Long-term Relationships: Static Gains and Dynamic Inefficiencies*

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Abstract

Do contractual frictions matter when firms are engaged in repeated interactions? This paper argues that long-term relationships, which allow firms to (partly) overcome the static costs associated with low contractibility, will under certain circumstances create dynamic inefficiencies. We consider the repeated interaction between final good producers and intermediate input suppliers, where the provision of the intermediate input is noncontractible. A producer/supplier pair can be a good match or a bad match, with bad matches featuring lower productivity. This allows us to build a cooperative equilibrium where producers can switch suppliers and start cooperation immediately with new suppliers. Every period, one supplier has the opportunity to innovate, and in the baseline model, innovations are imitated after one period. We show that (i) innovations need to be larger to break up existing relationships in the cooperative equilibrium than in either a set-up where the input is contractible or when we preclude cooperation in long-term relationships, (ii) the rate of innovation in the cooperative equilibrium is lower than in the contractible case, and may even be lower than in the non-cooperative equilibrium and (iii) cooperation may reduce welfare. Next, we assume that the frontier technology diffuses slowly to suppliers (instead of after one period). In that case, for sufficiently slow diffusion, the innovation rate in the cooperative equilibrium may be higher than even in the contractible case. Finally, we show that cooperation may also increase relationship specific innovations.

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1 Introduction

Do contractual frictions matter when firms are engaged in repeated interactions? There is widespread evidence that firms rely on relational contracts in order to ensure cooperation from their partners when legal contracts cannot be fully enforced. Yet recent work in the growth and trade literature has emphasized the central role that formal contractual enforcement plays in shaping income and growth differences across countries. Is there something fundamental about relational contracts that prevents them from being a good substitute for effective institutions? In this paper, we argue that the establishment of relational contracts creates rigidities in existing relationships, which are detrimental to economic growth. Firms engaged in relational contracting (that is, in implicit agreement relying on mutual trust) may be reluctant to switch to a new potential partner with a better technology, if the expected level of cooperation with this potential new partner is lower than with a long-standing one. In this case, the market size for a potential innovator is reduced, which in turn, reduces the incentive to innovate. Then, relational contracts are a poor substitute for good institutions, because they turn contractibility issues from a static problem of inefficient allocation of resources into a dynamic problem of inefficient development of technologies. This paper clarifies this trade-off but also derives conditions under which it does not apply. We focus on growth and innovation, but more generally, relational contracts, requiring long-term relationships, can come at odds with economic efficiency, whenever the economy would benefit from flexible relationships (for instance in the adjustment to macroeconomic shocks).

We have in mind an industry with the following characteristics: (i) production requires the participation of producers and suppliers, where the suppliers provide complex inputs designed specifically for the final good producer, (ii) suppliers are competing with each other, and (iii) innovations allow them to "escape competition" and to increase their market share at the expense of their competitors. In a non-repeated framework, non-contractibility of the intermediate input typically creates an ex post hold-up situation leading to underinvestment by the supplier as in Grossman and Hart (1986). In a repeated framework, if producer-supplier pairs were in isolation from each other, classic trigger strategies would easily allow for higher level of investment than in a one shot-interaction. However, if producers can switch suppliers and start cooperating with a new partner costlessly, achieving cooperation in the first place is more difficult, as producers could deviate without being punished. What makes cooperation possible in our set-up is the existence of good and bad matches between producers and suppliers, where good matches are characterized by a higher productivity level. The nature of the match is unknown to both parties before they start working together and it does not change over time. If a match turns out to be good, the value of the relationship in the following period is higher than the expected value of a new relationship. The supplier can capture the rents associated with this difference in values if cooperation with the producer continues, which induces him to invest more than his short-run interest would dictate. We consider this "cooperative" equilibrium as a model of an economy with poor contractibility but where relational contracts are widespread. We contrast this case with two other cases: an economy with the same poor contractibility, but where there is no cooperation in equilibrium (we refer to it as the "Nash case") and a setting in which inputs are fully contractible.

Every period, we let one supplier (the innovator) have the possibility to develop a new technology, which is imitated by her competitors after one period. Producers already engaged in a long-term relationship face a trade-off: switching to the innovator allows them to have access to a more productive technology, but at the risk of entering into a bad match. Entering into a bad match yields a lower productivity level no matter whether the input is contractible or not; but, when the input is noncontractible and when suppliers cooperate in good matches, bad matches are also characterized by more severe under-investment than good matches. Hence, bad matches become worse relative to good matches. This *worse bad match effect* is the main force behind our result that cooperation in a weak contractible setting magnifies rigidities in relationships.[1] In particular, in order for an innovator to capture a large share of the market, innovations have to be larger in the cooperative case than in the contractible or Nash cases. This result suggests that innovations spread more slowly in countries or sectors where relational contracts are widespread and enforcement of formal contracts difficult, relative to countries or sectors where enforcement of formal contracts is easy or where firms do not engage in relational contracts.

The insight that cooperation cannot be established if there is no cost of switching partners is also central to the analysis of both Kranton (1996) and Ghosh and Ray (1996), who both study the type of equilibria that can be sustained in models without full contractibility. In Kranton, the cost arises from the choice of an equilibrium in which cooperation is precluded for a number of periods initially. Ghosh and Ray reject such equilibria by imposing a condition of "bilateral rationality" requiring that two players always initiate a relationship with the highest level of cooperation. We impose an analogous condition. The cost in Ghosh and Ray is similar to ours and arises from a chance of matching with an "impatient" player who will under no circumstances engage in cooperation. Both papers feature cooperation and the source of the cost is not crucial for the possibility of establishing cooperation; an exogenous fixed cost of switching would likewise allow for cooperation.

The novel feature of the present paper is that cooperation can introduce rigid relationships and thereby dynamic inefficiencies. This result is not trivial and depends on the initial source of switching costs. In particular, it would not follow in a model of exogenous fixed cost of

[1] In our baseline equilibrium the decision to switch or not is efficient in the sense of maximizing joint profits (the standard monopoly distortion is present), as a producer who switches to the innovator can resume cooperation with his old supplier if the innovator turned out to be a bad match, without any punishment from the old supplier.

switching—as then the cost of breaking up an existing relationship would be independent of the level of cooperation and relationships would not by themselves imply rigidity. In our setting, however, the cost of switching increases with cooperation, since bad matches "become worse" as only good matches feature cooperation.

Further, we show that innovation is always lower in the cooperative than in the contractible case and that it can be lower than in the Nash case. This suggests that innovation is reduced in countries or sectors with poor contractual environment, and that the development of relational contracts can have negative circumstances. In an extension, we also demonstrate that cooperation might lead the innovator to follow a riskier innovation strategy.

The last section of the paper investigates different forms of innovations. First, we relax the assumption that the innovator is imitated after one period and let imitation happens gradually. Then, the long-standing presence of firms with better technologies can weaken cooperation in existing relationships, so that for some parameter values, existing relationships are in fact easier to break in the cooperative than in the contractible or Nash cases. Finally, we study within relationships innovations, and show that cooperation enhances them relative to the Nash case, and may even enhances them relative to the contractible set-up. Combined with our first set of results, we interpret these as suggesting that cooperation in a weak contractible environment specifically deters some types of innovations: those that are general and get easily imitated (maybe because of weak IPRs).

Our paper relates to two main topics in the literature: the possibility of building relationships under imperfect contractibility and the impact of institutions, in particular contractibility, on macroeconomic outcomes. A large body of theoretical literature addresses the question of building relationships in the presence of contractual incompleteness: the repetition of the same interaction can give rise to equilibria of the Folk theorem type, where parties cooperate and provide more effort (or investment) than they would in a one-shot interaction. Macaulay (1963) is the first paper to show that interactions between firms in most markets are repeated and that firms are engaged in relational contracts. In the law literature, the theory of relational contracts was first developed by MacNeil (1974), followed in particular by Ellickson (1991) and MacLeod (2006). Kreps (1996) argues that informal agreements can be superior to explicit contracts when writing an explicit contract is costly.[2] Dixit (2004) is closely related to our paper as he analyzes the type of informal institutions that emerge when the judiciary system of a country is not well developed (a famous example based on repeated interactions is Greif (1993)'s Maghribi traders). More recently, the importance of relational contracts in developing countries has been highlighted by Banerjee and Duflo (2000) who show that in the Indian software industry, reputation of firms matter for the kind of contracts they are offered.[3] The

[2]Klein (1996) shows that court enforcement and reputation mechanisms are complement in determining the range within which contractual relationships are self-sustained.

[3]Allen, Qian, and Qian (2005), Allen, Chakrabarti, De, Qian and Qian (2006) and Allen, Chakrabarti,

interaction between formal and informal institutions in the context of the interaction between a firm and an employee was analyzed in MacLeod and Malcomson (1989) who demonstrated that splitting payment into a contractible wage and noncontractible bonus can increase efficiency and the resulting contract resembles existing wage contracts. Baker, Gibbons, and Murphy (1994) analyze a similar setup and demonstrate that sometimes a signal better correlated to the actual performance (which can be interpreted as a proxy for more developed institutions) can prevent the formation of more efficient implicit contracts.

Closer to our work, Board (2011) considers a simple hold-up problem where a principal invests in a supplier for the provision of an input. There are several suppliers and investment costs are stochastic. To prevent hold-up, a principal and a set of suppliers enter a relational contract where the principal is biased towards the suppliers with whom she has already worked with (the "insiders"). This implies that an outsider with a better technology is not systematically chosen, in line with our results. Nevertheless, our paper goes further in several dimensions. First, we analyze how rigidities affect the incentives to innovate. Second, in our set-up, cooperation can be welfare reducing, which it never is in his paper. Third, we also emphasize situations where the establishment of long-term relationships does not create rigidities.

Two papers provide evidence for some of the assumptions of our model. First, Brown, Falk and Fehr (2004) ran experiments showing the endogenous emergence of long-term relationships in the absence of third party enforcement. They showed that low effort was punished by the termination of the relationship, and that in successful relationships, effort was high from the very beginning. Our cooperative equilibrium shares these features. The paper closest in spirit to ours is the empirical investigation of Johnson, McMillan and Woodruff (2002). The authors are interested in the impact of courts' efficiency on the extent to which firms grant trade credit to each other (which is viewed as a proxy for the level of trust between firms). They use a firms survey conducted in several Eastern European countries and show that in ongoing relationships, the belief in the efficiency of the court had very little impact on the level of trade credit, which suggests that firms engage in relational contracts. However, it matters a lot at the beginning of a relationship and for firms' incentives to try out new suppliers. Our model shares the same features, and may then be understood as a rationalization of their results.

There exists a large literature on the impact of institutions, particularly contractibility, on growth and development.[4] Acemoglu, Antràs and Helpman (2007) have shown that countries with weaker legal institutions adopt inferior technologies and develop a comparative advantage in sectors where there is more substitutability across inputs. Boehm (2013) structurally estimates the impact of weak contractibility on productivity across countries using a general

De, Qian, and Qian (2008) show in related papers that in India and China long-term relationships provide a successful way of financing firms.

[4] For instance, Acemoglu, Aghion and Zilibotti (2003) show that institutions that favor the establishment of long-term relationships between firms and managers are appropriate far from the frontier but turn out to be a burden close to it. Bonfiglioli and Gancia (2014) present a similar trade-off.

equilibrium model where firms face a choice between producing inputs in-house and outsourcing. These papers, however, do not allow for the establishment of relational contracts. Cowan and Neut (2007) show empirically, that productivity is relatively larger in countries with good legal enforcement in sectors with a more complex intermediate structure, and, similarly, Nunn (2007) show that these countries develop a comparative advantage in sectors that rely more on relation-specific investments.[5]

The literature on incomplete contracts and macroeconomics includes Francois and Roberts (2003), who study the impact of growth on contractual arrangements (some of our results point towards feedback effects where the frequency and the type of innovation affect the extent of cooperation between business partners) as well as Caballero and Hammour (1998) who relate incomplete contracts to the amplification of macroeconomics. Finally, the related idea that long-term relationships between producers and suppliers can be a barrier to entry was formalized in Aghion and Bolton (1987), who show that when an incumbent faces entry by potential competitors with superior technology, she will sign long-term contract that reduces the risk of entry. In our set-up, however, the relationship is of a different nature as the contract is implicit and we rule out explicit contracts that would last more than a single period.

We start out by introducing the basic model in section 2, where we describe the cooperative equilibrium that we study. Section 3 shows that cooperation leads to rigid relationships. Section 4 studies the effect of cooperation on the rate of innovation. Section 5 presents some extensions. Section 6 discusses different types of innovations for which cooperation may not be as much an impediment to innovation. Section 7 concludes. The proofs of the main results are available in Appendix A and the remaining proofs in the Online Appendix (Appendix B).

2 Model and cooperative equilibrium

In this section we develop a model of repeated interaction between final good producers (he) and intermediate input suppliers (she) in general equilibrium, where some producer-supplier matches are exogenously more productive than others. We first show that when the provision of the input is noncontractible, the classic hold-up problem arises and suppliers have an incentive to underinvest in an one-shot interaction. Then we introduce exogenous innovation and let the game be repeated, and define a cooperative equilibrium, where the prospect of continuing the relationship in the following period provides suppliers in a good match with an incentive to invest more than they would do in the one-shot interaction.

[5] In the trade literature, Rauch (1999) shows the importance of networks in shaping trade, especially for more differentiated products.

2.1 Preferences and production

We consider a quasi-general equilibrium model where consumers consume only two types of final goods: a set of differentiated goods (denoted c_i) of measure 1, and a homogenous outside good (denoted C_o). Aggregate preferences are given by a representative agent with utility:

$$U = \sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t} \left(C_{o,t} + \frac{\sigma}{\sigma - 1} \int_0^1 c_{jt}^{\frac{\sigma-1}{\sigma}} \, dj \right), \tag{1}$$

where ρ denotes the discount rate of the consumers. For clarity we will drop the subscripts t when this does not lead to confusion.

The outside good is produced at constant returns to scale one for one with labor and we normalize its price to 1, such that the wage in the economy is also equal to 1 (we consider parameter values such that the outside good always remains active).[6] All the action in the model takes place in the production of differentiated goods. The demand for a variety j (c_j) and the quantity of variety j produced (q_j), can be written as a function solely of its own price:[7]

$$q_j = c_j = p_j^{-\sigma}. \tag{2}$$

Production of the differentiated good requires the joint work of two types of agents, final good producers and intermediary input suppliers. The production of each variety is associated with the essential input of the final good producer who has the monopoly right over that variety (so there is a mass 1 of final good producers). Final good producers die with a probability δ^D every period and are replaced with new ones. Moreover, every period, each final good producer must hire a single intermediate input supplier. There is a mass 1 of intermediate input suppliers, who are infinitely lived. We could equally well have assumed that the intermediate good suppliers die with probability δ^D. Each intermediate input supplier can supply any number of final good producers without decreasing returns to scale.

More specifically, if the monopolist j hires the supplier k, the production technology is linear in the quantity of good quality inputs provided by the supplier:

$$q_j = (\theta_{jk} A_k)^{\frac{1}{\sigma-1}} X, \tag{3}$$

[6] Of course, as growth in the differentiated sector takes place, the differentiated sector eventually becomes so productive, that the consumption of the homogenous good is driven to 0. Technically, what we present here is an approximation, which is valid only as long as the productivity of the differentiated sector remains sufficiently low. Alternatively, we can assume that the productivity of the homogenous good grows at the rate of the technological frontier (through knowledge externality), in which case, what we present is not an approximation but the exact solution. Nothing of substance depends on this.

[7] The functional form of the utility function allows us to avoid general equilibrium effects going through the wages (thanks to the presence of the homogenous good) or the price index (as the elasticity of substitution between the varieties is equalized with the price elasticity of the CES aggregator). These general equilibrium features would complicate the analysis without changing any of our central results.

where θ_{jk} is a match specific and verifiable permanent level of productivity, A_k is the productivity of the intermediate input supplier k and X is the quantity of intermediate inputs of good quality provided by the supplier (technically, we should refer to $(\theta_{jk}A_k)^{\frac{1}{\sigma-1}}$ as the productivity, but, throughout the paper we make the abuse of language of referring to $\theta_{jk}A_k$ as productivity). Producing one good quality intermediate input requires one unit of the homogenous good, but the supplier can also produce an intermediate input of bad quality at 0 cost, which has no value in production. The match specific level of productivity θ_{jk} can take two values: $\theta_{jk} = 1$ in good matches, and $\theta_{jk} = \theta < 1$ in bad matches. The quality of a match is revealed to both the supplier and the producer only once they start working together (but before the supplier has to incur any investment) and is permanent.[8] Once a match has been chosen (and the match specific productivity determined) a period has to pass before they can form new relationships. We denote by b the probability that a new producer/supplier pair turns out to be a bad match. The supplier's level of productivity A_k is independent of the producer.

Throughout the paper we normalize the amount of good quality inputs provided by the supplier by the productivity of the relationship $\theta_{jk}A_k$, and denote it x (so that $x \equiv X/(\theta_{jk}A_k)$). We refer to x as the normalized amount of good quality input or the normalized investment level (as bad quality inputs are produced costlessly). We can then express revenues as $\theta_{jk}A_kR(x)$, where $R(x)$ are the normalized revenues ($R(x) \equiv x^{\frac{\sigma-1}{\sigma}}$), and joint profits as $\theta_{jk}A_k\Pi(x)$ where $\Pi(x)$ are the normalized joint profits $\Pi(x) \equiv x^{\frac{\sigma-1}{\sigma}} - x$.

2.2 Contractual incompleteness

We model contractual incompleteness in a standard fashion (a simpler version of Grossman and Hart, 1986): contractual incompleteness is the source of a classic hold-up problem. More specifically, an input is specific to a particular producer and is useless to any other agent in the economy, and, once a producer has chosen to work with a supplier, he cannot find any other supplier that period. Therefore once the two parties have decided to start working together, the set-up becomes one of bilateral monopoly. We briefly consider the one shot interaction in order to show the inefficiencies that repeated interactions can overcome.

If the input is contractible, the court can verify whether the input provided is of good or bad quality. The producer and the supplier sign a contract that maximize joint profits, so that the normalized quantity of good quality inputs is at the first best level (m) given by:

$$m \equiv \arg\max_x R(x) - x = ((\sigma-1)/\sigma)^{\sigma}.$$

If the input is noncontractible, the court cannot verify whether the input provided is of good

[8] As explained in section 3.3, this is not a crucial assumption: the logic of our results would carry through if the type of a match was only revealed after the first investment has occurred.

or bad quality.[9] There is a standard double hold-up problem: as contracts are unenforceable the stipulations of a given contract are irrelevant and the producer can claim that the inputs are of bad quality and refuse to pay, while a supplier can costlessly deliver low quality inputs (since the producer cannot find a new supplier at that point). Therefore, any contract specifying the amount of inputs of good quality to be provided is worthless. We make the classic assumption that the revenues are then shared through ex-post Nash Bargaining, and we denote by $\beta \in (0, 1)$ the bargaining power of the supplier. Therefore, providing the first best normalized amount of investment m is no longer in the interest of the supplier, as she bears the full cost of the investment but is only paid a share β of the revenues. Instead the supplier chooses to provide the amount of good quality input that maximize her ex-post profits and provides the "Nash" normalized level of investment n, given by:

$$n \equiv \arg\max_{x} \beta R(x) - x = \beta^{\sigma} m.$$

As in any standard model, there is underinvestment: $n < m$.[10]

Before the producer and the supplier start working together, an ex-ante cash transfer can be exchanged. If all suppliers are identical ($A_k \equiv 1$ for all k), they will break-even: therefore, in the contractible case, the ex-ante transfer from the supplier to the producer is equal to $t = (1 - b + b\theta)(\beta R(m) - m)$, and in the noncontractible setting it is equal to $t = (1 - b + b\theta)(\beta R(n) - n)$.

2.3 Innovation

We focus on "Schumpeterian" innovations where firms can improve the quality of their products to capture larger market shares (see Aghion, Akcigit and Howitt, 2015, for the relevance of Schumpeterian growth theory). We study within relationship innovations where a firm cooperate with its clients to improve the quality of the final product in section 6.2. For the moment, we take the innovation decision as given and assume that an innovation happens with probability $\delta^I \in (0, 1)$. As a main argument of the paper is that long-term relationships can reduce innovations, we endogenize the rate in section 4. When innovation occurs one of the suppliers gets access to a technology $\gamma > 1$ times more productive than the previous frontier

[9] We take contract incompleteness as a fact and do not model the informational assumption for this incompleteness to exist. We make the classic assumption that revenues and expenditures of the parties are non verifiable and therefore cannot be part of a contract. As argued in Malcomson (2011) and demonstrated more generally in Evans (2008) varifiable delivery can often reestablish efficiency: if the parties agree on a price from producer to supplier of $\theta R(m)$ but give the producer the right to refuse acceptance, both delivery of m by supplier and payment by producer are individually optimal in a one-shot game and efficency can be restored. Although these issues are interesting the focus here is on the ability of relationships to restore efficiency, and we sidestep them by assuming that any delivery is unverifiable as well.

[10] Underinvestment not only decreases profits but further reduces welfare: in the full contractibility case, the monopoly distortion already leads to a production of differentiated goods lower than the welfare maximizing level, incomplete contractibility aggravates this initial monopoly distortion.

technology, but, after a single period all suppliers have access to the new technology. This assumption simplifies the problem and may reflect a situation with poor intellectual property rights (IPRs). Section 6.1 relaxes it and solves for the case where the innovator is progressively imitated. We denote by A the current frontier level of technology, so that, in periods without innovation all suppliers use technology A, and, in periods with innovation only the innovator uses the frontier technology while the other suppliers use $\gamma^{-1}A$.

2.4 Timeline

The overall timeline within each period is as follows

1. Final good producers die with probability δ^D and a mass δ^D of new final good producers are born.

2. Innovation occurs with probability δ^I. If innovation occurs one supplier has access to a technology $\gamma > 1$ times more productive.

3. Each supplier makes a take-it-or-leave-it offer of an ex-ante transfer t to each producer. In the contractible case, she also commits to an amount of good quality input conditioned on the quality of the match.

4. Each producer chooses his supplier and the transfer t from the supplier to the producer is paid.

5. The type of the match is revealed if the two parties are interacting for the first time (it is already known otherwise).

6. The supplier decides on how much good quality input to provide in the noncontractible case.

7. Revenues are shared between the producer and the supplier through ex post Nash bargaining where the supplier has a weight of β.

Note that every stage game has three moves: in phase 3 suppliers make their offers for the ex-ante transfer, in phase 4 producers choose a supplier, and in phase 6 suppliers undertake the investment. The assumption of suppliers making take it or leave it offers to producers when deciding on the ex-ante transfer (Bertrand competition) simplifies matters, but is not necessary. We could extend the model to include ex ante Nash bargaining over surplus and allow producers to pay a (noncontractible) bonus to the supplier if she cooperates without affecting the incentive constraints in the following. A similar result is demonstrated in MacLeod and Malcomson (1989) (part 3 of Proposition 1) and we leave out a formal proof here.

2.5 Building a cooperative equilibrium

In the following we will consider three different setups: a situation where the input is fully contractible and the first best level of investment can be achieved even in a one-shot interaction (we refer to this as *the contractible case*), a case where the quality and delivery of the input is noncontractible and we allow for cooperation as described below (*the cooperative case*), and finally a case where the quality and delivery of the input is noncontractible and we preclude any form of cooperation (i.e. actions in one period cannot be conditioned on actions in previous periods, we call this *the Nash case*). It is important to be clear about the relations between these cases. Whereas the cooperative and Nash cases are two different equilibria in the same environment of noncontractibility, the contractible case is derived under different environmental assumptions. In principle agents could switch between cooperate and Nash equilibria. The comparison between the cooperative and contractible cases is still interesting because it makes clear that even though cooperation can increase investment to the first best it still does not achieve the same welfare as full contractibility.

We stress that the contractible environment is still a world of limited contractibility: we do not allow for contracts across periods or between more than two parties. Instead contractible refers here solely to the provision of the input. Therefore the equilibrium in the contractible case need not achieve the overall first best, but the investment level is always at the first best level m (for every history of the game). In this setting, a new born producer switches suppliers until he finds a good match. Once he has found one, he sticks to her in periods without innovation, and, because of Bertrand competition, the good match supplier offers an ex-ante transfer that allows her to capture the entire surplus of the ongoing relationship over any new relationship. In periods with innovation the producer optimally decides whether he should switch to the innovator (we study this in section 3.1). If the innovator turns out to be a bad match, the producer resumes working with his previous good match supplier in the following period. When we preclude cooperation (the Nash case) the same equilibrium exists but with input investment of n instead of the first best.

We now turn to the characterization of equilibria where the input is noncontractible and we allow for cooperation. As is typically the case in models of this type, there is a continuum of SPNEs featuring some level of cooperation between a producer and a supplier (that is equilibria where investment is higher than in the one shot interaction), and we restrict attention to a particular class of equilibria as described in the following.

We denote by $H_t^n(j,k)$ the set of histories of the game after t repetitions just after phase 5 has occurred (just after the type has been revealed) when producer j and supplier k are matched for the first time and supplier k has turned out to be a good match. We impose a *symmetry and information* condition:

Condition 1 ***Symmetry and Information (SI)*** *i) For any history belonging to* $\underset{k}{\cup} H_t^n(j,k)$ *where the supplier* k *has access to the frontier technology, the path of normalized investment undertaken in the following histories by the new supplier* k *are the same, and the decision of the producer to continue the relationship with the supplier* k *or not is the same; similarly for any history belonging to* $\underset{k}{\cup} H_t^n(j,k)$ *where the supplier* k *does not have access to the frontier technology; ii) the strategies played with one producer are independent of the history of the game played with other producers; iii) if a supplier has been chosen by the producer, her normalized investment is independent of the ex-ante transfer paid by the supplier;*

Part i) is a symmetry condition. Provided that the supplier has access to the frontier technology, every new good match relationship is identical in terms of the level of normalized investment and of the producer' decision to retain the supplier or not (both on and off the equilibrium path). In particular, if a producer starts a relationship with the innovator and the innovator turns out to be a good match, the outcome is symmetric to the case where the producer started his first relationship. We cannot however require that the strategies are identical, because, in general, the ex-ante transfer exchanged depend on whether the producer knows a good match supplier or not. This condition rules out equilibria where there is never cooperation with the innovator even if she is a good match.[11] Part ii) allows us to keep the strategies with other producers independent, so, for instance, producers cannot coordinate on punishing a supplier. Part iii) is necessary to ensure that the supplier gets the full value of the relationship when the first best is achieved. Otherwise it is possible to build equilibria where part of the surplus of a relationship would go to the producer, despite Bertrand competition. It should be clear that conditions i) and ii) avoid equilibria where players could coordinate their actions on histories that should have no direct impact on their interactions. Such restrictions would necessarily operate in an alternative environment where we directly restricted the information available to the players. Condition iii) does not affect any of our results but simplifies the exposition.

Consider a situation in which a new innovation takes place and a producer decides to break-up an existing relationship and try out the new supplier. Should the supplier consider this a deviation and punish the producer if he comes back? This matters both for the decision of whether to come back and for the division of profits in the new relationship. In the main section of the paper we impose a "forgiveness condition" and allow the resumption of cooperation.

Condition 2 ***Forgiveness****. The strategy played by a good match supplier at time* t*, is the same when the producer has worked with the supplier at time* $t-1$ *and when the producer has worked with an innovator but the innovator turned out to be a bad match.*

[11] Without this condition it would be possible to build equilibria where the path of investment levels will be systematically lower with new supplier than with the first supplier.

As we will discuss later, this condition ensures that the decision of whether to switch or not is jointly efficient. We consider the opposite assumption—where a supplier systematically punishes a producer if he switches supplier, no matter what happens with the new supplier—in subsection 5.1 and demonstrate that under quite general conditions the qualitative results are the same and that the inability to revert back adds an additional source of rigidity from cooperation.

In every period, the producer has to choose between continuing a relationship or switching to a new supplier. The difficulties of generating cooperation when players can switch partners at will is well known: If partners can costlessly start new relationships, the threat of retaliation from the current partner does not carry any force and the cost from not cooperating is non-existing. This is the interest of Kranton (1996) and Ghosh and Ray (1996). Kranton (1996) demonstrates that in a setting with identical agents and costless switching between partners any equilibrium featuring more cooperation than a one shot interaction cannot be "pair-wise enforceable": any equilibrium with cooperation requires some initial cost of a new relationship from lower initial cooperation, but when two new partners first meet they could credibly agree to skip the initial low level of cooperation and the equilibrium unravels. Both Ghosh and Ray (1996) and Kranton (1996) build equilibria that overcome this by introducing impatient players who will never cooperate. The existence of such players serve as an expected cost of establishing a new relationship and enables cooperation. The presence of good and bad matches in the current setup is analogous and following the language of Ghosh and Ray (1996) we impose a "bilateral rationality" condition. Denoting respectively by $V^{p,j}(\sigma)$ and $V^{s,k}(\sigma)$ the values of producer j and supplier k, when the profile of strategy is σ, we define the bilateral rationality condition as follows.

Condition 3 ***Bilateral rationality***. *At any history $h_t \in H_t^n(j,k)$, $\sigma|h_t$ is such that there is no $\sigma' = \left(\sigma'_j|h_t, \sigma'_k|h_t, \sigma_{-k}|h_t\right)$ (where σ_{-k} denotes the profile of the other suppliers) where $\sigma'_j|h'_t = \sigma_j|h'_j$ for all histories $h'_t \in H_t^n(j,k')$ $(k \neq k')$, σ'* satisfies condition 2, *and neither player j nor player k have an incentive to deviate from σ', such that $V^{p,j}(\sigma') + V^{s,k}(\sigma') > V^{p,j}(\sigma) + V^{s,k}(\sigma)$.*

The "bilateral rationality" means that a new pair chooses strategies that maximize their joint value under the condition that the strategy of the producer with a new good match is given (the producer is expected to renegotiate his strategies once he has found a new good match), strategies are enforceable (neither the producer nor the supplier have an incentive to deviate), and the forgiveness condition is not violated. This condition rules out "collusive" behavior by suppliers: in a good match, suppliers are willing to cooperate as much as possible right away.[12] This condition should not be confused with a "renegotiation-proof" condition. If

[12] In general the strategies that maximize the joint values may let cooperation resume at a lower level if the

one of the players deviates from the prescribed strategies a punishment phase is allowed even if it yields lower profits. Finally, we impose:

Condition 4 ***No investment in bad matches****. Normalized investment levels in bad matches are given by the Nash investment level, n.*

One could imagine cooperation in some bad match relationships, although an equilibrium with symmetric cooperation in all bad matches is impossible as there would be no punishment for deviation. For some parameter values, however, a mixed strategy equilibrium is possible. Allowing for such would alter little in our general analysis, but would complicate both exposition and notation. We therefore restrict attention to the only pure strategy symmetric equilibrium, which requires no investment in bad matches. This condition can be seen as an additional symmetry condition. If the productivity level θ is sufficiently low, this condition is automatically met as a producer would continue to search for a new supplier regardless of whether cooperation in bad matches is possible or not. We demonstrate the following proposition (the proof is contained in Appendices A and B.1).

Proposition 1 *a) There exists a symmetric SPNE satisfying conditions 1-4 b) In any symmetric SPNE satisfying conditions 1-4, there are two constants $x^*, y^* \in (n, m]$ such that normalized investment in good matches is x^* when the supplier uses the frontier technology and y^* otherwise.*

Further, as shown in Appendix B.1, any symmetric SPNE satisfying conditions 1-4 has the following structure: A newborn producer switches suppliers until he finds a good match. Once he has found a good match, he sticks to her in periods without innovation (and investment level is at x^*); in periods with innovation, he optimally decides between switching to the innovator or staying with the old good match (who now invest y^*). This decision depends on parameters and is the subject of the next section. If he switches and the innovator turns out to be a good match, he behaves with this supplier as he did with the previous good match supplier, but if the match turns out to be bad he will revert to the previous supplier and resume cooperation (allowed for by the forgiveness condition). Ex-ante transfers and Bertrand competition ensure that the supplier who offers the relationship of highest value captures the surplus of that relationship over any other relationship for the producer.

The exact structure of the equilibrium depends on parameters. We now analyze how cooperation arises in the equilibrium, and, without loss of generality, we focus on the case

producer switches to the innovator and the innovator turned out to be a bad match. On one hand, if the supplier punishes a producer, she loses the possibility to keep working at a high level with the producer if the innovator turns out to be bad. On the other hand, if the supplier were to reduce cooperation in case of a switch, she may prevent the switch from happening, which can increase the incentive to cooperate in the first place. This section and subsection 5.1, therefore cover the two extreme cases.

where after a deviation in a good match the producer looks for a different supplier (instead of staying with a good match supplier playing the Nash level of investment, which is the alternative).

When the supplier makes her investment decision, the ex-ante transfer is already paid. However, she has yet to receive the ex-post profits from the investment: a share β of the revenues minus the cost. Therefore, the supplier has a short run incentive to deviate from investing an amount x, by investing the Nash level n, which maximizes her ex post profits. Her gain for this period would then be given by $\varphi(x) A_k$, where A_k is the technology used by the supplier, with:

$$\varphi(x) \equiv (\beta R(n) - n) - (\beta R(x) - x). \tag{4}$$

There is, however, a long run cost from deviating. If the supplier deviates from the equilibrium investment level, the producer will switch supplier in the next period, so the continuation value is 0.

If the supplier does not deviate, her continuation value is positive. If there is no innovation in the following period, the producer continues working with the supplier and captures a value $V_1^s A_{t+1}$ (where A_{t+1} is the frontier technology in the following period), which corresponds to the surplus of a relationship with a good match supplier (of total value $V_1^T A_{t+1}$) over starting a new relationship (of total expected value $V_0^T A_{t+1}$). We denote the value of the old supplier if an innovation occurs by $W_1^s A_{t+1}$. When the producer does not switch to the innovator $W_1^s A_{t+1}$ is the surplus of a relationship with an outdated good match supplier of value $W_1^T A_{t+1}$, over the expected value of a new relationship with the innovator $V_0^T A_{t+1}$, when the producer switches to the innovator, the expected value of the old supplier is still positive, because if the innovator turns out to be a bad match the producer will revert and resume their cooperation.

In equilibrium where suppliers play x^* and y^*, the incentive constraint of the supplier in a period without innovation is therefore:

$$\varphi(x^*) \leq \frac{1-\delta^D}{1+\rho}\left(\left(1-\delta^I\right) V_1^s + \delta^I \gamma W_1^s\right), \tag{5}$$

as in the next period the frontier technology will be the same as the current technology of the supplier if there is no innovation and will be γ times higher otherwise. Similarly in a period with innovation, the incentive constraint of an outdated good match supplier can be written as:

$$\gamma^{-1}\varphi(y^*) \leq \frac{1-\delta^D}{1+\rho}\left(\left(1-\delta^I\right) V_1^s + \delta^I \gamma W_1^s\right), \tag{6}$$

as the technology used by the supplier is currently γ times less productive than the frontier technology.

In an equilibrium where cooperation is as high as possible, investment levels maximize joint profits under these incentive constraints. V_1^s and W_1^s themselves depend on the equilibrium levels of investment in good matches x^*, y^*, such that x^* and y^* are a solution to a fixed point problem. In subsection 3.2, we derive the fixed point problem and some properties of x^* and y^* (the expressions can also be found in Appendix A.1).

Overall, in this equilibrium, the incumbent supplier has the advantage that the nature of the match has been revealed. The advantage from the realization of being a good match acts as a fixed cost that pushes the producer to stick to the same supplier, and allows the supplier to capture the associated rents. It is the prospect of capturing these rents that induce cooperation in the first place. Crucially, this fixed cost interacts naturally with the incomplete contractibility: in a situation with incomplete contractibility, there is no cooperation in bad matches, as bad matches have no prospect. Hence bad matches are "even worse" relatively to good matches in the cooperative case than in the contractibility or the no cooperation (Nash) cases.

3 Cooperation and rigidity of relationships

In the previous section, we derived an equilibrium where cooperation in long term relationships mitigates the under-investment problem associated with contractual incompleteness. We now turn to a potential downside of this, namely that cooperation makes relationships more rigid. In the first subsection, we analyze whether a producer in a good relationship would be willing or not to switch to an innovator, and establish that cooperation makes it harder for an innovator to break into the market. In the second subsection, we derive how the level of cooperation depends on parameters and in particular how it depends on innovation itself. In the third subsection we discuss alternative set-ups and the generality of our results.

3.1 To switch or not to switch

When an innovator comes in with a superior technology, producers who have not already found a good match necessarily try the innovator since she has the same probability of being a bad match as any other supplier, but a better technology. Let us then focus on a producer who knows a good match supplier and therefore faces a trade-off. On the one hand, staying with his current good match delivers the certainty of being in a good match. On the other hand, switching gives access to the innovator's better technology, but at the risk of engaging in a bad match.

We first consider the contractible case. The technological advantage of the innovator lasts for only one period and, if the innovator turns out to be a bad match, the producer can revert to his old supplier (who remains a good match). Therefore, the producer switches to the innovator if and only if the expected productivity of the innovator is higher than the productivity of an outdated good match, that is, if and only if:

$$1 - b + b\theta > \gamma^{-1}, \text{ or equivalently, } \gamma > \gamma^{con} \equiv (1 - b + b\theta)^{-1}. \tag{7}$$

With probability $(1 - b)$ the innovator is a good match, with probability b he is a bad match, but the technology of the old good match supplier is γ times less productive. The Nash case is exactly identical except that normalized investment are always at the Nash level n, so that a producer previously in a good match switches to the innovator if and only if $\gamma > \gamma^{Nash} = \gamma^{con}$.

We now turn to the cooperative equilibrium built before. Because of Bertrand competition, a producer previously in a good match relationship switches to the innovator, if and only if the expected value of joint profits with the innovator is higher than with the old supplier. As the old supplier imitates the innovator's technology after one period, and as the producer can resume cooperation with the old supplier without any cost if the innovator turns out to be a bad match (as stipulated by the "forgiveness condition"), the decision to switch depends only on the difference in expected profits in the first period. The innovator is a good match with probability $(1 - b)$, in which case she invests x^*, and a bad match with probability b, in which case she invests n, while the old good match supplier invests y^* and her technology is γ times less productive, therefore we obtain (formal proof in Appendix A.1):

Lemma 1 *Producers previously in a good match switch to the innovator if and only if*

$$(1 - b)\,\Pi(x^*) + b\theta\Pi(n) > \gamma^{-1}\Pi(y^*). \tag{8}$$

We can rewrite (8) as:

$$1 - b + b\theta\left(\Pi(n)/\Pi(x^*)\right) > \gamma^{-1}\left(\Pi(y^*)/\Pi(x^*)\right). \tag{9}$$

Cooperation occurs in good matches so $x^* > n$, moreover, as explained below, $y^* \geq x^*$, therefore $\Pi(n)/\Pi(x^*) < 1$ and, $\Pi(y^*)/\Pi(x^*) \geq 1$, so that (7) is more easily satisfied than (9), which gives us:

Proposition 2 *(i) The parameter set for which innovators capture the whole market in the cooperative case is strictly smaller than the parameter set for which innovators capture the whole market in the contractible or the Nash cases. (ii) In particular, the minimum technological leap required for an innovator to capture the whole market in the cooperative case (γ^{coop}) is higher than that in the contractible or Nash cases: $\gamma^{coop} > \gamma^{con} = \gamma^{Nash}$.*

Proposition 2 delivers the first important message of the paper: in a context of weak contractibility, cooperation makes it more difficult to break up existing relationships. Because of the existence of bad matches, for γ sufficiently close to 1, innovations are not adopted by suppliers in good matches, but the threshold for adoption is higher in the cooperative case than in the contractible or Nash cases.

The intuition behind this result arises from two effects. The first effect is a *worse bad matches effect*: a bad match is more costly relative to a good match in the cooperative case. In this case, bad matches not only involve an inherently lower productivity level, they also involve less investment as both parties realize that the relationship will come to an end in the following period. This is captured by the term $\Pi(n)/\Pi(x^*)$ in (9).[13] This reflects that switching to the innovator is a more risky activity when the producer is engaged in a relationship.

The second effect is an *encouragement effect*, namely the fact that cooperation is (weakly) higher when using the outdated technology, than it is when using the frontier technology (that is, $y^* \geq x^*$, which affects the decision to switch or not through the term $\Pi(y^*)/\Pi(x^*)$ in (9)). The reason is that the incentive to deviate in a good match, is scaled by the technology currently used by the supplier, which is higher with the innovator than with the old supplier, whereas the reward from cooperation is scaled by the technology available in the next period which is the same for the innovator and the old supplier since imitation occurs after one period (as emphasized by (5) and (6)). In other words, the opportunity to imitate the frontier technology in the following period encourages outdated suppliers to provide a larger effort, partly compensating the fact that they are using an outdated technology. This encouragement effect is very strong here as imitation occurs after only one period, but some form of this effect will always be present as long as the supplier has a positive probability to eventually get access to the frontier technology (see also section 6.1).

So far, however, there is no welfare cost of cooperation and welfare in the cooperative equilibrium is necessarily higher than in the Nash case.[14] More rigid relationships will have a negative impact on welfare only when the rate of innovation itself is endogenized in section 4. Nevertheless, Proposition 2 (ii) states that innovation would be immediately adopted by all firms when $\gamma \in (\gamma^{con}, \gamma^{coop})$ in the contractible and Nash cases but not the cooperative case, so that we get:

Corollary 1 *When innovation $\gamma \in (\gamma^{con}, \gamma^{coop})$, there is more technological differences across firms in the cooperative case than in the contractible or Nash cases.*

Moreover, note that γ^{coop} depends on the rate of innovation δ^I through (x^*, y^*), but because

[13] Recall that for θ sufficiently small, cooperation in bad matches is necessarily impossible. For θ not small enough, the fact that there is no cooperation in bad matches comes from the condition on the equilibrium. However, even if a pair were to deviate and start cooperating, the level of normalized investment would be lower than in good matches, and so even this "cooperative" bad match would be relatively worse, than in the contractible or Nash cases.

[14] From a welfare point of view at given rate of innovation, producers switch to the innovator "too much". Bad matches are even more detrimental to welfare than to profits, as final good producers are monopolists (the level of normalized investment that maximizes welfare is higher than m), and switching to the innovator inevitably involves more bad matches.

γ^{con} does not, Proposition 2 (ii) will remain true with endogenous innovation and different rates of innovation for the cooperative, Nash and contractible cases. Therefore, the previous corollary predicts that firms are less likely to adapt innovations in countries with poor contractibility institutions and high level of cooperation/trust than in countries with good institutions or poor institutions but very low level of cooperation/trust.

3.2 Determining the level of cooperation

We now describe in detail the cooperative equilibrium when, after a deviation, a producer would always rather try a new supplier than keep working with the supplier with whom the deviation occurred (that supplier would invest the Nash level n). In Appendix A.1, we derive that the right hand side of both incentive constraints (5) and (6) is given by:

$$\begin{aligned} &\frac{1-\delta^D}{1+\rho}\left(\left(1-\delta^I\right)V_1^s+\delta^I\gamma W_1^s\right) \qquad (10)\\ &=\frac{1-\delta^D}{1+\rho-b\left(1-\delta^D\right)\delta^I\gamma}\left(\begin{array}{c}\left(1-\delta^I\right)b\frac{\left(\left(1+\rho-b\left(1-\delta^D\right)\delta^I\gamma\right)\left(\Pi(x^*)-\theta\Pi(n)\right)+b\left(1-\delta^D\right)\delta^I\left(\Pi(y^*)-\theta\Pi(n)\right)\right)}{1+\rho-b\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)}\\ +\delta^I\gamma\left(\frac{1}{\gamma}\Pi\left(y^*\right)-\left(\left(1-b\right)\Pi\left(x^*\right)+b\theta\Pi\left(n\right)\right)\right)^+\end{array}\right),\end{aligned}$$

where $X^+\equiv\max(X,0)$. The first term comes from the difference between the value of an ongoing relationship and the value of starting a new relationship in a period without innovation (this term is proportional to V_1^s, the value of a good match supplier in a period without innovation). This difference is a weighted sum between the difference in profits between a good match and bad match in periods without innovation $(\Pi(x^*)-\theta\Pi(n))$ and in periods with innovation $(\gamma^{-1}(\Pi(y^*)-\theta\Pi(n)))$. The factor b in front of the fraction reflects that a new supplier is a bad match with probability b. In periods with innovation, the difference between the profits with the old supplier and the profits with the innovator is given by $\frac{1}{\gamma}\Pi(y^*)-((1-b)\Pi(x^*)+b\theta\Pi(n))$. If this difference is positive, it contributes to the value of the old supplier, hence the second term. Comparative statics are then given by the following proposition and remark (proofs in Appendix B.2).

Proposition 3 *(i) The investment levels (x^*,y^*) weakly increase with the number of bad matches, b, and decrease with the relative productivity of bad matches, θ, the discount rate, ρ, and the probability of death δ^D; (ii) when the innovator captures the entire market, the investment levels (x^*,y^*) increase in the size of innovations γ.*

Remark 1 When a producer would always rather try a new supplier than work with a noncooperative good match supplier, and the innovator captures the entire market, the investment levels (x^*,y^*) decrease with the rate of innovation δ^I provided that innovations are not too large ($\gamma b\left(1-\delta^D\right)\left(2-\delta^I\right)<1+\rho$ is a sufficient condition).

Proof. Online Appendix. ■

How much suppliers cooperate depends on how bad the alternative option is. Therefore if the probability of a bad match, b is higher, or if they are more severe (low θ), a good relationship will have more value, and the potential for cooperation is higher. A higher value of the future (lower ρ and δ^D) have the same effect. This follows directly from (10) in the specific case where a producer does not work again with a good match supplier who has stopped cooperating. Furthermore, we get that when the innovator captures the entire market ($\gamma > \gamma^{coop}$) large innovations favor cooperation. The reason is that larger innovations lead to a higher growth rate, which increases the expected value a supplier can capture by cooperating, favoring more investment in good matches. If the innovator does not capture the entire market then larger innovations also reduce the value a good match supplier can capture in periods with innovation.

Finally the effect of the rate of innovation is in general ambiguous, even when the innovator captures the entire market. More frequent innovations will have three effects on investment levels: (i) a positive effect through a higher growth rate, (ii) a negative effect through a higher probability of ending the relationship, and (iii) a further negative effect which reflects that the benefit of being in a good match over a random match is higher in periods without innovation (and this benefit is precisely what drives the incentive to cooperate). For sufficiently small innovations, effect (ii) dominates effect (i), and therefore more frequent innovations will lower the level of cooperation. We can compare this result to Francois and Roberts (2003), who show that an increase in innovation can push firms towards providing short-term contract arrangements instead of implicit guarantees of lifetime employment to their workers. In our model, the same idea is captured by the possible decrease in cooperation following an increase in the rate of innovation.

3.3 Alternative set-ups

We now analyze the generality of Proposition 2 by discussing alternative setups—under the maintained assumption that the technological advantage of the innovator is short-lived (see subsection 6.1 for the alternative case). What drives our result is the fact that switching to the innovator becomes less attractive in the cooperative equilibrium. If the supplier turns out to be a bad match not only is productivity lower, but so is cooperation as well and switching to an innovator therefore becomes more risky. This is consistent with the findings of Johnson et al. (2002) that the belief in the efficiency of the court matters for the level of trust between firms at the beginning of a new relationship, but much less later.

This would be the case in a wide range of models. To generate cooperation in an equilibrium where parties can change partners at will, there must be a cost of switching from one partner to another (here, the risk of finding a bad match). In many set-ups this cost interacts with incomplete contractibility to generate a lower level of cooperation at the beginning of a relationship. This would also be the case if we had assumed that the type of a match was only revealed after the first investment has occurred, since then cooperation in the first period of a relationship would lie between the Nash level and the level in a match which is known to be good. Similarly, in models where suppliers differ in their discount rate, or in models with relationship-specific human capital, the (expected) level of cooperation in a new relationship will be lower than in an established one. In fact, a low level of cooperation at the beginning of a relationship itself can be the source of the fixed cost—as in the first model of Kranton (1996), but such equilibrium relies on some collusive behavior by the suppliers which we ruled out with the bilateral rationality condition.

Nevertheless, the result that cooperation creates rigidities is not straightforward. Consider an alternative set-up without good and bad matches but where there is a fixed cost of switching suppliers fA. Then, provided that the fixed cost is sufficiently large, the first best investment level can be achieved in the cooperative equilibrium and the producer switches to the innovator as soon as $(\gamma - 1)\,\Pi\,(m) \geq f$ in both the contractible and cooperative cases, but he switches if $(\gamma - 1)\,\Pi\,(n) \geq f$ in the Nash case, that is for higher innovation sizes γ. There (and in contrast with our set-up), the relative cost of switching does not increase with cooperation.

Finally, even in the current set-up, could society do better? The answer is yes, but only with collusion by suppliers. To see this, consider a model without innovation. One can build an equilibrium where cooperation occurs only when the producer meets a good match for the first time, and where there is no cooperation should a deviation occur and the producer finds a new good match. Then, the value of the first good match relationship over a new relationship is higher—since there is never going to be cooperation again—and therefore cooperation (on path) is higher. However, this requires that a new good match supplier punishes a producer for a deviation that occurred with the first good match supplier. Similarly, if outdated suppliers agree not to cooperate with potential producers in periods where innovation occurs, they can push producers to try out the innovator in the first period, and relationships end up being even less rigid than in the contractible case. Nevertheless, this does not fit the description of a competitive industry, and is difficult to generalize in a set-up with imperfect information (for instance if suppliers do not know whether a producer knows a good match or not, whether an innovation has occurred or not).

4 Endogenous innovation

Subsection 3.1 showed that cooperation creates rigidity in long-term relationships. We now turn to the issue of how this rigidity can be the source of dynamic inefficiencies. We study how the equilibrium rate of innovation δ^I is determined and show that the rate of innovation is reduced with noncontractibility, and may be further reduced by cooperation.

4.1 Rate of innovation

We now model the innovation effort. The crucial element of interest is the value of a new innovation which depends on whether the input is contractible or not, and if not, on whether cooperation occurs or not. The higher is the value the higher is the incentive to invest. We choose a particular simple setting to demonstrate this, but it should be clear that our results hold more generally.

Every period one supplier gets a new idea. This idea turns into a useful innovation with probability δ^I if the potential innovator invests $A\psi\left(\delta^I\right)$ (where A is the frontier technological level before innovation occurs), and ψ is a convex function with $\psi(0) = 0$, $\psi'(0) = 0$ and $\lim_{\delta^I \to 1} \psi'\left(\delta^I\right) = \infty$. The size of innovation γ is a constant. Because the probability that the potential innovator has already made a successful innovation is infinitesimal, the market share of the potential innovator is infinitesimal, so that, for all purposes the potential innovator is an entrant. In this subsection, we compare the rate of innovation in the three different cases: *contractible*, *Nash* and *cooperative*.

Thanks to Bertrand competition the innovator captures the entire value of a relationship with her over the second best option of the producer.[15] Recall that the technological advantage of the innovator lasts for only one period, and that, in the cooperative case, good match suppliers resume cooperation if the producer switches to the innovator and the innovator turns out to be a bad match. The difference between the value of a relationship with the innovator and the value of a relationship with the best alternative, is then simply equal to the difference in profits in the first period. We denote by $V_{I,K}^{s,t}$ the value captured by the innovator (normalized by the frontier productivity level) from a relationship with a producer, who knows a good match supplier ($t = g$), or who does not know any good match supplier ($t = b$), for the contractible ($K = con$), the Nash ($K = Nash$) and the cooperative cases ($K = coop$). We get that, in the contractible case, the value captured by the innovator from a relationship with a producer previously not in a good match is given by:

$$V_{I,con}^{s,b} = (1 - b + b\theta)\left(1 - \gamma^{-1}\right)\Pi(m), \tag{11}$$

while the value captured by the innovator from a relationship with a producer previously in a good match is given by:

$$V_{I,con}^{s,g} = \left(1 - b + b\theta - \gamma^{-1}\right)^{+}\Pi(m). \tag{12}$$

The situation of producers previously in a good match has been analyzed in (7). The reasoning is similar for the other producers: joint expected profits are same with the innovator and any other supplier except in the first period where they are γ times higher with the innovator; and

[15] If instead of Bertrand competition, we had assumed ex-ante Nash Bargaining, the innovator would capture only part of the difference, but as long as he captures a postive part, the results of this subsection carry through.

Bertrand competition allows the innovator to capture all the surplus of a relationship with her over any other relationship. Similarly, for the Nash case, we get:

$$V_{I,Nash}^{s,b} = (1-b+b\theta)\left(1-\gamma^{-1}\right)\Pi(n) \text{ and } V_{I,Nash}^{s,g} = \left(1-b+b\theta-\gamma^{-1}\right)^{+}\Pi(n). \tag{13}$$

Finally, in the cooperative case, we get:

$$V_{I,coop}^{s,b} = (1-b)\left(\Pi(x^*) - \gamma^{-1}\Pi(y^*)\right) + b\theta\left(1-\gamma^{-1}\right)\Pi(n), \tag{14}$$

$$V_{I,coop}^{s,g} = \left((1-b)\Pi(x^*) + b\theta\Pi(n) - \gamma^{-1}\Pi(y^*)\right)^{+}. \tag{15}$$

The case of producers previously in good matches was analyzed in (8). The case of producers previously not in good matches follows the same logic. The innovator captures the difference in expected profits between a relationship with her and starting a relationship with an outdated supplier. If the producer were to do so, expected joint profits would be the same except in the first period where the outdated supplier would be γ times less productive, and would invest y^* instead of x^* if she were a good match.

In equilibrium, the steady-state fraction of firms previously not in a good match is constant, independent of the rate of innovation and given by $\omega = \delta^D / \left(1 - \left(1-\delta^D\right)b\right)$.[16] Hence, assuming that the steady state has been reached, the innovator solves the problem:

$$\max_{\widehat{\delta}} \gamma\widehat{\delta}\left[\omega V_{I,K}^{s,b}\left(\delta^I\right) + (1-\omega)V_{I,K}^{s,g}\left(\delta^I\right)\right] - \psi\left(\widehat{\delta}\right), \tag{16}$$

for $K = C$, $Nash$, NC. The first order condition uniquely defines the equilibrium rate of innovation in the contractible case (δ^{con}), and in the Nash case (δ^{Nash}). In the cooperative case, the value of the innovator depends on the equilibrium rate of innovation, so any fixed point of the first order condition would be a solution to the problem, we consider the highest one and denote it δ^{coop}.[17] A higher expected reward from innovation leads to a higher rate of innovation. First, note that for certain parameter values, ie. $\gamma \in (\gamma^{con}, \gamma^{coop})$,[18] an innovation is only sufficiently profitable to justify switching for the contractible and Nash cases, but not for the cooperative case, that is $V_{I,coop}^{s,g} = 0$ and $V_{I,Nash}^{s,g}, V_{I,con}^{s,g} > 0$. More generally, applying that $m \geq y^* \geq x^* > n$, to equations (11), (12), (13), (14) and (15), we easily get $V_{I,con}^{s,g} > V_{I,coop}^{s,g}$, $V_{I,con}^{s,b} > V_{I,coop}^{s,b}$ and that $V_{I,coop}^{s,g}$ can be higher or lower than $V_{I,Nash}^{s,g}$.[19] This gives the following proposition.

[16] ω is the share of firms that know a good match supplier willing to cooperate with them. It does not depend on the rate of innovation, because when an innovation occurs, producers do not lose the possibility to cooperate with their old supplier.

[17] We could also assume that ψ is sufficiently convex to rule out the possibility of multiple equilibra.

[18] Note, that although we denote by γ^{coop} the size of innovation necessary for switching in the cooperative case in both this section and the preceding, they are mathematically different objects. In the preceding section, the rate of innovation, δ^I was exogenous and γ^{coop} was a function of δ^I. In this section, δ^I is a choice variable so γ^{coop} is no longer a function of δ^I. Not making this explicit in the text should not lead to confusion.

[19] One can prove that $V_{I,coop}^{s,b} > V_{I,Nash}^{s,b}$: the value captured by the innovator from producers who are not in an ongoing relationship is higher in the cooperative case than in the Nash case.

Proposition 4 *The highest equilibrium rate of innovation in the cooperative case is lower than the rate of innovation in the contractible case, and may even be lower than the rate of innovation in the Nash case:* $\delta^{coop}, \delta^{Nash} < \delta^{con}$ *but* $\delta^{coop} \lesseqgtr \delta^{Nash}$.

The crucial element of Proposition 4 is the comparison between the value of innovating in the cooperative and Nash cases. This is the result of three effects. The *worse bad match* effect reduces the expected gain from innovation as the lower productivity of a bad match will be further increased by the lack of cooperation and the *encouragement* effect induces more cooperation from the existing supplier which reduces the gain from switching. As the innovator must compensate the producer for this, the potential reward to the innovator is reduced. In addition to these two effects, there is an additional opposite *scale effect*: a higher level of cooperation with frontier good matches increases profitability should the innovator turn out to be a good match which increases the incentive to innovate. This explains why δ^{coop} may be higher or lower than δ^{Nash}. Comparing the contractible case to the cooperative one, all effects go in the same direction which is why $\delta^{coop} < \delta^{con}$.

Interestingly, if $\gamma \in (\gamma^{con}, \gamma^{coop})$, relationships never break up in the cooperative case (unless the producer dies) but do so in the contractible case. If $\gamma > \gamma^{coop}$, innovations break up relationships in both cases, but as innovations are more frequent in the contractible case, relationships still last longer in the cooperative case. Therefore the model predicts that as long as cooperation occurs, relationships should last longer in countries where contracts are poorly enforced.

As innovation is already too low from a welfare perspective because of "standard" externalities of imitation and building on the shoulders of giant, a lower rate of innovation can easily translate into lower welfare, so that we get:

Corollary 2 *Welfare is always lower with incomplete contractibility than with complete contractibility, and cooperation may increase or decrease welfare.*

In other words, if cooperation decreases the rate of innovation relative to the Nash case, it is possible that this lower rate of innovation leads to lower welfare, despite the higher level of investment in relationships. The fact that the rate of innovation is inefficient to start with is essential to get this result. Relationships make the profitability of a new innovation smaller for the innovator, but that loss in itself cannot outweigh the benefit of higher investment that comes from the relationship. It is only because innovation is already too low (such that a further reduction lowers welfare for society as a whole) that relationships can decrease overall welfare.

Consider alternatively a setup in which an innovation is temporary such that the innovator returns to the old technology after one period and no imitation is possible (such that both imitation and 'building on the shoulders of giants has been precluded').[20] In such a case, private and social benefits of an innovation are equal. All of our results—except corollary 2—would still hold.

5 Extensions

Here we present some extensions of the baseline set-up. First, we consider an alternative cooperative equilibrium where the supplier punishes the producer (by refusing to engage in future cooperation) if he switches to the innovator no matter what type the innovator turns out to be. Then, we discuss how our results depend on the distance to the technological frontier, before turning to a case where the supplier can choose the size of the innovation and demonstrating that relationships might induce the producer to engage in more "radical" innovations to break through existing relationships.

5.1 Case where suppliers systematically punish producers who switch to the innovator

Here we describe an *alternative cooperative equilibrium* where the supplier refuses to reengage in cooperation if the producer switches to the innovator (that is we replace the "forgiveness condition", condition 2, by its opposite, a "punishing" condition which specifies that after a deviation towards the innovator, the previous supplier always plays the Nash level of investment). For the sake of simplicity, we focus on parameters value for which a producer would rather switch supplier than stay with a non cooperative good match. We also assume that when innovators decide on how much to invest, they are unaware of when the last innovation occurred. We then prove in Appendix A.2, the following proposition.

Proposition 5 *(i) The parameter set for which innovators capture the whole market in the alternative cooperative case is strictly smaller than the parameter set for which innovators capture the whole market in the contractible or the Nash cases; in particular, the minimum technological leap required for an innovator to capture the whole market in the alternative cooperative case (γ^{coop2}) is higher than that in the contractible or Nash cases ($\gamma^{con}, \gamma^{Nash}$): $\gamma^{coop2} > \gamma^{con} = \gamma^{Nash}$. (ii) For ρ small enough ($\rho < \left(\gamma/\delta^{coop2} - 1\right)\left(1 - b\left(1 - \delta^{D}\right)\right) + b\left(1 - \delta^{D}\right)\delta^{coop2}\left(\gamma - 1\right)$ is a sufficient condition), the highest equilibrium rate of innovation in the alternative cooperative case is lower than the rate of innovation in the contractible case, and may even be lower than the rate of innovation in the Nash case.*

This proposition stipulates that our results carry through in this alternative equilibrium. This is not surprising and in some sense the results are reinforced. Indeed, if a producer

[20] Technically, to ensure efficient innovation it would still be necessary to implement a subsidy to the production of the final good in order to get rid of the existing monopoly distortion.

switches to the innovator, and the innovator turns out to be a bad match, the producer would have to suffer additional losses in the periods following innovation as he would have to keep looking for a good match, since the previous one would have stopped cooperating. This *loss of cooperation effect* pushes towards more rigid relationships in the cooperative case than in the contractible or Nash cases. In Appendix A.2, we show that producers would switch to the innovator if and only if

$$(1-b)+b\theta\frac{\Pi(n)}{\Pi(x^*)}-\frac{\left(1-\delta^D\right)b^2\left(\left(1-\delta^I\right)\left(1-\theta\frac{\Pi(n)}{\Pi(x^*)}\right)+\delta^I\left(\frac{\Pi(y^*)}{\Pi(x^*)}-\theta\frac{\Pi(n)}{\Pi(x^*)}\right)\right)}{1+\rho-b\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)}>\gamma^{-1}\frac{\Pi(y^*)}{\Pi(x^*)}. \tag{17}$$

The third term in (17) (which is absent in (9)) reflects the loss of cooperation effect. It is equal to the loss in expected profits from the risk of having to look for a new supplier in the subsequent periods, scaled by profits in a good match when no innovation arises ($\Pi(x^*)$).[21] Therefore, Proposition (2) carries through.[22]

In the cooperative case, when innovations are sufficiently large to break-up existing relationships, the share of producers who are not in an ongoing good match relationship depends on when the last innovation occurred. This is why to ensure a steady state rate of innovation, we consider that innovators ignore when this happened. The expected share of producers not in an ongoing good match relationship (ω) becomes a weakly increasing function of the equilibrium rate of innovation (as more innovation translates into a higher break-up rate of good match relationships).[23] This leaves significant room for multiple equilibria: for instance, there could be an equilibrium where innovation is scarce, so that most producers have found a good match supplier and cooperation is widespread, and another equilibrium, where innovation is frequent, many producers are not engaged in a good match relationship and cooperation is rare.

As before the scale effect pushes towards more innovation in the cooperative case than in the Nash case, but towards less innovation than in the contractible case. The encouragement effect, the worse bad match effect and now the loss of cooperation effect, by making relationships more rigid in the cooperative case, push towards less innovation in the cooperative case than in both the Nash and contractible cases. There is however a counteracting *general equilibrium effect*:

[21] The term is easy to interpret: when a producer is looking for a new supplier (which happens only if the innovator turned to be a bad match, that is with probability b), there is a probability b that the new supplier is a bad match, in which case profits are given by $\theta\Pi(n)$ instead of $\Pi(x^*)$ in a period without innovation (which happens with probability $1-\delta^I$), and instead of $\Pi(y^*)$ in a period with innovation (which happens with probability δ^I), this difference in profits is appropriately discounted in (17).

[22] Unfortunately, it is hard to compare the threshold γ^{coop} and γ^{coop2}. The loss of cooperation effect pushes towards $\gamma^{coop2}>\gamma^{coop}$, however, x^* can be lower than in the baseline model reducing the impact of the worse bad match effect.

[23] The share of producers previously not in a good match is given by $\frac{\delta^D+b\delta^I\left(1-\delta^D\right)}{1-b\left(1-\delta^D\right)\left(1-\delta^I\right)}$ when $\gamma>\gamma^{NC}$ but by $\frac{\delta^D}{1-b\left(1-\delta^D\right)}$ when $\gamma<\gamma^{NC}$ (it is discontinuous).

when innovations are sufficiently large to break up existing relationships, there will be more producers not in an ongoing good match relationship in the cooperative than in both the contractible and Nash cases. As an innovator captures more value from producers who are not in an ongoing good match relationship, this is a force pushing towards more innovation in the cooperative than in the Nash but also contractible cases. This last force is dominated by the loss of cooperation effect alone for a sufficiently low discount ρ.[24] This is because a lower discount rate makes the loss-of-cooperation effect more potent (since that effect depends on future outcomes).

Loss of good matches in the contractible and Nash cases. It may be the case that even in the contractible or Nash cases, a producer cannot resume working with a supplier after the relationship was halted, either because the two parties suffer a utility loss, or because the producer forgets the identity of good matches once he has stopped working with them. Under this scenario, switching to an innovator involves losing a good match supplier also for the contractible and Nash cases, so that a producer switches if and only if:

$$1 - b + b\theta - \frac{\left(1-\delta^D\right) b^2 \left(1-\theta\right)}{1+\rho - b\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)} > \gamma^{-1}. \tag{18}$$

Therefore, even though the loss of cooperation effect now applies in all cases, it is still the case that the parameter space for which a switch occurs is larger in the contractible or Nash cases than in the cooperative case, because cooperation is relatively lower in bad matches ($n < x$) and relatively higher in outdated good matches ($y \geq x$). For a given innovation rate the share of producers who do not know a good match at the beginning of a period in steady-state is the same in all cases (there is no general equilibrium effect). Therefore, the highest equilibrium innovation rate is lower in the cooperative case than in the contractible one (with no condition), and may be higher or lower than in the Nash case.

5.2 Cooperation and expanding varieties

The main proposition of the paper is that cooperation can be a poor substitute for contractibility by introducing dynamic inefficiencies. Here we argue that the importance of this effect may depend on the state of development of the economy and that cooperation might provide a better substitute for countries in phases of rapid development.

Consider an alternative model where the mass of final good producers is increasing. This could represent horizontal innovation, population growth or periods of increasing outsourcing (interpreting the new final good producers as foreign firms who decide to start acquiring their inputs from the country of study). Every period there will be a large mass of newborn producers, who will not already be engaged in ongoing relationships. Since cooperation raises

[24] This condition will be satisfied for reasonable parameter values since $\gamma > \gamma^{coop2} > (1-b+b\theta)^{-1}$ is necessary for the general equilibrium effect to exist, and δ^{coop2} is small.

the amount of profits that an innovator can make from supplying producers who are not in good matches ($V_{I,coop}^{s,b} > V_{I,Nash}^{s,b}$), it becomes more likely that cooperation raises the rate of innovation relative to the Nash case ($\delta^{coop} > \delta^{Nash}$ becomes more likely).

5.3 Choosing the type of innovation

In the cooperative case, innovators may be willing to react to the larger rigidity in existing relationships (and so smaller market size), by pursuing riskier innovation strategies. To study this in the simplest possible framework, we focus on a discrete choice of two innovation regimes: regime 1 (γ_1, δ_1) features small but frequent innovations, while regime 2 features large but rare innovations $(\gamma_2 > \gamma_1, \delta_2 < \delta_1)$. We investigate when regime 1 is an equilibrium, that is assuming that the economy is in regime 1, under which conditions an innovator has no incentive to switch to regime 2, for the contractible, Nash and cooperative cases.[25]

More specifically, for given $(\gamma_1, \delta_1, \gamma_2)$ we compute the highest innovation rate in regime 2 (the infrequent regime), δ_2, that would make the innovator stick to the regime 1, in the contractible (δ_2^{con}), Nash (δ_2^{Nash}) and cooperative (δ_2^{coop}) cases. The inequality $\delta_2^{con} > \delta_2^{coop}$ can be interpreted as the innovator having a higher incentive to pursue larger but scarcer innovations in the cooperative case than in the contractible case. We easily get $\delta_2^{Nash} = \delta_2^{con}$, as the reward from innovation is just scaled up by a factor $\Pi(m)/\Pi(n)$ in the full contractible case, the choice of regime by the innovator is not affected. Hence, it is only the existence of cooperation in long-term relationships that affects the relative reward from different regimes of innovations. We focus on the case where $\gamma_1 \in (\gamma^{con}, \gamma_1^{coop})$, that is, in the cooperative case, the innovator does not capture the entire market under regime 1 (γ_1^{coop} is the threshold when the innovator captures the market for an innovation rate δ_1). As shown in Appendix B.1, there is a threshold $\widetilde{\gamma}^{coop}$ such that for $\gamma_2 > \widetilde{\gamma}^{coop}$, the innovator could capture the entire market by switching to regime 2.[26] Denoting by (x_1, y_1) the equilibrium investment levels in regime 1, we obtain the following proposition (proof in Appendix B.3).

Proposition 6 *Consider the case where innovations are large enough to break up existing relationships in regime 1 in the contracible or Nash cases but not in the cooperative case,* $\gamma_1 \in (\gamma^{con}, \gamma_1^{coop})$*, then:*

(i) for sufficiently large innovations in regime 2, γ_2*, then the innovator has a larger incentive to switch to regime 2 in the cooperative case than in the contractible or Nash cases:* $\delta_2^{Nash} = \delta_2^{cont} > \delta_2^{coop}$*.*

[25] In a fully endogenous setting, the choice of the size and rate of innovation would depend on how the cost function ψ depend on both δ and γ. Considering the choice between 2 regimes allows us to abstract from this issue.

[26] In general this threshold is different from γ_2^{coop}, the threshold at which the innovator captures the market with an innovation rate δ_2, since here we look at a one shot deviation only, so that in future periods, the innovation rate is assumed to still be δ_1.

(ii) if $y_1 = m$, $\gamma_2 > \widetilde{\gamma}^{coop} + \omega\left(\gamma^{con} - 1\right)\left(\widetilde{\gamma}^{coop} - \gamma_1\right) / \left(\gamma_1 - \gamma^{con}\right)$ is a sufficient condition for $\delta_2^{Nash} = \delta_2^{cont} > \delta_2^{coop}$, it is also a necessary condition when $x_1 = m$.

Hence for γ_2 sufficiently large, the innovator has a higher incentive to switch to regime 2 in the cooperative than in the contractible or Nash cases. The intuition is that in the cooperative case, switching allows him to capture the whole market, whereas in the two other cases, he already captures the whole market in regime 1 (the threshold needs to be higher than $\widetilde{\gamma}^{coop}$ because when γ_2 is slightly above $\widetilde{\gamma}^{coop}$, the profits captured by the innovator from producers previously in good matches in the cooperative case are very small). The expression of the threshold when cooperation is sufficiently high ($y_1 = m$), $\widetilde{\gamma}^{coop} + \omega\left(\gamma^{con} - 1\right)\left(\widetilde{\gamma}^{coop} - \gamma_1\right) / \left(\gamma_1 - \gamma^{con}\right)$, is interesting by itself. It is increasing in ω: if most producers are in a good match relationship, (ω low), switching in the cooperative case involves capturing most of the market size, whereas it is already captured in the contractible case. The threshold is also decreasing in γ_1: if γ_1 is close to $\widetilde{\gamma}^{coop}$, switching to the regime 2 becomes more interesting in the cooperative case than in the contractible case. Therefore, introducing cooperation in long-term relationships may lead to the surprising result, that when contractibility is weak, although the rate of innovation is reduced, the size of an innovation may be larger, as long as cooperation arises.

6 When cooperation may not reduce innovation

So far we have demonstrated that cooperation in a weak contractible environment slows down technology adoption and innovation. Yet, the type of innovation that we considered was quite specific: we focused on general innovations that spread quickly. In this section, we demonstrate that other types of innovation may not suffer as much from the establishment of relational contracts. First, we analyze the case of an innovation that diffuses slowly, and second the case of relationship specific innovations.

6.1 Slow diffusion of innovations

Here we assume that instead of getting access to the frontier technology one period after an innovation has occurred, each outdated supplier gets access to the frontier technology with probability $\Delta \in (0, 1]$ in periods with no innovation and catch up with the previous frontier technology in periods with innovation. For simplicity, we assume that when a producer and a supplier have worked together but then stopped doing so, they can never resume any relationship even in the contractible or Nash case (as suggested at the end of section 5.1, this could result from a strong utility loss from resuming a broken relationship).[27]

[27] Without this assumption, the outside option of a producer depends on the number of good matches that he knows and on the number of periods since the last innovation, making the analysis significantly more complicated in the contractible or Nash cases without adding much economic insight.

In periods without innovation, there will be a continuum of suppliers with access to the frontier technology (the last innovator and the firms which have successfully imitated the frontier technology). Therefore a good match supplier with the frontier technology will only be able to capture the surplus of a relationship with her over starting a new relationship with another frontier firm (that is, at time $t+1$, she captures $V_1^s A_{t+1} = \left(V_1^T - V_0^T\right) A_{t+1}$). If an innovation occurs, an outdated good match supplier will capture the potential surplus of a relationship with her over starting a new relationship with the innovator (that is, she captures $W_1^s A_{t+1} = \max\left(\left(W_1^T - V_0^T\right) A_{t+1}, 0\right)$); for parameter values such that the producer switches to the innovator, this surplus is 0. Therefore the continuation value for a good match supplier with the frontier technology at time t is given by $\frac{1-\delta^D}{1+\rho}\left(\left(1-\delta^I\right) V_1^s + \delta^I \gamma W_1^s\right) A_{t+1}$. This continuation value is the same whether there is only one firm with the frontier technology at time t or a continuum. It is the prospect of capturing this continuation value for the supplier that makes cooperation possible in the first place. Therefore, in the cooperative equilibrium, there is a unique level of normalized investment undertaken by a good match supplier (denoted x^* as before), which must still satisfy the IC constraint (5).

Consider now the case of an outdated good match at time t. As before, we denote by y^* her level of normalized investment. In period $t+1$, this good match supplier will become a good match supplier with the frontier technology with probability Δ, otherwise she stays a good match supplier with an outdated technology. Therefore, in the cooperative equilibrium, the IC constraint for an outdated good match is given by:

$$\gamma^{-1}\varphi\left(y^*\right) \leq \frac{1-\delta^D}{1+\rho}\left(\left(1-\delta^I\right)\Delta V_1^s + \left(\left(1-\delta^I\right)(1-\Delta) + \delta^I\gamma\right) W_1^s\right). \tag{19}$$

As before, the encouragement effect pushes towards a higher level of cooperation in outdated relationships than in frontier relationships (the term γ^{-1} on the LHS of (19) pushes for $y^* \geq x^*$). Yet, for $\Delta < 1$, the RHS in (19) is also lower than the RHS in (5) since $V_1^s > W_1^s$, which pushes towards a lower level of cooperation in outdated relationships ($y^* \leq x^*$). This occurs because starting a new relationship with a frontier supplier is a more interesting outside option for a producer who is working with an outdated supplier than for one who is working with a frontier supplier. Therefore, we refer to this effect as the "outside option" effect. Overall the relationship between x^* and y^* is ambiguous and the arrival of an innovation may weaken cooperation in established relationship.

In Appendix B.4, we show that producers switch to the innovator in the cooperative case if and only if

$$\begin{aligned} &1 - b + b\theta\frac{\Pi(n)}{\Pi(x^*)} - \frac{b^2\left(1-\delta^D\right)\left(\left(1-\delta^I\right)\left(1-\theta\frac{\Pi(n)}{\Pi(x^*)}\right) + \delta^I\left(\frac{\Pi(y^*)}{\Pi(x^*)} - \theta\frac{\Pi(n)}{\Pi(x^*)}\right)\right)}{1+\rho-b\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)} \\ &> \gamma^{-1}\frac{\Pi(y^*)}{\Pi(x^*)} - (1-\Delta) K\left(1-\gamma^{-1}\frac{\Pi(y^*)}{\Pi(x^*)}\right), \end{aligned} \tag{20}$$

with $K > 0$.[28] This expression is the same as (17) except for the last term on the RHS. That term captures the loss experienced by a producer who stays with an outdated good match supplier (generating profits $\gamma^{-1}\Pi(y^*)$) relative to switching to a frontier good match supplier (with profits $\Pi(x^*)$) in all periods until either the technology diffuses (which happens with probability Δ), or another innovation occurs (which happens with probability δ^I). Everything else equal, slow diffusion of innovation (a low Δ) encourages producers to switch to the innovator. In the contractible and Nash cases on the other hand, the producer switches suppliers when:

$$1 - b + b\theta - \frac{b^2\left(1-\delta^D\right)(1-\theta)}{1+\rho-b\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)} > \gamma^{-1} - (1-\Delta)\,K\left(1-\gamma^{-1}\right). \qquad (21)$$

Comparing these two expressions reveals that, as before, whether a switch occurs less or more easily in the cooperative than in the contractible and Nash cases depend on the different cooperation levels with a frontier good match (x^*), an outdated good match (y^*) or a bad match (n).[29] As before, $x^* > n$, and this "worse bad match effect" encourages a producer to stick to an outdated supplier because of the loss which he would experience if he switches and the innovator turns out to be a bad match (both during the current period and the following ones as he looks for a new good match). Whenever the encouragement effect dominates the outside option effect, the level of cooperation is higher in outdated than frontier good matches ($y^* \geq x^*$), which also encourages producers to stick to an outdated supplier as the loss is relatively lower in this period and in future periods until the technology diffuses. Otherwise, the outside option effect might induce producers to switch more easily to the innovator in the cooperative than in either the contractible or Nash cases (so that Proposition 2 would no longer hold).[30] A larger share of bad matches b makes the worse bad match effect more potent, so that in Appendix B.4 we derive that for δ^I sufficiently small, $b\theta/\left(1+\rho-b\left(1-\delta^D\right)\right) > \gamma^{-1}/\left(1+\rho-\left(1-\delta^D\right)(1-\Delta)\right)$, is a sufficient condition for cooperation to act as a barrier to entry (the parameter space for which the innovator captures the entire market is lower in the cooperative equilibrium than in the Nash or contractible cases).

Endogenizing the innovation rate in this set-up can be done as in section 5.1. As before, the scale effect pushes towards more innovation in the contractible than in the cooperative case and more innovation in the cooperative than in the Nash case. As in section 5.1, the share of producers who are not in an ongoing relationship with a good match at the beginning

[28] We define $K \equiv \frac{\left(1-\delta^D\right)\left(1-\delta^I\right)}{1+\rho-\left(1-\delta^D\right)\left(1-\delta^I\right)(1-\Delta)}\left(1+\frac{b^2\left(1-\delta^D\right)\delta^I\gamma}{1+\rho-b\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)}\right)$.

[29] If the producer could resume a relationship with a supplier after it has been broken in the contractible or Nash cases, there would be no loss of cooperation effect term in (21).

[30] We ruled out the case with no diffusion $\Delta = 0$, which makes the problem significantly more complicated as then the level of cooperation in outdated firms will depend on whether the producer knows if the innovator is a good match or not. We did consider this case in a previous version of the paper and the results are available upon request. The logic of this section carries through: the worse bad match effect is still at work, but the outside option effect more easily dominates the encouragement effect.

of a period is a function of the innovation rate in the cooperative case, but now also in the contractible and Nash cases.[31] As long as $y^* \geq x^*$, the conclusions of Proposition 4 still hold, but if the outside option effect is strong enough, innovation might be higher in the cooperative than in the contractible case now. The following remark summarizes our results.

Remark 2 *(i) When innovation diffuses slowly ($\Delta \in (0,1)$), the level of cooperation in outdated good matches may be higher or lower than in frontier good matches ($y^* \lesseqgtr x^*$). (ii) The minimum technological leap to break existing relationship may be higher or lower in the cooperative than in the contractible and Nash cases ($\gamma^{coop} \lesseqgtr \gamma^{cont} = \gamma^{Nash}$ for given δ^I). If δ^I is small and $b\theta / \left(1+\rho-b\left(1-\delta^D\right)\right) > 1/\left(1+\rho-\left(1-\delta^D\right)(1-\Delta)\right)$, then it is harder to break existing relationships in the cooperative case $\gamma^{coop} > \gamma^{cont} = \gamma^{Nash}$. (iii) The highest equilibrium innovation rate in the cooperative case may be higher or lower than in the contractible or Nash cases ($\delta^{coop} \lesseqgtr \delta^{cont}$ and $\delta^{coop} \lesseqgtr \delta^{Nash}$), but if $y^* \geq x^*$, it is lower than in the contractible case $\delta^{coop} < \delta^{cont}$.*

Therefore our earlier results are generalized to this case but only if innovations diffuse sufficiently rapidly. How fast innovations diffuse depend on technological and institutional characteristics, for instance weak intellectual property rights may favor rapid technological diffusion. Then, our results suggest that weak contractibility is particularly damaging to an economy where IPRs are poorly enforced.

6.2 Innovations within relationships

In this section we show that the establishment of a cooperative equilibrium in a weak contractibility environment may encourage relationship-specific innovations. We depart from our baseline model in two ways. First, the technology level A_t is now kept constant (normalized to 1). Second, in a good match, a producer and a supplier can jointly undertake a relationship-specific innovation which increases the relationship-specific productivity of the supplier from 1 to $\gamma > 1$. We refer to a relationship where this innovation has succeeded as an "augmented match". To become an augmented match with probability δ^I, the producer and suppliers have to spend $\psi\left(\delta^I\right)$ units of final good. Whether the innovation succeeds or not is revealed at the beginning of the period, and the investment is undertaken after the nature of the match is revealed. In order to focus on the issue of contractibility of the intermediate input provided by the supplier (which we could also interpret as effort), we assume that the R&D investment $\psi\left(\delta^I\right)$ is contractible, so that its level is chosen so as to maximize the joint expected value of the relationship.

[31] Therefore, this general equilibrium effect no longer systematically pushes towards more innovation in the cooperative than contractible and Nash cases. Instead it pushes towards more innovation when producers switch to innovators. It may also still generate multiple equilibria.

As before, producers switch suppliers until they find a good match. In the cooperative case, good match suppliers are willing to cooperate as mush as possible until a deviation occurs. We focus on parameters such that if cooperation ceases, the producer's best option is always to look for a new good match (alternatively we may assume as above that there is a strong utility cost of resuming a broken relationship). We denote the normalized level of investment in a good match as y, which is constant as long as a good match does not get augmented. Once a match has become augmented, R&D investments are no longer necessary, and in the cooperative equilibrium, the level of cooperation changes to x (also constant). Therefore the equilibrium is fully characterized by the equilibrium values: δ^I, y^* and x^*. The structure of the equilibrium in the contractible and Nash cases is similar but the investment levels are fixed at m and n respectively. In Appendix B.5, we solve for the equilibrium and demonstrate the following proposition.

Proposition 7 *The equilibrium innovation rate in the cooperative case δ^{coop} satisfies*

$$\left(\frac{\rho+\delta^D}{1-\delta^D}+\delta^{coop}\right)\psi'\left(\delta^{coop}\right)-\psi\left(\delta^{coop}\right)=\left(\gamma-\frac{\Pi\left(y^*\right)}{\Pi\left(x^*\right)}\right)\Pi\left(x^*\right), \tag{22}$$

while in the contractible case and Nash cases it satisfies

$$\left(\frac{\rho+\delta^D}{1-\delta^D}+\delta^I\right)\psi'\left(\delta^I\right)-\psi\left(\delta^I\right)=\left(\gamma-1\right)\Pi\left(z\right), \tag{23}$$

with $z=m$ in the contractible case and $z=n$ in the Nash case. If $\gamma\leq\left(1+\rho\right)\left(1-b\right)/\left(b\left(1-\delta^D\right)\right)$, then $x^*\geq y^*$, so that $\delta^{coop}>\delta^{Nash}$ and $\delta^{coop}\lesseqgtr\delta^{cont}$.

The innovation rate depends on the difference in the profits of an augmented match relative to a good match. Therefore, the comparison between the innovation rates in the cooperative case and the two other cases is governed as before by a scale effect and by the relative levels of cooperation within augmented relative to good matches (x^* and y^*). The comparison between x^* and y^* is generally ambiguous. On one hand an encouragement effect pushes towards more cooperation when productivity is lower, because the prospect of an innovation increases the reward from cooperation relative to the incentive to deviate. On the other hand, the value that a supplier captures in an augmented match is larger than the one she captures in a good match (as in both cases the outside option is to start a new relationship). The proposition provides a sufficient condition under which the encouragement effect is dominated, resulting in a higher level of cooperation in augmented matches. This directly implies that cooperation fosters innovation relative to the Nash level. It may even lead to higher levels of innovation than in the contractible case ($\delta^{coop}>\delta^{cont}$ occurs for instance if $x^*=m$ but $y^*<m$).

7 Conclusion

In this paper we show that the development of implicit contracts in a context of poor contractibility is a poor substitute for strong institutions for the development of broad, general purpose innovations, particularly when innovations get easily imitated. In a nutshell, our argument goes as follows: incomplete contractibility leads firms to engage in cooperative long-term relationships, which can help overcome the classic underinvestment issue associated with the lack of contractibility. However, it is only in relationships which are a good fit, that is, in relationships where parties understand that they are going to keep working together for a long time, that cooperation is sustainable in the first place. Consequently, switching to a new supplier becomes a riskier activity because if the new supplier is a bad fit, cooperation will not take place. More rigid relationships, in return, slow down the process of creative destruction. Once innovation decisions are endogenized, the dynamic growth costs can be sufficiently large to overcome the static gains of cooperation. Our model predicts that with poor formal institutions but informal contracting, relationships are more rigid and last longer and innovation spreads is scarcer than with good formal institutions. In addition innovations may be larger when they happen.

The negative effects of cooperation can be attenuated in countries far from the technological frontier, or if imitation is slow as in this case the long-standing presence of firms with better technologies can weaken cooperation within established relationship where the supplier has only access to an outdated technology. In addition, cooperation may boost relationship-specific innovations to the detriment of general purpose innovations.

An interesting extension to our analysis would be to include foreign outsourcing as issues of incomplete contractibility and long-term relationships may be even more stringent when a firm is dealing with a supplier in a different country, as the firm may be less familiar with the local judicial system. More generally, the idea that the cooperation in long-term relationships can lead to dynamic inefficiencies can be exploited in different contexts, varying both the reason for why firms develop long-term relationships and the source of the dynamic inefficiency. For instance, long-term relationships may prevent firms from adjusting quickly enough to business cycles or trade shocks.

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A Main Appendix

A.1 Cooperative equilibrium characterization

We prove part a) of Proposition 1 and lemma 1. The proof proceeds as follows

1) We conjecture an equilibrium in particular with the two constants x^* and y^*,

2) We show lemma 1 in the cooperative case,

3) We derive the general form of the IC constraint

4) We derive the special form of the IC constraint for the parameter set where a producer will always rather look for a new supplier if any deviation occurs.

5) We refer the reader to the Online Appendix on blog.iese.edu/olsen/research where we show that the equilibrium exists, derive the IC constraint under alternative parameter sets and show that any equilibrium must be of the same form as described in step 1 below (part b of the proposition).

A.1.1 Step 1. Description of the equilibrium

The incentive constraint must be of the following form:

After a history of h_t when a good match supplier makes her investment decision she can invest n instead of the prescribed $z(h_t)$, which would increase ex-post profits this period by $\varphi(z(h_t))A_k(h_t)$, where $A_k(h_t)$ is the technology of supplier and

$$\varphi(z) \equiv \beta R(n) - n - (\beta R(z) - z)$$

Denoting by $I \in \{0,1\}$ either no new innovation ($I = 0$) or a new innovation ($I = 1$) we can express the incentive constraint as:

$$\varphi(z(h_t))A_k(h_t) \leq \frac{1-\delta^D}{1+\rho}\times \tag{A.1}$$

$$\left(\begin{array}{c} \left(1-\delta^I\right)V^{s,k}\left(h_t \cup \{z(h_t)\} \cup \{I=0\}\right) + \delta^I V^{s,k}\left(h_t \cup \{z(h_t)\} \cup \{I=1\}\right) \\ -\left(\left(1-\delta^I\right)V^{s,k}\left(h_t \cup \{n\} \cup \{I=0\}\right) + \delta^I V^{s,k}\left(h_t \cup \{n\} \cup \{I=1\}\right)\right) \end{array} \right),$$

where $V^{s,k}(h)$ denotes the value of the supplier after history h (The continuation value after a deviation other than n could be different, but the producer has no reason not to punish any deviation in the same way so we focus on the incentive not to play n).

We conjecture an equilibrium with the following restrictions on strategies (in the Online Appendix we demonstrate that all equilibria satisfying conditions 1 - 4 must take this form).

1. Investment levels in all good matches are given by two constant x^* and y^*, where the former is undertaken when the supplier has access to the best technology and the latter when he does not, as long as no deviation has occurred in the relationship between the producer and the supplier;

2. Investment levels are at the first best level if possible and otherwise the IC constraint binds;

3. Producers stay with the same supplier until an innovation or a deviation occurs, if an innovation occurs, the producer may or may not switch, but if he switches and the innovator turned out to be a bad match, he goes back to his old supplier;

4. Producers are just indifferent between choosing the supplier they are supposed to work with on equilibrium path and choosing the "second best" supplier, the "second best" supplier is just indifferent between being chosen and not being chosen by the producer;

5. If a supplier deviates once, investment is at the Nash level in any further interaction, and - without loss of generality - if the producer deviates (by switching to another supplier who is not an innovator that turned out to be a bad match) investment is also at the Nash level in any future interaction.

It is not complicated to demonstrate that an equilibrium with these characteristics will satisfy conditions 1 - 4. But we still have two things to show, first why is it possible to satisfy part a) of condition 1 (that is why is it possible that the level of investment will be the same when the producer knows several good matches and when he does not), and second that such an equilibrium always exist). We proceed to prove lemma 1

A.1.2 Step 2. Proof of lemma 1

We consider a producer who knows a good match supplier with whom no deviation has occurred and we study whether the producer would want to switch to the innovator or not.[32] We normalize values of firms by the level of the frontier technology and use the notations V_i^z to denote the value of a producer ($z = p$), a supplier ($z = s$) or the total value of the producer and the supplier ($z = T$) in a relationship which is new $i = 0$ or old $i = 1$ in a period without innovation. In a period with innovation we will similarly use W_i^z to denote the value with an outdated supplier and $V_I^{z,p}$ to denote the value with the innovator (where $p = g$ is used if the producer was in a good match - who did not deviate- in the previous period and $p = b$ is used otherwise). When an innovation occurs the value of a good match supplier does not fall to 0, as the producer may come back to the supplier if the innovator turns out to be a bad match. We denote the expected value of such a supplier by V_A^s.

The innovator and the old supplier enter in Bertrand Competition, the old supplier would be willing to offer a transfer that would guarantee herself at least V_A^s in order to keep the

[32] This analysis always applies on path. Off path it also applies except when the producer already knows a good match with whom a deviation has occurred and the value of a relationship with the innovator is lower than the value of staying with this good match.

producer, hence SPNE requires that:

$$W_1^s \geq V_A^s. \tag{A.2}$$

Moreover Bertrand Competition ensures that the supplier with whom the relationship is the highest captures the entire benefit of the relationship over the second best one, hence the value of the producer whether he switches supplier or not is the same:[33]

$$V_I^{p,g} = W_1^p. \tag{A.3}$$

The producer ends up switching if the highest amount that the innovator can offer is higher than the highest amount that the old supplier can offer, that is if if the total value of the producer and the innovator ($V_I^{T,g}$) is higher than the surplus value of the old relationship ($W_1^T - V_A^s$).[34]

$$V_I^{T,g} > W_1^T - V_A^s. \tag{A.4}$$

The total value of a relationship with the old supplier is given by:

$$W_1^T = \frac{1}{\gamma}\Pi\left(y^*\right) + \frac{1-\delta^D}{1+\rho}\left(\left(1-\delta^I\right)V_1^T + \delta^I\gamma W_1^T\right). \tag{A.5}$$

If the producer sticks with the old supplier, the relationship will produce profits $\frac{1}{\gamma}\Pi\left(y^*\right)$ in the first period, in the next period if no innovation occurs, the total value is given by V_1^T, and if innovation occurs the old supplier captures W_1^s and the producer would get $V_I^{p,g}$ if he switches and W_1^p otherwise, but the two are equal (thanks to (A.3)) so total value is γW^T.

Similarly the total value of a relationship with the innovator is given by:

$$\begin{aligned} V_I^{T,g} = {} & (1-b)\,\Pi\left(x^*\right) + (1-b)\,\frac{1-\delta^D}{1+\rho}\left(\left(1-\delta^I\right)V_1^T + \delta^I\gamma W_1^T\right) \\ & + b\theta\Pi\left(n\right) + b\frac{1-\delta^D}{1+\rho}\left(\left(1-\delta^I\right)V_1^p + \delta^I\gamma W_1^p\right). \end{aligned} \tag{A.6}$$

With probability $1-b$ the relationship turns out to be good delivering profits $\Pi\left(x^*\right)$ in the first period and with continuation value V_1^T if no innovation occurs and W_1^T if innovation occurs. With probability b, the relationship turns out to be a bad match, the continuation value for the supplier is then zero, and the producer goes back to his old good match supplier, so that his value is V_1^p if no innovation occurs and W_1^p if innovation occurs.

[33] Condition 1 plays a role here. Indeed, if the producer does not switch and the first best is achieved, the producer can capture more than $V_I^{T,g}$ -by having strategies where cooperation would cease if the supplier were to ask for an ex-ante transfer lower than the equilibrium value and were still chosen. Condition 1 precisely rules out such scheme. Regardless, this does not affect the analysis for whether a producer switches or not. - When the first best is not reached, such schemes are already ruled out by the bilateral rationality condition, as granting the largest share to the supplier guarantees the largest amount of cooperation.

[34] Technically this is derived under the condition that the value a good match old supplier is willing to offer is (weakly) higher than the value another outdated supplier would be willing to offer. We show in part 3 that this is necessarily true.

This leaves us with the expected value to the supplier from the possibility that the producer returns, V_A^s as the only missing element. If the producer switches, the current profits enjoyed by the old supplier are zero, but with probability b, the innovator will turn out to be a bad match, in which case the old supplier will get V_1^s if no innovation occurs and W_1^s if innovation occurs, hence:

$$V_A^s = \frac{1-\delta^D}{1+\rho} b \left(\left(1-\delta^I\right) V_1^s + \delta^I \gamma W_1^s \right). \tag{A.7}$$

Now combining (A.5), (A.6) and (A.7) one gets:

$$V_I^{T,g} - \left(W_1^T - V_A^s\right) = (1-b)\,\Pi\left(x^*\right) + b\theta\Pi\left(n\right) - \frac{1}{\gamma}\Pi\left(y^*\right), \tag{A.8}$$

which proves lemma 1.

A.1.3 Step 3. The general form of the incentive constraint

Using the notation and logic of the proceeding section we can rewrite (A.1) as:

$$\begin{aligned}
\varphi\left(x^*\right) &\leq \frac{1-\delta^D}{1+\rho} \left(\left(1-\delta^I\right) V_1^s + \delta^I \gamma W_1^s - \left(\left(1-\delta^I\right) V_N^s + \delta^I \gamma W_N^s \right) \right), && \text{(A.9)}\\
\gamma^{-1}\varphi\left(y^*\right) &\leq \frac{1-\delta^D}{1+\rho} \left(\left(1-\delta^I\right) V_1^s + \delta^I \gamma W_1^s - \left(\left(1-\delta^I\right) V_N^s + \delta^I \gamma W_N^s \right) \right),
\end{aligned}$$

where V_N^s and W_N^s are the value the supplier would get if he deviates (and investment would then be given by the Nash level), in periods where, respectively there is no innovation and there is innovation. If the supplier cooperates, her value in the following period is given by V_1^s if there is no innovation and W_N^s if there is innovation. The factor γ^{-1} on the LHS of the second IC constraint comes from the fact that the technology of the outdated supplier is only $\gamma^{-1}A$. We will refer to a good match supplier who has ceased to cooperate as a "deviator".

Combining (A.2), (A.3) and (A.8), we then get (still as long as switching to the innovator is a better option than switching to a potential deviator when the producer knows one):

$$W_1^s = V_A^s + \left(\frac{1}{\gamma}\Pi\left(y^*\right) - \left((1-b)\,\Pi\left(x^*\right) + b\theta\Pi\left(n\right) \right) \right)^+, \tag{A.10}$$

where $X^+ \equiv \max\{X, 0\}$

Using equation (A.7) and (A.10) we get:

$$\begin{aligned}
&\left(1-\delta^I\right) V_1^s + \delta^I \gamma W_1^s && \text{(A.11)}\\
&= \frac{1+\rho}{1+\rho-b\left(1-\delta^D\right)\delta^I\gamma} \left(\left(1-\delta^I\right) V_1^s + \delta^I \gamma \left(\frac{1}{\gamma}\Pi\left(y^*\right) - \left((1-b)\,\Pi\left(x^*\right) + b\theta\Pi\left(n\right) \right) \right)^+ \right),
\end{aligned}$$

Finally note that V_1^T must satisfy:

$$V_1^T = \Pi\left(x^*\right) + \frac{1-\delta^D}{1+\rho} \left(\left(1-\delta^I\right) V_1^T + \delta^I \gamma W_1^T \right), \tag{A.12}$$

which combined with (A.5) leads to:

$$V_1^T = \frac{\left(1+\rho-\left(1-\delta^D\right)\delta^I\gamma\right)\Pi\left(x^*\right)+\left(1-\delta^D\right)\delta^I\Pi\left(y^*\right)}{1+\rho-\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)}. \tag{A.13}$$

If the producer does not already know a deviator, we necessarily get through Bertrand competition:

$$V_1^p = V_0^{T,n} \text{ and } V_1^s = V_1^T - V_0^{T,n}, \tag{A.14}$$

where $V_0^{T,n}$ is the value of starting a new relationship when the producer knows a deviator. Indeed, the outside option for the producer is to start a new relationship, but should he do so, he would now know a deviator, namely the good match he was previously working with. If the producer knows a deviator, then his second best option will either be to resume a relationship with the deviator or to start a new relationship, now knowing two deviators, so that we get, through Bertrand Competition:

$$V_1^s = V_1^T - \max\left(V_N^T, V_0^{T,n}\right), \tag{A.15}$$

where V_N^T denotes the joint value of a relationship with the deviator.

As mentioned in the text, depending on parameters, there is a number of different cases to consider. In order to save space we will consider only the case where in case of a deviation the producer will always seek out a new producer. The other cases are considered in the Online Appendix. The main results of the paper hold in all cases.

A.1.4 Step 4 in a special case: When a deviation always leads the producer to try out a different supplier

Assume that in periods without innovation, the producer would always rather try out a new supplier than a deviator, and, in periods with innovation, should the producer would prefer both the innovator or an outdated new supplier than an (outdated) deviator. That is, we assume:

$$V_N^T < V_0^T \text{ and } W_N^T < W_0^T. \tag{A.16}$$

and we need not index V_0^T and W_0^T by n as whether a producer knows a deviator or not is now irrelevant. Note that, as the producer, will never return to a supplier the value from deviating is zero (Such that for the two rightmost terms in equation (A.9) $\left(1-\delta^I\right)V_N^s+\delta^I\gamma W_N^s=0$). We can therefore focus on $\left(1-\delta^I\right)V_1^s+\delta^I\gamma W_1^s$ which is given by equation (A.11). Using that $V_1^s = V_1^T - V_0^T$ (from equation A.14) we use that by definition:

$$V_0^T = (1-b)V_1^T + b\theta\Pi(n) + b\frac{1-\delta^D}{1+\rho}\left(\left(1-\delta^I\right)V_0^T+\delta^I\gamma W_0^T\right), \tag{A.17}$$

such that with the aid of equation (A.13) we need only an expression for the total value of starting a new relationship with an outdated supplier. This is given by the value V_0^T minus the expected loss from lower productivity:

$$W_0^T = V_0^T - (1-b)\left(\Pi(x^*) - \gamma^{-1}\Pi\left(y^*\right)\right) - b\theta\left(1-\gamma^{-1}\right)\Pi\left(n\right).$$

Combining these expressions we find

$$\left(1-\delta^I\right)V_1^s + \delta^I\gamma W_1^s =$$

$$\frac{1+\rho}{1+\rho-b\left(1-\delta^D\right)\delta^I\gamma}\left(\begin{array}{c} \left(1-\delta^I\right)\frac{b\left(\left(1+\rho-b\left(1-\delta^D\right)\delta^I\gamma\right)\left(\Pi(x^*)-\theta\Pi(n)\right)+b\left(1-\delta^D\right)\delta^I\left(\Pi(y^*)-\theta\Pi(n)\right)\right)}{1+\rho-b\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)} \\ +\delta^I\gamma\left(\frac{1}{\gamma}\Pi\left(y^*\right)-\left(\left(1-b\right)\Pi\left(x^*\right)+b\theta\Pi\left(n\right)\right)\right)^+ \end{array}\right),$$

which combined with the incentive constraint in equation (A.9) leads to equation (10) in the main text. We derive the incentive constraints for the exhaustive set of cases in Appendix B.1. Finally, to show the existence of the equilibrium, we need to show that the investment levels x^* and y^* exist, which we do for all possible cases.

A.2 Proof of Proposition 5

In this appendix we consider the case where the strategy of suppliers is to punish the producer - by playing the Nash strategy - if we he switches to an innovator that turns out to be a bad match. We derive expression (17) in the special case in which case the expected value of a new relationship is higher than remaining with a supplier who is punishing by investing the Nash level, such that if the innovator turns out to be a bad match the producer will seek out a new supplier rather than stick with the old one.

Compare to the situation in Appendix A.1, if the producer switches the old supplier lose all its value, hence $V_A^s = 0$. The producer will now switch if and only if:

$$V_I^{T,g} > W_1^T, \tag{A.18}$$

that is the total value of a new relationship with the innovator is higher than the total value of a relationship with the old supplier instead of (A.4). If the innovator turns out to be a bad match, the producer will try another new supplier in the following period, so the total value of the relationship with the innovator does not depend on whether the producer already knew a good match or not:

$$V_I^{T,g} = V_I^{T,b} = V_0.$$

Equation (A.6) is replaced by:

$$V_I^{T,g} = V_0^T = (1-b)\Pi\left(x^*\right) + (1-b)\frac{1-\delta^D}{1+\rho}\left(\left(1-\delta^I\right)V_1^T + \delta^I\gamma W_1^T\right) \tag{A.19}$$

$$+ b\theta\Pi\left(n\right) + b\frac{1-\delta^D}{1+\rho}\left(\left(1-\delta^I\right)V_0^T + \delta^I\gamma W_0^T\right). \tag{A.20}$$

Analogous to the analysis in Appendix A.1 we use that (A.5), (A.7), and (A.12) still hold to get:

$$V_I^{T,g} - W_1^T = (1-b)\,\Pi(x^*) + b\theta\Pi(n) - \gamma^{-1}\Pi(y^*) - \frac{b^2\left(1-\delta^D\right)\left(\left(1-\delta^I\right)\left(\Pi(x^*) - \theta\Pi(n)\right) + \delta^I\left(\Pi(y^*) - \theta\Pi(n)\right)\right)}{1+\rho-b\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)}, \tag{A.21}$$

so that a producer in a good match will switch to the innovator if and only if (17) holds, which defines a γ^{coop2}. Note, that equation $(A.21)$ differs from equation (8) only in the last term, and using Lemma 1 it follows that $\gamma^{coop2} > \gamma^{con} = \gamma^{Nash}$.

To show that the incentive to innovate is lower we need the fraction of the firms that are in good matches. In all cases, a producer in a bad match switch. If $\gamma < \gamma^{coop2}$ then only producers in bad matches in the cooperate equilibrium will switch, implying that in steady state (weakly) more producers will be in good matches in the cooperative equilibrium than in the contractible equilibrium. As the extra benefit for the innovator from contractibility is higher for good matches than bad matches, it follows that the incentive to innovate is higher in the contractible case, $\delta^{coop2} < \delta^{con}$.

Now, consider the case where $\gamma > \gamma^{coop2}$, such that good matches remain with the same producer when innovation takes place. Use the fact that in the contractible case a fraction $\tilde{\omega}^c = \frac{\delta^D}{1-b(1-\delta^D)}$ of producers will not be in good relationships, whereas in the noncontractible case a fraction $\tilde{\omega}^{nc} = \frac{\delta^D + b\delta^I\left(1-\delta^D\right)}{1-b(1-\delta^D)(1-\delta^I)}$ will not be in a good relationship. Inserting into the expressions for expected profits in the contractible and noncontractible case, respectively:

$$\left(\tilde{\omega}^c(\gamma-1)(1-b+b\theta) + (1-\tilde{\omega}^c)\left((1-b+b\theta)\gamma - 1\right)\right)\Pi(m)$$

Cooperative case:

$$\begin{aligned}&\tilde{\omega}^{nc}\left((1-b)\left(\gamma\Pi(x^*) - \Pi(y^*)\right) + b\theta(\gamma-1)\Pi(n)\right)\\&+(1-\tilde{\omega}^{nc})\left(\gamma(1-b)\Pi(x^*) + \gamma b\theta\Pi(n) - \Pi(y^*)\right)\\&-(1-\tilde{\omega}^{nc})\frac{\gamma\left(1-\delta^D\right)b^2\left(\left(1-\delta^I\right)\left(\Pi(x^*) - \theta\Pi(n)\right) + \delta^I\left(\Pi(y^*) - \theta\Pi(n)\right)\right)}{1+\rho-b\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)},\end{aligned}$$

straightforward, but somewhat tedious algebra demonstrates that the condition $\delta^{coop2}\left(1+\rho-b\left(1-\delta^D\right)\left(1+\delta^{coop2}(\gamma-1)\right)\right) < \gamma\left(1-b\left(1-\delta^D\right)\right)$ is sufficient to ensure that the incentive to innovate is lower: $\delta^{coop2} < \delta^{con}$.