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# Asset Prices with Non-Permanent Shocks to Consumption\*

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## Abstract

Most standard asset-pricing models assume that all shocks to consumption are permanent. We relax this assumption and allow also for non-permanent shocks. In our specification, the long-run mean of consumption growth is constant; consumption levels are subject to short-run deviations from their long-run trend. The implications of our model are dramatically different from those obtained in the prior literature. A canonical and parsimonious asset pricing model with CRRA preferences and non-permanent shocks can reproduce the equity premium, high return volatility and return predictability with a coefficient of relative risk aversion below ten. This finding suggests that non-permanent shocks can play an important role in explaining asset pricing puzzles.

**Keywords:** Asset prices, equity premium, unit root, non-permanent shocks.

**JEL codes:** G11, G12.

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# 1 Introduction

It is a well-understood fact in the asset-pricing literature that if shocks to consumption are permanent—so consumption is a random walk—then the behavior of U.S. stock prices are difficult to reconcile with a parsimonious model of agents with constant relative risk aversion (CRRA). Expected returns are too high, stock prices are too volatile, and future returns are too predictable to be generated by such a model given the low volatility of consumption growth in the data. Thus, the general thrust of asset-pricing models has been towards more complex elements such as models with external habit or long-run risk, or towards disaster risk; see, among other papers, Campbell and Cochrane (1999), Wachter (2006), and Santos and Veronesi (2010); Bansal and Yaron (2004), Hansen, Heaton, and Li (2008), and Bansal and Shaliastovich (2013); Barro (2006), Rodriguez (2006), and Nakamura, Steinsson, Barro, and Ursúa (2013).

But what if there are non-permanent shocks to consumption, shocks whose impact diminishes over time? Then, as we show in this paper, the aforementioned conclusions neatly reverse themselves. Even for moderate levels of risk aversion in a canonical and parsimonious model, stock prices are volatile, expected returns are high, and future stock returns are partially predictable.

The question of whether or not shocks to the economy are permanent has led to a long and controversial discussion. Different studies (Nelson and Plosser (1982), DeJong, Nankervis, Savin, and Whiteman (1992)) have come down on both sides of the issue. Distinguishing between permanent and temporary-yet-persistent consumption shocks is very difficult given the data we have. Fortunately, our results are not driven by the *absence* of a permanent shock, but by the *presence* of non-permanent shocks. A model with a mixture of permanent and non-permanent shocks exhibits similar behavior to one with non-permanent shocks alone.

We amass several pieces of evidence towards the importance of non-permanent shocks for asset pricing. We consider a representative-agent model with CRRA utility, with a single process for aggregate consumption. We use a general specification for aggregate consumption that experiences both permanent and non-permanent shocks. One special case of this model – with temporary shocks that only last for one period – permits an exact analytical solution. We exploit this case to show how including a non-permanent shock can produce both a high equity premium and volatile returns with moderate levels of risk aversion.

We then calibrate a parsimonious model with a single shock to U.S. consumption and return data. The shock is persistent, but not permanent. This makes the model trend-stationary—that is, in the absence of additional shocks the economy returns to a constant long-run growth trend. We choose three different empirical targets for our calibration exercise. In our base case, we use consumption data from 1889 to the present. Explaining the equity premium in post-war data is particularly challenging, so we also consider post-war data as a second target. Finally, an emerging literature (Savov (2011), Da and Yun (2011), Qiao (2013)) argues that mismeasurement in consumption has led to an artificially smooth consumption series, so we consider Savov’s proxy consumption series as a third empirical target. In all three cases, we are able to produce a high equity premium and volatile returns with much lower levels of risk aversion than would be required if the shock were permanent. The post-war NIPA consumption data does indeed require a somewhat higher level of risk aversion than the long sample, but the long sample and Savov’s

proxy consumption series lead to very similar results. We also show that adding an additional permanent shock in these calibrations does not materially change the results. This is consistent with our analytical results in that it is the presence of a non-permanent shock that drives the asset pricing results, and not the absence of a permanent shock.

Models with non-permanent consumption shocks are also able to generate many other time-series properties of asset prices with low levels of risk aversion. For example, in such a model most variation in the price-dividend ratio is driven by changes in expected returns rather than changes in expected dividends, consistent with Campbell and Shiller (1988a). Non-permanent shocks are also sufficient to generate return predictability and meet the Hansen-Jagannathan bounds.

Several authors have previously considered the asset pricing implications of trend-stationary consumption. Tallarini (2000) considers mean reversion in consumption for Epstein-Zin utility with the special case where the intertemporal elasticity of consumption (IES) is one, where the model is exactly solvable. That paper finds that for high levels of risk aversion mean reversion in consumption actually lowers the equity premium. DeJong and Ripoll (2007) estimates a model with trend-stationary dividends and consumption, but use a log-linear approximation that leads to a much smaller estimate of the equity premium. Rodriguez (2006) finds that trend-stationary consumption helps explain the volatilities of returns, but needs large possible permanent shocks (disaster states) to match the empirical equity premium. In particular in his calibrated model the probability of a drop in consumption of more than 25% exceeds 17% and the possibility of rare but large shocks to consumption explains a large fraction of the equity premium. Nakamura, Steinsson, Barro, and Ursúa (2013) incorporate large permanent and non-permanent rare disasters to consumption. When they occur, the non-permanent disasters cause an average drop of 11% in consumption per year and occur for six subsequent years (that is, an 11% drop each year, not a 11% drop over 6 years). The standard deviation of the disaster shocks is also quite large, which generates the risk of even larger losses in consumption. Our model contains no disaster shocks; instead the permanent and non-permanent shocks are calibrated to match the U.S. experience. Bansal, Kiku, and Yaron (2010) also find a large equity premium, but in a model that contains both short-term and long-term risks, as well as stochastic volatility. Alvarez and Jermann (2005) find, in interesting contrast to the equity market, that long-term bonds data suggests that shocks to consumption are permanent.

The remainder of this paper is organized as follows. In Section 2 we present the basic model and provide analytical solutions for the special case of one-period temporary shocks. Section 3 provides a description of the data sets and the consumption specifications for the baseline version of the model. In Section 4 we report results on the asset-pricing implications of the model and demonstrate their robustness. Finally, Section 5 concludes. In the Appendix, we derive the analytical results, describe the numerical solution method, and report additional results.

## 2 A Consumption-based Asset-pricing Model

We briefly describe the particular version of the standard Lucas (1978) asset-pricing model that we employ in this paper with both permanent and non-permanent shocks. We then consider a special case that permits closed-form solutions for asset prices. We use this case to illustrate the

impact of non-permanent shocks on asset prices.

## 2.1 Consumption and Asset Prices

We consider a standard Lucas (1978) infinite-horizon representative-agent asset-pricing model in discrete time,  $t = 0, 1, \dots$ . There is a single perishable consumption good in each period. The agent's consumption in period  $t$  is denoted by  $C_t$ . The agent has expected utility

$$E_0 \left[ \sum_{t=0}^{\infty} \beta^t u(C_t) \right],$$

with CRRA Bernoulli utility

$$u(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma}$$

and discount factor  $\beta$ . The stochastic discount factor,  $M_t$ , to price assets in this model is the well-known expression

$$M_{t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma}. \quad (1)$$

The logarithm of consumption,  $c_t = \log(C_t)$ , is the sum of two processes,  $g_t$  and  $x_t$ , which are the permanent and non-permanent components of consumption, respectively,

$$\begin{aligned} c_t &= g_t + x_t \\ g_t &= \bar{g} + g_{t-1} + \sigma_\nu \nu_t \\ x_t &= \rho_c x_{t-1} + \sigma_\epsilon \epsilon_t \\ \epsilon_t, \nu_t &\sim N(0, 1) \text{ i.i.d.} \end{aligned} \quad (2)$$

with  $\rho_c < 1$  and  $\bar{g}$  denoting the long-term expected growth rate of consumption.

This specification encompasses several processes that have been used in the economic literature. For  $\rho_c = 0$ ,  $\sigma_\epsilon = 0$ , consumption is a simple random walk with drift as considered in many papers; see, for example, Mehra (2006) or Tallarini (2000). For  $\sigma_\nu = 0$ , consumption is trend-stationary which is the second process analyzed in Tallarini (2000).

**Side Note.** We contrast our model with other complex consumption models in the literature. The long-run risk literature, beginning with Bansal and Yaron (2004), considers highly persistent shifts in the long-run mean of consumption *growth*. In our specification, the long-run mean of consumption growth is a constant, and the dynamics are driven by short-run deviations of consumption *levels* from their long-run trend ( $g_t$ ). To illustrate this feature of the specification, consider the logarithmic growth rates,  $\Delta c_t = c_t - c_{t-1}$ . Then

$$\Delta c_t = \bar{g} + (\rho_c - 1)x_{t-1} + \sigma_\epsilon \epsilon_t + \sigma_\nu \nu_t,$$

which shows that the growth rate process  $\{\Delta c_t\}$  is correlated with the process of prior deviations from trend,  $\{x_{t-1}\}$ .

Nakamura, Steinsson, Barro, and Ursúa (2013) also decompose growth into permanent and

non-permanent components, but both components feature large disasters. Those disasters cause an average drop of 11% in consumption per year and occur for six subsequent years (that is, an 11% drop each year, not a 11% drop over 6 years). The standard deviation of the disaster shocks is also quite large, which generates the possibility of huge losses in consumption. As a result, Nakamura, Steinsson, Barro, and Ursúa (2013) overestimate the volatility of consumption growth by a factor of 1.66. (3.5% in the data compared to 5.8% implied by the model) in the pre-war period. For post-war consumption, the difference is even larger with a factor of 2.78 (1.8% in the data compared to 5.0% implied by the model).

For our specification of the consumption process 2, the expected value, the standard deviation, and the first-order autocorrelation of consumption growth are as follows,<sup>1</sup>

$$E(\Delta c_t) = \bar{g} \quad (3)$$

$$\sigma(\Delta c_t) = \sqrt{\sigma_\nu^2 + \frac{2}{1 + \rho_c} \sigma_\epsilon^2} \quad (4)$$

$$\text{AC1}(\Delta c_t) = \left( \frac{\rho_c - 1}{\rho_c + 1} \sigma_\epsilon^2 \right) (\text{Var}(\Delta c_t))^{-1}. \quad (5)$$

The objective of this paper is to analyze the asset pricing implications of the model with the consumption process (2). In the first, baseline, version of the model we assume that there is no labor income and that there is a risky asset (“Lucas tree”) paying dividends equal to the aggregate consumption claim,

$$D_t = C_t,$$

each period. We consider the consequences of relaxing this assumption and including a separate dividend process in Section 4.4 below. In the analysis of the baseline model we frequently call the risky asset the aggregate consumption claim.

The return of a one-period bond that pays one unit of the consumption good is given by

$$R_{t+1}^f = \frac{1}{P_t^f} \quad (6)$$

where  $P_t^f$  is the price of the bond. The return on the aggregate consumption claim can be expressed in terms of the price-consumption ratio,

$$R_{t+1} = \frac{\left( \frac{P_{t+1}}{C_{t+1}} + 1 \right)}{\frac{P_t}{C_t}} \times \frac{C_{t+1}}{C_t}. \quad (7)$$

Using the general asset pricing equation

$$1 = E_t \{ M_{t+1} R_{t+1}^i \} \quad (8)$$

together with the stochastic discount factor from equation (1), the pricing equations for the two

<sup>1</sup>The derivations of the unconditional moments can be found in Appendix A.1. We use these analytical expressions in our analysis below to exactly match the moments of the underlying consumption process to the data.

assets read

$$P_t^f = \beta E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \right\} \quad (9)$$

$$\frac{P_t}{C_t} = \beta E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{1-\gamma} \left( \frac{P_{t+1}}{C_{t+1}} + 1 \right) \right\}. \quad (10)$$

To numerically compute solutions for asset prices and returns, we rewrite the model in terms of the stationary variable  $x_t = c_t - g_t$  and the change in the log growth rate  $g_{t+1} - g_t = \bar{g} + \sigma_\nu \nu_{t+1}$ . The pricing equation (9) for the riskless asset and the price-consumption ratio (10) for the infinitely-lived risky asset can then be expressed by

$$P_t^f = \beta E_t \left\{ e^{-\gamma(g_{t+1} - g_t + x_{t+1} - x_t)} \right\} \quad (11)$$

$$\frac{P_t}{C_t} = \beta E_t \left\{ e^{(1-\gamma)(g_{t+1} - g_t + x_{t+1} - x_t)} \left( \frac{P_{t+1}}{C_{t+1}} + 1 \right) \right\}, \quad (12)$$

respectively. Our calibration results lead us to consider models where  $\beta > 1$ . Kocherlakota (1990) first showed that in an economy with a positive growth rate,  $\bar{g} > 0$ , and sufficiently risk-averse investors,  $\gamma > 1$ , the discount factor  $\beta$  can, in fact, exceed one; the agent's utility maximization problem remains well-defined and a finite solution to the pricing equations (11) and (12) exists. In Appendix A.2, we extend this result and show that the agent's expected utility remains finite for  $\gamma > 1$  if

$$\beta < e^{(\gamma-1)\bar{g}}. \quad (13)$$

Piazzesi, Schneider, and Tuzel (2007) also consider values of  $\beta > 1$ .

## 2.2 Analytical Results: Permanent versus Temporary Shocks

As is well-known, if consumption simply follows a random walk, the model can be solved in closed form; see, for example, Mehra (2006). We can also solve the model for the more general consumption process (2) with both a permanent and a purely temporary shock ( $\rho_c = 0$ ). This version of the model simplifies to

$$\begin{aligned} c_t &= g_t + \sigma_\epsilon \epsilon_t \\ g_t &= \bar{g} + g_{t-1} + \sigma_\nu \nu_t \\ \epsilon_t, \nu_t &\sim N(0, 1) \text{ i.i.d.} \end{aligned} \quad (14)$$

For  $\sigma_\epsilon = 0$ , the model collapses to a simple random walk for log consumption. For  $\sigma_\nu = 0$ , log consumption is trend-stationary with persistence  $\rho_c = 0$ . (We consider non-zero  $\rho_c$  in our calibration results in Section 4.)

The key asset pricing moments are given by the following theorem.

**Theorem 1.** *Consider the model with the consumption process (14) exhibiting a temporary shock,  $\epsilon_t$ , and a permanent shock,  $\nu_t$ . Then the first and second unconditional moments of the risk-free*

rate and the expected return of the aggregate consumption claim are as follows,

$$E(R_t^f) = \frac{1}{\beta} e^{\gamma\bar{g} - \frac{1}{2}\gamma^2\sigma_\nu^2} \quad (15)$$

$$E((R_t^f)^2) = \frac{1}{\beta^2} e^{2\gamma\bar{g} - \gamma^2\sigma_\nu^2 + \gamma^2\sigma_\epsilon^2} \quad (16)$$

$$E(R_{t+1}) = e^{\frac{1}{2}\sigma_\nu^2 + \gamma^2\sigma_\epsilon^2 + \bar{g}} + \frac{1 - \beta e^{(1-\gamma)\bar{g} + \frac{1}{2}(1-\gamma)^2\sigma_\nu^2}}{\beta} e^{\gamma\bar{g} + \gamma\sigma_\nu^2 - \frac{1}{2}\gamma^2\sigma_\nu^2 + \gamma\sigma_\epsilon^2} \quad (17)$$

$$E(R_{t+1}^2) = e^{4\gamma^2\sigma_\epsilon^2 + 2\bar{g} + 2\sigma_\nu^2} + \frac{1}{X^2} e^{2\gamma^2\sigma_\epsilon^2 + 2\sigma_\epsilon^2 + 2\bar{g} + 2\sigma_\nu^2} + \frac{2}{X} e^{2.5\gamma^2\sigma_\epsilon^2 + \gamma\sigma_\epsilon^2 + 0.5\sigma_\epsilon^2 + 2\bar{g} + 2\sigma_\nu^2}, \quad (18)$$

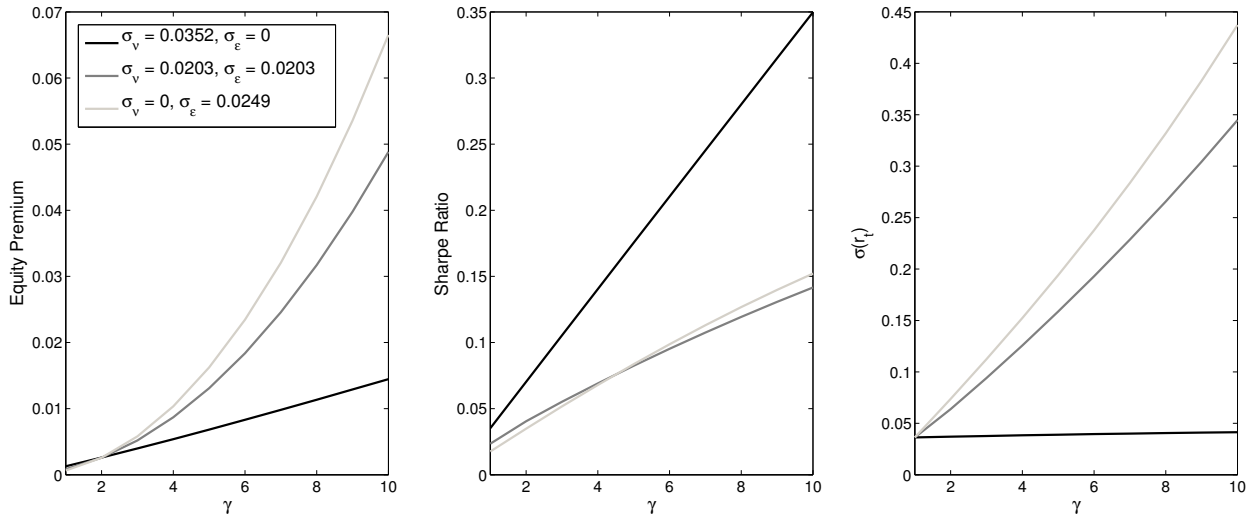
with the constant

$$X = \frac{\beta e^{(1-\gamma)\bar{g} + \frac{1}{2}(1-\gamma)^2(\sigma_\epsilon^2 + \sigma_\nu^2)}}{1 - \beta e^{(1-\gamma)\bar{g} + \frac{1}{2}(1-\gamma)^2\sigma_\nu^2}}.$$

**Proof.** See Appendix A.3.  $\square$

Since the analytical expressions in Theorem 1 are rather complex, we illustrate them for a particular set of parameter values. We fix the expected growth rate at  $E(\Delta c_t) = 0.020$  and the volatility of consumption growth at  $\sigma(\Delta c_t) = 0.0352$ . (We derived these values from one of our three data sets; see Section 3 for a description of the data sets and Table 1 for the parameter estimates.) Figure 1 shows the equity premium, Sharpe ratio, and volatility of the return on the aggregate consumption claim for different degrees of risk aversion  $\gamma$ . In all three graphs in

Figure 1: Equity Premium, Sharpe Ratio, and Return Volatility of the Consumption Claim



The graphs show the equity premium, Sharpe ratio, and volatility of the return of the aggregate consumption claim for different degrees of risk aversion  $\gamma$ . The average growth rate is  $E(\Delta c_t) = 0.020$  and the volatility of consumption growth is  $\sigma(\Delta c_t) = 0.0352$ . We consider three sets of results. In the first case, we only have the persistent shock  $\nu_t$  with  $\sigma_\epsilon = 0$  (black line). In the second case, we assume  $\sigma_\epsilon = \sigma_\nu$  (dark grey line) and in the third case we allow only for temporary shocks, so  $\sigma_\nu = 0$  (light grey line). For all cases  $\rho_c = 0$  and  $\beta = 0.99$ .



Figure 1, the light grey line shows the case of  $\sigma_\nu = 0$  when all consumption volatility comes from the temporary shock  $\epsilon_t$ . The black line shows the case of  $\sigma_\epsilon = 0$  when all consumption volatility comes from the permanent shock  $\nu_t$ . The dark grey line shows the intermediate case  $\sigma_\nu = \sigma_\epsilon$ . The temporary shock generates a much higher equity premium and stock volatility than the permanent shock, particularly for higher levels of risk aversion. On the contrary, the Sharpe ratio is larger for the permanent shock than for the temporary shock. The large Sharpe ratio for the permanent shock, however, is not a consequence of a high excess return but instead of a very low return volatility. For a permanent shock the standard deviation of returns on the aggregate consumption claim is below 5% for all  $\gamma \leq 10$ , so even a small equity premium of 1.45% has a Sharpe ratio of 0.35. In contrast, for  $\gamma = 10$  the temporary shock leads to a premium of 6.65% but a volatility of more than 40%.

If we calibrate the asset-pricing model to an equity premium of 5.63%, which is the estimate for the S&P 500 for the period 1889–2009 (see Section 3 for a discussion), we can match the equity premium with a single temporary consumption shock with  $\gamma = 9.22$ . With a single permanent shock and  $\beta = 0.99$ , there is no choice of gamma that matches the equity premium. (The maximum possible equity premium is about 4.5%.) With both a permanent and temporary shock,  $\gamma = 10.8$  matches the equity premium.

Figure 2 illustrates the price-dividend ratio as a function of the temporary shock. As risk aversion increases, the response to the shock becomes both larger and increasingly nonlinear. This demonstrates how a temporary shock is sufficient to generate interesting dynamics. The model with only a temporary shock and both temporary and permanent shocks show similar dynamic effects, while the model with only a permanent shock has, of course, no dynamics in the price-dividend ratio whatsoever.

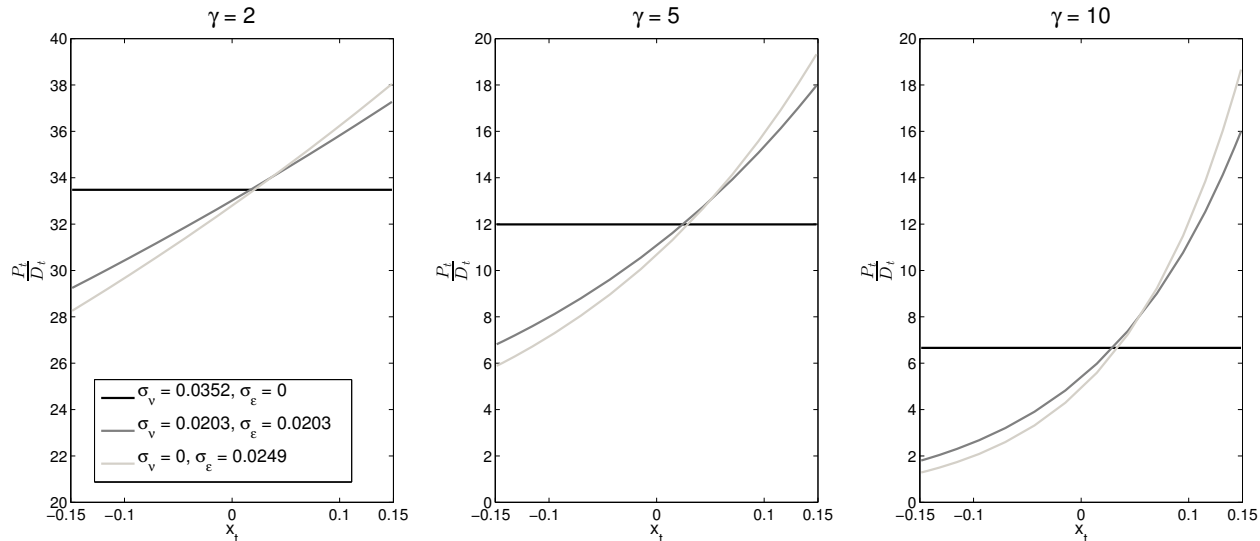
This completes our initial analysis of the asset pricing implications of our economic model. Obviously, the special case of  $\rho_c = 0$  does not reflect a property of actual market data but instead only serves as a benchmark to obtain a first impression of the different effects of permanent and non-permanent consumption shocks on asset prices. We find that temporary shocks produce significantly larger risk premia than permanent shocks and also increase the return volatilities of the assets. These properties come at the cost of much lower Sharpe ratios for temporary shocks than for permanent shocks.

Before we discuss more general results with  $\rho_c > 0$  in Section 4 below, we first describe the properties of market data that are relevant for a proper specification of the consumption process.

### 3 Data and Summary Statistics

We describe our data sources and report summary statistics. Then we provide results from unit root tests on the data series for consumption, dividends, and asset prices, respectively. Finally we report parameter estimates for a trend-stationary consumption process and for a random walk specification.

Figure 2: Price-Dividend Ratio as a Function of the State  $x_t$



The graph shows the price-dividend ratio  $\frac{P_t}{D_t}$  as a function of the state  $x_t$  for three different degrees of risk aversion  $\gamma \in \{2, 5, 10\}$ . The volatility of consumption growth is fixed at  $\sigma(\Delta c_t) = 0.0352$  and  $E(\Delta c_t) = 0.020$ . We consider three sets of results. In the first case, we only have the persistent shock  $\nu_t$  with  $\sigma_\epsilon = 0$  (black line). In the second case, we assume  $\sigma_\epsilon = \sigma_\nu$  (dark grey line) and in the third case we allow only for temporary shocks, so  $\sigma_\nu = 0$  (light grey line). For all cases  $\rho_c = 0$  and  $\beta = 0.99$ .

### 3.1 Consumption, Dividends, and Return Series

We use U.S. consumption data to calibrate the underlying consumption process (2). Parameters are chosen so that the resulting moments (3)–(5) match those of observed market data. We consider three different aggregate consumption series to examine the consequences for our results.

The first series we consider is the annual consumption data series constructed by Robert J. Shiller.<sup>2</sup> Consumption is aggregate per-capita real personal consumption from the National Income and Product Accounts. (Prior to 1929 this data series is not available, so Shiller uses estimates from Kendrick (1961).) We refer to this sample as the “Long Sample”.

The behavior of stock prices is particularly hard to explain in the post-World War II period (Grossman and Shiller (1981)). Therefore, we use as a second sample the Shiller consumption data from 1947–2009. Table 1 shows the mean, standard deviation, autocorrelation, and the correlation with the market portfolio of the consumption growth in the two time series. The moments of the two consumption series are in line with the values reported in Campbell and Cochrane (1999) or Guvenen (2009). The volatility in the post-war consumption series is significantly lower compared to the long sample. This property of the data is one of the reasons why it is so difficult to explain the large difference in equity and risk free returns in the post-war sample; see Grossman and Shiller (1981). Another source of difficulty is the positive autocorrelation in consumption growth rates.

An emerging literature (Savov (2011), Da and Yun (2011), Qiao (2013)) considers the consequences of mismeasurement in NIPA consumption. Triplett (1997) provides a critical look at

<sup>2</sup>The data set can be downloaded from <http://www.econ.yale.edu/~shiller/data.htm> (last accessed April 13, 2016).

Table 1: Empirical Moments of Consumption

	Long Sample	Post-War	Garbage
$E(\Delta c_t)$	0.0200 (0.0032)	0.0213 (0.0023)	0.0142 (0.0041)
$\sigma(\Delta c_t)$	0.0352 (0.0028)	0.0180 (0.0014)	0.0286 (0.0037)
$AC1(\Delta c_t)$	-0.0640 (0.1224)	0.2466 (0.1286)	-0.1438 (0.1747)
Corr. $R_m$	0.5613 (0.0631)	0.5454 (0.0887)	0.6016 (0.1110)

The table shows the mean  $E(\Delta c_t)$ , standard deviation  $\sigma(\Delta c_t)$ , autocorrelation  $AC1(\Delta c_t)$  and correlation with the market portfolio  $Corr. R_m$  of the different growth series. Bootstrapped standard errors from  $10^6$  simulations are provided in parentheses. The long sample consists of all real consumption from 1889–2009 and the post-war series from 1947–2009. The garbage data is available from 1960–2006.

how consumption is actually computed. Savov (2011) argues that the consumption estimates in the National Income and Product Accounts are artificially smooth. A smoothed series will have lower volatility than the true series, and the smoothing will introduce artificial positive autocorrelation, both of which make stock price dynamics harder to explain. Savov proposes municipal solid waste data collected by the U.S. Environmental Protection Agency as an alternative proxy for consumption. The logic is that consumption generates waste, so waste should be highly correlated with actual consumption. In response to Savov’s arguments, we also analyze the asset pricing implications of our model using the time series of garbage growth ranging from 1960–2006 as in Savov (2011).<sup>3</sup> We refer to this data series as “Garbage”. The rightmost column in Table 1 reports the summary measures for this time series.

Risky asset prices are again taken from the Shiller website. Starting from 1926, risky asset prices are given by the January level of the S&P 500 (or its predecessor indices), deflated by the January consumer price index (CPI-U). Dividends,  $D_t$ , are measured by the total amount of S&P 500 dividends in a year, deflated by the average CPI for that year. (Shiller again uses alternative sources to extend the data back to 1889. Stock data comes from Cowles and Associates (1939). The CPI-U series only extends back to 1913, so prior to that date Shiller uses the price index from Warren and Pearson (1935).) Table 2 reports the empirical moments of the dividends series for the respective time frames of our three data sets. Similarly, Table 3 reports the mean and standard deviation of the market return and the risk free rate (in percent) as well as the log price dividend ratio for the three different data sets.

### 3.2 Unit Root Statistics

Before we can analyze the asset-pricing model, we need to specify the parameters of the consumption process (2). This task forces us to confront the issue whether the consumption process has a unit root and to take a stand on the influence of non-permanent and permanent shocks on

<sup>3</sup>We thank Alexi Savov for making his data available to us.

Table 2: Empirical Moments of Dividends

	<b>Long Sample</b>	<b>Post-War</b>	<b>Garbage</b>
$E(\Delta d_t)$	0.0106 (0.0105)	0.0173 (0.0081)	0.0144 (0.0061)
$\sigma(\Delta d_t)$	0.1160 (0.0131)	0.0647 (0.0112)	0.0421 (0.0034)
$AC1(\Delta d_t)$	0.1379 (0.1127)	0.4470 (0.1677)	0.6836 (0.0855)

The table shows the mean  $E(\Delta d_t)$ , standard deviation  $\sigma(\Delta d_t)$ , and autocorrelation  $AC1(\Delta d_t)$  of the dividend growth for the three different data sets. Bootstrapped standard errors from  $10^6$  simulations are provided in parentheses. The long sample extends from 1889–2009 and the post-war series from 1947–2009. For the garbage series, the moments are for the period of 1960–2006.

Table 3: Empirical Moments of Financial Market Data

	$E(R_t)$	$\sigma(R_t)$	$E(R_t^f)$	$\sigma(R_t^f)$	$E(p_t - d_t)$	$\sigma(p_t - d_t)$
<b>Long Sample</b>	7.60	18.73	1.97	5.80	3.22	0.40
<b>Post-War</b>	7.92	16.60	1.84	2.65	3.42	0.44
<b>Garbage</b>	7.09	15.18	2.21	2.60	3.51	0.40

The table shows the mean and standard deviation of the market return and the risk free rate (in percent) as well as the log price dividend ratio for the three different data sets. The long sample extends from 1889–2009 and the post-war series from 1947–2009. For the garbage series, the moments are for the period of 1960–2006.

aggregate consumption.

The question as to whether shocks to the economy are permanent has led to a long and controversial discussion, ever since Nelson and Plosser (1982) first provided evidence that most macroeconomic time series have a unit root. DeJong and Whiteman (1991a), DeJong and Whiteman (1991b), DeJong and Whiteman (1991c), Kwiatkowski, Phillips, Schmidt, and Shin (1992) and DeJong, Nankervis, Savin, and Whiteman (1992) argue that for most macroeconomic time series the trend-stationarity hypothesis is much more likely than the unit root alternative. Perron (1989) and Andreou and Spanos (2003) present evidence that most macroeconomic time series are best represented by stationary fluctuations around a trend, with certain structural breaks, e.g., the 1929 crash or the 1973 oil price shock which both had persistent effects. Several authors (Christiano and Eichenbaum (1990), Cochrane (1991), Rudebusch (1993), and Diebold and Senhadji (1996)) observe that the presence or size of the persistent component is difficult to tease out with the data we have.

Clearly, this discussion in the literature is of great importance for the exact specification of the consumption process (2) in our model. Therefore, we conduct three common unit root tests for the time series of consumption, dividends, and asset prices for each of our three data sets. We employ the augmented Dickey-Fuller (ADF) (see Dickey and Fuller (1979)) and the Phillips and Perron (1988) (PP) test with the null hypothesis of a random walk with a constant and a drift, as well as the Kwiatkowski, Phillips, Schmidt, and Shin (1992) (KPSS) test with the null hypothesis of trend-stationarity. Table 4 provides test statistics and critical values for the three tests. We find strong empirical evidence for trend-stationarity in the dividend series for the long

Table 4: Test Statistics and Critical Values of the Unit Root Tests

		ADF-Test			PP-Test			KPSS-Test		
<b>Long Sample</b>	$c_t$	-2.32			-2.58			0.20		
	$d_t$	-4.15			-3.56			0.09		
	$p_t$	-2.73			-2.59			0.15		
<b>Post-War</b>	$c_t$	-2.38			-2.24			0.09		
	$d_t$	-4.20			-3.62			0.09		
	$p_t$	-1.95			-1.76			0.09		
<b>Garbage</b>	$c_t$	-1.81			-1.85			0.13		
	$d_t$	-2.34			-1.29			0.13		
	$p_t$	-1.35			-1.31			0.14		
<b>Critical Values</b>										
		1%	5%	10%	1%	5%	10%	1%	5%	10%
		-3.99	-3.43	-3.13	-4.04	-3.45	-3.15	0.216	0.146	0.119

*Test statistics for log consumption, dividends, and prices of the ADF-Test with a constant and a trend using one lag order, the PP-Test where the truncation lag parameter is set to  $\text{trunc}(4(T/100)^{0.25})$  and the KPSS-Test where the truncation lag parameter is set to  $\text{trunc}(\frac{10}{14}\sqrt{T})$  with  $T$  being the sample size. In the lower panel 1%, 5%, and 10% critical values are provided.*

sample and the post-war period, while it is not that obvious for consumption and prices.

The null hypothesis of trend-stationarity in the KPSS test cannot be rejected for any of the time series at the 1% significance level. So neither the hypothesis of a unit-root nor the hypothesis of trend-stationarity can be ruled out by the tests. Therefore we analyze the asset pricing implications of the trend-stationary model against the unit-root hypothesis (random walk model).

**Side Note.** To emphasize the point that the random walk and trend-stationary model are almost indistinguishable by the tests, we run the unit root tests on simulated data. We simulate  $n$  observations of  $c_t$  where  $n$  is the length of the corresponding data set (121 for the long sample, 63 for the post-war sample and 47 for the garbage sample). This is done 10'000 times. We report the median of the corresponding test statistics of the three tests as well as the standard deviations in parentheses. This is done for the case where consumption is trend-stationary and for the case where consumption is a random walk. Table 5 shows, that even in the simulated data, it is hard to distinguish the trend-stationary model from the random walk model. Looking at the results for the trend-stationary model ( $\sigma_\nu = 0$ ) we find that both, the ADF-Test and the PP-Test do not reject the null hypothesis of a random walk. We observe the same finding for the random walk model ( $\sigma_\epsilon = 0$ ) and a model where the permanent shock accounts for 40% of the total volatility in consumption growth,  $\frac{\sigma_\nu}{\sigma(\Delta c_t)} = 0.4$ . (This third model is of interest to us below.) For all three models, we also cannot reject the null hypothesis of trend-stationarity in the KPSS test at a 5% significance level. In addition we note that the standard deviations of the test statistics are quite large, which suggests that the sample is too small to dismiss either one of the two alternatives. These results are in line with previous Monte Carlo studies by Schwert (2002).

### 3.3 Parameter Estimates for Consumption Processes

The critical input in the asset-pricing model is the aggregate consumption process (2). In light of the results of the three unit root tests, we fit both a trend-stationary process and a random walk to the consumption time series for all three data sets. (In our robustness analysis below, we also consider models that contain both permanent and non-permanent shocks.)

#### 3.3.1 Trend-stationary Consumption

For  $\rho_c < 1$  the general (logarithmic) consumption specification (2) is trend-stationary when  $\sigma_\nu = 0$  and so,

$$\begin{aligned}
 c_t &= g_t + x_t \\
 g_t &= \bar{g} + g_{t-1} \\
 x_t &= \rho_c x_{t-1} + \sigma_\epsilon \epsilon_t \\
 \epsilon_t &\sim N(0, 1) \text{ i.i.d.}
 \end{aligned}
 \tag{19}$$

This log consumption process is composed of a AR(1) deviations from a deterministic linear trend. The smaller the coefficient  $\rho_c$ , the faster the process reverses to its linear trend. In the extreme case  $\rho_c = 0$ , the consumption process becomes a linear trend with white Gaussian noise. The larger the autocorrelation coefficient  $\rho_c$ , the more persistent is a shock  $\epsilon_t$  to consumption. In the extreme case  $\rho_c = 1$ , the consumption process ceases to be trend-stationary and instead has a

Table 5: Test Statistics and Critical Values of the Unit Root Tests for Simulated Consumption Data

		ADF-Test	PP-Test	KPSS-Test						
<b>Long Sample</b>	$\sigma_\nu = 0$	-2.89 (0.65)	-3.02 (0.65)	0.11 (0.043)						
	$\sigma_\epsilon = 0$	-2.18 (0.79)	-2.24 (0.79)	0.14 (0.054)						
	$\frac{\sigma_\nu}{\sigma(\Delta c_t)} = 0.4$	-2.70 (0.68)	-2.80 (0.69)	0.12 (0.048)						
<b>Post-War</b>	$\sigma_\nu = 0$	-2.40 (0.72)	-2.51 (0.70)	0.11 (0.030)						
	$\sigma_\epsilon = 0$	-2.18 (0.80)	-2.24 (0.80)	0.12 (0.034)						
	$\frac{\sigma_\nu}{\sigma(\Delta c_t)} = 0.4$	-2.35 (0.74)	-2.45 (0.72)	0.11 (0.032)						
<b>Garbage</b>	$\sigma_\nu = 0$	-2.85 (0.71)	-3.04 (0.66)	0.10 (0.023)						
	$\sigma_\epsilon = 0$	-2.18 (0.81)	-2.26 (0.80)	0.12 (0.028)						
	$\frac{\sigma_\nu}{\sigma(\Delta c_t)} = 0.4$	-2.66 (0.75)	-2.83 (0.70)	0.11 (0.025)						
		Critical Values								
		1%	5%	10%	1%	5%	10%	1%	5%	10%
		-3.99	-3.43	-3.13	-4.04	-3.45	-3.15	0.216	0.146	0.119

The table shows statistics for the same tests as in Table 4 but for simulated consumption data. For this purpose we simulate  $n$  observations of  $c_t$  where  $n$  is the length of the corresponding data set (121 for the long sample, 63 for the post-war sample and 47 for the garbage sample). We perform 10'000 such simulations. We report the median of the corresponding test statistics of the three tests as well as the standard deviations in brackets. This is done for the case where consumption is trend-stationary,  $\sigma_\nu = 0$  (parameter estimates for the consumption process are taken from Table 6), for the case where consumption is a random walk,  $\sigma_\epsilon = 0$  (parameter estimates are taken from Table 1 with  $\sigma_\nu = \sigma(\Delta c_t)$ ) and for the case where the permanent shocks account for 40% of the total volatility in consumption growth,  $\frac{\sigma_\nu}{\sigma(\Delta c_t)} = 0.4$ .

unit root and becomes a random walk with drift.

We estimate the three parameters  $\bar{g}$ ,  $\sigma_\epsilon$ , and  $\rho_c$  in the trend-stationary (“TS”) consumption process (19) for each of our three data sets (long sample, post-war, garbage). For this purpose, we first remove from each consumption time series the linear trend  $\bar{g} = E(\Delta c_t)$  to obtain the de-trended time series  $x_t = c_t - g_t$ . Then the estimate for the coefficient  $\rho_c$  is simply the correlation coefficient of the de-trended levels and its one-period lag and  $\sigma_\epsilon$  is the standard deviation of the residuals  $x_t - \rho_c x_{t-1}$ . Table 6 shows the point estimates as well as bootstrapped standard errors for the parameters for the different time series. Recall from Table 1 that the volatility of

Table 6: Parameter Estimates for the Trend-stationary (TS) Consumption Process

	$\bar{g}$	$\sigma_\epsilon$	$\rho_c$
<b>Long Sample</b>	0.0200 (0.0032)	0.0343 (0.0024)	0.9100 (0.0173)
<b>Post-War</b>	0.0213 (0.0023)	0.0175 (0.0012)	0.9259 (0.0165)
<b>Garbage</b>	0.0142 (0.0041)	0.0276 (0.0024)	0.7661 (0.0726)

*The table provides the point estimates for the parameters of the trend-stationary consumption process (19). Bootstrapped standard errors from  $10^6$  simulations are provided in parentheses. For each consumption time series,  $\bar{g} = E(\Delta c_t)$  denotes the linear trend. The estimate for the autocorrelation coefficient  $\rho_c$  is the correlation coefficient of the de-trended levels  $x_t = c_t - g_t$  and its one-period lag.  $\sigma_\epsilon$  is the standard deviation of the residuals  $x_t - \rho_c x_{t-1}$ . The long sample consists of all real consumption in the Shiller data set from 1889–2009 and the post-war series from the same data set for 1947–2009. The garbage data of Savov (2011) is available from 1960–2006.*

consumption growth  $\sigma(\Delta c)$  in the post-war series (1947–2009) and the garbage series (1960–2006) is much lower than in the long sample (1889–2009). As a result, the estimates for the standard deviation  $\sigma_\epsilon$  is considerably larger for the long sample than for the post-war period and for the garbage data series. The estimates for the autocorrelation parameter  $\rho_c$  indicate that shocks to consumption are much less persistent for consumption based on garbage data than for the other two data series. In other words, the garbage data series exhibits the fastest reversion to the long-run trend.

### 3.3.2 Random Walk Consumption

In addition to the trend-stationary consumption specification, we also consider a model of i.i.d. consumption growth, so consumption has a unit root. Setting  $\sigma_\epsilon = 0$  and  $\rho_c = 0$ , the general specification (2) simplifies to the random walk (“RW”)

$$\begin{aligned}
 c_t &= g_t \\
 g_t &= \bar{g} + g_{t-1} + \sigma_\nu \nu_t \\
 \nu_t &\sim N(0, 1) \text{ i.i.d.}
 \end{aligned}
 \tag{20}$$



For each consumption time series the parameter estimates for the random walk model are  $\bar{g} = E(\Delta c_t)$  and  $\sigma_\nu = \sigma(\Delta c_t)$  from Table 1, respectively.

## 4 Asset Pricing Implications

We first present the main asset pricing implications of the trend-stationary model and contrast them with those of the random walk model. Subsequently we demonstrate the robustness of the results by examining the implications of various modifications of the consumption process. And finally we analyze the return predictability of the trend-stationary model.

### 4.1 Calibration of the Trend-stationary Model

We calibrate the discount factor  $\beta$  and the risk-aversion coefficient  $\gamma$  so that the asset prices in the trend-stationary model match the empirical values of the risk-free rate and the equity premium. Table 7 compares the resulting summary measures of the trend-stationary model to the empirical moments found in the data. The first two rows of the table show the necessary values of  $\gamma$  and

Table 7: Summary Measures of the Trend-stationary Model Compared to the Data

	Long Sample		Post-War		Garbage	
	Data	Model	Data	Model	Data	Model
$\gamma$		7.70		16.5		8.24
$\beta$		1.10		1.34		1.08
$E(R_t)$	7.60	7.60	7.92	7.92	7.09	7.09
$\sigma(R_t)$	18.73	23.02	16.60	22.81	15.18	23.92
$E(R_t^f)$	1.97	1.97	1.84	1.84	2.21	2.21
$\sigma(R_t^f)$	5.80	5.85	2.65	5.76	2.60	8.48
EP	5.63	5.63	6.08	6.08	4.87	4.87
SR	0.30	0.24	0.37	0.27	0.32	0.20
$E(p_t - d_t)$	3.22	3.54	3.42	3.52	3.51	3.54
$\sigma(p_t - d_t)$	0.40	0.42	0.44	0.50	0.40	0.29
AC1( $p_t - d_t$ )	0.92	0.91	0.94	0.93	0.93	0.90

*The table compares implied model moments of the trend-stationary model with the empirical moments found in the data. The parameter estimates for the trend-stationary model are given in Table 6. The parameters  $\gamma$  and  $\beta$  are calibrated to match the risk-free rate and the equity premium.*

$\beta$  for which the asset prices generated by the model match the observed risk-free rate and the equity premium. For the long sample the values are  $\gamma = 7.70$  and  $\beta = 1.10$ , for the post-war series they are  $\gamma = 16.5$  and  $\beta = 1.34$ , and for the garbage data they are  $\gamma = 8.24$  and  $\beta = 1.08$ . Our findings are consistent with a recent strand of literature considering  $\beta > 1$ .

All three pairs of  $\beta$  and  $\gamma$  satisfy the condition (13) for the existence of equilibrium when the discount factor exceeds one. The lower volatility and positive autocorrelation in the post-war data leads to a higher implied risk aversion.

For the long sample, the trend-stationary model does a very good job matching the data. The model generates close estimates for the volatility of the risk-free rate as well as all three summary measures for the price-dividend ratio. It slightly overestimates the return volatility

for the aggregate consumption claim and thus underestimates the Sharpe ratio. The results are almost as good for the post-war data and the garbage data. In addition to overestimating the return volatility, the model also overestimates the volatility of the risk-free rate, which has been very low in the post-war period.

We next document that all estimates from the trend-stationary model, even those that are a bit off, are much closer to the data than the estimates from the calibrated random walk model. Table 8 reports the necessary values of  $\gamma$  and  $\beta$  for which the asset prices generated by the random walk model match the observed risk-free rate and the equity premium. The necessary values for

Table 8: Summary Measures of the Random Walk Model Compared to the Data

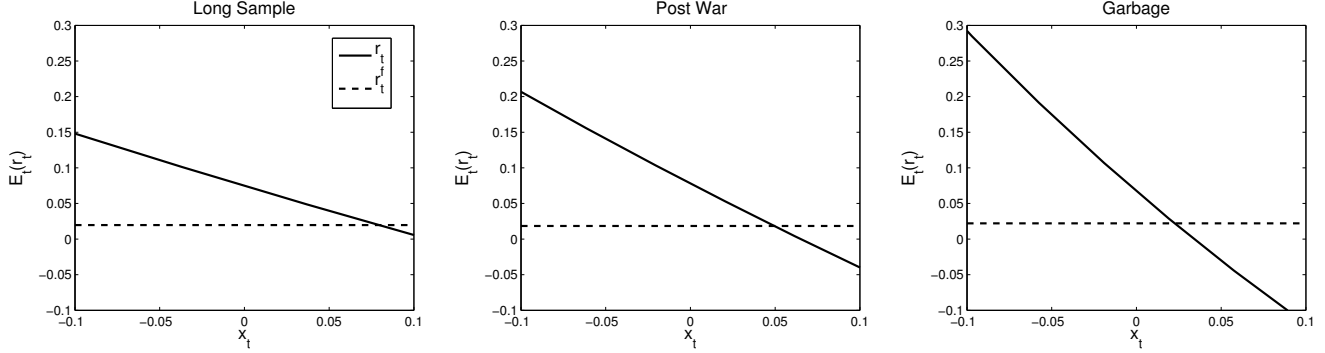
	Long Sample		Post-War		Garbage	
	Data	RW	Data	RW	Data	RW
$\gamma$		43.9		179		57
$\beta$		0.7175		0.2475		0.5820
$E(R_t)$	7.60	7.60	7.92	7.92	7.09	7.09
$\sigma(R_t)$	18.73	3.79	16.60	1.94	15.18	3.06
$E(R_t^f)$	1.97	1.97	1.84	1.84	2.21	2.21
$\sigma(R_t^f)$	5.80	0	2.65	0	2.60	0
EP	5.63	5.63	6.08	6.08	4.87	4.87
SR	0.30	1.49	0.37	3.12	0.32	1.59
$E(p_t - d_t)$	3.46	2.91	3.42	2.87	3.46	2.89
$\sigma(p_t - d_t)$	0.40	0	0.44	0	0.41	0
AC1( $p_t - d_t$ )	0.91	0	0.94	0	0.93	0

*The table compares implied model moments of the random walk model with the empirical moments found in the data. The parameter estimates for the trend-stationary model are given in Table 1. The parameters  $\gamma$  and  $\beta$  are calibrated to match the risk-free rate and the equity premium.*

the risk-aversion coefficient are much larger than for the trend-stationary model; in fact, they are unreasonably large, particularly the value of 179 for the post-war data. The need to match the risk-free rate with such high coefficients of risk-aversion requires very low betas. The random walk model delivers a return volatility (for the aggregate consumption claim) that is much too small. As a result the estimated Sharpe ratios are much too large. The random walk model also underestimates the price-dividend ratio. And as is well-known, the model cannot generate volatility of the risk-free rate and of the price-dividend ratio.

We next provide some intuition for the successful predictive performance of the canonical and parsimonious asset-pricing model with the trend-stationary consumption process. Figure 3 shows conditional expected returns of the risky and riskless asset given the state  $x_t$  for the three different data sets. We observe that expected excess returns are monotonically decreasing in the state  $x_t$ ; put differently, expected excess returns are higher than average when the economy is below the trend (negative  $x_t$ ) and lower than average when it is above the trend (large  $x_t$ ). This result is in line with the empirical finding that expected returns are large in recessions and low in economic booms. Changes in expected returns are actually the main driver for most of the asset pricing dynamics as Shiller (1981) has shown (instead of changes in expected dividend growth, as had been assumed previously). For a more in-depth analysis of these facts in the context of our model

Figure 3: Conditional Expected Returns in the Trend-stationary Model



The graphs show the conditional expected returns of the risky and riskless asset given the state  $x_t$ . Results are calculated using the calibrations reported in Table 7.

we now decompose the volatility of the price-dividend ratio generated by the model. In fact, we can show that just like in the data most of the variation in the price-dividend ratio is generated by changes in expected returns.

## 4.2 Volatility Tests

It is well known that stock prices move far more than can be explained by changes in expected dividend growth (see Shiller (1981)) and most of the price dynamics are driven by changes in expected returns. To demonstrate that the trend-stationary model captures this fact, we run a volatility test as in Cochrane (1992). The test is based on a first-order approximation of the return identity  $R_{t+1} = (P_{t+1} + D_{t+1})/P_t$  which implies that

$$\text{var}(p_t - d_t) \approx \sum_{i=1}^{\infty} \xi^i \text{cov}(p_t - d_t, -r_{t+i}) + \sum_{i=1}^{\infty} \xi^i \text{cov}(p_t - d_t, \Delta d_{t+i}) \quad (21)$$

where  $\xi = (P/D)/(1 + P/D)$ . The linearization is around the point  $P/D$ ; in our model, this point is the sample mean of the price-dividend ratio. So variations in the price-dividend ratio can only exist, if this ratio has predictive power for either returns or dividend growth or both. Following Campbell and Cochrane (1999), we use 15 years of covariances to approximate the two (infinite) sums on the right-hand side of the expression (21). Table 9 reports both empirical results and model predictions for all three data sets.

In the data we find that almost all variation in the price-dividend ratio is driven by changes in expected returns, while the changes attributed to expected dividends are rather small. For the trend-stationary model we find about the same patterns for all three calibrations with most of the variations in the price-dividend ratio coming from changes in expected returns.

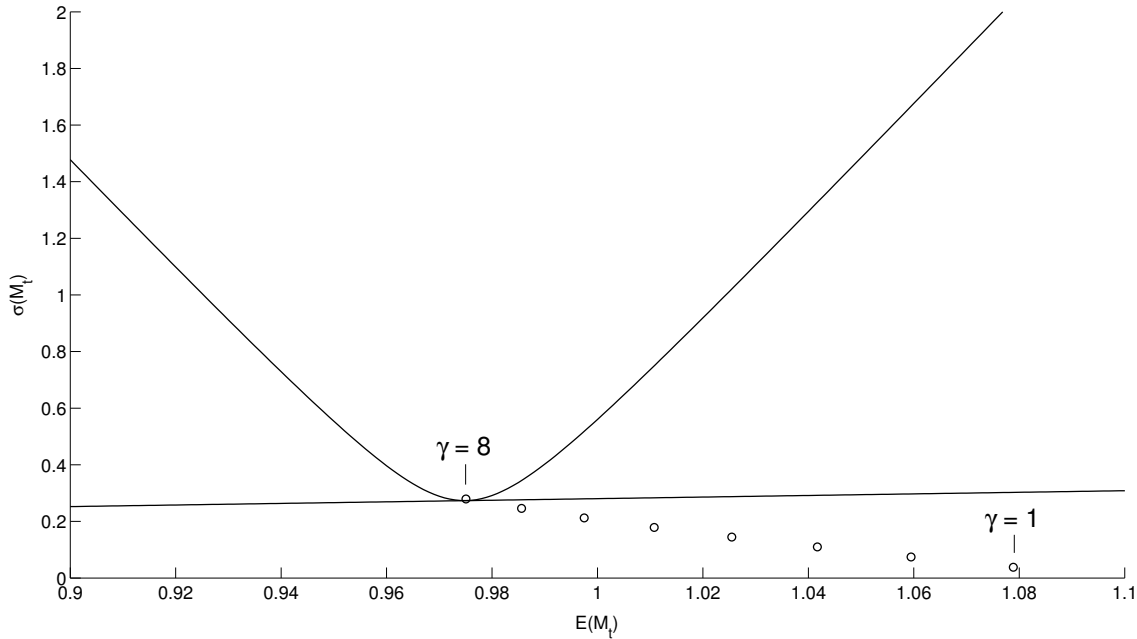
A related challenge for asset-pricing models is the Hansen and Jagannathan (1991) bound. To generate a high Sharpe ratio, the stochastic discount factor must be very volatile. Figure 4 illustrates the relationship between the Hansen-Jagannathan bound and the trend-stationary model SDF for the full sample. For  $\gamma$  around 8 the bound is met. (The corresponding figures for post-war and garbage data are shown in Appendix C.)

Table 9: Variance Decompositions

	Data		Model	
	Returns	Dividends	Returns	Dividends
<b>Long Sample</b>	0.7665	-0.0922	0.9719	-0.1283
<b>Post-War</b>	1.1714	-0.2024	0.8597	-0.0532
<b>Garbage</b>	0.6587	0.1316	1.1088	-0.1351

The table shows shares of the variance in the price-dividend ratio that are explained by returns and dividends for the three different data sets. The shares are calculated as  $\frac{\sum_{i=1}^{15} \xi^i \text{cov}(p_t - d_t, -r_{t+i})}{\text{var}(p_t - d_t)}$  and  $\frac{\sum_{i=1}^{15} \xi^i \text{cov}(p_t - d_t, \Delta d_{t+i})}{\text{var}(p_t - d_t)}$ , respectively.

Figure 4: Hansen-Jagannathan Bounds



The graph shows the mean and standard deviation of the pricing kernel  $M_t$  implied by the trend-stationary model as well as the Hansen-Jagannathan bounds for different degrees of risk aversion  $\gamma$ . Each circle represents an increase of  $\gamma$  of one, starting at  $\gamma = 1$  in the right lower corner. Parameter estimates for consumption are taken from the long sample; see Table 1 with the calibration from Table 7, with  $E(\Delta c_t) = 0.020$  and  $\sigma(\Delta c_t) = 0.0352$ ,  $\rho_c = 0.91$ , and  $\beta = 1.1$ .

### 4.3 Robustness Checks on the Consumption Process

We perform a series of robustness checks on the parameters of the consumption process. These checks not only demonstrate the robustness of our results but also help us to develop more intuition for the features of the consumption process that drive the asset pricing implications of the trend-stationary model.

#### 4.3.1 Model with both Permanent and Non-Permanent Shocks

The consumption process (19) in the trend-stationary model has a deterministic linear trend with AR(1) deviations; there are no permanent shocks to the growth rate  $g_t$  since  $\sigma_\nu = 0$ . As a first robustness check, we now add a permanent shock to the trend-stationary model. Recall from Equation (4) the analytical relationship between the consumption growth volatility  $\sigma(\Delta c_t)$  and the standard deviations of the two shocks  $\epsilon_t$  and  $\nu_t$ ,

$$\sigma(\Delta c_t) = \sqrt{\sigma_\nu^2 + \frac{2}{1 + \rho_c} \sigma_\epsilon^2}.$$

We now vary  $\sigma_\nu$  and adjust  $\sigma_\epsilon$  to hold  $\sigma(\Delta c_t)$  constant. We hold the autocorrelation parameter  $\rho_c$  constant at its value reported in Table 6. In addition, we keep the calibrated values for the risk-aversion coefficient  $\gamma$  and the discount factor  $\beta$ . Note that we deliberately do not recalibrate the model because we want to examine how the summary measures respond to changes in the magnitude of the permanent shock for fixed preferences.

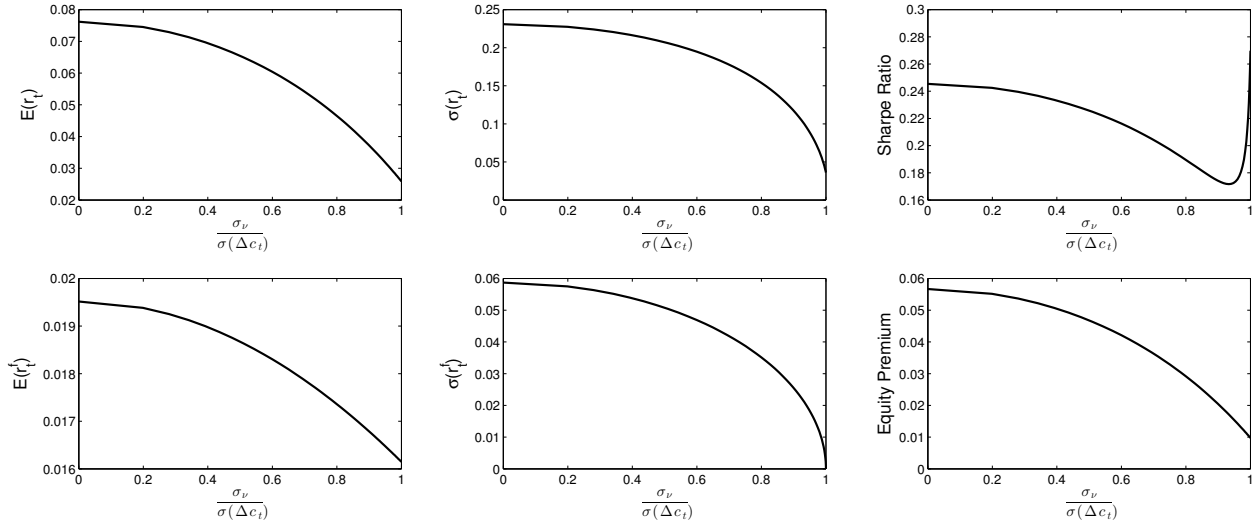
Figure 5 shows the influence of the permanent shock  $\sigma_\nu$  on several asset pricing characteristics for the long sample. (Appendix C shows the corresponding figures for the post-war and garbage data.) On the horizontal axis we report the share of the permanent shock  $\sigma_\nu$  in total consumption growth volatility  $\sigma(\Delta c_t)$ —that is, the ratio  $\sigma_\nu/\sigma(\Delta c_t)$ . At the left end of the horizontal axis, it holds that  $\sigma_\nu = 0$ , and so all volatility stems from the (non-permanent) shock  $\epsilon_t$ . This case corresponds to the pure trend-stationary model, recall the results in Table 7. At the right end of the horizontal axis, all consumption volatility comes from the permanent shock, so  $\sigma(\Delta c_t) = \sigma_\nu$  and  $\sigma_\epsilon = 0$ . This case correspond to the pure random walk model.

Observe that all six curves in Figure 5 are rather flat as long as the share  $\sigma_\nu/\sigma(\Delta c_t)$  is less than 40%. For example, the equity premium remains above 5% in this range. So, the reported summary measures do not change significantly as long as the share of the permanent shock in the consumption volatility remains below 40%. Put differently, the asset pricing implications of the trend-stationary model are very robust to the inclusion of a permanent component in the consumption growth volatility. Non-permanent shocks to consumption growth, as long as they are sufficiently large, drive the asset pricing implications of the model.

#### 4.3.2 Effects of the Autocorrelation Parameter $\rho_c$

As a second robustness check, we now examine the influence of the autocorrelation parameter  $\rho_c$ . Recall that the larger  $\rho_c$ , the more persistent is a shock  $\epsilon_t$  to consumption. (In the extreme case  $\rho_c = 1$ , the consumption process ceases to be trend-stationary and instead has a unit root and becomes a random walk with drift.) We hold the consumption growth volatility constant—that

Figure 5: Asset Pricing Effects of Adding Permanent Shocks  $\sigma_\nu$  to the Trend-stationary Model for the Long data set



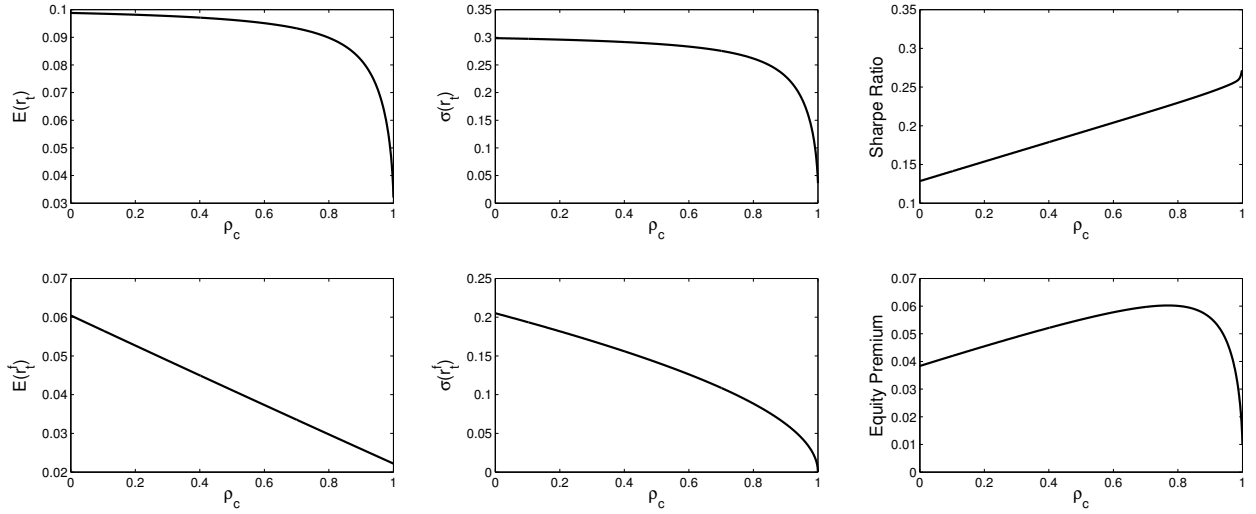
The graphs show the expected return of the aggregate consumption claim, its volatility, the Sharpe ratio, the risk-free rate, its volatility, and the equity premium as a function of the share of the permanent shock  $\sigma_\nu$  in the total volatility of consumption growth  $\sigma(\Delta c_t)$ . (All other volatility comes from the non-permanent shock  $\sigma_\epsilon$ .) Parameter estimates for consumption are taken from the long sample; see Table 1 with the calibration from Table 7, so  $E(\Delta c_t) = 0.020$  and  $\sigma(\Delta c_t) = 0.0352$ ,  $\rho_c = 0.91$ ,  $\gamma = 7.7$  and  $\beta = 1.1$ .

is, a change in  $\rho_c$  implies a change in the standard deviation  $\sigma_\epsilon$  to hold  $\sigma(\Delta c_t)$  constant according to Equation (4). We maintain the calibrated values for the risk-aversion coefficient  $\gamma$  and the discount factor  $\beta$  from Table 7.

Figure 6 shows the expected return of the aggregate consumption claim, its volatility, the Sharpe ratio, the expected return of the risk free asset, its volatility, and the equity premium as a function of the autocorrelation coefficient in consumption,  $\rho_c$ . For all results, the expected value and volatility of consumption growth are fixed at the empirical values of the long sample, so  $E(\Delta c_t) = 0.020$  and  $\sigma(\Delta c_t) = 0.0352$ . (In Appendix C we show the corresponding figures for the post-war and garbage data series.) We observe that the return volatility of the aggregate consumption claim and the equity premium are very sensitive to changes in the autocorrelation coefficient  $\rho_c$  when  $\rho_c$  is close to one. The equity premium and both the average and the volatility of the return of the aggregate consumption claim sharply increase for small deviations from the random walk case ( $\rho_c = 1$ ). We also observe that the Sharpe ratio is increasing in the autocorrelation coefficient  $\rho_c$  while the equity premium peaks at around 0.8. So, a large level of  $\rho_c$  is desired to obtain a high Sharpe ratio while somewhat lower values increase the equity premium and generate more volatility in the asset returns. These findings hold qualitatively for all three data sets; see Figures 13 and 14 in Appendix C.

In sum, the robustness check with regard to the autocorrelation coefficient  $\rho_c$  stresses the importance of trend-stationarity of the consumption process to generate realistic pricing implications in our canonical and parsimonious asset-pricing model. Even a modest deviation from the random walk model ( $\rho_c \in [0.8, 0.95]$ ) leads to substantially improved asset-pricing implications of

Figure 6: Asset Pricing Effects of the Autocorrelation Coefficient  $\rho_c$  in Consumption for the Long data set



The graphs show the expected return of the aggregate consumption claim, its volatility, the Sharpe ratio, the risk-free rate, its volatility, and the equity premium as a function of the autocorrelation coefficient in consumption,  $\rho_c$ . The underlying consumption process is from the long sample; see Table 1, so  $E(\Delta c_t) = 0.020$  and  $\sigma(\Delta c_t) = 0.0352$ . The results are computed using the parameter estimates from Table 7 ( $\gamma = 7.7$ ,  $\beta = 1.1$ ). ( $\sigma_\nu = 0$ ).

the model.

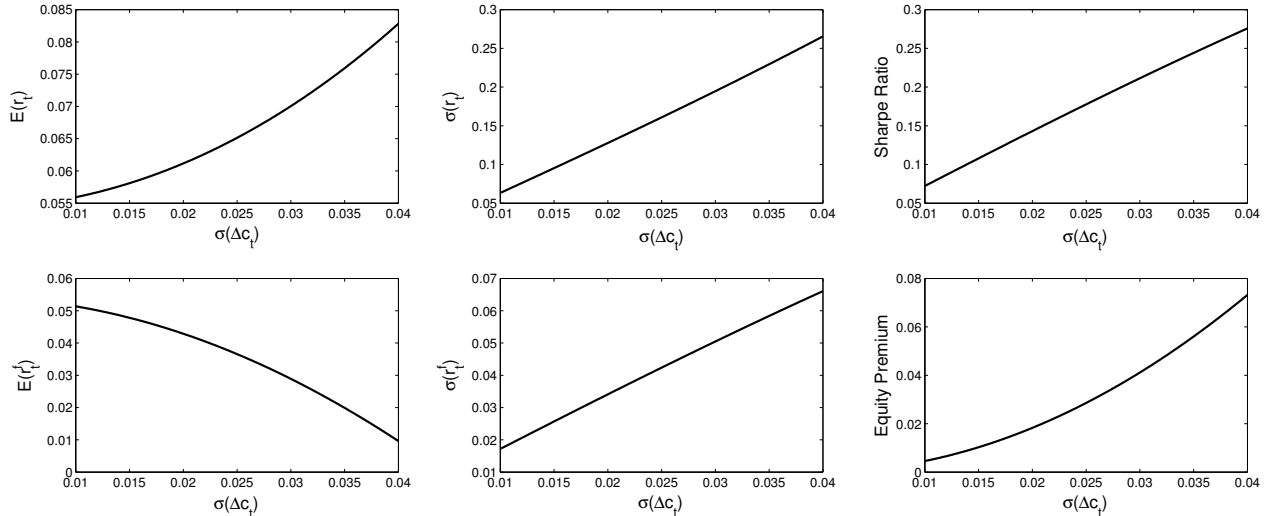
### 4.3.3 Effects of the Consumption Growth Volatility $\sigma(\Delta c_t)$

The volatility of consumption growth,  $\sigma(\Delta c_t)$ , plays a critical role in the pricing implications of consumption-based asset-pricing models. Many papers have pointed out that this volatility in the data is too small to generate non-trivial risk premia in such models; see, among other papers, Grossman and Shiller (1981) or Mehra and Prescott (1985).

As a third robustness check, we vary  $\sigma(\Delta c_t)$  in the interval  $[0.01, 0.04]$ . This range encompasses the estimated values for all three data sets; see Table 1. Once again we use Equation (4) to adjust  $\sigma_\epsilon$  while holding  $\rho_c = 0.91$  and  $\sigma_\nu = 0$  constant. We do not recalibrate the model but maintain the values for  $\gamma$  and  $\beta$  from the initial calibration; see Table 7. Figure 7 shows the usual summary measures as a function of the volatility  $\sigma(\Delta c_t) \in [0.01, 0.04]$  in a trend-stationary model based on the long sample, so for the statistical parameters  $\bar{g} = E(\Delta c_t) = 0.02$ ,  $\rho_c = 0.91$ , and the model parameters  $\gamma = 7.7$  and  $\beta = 1.1$ . (Appendix C shows the corresponding figures for the post-war and garbage data series.) While the expected return of the aggregate consumption claim increases considerably with  $\sigma(\Delta c_t)$  in the range  $[0.01, 0.04]$ , the risk-free rate decreases in  $\sigma(\Delta c_t)$ . As a result, the equity premium increases rather swiftly in the consumption volatility  $\sigma(\Delta c_t)$  in the range  $[0.01, 0.04]$ . Also the return volatilities of the aggregate consumption claim and the risk-free rate are increasing in  $\sigma(\Delta c_t)$ .

Our observations underline the important role of the consumption growth volatility for the asset pricing implications of our canonical and parsimonious model. Rather small increases in the

Figure 7: Summary Measures for the TS Model as a Function of the Consumption Growth Volatility for the Long data set



The graph shows the expected return of the aggregate consumption claim, its volatility, the Sharpe ratio, the expected risk free rate, its volatility and the equity premium as a function of the standard deviation of consumption growth  $\sigma(\Delta c_t)$ . The underlying consumption process is from the long sample; see Table 1, so  $E(\Delta c_t) = 0.020$  and  $\rho_c = 0.91$ . The results are computed using the parameter estimates from Table 7 ( $\gamma = 7.7$ ,  $\beta = 1.1$ ). ( $\sigma_\nu = 0$ ).

volatility of consumption growth, particularly above a value of 0.03, have a considerable impact on asset prices in the trend-stationary model.

#### 4.3.4 Robustness: Mehra and Prescott (1985) Sample Period

In our final robustness check, we re-evaluate the trend-stationary model for the sample restricted to 1889–1978—as used in Mehra and Prescott (1985)—to rule out the possibility that our results are solely driven by new data that was not available to Mehra and Prescott in 1985. In Table 10 we show the parameter estimates for the trend-stationary model as well as the data and model moments for the time period from 1889–1978. As in Table 7, we calibrate  $\gamma$  and  $\beta$  to match the risk-free rate and the equity premium for each subsample.

We observe that for the time period from 1889–1978, we can match the risk-free rate and the equity premium with a risk aversion of 6.76 and a discount factor of 1.09. These estimates are slightly smaller compared to the parameters reported for the full sample from 1889–2009 (7.70 and 1.09, respectively; see Table 7). Hence, our results do not rely on additional data that was not available to Mehra and Prescott in 1985. The table also shows the results for the model used in Mehra and Prescott (1985) where consumption growth follows an AR(1) process.<sup>4</sup> We observe that with the Mehra-Prescott model a risk aversion of 19 and a discount factor of 1.10 is needed to jointly match the risk free-rate and equity premium. We also show results where we match

<sup>4</sup>The results are computed using the closed-form solution from Burnside (1998). The mean, standard deviation and persistence of consumption growth can be obtained from equations (3)–(5) using the parameter estimates of the trend-stationary model and are given by 0.0203, 0.039, and -0.06, respectively.



the equity premium with  $\beta = 0.99$  and  $\beta = 0.96$ . We observe that the equity premium can still be matched with a risk aversion well below ten for the trend-stationary model, while the original model specification as in Mehra and Prescott (1985) requires a risk aversion of more than 17. So a discount factor larger than one alone does not provide a solution for the equity premium puzzle for the Mehra and Prescott data. It is the persistent, trend-stationary shocks that explain the premium and the discount factor larger than one is only required to match the low risk-free rate.

Table 10: Summary Measures of the Trend-stationary Model for the Mehra and Prescott (1985) Data

	1889–1978						
	Data	TS-1	TS-2	TS-3	MP-1	MP-2	MP-3
$\bar{g}$	0.0203						
$\sigma_\epsilon$	0.0385						
$\rho_c$	0.8874						
$\gamma$		6.76	8.41	8.73	19	17.68	17.47
$\beta$		1.09	0.99	0.96	1.10	0.99	0.96
$E(R_t)$	7.50	7.50	19.91	23.79	7.50	19.11	22.83
$\sigma(R_t)$	19.51	24.60	20.92	20.58	9.57	9.96	10.10
$E(R_t^f)$	1.64	1.64	14.05	17.93	1.64	13.25	16.96
$\sigma(R_t^f)$	6.47	6.48	9.03	9.70	4.48	4.77	4.87
EP	5.86	5.86	5.86	5.86	5.86	5.86	5.86
SR	0.30	0.24	0.28	0.28	0.62	0.59	0.59
$E(p_t - d_t)$	3.08	3.69	1.89	1.66	3.00	1.81	1.61
$\sigma(p_t - d_t)$	0.25	0.40	0.29	0.28	0.04	0.04	0.04
AC1( $p_t - d_t$ )	0.78	0.89	0.89	0.89	-0.06	-0.06	-0.06

The table compares implied model moments of the trend-stationary model with the empirical moments found in the data for the time series from 1889–1978 (as used in Mehra and Prescott (1985)). TS-1 shows the results for the trend-stationary model where the parameters  $\gamma$  and  $\beta$  are calibrated to jointly match the risk-free rate and the equity premium. TS-2 and TS-3 show the results for  $\beta = 0.99$  and  $\beta = 0.96$  where  $\gamma$  is calibrated to match the equity premium. MP-1, MP-2 and MP-3 show the corresponding results obtained using the model specification in Mehra and Prescott (1985) where consumption growth follows an AR(1) process. The mean, standard deviation and persistence of consumption growth can be obtained from equations (3)–(5) using the parameter estimates of the trend-stationary model and are given by 0.0203, 0.039, and -0.06, respectively.

This completes our series of robustness checks on the parameters in the consumption process. In the next step of our analysis, we document the robustness of the asset pricing implications of the trend-stationary model when dividends and consumption are allowed to be distinct processes.

#### 4.4 Dividend and Labor Income

We now turn to a generalization of our model to emphasize the robustness of our results with regard to different model specifications. We abandon the assumption of dividends being a fixed fraction of consumption and instead allow for time variation in the shares of financial and labor income in aggregate consumption. This model extension is motivated by recent results in the finance literature. For example, Longstaff and Piazzesi (2004) find that explicitly including dividend payments by the corporate sector in the analysis of macroeconomic cash flows has strong

effects on asset prices.<sup>5</sup> Our model specification here is a linear, discrete-time version of the model by Santos and Veronesi (2006) with the assumption of consumption being trend-stationary (instead of consumption growth being stationary). Santos and Veronesi (2006) analyze the impact of time variations in the share of labor income in aggregate consumption on the predictability of stock returns. We build on their results and analyze the effects on expected returns and the equity premium. We follow DeJong and Ripoll (2007) who state that labor endowments, dividends, prices, and consumption follow a balanced growth path with an annual common mean growth rate. DeJong and Ripoll (2007) built on the results of Shiller (1981) who assumes that dividends and prices are trend-stationary. DeJong (1992) provides empirical support for this assumption.

In the new model specification, we allow for time variation in the share of financial income in aggregate consumption,

$$C_t = D_t + E_t,$$

where  $E_t$  describes all non-dividend income. We write the model in terms of the non-dividend to dividend income ratio to ensure stationarity,

$$\Phi_t = \frac{E_t}{D_t}$$

and, alternatively, in logs

$$\phi_t = e_t - d_t.$$

For the description of equilibria, we employ two state variables, detrended consumption  $x_t = c_t - g_t$  and the log non-dividend to dividend income ratio  $\phi_t = e_t - d_t$ . The claim on the aggregate dividend stream can now be priced in terms of the price-consumption ratio, see Equation (10),

$$\begin{aligned} \frac{P_t}{C_t} &= \beta E_t \left\{ e^{(1-\gamma)(g_{t+1}-g_t+x_{t+1}-x_t)} \left( \frac{P_{t+1}}{C_{t+1}} + \frac{D_{t+1}}{C_{t+1}} \right) \right\} \\ &= \beta E_t \left\{ e^{(1-\gamma)(g_{t+1}-g_t+x_{t+1}-x_t)} \left( \frac{P_{t+1}}{C_{t+1}} + \frac{1}{1+e^{\phi_{t+1}}} \right) \right\}, \end{aligned} \quad (22)$$

with the resulting one-period return

$$\begin{aligned} R_{t+1} &= \frac{\left( \frac{P_{t+1}}{C_{t+1}} + \frac{D_{t+1}}{C_{t+1}} \right)}{\frac{P_t}{C_t}} \times \frac{C_{t+1}}{C_t} \\ &= \frac{\left( \frac{P_{t+1}}{C_{t+1}} + \frac{1}{1+e^{\phi_{t+1}}} \right)}{\frac{P_t}{C_t}} \times e^{(g_{t+1}-g_t+x_{t+1}-x_t)}. \end{aligned} \quad (23)$$

We refer to this specification of the model as the DC (Dividend Claim) model.

<sup>5</sup>Longstaff and Piazzesi (2004) consider a model where consumption growth and the share of financial income in aggregate consumption are continuous time jump-diffusion processes. They assume a 1% probability of a 10% decline in consumption and a 90% decline in dividends. Their low estimates of the equity premium can be explained by their unit root assumption in consumption. As we see below, the trend-stationary alternative produces significantly larger premia even without the large shocks.

#### 4.4.1 Estimation of the Dividend Claim Model

To calibrate the model, we use data on observed dividends and consumption. In line with Longstaff and Piazzesi (2004), endowment income is calculated as the difference between aggregate consumption and observed dividends scaled, so that the share of dividends in aggregate consumption is 4% on average.<sup>6</sup> Longstaff and Piazzesi (2004) use a measure they call ‘imputed’ dividends to account for the fact that corporations tend to artificially smooth dividends over time, so their measure is more volatile than the reported dividends that we use. As the volatility of dividends positively affects the equity premium, our estimate is rather conservative and we restrict ourselves to the lower bounds of expected returns and volatilities. We assume that the state variables  $x_t$  and  $\phi_t$  follow two correlated AR(1) processes given by<sup>7</sup>

$$\begin{aligned} x_t &= (1 - \rho_c)\mu_x + \rho_c x_{t-1} + \epsilon_{x,t} \\ \phi_t &= (1 - \rho_\phi)\mu_\phi + \rho_\phi \phi_{t-1} + \epsilon_{\phi,t} \end{aligned} \tag{24}$$

with  $\epsilon_{x,t}, \epsilon_{\phi,t} \sim N(0, \Sigma_{\mathbf{x},\phi})$ . We estimate the model using simple OLS regressions. Table 11 reports the parameter estimates for this model for all three data sets.

Table 11: Parameter Estimates for the Pricing of the Dividend Claim

	$\mu_x$	$\mu_\phi$	$\rho_c$	$\rho_\phi$	$\sigma_c$	$\sigma_\phi$	$cov_{c,\phi}$	$\bar{g}$
<b>Long Sample</b>	2.6208	3.5005	0.9100	0.9618	0.0344	0.1134	-0.0002	0.0200
<b>Post-War</b>	2.6336	3.4632	0.9259	0.9841	0.0177	0.0634	-0.0000	0.0213
<b>Garbage</b>	0.2646	3.1873	0.7661	0.9545	0.0269	0.0471	0.0006	0.0142

*Parameter estimates using least squares estimation for Equation (24) for the three different data sets.*

#### 4.4.2 Pricing the Market Portfolio

We calibrate the discount factor  $\beta$  and the risk-aversion coefficient  $\gamma$  again so that the asset prices in the new model with dividend and labor income match the empirical values of the risk-free rate and the equity premium. Table 12 reports the calibrated values of the two parameters as well as the summary measures for the pricing of the dividend claim for the three different data sets.<sup>8</sup> Broadly speaking, the results are qualitatively very similar to those for the benchmark model with only the aggregate consumption claim in Section 4.1. The calibrated values of the model parameters  $\gamma$  and  $\beta$  are close to those reported in Table 7 for the benchmark model. We again find values for the risk aversion coefficient  $\gamma$  that are much smaller than the values commonly found in the economics literature. The new model shows also a similar performance to the benchmark model in matching the empirical moments in the data. The model matches the mean, standard

<sup>6</sup>We also varied this share but it didn’t affect our results much.

<sup>7</sup>We also tried a VAR(1,1) specification instead of two correlated AR(1) processes for detrended consumption  $x_t$  and the non-dividend to dividend income ratio  $\phi_t$ , but this change in the underlying process has no significant influence on the results. The results for the VAR(1,1) specification are shown in Appendix C.

<sup>8</sup>The corresponding table for the VAR(1,1) specification instead of two correlated AR(1) processes for detrended consumption  $x_t$  and the non-dividend to dividend income ratio  $\phi_t$  is shown in Appendix C.

Table 12: Summary Measures of the Second Model Compared to the Data

	Long Sample		Post-War		Garbage	
	Data	Model	Data	Model	Data	Model
$\gamma$		7.58		16.2		8.68
$\beta$		1.10		1.33		1.08
$E(R_t)$	7.60	7.60	7.92	7.92	7.09	7.09
$\sigma(R_t)$	18.73	23.83	16.60	23.47	15.18	23.59
$E(R_t^f)$	1.97	1.97	1.84	1.84	2.21	2.21
$\sigma(R_t^f)$	5.80	5.78	2.65	5.73	2.60	8.71
EP	5.63	5.63	6.08	6.08	4.87	4.87
SR	0.30	0.24	0.37	0.26	0.32	0.21
$E(p_t - d_t)$	3.22	3.62	3.42	3.54	3.51	3.52
$\sigma(p_t - d_t)$	0.40	0.47	0.44	0.51	0.40	0.35
AC1( $p_t - d_t$ )	0.92	0.92	0.94	0.93	0.93	0.82

The table compares implied model moments of the pricing of the dividend claim with the empirical moments found in the data. The parameter estimates for the trend-stationary dividend and income processes are given in Table 11. The model parameters  $\gamma$  and  $\beta$  are calibrated to match the risk-free rate and the equity premium.

deviation and first order autocorrelation of the log price-dividend ratio for all three data sets pretty well. As in the benchmark model, the model slightly overestimates the volatility of returns to the dividend claim compared to the volatility we find in the data. In sum, the distinction between dividend and labor income in the new model does not lead to better estimates than the benchmark model with only the aggregate consumption claim.

#### 4.5 Long-Horizon Predictability

Empirical evidence (Campbell and Shiller (1988a), Fama and French (1988)) suggests that over long horizons the price-dividend ratio predicts cumulative returns: high values predict low future returns, and vice versa. A pure random walk CRRA model of consumption and dividends has a constant price-dividend ratio, so it cannot reproduce such predictions.

We test the ability of transient shocks to generate this long-horizon predictability, by comparing the results from simulated model data against the empirical results. In each case, we consider the following specification,

$$R_{t+h} = \hat{\alpha} + \hat{\beta}_1(p_t - d_t) + \epsilon_{t+h},$$

where  $R_{t+h}$  is the cumulative return over  $h$  periods.

Table 13 shows the results for long-horizon regressions. We consider both the trend-stationary consumption claim model of Section 4.1, as well as the separate dividend claim model of Section 4.4. (The parameters are those specified in Table 7 for the pricing of the consumption claim (model CC) and Table 12 for the pricing of the dividend claim (model DC).)

In the data we find the standard patterns documented by Campbell and Shiller (1988b). Coefficients are negative, so high price-dividend ratios today imply low future returns. The  $R^2$  are low for short-term predictions but grow rapidly for longer horizons. The trend-stationary models qualitatively reproduces these findings. For all three data series, the model produces  $R^2$

Table 13: Predictability of Stock Returns

		$h = 1$		$h = 3$		$h = 5$		$h = 8$	
		$R^2$	$\hat{\beta}_1$	$R^2$	$\hat{\beta}_1$	$R^2$	$\hat{\beta}_1$	$R^2$	$\hat{\beta}_1$
<b>Long Sample</b>	Data	0.0317	-0.0880	0.0644	-0.1962	0.1048	-0.3268	0.1598	-0.5310
	CC	0.0653	-0.1398	0.1725	-0.4433	0.2541	-0.7759	0.3416	-1.3287
	DC	0.0456	-0.1106	0.1192	-0.3479	0.1740	-0.6070	0.2295	-1.0358
<b>Post-War</b>	Data	0.1055	-0.1253	0.1920	-0.2800	0.2779	-0.4667	0.3642	-0.7363
	CC	0.0598	-0.1107	0.1605	-0.3590	0.2394	-0.6454	0.3257	-1.1506
	DC	0.0536	-0.1049	0.1421	-0.3387	0.2100	-0.6074	0.2885	-1.0807
<b>Garbage</b>	Data	0.0336	-0.0684	0.0823	-0.1930	0.1385	-0.3493	0.1547	-0.6122
	CC	0.1351	-0.3099	0.3076	-0.8237	0.4023	-1.2278	0.4706	-1.7058
	DC	0.1359	-0.2594	0.3104	-0.6925	0.4022	-1.0356	0.4649	-1.4397

The table shows the  $R^2$  statistic and the slope coefficient  $\hat{\beta}_1$  of regressing cumulative stock returns on the log price-dividend ratio for different time horizons  $h$  (in years) for the three different data sets. Parameter estimates for the models are chosen as in Table 7 and Table 12.

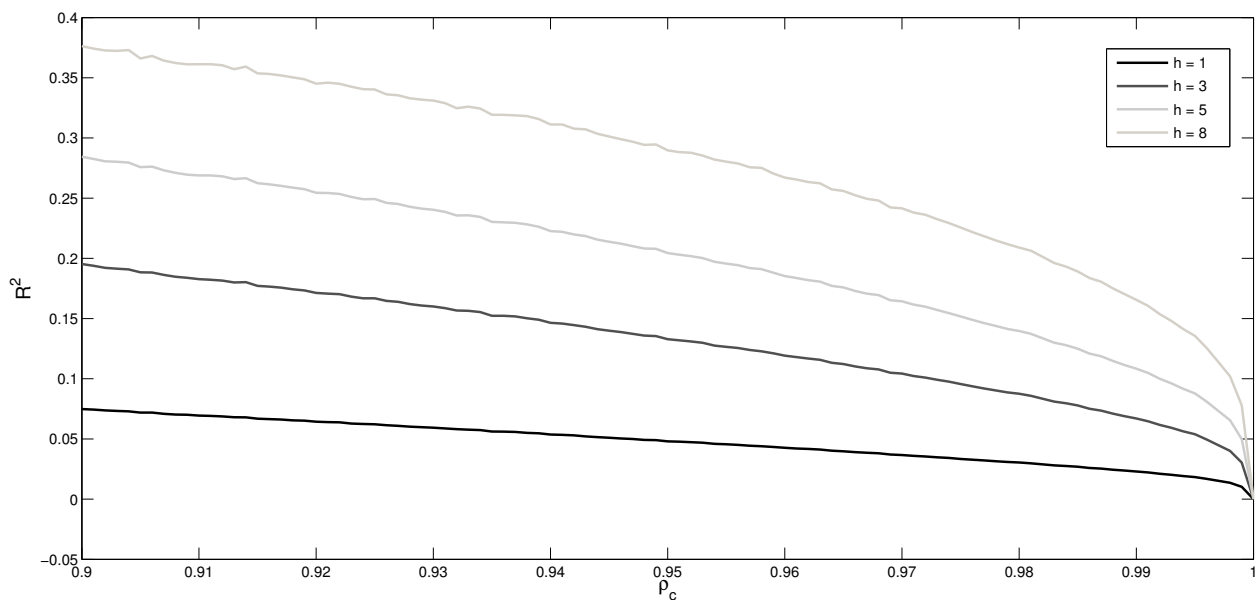
statistics that are increasing in the horizon of the prediction. The  $R^2$  statistics are somewhat too large for the long sample and the garbage data series and slightly too low for the post-war data.

To illustrate how non-permanent shocks help generate return predictability, we illustrate in Figure 8 the influence of the autocorrelation coefficient in consumption,  $\rho_c$ , on the predictability of stock returns (as measured by  $R^2$ ). Results are shown for the long data set. When changing  $\rho_c$  we adjust  $\sigma_\epsilon$  to fix  $\sigma(\Delta c_t)$  at its empirical estimate. At  $\rho_c = 1$ , the random walk case, return predictability disappears, while it grows quickly as  $\rho_c$  decreases below 1. Corresponding graphs for the post-war and the garbage data set are shown in Appendix C.

## 5 Conclusion

The macroeconomic data on the U.S. economy is consistent with non-permanent shocks that dissipate over time. In this paper we document the significance of non-permanent shocks to consumption for consumption-based asset pricing. For a simple theoretical model where some consumption shocks are permanent and other shocks are not, the non-permanent shock dominates the dynamics of asset prices. We also numerically calibrate a standard CRRA consumption model with non-permanent shocks using U.S. data, and find that we can match the equity premium and risk-free rate with moderate levels of risk aversion. This same model also generates many of the features of stock prices that have been considered puzzles. Asset prices in the model are very volatile, consistent with the excess volatility puzzle (they are actually somewhat more volatile than in the actual data). Consistent with the variance decomposition of Campbell and Shiller, changes in the model's price-dividend ratio are largely driven by changes in model expected returns, which are large, rather than changes in expected dividends, which are small. High price-dividend ratios today predict low expected returns in the future, consistent with the return predictability literature.

Figure 8: Influence of the Autocorrelation Coefficient  $\rho_c$  on the Predictability of Stock Returns for the Long data set



The graph shows the predictability of stock returns  $R^2$  for different forecasting horizons  $h$  as a function of the autocorrelation coefficient in consumption,  $\rho_c$ . The underlying consumption process is from the long sample—see Table 1—so  $E(\Delta c_t) = 0.020$  and  $\sigma(\Delta c_t) = 0.0352$ . The results are computed using the parameter estimates from Table 7 ( $\gamma = 7.7$ ,  $\beta = 1.1$ ). ( $\sigma_\nu = 0$ ).

From time-series evidence alone, it is difficult to distinguish between shocks to consumption that are permanent from those that are very persistent, or even a mix of non-permanent and permanent shocks. Our findings show that the presence of non-permanent shocks have a dramatic impact on asset pricing dynamics, making many of the empirical aggregate stock market puzzles less puzzling. Permanent shocks have a much smaller impact. Even in a model with both non-permanent and permanent shocks, the non-permanent shocks generate most of the dynamics. Together, these results suggest that non-permanent shocks to consumption are an important, yet largely overlooked, mechanism to explain puzzles in asset pricing.

## Appendix

### A Analytical Results

We provide proofs for the theoretical results stated in the paper.

#### A.1 The Unconditional Moments of Consumption Growth

We derive the unconditional moments of consumption growth (3)–(5). Log consumption growth is given by

$$\Delta c_t = \bar{g} + (\rho_c - 1)(x_{t-1}) + \sigma_\epsilon \epsilon_t + \sigma_\nu \nu_t, \quad (25)$$

with  $E(x_t) = 0$  and  $\text{Var}(x_t) = \sigma_\epsilon^2 / (1 - \rho_c^2)$ . Thus,

$$E(\Delta c_t) = \bar{g} \quad \text{and} \quad \sigma(\Delta c_t) = \sqrt{\sigma_\nu^2 + \frac{2}{1 + \rho_c} \sigma_\epsilon^2}.$$

The auto-covariance of log consumption growth  $\text{cov}(\Delta c_t, \Delta c_{t-1})$  is given by

$$\begin{aligned} \text{cov}(\Delta c_t, \Delta c_{t-1}) &= E(\Delta c_t \Delta c_{t-1}) - E(\Delta c_t)^2 \\ &= \bar{g}^2 + (\rho_c - 1)^2 E(x_t x_{t-1}) + (\rho_c - 1) E(x_t \sigma_\epsilon \epsilon_t) - \bar{g}^2 \\ &= \frac{\rho_c - 1}{\rho_c + 1} \sigma_\epsilon. \end{aligned} \quad (26)$$

The resulting first-order autocorrelation is  $\text{AC1}(\Delta c_t) = \frac{\text{cov}(\Delta c_t, \Delta c_{t-1})}{\sigma(\Delta c_t)^2}$ .

#### A.2 Equilibria in Growth Economies with Discount Factors $\beta > 1$

We derive Condition (13) under which the agent's expected utility,

$$\frac{1}{1 - \gamma} E_t \left[ \sum_{k=0}^{\infty} \beta^k C_{t+k}^{1-\gamma} \right],$$

remains finite for any starting values  $g_t, x_t$  of the consumption process (2). The two processes  $g_t$  and  $x_t$  are independent and thus

$$E_t \left( C_{t+k}^{1-\gamma} \right) = E_t \left( e^{(1-\gamma)(g_{t+k}+x_{t+k})} \right) = E_t \left( e^{(1-\gamma)g_{t+k}} \right) E_t \left( e^{(1-\gamma)x_{t+k}} \right).$$

Trivially,  $E_t \left( C_{t+k}^{1-\gamma} \right)$  is positive for all  $k = 0, 1, 2, \dots$ . However, we can find a much tighter lower bound. Note that  $E_t(g_{t+k}) = g_t + \bar{g}k$  and so due to the convexity of the exponential function and Jensen's inequality,

$$E_t \left( e^{(1-\gamma)g_{t+k}} \right) \geq e^{(1-\gamma)(g_t + \bar{g}k)}.$$

Similarly, note that  $E_t(x_{t+k}) = \rho_c^k x_t$  and so again by Jensen's inequality,

$$E_t \left( e^{(1-\gamma)x_{t+k}} \right) \geq e^{(1-\gamma)\rho_c^k x_t}.$$

And so we obtain the lower bound

$$E_t \left( C_{t+k}^{1-\gamma} \right) \geq e^{(1-\gamma)(g_t + \rho_c^k x_t)} e^{(1-\gamma)(\bar{g}k)}.$$

Next, for  $1 - \gamma < 0$  we obtain the inequality

$$\frac{1}{1-\gamma} \sum_{k=0}^{\infty} E_t \left[ \beta^k C_{t+k}^{1-\gamma} \right] \leq \frac{1}{1-\gamma} \sum_{k=0}^{\infty} \left( e^{(1-\gamma)(g_t + \rho_c^k x_t)} \right) \left( \beta e^{(1-\gamma)\bar{g}} \right)^k.$$

The first exponential term in the product on the right-hand side has a uniform upper bound for all  $k = 0, 1, 2, \dots$ , since  $\rho_c \in [0, 1]$ . Therefore, the right-hand side is absolutely convergent as long as

$$\beta e^{(1-\gamma)\bar{g}} < 1.$$

Under this condition, the series on the left-hand side of the inequality is absolutely convergent as well. Therefore, Fubini's Theorem implies that

$$\frac{1}{1-\gamma} E_t \left[ \sum_{k=0}^{\infty} \beta^k C_{t+k}^{1-\gamma} \right] = \frac{1}{1-\gamma} \sum_{k=0}^{\infty} E_t \left[ \beta^k C_{t+k}^{1-\gamma} \right].$$

This completes the proof that an equilibrium exists if  $\gamma > 1$  and  $\beta e^{(1-\gamma)\bar{g}} < 1$ .

### A.3 Proof of Theorem 1

We prove Theorem 1 by deriving the expressions (15)–(18). Recall the consumption process (14),

$$\begin{aligned} c_t &= g_t + \sigma_\epsilon \epsilon_t \\ g_t &= \bar{g} + g_{t-1} + \sigma_\nu \nu_t \\ \epsilon_t, \nu_t &\sim N(0, 1) \text{ i.i.d.} \end{aligned}$$



with the mean growth rate  $\bar{g}$  of consumption. The time  $t$  conditional expectation of  $C_{t+1}^a = e^{act}$  is then given by

$$E_t(C_{t+1}^a) = e^{a(\bar{g}+g_t)+\frac{1}{2}a^2\sigma_\epsilon^2+\frac{1}{2}a^2\sigma_\nu^2}.$$

### A.3.1 Return and Volatility of the Risk Free Rate

Equation (9) determines the price of risk-free one-period bond and implies

$$\begin{aligned} P_t^f &= \beta E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \right\} \\ &= \beta e^{-\gamma\bar{g}+\gamma\sigma_\epsilon\epsilon_t+\frac{1}{2}\gamma^2\sigma_\epsilon^2+\frac{1}{2}\gamma^2\sigma_\nu^2}. \end{aligned}$$

The resulting expected one-period return is then

$$E(R_t^f) = \frac{1}{\beta} e^{\gamma\bar{g}-\frac{1}{2}\gamma^2\sigma_\nu^2}.$$

For the volatility of the risk-free rate we first derive

$$E((R_t^f)^2) = \frac{1}{\beta^2} e^{2\gamma\bar{g}-\gamma^2\sigma_\nu^2+\gamma^2\sigma_\epsilon^2}$$

to obtain

$$\sigma(R_t^f) = \frac{1}{\beta} e^{\gamma\bar{g}-\frac{1}{2}\gamma^2\sigma_\nu^2} \left( e^{\gamma^2\sigma_\epsilon^2} - 1 \right)^{\frac{1}{2}}.$$

### A.3.2 Return of the Infinitely-Lived Asset

Rewriting Equation (10) we obtain

$$\frac{P_t}{C_t^\gamma} = \beta E_t \left\{ \frac{P_{t+1}}{C_{t+1}^\gamma} + C_{t+1}^{1-\gamma} \right\},$$

which, after dividing both sides by  $e^{(1-\gamma)g_t}$ , yields

$$\frac{P_t}{C_t^\gamma e^{(1-\gamma)g_t}} = \beta E_t \left\{ \frac{P_{t+1}}{C_{t+1}^\gamma e^{(1-\gamma)g_{t+1}}} e^{(1-\gamma)(\bar{g}+\sigma_\nu\nu_{t+1})} + \frac{C_{t+1}^{1-\gamma}}{e^{(1-\gamma)g_t}} \right\}.$$

Now we employ a guess-and-verify approach. Define  $X \doteq \frac{P_t}{C_t^\gamma e^{(1-\gamma)g_t}}$  and assume that  $X$  is constant. Then

$$X = \beta E_t \left\{ X e^{(1-\gamma)(\bar{g}+\sigma_\nu\nu_{t+1})} + \frac{C_{t+1}^{1-\gamma}}{e^{(1-\gamma)g_t}} \right\}$$

which implies

$$X = \frac{\beta e^{(1-\gamma)\bar{g}+\frac{1}{2}(1-\gamma)^2(\sigma_\epsilon^2+\sigma_\nu^2)}}{1 - \beta e^{(1-\gamma)\bar{g}+\frac{1}{2}(1-\gamma)^2\sigma_\nu^2}}.$$

Note that the right-hand side is indeed independent of time and thus a constant. Using the definition of  $X$ , we can express the one-period return of the infinitely-lived asset as

$$\begin{aligned} R_{t+1} &= \frac{P_{t+1} + D_{t+1}}{P_t} - 1 \\ &= \frac{C_{t+1}^\gamma}{C_t^\gamma} e^{(1-\gamma)(\bar{g} + \sigma_\nu \nu_{t+1})} + \frac{C_{t+1}}{XC_t^\gamma} e^{-(1-\gamma)g_t}. \end{aligned}$$

Observing that

$$\frac{C_{t+1}^\gamma}{C_t^\gamma} = e^{\gamma(\bar{g} + \sigma_\nu \nu_{t+1} + \sigma_\epsilon \epsilon_{t+1} - \sigma_\epsilon \epsilon_t)}$$

and taking conditional expectations yields

$$E_t(R_{t+1}) = e^{\bar{g} + \frac{1}{2}\sigma_\nu^2 + \frac{1}{2}\gamma^2\sigma_\epsilon^2 - \gamma\sigma_\epsilon\epsilon_t} + \frac{e^{\bar{g} + \frac{1}{2}\sigma_\nu^2 + \frac{1}{2}\sigma_\epsilon^2 - \gamma\sigma_\epsilon\epsilon_t}}{X}$$

which in turn results in an unconditional expected return of

$$E(R_{t+1}) = e^{\bar{g} + \frac{1}{2}\sigma_\nu^2 + \gamma^2\sigma_\epsilon^2} + \frac{1 - \beta e^{(1-\gamma)\bar{g} + \frac{1}{2}(1-\gamma)^2\sigma_\nu^2}}{\beta} e^{\gamma\bar{g} + \gamma\sigma_\nu^2 - \frac{1}{2}\gamma^2\sigma_\nu^2 + \gamma\sigma_\epsilon^2}.$$

Finally, Equation (18) follows from the second moment of the return,

$$E(R_{t+1}^2) = e^{4\gamma^2\sigma_\epsilon^2 + 2\bar{g} + 2\sigma_\nu^2} + \frac{1}{X^2} e^{2\gamma^2\sigma_\epsilon^2 + 2\sigma_\epsilon^2 + 2\bar{g} + 2\sigma_\nu^2} + \frac{2}{X} e^{2.5\gamma^2\sigma_\epsilon^2 + \gamma\sigma_\epsilon^2 + 0.5\sigma_\epsilon^2 + 2\bar{g} + 2\sigma_\nu^2},$$

and the standard deviation  $\sigma(R_t) = (E(R_t^2) - E(R_t)^2)^{\frac{1}{2}}$  of the aggregate consumption claim. This completes the derivations of the statements in Theorem 1.

## B Numerical Solution Method

We briefly describe the numerical solution approach for the baseline version of the economic model. The solution method relies on quadrature and projection techniques; see Judd (1992) and Judd (1998).

Equations (6) and (11) determine the risk-free rate in the baseline model. Similarly, Equations (7) and (12) determine the return of the long-lived risky asset. Both the price  $P_t^f$  of the riskless one-period asset and the price-consumption ratio  $P_t/C_t$  of the infinitely-lived risky asset depend on the detrended consumption  $x_t$  which serves as the endogenous state variable for the model. The state space is the interval given by  $\pm 6$  standard deviations around the unconditional mean of detrended consumption. On this state space we approximate the functions  $P_t^f$  and  $P_t/C_t$  by Chebyshev polynomials of degree 18 and use the Galerkin projection method to find the best approximation. (We checked approximations up to degree 32 on a state space as wide as  $\pm 20$  standard deviations around the steady state and obtained effectively identical solutions.) We solve the integrals arising due to the Galerkin projection by Gauss-Chebyshev quadrature.<sup>9</sup>

<sup>9</sup>We also solved the model by using the discretization methods of Tauchen (1986) or Tauchen and Hussey (1991). We obtain almost identical results but find that these methods require many more nodes than the Galerkin method to deliver a good approximation of the solution.

The conditional expectations operator  $E_t$  in Equations (11) and (12) is a two-dimensional integral over  $x_{t+1}$  and  $g_{t+1}$ —that is, an integral over  $\epsilon_{t+1}$  and  $\nu_{t+1}$ . We approximate this double integral by two-dimensional Gauss-Hermite quadrature. As a result we obtain two linear systems of equations in the coefficients of the Chebyshev polynomials which are straightforward to solve. The respective solutions yield approximations of the functions  $P_t^f$  and  $P_t/C_t$  on the state space.

Finally we can compute returns. For the average risk-free rate and its standard deviation, we integrate  $1/P_t^f$  using Gauss-Hermite quadrature on the unconditional distribution of the state variable. Equation (7) provides the conditional return of the aggregate consumption claim given the state  $x_t$ . We first compute conditional expected returns by a two-dimensional Gauss-Hermite quadrature, again over  $x_{t+1}$  and  $g_{t+1}$  on the Gauss-Hermite nodes of the unconditional distribution of  $x_t$ . Once the conditional returns have been computed, we can calculate unconditional returns by one-dimensional Gauss-Hermite quadrature. (Alternatively, we could simulate the economy for hundreds of thousands of periods and report simulated returns. That approach yields the same results but requires much longer running times.)

For the return predictability, we simulate the underlying processes over 1,000,000 periods and calculate the corresponding prices and returns.

## C Additional Tables and Graphs

In Section 4.4.1 we assume that the state variables  $x_t$  and  $\phi_t$  follow two correlated AR(1) processes. Alternatively, we also tried a VAR(1,1) specification for the two state variables. For such a specification, Table 14 reports the summary measures of asset prices for the dividend claim model.

Table 14: Summary Measures of the Second Model Compared to the Data – VAR(1,1) Specification

	Long Sample		Post-War		Garbage	
	Data	Model	Data	Model	Data	Model
$\gamma$		7.00		17.90		8.70
$\beta$		1.10		1.37		1.08
$E(R_t)$	7.60	7.60	7.92	7.92	7.09	7.09
$\sigma(R_t)$	18.73	26.72	16.60	22.31	15.18	23.57
$E(R_t^f)$	1.97	1.97	1.84	1.84	2.21	2.21
$\sigma(R_t^f)$	5.80	6.82	2.65	7.41	2.60	8.88
EP	5.63	5.63	6.08	6.08	4.87	4.87
SR	0.30	0.21	0.37	0.27	0.32	0.21
$E(p_t - d_t)$	3.22	3.87	3.42	3.47	3.51	3.53
$\sigma(p_t - d_t)$	0.40	0.55	0.44	0.50	0.40	0.36
AC1( $p_t - d_t$ )	0.92	0.93	0.94	0.94	0.93	0.83

The table compares implied model moments of the pricing of the dividend claim with the empirical moments found in the data. The table shows the same summary statistics as table 12, but instead of two correlated AR(1) processes we assume an VAR(1,1) process for detrended consumption  $x_t$  and the non dividend to dividend income ratio  $\phi_t$ . Again, the model parameters  $\gamma$  and  $\beta$  are calibrated to match the risk-free rate and the equity premium.

Figures 4–8 show the asset pricing effects of parameter changes for the long sample. For completion, we include the corresponding figures for the post-war data and the garbage series.

Figure 4 in Section 4.1 shows the Hansen-Jagannathan bounds for the long sample. Figures 9 and 10 are the corresponding figures for the post-war and the garbage data, respectively.

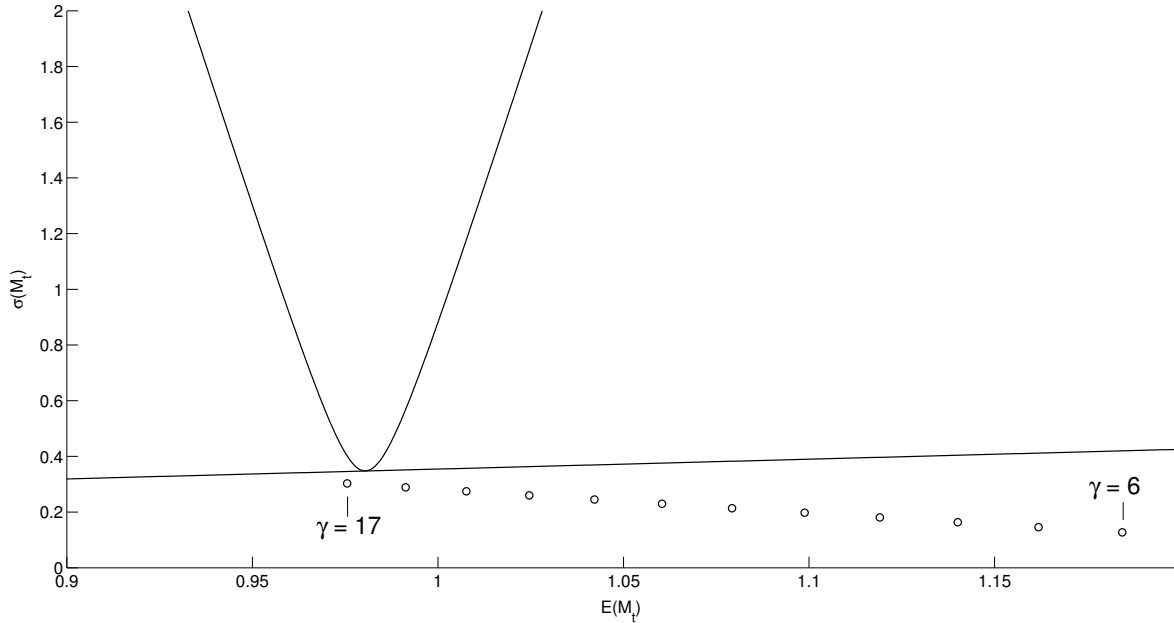
Figure 5 in Section 4.3.1 shows the effects of adding a permanent shock to the trend-stationary model for the long sample. Figures 11 and 12 are the corresponding figures for the post-war and the garbage data, respectively.

Figure 6 in Section 4.3.2 shows the effects of changes in the autocorrelation coefficient  $\rho_c$  for the long sample. Figures 13 and 14 are the corresponding figures for the post-war and the garbage data, respectively.

Figure 7 in Section 4.3.3 shows the effects of changes in the consumption growth volatility  $\sigma(\Delta c_t)$  for the long sample. Figures 15 and 16 are the corresponding figures for the post-war and the garbage data, respectively.

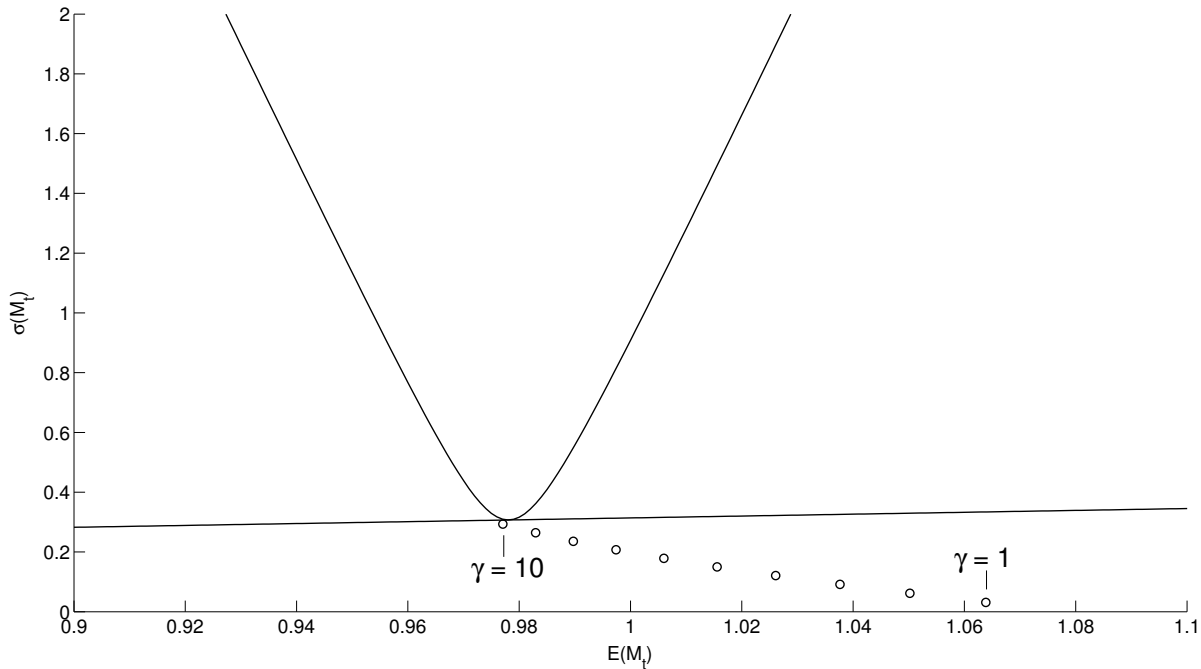
Figure 8 in Section 4.5 shows the influence of the autocorrelation parameter  $\rho_c$  on the predictability of returns for the long sample. Figures 17 and 18 are the corresponding figures for the post-war and the garbage data, respectively.

Figure 9: Hansen-Jagannathan Bounds for Post-War Data



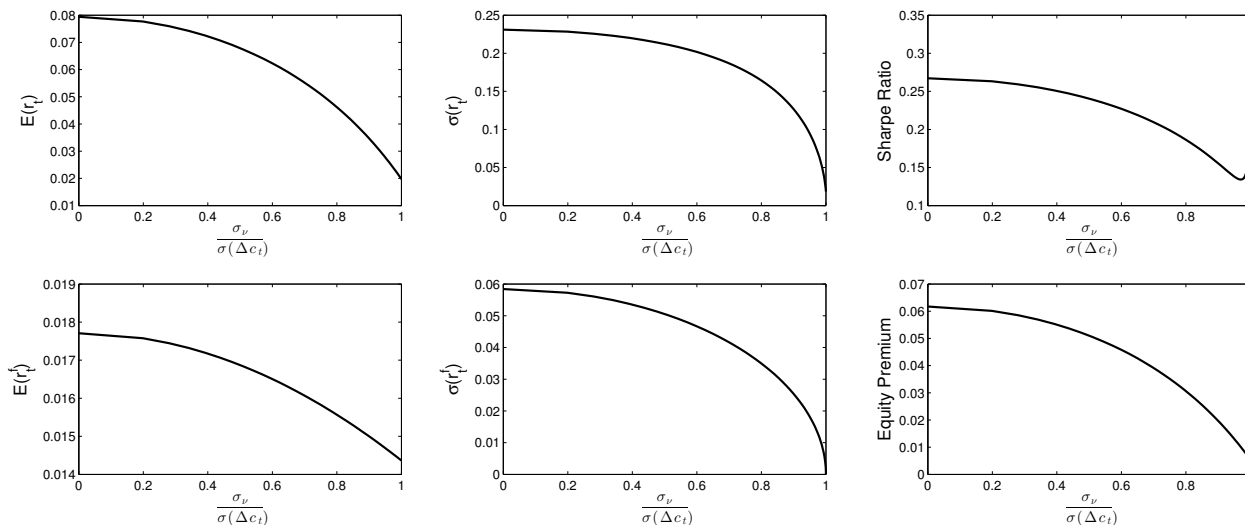
The graph shows the mean and standard deviation of the pricing kernel  $M_t$  implied by the trend-stationary model as well as the Hansen-Jagannathan bounds for different degrees of risk aversion  $\gamma$ . Each circle represents an increase of  $\gamma$  of one, starting in the right lower corner. Parameter estimates for consumption are taken from the post-war sample—see Table 1—with the calibration from table 7, so  $E(\Delta c_t) = 0.0213$  and  $\sigma(\Delta c_t) = 0.0180$ ,  $\rho_c = 0.9259$ , and  $\beta = 1.34$ .

Figure 10: Hansen-Jagannathan Bounds for Garbage Data



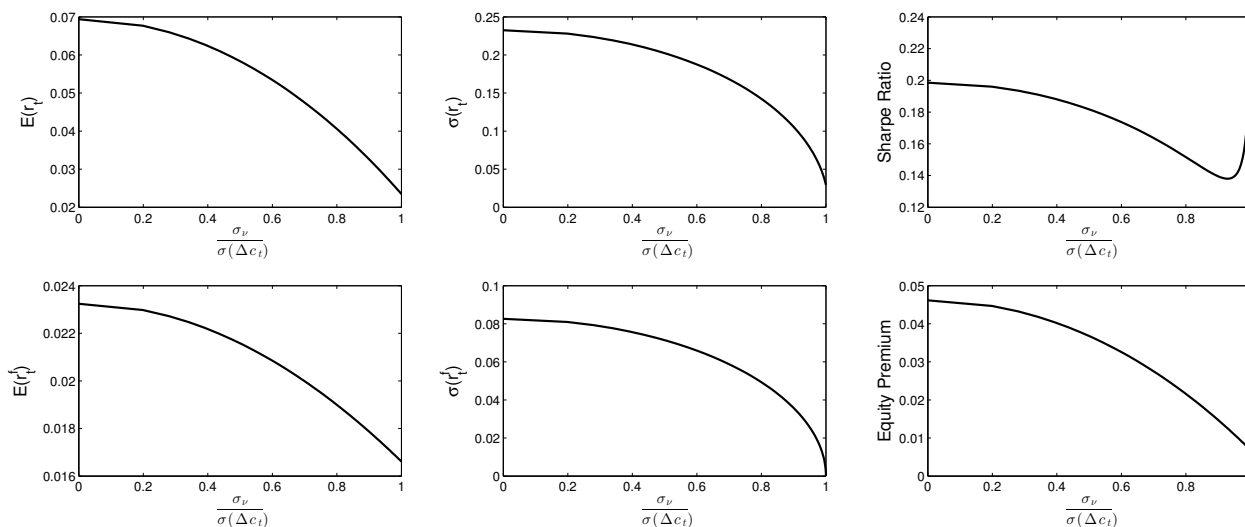
The graph shows the mean and standard deviation of the pricing kernel  $M_t$  implied by the trend-stationary model as well as the Hansen-Jagannathan bounds for different degrees of risk aversion  $\gamma$ . Each circle represents an increase of  $\gamma$  of one, starting in the right lower corner. Parameter estimates for consumption are taken from the garbage sample—see Table 1—with the calibration from table 7, so  $E(\Delta c_t) = 0.0142$  and  $\sigma(\Delta c_t) = 0.0286$ ,  $\rho_c = 0.7661$ , and  $\beta = 1.08$ .

Figure 11: Asset Pricing Effects of Adding Permanent Shocks  $\sigma_\nu$  to the Trend-stationary Model for the Post-War Data Set



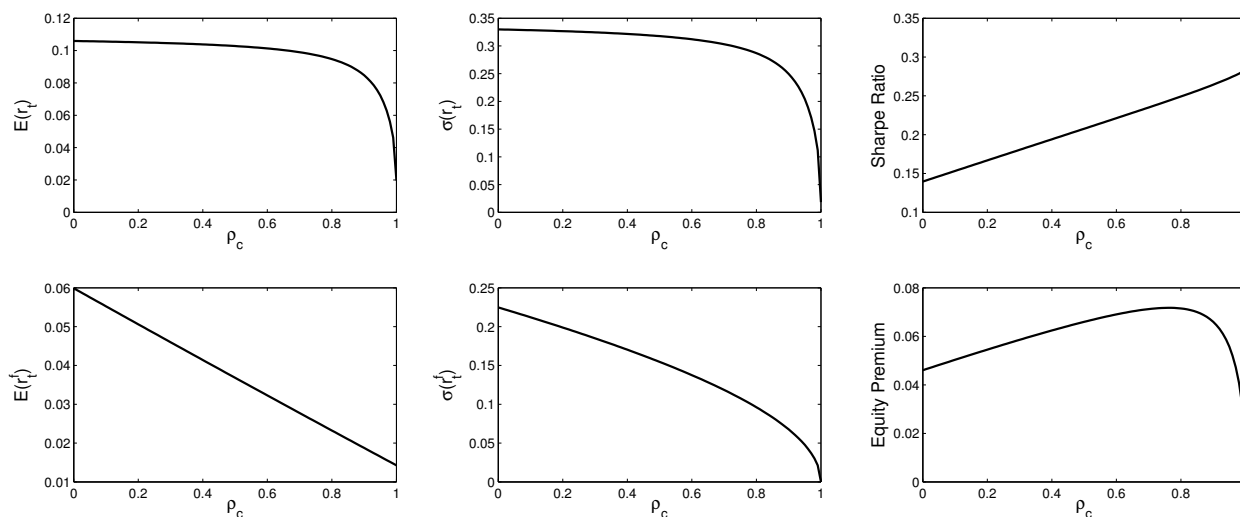
The graphs show the expected return of the aggregate consumption claim, its volatility, the Sharpe ratio, the risk-free rate, its volatility, and the equity premium as a function of the share of the permanent shock  $\sigma_\nu$  in the total volatility of consumption growth  $\sigma(\Delta c_t)$  (All other volatility comes from the non-permanent shock  $\sigma_\epsilon$ ). Parameter estimates for consumption are taken from the post-war sample—see Table 1—with the calibration from table 7, so  $E(\Delta c_t) = 0.0213$  and  $\sigma(\Delta c_t) = 0.018$ ,  $\rho_c = 0.9259$ ,  $\gamma = 16.5$ , and  $\beta = 1.34$ .

Figure 12: Asset Pricing Effects of Adding Permanent Shocks  $\sigma_\nu$  to the Trend-stationary Model for the Garbage Data Set



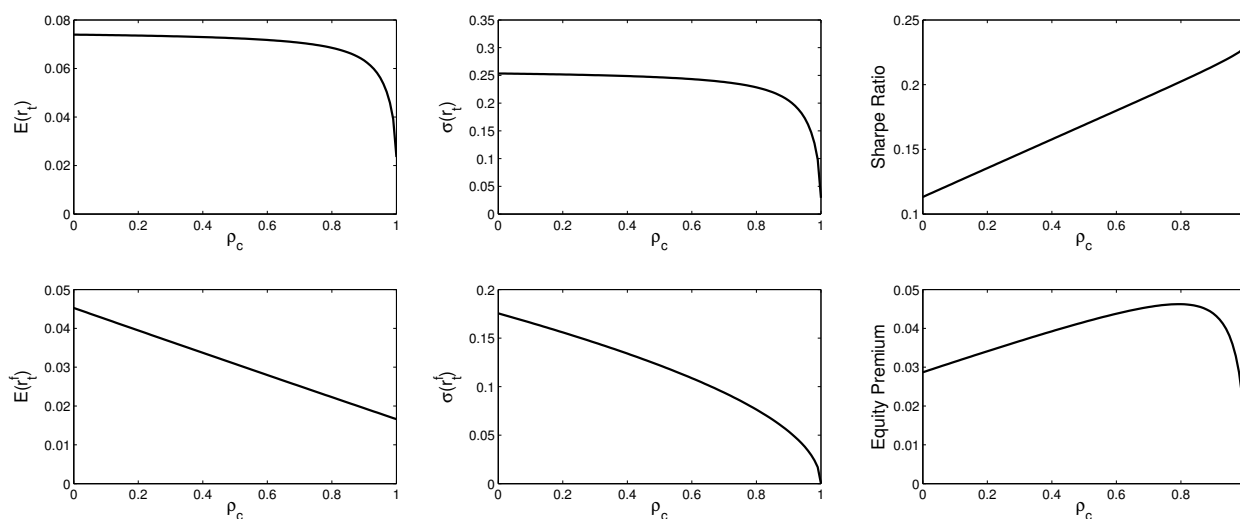
The graphs show the expected return of the aggregate consumption claim, its volatility, the Sharpe ratio, the risk-free rate, its volatility, and the equity premium as a function of the share of the permanent shock  $\sigma_\nu$  in the total volatility of consumption growth  $\sigma(\Delta c_t)$  (All other volatility comes from the non-permanent shock  $\sigma_\epsilon$ ). Parameter estimates for consumption are taken from the garbage sample—see Table 1—with the calibration from table 7, so  $E(\Delta c_t) = 0.0142$  and  $\sigma(\Delta c_t) = 0.0286$ ,  $\rho_c = 0.7661$ ,  $\gamma = 8.24$ , and  $\beta = 1.08$ .

Figure 13: Asset Pricing Effects of the Autocorrelation Coefficient  $\rho_c$  in Consumption for the Post-War data set



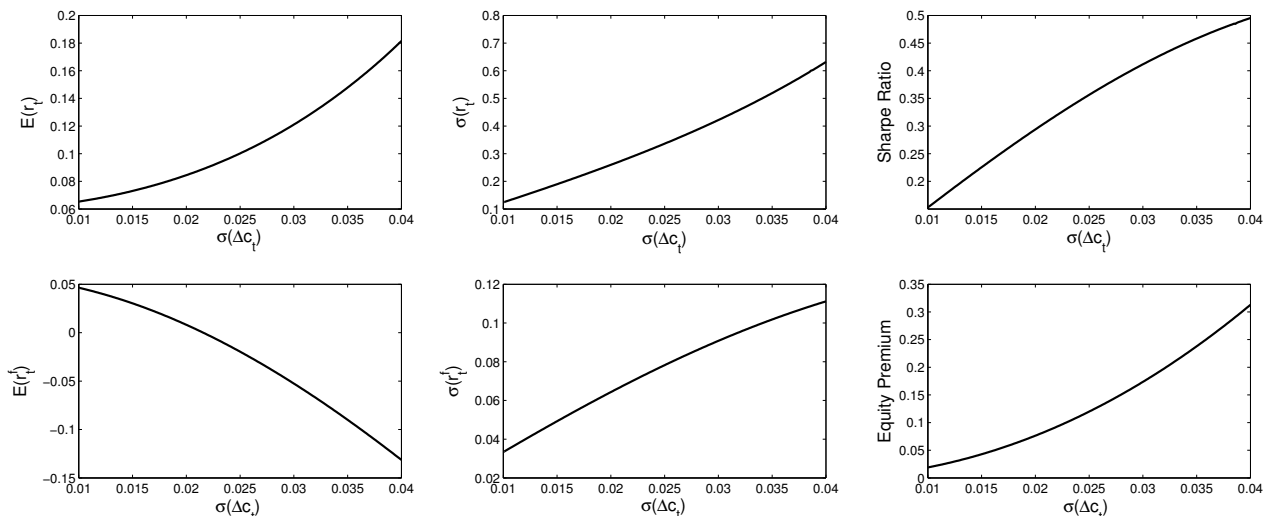
The graphs show the expected return of the aggregate consumption claim, its volatility, the Sharpe ratio, the risk-free rate, its volatility, and the equity premium as a function of the autocorrelation coefficient in consumption,  $\rho_c$ . The underlying consumption process is from the post-war sample—see Table 1—so  $E(\Delta c_t) = 0.0213$  and  $\sigma(\Delta c_t) = 0.0180$ . The results are computed using the parameter estimates from table 7 ( $\gamma = 16.5$ ,  $\beta = 1.34$ ). ( $\sigma_\nu = 0$ ).

Figure 14: Asset Pricing Effects of the Autocorrelation Coefficient  $\rho_c$  in Consumption for the Garbage data set



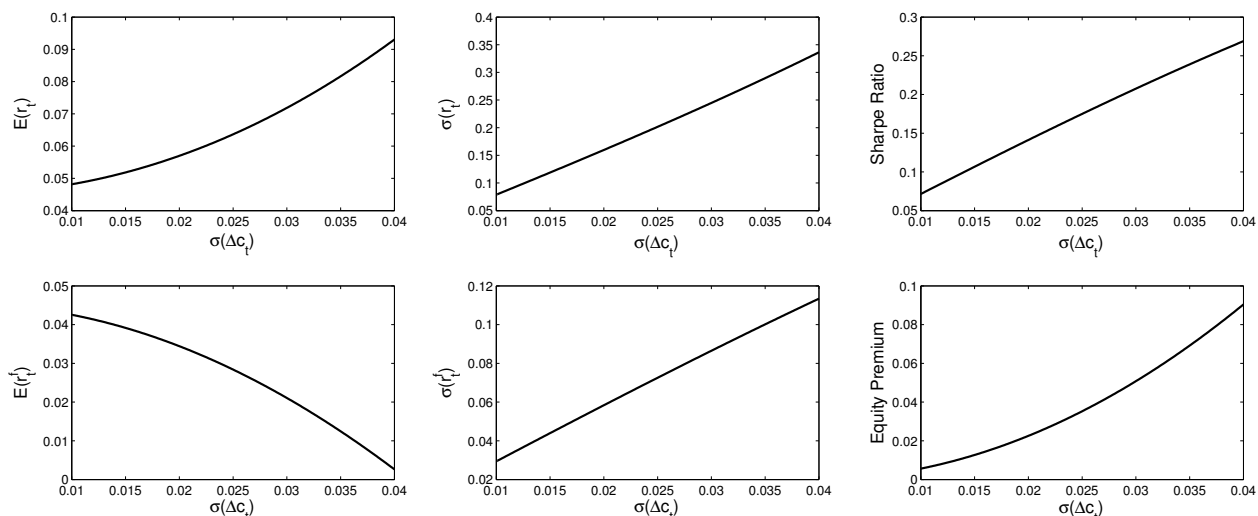
The graphs show the expected return of the aggregate consumption claim, its volatility, the Sharpe ratio, the risk-free rate, its volatility, and the equity premium as a function of the autocorrelation coefficient in consumption,  $\rho_c$ . The underlying consumption process is from the garbage data sample—see Table 1—so  $E(\Delta c_t) = 0.0142$  and  $\sigma(\Delta c_t) = 0.0286$ . The results are computed using the parameter estimates from table 7 ( $\gamma = 8.24$ ,  $\beta = 1.08$ ). ( $\sigma_\nu = 0$ ).

Figure 15: Summary Measures for the TS Model as a Function of the Consumption Growth Volatility for the Post-War Sample



The graph shows the expected return of the aggregate consumption claim, its volatility, the Sharpe ratio, the expected risk free rate, its volatility, and the equity premium as a function of the standard deviation of consumption growth  $\sigma(\Delta c_t)$ . The underlying consumption process is from the post-war sample—see Table 1—so  $E(\Delta c_t) = 0.0213$  and  $\rho_c = 0.9259$ . The results are computed using the parameter estimates from table 7 ( $\gamma = 16.5$ ,  $\beta = 1.34$ ). ( $\sigma_\nu = 0$ ).

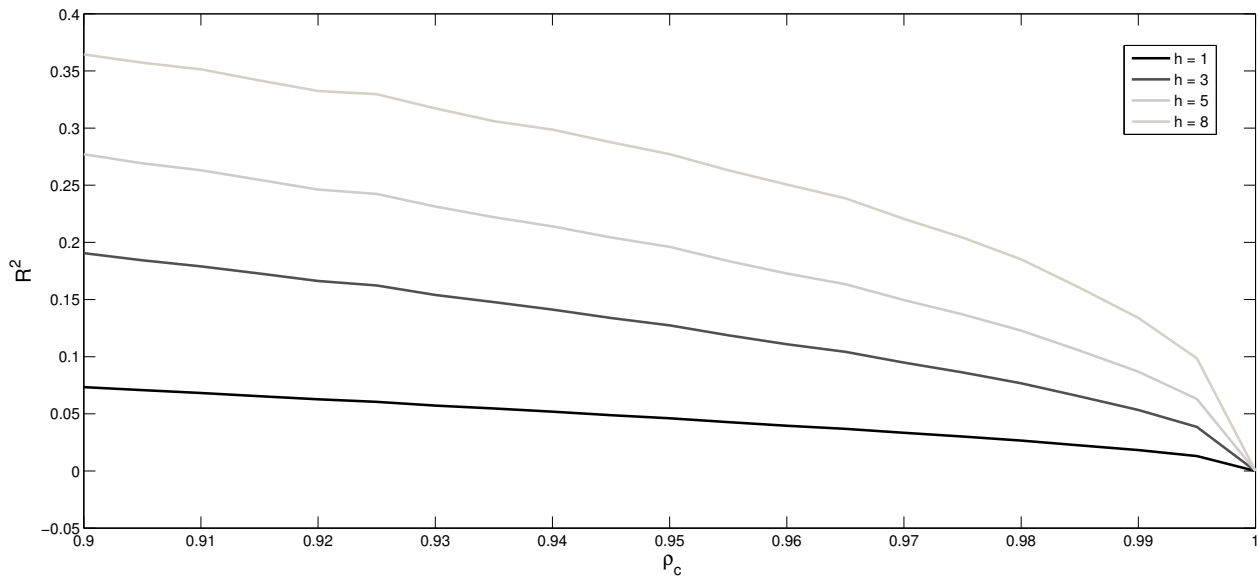
Figure 16: Summary Measures for the TS Model as a Function of the Consumption Growth Volatility for the Garbage Sample



The graph shows the expected return of the aggregate consumption claim, its volatility, the Sharpe ratio, the expected risk free rate, its volatility, and the equity premium as a function of the standard deviation of consumption growth  $\sigma(\Delta c_t)$ . The underlying consumption process is from the garbage sample—see Table 1—so  $E(\Delta c_t) = 0.0142$  and  $\rho_c = 0.7661$ . The results are computed using the parameter estimates from table 7 ( $\gamma = 8.24$ ,  $\beta = 1.08$ ). ( $\sigma_\nu = 0$ ).

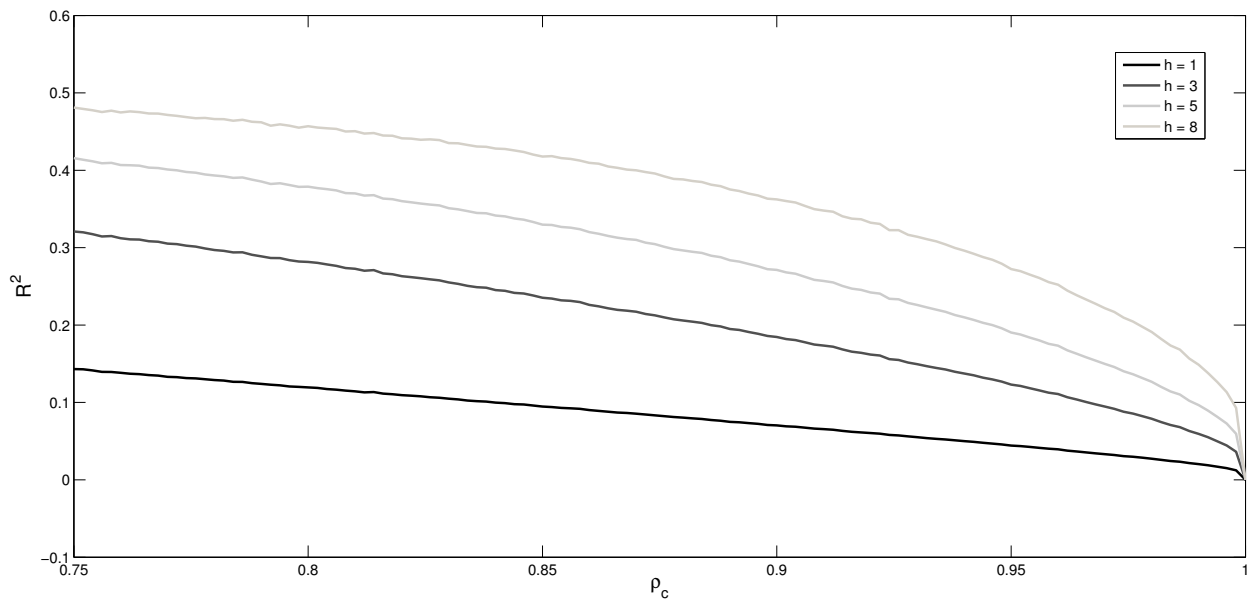


Figure 17: Influence of the Autocorrelation Coefficient  $\rho_c$  on the Predictability of Stock Returns for the Post-War data set



The graph shows the predictability of stock returns  $R^2$  for different forecasting horizons  $h$  as a function of the autocorrelation coefficient in consumption,  $\rho_c$ . The underlying consumption process is from the post-war sample—see Table 1—so  $E(\Delta c_t) = 0.0213$  and  $\sigma(\Delta c_t) = 0.0180$ . The results are computed using the parameter estimates from Table 7 ( $\gamma = 16.5$ ,  $\beta = 1.34$ ). ( $\sigma_\nu = 0$ ).

Figure 18: Influence of the Autocorrelation Coefficient  $\rho_c$  on the Predictability of Stock Returns for the Garbage data set



The graph shows the predictability of stock returns  $R^2$  for different forecasting horizons  $h$  as a function of the autocorrelation coefficient in consumption,  $\rho_c$ . The underlying consumption process is from the garbage sample—see Table 1—so  $E(\Delta c_t) = 0.0142$  and  $\sigma(\Delta c_t) = 0.0286$ . The results are computed using the parameter estimates from Table 7 ( $\gamma = 8.24$ ,  $\beta = 1.08$ ). ( $\sigma_\nu = 0$ ).

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