ , implying that the network monopolist's incentive to invest is decreasing in the amount of sabotage undertaken, leads to  $d\theta/ds < 0$ , so that the quality level must be lower than  $\theta^I$ . As to Propositions 2 and 3, allowing for cost-increasing sabotage will affect the slope of both the welfare function and the menu equation, but not the sufficient conditions for welfare-improving changes in the access charge.

## 6.2 Downstream Quality

So far, we have limited ourselves to analyzing the network aspect of service quality. Particularly, we have modeled quality as a demand enhancing service attribute, set by a monopolistic network owner for the network as a whole, and thereby applying uniformly to all downstream firms. Frequently however, the quality of network services as perceived by the consumer depends not only on the quality of the network, but also on quality attributes set individually by the downstream firms. Thus, downstream firms can distinguish themselves from competitors through their choice of service quality, and hence will compete for consumers not only in prices (or quantities) but also in service quality.

Assuming that firms are competing in prices and quality amounts to assuming differentiated Bertrand competition. More specifically, for given network quality  $\theta$ , let each downstream firm  $i = 1, \dots, n$ , face a demand function  $D_i(\mathbf{p}, \mathbf{z}, \theta)$  where  $\mathbf{p} = (p_1, \dots, p_n)$  and  $\mathbf{z} = (z_1, \dots, z_n)$  denote the vectors of retail prices and quality levels, respectively. Without additional (non-trivial) assumptions on the interplay of  $\theta$  and  $\mathbf{z}$ , the effects of deregulation on the sets of feasible price-quality bundles under each regime remain ambiguous. However, we believe that studying a richer framework along these lines will be useful for a better understanding of the price-quality tradeoffs associated with deregulating network industries.

## 7 Conclusions

Our analysis has produced a number of results that are of relevance for regulatory policy. First, our model's most striking message is that, starting from a situation where an integrated monopolist is in charge of network quality while operating in the downstream market, any policy aimed at opening the downstream market to competitors (and possibly banning the incumbent from that market) cannot simultaneously lead to both lower consumer prices and higher network quality. This implies that any regulatory strategy of breaking up an integrated network monopoly with the explicit goal of lowering consumer prices must be well aware that it can do so only at the cost of sacrificing network quality.

Second, although no regime dominates the integrated monopoly in terms of both price and quality, there is scope for welfare-improving deregulation. Both liberalizing and separating an integrated network monopoly are capable of generating welfare improvements by trading off network quality against lower consumer prices in a socially desirable way (although separation can do so only in conjunction with sufficiently strong downstream competition).

Third, our analysis shows that the realization of welfare gains depends in a delicate way on the level of access charges set by the regulatory authority. Here, the advantage of liberalization over separation is that in the former market configuration, the regulator only runs the risk of setting the access charge too low to generate welfare gains, whereas high access charges will 'at worst' lead back to the integrated monopoly by driving competitors out of the market.

Of course, one might argue that these policy implications are driven by the assumption that regulatory authorities dispose of a rather limited set of instruments. If, for instance, network quality was contractible, almost any level of quality could be implemented. While we have argued that the assumption of non-contractible network quality approximates regulatory practice fairly well, under a slightly wider interpretation our model may also be used to make a different point: If network



quality may be verified, contractually specified, and enforced, regulators should consider this a real alternative to breaking up an integrated network monopoly, regulating access charges, and letting the market do the rest—even if the regulation of quality itself is costly.

## Appendix

### A.1 The Constrained Welfare Optimum

If the social planner aims at ensuring the firm a nonnegative profit, he chooses output and quality so as to

$$\max_{p, \theta} W(p, \theta) \quad \text{s.t.} \quad \pi(p, \theta) = pD(p, \theta) - K(\theta) \geq 0.$$

Assuming that the constraint holds with equality, the Lagrangian for this problem is given by

$$L(p, \theta, \lambda) = \int_p^\infty D(\tilde{p}, \theta) d\tilde{p} + pD(p, \theta) - K(\theta) + \lambda [pD(p, \theta) - K(\theta)].$$

At an interior solution, the constrained optimum  $(p_c^*, \theta_c^*)$  solves

$$\begin{aligned} L_p &= pD_p + \lambda [D(p, \theta) + pD_p] = 0 \\ L_\theta &= \int_p^\infty D_\theta d\tilde{p} + pD_\theta - K' + \lambda [pD_\theta - K'] = 0 \\ L_\lambda &= pD(p, \theta) - K(\theta) = 0. \end{aligned}$$

### A.2 Proof of Proposition 2

Rewriting the slope of the line representing feasible price-quality bundles under regime  $\rho$  yields

$$\left. \frac{dp}{d\theta} \right|_{(p, \theta)}^\rho = \left. \frac{dp/da}{d\theta/da} \right|_{(p, \theta)}^\rho,$$

i.e., the slope reflects the effects of a marginal change in  $a$  on both  $p$  and  $\theta$ . By marginally reducing the access charge (and thus network quality  $\theta$ ), welfare is increased if and only if

$$\left. \frac{dW}{d\theta} \right|_{(p, \theta)} = W_p \left. \frac{dp}{d\theta} \right|_{(p, \theta)}^\rho + W_\theta < 0.$$

Partially differentiating the welfare function (1) with respect to  $p$  and  $\theta$  yields  $W_p = pD_p$  ( $< 0$ , by [A1]) and  $W_\theta = \int_p^\infty D_\theta(p, \theta)d\tilde{p} + pD_\theta(p, \theta) - K'(\theta)$ . Rewriting the above condition yields

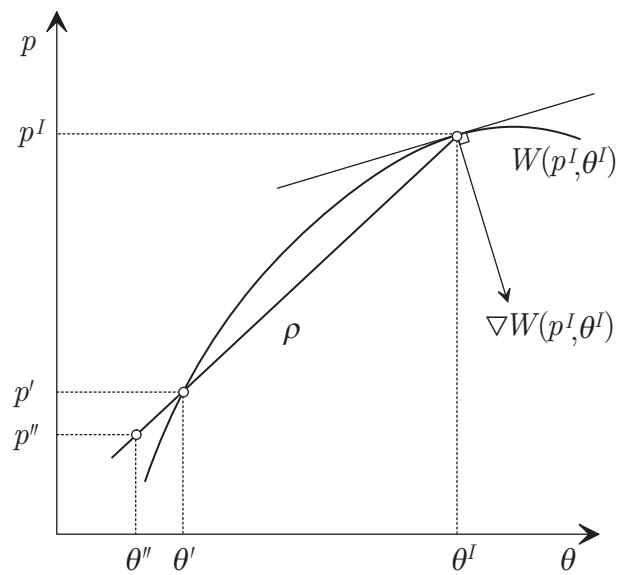
$$\left. \frac{dp}{d\theta} \right|_{(p,\theta)}^\rho > -\frac{W_\theta}{W_p} \equiv \left. \frac{dp}{d\theta} \right|_{(p,\theta)}^W = -\frac{\int_p^\infty D_\theta d\tilde{p} + pD_\theta - K'}{pD_p}.$$

This completes the proof.

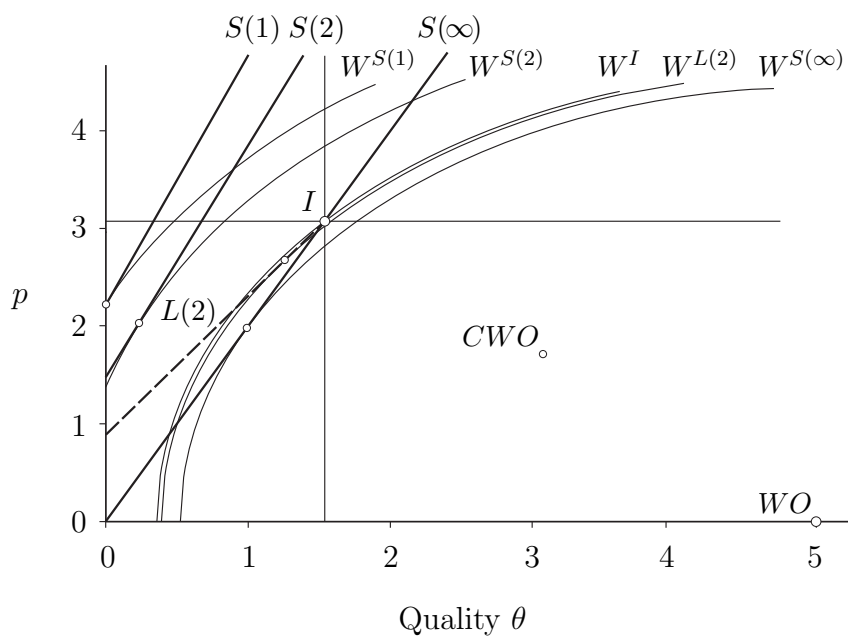
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**Figure 1:** Welfare Effects of Reducing the Access Charge.



**Figure 2:** The Linear Demand Example ( $\alpha = 4, \beta = 0.9$ ).

**Table 1:** Prices, Qualities, and Menu Equations.

| Regime      | Retail price $p$   | Quality $\theta$                                   | Menu Equation   |
|-------------|--|--|---|
| $WO$        | 0  | $\frac{\alpha}{2\beta-1}$                          | –   |
| $CWO$       | $\frac{\alpha}{(4\beta-1)\beta}$                             | $\frac{2\alpha}{4\beta-1}$                         | –   |
| $I$         | $\frac{2\alpha}{4\beta-1}$                                   | $\frac{\alpha}{4\beta-1}$                          | –   |
| $S(n)$      | $\frac{2(n+1)\alpha+an[1+2(n+1)\beta]}{2(n+1)^2\beta}$       | $\frac{a}{2} \frac{n}{(n+1)}$                      | $p = \frac{\alpha}{(n+1)\beta} + \frac{1+2(n+1)\beta}{(n+1)\beta} \theta$ |
| $S(\infty)$ | $a$  | $a/2$  | $p = 2\theta$   |
| $L(n)$      | $\frac{2(n+1)\alpha+a(n-1)[1+2(n+1)\beta]}{2(n+1)^2\beta-2}$ | $\frac{2\alpha+(n+3)(n-1)a\beta}{2(n+1)^2\beta-2}$ | $p = \frac{\alpha}{(n+3)\beta} + \frac{1+2(n+1)\beta}{(n+3)\beta} \theta$ |