Year: 2017

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DOI: https://doi.org/10.1142/S0218271817410140

Originally published at:
Jetzer, Philippe (2017). General relativity tests with space clocks in highly elliptic orbits. International Journal of Modern Physics D, 26(05):1741014.
DOI: https://doi.org/10.1142/S0218271817410140

International Journal of Modern Physics D
Vol. 26, No. 5 (2017) 1741014 (9 pages)

DOI: 10.1142/S0218271817410140

General relativity tests with space clocks in highly elliptic orbits

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Received 7 November 2016
Accepted 21 December 2016
Published 20 March 2017

The test of the Einstein Equivalence Principle (EEP) is of crucial importance as a deviation from it could hint to quantum effects in gravity or to unification with the other fundamental forces. One aspect of EEP is the local position invariance (LPI), which can be tested by measuring the gravitational red-shift. As an example of a possible space mission which could test the EEP , we will discuss a recently proposed satellite experiment, Einstein Gravitational RedShift Probe (E-GRIP), with the aim to test LPI using an hydrogen maser atomic clock on a highly elliptic orbit around Earth and compare the on-board clock to clocks located on Earth via a microwave link.

Keywords: Relativity and gravitation; experimental tests of gravitational theories.

PACS Number: 04.80.Cc

1. Introduction

Einstein's theory of general relativity (GR) is a cornerstone of our current description of the physical world. It is used to understand the flow of time in the presence of gravity, the motion of bodies from satellites to galaxy clusters, the propagation of electromagnetic waves in the presence of massive bodies, the evolution of stars, and the dynamics of the universe as a whole. Although very successful so far, GR as well as numerous other alternative or more general theories of gravitation are classical theories.[1] It is believed that GR is the classical limit of a quantum theory of gravity that has not yet been discovered just like electromagnetism is the classical limit of quantum electrodynamics. As such, many theories propose concepts that ultimately aim to bridge the gap between GR and quantum mechanics. They all lead to tiny, yet-to-be-detected violations of the fundamental principles of GR that should guide us towards building a theory that unifies all fundamental forces of

*On behalf of the E-GRIP team.

Nature. A full understanding of gravitation will require experiments that determine the relationship of gravity to the quantum world. These experiments have repercussions over a wide array of physical phenomena ranging from particle physics to nuclear physics to our understanding of galaxies and of the universe as a whole.

Multiple tests of GR have, until recently, been possible only with 'natural laboratories' like the binary pulsar systems. Binary pulsars are effectively clocks in nearly Keplerian orbits for which the main observable is the arrival times of tick signals. Improved spacecraft technology now enables similar relativity tests, but with artificial satellites.[2,3] The most precise gravitational time dilation measurement in Earth's gravity field dates back to 1976, when Gravity Probe A (GP-A)[4] performed gravitational redshift measurements to an accuracy of $\sim 10^{-4}$. ESA's ACES mission, which shall be brought to the International Space Station in 2018, aims to improve this measurement by approximately two orders of magnitude.

The proposed Einstein Gravitational RedShift Probe (E-GRIP) satellite will, as a main payload, carry a hydrogen maser atomic clock on an elliptic orbit around Earth and compare the on-board clock to clocks located on Earth via a microwave link (MWL). E-GRIP primary goal is to test the Earth field redshift with an accuracy of 2×10^{-6}, taking also advantage of the frequency modulation along the highly elliptic orbit. A further goal is to measure the time dilation due to Sun's and Moon's gravitational fields with an accuracy of 5×10^{-5} and 1×10^{-2}, respectively, thereby testing its independence on the nature of the mass producing the gravitational field. The latter measurements are performed by comparing ultra-precise ground clocks, linked via the MWL on-board of E-GRIP. The time dilation measurements are performed by comparing ultra-precise ground clocks as they rotate with the Earth with the E-GRIP satellite hereby used as a relay station.

Additionally, E-GRIP provides a wealth of science for time and frequency metrology and geodesy. The realization of highly accurate spacetime reference frames thereby bridges the gap between fundamental physics and practical applications. Improved time and frequency transfer has direct impact on a variety of studies on climate change, sea level rise, and on geophysics, in general. The geodetic aspect of E-GRIP consists in performing the comparison of the frequencies of (identical) clocks located in nearly arbitrary locations on the Earth. The result of the comparison yields the difference in the gravitational potential. This so-called relativistic geodesy provides a fundamentally new approach to geodesy, complementing and extending ground and space-based geodesy based on gravitational acceleration.

2. Geodesics and Field Equations

Einstein's theory was summarized by Wheeler as *Spacetime tells matter how to move, matter tells spacetime how to curve.* The first statement refers to the Einstein equivalence principle (EEP), which states that:

(1) free fall under gravity is the same for all substances (the weak equivalence principle or WEP, also known as the Universality of Free Fall or UFF), while

(2) for freely falling observers, special relativity applies locally (Local Lorentz Invariance or LLI) and
(3) the preceding properties apply at all times and places (Local Position Invariance or LPI).

A consequence of the EEP is that gravity causes bodies to move along geodesics (photons along null geodesics) of a Riemannian spacetime. In other words, the EEP implies that gravity is a metric theory. The second part of Wheeler's epigram refers to Einstein's field equations, which relate the metric to the mass, energy and momentum. The metric enables us to measure distances by encoding the space curvature, and *the gravitational field* is implied from the metric tensor. Many experiments could test different aspects of Einstein gravity. One could test the WEP or LLI or LPI in isolation, or test whether gravity is indeed a metric theory, or consequences of the full theory.

Tests of the EEP are often viewed as tests of the universal coupling of gravity to all nongravitational fields of the Standard Model of particle physics.[5] Violations occur when the coupling is dependent on some attribute of the nongravitational fields at hand that may be different for different test bodies, e.g. electromagnetic charge, nuclear charge, total spin, nuclear spin, quark flavor, lepton number, etc. EEP tests check if the metric and geodesic structure of gravity coincide with that predicted by GR, where the Levi-Civita connection is the only allowed affine structure. In other theories such as the metric-affine (Palatini) or purely affine theories of gravity, the EEP does not strictly hold.[6]

Exploring all possibilities of such anomalous couplings is the fundamental aim of experimental tests of the EEP. Note also that in any particular experimental situation, symmetry requires that such anomalous couplings be not only a function of the composition of the test body, but also of the mass which is the source of the gravitational field. As a consequence, the widest possible range of source and test body configuration needs to be explored when testing the different aspects of EEP, and this is one of the aims of E-GRIP, which will test for EEP violation in the gravitational fields of the Earth, the Sun and the Moon. Table 1 summarizes the tests of LPI in the gravitational fields of the Earth, the Sun and the Moon that can be performed by E-GRIP, as well as ACES and the Galileo 5 and 6 satellites as comparison.

Table 1. Overview of LPI tests for E-GRIP, ACES and Galileo 5 and 6 satellites.

Tests of LPI with E-GRIP	Fractional uncertainty: E-GRIP	ACES	Galileo 5 and 6
Earth Gravitational redshift (EEP Earth)	2×10^{-6} (goal: 4×10^{-7})	3×10^{-6}	3–4 $\times 10^{-5}$
Solar Gravitational redshift (EEP Sun)	5×10^{-5}	—	—
Lunar Gravitational redshift (EEP Moon)	1×10^{-2}	—	—

3. Why Would the EEP be Violated?

It has already been pointed out that the EEP is in fact rather *unnatural* in the sense that it renders gravity so different from other interactions because the corresponding universal coupling implies that gravitation is a geometrical attribute of spacetime itself rather than a field over spacetime like all other known interactions. Einstein himself initially called it the *hypothesis of equivalence* before elevating it to a *principle* once it became clear how central it was in the generalization of special relativity to include gravitation. This shows how surprising it is in fact that such a hypothesis should be satisfied at all, let alone down to the uncertainties of present day tests. Therefore, rather than asking why the EEP should be violated, the more natural question to ask is why no violation has been observed yet. Indeed, most attempts at quantum gravity and unification theories lead to a violation of the EEP,[7–12] which in general have to be handled by some tuning mechanism in order to make the theory compatible with existing limits on EEP violation. Violations of the inverse square law will also be detected by certain EEP tests (e.g. redshift tests), allowing for a much richer phenomenology with different distance dependences and anomalous couplings. Therefore, not only do we expect a violation of EEP at some level, but the nonobservation of such a violation with improving uncertainty is already one of the major experimental constraints for the development of new theories in the quest for quantum gravity and unification. This makes experimental tests of EEP in all its aspects one of the most essential enterprises of fundamental physics today. This is then the main motivation of many experiments like MICROSCOPE, the planned ACES, to be launched in 2018, and the proposed satellites, like STE-QUEST[13,14] or E-GRIP, in fundamental physics. MICROSCOPE has been successfully launched on April 2016 and its aim is to test the WEP about 100 times more accurately than known today. The best limit today on the Eötvös parameter is about 10^{-13}.

It is interesting to note that experimental constraints for EEP violations at low energy are rather closely related to present day physics at the very small scale (particle physics) and the very large scale (cosmology). In particle physics, the Standard Model requires a number of dimensionless coupling constants to be *put in* by hand, which seems somewhat arbitrary and is not very satisfactory.[15] One of the aims of theoretical developments is then to replace these constants by some dynamical field that provides the coupling constants (e.g. moduli fields in string theory, dilaton, etc.), similar to the Higgs field giving rise to the mass of fundamental particles. As a consequence, the coupling constants become dynamical quantities that vary in spacetime (e.g. spacetime variation of the fine structure constant), which necessarily leads to violations of the EEP (violation of LPI, but also of WEP/UFF, and LLI). However, the resulting phenomenological consequences are such, that in most approaches one requires some mechanism to stabilize these fields in order to be compatible with present day constraints from EEP tests.[7,8] Although no firm predictions exist, this makes the discovery of the effect of such fields (e.g. EEP violation) a distinct possibility.[15] Most such additional fields are scalar fields, and the experimental confirmation of the Higgs boson has thus lent strong credibility to their existence, as the Higgs is the first fundamental scalar field observed in nature. It is thus likely that additional long and/or short range scalar fields exist, as postulated by many unification theories, and EEP tests are one of the most promising experimental means for their observation.

At the other extreme, in cosmology, most models for Dark Energy (DE) are also based on long-range scalar fields that, when considered in the context of particle physics, are nonuniversally coupled to the fields of the Standard Model.[16] As a consequence, one would expect EEP violations from such fields at some level, which might be detectable by experiments like E-GRIP thus shedding light on the DE content of the universe from a completely different angle. Similarly, long-range scalar fields coupled to Dark Matter (DM) have been investigated as a possible source of EEP violations,[17] which again provides a very appealing route towards independent confirmation of DM, making it more tangible than only a hypothesis for otherwise unexplained astronomical observations.

4. E-GRIP Test of LPI: Earth, Sun and Moon Redshifts

The space hydrogen maser on board of E-GRIP is expected to reach an uncertainty in the Earth field redshift test of 2×10^{-6} accuracy, which will allow an independent verification of the ACES redshift measurement, by a different method, i.e. frequency modulation along the orbit. However, a slightly improved performance of the space hydrogen maser could even lead to a further improvement compared to ACES by a factor of about 5. In the meantime, it has been decided to use the Galileo satellites 5 and 6, due to their large eccentricity, to perform the gravitational redshift measurement. It is expected to reach, after one year of integration of the data, an uncertainty around $(3-4)\times10^{-5}$, about a factor 5 better than achieved with GP-A.[18]

The second primary goal is a precise measurement of the time dilation due to the Sun's and the Moon's gravitational fields, thereby testing for the independence of the time dilation on the nature of the mass producing the gravitational field. The measurement is performed by comparing ultra-precise ground clocks as they rotate with the Earth. The E-GRIP satellite is hereby used as a relay station. The accuracy improvements for this secondary goal will be of about a factor of 20 as compared to ACES. E-GRIP will be able to compare distant ground clocks using the MWL in common-view mode. In the common-view technique, two ground clocks are simultaneously compared to the space clock. The difference of simultaneous measurements provides then a direct comparison of the two clocks on the ground. For a detailed discussion, we refer to Ref. 19. Given the E-GRIP orbit, common-view contacts e.g. between USA and Europe, Europe and Japan/China, Japan/China and USA have uninterrupted durations longer than 10 h with each of them repeated every two days.

With the Sun as the source of the anomalous gravitational coupling, the measured frequency ratio of the two clocks can be written as

$$\frac{\nu_T}{\nu_B} = 1 - \frac{1}{c^2}\left((U_B - U_T) + \frac{v_B^2 - v_T^2}{2} + (\alpha_B U_B - \alpha_T U_T)\right) + \Delta, \qquad (1)$$

where U_B and U_T are the solar Newtonian gravitational potentials at the two locations of the ground clocks and v_B and v_T are the corresponding velocities in a solar system barycentric reference frame. The LPI violating parameters α_B and α_T depend on the type of transition used in the respective clocks, and Δ represents all corrections due to the other solar system bodies (including the Earth) assumed to behave normally, as well as higher order correction terms.

An essential point to note is that, in the absence of an LPI violation ($\alpha_B = \alpha_T = 0$), the leading part in Eq. (1) is equal to zero (up to small tidal correction terms in Δ and constant terms from the Earth field). This is a direct consequence of the EEP, as the Earth is freely falling in the Sun field.[19–21] The LPI test in the Sun field is thus a null-test, verifying whether the measured frequency ratio is equal to the expected value, i.e. $1 + \Delta$ in this example.

In general, the types of clocks used at the different ground stations may be of different types so $\alpha_B \neq \alpha_T$. In the following, we will assume for simplicity clocks of the same type which simplifies the LPI violating term in Eq. (1) to $\alpha(U_B - U_T)$, with the aim of the experiment being the measurement of α. More precisely, the experiment will measure the time evolution of the ratio ν_T/ν_B, which again should be zero in GR (up to correction terms), but will evolve in time if the LPI violating parameters are nonvanishing because of the time evolution of $(U_B - U_T)$, mainly related to the rotation of the Earth. The time evolution of $(U_B - U_T)/c^2$, will be predominantly periodic with a diurnal period and peak-to-peak amplitude of about 1×10^{-12}. Then, the determination of the LPI parameters boils down to a search of a periodic signal with known frequency and phase in the clock comparison data. The procedure for the LPI test in the Moon field is identical to the Sun field test described above. The difference is that the frequency and phase of the signal that one searches for are different and that the sensitivity is decreased by a factor ≈ 175 (see below).

With the onboard clock of E-GRIP, it will be possible to perform also an LPI test in the field of the Earth. In this case, the MWL (or optical) link is used to compare the onboard clock to ground clocks. The frequency ratio can be written as

$$\frac{\nu_{\mathrm{EGRIP}}}{\nu_B} = 1 - \frac{1}{c^2}\left((U_B - U_{\mathrm{EGRIP}}) + \frac{v_B^2 - v_{\mathrm{EGRIP}}^2}{2} + (\alpha_B U_B - \alpha_{\mathrm{EGRIP}} U_{\mathrm{EGRIP}})\right) + \Delta, \qquad (2)$$

where U_B and U_{EGRIP} are the Earth Newtonian gravitational potentials at the locations of the ground clock and the onboard clock, and v_B and v_{EGRIP} are the corresponding velocities in a geocentric reference frame. The LPI violating parameters α_B and α_{EGRIP} depend on the type of transition used in the respective clocks, and Δ is defined as above.

The main difference with respect to the Sun LPI test above is that the ground clocks are not freely falling in the field of the Earth, so even in the absence of an LPI violation the frequency ratio is not zero and varying in time with the eccentric orbit of E-GRIP. The test then compares the theoretically calculated frequency ratio (from the knowledge of the E-GRIP orbit and the ground station locations) to the actually measured one. This leads to two methods for the measurement, one based on the accuracy of the clocks (so-called DC measurement) that searches for an offset with respect to the expected value, and one based on the periodic variation due to the orbit eccentricity (so-called AC measurement) that searches for the time varying signature and thus relies on the clock stability. The former is carried out mainly when the satellite is at apogee (when the LPI violating term in Eq. (2) is largest), the latter uses measurements over the full orbit.

LPI was challenged by various null-tests and direct tests.[22] The latter set limits directly on the parameter α_i for the relevant transition, e.g. the H-maser experiment of 1979[4] sets a limit on α_H for the hydrogen hyperfine transition. 'Null Redshift' experiments typically consist of two co-located clocks of different type in the same laboratory whose relative frequency is monitored as the local gravitational potential varies in time. Thus, one measures $(\alpha_i - \alpha_j)U/c^2$ for two clocks of type i and j and sets a limit on the difference $(\alpha_i - \alpha_j)$. The most precise such test at present sets a limit of $(\alpha_{\mathrm{Rb}} - \alpha_{\mathrm{Cs}}) = (0.11 \pm 1.0) \times 10^{-6}$ for the Rb versus. Cs hyperfine transitions,[23] using the annual variation of the solar potential in the laboratory due to the eccentricity of the Earth's orbit. Depending on the underlying model, the difference $(\alpha_i - \alpha_j)$ might be much smaller than the individual values, especially when similar transitions are used (both hyperfine or both electronic, i.e. optical), so direct tests are necessary and complementary to co-located tests, which is one of the main drivers for experiments like ACES or E-GRIP. In the E-GRIP LPI test, a nonzero signal will be observed, no matter what the actual values of α_T and α_B in Eq. (1), are, provided at least one of them is nonzero, because of the different temporal variation of U_B and U_T. This is not the case in null-tests with co-located clocks, where one necessarily has $U_i = U_j$ and thus a signal can only be detected if $\alpha_i \neq \alpha_j$, which is not the case for E-GRIP. Finally, all Sun LPI science objectives also apply to a test with the Moon as the source mass.

E-GRIP will carry out a direct LPI test in the Moon field using the same methods (and data) as the test in the Sun field described above. Note that the two putative signals can be easily decorrelated in the data due to the different frequency and phase. The sensitivity of E-GRIP to a possible violation of LPI sourced by the Moon is then simply given by a reduction factor with respect to the Sun effect of

$$\frac{\frac{M_{\mathrm{Sun}}}{d_{\mathrm{Sun}}^2}}{\frac{M_{\mathrm{Moon}}}{d_{\mathrm{Moon}}^2}} = 175. \tag{3}$$

In the baseline configuration, the measurement uncertainties of the MWL and the ground clocks should allow a detection of any nonzero value of the LPI violating parameter α sourced by the Moon.

5. Conclusions

Clock tests as described above are sometimes interpreted as searches for a space-time variation of fundamental constants, in particular those of the Standard Model (fine structure constant, electron, proton and quark masses, QCD mass scale, etc.). Such an interpretation is, however, model-dependent (one assumes the validity of the Standard Model of particle physics to describe atomic transitions). In order to best constrain all possible variations of constants, the comparison of as many different transitions as possible is essential. Comparisons of ground clocks based on different types of transitions repeated during the E-GRIP mission will provide a wealth of data to search for temporal variations of fundamental constants, the fine structure constant α and the electron-to-proton mass ratio μ in particular. Different clock transitions have different dependency on fundamental constants. Therefore, the results of crossed frequency comparisons repeated in time provides a clear interpretation of any observed drift over time and imposes unambiguous limits on time variations of fundamental constants.

A weakness of the E-GRIP proposal is the fact that the development of the MWL is still under way and at present it is not clear when it will be possible to achieve a MWL with the required performances. Besides the MWL, all other components of the satellite, and in particular the space hydrogen maser, are available and should thus not be critical. E-GRIP will be able to achieve an uncertainty in the Earth field redshift test comparable to ACES. Nonetheless, the two measurements are complementary due to the very different orbit: ACES is on the International Space Station at about 400 km hight on a circular orbit, whereas E-GRIP is on a very elliptic orbit, thus leading to a frequency modulation of the in the signal. Moreover, a slight improved performance of the space hydrogen maser could lead to a further improvement compared to ACES by a factor of about 5. Given the importance of the measurement, especially in case of a positive signal, an independent measurement based on a somewhat different method would certainly be very welcome.

Tests of GR, in particular of the different aspects of EEP, are now entering a new era thanks to the various space missions, either already in orbit or in an advanced phase of preparation. New missions are proposed and certainly some of them will be eventually implemented. These tests are important in order to check the validity of GR but might also open new windows in the direction of the unification of all forces or towards a quantum version of GR. The next years will thus see an important increase towards such activities both experimental and theoretical. The recent discovery of gravitational waves by the LIGO detectors[24] opens also a new way to test GR besides of opening a new window for the observation of the universe. LISA, thanks to the impressive performance of LISA Pathfinder,[25] is now strongly pushed forward by ESA, with some NASA participation, and will become the first gravitational wave observatory in space. Given all these developments, the study of GR is entering a new phase where there will be a wealth of experimental data and observations. This will certainly allow to make important progress and perhaps new discoveries.

References

1. C. M. Will, *Theory and Experiment in Gravitational Physics,* 2nd edn. (Cambridge University Press, Cambridge, New York, 1993)
2. R. Angélil *et al.*, *Phys. Rev. D* **89** (2014) 064067.
3. A. Schärer *et al.*, *Phys. Rev. D* **90** (2014) 123005.
4. R. F. C. Vessot *et al.*, *Phys. Rev. Lett.* **45** (1980) 2081.
5. T. Damour and J. F. Donoghue, *Phys. Rev. D* **82** (2010) 084033.
6. S. Capozziello *et al.*, *Phys. Rep.* **509** (2011) 167.
7. T. R. Taylor and G. Veneziano, *Phys. Lett. B* **213** (1988) 450.
8. T. Damour and A. M. Polyakov, *Nucl. Phys. B* **423** (1994) 532.
9. S. Dimopoulos and G. Giudice, *Phys. Lett. B* **379** (1996) 105.
10. I. Antoniadis *et al.*, *Nucl. Phys. B* **516** (1998) 70.
11. V. A. Rubakov, *Phys.-Usp.* **44** (2001) 871.
12. R. Maartens and K. Koyama, *Living Rev. Relativ.* **13** (2010) 5.
13. B. Altschul *et al.*, *Adv. Space Res.* **55** (2015) 501.
14. D. N. Aguilera *et al.*, *Class. Quantum Grav.* **31** (2014) 115010.
15. T. Damour, *Class. Quantum Grav.* **29** (2012) 184001.
16. J. Khoury and A. Weltman, *Phys. Rev. Lett.* **93** (2004) 171104.
17. S. M. Carroll *et al.*, *Phys. Rev. Lett.* **103** (2009) 011301.
18. P. Delva *et al.*, *Class. Quantum Grav.* **32** (2015) 232003.
19. P. Wolf and L. Blanchet, *Class. Quantum Grav.* **33** (2016) 035012.
20. N. Ashby and M. Weiss, arXiv:1307.6525 (2013).
21. B. Hoffmann, *Phys. Rev.* **121** (1961) 337.
22. C. M. Will, *Living Rev. Relativ.* **17** (2014) 4.
23. J. Guena *et al.*, *Phys. Rev. Lett.* **109** (2012) 080801.
24. LIGO Collab. (B. P. Abbott *et al.*), *Phys. Rev. Lett.* **116** (2016) 061102.
25. LISA Pathfinder Collab. (M. Armano *et al.*), *Phys. Rev. Lett.* **116** (2016) 231101.