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Voluntary Disclosure in Asymmetric Contests

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Voluntary Disclosure in Asymmetric Contests*

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Abstract This paper studies the incentives for interim voluntary disclosure of verifiable information in probabilistic all-pay contests. Provided that the contest is *uniformly asymmetric*, full revelation is the unique perfect Bayesian equilibrium outcome. This is so because the weakest type of the underdog will try to moderate the favorite, while the strongest type of the favorite will try to discourage the underdog—so that the contest unravels. Self-disclosure is optimal even though a weak favorite or strong underdog may be induced to raise their efforts, i.e., show "dominant" or "defiant" reactions. To avert Pareto inferior unraveling, the favorite may prefer to shut down communication, but this is never the case for the underdog. We also consider partial information release, cheap talk, Bayesian persuasion, information design, correlation, and continuous types. Applications are discussed. The proofs employ novel arguments in monotone comparative statics and an improved version of Jensen's inequality.

Keywords Asymmetric contests · Incomplete information · Disclosure · Strategic complements and substitutes · Dominance and defiance · Bayesian persuasion · Jensen's inequality

JEL Classification C72 Non-cooperative Games · D74 Conflict, Conflict Resolution, Alliances, Revolutions · D82 Asymmetric & Private Information · J71 Discrimination

*) This version supersedes the earlier draft "Voluntary Disclosure in Unfair Contests" which was circulated when the second author still used her maiden name Grünseis. Valuable comments by the Editor and two referees helped us to improve the paper. This work has further benefited from conversations with David Austen-Smith, Mikhail Drugov, Jörg Franke, Dan Kovenock, Wolfgang Leininger, Jingfeng Lu, Meg Mayer, Alessandro Pavan, Dmitry Ryvkin, Tarun Sabarwal, Aner Sela, Curtis Taylor, Karl Wärneryd, Juuso Välimäki, Xavier Vives, and Junjie Zhou. The paper has been presented at the Oligo Workshop in Piraeus, at PET in Hue, Vietnam, at the SAET conference in Taipei, as well as to seminar audiences in Bath, Zurich, and at UCLA.

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1. Introduction

In numerous economic environments, information is a key determinant for the fierceness of a subsequent struggle or competitive interaction. Especially if the disclosure of evidence is apt to clarify contestants' relative strength in a definite way, competition may soften quite suddenly and dramatically. For instance, in a patent race, firms may choose to inform competitors about a "disruptive" advancement so as to establish a role as a leader in innovation and to discourage competitors. In litigation, parties try to avoid going to court by sharing hard evidence such as expert reports and deposition protocols at an early stage. Candidates for top positions may find themselves "designated" in a theoretically much larger field of competitors. In social and biological conflict, the presentation of phenotype "indices" helps mitigating or even avoiding physical fights. Last but not least, military conflict may be ended using a white flag. On the other hand, voluntary disclosure may be less commonly observable in those cases when the release of information would only tighten competition.

In this paper, we extend the standard model of a probabilistic contest (Rosen, 1986; Dixit, 1987) by allowing for pre-play communication of verifiable information (Okuno-Fujiwara et al., 1990; van Zandt and Vives, 2007; Hagenbach et al., 2014). Contestants are assumed to differ in their marginal cost of effort, which is private information for them. However, at a stage preceding the contest, any player may interim, i.e., subsequent to having observed her type, choose to disclose that information to her opponent. The focus of the present paper lies on contests that are *uniformly asymmetric* in the sense that one of the two players is, subject to activity, interim always strictly more likely to win than the other player.[1] We identify a condition on the primitives of the model that guarantees that the contest is uniformly asymmetric. As will be discussed, this condition is consistent with both asymmetric technologies (e.g., O'Keeffe et al., 1984; Meyer, 1992; Franke et al., 2014) and heterogeneous type distributions (e.g., Amann and Leininger, 1996; Maskin and Riley, 2000).

In this type of framework, we evaluate the incentives of individual types of each player to voluntarily disclose their private information. Moreover, we characterize the perfect Bayesian equilibrium of the resulting two-stage game. Our main result says that, provided that the contest is uniformly asymmetric, the *only* outcome of the revelation game consistent with the assumption of perfect Bayesian rationality is the one in which all the privately held information is unfolded

[1] The formal definition will be given in Section 3. While the focus lies on uniformly asymmetric contests, we also explore more broadly the scope of the strong-form disclosure principle in other contests.

prior to the contest. Thus, we find general conditions under which the strong-form disclosure principle applies to a standard contest setting. This may be of interest because probabilistic all-pay contests do not satisfy usual conditions sufficient for the disclosure principle in the *strong form*, according to which any perfect Bayesian or sequential equilibrium entails full revelation.[2]

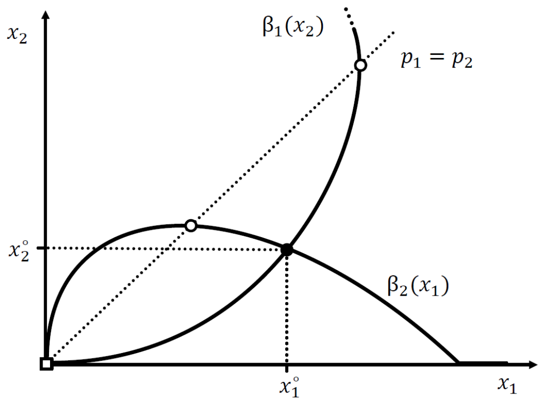

Figure 1. Best-response curves in an asymmetric contest of complete information.

The analysis revisits Dixit's (1987, p. 893) observation that, in an asymmetric contest of complete information, the player more likely to win, the *favorite*, has a best-response function that is locally strictly increasing at the equilibrium, whereas the player less likely to win, the *underdog*, has a best-response function that is locally strictly declining at the equilibrium. For example, in Figure 1, at the equilibrium (x_1°, x_2°), player 1's best-response function $\beta_1 \equiv \beta_1(x_2)$ is strictly increasing, while player 2's best-response function $\beta_2 \equiv \beta_2(x_1)$ is strictly declining. We extend Dixit's (1987) observation to a setting with incomplete information. Thus, also in our setting, there will be a favorite whose best-response mapping is locally strictly increasing at the equilibrium, and an underdog whose best-response mapping is locally strictly declining at the equilibrium. There are, however, important differences. Specifically, with incomplete information, strategy spaces are *multi-dimensional*, reflecting the fact that each type of a given player may choose a different effort level. Moreover, the comparative statics with respect to changes in the information structure is in general monotone *for one player only*, which precludes the use of existing methods.[3]

[2] The main complication is that best-response mappings are not monotone in a contest. Cf. Denter et al. (2014) and Kovenock et al. (2015).

[3] Probabilistic contests of incomplete information have been studied for some time. Rosen (1986, fn. 7) still complained that "few analytical results" were available. Early papers on the topic include, e.g., Linster (1993) and

While the proof of the unraveling result is not entirely straightforward, there is a simple story. Specifically, in view of the high effort level to be expected from a favorite that is left to speculate about the underdog's true ability, the weakest type of the underdog will have a strict incentive to self-disclose, so as to moderate the favorite. But once this is accounted for, any silent type of the underdog will be confronted with an even higher effort of the favorite. The weakest of those remaining types will therefore choose to disclose her private information, too. As a result, there is an unraveling on the underdog's side. However, in the resulting contest with one-sided incomplete information, the unraveling continues on the side of the favorite. Indeed, the respective strongest type of the favorite has a strict incentive to self-disclose, so as to discourage the underdog. In the end, full revelation of all private information is inevitable.

To evaluate welfare implications, we compare full revelation with a benchmark outcome in which players do not have the option to disclose their private information. For a more structured environment with one-sided incomplete information and an unbiased lottery technology, we show that the unraveling is ex-ante strictly undesirable for a privately informed underdog. In other words, full revelation obtains in this case just because the underdog has ex ante no means of committing herself to not reveal her type.[4] We go on and show that, depending on parameters, the unraveling may even lead to a strictly Pareto inferior outcome. We call this outcome the "disclosure trap." However, such possibilities are not universal, i.e., there are examples in which a privately informed contestant will appreciate disclosure not only interim, i.e., when being of the marginal type, but also from an ex-ante perspective. Still, there are instances in which the favorite, to avert the Pareto inefficiencies resulting from the unraveling, would prefer to shut down communication. As we also show, however, this is never the case for the underdog.

The analysis is extended in a variety of ways. First, we check the robustness of the findings by considering the possibility of partial information releases, such as through partitional disclosures of the state space, randomized signals, and noisy signals. The key finding for noisy signals, e.g., is that the "all-or-nothing" nature of disclosure matters for marginalized types only, who may need

Baik and Shogren (1995). The general framework with one-sided and two-sided private valuations is due to Hurley and Shogren (1998a, 1998b). Wärneryd (2003) observed that the uninformed player in a common-value setting is more likely to win than the informed player. Malueg and Yates (2004) analyzed a symmetric two-player Tullock contest with two equally likely, but possibly correlated types. Schoonbeek and Winkel (2006) noted that individual types may remain inactive. General results on the existence and uniqueness of Bayesian equilibrium have been obtained by Einy et al. (2015) and Ewerhart and Quartieri (2020), in particular.

[4] The proof of this result relies on an improved version of Jensen's inequality, which in turn is derived using the theory of moment spaces (Dresher, 1953).

a substantial incentive for becoming active. For all other types, however, marginal incentives resulting from noisy signals tend to be strong enough to trigger voluntary disclosure. Next, we look at cheap talk. In line with existing results for sender-receiver games in which interests are not sufficiently aligned, we find that "babbling" is a necessity under one-sided incomplete information. Under two-sided incomplete information, we can still show that complete separation is impossible. Thus, verifiable evidence indeed matters. We go on and study Bayesian persuasion. For the case where an informed player has commitment power, we identify the optimal signal. We also study a variety of design problems, complementing the analysis of Zhang and Zhou (2016). Finally, we discuss the cases of correlated and continuous type distributions, where especially the case of positive correlation helps sharpen the intuitions underlying our main result.

The economics literature has a long tradition of studying incentives for the voluntary disclosure of private information. Seminal contributions by Grossman (1981) and Milgrom (1981) pointed out that sellers, as a consequence of unraveling, will it find hard to withhold verifiable information about the quality of their products. This argument, often referred to as the *disclosure principle*, has since shaped the theoretical discussion about the pros and cons of disclosure regulation, as is reflected by a very large body of literature.[5]

The present paper falls into the recent and quickly expanding literature concerned with the disclosure of *verifiable* information in contests.[6] Research in this literature has tended to focus on either ex-ante voluntary disclosure, optimal disclosure policies, or interim voluntary disclosure.[7] Ex-ante voluntary disclosure in probabilistic contests has been studied by Denter et al. (2014), in particular. Assuming a probabilistic contest technology with one-sided incomplete information, they showed that a "laissez-faire" policy regarding the informed player's ex-ante disclosure decision leads to lower expected lobbying expenditures than a policy of mandatory disclosure.[8] The second topic, optimal disclosure policies in contests, has recently seen a strong development. In particular,

[5] In addition to the contributions already mentioned, see Verrecchia (1983), Dye (1985), Shin (1994), Seidmann and Winter (1997), Benoît and Dubra (2006), and Giovannoni and Seidmann (2008), for instance. Milgrom (2008) or Dranove and Jin (2010) offer surveys.

[6] Another form of pre-play communication, not considered in the present paper, is the costly signaling of unverifiable information. See, e.g., Katsenos (2010), Slantchev (2010), Fu et al. (2013), Heijnen and Schoonbeek (2017), and M. Yildirim (2017). We do, however, discuss cheap talk.

[7] Numerous additional research questions, related to learning, feedback, and motivation, for example, arise in the analysis of dynamic contests of incomplete information. Such research questions have been dealt with in papers by Clark (1997), H. Yildirim (2005), Krähmer (2007), Münster (2009), Zhang and Wang (2009), Aoyagi (2010), Ederer (2010), and Goltsman and Mukherjee (2011), for instance.

[8] Relatedly, Wu and Zheng (2017) considered a symmetric two-player lottery contest with two equally likely, independently drawn types for each player. In this framework, they showed that ex-ante disclosure decisions are fully revealing if and only if the two possible type realizations are sufficiently close to each other.

effort-maximizing disclosure policies have been characterized by Zhang and Zhou (2016) and Serena (2017) for probabilistic technologies, and by Fu et al. (2014), Chen et al. (2017) and Lu et al. (2018) for deterministic technologies.[9] The present analysis is concerned, however, with the third topic, i.e., the interim voluntary disclosure in contests. As far as we know, there is only one paper that has dealt with this issue on a comparable level of generality.[10] Specifically, Kovenock et al. (2015) showed that, regardless of whether valuations are private or common, the interim information sharing game followed by an all-pay auction admits a perfect Bayesian equilibrium in which no player ever shares her private information. The present analysis is complementary to that of Kovenock et al. (2015) in the sense that, instead of the all-pay auction, we consider a probabilistic contest. Overall, the review of the literature suggests that the specific research question pursued in the present paper, viz. the analysis of incentives for the interim voluntary disclosure of hard evidence in ex-ante asymmetric contests with probabilistic technologies and two-sided incomplete information, has not been addressed in prior work.[11]

The remainder of this paper is structured as follows. Section 2 introduces the set-up. The main result is stated in Section 3. Section 4 discusses contestants' incentives for interim voluntary disclosure. In Section 5, we provide examples for nonmonotone reactions to self-disclosure. The scope of the strong-form disclosure principle is discussed in Section 6. Section 7 discusses commitment and welfare issues. Partial information release is allowed for in Section 8. Section 9 considers cheap talk. Section 10 studies Bayesian persuasion and information design. Correlation between types is allowed for in Section 11. An extension to continuous type spaces is outlined in Section 12. Section 13 discusses applications. Section 14 concludes. Technical material has been relegated to two appendices. Appendix A presents auxiliary results, while Appendix B contains proofs and other material omitted from the body of the paper.

[9] Dubey (2013) studied a set-up with two-sided incomplete information about a binary type distribution and two effort levels. Assuming that abilities are sufficiently dispersed, he showed that incomplete (complete) information engenders more effort if the prize is high (low). Einy et al. (2017) studied the value of public information in Tullock contests with nonlinear costs. Optimal disclosure policies have been analyzed also in models of population uncertainty. See Münster (2006), Myerson and Wärneryd (2006), Lim and Matros (2009), Fu et al. (2011), Feng and Lu (2016), and Fu et al. (2016), among others.

[10] Epstein and Mealem (2013) considered a lottery contest with one-sided incomplete information, and characterized the perfect Bayesian equilibrium outcome in the case of two possible type realizations. While they considered also an extension to more than two types, they did not characterize the perfect Bayesian equilibrium in that case.

[11] In general, signals may have multiple audiences. E.g., in Board's (2009) model, a direct benefit from disclosure on the consumer side is balanced by the firm against the cost of tighter competition. The present paper, however, focuses on the informational exchange between contestants in the absence of informational externalities.

2. Set-up

Considered is an interaction over two stages, referred to as revelation stage and contest stage. The modeling follows the literature on pre-play communication (Okuno-Fujiwara et al., 1990; van Zandt and Vives, 2007; Hagenbach et al., 2014). We start with the contest stage, and continue with the relevation stage.

2.1 The contest stage

Two players (or teams) $i = 1, 2$ exert efforts so as to increase their respective probability of winning a contested prize that is commonly valued at $V > 0$. Contestant i's effort (or bid) is denoted by $x_i \geq 0$. It is assumed that player i's payoff may be written as

$$\Pi_i(x_1, x_2; c_i) = p_i(x_1, x_2)V - c_i x_i, \tag{1}$$

where $p_i(x_1, x_2)$ denotes i's probability of winning, and $c_i > 0$ contestant i's marginal cost of effort. Without loss of generality, the value of the prize will be normalized to $V = 1$. Following Rosen (1986), we assume that

$$p_i(x_1, x_2) = \begin{cases} \dfrac{\gamma_i h(x_i)}{\gamma_1 h(x_1) + \gamma_2 h(x_2)} & \text{if } x_1 + x_2 > 0 \\ \gamma_i/(\gamma_1 + \gamma_2) & \text{if } x_1 + x_2 = 0, \end{cases} \tag{2}$$

where $\gamma_1 > 0$ and $\gamma_2 > 0$ are parameters, while $h : \mathbb{R}_+ \to \mathbb{R}_+$ is a continuous function that is twice continuously differentiable at positive bid levels, with $h(0) = 0$, $h' > 0$, and $h'' \leq 0$.[12]

This set-up includes, as an important special case, the example of the biased *Tullock contest* (Tullock, 1975; Leininger, 1993; Clark and Riis, 1998), where the production function is given by $h(z) = h^{\text{TUL}}(z; r) \equiv z^r$ for an arbitrary parameter $r \in (0, 1]$. The *lottery contest* corresponds to the case $r = 1$.

Each player i's type is private information and drawn ex-ante from a probability distribution

[12] Relaxing the assumption of a concave production function would take us away from the main focus of this paper. In contrast, the extension to player-specific production functions is easily accomplished, yet does not yield additional insights.

over the finite[13] set $C_i = \{c_i^1, \ldots, c_i^{K_i}\}$, where $K_i \geq 1$, and

$$\underline{c}_i \equiv c_i^1 < \ldots < c_i^{K_i} \equiv \overline{c}_i \qquad (i \in \{1, 2\}). \tag{3}$$

Thus, $\underline{c}_i$ denotes the most efficient, or strongest type, while $\overline{c}_i$ denotes the least efficient, or weakest type of player i. The ex-ante probability of type c_i^k is denoted by $q_i^k \equiv q_i(c_i^k)$, for $k \in \{1, \ldots, K_i\}$, with $q_i^k > 0$. It will initially be assumed that players' types are independent.[14]

A *bid schedule* for player $i \in \{1, 2\}$ is a mapping $\xi_i : C_i \to \mathbb{R}_+$. The set of i's bid schedules is denoted as X_i. A pair of bid schedules $\xi^* = (\xi_1^*, \xi_2^*) \in X_1 \times X_2$ is a *Bayesian equilibrium* if, for any type $c_i \in C_i$ of any player $i \in \{1, 2\}$, the effort level $x_i = \xi_i^*(c_i)$ maximizes type c_i's expected payoff $E_{c_j}[\Pi_i(x_i, \xi_j^*(c_j); c_i)]$, where $E_{c_j}[.]$ denotes the expectation over the realizations of $c_j \in C_j$, with $j \neq i$. Following Schoonbeek and Winkel (2006), a type $c_i \in C_i$ that chooses an equilibrium effort $\xi_i^*(c_i) > 0$ ($\xi_i^*(c_i) = 0$) will be called *active* (*inactive*). The discontinuity of the payoff functions at the origin implies that, at any Bayesian equilibrium, both players are necessarily active with positive probability.[15] By the same token, at least one player will have to be active with probability one.

Lemma 1. *The contest stage admits a unique Bayesian Nash equilibrium.*[16]

Proof. See Appendix B. □

In the special cases of complete and one-sided incomplete information, the following notation will be used. If $(c_1, c_2) = (c_1^\circ, c_2^\circ)$ is public, then i's equilibrium strategy will be written as $x_i^\circ = x_i^\circ(c_1^\circ, c_2^\circ)$. Further, if player i's type $c_i = c_i^\#$ is public, while player j's type, with $j \neq i$, remains uncertain, then equilibrium strategies will be written as $x_i^\# = x_i^\#(c_i^\#)$ for player i and $\xi_j^\# = \xi_j^\#(.; c_i^\#)$ for player j, so that $\xi_j^\#(c_j; c_i^\#)$ is type c_j's equilibrium effort.[17]

[13] An extension to continuous type distributions is outlined in Section 12.

[14] For an extension to correlated type distributions, see Section 11.

[15] To see this, suppose that one player is active with probability zero. Then, any sufficiently small positive bid is a better response than the zero bid, but any positive bid is suboptimal. Hence, there is no best response in this case.

[16] Lemma 1 extends to mixed strategies. Indeed, since each player is active with positive probability, and payoffs functions are own-bid l.s.c. at the origin, expected payoffs against the opponent's equilibrium strategy are strictly concave over $\mathbb{R}_+$, so that it is suboptimal to randomize strictly.

[17] For completeness, we mention that our set-up is isomorphic to a model in which costs are commonly known but valuations are private information for the contestants (e.g., Hurley and Shogren, 1998b). This can be seen by normalizing payoff functions in the agent-normal form of the Bayesian contest game.

2.2 The revelation stage

At a stage preceding the contest, players are given the opportunity to simultaneously and independently disclose their marginal costs. Initially, it will be assumed that private information cannot be misrepresented. Further, we assume that the decision to self-disclose does not lead to any direct costs.[18]

Ex-ante probability distributions are updated in response to the observation of verifiable information, and otherwise according to Bayes' rule whenever possible. Note that off-equilibrium beliefs may arise, but only in the distinct case where a player chooses to conceal her private information even though the equilibrium strategy entails disclosure by all types of that player.[19]

In any case, the contest stage begins with a pair of well-defined posterior *beliefs* $(\mu_1, \mu_2) \in \Delta(C_1) \times \Delta(C_2)$, where $\Delta(C_i) = \{\mu_i : C_i \to [0,1] \text{ s.t. } \sum_{k=1}^{K_i} \mu_i(c_i^k) = 1\}$ denotes the set of beliefs about player $i \in \{1,2\}$. Ignoring zero-probability types, a unique Bayesian equilibrium exists by Lemma 1. In particular, the expected continuation payoff from the contest stage is well-defined for any type $c_i \in C_i$, for $i \in \{1,2\}$.[20] By a (reduced-form) *perfect Bayesian equilibrium*, we mean sets $S_1 \subseteq C_1$, $S_2 \subseteq C_2$ of revealing types, and off-equilibrium beliefs $\mu_i^0 \in \Delta(C_i)$ for any $i \in \{1,2\}$ with $S_i = C_i$, such that

$$E_{c_j}[\Pi_i(x_i^{\#}, \xi_j^{\#}(c_j); c_i)] \geq E_{c_j}[\Pi_i(x_i, \xi_j^*(c_j); c_i)] \qquad (x_i \geq 0), \tag{4}$$

for any $c_i \in S_i$, and

$$E_{c_j}[\Pi_i(\xi_i^*(c_i), \xi_j^*(c_j); c_i)] \geq E_{c_j}[\Pi_i(x_i^{\#}, \xi_j^{\#}(c_j); c_i)], \tag{5}$$

for any $c_i \in C_i \backslash S_i$. Note that we dropped, for convenience, the reference to the prior disclosure decisions in the notation of the equilibrium bids.[21]

[18] Introducing costs for disclosing information would not change our conclusions, provided those are not too large compared to the benefits of self-disclosure identified below.

[19] A formal account of belief updating is provided in Appendix B.

[20] This is obvious for any $c_i \in C_i$ with $\mu_i(c_i) > 0$. Should, however, a type c_i deviate by not disclosing so that $\mu_i(c_i) = 0$, then there may not be a best response if the thereby deluded opponent plays zero with positive probability. In that case, we replace the continuation payoff by the supremum payoff feasible for c_i.

[21] Type-dependent signal spaces and continuous strategy sets preclude a direct reference to the standard definition of a perfect Bayesian equilibrium in a multi-stage game with observable actions (Fudenberg and Tirole, 1991, p. 331). Otherwise, however, the definition is standard.

3. The unraveling theorem

This section is central to our analysis. We start by defining what we call uniformly asymmetric contests. We then provide a sufficient condition for a contest to be uniformly asymmetric. Finally, we present the main result of this paper.

3.1 Uniformly asymmetric contests[22]

The following definition extends Dixit's (1987) concept of asymmetry to contests of incomplete information.

Definition 1. *A probabilistic contest of incomplete information will be called* ***uniformly asymmetric*** *if, for any pair of posterior beliefs $\mu_1 \in \Delta(C_1)$, $\mu_2 \in \Delta(C_2)$, at the contest stage*

(i) all types $c_1 \in \text{supp}(\mu_1)$ are active; and

(ii) if all types $c_2 \in \text{supp}(\mu_2)$ are active as well, then

$$p_1(\xi_1^*(c_1), \xi_2^*(c_2)) > \frac{1}{2} > p_2(\xi_1^*(c_1), \xi_2^*(c_2)) \qquad (c_1 \in \text{ supp}(\mu_1); c_2 \in \text{ supp}(\mu_2)). \tag{6}$$

Here, as usual, $\text{supp}(\mu_i) = \{c_i \in C_i : \mu_i(c_i) > 0\}$ denotes the *support* of player i's posterior belief μ_i, for $i \in \{1, 2\}$. Thus, in a uniformly asymmetric contest, two properties hold regardless of posterior beliefs at the contest stage. First, player 1 is active with probability one. Second, provided that player 2 is also active with probability one, player 1 is interim always (i.e., for all type realizations) more likely to win than player 2.[23]

If the contest is of complete information (i.e., if $K_1 = K_2 = 1$), then being uniformly asymmetric is equivalent to what Dixit (1987) called an asymmetric contest. Henceforth, we will refer to player 1 as the *favorite* and to player 2 as the *underdog*.

3.2 A sufficient condition

In this section, we derive a condition on the primitives of the model that is sufficient for a contest to be uniformly asymmetric. While this assumption is strong, it will allow us to capture a very clear and robust intuition.

[22] We dropped the term "unfair" because, as pointed out by a referee, it had been used before with a different meaning (e.g., Feess et al., 2008; Epstein el al., 2013).

[23] To understand why the activity of all types of player 2 is presupposed in property (ii) of the definition, it should be noted that an inactive type of player 2 may, in general, dilute the marginal incentives of a strong player 1 so much that the probability ranking (6) could easily break down.

Assumption 1. *The production function h has a finite curvature $\underline{\rho}$.*[24] *Moreover, the net bias $\gamma \equiv \gamma_2/\gamma_1$ satisfies*

$$\gamma < \gamma^* \equiv \frac{\pi_1 + 2\pi_2 - 2}{2 - \pi_1} \cdot \begin{cases} \sigma & \text{if } \sigma \leq 1 \\ \sigma^{1/\underline{\rho}} & \text{if } \sigma > 1, \end{cases} \tag{7}$$

where $\sigma = \underline{c}_2/\overline{c}_1$, and $\pi_i = \sqrt{\underline{c}_i/\overline{c}_i}$ for $i \in \{1, 2\}$.

Assumption 1 is a joint restriction on several parameters. The ratio σ captures player 1's *lowest relative resolve* (cf. Hurley and Shogren, 1998a, 1998b). E.g., if player 1 is interim always more efficient than player 2, then σ strictly exceeds one and corresponds to the worst-case relative cost advantage of player 1 compared to player 2. The parameter π_i reflects the *predictability* of player i's marginal cost, where $i \in \{1, 2\}$. A predictability equal to one (strictly lower than one) for a player corresponds to complete information (incomplete information) about her type. The *net bias* γ has an immediate interpretation, being weakly below one (weakly above one) meaning that the contest technology is weakly biased against player 2 (against player 1). Finally, the *curvature* $\underline{\rho}$ measures the degree of noise in the contest technology, with a higher (lower) $\underline{\rho}$ corresponding to more (less) noise.[25]

The comparative statics of the cut-off value for the bias, γ^*, is straightforward. When positive, γ^* is strictly increasing in each of the three parameters π_1, π_2, and σ, as well as monotone declining in $\underline{\rho}$. Thus, the assumption is more likely to hold when the net bias discriminates more strongly against player 2, when marginal costs are more predictable, when player 1's lowest relative resolve is larger, or when the production function has a lower curvature. As a result of the comparative statics, we see that if Assumption 1 holds for a given contest of incomplete information, changes to the information structure caused by pre-play disclosure decisions cannot invalidate it. For instance, if the assumption holds for type sets C_1 and C_2, then it holds also for any pair of nonempty subsets of those. Indeed, if either C_1 or C_2 is substituted by a nonempty subset, then the parameters π_1, π_2, and σ all increase weakly, so that the cut-off value for the bias, γ^*, increases weakly. A similar remark applies to updating of beliefs.[26]

[24] The curvature $\underline{\rho} = \underline{\rho}(h)$ corresponds to the smallest ρ for which h is *ρ-convex* (cf. Anderson and Renault, 2003). In the Tullock case, $\underline{\rho}(h^{\text{TUL}}) = 1/r$. In the lottery case, $r = 1$, and hence $\underline{\rho} = 1$. For background on generalized concavity, see Caplin and Nalebuff (1991a, 1991b).

[25] It should also be noted that the specific form of inequality (7) has been derived from the proof of Lemma 2 below and thus constitutes a sufficient but not necessary condition for the contest to be uniformly asymmetric.

[26] It is also noteworthy that Assumption 1 does not impose any activity conditions. In general, corner solutions are

In the limit case of complete information and symmetric costs (i.e., $\underline{c}_1 = \overline{c}_1 = \underline{c}_2 = \overline{c}_2$), Assumption 1 says that the technology is biased against player 2 (i.e., $\gamma_2 < \gamma_1$). Further, the case of a biased contest with ex-ante symmetric type distributions (i.e., $\underline{c}_1 = \underline{c}_2 \leq \overline{c}_1 = \overline{c}_2$), as discussed, e.g., by Drugov and Ryvkin (2017), is not generally excluded by Assumption 1.[27]

Clearly, with Assumption 1 in place, player 1 is in a quite strong position relative to player 2. As the following result shows, this implies that the contest, even though of incomplete information, is structurally similar to the complete-information contest considered by Dixit (1987).

Lemma 2. (Sufficient condition) *Any incomplete-information contest that satisfies Assumption 1 is uniformly asymmetric.*

Proof. See Appendix B. □

3.3 Main result

We will use the term *full revelation* to characterize the perfect Bayesian equilibrium, or the perfect Bayesian equilibrium outcome, in which all types disclose their private information. The main result of the present paper is the following.

Theorem 1. (Strong-form disclosure principle) *In any uniformly asymmetric contest with pre-play communication of verifiable information, full revelation is the unique perfect Bayesian equilibrium outcome.*

Proof. See Appendix B. □

Theorem 1 states that the strong-form disclosure principle applies to any uniformly asymmetric contest.

It is not hard to see that self-disclosure by all types is actually a perfect Bayesian equilibrium. Indeed, it suffices to specify off-equilibrium beliefs so that a player that surprises her opponent by concealing her private information is deemed being the worst-case type, i.e., the type that no other type would like to masquerade as.[28] In our setting, the worst-case types are the most efficient

known to be consistent with the existence of a perfect Bayesian equilibrium with no revelation of private information (Okuno-Fujiwara et al., 1990, Ex. 4). In our framework, however, this problem does not occur.

[27] Indeed, in this case, $\gamma^* = \frac{(3\pi-2)\pi^2}{2-\pi}$, with $\pi \equiv \pi_1 = \pi_2 = \sqrt{\sigma}$. For example, for $\pi = 0.8$, we get $\gamma^* = 0.21$. However, as noted by a referee, in that case the additional assumption $\sigma > 4/9$ is needed to fulfill $\gamma < \gamma^*$.

[28] This useful terminology is, of course, borrowed from Seidmann and Winter (1997) and Hagenbach et al. (2014).

underdog and the least efficient favorite, respectively. Provided that off-equilibrium beliefs are specified in that skeptical way (Milgrom, 2008), it is clearly optimal for all types of both players to stick to self-disclosure.

Next, it should be noted that uniqueness is claimed for the equilibrium outcome rather than for the disclosure strategies. The reason is simple. To reveal all private information, it would suffice that, for each player, all types except one disclose their private information. If the residual type that does not disclose happens to be the worst-case type (on at least one side of the contest), then we have another perfect Bayesian equilibrium. However, the resulting multiplicity is trivial if all equilibria are equivalent in terms of the outcome, which is precisely what the theorem says. An overview discussion of the proof of the uniqueness claim in Theorem 1 will be provided in the next section.

It is instructive to relate Theorem 1 to the literature on unraveling in auctions with interdependent valuations (Benoît and Dubra, 2006; Tan, 2016). There, incentives to reveal a private signal are typically strongest at the bottom of the signal support for the common-value component. For instance, should a bidder in an auction of art privately learn that a purportedly original painting is not authentic, then she may have an incentive to share that information with the other bidders. Thus, as in the present paper, voluntary disclosure is more likely to occur when it reduces the opponent's incentives for bidding too aggressively.[29]

4. Understanding the unraveling result

This section discusses the mechanics underlying Theorem 1. First, the disclosure decision of the weakest type of the underdog is dealt with. Subsequently, we consider the disclosure decision of the strongest type of the favorite.

4.1 Benefits of self-disclosure for the underdog

In this subsection, we study the incentives of the weakest type of the underdog to disclose her type, given a candidate equilibrium in which all types conceal their information. Consider, consequently, a contest with incomplete information in which player 2 has at least two possible type realizations. Let $\xi^* = (\xi_1^*, \xi_2^*)$ denote the Bayesian equilibrium at the contest stage. For the weakest type of the underdog $\overline{c}_2$, the probability of winning and the expected payoff are given by $p_2^*(\overline{c}_2) =$

[29] In particular, we conjecture that allowing for a common-value signal in the present set-up would lead to similar conclusions as the literature on unraveling in auctions has identified.

$E_{c_1}[p_2(\xi_1^*(c_1), \xi_2^*(\overline{c}_2))]$ and $\Pi_2^*(\overline{c}_2) = E_{c_1}[\Pi_2(\xi_1^*(c_1), \xi_2^*(\overline{c}_2); \overline{c}_2)]$, respectively. Consider, next, the Bayesian equilibrium $(\xi_1^\#, x_2^\#)$ in the contest with one-sided incomplete information that results when the weakest type of the underdog reveals her type. Then, type $\overline{c}_2$'s probability of winning and expected payoff are given by $p_2^\# = E_{c_1}[p_2(\xi_1^\#(c_1), x_2^\#)]$ and $\Pi_2^\# = E_{c_1}[\Pi_2(\xi_1^\#(c_1), x_2^\#; \overline{c}_2)]$, respectively. The following result summarizes the comparative statics of the equilibrium at the contest stage with respect to the disclosure decision by the weakest type of the underdog.

Proposition 1. (Self-disclosure by the weakest type of the underdog) *Suppose that, in a uniformly asymmetric contest, the underdog has at least two possible type realizations. Then, a unilateral disclosure by the weakest type of the underdog, $\overline{c}_2$,*

(i) induces type $\overline{c}_2$ to strictly raise her effort, i.e., $x_2^\# > \xi_2^*(\overline{c}_2)$;

(ii) strictly raises type $\overline{c}_2$'s interim probability of winning, i.e., $p_2^\# > p_2^*(\overline{c}_2)$ (even against any given type of player 1); and

(iii) strictly raises type $\overline{c}_2$'s expected payoff, i.e., $\Pi_2^\# > \Pi_2^*(\overline{c}_2)$.

Proof. See Appendix B. □

Thus, after revealing her relative weakness, the weakest type of the underdog behaves as if gaining confidence. She bids more aggressively and wins with a strictly higher probability. Moreover, the disclosure is always strictly beneficial for her. In the proof of the unraveling result, we will actually need only part (iii) of Proposition 1. However, as explained below, parts (i) and (ii) are crucial steps that need to be made in order to derive part (iii).

Note that the conclusions of Proposition 1 are immediate for any type of the underdog that is inactive in ξ^*. Indeed, disclosure is the only way for such types to ensure an active participation, a positive probability of winning, and a positive expected payoff. Thus, Proposition 1 shows that the weakest type of the underdog has an incentive to disclose her type even when she foresees herself being active after concealment. Through repeated application of Proposition 1, the underdog's side of the contest equilibrium is seen to unravel. Indeed, all types of the underdog except the worst-case type, when in the position of the weakest type that is foreseen to conceal, will find it strictly optimal to voluntarily disclose their private information. Thus, incomplete information is effectively one-sided in any perfect Bayesian equilibrium.

Next, we provide an overview over the proof of Proposition 1. Given two bid schedules $\xi_i, \widehat{\xi}_i \in X_i$, we write $\xi_i \succeq_i \widehat{\xi}_i$ if $\xi_i(c_i) \geq \widehat{\xi}_i(c_i)$ holds for any $c_i \in C_i$. Thus, $(X_i, \succeq_i)$ is the set of bid schedules equipped with the product order. As usual, we will write $\xi_i \succ_i \widehat{\xi}_i$ if $\xi_i \succeq_i \widehat{\xi}_i$ and there is $c_i \in C_i$ such that $\xi_i(c_i) > \widehat{\xi}_i(c_i)$.[30] Denote by $X_j^* \subseteq X_j$ the set of bid schedules ξ_j for player $j \in \{1, 2\}$ that admit, for any type $c_i \in C_i$ of player $i \neq j$, a unique maximizer $x_i \equiv \widetilde{\beta}_i(\xi_j; c_i) \in \mathbb{R}_+$ of the expected payoff function $x_i \mapsto E_{c_j}[\Pi_i(x_i, \xi_j(c_j); c_i)]$. Given $\xi_j \in X_j^*$, the bid schedule $\beta_i(\xi_j) = \widetilde{\beta}_i(\xi_j; \cdot) : C_i \to \mathbb{R}_+$ will be called the *best-response bid schedule* against ξ_j. In Appendix A, it is shown that, for any $\xi_j \in X_j^*$, the best-response bid schedule $\beta_i(\xi_j)$ is weakly declining in the type, and strictly so at positive bid levels. The mapping $\beta_i : X_j^* \to X_i$ that maps a given bid schedule ξ_j^* of player j to player i's best-response bid schedule against ξ_j^* will be referred to as player i's *best-response mapping.* In the case of complete information, the best-response mapping satisfies monotonicity properties only on a strict subset of the opponent's strategy space.[31] This is likewise so in the case of incomplete information, and we characterize suitable *domain restrictions* sufficient for strict monotone comparative statics in Apppendix B.

The fact that the weakest type of the underdog raises her effort after self-disclosure may be unexpected. To understand this point, suppose that, instead of strictly raising her effort, the weakest type of the underdog were to weakly lower her effort after disclosure, i.e., $x_2^\# \leq \xi_2^*(\overline{c}_2)$, as shown in the diagram on the right-hand side of Figure 2. Consider now the *flat* bid schedule $\psi_2(x_2^\#) \in X_2$ that prescribes an effort of $x_2^\#$ for each type $c_2 \in C_2$ of the underdog. Then, since there are at least two types for the underdog, and since the equilibrium bid schedule ξ_2^* is strictly declining, we get $\xi_2^* \succ \psi_2(x_2^\#)$. From the strict monotonicity of player 1's best-response mapping, after checking domain conditions, we therefore obtain $\xi_1^* = \beta_1(\xi_2^*) \succ \beta_1(\psi_2(x_2^\#)) = \xi_1^\#$, as shown in the diagram on the left-hand side of Figure 2. Applying now the strictly declining best-response mapping of the weakest type of the underdog, checking also here the domain condition, we arrive at $\xi_2^*(\overline{c}_2) = \widetilde{\beta}_2(\xi_1^*; \overline{c}_2) < \widetilde{\beta}_2(\xi_1^\#; \overline{c}_2) = x_2^\#$, which yields the desired contradiction. Thus, the weakest type of the underdog indeed gains in confidence after self-disclosure.

[30] The subscript i in $\succeq_i$ and $\succ_i$ will be dropped whenever there is no risk of ambiguity.

[31] See Dixit (1987). The comparative statics of complete-information contests has been studied by Jensen (2016) and Gama and Rietzke (2017), in particular.

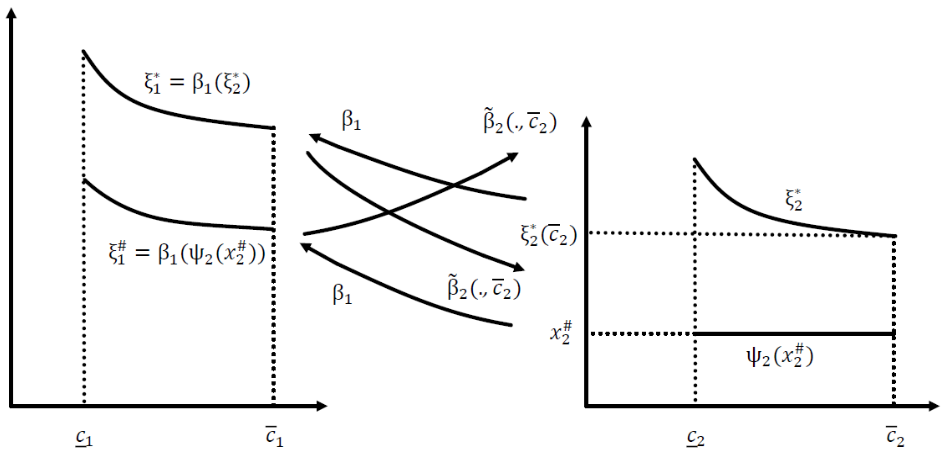

Figure 2. A monotonicity argument.

Based upon this fact, it can be shown that self-disclosure strictly raises also the probability of winning for the weakest type of the underdog. Ultimately, this is a consequence of what we call the Stackelberg monotonicity of the complete-information model (see Appendix A). By this, we mean the fact that an increase of player i's bid, subject to an optimal response by the opponent j, always raises player i's winning probability (and strictly so in the interior). Intuitively, a higher effort is rewarded in terms of higher winning probabilities.[32] Applied to the present situation, this says that a Stackelberg-leading player 2 that raises her bid from $\xi_2^*(\overline{c}_2)$ to $x_2^\#$ strictly increases her probability of winning. But type $\overline{c}_2$'s probability of winning with her bid $\xi_2^*(\overline{c}_2)$ in the Stackelberg setting is already strictly higher than in the Bayesian equilibrium under two-sided incomplete information, because player 1's best-response bid schedule against the leader's bid $\xi_2^*(\overline{c}_2)$ is strictly lower than ξ_1^* in the product order. Combining these two insights, it follows that indeed, the probability of winning for the weakest type of the underdog rises strictly subsequent to self-disclosure. In fact, this is so even for any given type of the favorite.

Finally, we check that the weakest type of the underdog has a strict incentive to disclose her private information. The proof we managed to come up with exploits, in the spirit of the envelope theorem, type $\overline{c}_2$'s first-order condition in order to rewrite her expected payoff from the contest as a monotone function of ex-post winning probabilities and bids. Given parts (i) and (ii) of Proposition 1, this suffices to prove the claim. Unfortunately, however, developing a simple

[32]This property, for which we did not find a suitable reference, may be seen as an analogue of Dixit's (1987, Eq. 8) precommitment result. However, in contrast to that result, the Stackelberg monotonicity property holds regardless of contestants' relative strengths.

intuition for part (iii) seems to be more intricate.

4.2 Benefits of self-disclosure for the favorite

From the previous section, we know that, in any perfect Bayesian equilibrium, the type of the underdog is public information at the contest stage. Let $c_2^{\#}$ denote the commonly known cost type of the underdog. Given this setting with one-sided incomplete information, we will study the incentive of the strongest type of the favorite to disclose her private information to the underdog.

If type $\underline{c}_1$ decides to conceal her private information, then the ensuing contest is of one-sided incomplete information, with equilibrium efforts $\xi_1^{\#}(\underline{c}_1) \equiv \xi_1^{\#}(\underline{c}_1; c_2^{\#})$ and $x_2^{\#} \equiv x_2^{\#}(c_2^{\#})$. Type $\underline{c}_1$'s probability of winning and expected payoff are consequently given by $p_1^{\#} = p_1(\xi_1^{\#}(\underline{c}_1), x_2^{\#})$ and $\Pi_1^{\#} = \Pi_1(\xi_1^{\#}(\underline{c}_1), x_2^{\#}; \underline{c}_1)$, respectively. If, however, type $\underline{c}_1$ decides to disclose her private information, then the ensuing contest is of complete information, with equilibrium efforts $x_i^{\circ} \equiv x_i^{\circ}(\underline{c}_1, c_2^{\#})$, for $i = 1, 2$. In that case, type $\underline{c}_1$'s probability of winning and expected payoff are given by $p_1^{\circ} = p_1(x_1^{\circ}, x_2^{\circ})$ and $\Pi_1^{\circ} = \Pi_1(x_1^{\circ}, x_2^{\circ}; \underline{c}_1)$, respectively. The following result summarizes the comparative statics of the one-sided incomplete-information contest with respect to a revelation by the strongest type of the favorite.

Proposition 2. (Self-disclosure by the strongest type of the favorite) *Suppose that, in a uniformly asymmetric contest, the type of the underdog is public information, while the favorite has at least two possible type realizations. Then, a unilateral disclosure by the strongest type of the favorite, $\underline{c}_1$,*

(i) induces the underdog to strictly lower her effort, i.e., $x_2^{\circ} < x_2^{\#}$;

(ii) allows type $\underline{c}_1$ to strictly lower her effort, i.e., $x_1^{\circ} < \xi_1^{\#}(\underline{c}_1)$;

(iii) strictly raises type $\underline{c}_1$'s probability of winning, i.e., $p_1^{\circ} > p_1^{\#}$; and

(iv) strictly raises type $\underline{c}_1$'s expected payoff, i.e., $\Pi_1^{\circ} > \Pi_1^{\#}$.

Proof. See Appendix B. □

Thus, if the type of the underdog is public, then the self-revelation by the strongest type of the favorite discourages the underdog. As a result, the strongest type of the favorite exerts a lower effort, but still wins with higher probability. Clearly then, she finds it strictly optimal to reveal her private information to the underdog. The proof of Proposition 2 employs the same methods that have been used before. However, given that informational incompleteness is one-sided, the argument is of course much simpler in this case.[33]

As shown in Appendix B, an iterated application of Proposition 2 implies that also the favorite's side unravels. Thus, in a uniformly asymmetric contest, full revelation is the only outcome consistent with the assumption of perfect Bayesian rationality. But, as already discussed, disclosure by all types of both players is indeed a perfect Bayesian equilibrium of the contest with pre-play communication, which completes the proof of the uniqueness part of Theorem 1.[34]

5. Nonmonotone reactions to disclosure

This section presents two examples, intuitively capturing "dominant" and "defiant" reactions to one-sided disclosure.[35] The main motivation for having this section is to make transparent in what sense our contribution is novel and important, and why analyzing disclosure in probabilistic contests causes unexpected difficulties that require new methods. The examples arise in a natural way from the comparative statics of the Bayesian equilibrium at the contest stage with respect to changes in the information structure.

5.1 Equilibrium responses to the underdog's self-disclosure

While self-disclosure by the weakest type of the underdog tends to have an overall moderating effect on the favorite, some types of the favorite may actually respond by bidding higher.

Example 1. ("Dominant reaction")[36] Consider the set-up specified in Table I. As can be seen, after the self-disclosure by type $\overline{c}_2 = c_2^2$ of the underdog, the weak type $\overline{c}_1 = c_1^2$ of the favorite raises her effort.

[33] Part (ii) of Proposition 2 can actually be shown to hold also in the case of two-sided incomplete information, using an argument similar to the one used for Proposition 1. Beyond this observation, however, the analogy is incomplete. In fact, we conjecture that parts (iii) and (iv) of Proposition 2 do not generalize to a setting with two-sided incomplete information.

[34] Theorem 1 continues to hold when the revelation stage is replaced by a sequential-move model in which the disclosure decision is made first by the favorite. But also in the case where the underdog moves first, we have found (by a definite result for the lottery contest, and an extensive numerical search for more general technologies) that full revelation remains the unique perfect Bayesian equilibrium outcome. Thus, even if disclosure decisions are made sequentially, it does not seem possible for players in a uniformly asymmetric contest to escape the logic of the unraveling result.

[35] Of course, we acknowledge the crucial role played by emotions in many real-world contests and tournaments (e.g., Kräkel, 2008).

[36] All the numerical examples in this paper are based on the unbiased lottery contest.

Player 1		Player 2	
$c_1^1 = 0.1$	$c_1^2 = 0.2$	$c_2^1 = 3.7$	$c_2^2 = 7.8$
$q_1(c_1^1) = 0.5$	$q_1(c_1^2) = 0.5$	$q_2(c_2^1) = 0.05$	$q_2(c_2^2) = 0.95$
Assumption 1 holds: $\gamma = 1 < \gamma^* = 1.2103$			
$\xi_1^*(c_1^1) = 0.1592$	$\xi_1^*(c_1^2) = 0.1042$	$\xi_2^*(c_2^1) = 0.0572$	$\xi_2^*(c_2^2) = 0.0011$
$\vee$	$\wedge$		$\wedge$
$\xi_1^\#(c_1^1) = 0.1495$	$\xi_1^\#(c_1^2) = 0.1051$	$-$	$x_2^\# = 0.0023$

Table I. Equilibrium bids before and after the underdog's self-disclosure.

Example 1 shows that the self-disclosure by the weakest type of the underdog need not cause a generally soothing shift in the favorite's bid schedule. Indeed, in response to learning that the underdog is weak, only the strong type of the favorite decreases her bid, whereas the weak type of the favorite raises her bid, as if being challenged. For intuition, note that there are two countervailing effects. On the one hand, following the self-disclosure by the weakest type of the underdog, the favorite's belief regarding the underdog's type collapses and henceforth assigns probability one to the weakest type of the underdog. Clearly, this induces all of the favorite's types to *lower* their respective bids. On the other hand, the weakest type of the underdog will raise her bid after having disclosed her type, which induces all of the favorite's types to likewise *raise* their respective bids. Since the two effects have opposite signs, the overall effect of the underdog's self-disclosure on the bid of a given type of the favorite is, in general, ambiguous.

Despite this flexibility, the model does impose some structure of the favorite's reaction. First, not all types of the favorite may simultaneously raise their bids in response to the self-disclosure by the weakest type of the underdog.[37] Indeed, this would be incompatible with our earlier conclusion that the weakest type of the underdog necessarily raises her bid. Second, even a dominant reaction of the favorite will never be strong enough to press the probability of winning for the weakest type of the underdog weakly below her probability of winning under concealment.

5.2 Equilibrium responses to the favorite's self-disclosure

[37] See, however, the discussion of positive correlation in Section 11.

The following example demonstrates that, in analogy to the case just considered, a type of the underdog may actually raise her effort after the favorite's attempt to discourage her.

Example 2. ("Defiant reaction") Consider the set-up specified in Table II. It can be seen that, in response to the favorite's attempt to discourage the underdog, only the two weaker types of the underdog lower their respective efforts, whereas the strongest type of the underdog actually raises her effort.

Player 1		Player 2		
$c_1^1 = 0.2$	$c_1^2 = 0.6$	$c_2^1 = 7.5$	$c_2^2 = 11$	$c_2^3 = 11.5$
$q_1(c_1^1) = 0.5$	$q_1(c_1^2) = 0.5$	$q_2(c_2^1) = 0.1$	$q_2(c_2^2) = 0.1$	$q_2(c_2^3) = 0.8$
Assumption 1 holds: $\gamma = 1 < \gamma^* = 1.6914$				
$\xi_1^*(c_1^1) = 0.1195$	$\xi_1^*(c_1^2) = 0.0647$	$\xi_2^*(c_2^1) = 0.0205$	$\xi_2^*(c_2^2) = 0.0031$	$\xi_2^*(c_2^3) = 0.0014$
$\vee$		$\wedge$	$\vee$	$\vee$
$x_1^\# = 0.0872$	–	$\xi_2^\#(c_2^1) = 0.0206$	$\xi_2^\#(c_2^2) = 0.0018$	$\xi_2^\#(c_2^3) = 0.0000$

Table II. Equilibrium bids before and after the favorite's self-disclosure.

In fact, the example illustrates another possibility, viz. that a type of the underdog may become so discouraged that she decides to exert zero effort.[38]

5.3 Games of strategic heterogeneity

In parameterized games of strategic heterogeneity (Monaco and Sabarwal, 2016; Barthel and Hoffmann, 2019), strategy spaces are multi-dimensional, and payoff functions allow for strategic complements and substitutes at the same time. Under suitable constraints on bids, the incomplete-information contests considered in the present paper would indeed satisfy the definition. Moreover, the monotone comparative statics of the contest stage with respect to changes in the information structure conducted above clearly draws on intuitions suggested by that literature. Quite notably,

[38] This possibility might be reminiscent of the *drop-out* identified by Parreiras and Rubinchik (2010). However, in their setting, drop-out is caused by the presence of additional players, whereas in our setting, marginalization is caused by disclosed information.

however, existing conditions do not apply to our model. As Examples 1 and 2 have shown, the relevant comparative statics of the Bayesian equilibrium is, in general, monotone for one player only. In contrast, Monaco and Sabarwal's (2016) conditions, like any of the conditions in the literature that we are aware of, imply the monotone comparative statics of the entire equilibrium profile. In fact, the contraction-mapping approach underlying Monaco and Sabarwal's (2016, Th. 5) result need not go through when the contest is too asymmetric. The problem is that, as noted by Wärneryd (2018) in a different context, the iteration of the best response in an asymmetric contest need not be a contraction. Figure 1 in the Introduction illustrates the steepness in the favorite's best response for the case of complete information, but the situation is similar under incomplete information. Therefore, we indeed cannot make use of existing methods.[39]

6. The scope of the strong-form disclosure principle

As explained in the previous section, the nonmonotonicity of the comparative statics with respect to the release of evidence complicates the analysis, which makes it difficult to make predictions about what happens in contests that are not uniformly asymmetric. Some limited conclusions are feasible, however. We start with the case of one-sided incomplete information, and then turn to the case of two-sided incomplete information.

6.1 One-sided incomplete information

If only one player is privately informed, then the strong-form disclosure principle does not require any strong assumption.

Theorem 2. (Generic strong-form disclosure) *Consider a Tullock contest with one-sided incomplete information and generic cost types. Then, the conclusion of Theorem 1 continues to hold true.*

Proof. See Appendix B. □

This result is derived using a different strategy of proof than before. Specifically, for one-sided asymmetric information, marginal analysis via total differentiation of first-order conditions proves tractable, as we show more formally in Section 8. The argument is not entirely mechanic though,

[39] In Appendix B, we provide a numerical example that shows that the iteration of the best response in Example 1 is not a contraction.

mainly because the bid schedule of the informed player may tilt in response to the gradual release of information.

The assumption of cost genericity might need some explanation. The reason for possible exceptions is that the best-response mapping is hump-shaped, so that in the special case with $K_2 = 2$ (and only in that case), there may be two types of the informed player that choose the same bid level. In the proof of Theorem 2, the only use of the Tullock assumption is to show that, generically, two different types $c_2 \in C_2$ use different bids.[40]

In the remainder of this section, we present a collection of examples that illustrate how the conclusion of Theorem 1 may fail if the contest is not uniformly asymmetric. I.e., we will find parameter domains in which the contest with pre-play communication admits additional perfect Bayesian equilibria in which full disclosure is not achieved. Given Theorem 2, all our examples will feature two-sided incomplete information.

6.2 Two-sided asymmetric information

We start with the case of symmetric type distributions.

Proposition 3. (Symmetric type distributions) *Consider an ex-ante symmetric lottery contest with two equally likely type realizations. Then the strong-form disclosure principle applies if* $c_1^2/c_1^1 = c_2^2/c_2^1 < \frac{6\sqrt{31}+31}{5} \approx 12.8$. *Otherwise, i.e., if the inequality does not hold, then there exists a perfect Bayesian equilibrium in which no type discloses her private information.*

Proof. See Appendix B. □

Thus, if the types are relatively different, then the weak type has no incentive to reveal her private information because it would inform the opponent's weak type that there is no strong type around. If the types are close, however, then the mollifying effect on the high type from disclosure would be stronger than the challenging effect on the low type.

In general, i.e., when private information is two-sided, and type distributions are not identical, the incentive to reveal one's type follows a simple intuition, which however proves elusive when one tries to capture it in a formal way. Intuitively, a type has an incentive to reveal her private information if, by doing so, the probability that some type of the opponent feels challenged declines. Challenged type are types that bid in a similar range. However, facing a challenging

[40] It may therefore be speculated that Theorem 2 holds much more generally.

type of the opponent does not, in general, imply that marginal costs are closeby. Rather, it is the overall picture of forces that matters. We show this by displaying two examples where the strong-form disclosure principle breaks down, in the absence of uniform asymmetry.

Example 3. (Straddling type distributions) We say that type distributions are *straddling* if $c_1^1 < c_2^1 < c_1^2 < c_2^2$. An example is the contest specified in Table III. As can be seen, no type has an incentive to reveal her private information. Therefore, the strong-form disclosure principle does not hold in this example.

Player 1		Player 2	
$c_1^1 = 1$ $q_1(c_1^1) = 0.5$	$c_1^2 = 5.5$ $q_1(c_1^2) = 0.5$	$c_2^1 = 2$ $q_2(c_2^1) = 0.9$	$c_2^2 = 6.5$ $q_2(c_2^2) = 0.1$
Assumption 1 does not hold: $\gamma = 1 > \gamma^* = -0.1073$			
$\xi_1^*(c_1^1) = 0.2090$ $\Pi_1^*(c_1^1) = 0.4825$	$\xi_1^*(c_1^2) = 0.0357$ $\Pi_1^*(c_1^2) = 0.0926$	$\xi_2^*(c_2^1) = 0.1031$ $\Pi_2^*(c_2^1) = 0.3303$	$\xi_2^*(c_2^2) = 0.0265$ $\Pi_2^*(c_2^2) = 0.0970$
$x_1^\# = 0.2066$ $\Pi_1^\# = 0.4719$	–	$\xi_2^\#(c_2^1) = 0.1147$	$\xi_2^\#(c_2^2) = 0.0000$
–	$x_1^\# = 0.0440$ $\Pi_1^\# = 0.0916$	$\xi_2^\#(c_2^1) = 0.0988$	$\xi_2^\#(c_2^2) = 0.0342$
$\xi_1^\#(c_1^1) = 0.2171$	$\xi_1^\#(c_1^2) = 0.0343$	$x_2^\# = 0.1015$ $\Pi_2^\# = 0.3299$	–
$\xi_1^\#(c_1^1) = 0.1421$	$\xi_1^\#(c_1^2) = 0.0437$	–	$x_2^\# = 0.0294$ $\Pi_2^\# = 0.0956$

Table III. Straddling type distributions.

In this example, player 2's strong type c_2^1, which is sandwiched between player 1's types, is relatively likely to occur. This keeps player 1's strong type c_1^1 from revealing her private information, since player 2's strong type c_2^1 will show a defiant reaction, while the marginalizing effect on player 2's weak type c_2^2 is unlikely to matter. Neither discloses player 1's weak type c_1^2. Even though there is a mollifying effect on player 2's strong type, the defiant reaction of player 2's weak type c_2^2, which is similar in strength to c_1^2, is definitely unwanted. The intuition for player 2's types is similar.

Example 4. (Nested type distributions) Next, we consider the case where type distributions are *nested*, i.e., where $c_1^1 < c_2^1 < c_2^2 < c_1^2$. In the example shown in Table IV, no type of any player has an incentive to reveal her private information.

Player 1		Player 2	
$c_1^1 = 1$ $q_1(c_1^1) = 0.7$	$c_1^2 = 7$ $q_1(c_1^2) = 0.3$	$c_2^1 = 2$ $q_2(c_2^1) = 0.9$	$c_2^2 = 6$ $q_2(c_2^2) = 0.1$
Assumption 1 does not hold: $\gamma = 1 > \gamma^* = -0.0823$			
$\xi_1^*(c_1^1) = 0.2067$ $\Pi_1^*(c_1^1) = 0.4901$	$\xi_1^*(c_1^2) = 0.0233$ $\Pi_1^*(c_1^2) = 0.0575$	$\xi_2^*(c_2^1) = 0.0999$ $\Pi_2^*(c_2^1) = 0.2716$	$\xi_2^*(c_2^2) = 0.0230$ $\Pi_2^*(c_2^2) = 0.0812$
$x_1^\# = 0.2066$ $\Pi_1^\# = 0.4719$	–	$\xi_2^\#(c_2^1) = 0.1148$	$\xi_2^\#(c_2^2) = 0.0000$
–	$x_1^\# = 0.0348$ $\Pi_1^\# = 0.0496$	$\xi_2^\#(c_2^1) = 0.0930$	$\xi_2^\#(c_2^2) = 0.0375$
$\xi_1^\#(c_1^1) = 0.2144$	$\xi_1^\#(c_1^2) = 0.0208$	$x_2^\# = 0.0968$ $\Pi_2^\# = 0.2711$	–
$\xi_1^\#(c_1^1) = 0.1409$	$\xi_1^\#(c_1^2) = 0.0353$	–	$x_2^\# = 0.0288$ $\Pi_2^\# = 0.0807$

Table IV. Nested type distributions.

7. Commitment and welfare issues

In this section, we discuss the desirability of full revelation from an ex-ante perspective. First, we study the impact of unraveling on the ex-ante utility of an informed contestant. Then, we explore the social desirability of voluntary disclosure. Finally, we address the question if a contestant would wish to shut down communication.

7.1 A commitment problem

While self-disclosure may be individually rational for some type of some player, other types of the same player might subsequently suffer from an increased level of competition. It turns out that this is indeed feasible. More specifically, it will be shown in this section that the unraveling may lead to higher ex-ante levels of rent dissipation for both players and to a lower ex-ante probability of winning for the underdog.

To illustrate this point, we will compare the equilibrium scenario of full revelation (FR) with the hypothetical benchmark of mandatory concealment (MC). Let $\mathbf{C}^{\mathrm{FR}} = E[c_1 x_1^\circ(c_1, c_2) + c_2 x_2^\circ(c_1, c_2)]$ and $\mathbf{C}^{\mathrm{MC}} = E[c_1 \xi_1^*(c_1) + c_2 \xi_2^*(c_2)]$, respectively, denote total expected costs under full revelation and under mandatory concealment.[41] Further, for $i \in \{1, 2\}$, let $p_i^{\mathrm{FR}} = E\left[p_i(x_1^\circ(c_1, c_2), x_2^\circ(c_1, c_2))\right]$ and $p_i^{\mathrm{MC}} = E\left[p_i(\xi_1^*(c_1), \xi_2^*(c_2))\right]$ denote player i's ex-ante probability of winning under full revelation and under mandatory concealment. Finally, likewise for $i \in \{1, 2\}$, let $\Pi_i^{\mathrm{FR}} = p_i^{\mathrm{FR}} - E[c_i x_i^\circ(c_1, c_2)]$ and $\Pi_i^{\mathrm{MC}} = p_i^{\mathrm{MC}} - E_{c_i}[c_i \xi_i^*(c_i)]$ denote player i's ex-ante expected payoff under full revelation and mandatory concealment. A specific setting that allows to draw some clear-cut conclusions is assumed in the following result.

Proposition 4. (Commitment problem) *Consider a uniformly asymmetric, unbiased lottery contest. Suppose that the type of the favorite is public information, whereas the underdog has at least two possible type realizations. Assume also that, under mandatory concealment, all types are active. Then,*

(i) $\mathbf{C}^{\mathrm{FR}} > \mathbf{C}^{\mathrm{MC}}$ *(in both cases, expected costs split evenly between the players);*[42]

(ii) the underdog's (the favorite's) ex-ante probability of winning is strictly lower (strictly higher) under full revelation than under mandatory concealment, i.e., $p_2^{\mathrm{FR}} < p_2^{\mathrm{MC}}$ $(p_1^{\mathrm{FR}} > p_1^{\mathrm{MC}})$*; and*

(iii) the ex-ante payoff for the underdog is strictly lower under full revelation than under mandatory concealment, i.e., $\Pi_2^{\mathrm{FR}} < \Pi_2^{\mathrm{MC}}$.[43]

Proof. See Appendix B. □

The result above shows that the option to disclose private information may be undesirable for a contestant. Intuitively, there is an externality that the disclosing marginal type imposes upon the silent submarginal types. The externality is a virtual one only, because two type realizations of the same contestant never coexist. Notwithstanding, the inability to commit leads to a situation in which the privately informed player loses in expected terms by the unraveling.

[41] $E[.] = E_{c_1, c_2}[.]$ denotes the ex-ante expectation.

[42] Thus, the effort of the favorite is strictly higher under full revelation than under mandatory concealment. The expected effort of the underdog, however, may either rise or fall, depending on parameters.

[43] The payoff comparison for the favorite is ambiguous, i.e., depending on parameters, it may be that $\Pi_1^{\mathrm{FR}} \geq \Pi_1^{\mathrm{MC}}$, or as in Example 5 below, that $\Pi_1^{\mathrm{FR}} < \Pi_1^{\mathrm{MC}}$.

7.2 Pareto-inferior unraveling

The following example illustrates the possibility that full revelation may actually be ex-ante undesirable for both contestants.

Example 5. (**"Disclosure trap"**) The setting specified in Table V satisfies the assumptions of Proposition 4, and hence, illustrates the conclusions of the proposition. More importantly, it can be seen that the unraveling leads the contestants into a strictly Pareto inferior outcome.

Player 1		Player 2	
$c_1^1 = 1$ $q_1(c_1^1) = 1$		$c_2^1 = 2$ $q_2(c_2^1) = 0.5$	$c_2^2 = 3$ $q_2(c_2^2) = 0.5$
Assumption 1 holds: $\gamma = 1 < \gamma^* = 1.2660$			
$x_1^\# = 0.2020$		$\xi_2^\#(c_2^1) = 0.1158$	$\xi_2^\#(c_2^2) = 0.0575$
$x_1^\circ = 0.2222$		$x_2^\circ = 0.1111$	–
$x_1^\circ = 0.1875$		–	$x_2^\circ = 0.0625$
$\mathbf{C}^{\mathrm{FR}} = 0.4097$	$>$	$\mathbf{C}^{\mathrm{MC}} = 0.4040$	
$p_1^{\mathrm{FR}} = 0.7083 > p_1^{\mathrm{MC}} = 0.7071$		$p_2^{\mathrm{FR}} = 0.2917 < p_2^{\mathrm{MC}} = 0.2929$	
$\Pi_1^{\mathrm{FR}} = 0.5035 < \Pi_1^{\mathrm{MC}} = 0.5050$		$\Pi_2^{\mathrm{FR}} = 0.0868 < \Pi_2^{\mathrm{MC}} = 0.0909$	

Table V. Equilibrium bids under full revelation and mandatory concealment.

Thus, in contrast to the more common situation in which the receiver in a persuasion game, such as an employer, a consumer, or a health insurer, tends to benefit from the unraveling, sometimes even unduly so, this need not be the case in a contest.

7.3 Shutting down communication[44]

So far, we assumed that, if one player discloses, the other player automatically gets informed, and this is commonly known. But in some situations, it may be possible to publicly commit to ignore any information provided by one's opponent.

Proposition 5. (The underdog never shuts down communication) *Consider a uniformly asymmetric, unbiased lottery contest. Suppose that the type of the underdog is public information,*

[44] We are grateful to a referee for suggesting this extension to us.

whereas the favorite has at least two possible type realizations. Then, the underdog's ex-ante expected payoff is strictly higher under full revelation than under mandatory concealment, i.e., $\Pi_2^{\mathrm{FR}} > \Pi_2^{\mathrm{MC}}$.

Proof. See Appendix B. □

Thus, the underdog would never prefer to publicly announce to ignore any information received. The intuitive force behind this result is that the underdog can better target her effort, so that the ex-ante winning probability increases. An analogous result for the favorite is not true, however. Indeed, Example 5 above shows that the favorite may benefit from committing to ignore any information released by the underdog.[45]

8. Partial information

So far, we assumed that disclosure is "all-or-nothing". However, in many cases, contestants may have more control over the information they choose to disclose than what has been assumed so far. In this section, we therefore explore the value of partial information release. Specifically, we will discuss partitional disclosures, randomized revelations, noisy signals with one-sided incomplete information, and noisy signals with two-sided incomplete information.

8.1 Partitional disclosures of the state space

In a model with pre-play partitional disclosure of the state space, Hagenbach et al. (2014) identified necessary and sufficient conditions for the existence of a fully revealing sequential equilibrium with "extremal" off-equilibrium beliefs that implements a given Nash equilibrium action profile on and off the equilibrium path. Interestingly, our main result continues to hold provided that contestants' message correspondences each contain an *evidence base*. For example, the disclosure decision might alternatively establish an upper bound for the favorite's cost parameter and a lower bound for the underdog's cost parameter, respectively. It should be immediate that the unraveling argument underlying Theorem 1 extends, in a rather straightforward way, to the more general framework of partitional disclosures. Thus, we complement the analysis of Hagenbach et al. (2014) by providing conditions sufficient for the uniqueness of the perfect Bayesian equilibrium outcome in the special case of probabilistic all-pay contests.

[45] The value of commitment power will be further analyzed in Section 10.

8.2 Randomized revelations

Theorem 1 has been formulated in a framework in which players choose pure strategies at the revelation stage. But little changes if randomized strategies are allowed. This is especially easy to see for the claim that full revelation is a perfect Bayesian equilibrium. Indeed, if a randomized strategy at the revelation stage were to be a profitable deviation, then one of the pure strategies in its support must be a profitable deviation, too, which is impossible by Theorem 1. Neither the uniqueness of the equilibrium outcome is affected. To see this, consider for example the decision of the weakest type of the underdog. Any pure decision regarding disclosure will lead to a lottery over continuation payments, depending on the various scenarios created by the favorite's randomizations. The point to note is that, for any such scenario separately, the logic of Proposition 1 applies. As the weak type of the underdog benefits from self-disclosure in each of these scenarios, she also benefits from self-disclosure in expectation. Moreover, even if the probability of not disclosing her private information was small, there is still a gain for her when switching to self-disclosure with probability one. As before, this argument extends to the other types of the underdog and subsequently to the favorite's side. Thus, Theorem 1 does extend to arbitrary mixed strategies at the revelation stage and, in fact, using our earlier remarks regarding the uselessness of randomization at the contest stage, even in the extensive form.

8.3 Noisy information: one-sided incomplete information

Suppose that player 1's type $c_1^{\#}$ is public, while player 2's type $c_2 \in C_2 = \{\underline{c}_2 \equiv c_2^1, \ldots, c_2^{K_2} \equiv \overline{c}_2\}$ is private with $K_2 \geq 2$. By a *marginal piece of evidence,* we mean a (K_2)-dimensional vector $\delta_2 = (\delta_2^1, \ldots, \delta_2^{K_2})$ such that $\sum_{k=1}^{K_2} \delta_2^k = 0$. The intuition is that δ_2 turns i's prior belief $q_2 \in \Delta(C_2)$ about player 2's type into a nearby posterior $\widetilde{q}_2 \in \Delta(C_2)$ such that $\widetilde{q}_2(c_2^k) = q_2(c_2^k) + \varepsilon\delta_2^k$, where $\varepsilon > 0$ is a small positive number. Given our assumption that all types have a positive ex-ante probability, adding a marginal piece of evidence for small enough $\varepsilon > 0$ will always be feasible in a comparative statics exercise.

Lemma 3. (Necessary and sufficient conditions) *Suppose that player 1's type $c_1^{\#}$ is public, while player 2's type $c_2 \in C_2 = \{\underline{c}_2 \equiv c_2^1, \ldots, c_2^{K_2} \equiv \overline{c}_2\}$ is private with $K_2 \geq 2$. Suppose also that all types of player 2 are active. Then, there exists a positive and strictly hump-shaped (which includes the possibility of strictly monotone increasing or strictly monotone decreasing) sequence $(\varphi^1, \ldots, \varphi^{K_2})$ such that, for any marginal piece of evidence δ_2, any type $c_2 \in C_2$ strictly prefers* *disclosing δ_2 over concealing δ_2 if and only if*

$$\begin{pmatrix} \delta_2^1 \\ \vdots \\ \delta_2^{K_2} \end{pmatrix} \cdot \begin{pmatrix} \varphi^1 \\ \vdots \\ \varphi^{K_2} \end{pmatrix} < 0. \tag{8}$$

Proof. See Appendix B. □

This lemma offers a necessary and sufficient condition for letting any type $c_2 \in C_2$ prefer to release a marginal piece of evidence δ_2. As will be noted, the condition does not depend on c_2, which means that all types have the same preference for disclosure. Obviously, this is due to the assumption of one-sided asymmetric information, which implies that a decline of $x_1^{\#}$ is equally desirable for all types of player 2.[46] Note also that Lemma 3 does not require that the contest is uniformly asymmetric. However, the interiority assumption is crucial. Indeed, inactive types may not have a strict incentive to disclose a marginal piece of evidence if it does not change their state of marginalization. Thus, marginalized types (with positive shadow costs) exhibit some inertia with respect to the release of a marginal piece of evidence.

Condition (8) gets a simple interpretation in the Tullock case, where it turns out that we may choose $\varphi^k = c_2^k \xi_2^{\#}(c_2^k)$, for $k \in \{1, \ldots, K_2\}$, to be the type-specific *equilibrium costs.* In an interior equilibrium, these costs indeed exhibit the hump-shape described in Lemma 3 as a consequence of the first-order condition. Thus, in the Tullock case, a marginal piece of evidence is preferred to be disclosed if, roughly speaking, it makes extremal types (i.e., those with the lowest equilibrium costs) more likely and central types (i.e., those with highest equilibrium costs) less likely.[47]

In addition to marginal analysis, it is also of interest to see players' incentives to release noisy signals (not to be confused with randomized revelations). This question is, in general, harder to address. The following result shows that the weakest type of the underdog, provided she is active, has always a strict incentive to send a noisy signal that corresponds to a first-order increase over her type space, provided that her own type will appear more likely.

Proposition 6. *Consider an unbiased lottery contest, and assume that the type $c_1^{\#}$ of player 1*

[46] This contrasts with the case of two-sided asymmetric information, dealt with below, where preferences regarding information release generally differ across types.

[47] In Appendix B, we use Lemma 3 to prove Theorem 2.

is public, while the type of player 2 is private information. Suppose that $\underline{c}_2 > c_1^{\#}$, and that type $\overline{c}_2$ is active. Then a FOSD shift in the type distribution of player 2 that makes $\overline{c}_2$ strictly more likely induces player 1 to strictly lower her effort $x_1^{\#}$.

Proof. See Appendix B. □

Here as well, the activity assumption is crucial to obtain the conclusion of voluntary disclosure. Note, however, that Proposition 6 does not admit an unraveling conclusion.

8.4 Noisy information: two-sided incomplete information

Finally, we study a setting with two-sided incomplete information. As a complete analysis is beyond the scope of the present paper, we focus on the case of an interior equilibrium in an unbiased lottery contest with two types per player.

Proposition 7. (Two-sided incomplete information) *Consider a uniformly asymmetric contest. Then, in response to a marginal increase by ε in the probability of the underdog's weak type c_2^2, we see that:*

(i) the bid of the strong favorite declines, i.e., $dx_1^1/d\varepsilon < 0$;

(ii) the bid of the weak favorite increases, i.e., $dx_1^2/d\varepsilon > 0$;

(iii) the expected payoff of the weak underdog increases, i.e., $d\Pi_2^*(c_2^2)/d\varepsilon > 0$.

Proof. See Appendix B. □

This proposition refines the analysis underlying Proposition 1, albeit in a special case. Thus, the previous assumption that signals are all-or-nothing is seen to be salient for marginalized types only. Provided that all types are active, that assumption does not seem to be crucial for obtaining the conclusion of Theorem 1.

9. Cheap talk[48]

Up to this point, we assumed that all evidence released by the contestants is verifiable. The present section considers the case of non-verifiable information (Crawford and Sobel, 1982). Thus, in contrast to the setting considered above, pre-play communication is now assumed to take the

[48] We are indebted to the Editor for suggesting this important extension.

form that contestants are simultaneously called upon to send costless, unverifiable messages after they observed their own types but before they choose their actions. As usual, this cheap-talk game admits a "babbling" equilibrium in which messages carry no information regarding the senders' types, and receivers ignore the messages. However, one may wonder if probabilistic contests admit equilibria in which one player reacts effectively to the other player's messages. Fey et al. (2007) studied cheap talk in games with two-sided incomplete information, and identified a role for complementarity vs. substitutability. For two-player all-pay auctions, Pavlov (2013) has shown that cheap talk has no impact on equilibria in deterministic all-pay auctions. While interesting, these contributions do not resolve our research question.

With one-sided incomplete information, all cheap talk in probabilistic contests is necessarily ineffective, regardless of distributional assumptions. This follows from the usual straightforward intuition. Indeed, suppose that messages m_1 and m_2 are sent with positive probability by two types c^1 and c^2 of the privately informed sender so that the equilibrium effort of the receiver is strictly lower for message m_1 than for m_2. Then, subject to activity,[49] both types strictly prefer sending m_1 over sending m_2. Thus, there cannot be an equilibrium in the cheap-talk game with one-sided incomplete information in which messages have any impact on equilibrium efforts. This delivers the next result.

Proposition 8. ("Babbling") *Consider a contest with one-sided incomplete information. Then, all unverifiable messages sent by the informed player are ignored by the uninformed player.*

Proof. See Appendix B. □

Thus, if just one player is in the possession of an informational advantage, cheap talk is necessarily ineffective.[50] Next, we study the conditions necessary for full separation.

Proposition 9. (Impossibility of full separation) *Consider an unbiased lottery contest with cheap talk pre-play communication. Then, any of the following conditions suffices to guarantee that full separation cannot be achieved as a perfect Bayesian equilibrium:*

(i) $K_1 = 1$ *and* $K_2 \geq 3$;

[49] If types can get marginalized, the argument complicates, but still goes through. See Appendix B.

[50] Notwithstanding, due to the hump-shape of the best-response mapping, messages may indeed carry information about types. E.g., if $c_1 = 1$, $\underline{c}_2 = \frac{1}{4}$, and $\overline{c}_2 = 4$, then we have $x_1^{\#} = (\frac{E[\sqrt{c_2}]}{c_1 + E[c_2]})^2 = (\frac{2q+(1-q)\frac{1}{2}}{1+4q+(1-q)\frac{1}{4}})^2 = \frac{4}{25}$, regardless of posterior beliefs. However, in equilibrium, the receiver would not make use of that information.

(ii) there is two-sided incomplete information, with $\bar{c}_1 \leq \underline{c}_2$*;*

(iii) each player has two types, with $\underline{c} \equiv \underline{c}_1 = \underline{c}_2$ *and* $\bar{c} \equiv \bar{c}_1 = \bar{c}_2$*;*

(iv) each player $i \in \{1,2\}$ *has a continuum of types distributed on* $[\underline{c}_i, \bar{c}_i]$ *according to some absolutely continuous and strictly increasing distribution function* $F_i \equiv F_i(c_i)$*, where* $0 \leq \underline{c}_i < \bar{c}_i$*.*[51]

Proof. See Appendix B. □

Part (i) should be intuitive. With full separation, we have a complete-information contest. Hence, player 1's equilibrium bid is positive and strictly hump-shaped in $c_2 \in C_2$. Therefore, given what we said above, at most two signals can be used in equilibrium. Part (ii) should also be intuitive. Case (iii) reveals one reason why full separation is difficult to obtain. In that case, weak types may prefer to *overstate* their abilities. Indeed, those types have little chances to win against a strong opponent, but it helps them a lot to impress the opponent's weak type, even if it means to entirely give up (i.e., self-marginalize) against the opponent's strong type. Only if the probability of being matched with a weak opponent is very low, then lying is no longer optimal for the weak type. Then, instead, *understatement* may become attractive for the efficient opponent. In case (iv), i.e., with continuous type distributions, the weakest type in the contest has always an incentive to overstate her ability.[52]

In his analysis of sender-receiver games with two-sided incomplete information, Seidmann (1990, Ex. 1) has shown that, in general, even if all types of the sender share the same preferences over pure effort choices by the receiver, the sender types' preferences regarding lotteries over efforts may differ. This property can then be used to construct fully separable equilibria. Even though the lottery contest with one-sided incomplete information has precisely this property, our results above show that fully separating equilibria in simple contests are not feasible.[53]

Wrapping up, what we learn from Propositions 8 and 9 is that verifiable information, as considered in the main part of the analysis, matters.

[51] For the modifications to the set-up necessary to deal with continuous type distributions, see Section 12.

[52] The strict incentive to overstate one's abilities relates Proposition 9 to the applied literature (e.g., Alfano and Robinson, 2014).

[53] As we checked numerically, Seidmann's (1990) argument does not seem to apply to Tullock contests with parameter $r \in (0, 1]$.

10. Bayesian persuasion and information design

Kamenica and Gentzkow (2011) considered a general setting with one sender and one receiver, and an unknown state of the world, where the sender precommits to a signal about the state of the world. Upon receiving the signal, the receiver rationally updates her belief about the state of the world and takes an action. Depending on whether the commitment power lies with a player or with the social planner, the approach is known as Bayesian persuasion or information design. In this section, we consider both problems for contests. Information design in contests has been studied before by Zhang and Zhou (2016), and we will review their contribution below.[54]

10.1 Bayesian persuasion

Consider a lottery contest with one-sided incomplete information, where the type $c_1^\#$ of player 1 is public. Then the expected payoff to an active type c_2 is given by

$$\Pi_2^\#(c_2|\mu_2) = \left(1 - \frac{\sqrt{c_2}E[\mathbf{1}_{\widetilde{c}_2 \text{ active}}\sqrt{\widetilde{c}_2}|\mu_2]}{c_1^\# + E[\mathbf{1}_{\widetilde{c}_2 \text{ active}}\widetilde{c}_2|\mu_2]}\right)^2, \tag{9}$$

where $\mathbf{1}_{\widetilde{c}_2 \text{ active}}$ is an indicator variable that equals one (zero) if $\xi_2^\#(\widetilde{c}_2; c_1^\#) > 0$ $(= 0)$ in the contest with one-sided incomplete information and type distribution μ_2 for player 2, and where $E[.|\mu_2]$ denotes the expectation given player 1's belief $\mu_2 \in \Delta(C_2)$ about 2's types at the contest stage.[55] The logic of marginalization is as follows. As the belief μ_2 gives too much weight to strong types, player 1 bids higher which induces weak types of player 2 to bid zero. The condition spelt out in the following lemma ensures that marginalization does not occur for a persuasion model with two types.

Lemma 4. (Interiority condition) *Suppose that $K_2 = 2$ and $c_1^\# + \underline{c}_2 > \sqrt{\underline{c}_2\overline{c}_2}$. Then, all types $c_2 \in C_2$ of player 2 are active, regardless of player 1's posterior belief μ_2.*

Proof. See Appendix B. □

Now, in the absence of communication, μ_2 simply corresponds to the ex-ante distribution $\{q_2(c_2^k)\}_{k=1}^{K_2}$. Bayesian persuasion allows player 2 to precommit to a signal, which induces a probability distri-

[54] Denter et al. (2014) compared no and full disclosure.

[55] The expected payoff of an inactive type is zero.

bution $\tau_2 \in \Delta(\Delta(C_2))$ over posterior beliefs $\mu_2 \in \Delta(C_2)$ that is subject to Bayes plausibility

$$\int \mu_2(c_2) d\tau_2(\mu_2) = q_2(c_2) \qquad (c_2 \in C_2). \tag{10}$$

Therefore, player 2's problem reads

$$\max_{\tau_2 \text{ s.t. } (10)} \int E_{c_2} \left[\Pi_2^{\#}(c_2|\mu_2) \right] d\tau_2(\mu_2), \tag{11}$$

where $E_{c_2}[.]$ denotes, as before, the expectation with respect to the prior distribution on C_2 given by q_2.

As a general solution of problem (11) is beyond the scope of the present analysis, we discuss a simple example with $K_2 = 2$. Then, with precommitment, the signal may lead to a probability distribution τ_2 over two distributions $\mu_2^{\text{A}}, \mu_2^{\text{B}} \in \Delta(C_2)$, with respective probabilities τ_2^A and τ_2^B satisfying

$$\tau_2^{\text{A}} \mu_2^{\text{A}}(c_2) + \tau_2^{\text{B}} \mu_2^{\text{B}}(c_2) = q_2(c_2) \qquad (c_2 \in C_2). \tag{12}$$

For instance, in the special case where $\overline{c}_2 > c_1^{\#} > \underline{c}_2$ then we might expect that player 2 benefits if, compared to the prior, μ_2^{A} is biased towards $\underline{c}_2$, while μ_2^{B} is biased towards $\overline{c}_2$. Intuitively, the positive effect of overstatement on the weak type's payoff would be combined with the likewise positive effect of understatement on the strong type's payoff.

Proposition 10. (Bayesian persuasion) *Consider an unbiased lottery contest where player 1's type $c_1^{\#}$ is public, while player 2's type $c_2 \in \{\underline{c}_2, \overline{c}_2\}$ is private with $\overline{c}_2 > \underline{c}_2$. Suppose also that the interiority assumption of Lemma 4 holds. Then there exists a threshold value $\chi^* \in [0,1]$ such that:*

(i) If $c_1^{\#} > \sqrt{\underline{c}_2 \overline{c}_2}$, then full disclosure is optimal;

(ii) if $c_1^{\#} = \sqrt{\underline{c}_2 \overline{c}_2}$, then any signal is optimal;

(iii) if $c_1^{\#} < \sqrt{\underline{c}_2 \overline{c}_2}$ and $q_2(\underline{c}_2) \leq \chi^*$, then full concealment is optimal;

(iv) if $c_1^{\#} < \sqrt{\underline{c}_2 \overline{c}_2}$ and $q_2(\underline{c}_2) > \chi^*$, then player uses a randomized signal with posterior beliefs satisfying $\mu_2^{\text{A}}(\underline{c}_2) = \chi^*$ and $\mu_2^{\text{B}}(\underline{c}_2) = 1$.

Proof. See Appendix B. □

The rough intuition for the underlying effects here is that a stronger uninformed contestant raises her efforts in response to uncertainty, whereas a weaker uninformed contestant lowers her efforts in response to uncertainty (cf. Hurley and Shogren, 1998a). With $c_1^\# > \sqrt{\underline{c}_2\overline{c}_2}$, player 1 is comparably weak, so it makes sense for player 2 to inform player 1. In the knife-edge case where $c_1^\# = \sqrt{\underline{c}_2\overline{c}_2}$, player 1 does not care about player 2's type as each type chooses the same bid level. Hence, any signal is optimal in that case. The situation gets more structured for $c_1^\# < \sqrt{\underline{c}_2\overline{c}_2}$, where player 1 is comparably strong. In that case, the signal will never be fully informative. Instead, either full concealment is optimal (if $q_2(\underline{c}_2) \leq \chi^*$), or player 2 optimally uses a randomized signal (if $q_2(\underline{c}_2) > \chi^*$). When a randomized signal is used, player 2 reveals her type when strong with a probability τ_2^{B} strictly smaller than one, but never reveals her type when weak. As we show in the proof of Proposition 10, player 2's expected payoff $\widehat{\Pi}_2(\mu_2)$ in a contest with posterior μ_2, considered as a function of m, is concave left of some cut-off value m^* and convex right of m^*. In Appendix B, we provide a numerical example that shows that the potentially unexpected case (iv) with $\chi^* > 0$ is indeed feasible.

10.2 Information design

Next, we assume that an informed contest designer chooses a signal so as to maximize some policy objective. Upon receiving the realization of the signal, the uninformed player updates her belief and the contest takes place. The following result characterizes the optimal signal for three specific policy objectives, viz. maximizing total expected efforts, maximizing total expected payoffs, and minimizing the expected quadratic distance of players' winning probabilities.

Proposition 11. (Information design) *Consider an unbiased lottery contest where player 1's type $c_1^\#$ is public, while player 2's type $c_2 \in \{\underline{c}_2, \overline{c}_2\}$ is private. Suppose also that the interiority assumption of Lemma 4 holds. Then, for a contest designer:*

(i) maximizing total expected efforts, full disclosure (full concealment, any signal) is optimal if $c_1^\# < \sqrt{\underline{c}_2\overline{c}_2}$ (if $c_1^\# > \sqrt{\underline{c}_2\overline{c}_2}$, if $c_1^\# = \sqrt{\underline{c}_2\overline{c}_2}$);

(ii) maximizing total expected payoffs, it is optimal to delegate the problem to player 2, i.e., to use the signal characterized in Proposition 10;

(iii) minimizing $E_{c_2}[(p_1 - p_2)^2]$, it is optimal to use the signal characterized in part (i).

Proof. See Appendix B. □

Part (i) says that, to maximize expected efforts, full disclosure is optimal if player 1 is comparably strong (i.e., if $c_1^{\#} < \sqrt{\underline{c}_2 \overline{c}_2}$), while full concealment is optimal if player 1 is comparably weak (i.e., if $c_1^{\#} > \sqrt{\underline{c}_2 \overline{c}_2}$), with any signal being optimal in the knife-edge case where $c_1^{\#} = \sqrt{\underline{c}_2 \overline{c}_2}$. Part (i) is a straightforward reformulation of a well-known result due to Zhang and Zhou (2016).[56] For parts (ii) and (iii), however, we have not found a suitable reference. Part (ii) is a statement about decentralization. Part (iii) may not be too surprising. Indeed, under the assumptions made, minimizing the expected quadratic distance turns out to be equivalent to maximizing total expected efforts.

However, there is an intuitive tension between part (iii) and the discussion in Section 7. Specifically, in a setting with a comparably strong player 1 in which both Assumption 1 holds and $c_1^{\#} < \sqrt{\underline{c}_2 \overline{c}_2}$, we find here that the optimal signal entails full disclosure, whereas Proposition 4(ii) implies mandatory concealment. To understand what is going on, note that Proposition 11 works with a quadratic distance of probabilities, whereas Proposition 4(ii) works with ex-ante winning probabilities. Therefore, the policy objective considered here, intuitively speaking, places overproportional weight on the most lopsided encounters, whereas the earlier discussion weights all encounters according to their ex-ante probability of occurrence, which explains the difference in conclusions.

An interesting policy objective is also the maximization of the expected highest bid. In general, that problem may be difficult. Imposing Assumption 1, however, the problem simplifies. Indeed, since player 1 is known to submit the highest bid, and ex-ante expected costs are identical for both bidders, the problem becomes equivalent to the one considered in part (i) above.

11. Correlation

In this section, we discuss how Theorem 1 extends to the case of correlated types. We start with a particularly clean case in which type distributions are negatively correlated.

Proposition 12. (Negative correlation) *Suppose that Assumption 1 holds and that the conditional belief $\mu_2(\cdot \mid c_2) \in \Delta(C_1)$ held by the underdog's type c_2 is first-order stochastically decreasing*

[56] In their case, however, private information is about valuations. Zhang and Zhou (2016) also offer an algorithm for solving the case with $K_2 \geq 3$ types. With more than two types, if the uninformed player is strong enough, full disclosure is optimal, otherwise pooling the highest two valuations together and fully separating the others maximizes total efforts. The paper points out the difficulties that arise in a setting with two-sided incomplete information, namely the multi-dimensional state of nature of both contestants' valuations which complicates the persuasion stage, the private information on two sides, where the simplifying step of the analysis of Kamenica and Gentzkow (2011) cannot be applied and lastly, the equilibrium characterization which is in general not available.

in $c_2 \in C_2$. Then, the conclusion of Theorem 1 continues to hold true.

Proof. See Appendix B. □

The intuition is simple. The assumption on conditional beliefs means that weaker types of the underdog are more pessimistic, in the sense that they deem stronger types of the favorite more likely. As a result, the best-response bid schedule of the underdog remains strictly declining in the interior, so that the conclusions of the crucial Proposition 1 continue to hold true. Moreover, once the side of the underdog has unraveled, any ex-ante correlation will be resolved, so that full separation obtains as before via Proposition 2.

For similar reasons, the strong-form disclosure principle holds for general forms of correlation provided that the degree of correlation is small enough. For strongly positively correlated types, however, the situation may complicate. In fact, the literal conclusions of Proposition 1 may break down, as the following example illustrates.

Example 6. (**Positive correlation**) Consider the contest specified in Table VI. Shown are the equilibrium bids with and without disclosure by the weak type of the underdog. As can be seen, disclosure induces both types of the favorite to bid higher. Hence, the weak type of the underdog does not benefit from disclosing her type.

Player 1		Player 2	
	$c_2^1 = 5.5$	$c_2^2 = 6$	
$c_1^1 = 1$	0.45	0.05	
$c_1^2 = 1.5$	0.05	0.45	

Assumption 1 holds: $\gamma = 1 < \gamma^* = 2.265$

$\xi_1^*(c_1^1) = 0.1318$	$\xi_1^*(c_1^2) = 0.1056$	$\xi_2^*(c_2^1) = 0.0242$	$\xi_2^*(c_2^2) = 0.0262$
$\wedge$	$\wedge$		$\vee$
$\xi_1^\#(c_1^1) = 0.1353$	$\xi_1^\#(c_1^2) = 0.1057$	–	$x_2^\# = 0.0260$

Table VI. Positive affiliation.

The logic of the example is that, without disclosure, the strong type of the favorite expects meeting the strong type of the underdog that paradoxically bids lower than the weak type of the underdog. Therefore, with disclosure, the strong type of the favorite raises her bid. In contrast, disclosure does not substantially change the belief of the favorite's weak type, but she expects meeting the weak type with somewhat higher probability, which makes her bid higher.

Notwithstanding the complication illustrated by Example 6, there is a simple intuition suggesting that the strong-form disclosure principle "generically" continues to hold with any type of correlation. The point to note is that, even though the weakest type of the underdog need not have a strict incentive to reveal her private information, the type of the underdog with the lowest bid will always have such a strict incentive—unless we are in the unlikely scenario in which positive correlation induces *all* types of the underdog to choose the same bid. The reason is that the proof of Proposition 1 does not depend on the natural ordering of cost types. Thus, even though the literal conclusions of Proposition 1 need not hold with arbitrary correlation, intuition suggests that a straightforward variant of the proposition should oftentimes remain valid, in which case an unraveling would be the necessary consequence.

12. Continuous type distributions

Benoît and Dubra (2006) have derived a general unraveling result for auctions and other Bayesian games that allows for multiple players and metric types spaces. In this section, we use their arguments to extend Theorem 1 to the case of continuous type distributions.[57] In contrast to the assumptions made so far, we will assume now that for $i \in \{1, 2\}$, player i's marginal cost is drawn from an interval $[\underline{c}_i, \overline{c}_i]$, with $0 < \underline{c}_i < \overline{c}_i$, according to some continuous distribution function F_i. Note that Definition 1 extends to this case in a straightforward way, with the probability ranking property (6) required for any pair of cost realizations in the support of players' posterior beliefs. Thus, we may certainly consider a uniformly asymmetric contest with continuous type distributions. The sufficient condition reflected in Lemma 2 extends to this setting. As a solution concept for the contest stage, and thereby to define expected payoffs from the strategic interaction following pre-play communication, we use pure-strategy Nash equilibrium (Ewerhart, 2014). The reduced-form definition of perfect Bayesian equilibrium remains unchanged. However, to ensure

[57] Contests with continuous type distributions have been considered, in particular, by Fey (2008), Ryvkin (2010), Wasser (2013a, 2013b), and Ewerhart (2014).

continuity properties of type-specific payoffs, we will have to restrict attention to the special case of the lottery contest.

Proposition 13. (Continuous type distributions) *Consider a uniformly asymmetric lottery contest with continuous and independent type distributions. Then, in any perfect Bayesian equilibrium of the contest with pre-play communication of verifiable information, both contestants' types are almost surely revealed at the contest stage.*

Proof. See Appendix B. □

13. Applications

In this section, we review our introductory examples and relate our findings back to those.

13.1 Research and development

Patent races can be financially exhausting. To stay in the race, firms must have a realistic chance of winning. Drop-outs of followers may be caused, in particular, by the disclosure of a breakthrough by the leader in the race. According to Fudenberg et al. (1983), the decision between surrender and continuing in an ongoing contest depends on the possibility that a firm can "leapfrog" and become the favorite in the race. This possibility is closely related to our analysis above. Specifically, disclosure of breakthroughs by the leader may signal that leapfrogging is no longer possible for the relatively weaker competitors, inducing them to exit the race, as in Example 2. The delegation conclusion in Proposition 11 suggests that a non-interventionalist position may be optimal.

13.2 Pretrial discovery and settlement

Especially in the US, the outlook on the tremendous costs of going to court sets clear incentives for parties of legal dispute to find agreeable terms at an early stage (Shavell, 1989). This is particularly the case if, as suggested by Rosen (1988, p. 80), "judgement is shared by both parties that one has the winning hand, and some of the costs of litigation are avoided in that case." Unraveling as captured by Theorems 1 and 2 is then to be expected especially on the side of the stronger party. Anecdotal evidence suggests a strong link between the extent of information revelation feasible during the pretrial phase and the symmetry or asymmetry of the conflict. If information is not shared voluntarily, parties can obtain evidence from the other side by means of discovery devices such as interrogatories, requests, and depositions.

13.3 Promotion tournaments

Our theory might also help to explain why, in certain professional environments, there is sometimes only a single candidate in an election or promotion. In general, this may be due to the exchange of information (both verifiable and non-verifiable) between the candidates that ultimately induce all but one candidate to back off. In firms or authoritarian systems, successors for positions may be *designated* long before the start of the position.[58] To stop communication between candidates, designers of elections or promotion tournaments may therefore publicly announce to treat applications confidential as long as possible, in line with our discussion of Proposition 4 and Example 5.

13.4 Social and biological conflict

The empirical literature on social conflict (e.g., Hand, 1986) has identified signaling conventions that resolve or avoid physical conflicts in dyadic social relationships depending on their nature. For conflicts arising within dominance or subordination relationships, signals tend to be placating or acquiescent. Within egalitarian or unresolved relationships, however, there may either be no signals, or signals may indicate relative desire for some item on a case-by-case basis. Theorem 1 is in line with voluntary signaling in the first case, and our discussion of Proposition 3 and Examples 3 and 4 with the ambiguity in the second case. Thus, the patterns underlying these empirical observations are indeed reflected in our theory. A conceptual discussion of dominance and defiance, with numerous examples from politics and history, can be found in Caygill (2013).

13.5 Military crisis and war

Our analysis may have implications for the role of communication as a means to avert military crisis. Fearon (1995) uses historical case studies to argue that private information about resolve and ability, incentives to hide or misrepresent these, and the lack of commitment power are important ingredients for a rationalist explanation for why war comes about. Acts of supplication (Pedrick, 1982; van Kleef et al., 2006) may be seen as underdog tactics to avert imminent conflict. Atomic weapon tests (Beardsley and Asal, 2009), the intimidation of witnesses (Maynard, 1994),

[58] E.g., Chen (2003) mentions the early designation of a successor in Imperial China as a tactic against sabotage by weaker competitors.

the announcement and use of military high-tech (such as chemical nerve agents against Russian dissidents, pulsed microwave against European diplomats, supra-sonic missiles, etc.) are all examples of a show of force with clear intention to impress the other side. This is undesirable because it leads to more armament and international conflict. Witness protection programs effectively stop the communication. In more balanced settings, however, disclosure cannot be expected to take place voluntarily, with weapons require contracts regarding regular inspections.

14. Concluding remarks

In this paper, we have identified general and robust conditions under which a probabilistic contest with verifiable pre-play communication admits full disclosure as the unique perfect Bayesian equilibrium outcome. Given that the usual assumptions for the uniqueness of the fully revealing equilibrium outcome (Milgrom, 1981; Okuno-Fujiwara et al., 1990; Seidmann and Winter, 1997; van Zandt and Vives, 2007) fail to hold for contests, our results mean an extension of existing theory. In particular, the strong-form disclosure principle is more general than previously perceived.

The analysis conducted in this paper has shown that incentives for voluntary disclosure in contests depend on whether the release of information tends to tighten or ease competition. If the contest is uniformly asymmetric, then the logic is particularly transparent. The extremal weakest type of the underdog is never protected by a stronger type that would keep the favorite in check. Therefore, it is always beneficial for her to reveal her type. Conversely, the extremal strongest type of the favorite cannot hope to be mistaken for bidding lower than the underdog, so that disclosure is likewise beneficial for her. Consistent with this logic, Theorem 1 captures a situation in which parties to an imminent conflict are naturally inclined to disclose their private information. Theorem 2 requires one-sided asymmetric information only, regardless of roles. The logic here is that at least one of the extremal types, either the strongest or the weakest, will have a strict incentive to disclose her private information. However, if either the contest fails to be uniformly asymmetric or if the incomplete information is two-sided, then there may be a trade-off, since revealing one's type implies the risk that the opponent becomes an even match in terms of bid levels.

Superficially, our main conclusion is just the opposite of the corresponding finding for the all-pay auction. Specifically, Kovenock et al. (2015, Prop. 5) says that, with either private or common values, the interim information sharing game admits a perfect Bayesian equilibrium in which no firm ever shares its information. As noted by a referee, there is an important caveat here because the literature on all-pay auctions (Kovenock et al., 2015; Tan, 2016) has tended to focus on symmetric type distributions, while our analysis has focused on asymmetric contests. However, Proposition 3 suggests (in a somewhat non-robust way, though) that it is the contest technology that matters. Indeed, the difference in conclusions might mirror a more general fact, viz. that an all-pay auction that admits only equilibria in randomized strategies makes different demands on a player than a probabilistic contest that admits an equilibrium in pure strategies. For example, while the auction induces a hide-and-seek type of behavior for which keeping private information secret seems advisable, the probabilistic contest induces players to think in trade-offs, which may then entail even the voluntary disclosure of private information to the opponent. More work on the relationship of these two "battle modes" in contests is certainly desirable.

Our analysis took us naturally to a formalization of several intuitive concepts for which, to our knowledge, a flexible and all-encompassing framework in the realm of contest theory has been lacking so far. These concepts include strategic attempts of individual types to either moderate or discourage an opponent through pre-play communication of verifiable information, as well as the possibility nonmonotone, "dominant" or "defiant", reactions to such evidence. Clearly, these findings are a bit unexpected given the absence of behavioral elements in our framework.[59]

In addition to the purely game-theoretic analysis of pre-contest communication with verifiable and non-verifiable messages, we explored the role of disclosure policy and commitment power on individual payoffs and social welfare, covering also questions such as ex-ante optimal design of signals by an informed player or by a contest designer. Three policy conclusions seem to us as being most salient. First, it may sometimes be optimal to prohibit the exchange of information. This general theme of the anti-discrimination and diversity debate just reappears in our framework. Second, it may be optimal to shut down communication. For instance, a government might prefer closing the official diplomatic channels in foreign policy, so as to avoid piecemeal destruction of its ethical principles. Finally, it may be optimal to delegate disclosure policy to an informed player. For example, in research and development, centrally enforced disclosure (e.g., of data sets, software, and preliminary results put together by university members) may be counter-productive.

[59]The further analysis of such effects appears to us as a valuable route for future research. For example, it might be interesting to understand why nonmonotone reactions to information disclosure are more common for worst-case types.

Given constraints in tractability, however, some of these conclusions have been derived under somewhat restrictive assumptions. Needless to say, for example, it would be a great achievement to be able to say anything nontrivial about information design in contests with two-sided incomplete information. Another potential limitation of the present study is that our main result, while addressing a sort of natural question from the perspective of the existing literature, may not shed much light on traditional issues in competition policy such as the regulation of product quality disclosure or optimal standards for financial reporting. We also concede that the empirical validation of uniform asymmetry might remain challenging, even though suitable proxies for the ability parameter (such as number of experts, size of research budgets, number of tanks, etc.) may often be readily available. Notwithstanding, we believe that the theoretical advancement made in this paper is important not only because, judging from our conversations, the mechanics of the model are broadly in line with how people tend to think about the topic, but also because our conclusions indeed seem to capture anecdotal evidence, e.g., from pre-trial discovery, promotion tournaments, and conflict resolution. In addition, the methods developed in the course of the analysis might prove useful for tackling further areas of application which rely on similar mechanics but work under different assumptions.

Appendix A. Auxiliary results

In this Appendix, we state and prove a number of auxiliary results. Lemma A.1 collects some properties of a transformation introduced by Wärneryd (2003).[60] Lemma A.2 establishes a basic monotonicity property of the best-response bid schedule. Lemma A.3 provides bounds on the bid distributions. Lemma A.4 establishes the Stackelberg monotonicity property of the complete-information contest. Finally, Lemma A.5 offers an extension of Jensen's inequality.

Lemma A.1 (Wärneryd's transformation) *Let* $\Phi(z) = h(z)/h'(z)$, *for any* $z > 0$. *Then, the following holds true:*

(i) $\lim_{z \searrow 0} \Phi(z) = 0$;

(ii) $1 \leq \Phi' \leq \underline{\rho}$;

(iii) $(d \ln h)/(d \ln \Phi) = 1/\Phi'$;

(iv) if $x_i > 0$, *then player* i*'s best-response mapping in the complete-information contest is differentiable with*

$$\frac{dx_i}{dx_j} = \frac{\Phi(x_i)}{\Phi(x_j)} \frac{2p_i - 1}{\Phi'(x_i) - 1 + 2p_i}, \tag{13}$$

where $i, j \in \{1, 2\}$ *with* $j \neq i$, *and* $p_i = p_i(x_1, x_2)$.

Proof. (i) By assumption, h is differentiable in the interior of the strategy space, with h' positive and declining. Hence, $\lim_{z \to 0} h'(z) \in (0, \infty]$. Moreover, by continuity, $\lim_{z \to 0} h(z) = 0$. The claim follows. (ii) Note first that $\Phi' = 1 - (hh''/(h')^2) \geq 1$ by the concavity of h. To see that $\Phi' \leq \underline{\rho}$, take some $\rho > \underline{\rho}$ such that h^ρ is convex. Then, in the interior of the strategy space,

$$\rho(\rho - 1)h^{\rho - 2} \left(h'\right)^2 + \rho h^{\rho - 1} h'' \geq 0. \tag{14}$$

Recall that $\underline{\rho} \geq 1$. Hence, $\rho > 1$. Dividing (14) by $\rho h^{\rho-2}(h')^2 > 0$, and rearranging, one obtains $\Phi' \leq \rho$. Taking the limit $\rho \to \underline{\rho}$, the claim follows. (iii) A straightforward calculation shows that

$$\frac{d \ln h(z)}{d \ln \Phi(z)} = \left(\frac{dh(z)}{h(z)}\right) \bigg/ \left(\frac{d\Phi(z)}{\Phi(z)}\right) = \frac{h'(z)dz}{h(z)} \cdot \frac{\Phi(z)}{\Phi'(z)dz} = \frac{1}{\Phi'(z)} \qquad (z > 0), \tag{15}$$

[60] See also Inderst et al. (2007).

as claimed. (iv) The first-order condition characterizing the best response x_i reads

$$p_i(1-p_i) = c_i\Phi(x_i). \tag{16}$$

Total differentiation of (16) delivers

$$(1-2p_i)dp_i = c_i\Phi'(x_i)dx_i, \tag{17}$$

where

$$\begin{aligned} dp_i &= \frac{p_i(1-p_i)}{\Phi(x_i)}dx_i - \frac{p_i(1-p_i)}{\Phi(x_j)}dx_j &(18)\\ &= c_i dx_i - c_i\frac{\Phi(x_i)}{\Phi(x_j)}dx_j. &(19)\end{aligned}$$

Plugging the expression in (19) into (17) and collecting terms, we obtain (13). □

Lemma A.2 (Monotonicity of best-response bid schedules) *Let $\xi_j \in X_j^*$ and $c_i, \widehat{c}_i \in C_i$ for $i \neq j$ such that $c_i > \widehat{c}_i$. Then, $\widetilde{\beta}_i(\xi_j; c_i) \leq \widetilde{\beta}_i(\xi_j; \widehat{c}_i)$, where the inequality is strict if $\widetilde{\beta}_i(\xi_j; \widehat{c}_i) > 0$.*

Proof. Take an arbitrary bid schedule $\xi_j \in X_j^*$ of player j. The assertion is obvious for $\widetilde{\beta}_i(\xi_j; c_i) = 0$. Suppose instead that $x_i \equiv \widetilde{\beta}_i(\xi_j; c_i) > 0$. Then, the necessary first-order condition for type c_i implies

$$\frac{\partial E_{c_j}[p_i(x_i, \xi_j(c_j))]}{\partial x_i} = c_i. \tag{20}$$

We will show first that player i's marginal probability of winning, i.e., the left-hand side of Eq. (20), is strictly declining in i's bid. Indeed, because the best-response bid $\widetilde{\beta}_i(\xi_j; c_i)$ exists, there is a type $c_j \in C_j$ such that $\xi_j(c_j) > 0$. A straightforward calculation shows, therefore, that

$$\begin{aligned} &\frac{\partial^2 E_{c_j}[p_i(x_i, \xi_j(c_j))]}{\partial x_i^2} \\ &= \frac{\partial}{\partial x_i}E_{c_j}\left[\frac{\gamma_i\gamma_j h'(x_i)h(\xi_j(c_j))}{(\gamma_i h(x_i) + \gamma_j h(\xi_j(c_j)))^2}\right] &(21)\\ &= E_{c_j}\left[\frac{\gamma_i\gamma_j h(\xi_j(c_j))\left\{(\gamma_i h(x_i) + \gamma_j h(\xi_j(c_j)))h''(x_i) - 2\gamma_i(h'(x_i))^2\right\}}{(\gamma_i h(x_i) + \gamma_j h(\xi_j(c_j)))^3}\right] &(22)\\ &< 0, &(23)\end{aligned}$$

which proves the claim. There are now two cases. Assume first that $\widehat{x}_i > 0$. For this case, it is claimed that $\widehat{x}_i > x_i$. To provoke a contradiction, suppose that $\widehat{x}_i \leq x_i$. Then, since the marginal probability of winning for player i is strictly declining in i's bid,

$$\widehat{c}_i = \frac{\partial E_{c_j}[p_i(\widehat{x}_i, \xi_j(c_j))]}{\partial x_i} \geq \frac{\partial E_{c_j}[p_i(x_i, \xi_j(c_j))]}{\partial x_i} = c_i, \tag{24}$$

in conflict with $\widehat{c}_i < c_i$. Hence, $\widehat{x}_i > x_i$, as claimed. Assume next that $\widehat{x}_i = 0$, i.e., type $\widehat{c}_i$ finds it optimal to respond to ξ_j with a zero effort. But then, clearly, strictly higher marginal costs induce type c_i to do the same, i.e., $x_i = 0$. The lemma follows. □

Lemma A.3 (Bounds on the bid distributions) *Let $\xi^* = (\xi_1^*, \xi_2^*)$ be a Bayesian equilibrium in an incomplete-information contest such that both players are active with probability one. Then,*

$$\gamma_i h(\xi_i^*(\underline{c}_i)) \leq \frac{1}{\pi_i} \cdot \gamma_i h(\xi_i^*(\overline{c}_i)) + \frac{1-\pi_i}{\pi_i} \cdot \gamma_j h(\xi_j^*(\underline{c}_j)) \qquad (i, j \in \{1,2\} \text{ s.t. } j \neq i), \tag{25}$$

$$h(\xi_2^*(\overline{c}_2)) \leq \frac{1}{\widehat{\sigma}} \cdot h(\xi_1^*(\underline{c}_1)), \tag{26}$$

where

$$\widehat{\sigma} = \begin{cases} \sigma & \text{if } \sigma \leq 1 \\ \sigma^{1/\underline{\rho}} & \text{if } \sigma > 1. \end{cases} \tag{27}$$

Proof. Take an arbitrary type $c_i \in C_i$ of player i. Since, by assumption, $\xi_i^*(c_i) > 0$, the necessary first-order condition for type c_i holds, i.e.,

$$E_{c_j}\left[\frac{\gamma_i \gamma_j h'(\xi_i^*(c_i)) h(\xi_j^*(c_j))}{(\gamma_i h(\xi_i^*(c_i)) + \gamma_j h(\xi_j^*(c_j)))^2}\right] - c_i = 0, \tag{28}$$

where $j \neq i$. To prove the first claim, evaluate (28) at $c_i = \overline{c}_i$. Then, making use of Lemma A.2

and the concavity of h, we get

$$\overline{c}_i = E_{c_j}\left[\frac{\gamma_i\gamma_j h'(\xi_i^*(\overline{c}_i))h(\xi_j^*(c_j))}{(\gamma_i h(\xi_i^*(\underline{c}_i)) + \gamma_j h(\xi_j^*(c_j)))^2}\cdot\left(\frac{\gamma_i h(\xi_i^*(\underline{c}_i)) + \gamma_j h(\xi_j^*(c_j))}{\gamma_i h(\xi_i^*(\overline{c}_i)) + \gamma_j h(\xi_j^*(c_j))}\right)^2\right] \tag{29}$$

$$= E_{c_j}\left[\frac{\gamma_i\gamma_j h'(\xi_i^*(\overline{c}_i))h(\xi_j^*(c_j))}{(\gamma_i h(\xi_i^*(\underline{c}_i)) + \gamma_j h(\xi_j^*(c_j)))^2}\cdot\underbrace{\left(1+\frac{\gamma_i h(\xi_i^*(\underline{c}_i)) - \gamma_j h(\xi_i^*(\overline{c}_i))}{\gamma_i h(\xi_i^*(\overline{c}_i)) + \gamma_j h(\xi_j^*(c_j))}\right)^2}_{\text{monotone increasing in } c_j}\right] \tag{30}$$

$$\geq E_{c_j}\left[\frac{\gamma_i\gamma_j h'(\xi_i^*(\overline{c}_i))h(\xi_j^*(c_j))}{(\gamma_i h(\xi_i^*(\underline{c}_i)) + \gamma_j h(\xi_j^*(c_j)))^2}\right]\cdot\left(\frac{\gamma_i h(\xi_i^*(\underline{c}_i)) + \gamma_j h(\xi_j^*(\underline{c}_j))}{\gamma_i h(\xi_i^*(\overline{c}_i)) + \gamma_j h(\xi_j^*(\underline{c}_j))}\right)^2 \tag{31}$$

$$= E_{c_j}\left[\frac{\gamma_i\gamma_j h'(\xi_i^*(\underline{c}_i))h(\xi_j^*(c_j))}{(\gamma_i h(\xi_i^*(\underline{c}_i)) + \gamma_j h(\xi_j^*(c_j)))^2}\right]$$

$$\times\underbrace{\left(\frac{h'(\xi_i^*(\overline{c}_i))}{h'(\xi_i^*(\underline{c}_i))}\right)}_{\geq 1}\cdot\left(\frac{\gamma_i h(\xi_i^*(\underline{c}_i)) + \gamma_j h(\xi_j^*(\underline{c}_j))}{\gamma_i h(\xi_i^*(\overline{c}_i)) + \gamma_j h(\xi_j^*(\underline{c}_j))}\right)^2 \tag{32}$$

$$\geq \underline{c}_i\cdot\left(\frac{\gamma_i h(\xi_i^*(\underline{c}_i)) + \gamma_j h(\xi_j^*(\underline{c}_j))}{\gamma_i h(\xi_i^*(\overline{c}_i)) + \gamma_j h(\xi_j^*(\underline{c}_j))}\right)^2. \tag{33}$$

Dividing by $\underline{c}_i > 0$, and using $\pi_i = \sqrt{\underline{c}_i/\overline{c}_i}$, we obtain

$$\frac{\gamma_i h(\xi_i^*(\underline{c}_i)) + \gamma_j h(\xi_j^*(\underline{c}_j))}{\gamma_i h(\xi_i^*(\overline{c}_i)) + \gamma_j h(\xi_j^*(\underline{c}_j))} \leq \frac{1}{\pi_i}. \tag{34}$$

Inequality (25) follows. To prove the second claim, one multiplies type c_i's first-order condition (28) by $\Phi(\xi_i^*(c_i))$, and subsequently takes expectations. This yields

$$E_{c_i}[c_i\Phi(\xi_i^*(c_i))] = E_{c_1,c_2}\left[\frac{\gamma_1\gamma_2 h(\xi_1^*(c_1))h(\xi_2^*(c_2))}{(\gamma_1 h(\xi_1^*(c_1)) + \gamma_2 h(\xi_2^*(c_2)))^2}\right] \qquad (i = 1, 2), \tag{35}$$

where $E_{c_1,c_2}[.]$ denotes the ex-ante expectation. Exploiting the fact that equilibrium bid schedules are monotone declining (by Lemma A.2), and that $\Phi' > 0$, this implies

$$\underline{c}_2\Phi(\xi_2^*(\overline{c}_2)) \leq E_{c_2}[c_2\Phi(\xi_2^*(c_2))] = E_{c_1}[c_1\Phi(\xi_1^*(c_1))] \leq \overline{c}_1\Phi(\xi_1^*(\underline{c}_1)), \tag{36}$$

or, using that $\Phi(\xi_2^*(\bar{c}_2)) > 0$,

$$\frac{\Phi(\xi_1^*(\underline{c}_1))}{\Phi(\xi_2^*(\bar{c}_2))} \geq \frac{\underline{c}_2}{\bar{c}_1} = \sigma. \tag{37}$$

There are two cases. Assume first that $\xi_1^*(\underline{c}_1) \geq \xi_2^*(\bar{c}_2)$. Then, using $\Phi' \leq \underline{\rho}$ (see Lemma A.1), we obtain

$$\begin{aligned}
\ln\left(\frac{h(\xi_1^*(\underline{c}_1))}{h(\xi_2^*(\bar{c}_2))}\right) &= \int_{\xi_2^*(\bar{c}_2)}^{\xi_1^*(\underline{c}_1)} d\ln h(z) &(38)\\
&= \int_{\xi_2^*(\bar{c}_2)}^{\xi_1^*(\underline{c}_1)} \frac{d\ln h(z)}{d\ln\Phi(z)} d\ln\Phi(z) &(39)\\
&= \int_{\xi_2^*(\bar{c}_2)}^{\xi_1^*(\underline{c}_1)} \frac{1}{\Phi'(z)} d\ln\Phi(z) &(40)\\
&\geq \frac{1}{\underline{\rho}} \int_{\xi_2^*(\bar{c}_2)}^{\xi_1^*(\underline{c}_1)} d\ln\Phi(z) &(41)\\
&= \frac{1}{\underline{\rho}} \ln\left(\frac{\Phi(\xi_1^*(\underline{c}_1))}{\Phi(\xi_2^*(\bar{c}_2))}\right). &(42)
\end{aligned}$$

Using (37), this implies

$$h(\xi_2^*(\bar{c}_2)) \leq \frac{1}{\sigma^{1/\underline{\rho}}} \cdot h(\xi_1^*(\underline{c}_1)). \tag{43}$$

Assume next that $\xi_1^*(\underline{c}_1) < \xi_2^*(\bar{c}_2)$. Then using $\Phi' \geq 1$ (taken likewise from Lemma A.1) delivers

$$\ln\left(\frac{h(\xi_2^*(\bar{c}_2))}{h(\xi_1^*(\underline{c}_1))}\right) = \int_{\xi_1^*(\underline{c}_1)}^{\xi_2^*(\bar{c}_2)} \frac{d\ln\Phi(z)}{\Phi'(z)} \leq \int_{\xi_1^*(\underline{c}_1)}^{\xi_2^*(\bar{c}_2)} d\ln\Phi(z) = \ln\left(\frac{\Phi(\xi_2^*(\bar{c}_2))}{\Phi(\xi_1^*(\underline{c}_1))}\right). \tag{44}$$

Hence, in that case,

$$h(\xi_2^*(\bar{c}_2)) \leq \frac{1}{\sigma} \cdot h(\xi_1^*(\underline{c}_1)). \tag{45}$$

Thus, exploiting that $\underline{\rho} \geq 1$,

$$\begin{aligned}
h(\xi_2^*(\bar{c}_2)) &\leq h(\xi_1^*(\underline{c}_1)) \cdot \max\left\{\frac{1}{\sigma}, \frac{1}{\sigma^{1/\underline{\rho}}}\right\} &(46)\\
&= h(\xi_1^*(\underline{c}_1)) \cdot \begin{cases} 1/\sigma & \text{if } \sigma \leq 1 \\ 1/\sigma^{1/\underline{\rho}} & \text{if } \sigma > 1. \end{cases} &(47)
\end{aligned}$$

Clearly, this proves (26). □

Lemma A.4 (Stackelberg monotonicity) *Let* $x_2 > \widehat{x}_2 \geq 0$ *and* $c_1 \in C_1$ *such that* $x_1 = \widetilde{\beta}_1(\psi_2(x_2); c_1)$ *and* $\widehat{x}_1 = \widetilde{\beta}_1(\psi_2(\widehat{x}_2); c_1)$. *If* $\widehat{x}_1 > 0$ *then,*

(i) $p_2(x_1, x_2) > p_2(\widehat{x}_1, \widehat{x}_2)$, *and*

(ii) $\Pi_1(x_1, x_2; c_1) < \Pi_1(\widehat{x}_1, \widehat{x}_2; c_1)$*;*

Proof. (i) By assumption, $\widehat{x}_1 = \widetilde{\beta}_1(\psi_2(\widehat{x}_2); c_1) > 0$. Therefore, $x_2 > \widehat{x}_2$ implies $p_2(\widehat{x}_1, x_2) > p_2(\widehat{x}_1, \widehat{x}_2)$. Assume first that $x_1 \leq \widehat{x}_1$. Then, clearly, $p_2(x_1, x_2) \geq p_2(\widehat{x}_1, x_2)$ and, hence, $p_2(x_1, x_2) > p_2(\widehat{x}_1, \widehat{x}_2)$, as claimed. Assume next that $x_1 > \widehat{x}_1$. Then, the necessary first-order conditions associated with the respective optimality of $\widehat{x}_1$ and x_1 hold true. As for $\widehat{x}_1$, we find that

$$\frac{\gamma_1 h'(\widehat{x}_1)\gamma_2 h(\widehat{x}_2)}{(\gamma_1 h(\widehat{x}_1) + \gamma_2 h(\widehat{x}_2))^2} = c_1. \tag{48}$$

Multiplying by $\gamma h(\widehat{x}_2)/h'(\widehat{x}_1)$, with $\gamma = \gamma_2/\gamma_1$ as before, yields

$$(p_2(\widehat{x}_1, \widehat{x}_2))^2 = \frac{c_1 \gamma h(\widehat{x}_2)}{h'(\widehat{x}_1)}. \tag{49}$$

Similarly, one shows that the optimality of x_1 implies

$$(p_2(x_1, x_2))^2 = \frac{c_1 \gamma h(x_2)}{h'(x_1)}. \tag{50}$$

Recalling that h is strictly increasing and that h' is weakly declining, we see that $(p_2(x_1, x_2))^2 > (p_2(\widehat{x}_1, \widehat{x}_2))^2$. The claim follows.

(ii) As a consequence of the envelope theorem,

$$\begin{aligned}
\frac{d\Pi_1(\widetilde{\beta}_1(\psi_2(x_2); c_1), x_2; c_1)}{dx_2} &= \left.\frac{\partial \Pi_1(x_1, x_2; c_1)}{\partial x_2}\right|_{x_1 = \widetilde{\beta}_1(\psi_2(x_2); c_1)} && (51)\\
&= -\frac{\gamma_1 h(\widetilde{\beta}_1(\psi_2(x_2); c_1))\gamma_2 h'(x_2)}{(\gamma_1 h(\widetilde{\beta}_1(\psi_2(x_2); c_1)) + \gamma_2 h(x_2))^2} && (52)\\
&< 0. && (53)
\end{aligned}$$

Thus, player 1 indeed strictly benefits from the lowered effort of player 2. This proves the second claim and, hence, the lemma. □

Lemma A.5 (Improved Jensen's inequality)[61] *Let* $g : (1, \infty) \times (1, \infty) \to \mathbb{R}$ *be a twice continuously differentiable function with Hessian matrix*

$$H_g(x,y) = \begin{pmatrix} \frac{\partial^2 g(x,y)}{\partial x^2} & \frac{\partial^2 g(x,y)}{\partial x \partial y} \\ \frac{\partial^2 g(x,y)}{\partial y \partial x} & \frac{\partial^2 g(x,y)}{\partial y^2} \end{pmatrix}, \tag{54}$$

and let Y *be a nondegenerate random variable with finite support in* $(1, \infty)$*. If*

$$\left\{ x > 1, y \geq x^2, d_x > 0, d_y > 0, \frac{d_y}{d_x} > \frac{y-1}{x-1} \right\} \Rightarrow (d_x \; d_y) \left(H_g(x,y) \right) \begin{pmatrix} d_x \\ d_y \end{pmatrix} > 0, \tag{55}$$

then

$$E\left[g\left(Y, Y^2\right) \right] > g\left(E[Y], E[Y^2]\right). \tag{56}$$

Proof. By induction. Assume first that the random variable Y has precisely two possible realizations $y_1, y_2 \in (1, \infty)$. Without loss of generality, $y_1 < y_2$. Consider the auxiliary mapping $f : [0,1] \to \mathbb{R}^2$ defined through

$$f(t) = (1-t) \begin{pmatrix} y_1 \\ y_1^2 \end{pmatrix} + t \begin{pmatrix} y_2 \\ y_2^2 \end{pmatrix} \qquad (t \in [0,1]). \tag{57}$$

See Figure 3(a) for illustration. By assumption, g is strictly convex along the straight line described by f.[62] In particular, the composed mapping $g \circ f$ is strictly convex. Therefore, when t is considered a random variable that assumes the value $t = 0$ with probability $q_1 = \text{pr}(Y = y_1) > 0$ and the value $t = 1$ with probability $q_2 = 1 - q_1 = \text{pr}(Y = y_2) > 0$, then

$$\begin{aligned} E[g(Y, Y^2)] &= E[g(f(t))] & (58) \\ &> g(f(E[t])) & (59) \\ &= g\left(q_1 y_1 + (1-q_1) y_2, q_1 y_1^2 + (1-q_1) y_2^2\right) & (60) \\ &= g([E[Y], E[Y^2]). & (61) \end{aligned}$$

[61] This auxiliary result is used in the proof of Proposition 4. It also helped us to see through the analysis of sequentially taken disclosure decisions (see the extensions section). Alternative extensions of Jensen's inequality have been proposed by Pittenger (1990), Guljaš et al. (1998), and Liao and Berg (2017), in particular. However, those results do not render the payoff comparisons made in the proof of Proposition 4.

[62] To see this, let $x = (1-t)y_1 + ty_2 > 1$, $y = (1-t)y_1^2 + ty_2^2 \geq x^2$, $d_x = y_2 - y_1 > 0$, and $d_y = y_2^2 - y_1^2 > 0$. Then, $d_y/d_x = y_2 + y_1 > y_2 + 1 \geq (y-1)/(x-1)$, so that the precondition in (55) indeed holds true.

This proves the claim in the case that Y has two realizations. Suppose that the claim has been shown for $K \geq 2$ realizations, and assume that Y has $K+1$ realizations $y_1 < \ldots < y_{K+1}$, with respective probabilities $q_k = \text{pr}(Y = y_k) > 0$, where $k = 1, \ldots, K+1$. Consider the random variable Y' that attains value y_k, for $k = 2, \ldots, K+1$, with probability

$$q'_k = \frac{q_k}{1 - q_1} = \frac{q_k}{\sum_{\kappa=2}^{K+1} q_\kappa}. \tag{62}$$

Thus, Y' follows a conditional distribution after learning $Y \neq y_1$. In particular,

$$E[Y] = q_1 y_1 + (1 - q_1) E[Y'], \tag{63}$$

$$E[Y^2] = q_1 y_1^2 + (1 - q_1) E[(Y')^2]. \tag{64}$$

Moreover, by the induction hypothesis, inequality (56) holds for Y', i.e.,

$$E\left[g\left(Y', (Y')^2\right)\right] > g\left(E[Y'], E[(Y')^2]\right). \tag{65}$$

As above, we define an auxiliary mapping

$$\widetilde{f}(t) = (1-t)\binom{y_1}{y_1^2} + t\binom{E[Y']}{E[(Y')^2]} \qquad (t \in [0,1]). \tag{66}$$

Clearly, $E[(Y')^2] > E[Y']^2$. Therefore, as illustrated in Figure 3(b), the vector that directs from $\binom{y_1}{y_1^2}$ to $\binom{E[Y']}{E[(Y')^2]}$ is steeper than the vector that directs from $\binom{y_1}{y_1^2}$ to $\binom{E[Y']}{E[Y']^2}$. Hence, g is strictly convex also along the linear path described by $\widetilde{f}$.[63] Thus, $g \circ \widetilde{f}$ is strictly convex.

[63] Indeed, letting $x = (1-t)y_1 + tE[Y'] > 1$, $y = (1-t)y_1^2 + tE[(Y')^2] > x^2$, $d_x = E[Y'] - y_1 > 0$, and $d_y = E[(Y')^2] - y_1^2 > 0$, we see that $d_y/d_x = (E[(Y')^2] - y_1^2)/(E[Y'] - y_1) > (E[(Y')]^2 - y_1^2)/(E[Y'] - y_1) = E[Y'] + y_1 > 1 + y_1 = (y-1)/(x-1)$, so that the precondition in (55) holds true also in this case.

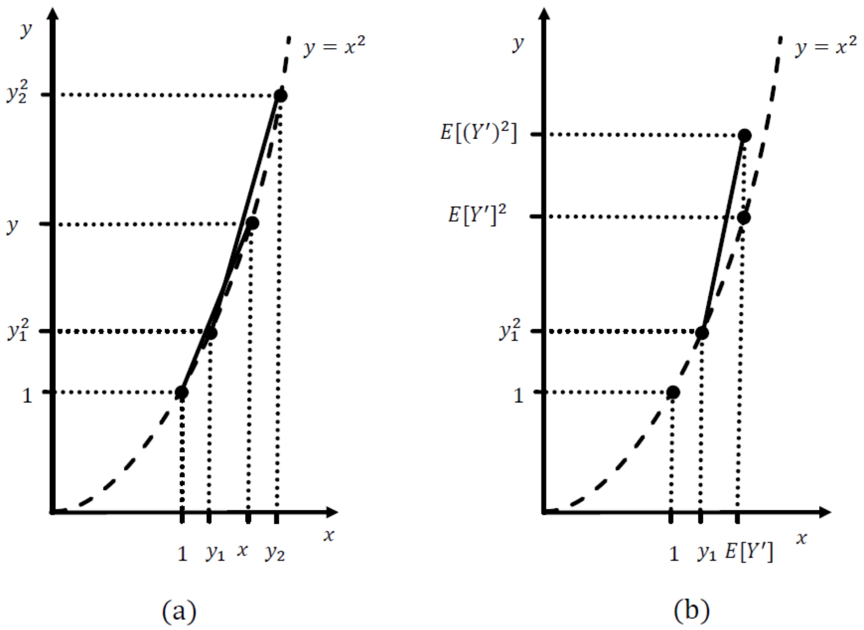

Figure 3. Improving upon Jensen's inequality.

Therefore, considering t as a random variable that assumes the value $t = 0$ with probability $q_1 = \text{pr}(Y = y_1) > 0$ and the value $t = 1$ with probability $1 - q_1 > 0$, relationships (63-66) imply

$$\begin{aligned} E[g(Y, Y^2)] &= q_1 g(y_1, y_1^2) + (1 - q_1)E[g(Y', (Y')^2] && (67) \\ &> q_1 g(y_1, y_1^2) + (1 - q_1)g\left(E[Y'], E[(Y')^2]\right) && (68) \\ &= E[g(\widetilde{f}(t))] && (69) \\ &> g(\widetilde{f}(E[t])) && (70) \\ &= g\left(q_1 y_1 + (1 - q_1)E[Y'], q_1 y_1^2 + (1 - q_1)E[(Y')^2]\right) && (71) \\ &= g(E[Y], E[Y^2]). && (72) \end{aligned}$$

Thus, the claim holds for $K + 1$ realizations. This completes the induction, and thereby, the proof of the lemma. □

Lemma A.6[64] *Let* $g : (0, 1) \times (0, 1) \to \mathbb{R}$ *be a twice continuously differentiable function with*

[64] This auxiliary result, a mirror image of Lemma A.5, is used in the proof of Proposition 5.

Hessian matrix

$$H_g(x,y) = \begin{pmatrix} \frac{\partial^2 g(x,y)}{\partial x^2} & \frac{\partial^2 g(x,y)}{\partial x \partial y} \\ \frac{\partial^2 g(x,y)}{\partial y \partial x} & \frac{\partial^2 g(x,y)}{\partial y^2} \end{pmatrix}, \tag{73}$$

and let Y be a nondegenerate random variable with finite support in $(0,1)$*. If*

$$\left\{ x, y \in (0,1), y \geq x^2, d_x > 0, d_y > 0, \frac{d_y}{d_x} < \frac{1-y}{1-x} \right\} \Rightarrow (d_x \ d_y)\left(H_g(x,y)\right)\begin{pmatrix} d_x \\ d_y \end{pmatrix} > 0, \tag{74}$$

then

$$E\left[g\left(Y, Y^2\right)\right] > g\left(E[Y], E[Y^2]\right). \tag{75}$$

Proof. By induction. Assume first that the random variable Y has precisely two possible realizations $y_1, y_2 \in (0,1)$. Without loss of generality, $y_1 < y_2$. Consider the auxiliary mapping $f : [0,1] \to \mathbb{R}^2$ defined through

$$f(t) = (1-t)\begin{pmatrix} y_1 \\ y_1^2 \end{pmatrix} + t\begin{pmatrix} y_2 \\ y_2^2 \end{pmatrix} \qquad (t \in [0,1]). \tag{76}$$

By assumption, g is strictly convex along the straight line described by f.[65] In particular, the composed mapping $g \circ f$ is strictly convex. Therefore, when t is considered a random variable that assumes the value $t = 0$ with probability $q_1 = \mathrm{pr}(Y = y_1) > 0$ and the value $t = 1$ with probability $q_2 = 1 - q_1 = \mathrm{pr}(Y = y_2) > 0$, then

$$\begin{aligned} E[g(Y, Y^2)] &= E[g(f(t))] & (77) \\ &> g(f(E[t])) & (78) \\ &= g\left(q_1 y_1 + (1-q_1)y_2, q_1 y_1^2 + (1-q_1)y_2^2\right) & (79) \\ &= g([E[Y], E[Y^2]]). & (80) \end{aligned}$$

This proves the claim in the case that Y has two realizations. Suppose that the claim has been shown for $K \geq 2$ realizations, and assume that Y has $K+1$ realizations $y_1 < \ldots < y_{K+1}$, with respective probabilities $q_k = \mathrm{pr}(Y = y_k) > 0$, where $k = 1, \ldots, K+1$. Consider the random

[65] To see this, let $x = (1-t)y_1 + ty_2 \in (0,1)$, $y = (1-t)y_1^2 + ty_2^2 \geq x^2$, $d_x = y_2 - y_1 > 0$, and $d_y = y_2^2 - y_1^2 > 0$. Then, $d_y/d_x = y_2 + y_1 < 1 + y_1 \leq (1-y)/(1-x)$, so that the precondition in (74) indeed holds true.

variable Y' that attains value y_k, for $k = 2, \ldots, K+1$, with probability

$$q'_k = \frac{q_k}{1-q_1} = \frac{q_k}{\sum_{\kappa=2}^{K+1} q_\kappa}. \tag{81}$$

Thus, Y' follows a conditional distribution after learning $Y \neq y_1$. In particular,

$$E[Y] = q_1 y_1 + (1-q_1)E[Y'], \tag{82}$$

$$E[Y^2] = q_1 y_1^2 + (1-q_1)E[(Y')^2]. \tag{83}$$

Moreover, by the induction hypothesis, inequality (56) holds for Y', i.e.,

$$E\left[g\left(Y', (Y')^2\right)\right] > g\left(E[Y'], E[(Y')^2]\right). \tag{84}$$

As above, we define an auxiliary mapping

$$\widetilde{f}(t) = (1-t)\begin{pmatrix} y_1 \\ y_1^2 \end{pmatrix} + t\begin{pmatrix} E[Y'] \\ E[(Y')^2] \end{pmatrix} \qquad (t \in [0,1]). \tag{85}$$

Clearly, $E[(Y')^2] > E[Y']^2$. Therefore, the vector that directs from $\binom{y_1}{y_1^2}$ to $\binom{E[Y']}{E[(Y')^2]}$ is steeper than the vector that directs from $\binom{y_1}{y_1^2}$ to $\binom{E[Y']}{E[Y']^2}$. Hence, g is strictly convex also along the linear path described by $\widetilde{f}$.[66] Thus, $g \circ \widetilde{f}$ is strictly convex. Therefore, considering t as a random variable that assumes the value $t = 0$ with probability $q_1 = \text{pr}(Y = y_1) > 0$ and the value $t = 1$ with probability $1 - q_1 > 0$, relationships (82-85) imply

$$E[g(Y, Y^2)] = q_1 g(y_1, y_1^2) + (1-q_1)E[g(Y', (Y')^2] \tag{86}$$

$$> q_1 g(y_1, y_1^2) + (1-q_1)g\left(E[Y'], E[(Y')^2]\right) \tag{87}$$

$$= E[g(\widetilde{f}(t))] \tag{88}$$

$$> g(\widetilde{f}(E[t])) \tag{89}$$

$$= g\left(q_1 y_1 + (1-q_1)E[Y'], q_1 y_1^2 + (1-q_1)E[(Y')^2]\right) \tag{90}$$

$$= g(E[Y], E[Y^2]). \tag{91}$$

[66] Indeed, letting $x = (1-t)y_1 + tE[Y'] \in (0,1)$, $y = (1-t)y_1^2 + tE[(Y')^2] > x^2$, $d_x = E[Y'] - y_1 > 0$, and $d_y = E[(Y')^2] - y_1^2 > 0$, we see that $d_y/d_x = (E[(Y')^2] - y_1^2)/(E[Y'] - y_1) > (E[(Y')]^2 - y_1^2)/(E[Y'] - y_1) = E[Y'] + y_1 > 1 + y_1 \geq (1-y)/(1-x)$, so that the precondition in (74) holds true also in this case.

Thus, the claim holds for $K+1$ realizations. This completes the induction, and thereby, the proof of the lemma. □

Appendix B. Proofs

This Appendix contains formal proofs and technical details omitted from the body of the paper.

Proof of Lemma 1. This is a special case of a result in Ewerhart and Quartieri (2020). The details are omitted. □

Formal description of the Bayesian updating. For each contestant $i \in \{1, 2\}$, there are three basic scenarios .

(i) Suppose first that player i discloses $c_i \in C_i$. Then, player i is believed to be of type c_i with probability one, i.e., $\mu_i(c_i) = 1$.

(ii) Next, suppose that player i does not disclose her type, and that player i's decision to not disclose is a possibility on the equilibrium path, i.e., $C_i \backslash S_i \neq \varnothing$. Then, c_i is expected to be in the set-theoretic complement of S_i. Hence, by Bayes' rule,

$$\mu_i(c_i) = \begin{cases} q_i(c_i)/\sum_{c_i' \in C_i \backslash S_i} q_i(c_i') & \text{if } c_i \in C_i \backslash S_i \\ 0 & \text{if } c_i \in S_i. \end{cases} \tag{92}$$

(iii) Finally, suppose that player i does not disclose her type, and that i's decision to not disclose is an off-equilibrium event, i.e., $C_i \backslash S_i = \varnothing$. Then, the belief about player i may be specified as an arbitrary probability distribution $\mu_i = \mu_i^0$ over C_i.

Proof of Lemma 2. Lemma 2 is derived by combining several inequalities, all of which are derived from the first-order conditions necessary for players' bid schedules to be mutual best responses. Property (ii) of Definition 1 will be checked first. Suppose that all types of both players are active. There are two cases.

Case A. Suppose first that $\text{Supp}(\mu_1) = C_1$ *and* $\text{Supp}(\mu_2) = C_2$. We make use of Lemma A.3. Letting $i = 2$ in (25) yields

$$\gamma_2 h(\xi_2^*(\underline{c}_2)) \leq \frac{1}{\pi_2} \cdot \gamma_2 h(\xi_2^*(\overline{c}_2)) + \frac{1-\pi_2}{\pi_2} \cdot \gamma_1 h(\xi_1^*(\underline{c}_1)). \tag{93}$$

Combining this with (26) delivers

$$\gamma_2 h(\xi_2^*(\underline{c}_2)) \leq \underbrace{\left\{\frac{1}{\pi_2}\cdot\frac{\gamma}{\widehat{\sigma}}+\frac{1-\pi_2}{\pi_2}\right\}}_{\equiv\alpha}\cdot\gamma_1 h(\xi_1^*(\underline{c}_1)), \tag{94}$$

where $\gamma = \gamma_2/\gamma_1$, as before. Letting $i = 1$ in (25), and plugging the result into (94) yields

$$\gamma_2 h(\xi_2^*(\underline{c}_2)) \leq \alpha\cdot\left\{\frac{1}{\pi_1}\cdot\gamma_1 h(\xi_1^*(\overline{c}_1)) + \frac{1-\pi_1}{\pi_1}\cdot\gamma_2 h(\xi_2^*(\underline{c}_2))\right\}. \tag{95}$$

To be able to solve for $\gamma_2 h(\xi_2^*(\underline{c}_2))$, we assume for the moment that

$$1-\alpha\frac{1-\pi_1}{\pi_1} > 0. \tag{96}$$

Then, rewriting (95), we obtain

$$\gamma_2 h(\xi_2^*(\underline{c}_2)) \leq \underbrace{\left\{\frac{\alpha\cdot\frac{1}{\pi_1}}{1-\alpha\cdot\frac{1-\pi_1}{\pi_1}}\right\}}_{\equiv\lambda}\cdot\gamma_1 h(\xi_1^*(\overline{c}_1)). \tag{97}$$

Thus, $\gamma_2 h(\xi_2^*(\underline{c}_2)) \leq \lambda\cdot\gamma_1 h(\xi_1^*(\overline{c}_1))$. We claim that inequality (96) holds. Indeed, starting with Assumption 1, we find that

$$\gamma \quad < \quad \frac{\pi_1+2\pi_2-2}{2-\pi_1}\cdot\widehat{\sigma} \tag{98}$$

$$\Leftrightarrow \quad \frac{\gamma}{\widehat{\sigma}}+1 < \frac{2\pi_2}{2-\pi_1} \tag{99}$$

$$\Leftrightarrow \quad \underbrace{\frac{(\gamma/\widehat{\sigma})+1}{\pi_2}}_{=\alpha+1} < \underbrace{\frac{2}{2-\pi_1}}_{=\frac{\pi_1}{2-\pi_1}+1} \tag{100}$$

$$\Leftrightarrow \quad \alpha < \frac{\pi_1}{2-\pi_1} \tag{101}$$

$$\Leftrightarrow \quad 1-\frac{\alpha(1-\pi_1)}{\pi_1} > \frac{\alpha}{\pi_1}. \tag{102}$$

Clearly, this implies (96). Moreover, it can be readily verified that (102) implies $\lambda < 1$. Therefore, $\gamma_2 h(\xi_2^*(\underline{c}_2)) \leq \gamma_1 h(\xi_1^*(\overline{c}_1))$. Using the monotonicity of equilibrium bid schedules (Lemma A.2

above), this proves

$$\gamma_2 h(\xi_2^*(c_2)) \leq \gamma_1 h(\xi_1^*(c_1)) \qquad (c_1 \in C_1, c_2 \in C_2). \tag{103}$$

Clearly this proves property (ii) in Definition 1 for the case that all types of both players conceal their private information.

Case B. Supp$(\mu_i) \subsetneq C_i$ *for some player* $i \in \{1, 2\}$. The conclusion remains valid even if not all types conceal. To understand why, note that disclosure by some types means that, in the relevant information set at the contest stage, the sets C_1 and C_2 are replaced by nonempty subsets, respectively. Therefore, player 1's lowest relative resolve $\sigma = \underline{c}_2/\overline{c}_1$ rises weakly. Given that the curvature $\underline{\rho} \geq 1$ stays unchanged, this implies that $\widehat{\sigma}(\sigma, \underline{\rho})$ rises weakly as well. Further, player 1 and 2's predictabilities π_1 and π_2 fall weakly, while the net bias γ stays the same. Therefore, Assumption 1 continues to hold, and the argument detailed under case A goes through as before. This concludes the proof of property (ii) of Definition 1.

It remains to verify property (i) of the definition of uniform asymmetry, i.e., that all types of player 1 are active. Suppose not. Then, all types of player 2 are active. Denote by $\varnothing \neq C_1^* \subsetneq C_1$ the set of active types of player 1, and by $q_1^* = \sum_{c_1 \in C_1^*} q_1(c_1)$ the ex-ante probability that player 1 is active. Then, since any positive bid wins against an inactive type with probability one, the corresponding terms in player 2's first-order condition vanish, so that

$$\sum_{c_1 \in C_1^*} q_1(c_1) \frac{\gamma_2 h'(\xi_2^*(c_2)) \gamma_1 h(\xi_1^*(c_1))}{(\gamma_1 h(\xi_1^*(c_1)) + \gamma_2 h(\xi_2^*(c_2)))^2} = c_2 \qquad (c_2 \in C_2). \tag{104}$$

In the *modified contest*, player 1's type set C_1 is replaced by the subset C_1^*, the probability distribution $q_1(.)$ is replaced by $q_1^*(c_1) = q_1(c_1)/q_1^*$, and player 2's type set C_2 is replaced by

$$\frac{C_2}{q_1^*} = \left\{ \frac{c_2}{q_1^*} \middle| c_2 \in C_2 \right\}. \tag{105}$$

Denote by $\xi_1^*|_{C_1^*}$ the restriction of the mapping $\xi_1^* : C_1 \to \mathbb{R}_+$ to C_1^*, and by $\xi_2^*|_{q_1^*} : \frac{C_2}{q_1^*} \to \mathbb{R}_+$ the bid schedule for player 2 in the modified contest that satisfies $\xi_2^*|_{q_1^*}(\frac{c_2}{q_1^*}) = \xi_2^*(c_2)$ for any $c_2 \in C_2$. We claim that $(\xi_1^*|_{C_1^*}, \xi_2^*|_{q_1^*})$ is a Bayesian equilibrium in the modified contest. Indeed, quite obviously, the first-order condition of any active type of player 1 holds in the modified contest.

Moreover, dividing (104) by $q_1^* > 0$, we get

$$\sum_{c_1 \in C_1^*} \frac{q_1(c_1)}{q_1^*} \frac{\gamma_2 h'(\xi_2^*(c_2))\gamma_1 h(\xi_1^*(c_1))}{(\gamma_1 h(\xi_1^*(c_1)) + \gamma_2 h(\xi_2^*(c_2)))^2} = \frac{c_2}{q_1^*} \qquad (c_2 \in C_2), \tag{106}$$

i.e., also the first-order condition of any type of player 2 holds in the modified contest. Since all types of both players are active in $(\xi_1^*|_{C_1^*}, \xi_2^*|_{q_1^*})$ and since, in addition, the expected payoff against a player that is always active is strictly concave in the own bid, this proves the claim, i.e., $(\xi_1^*|_{C_1^*}, \xi_2^*|_{q_1^*})$ is indeed a Bayesian equilibrium in the modified contest. Next, one notes that, since Assumption 1 holds for the original contest, Assumption 1 holds also for the modified contest (because π_1 and σ rise weakly, while γ, $\underline{\rho}$, and π_2 stay the same). From the first part of the proof, applied to the modified contest, it therefore follows that

$$\gamma_2 h(\xi_2^*(c_2)) \leq \gamma_1 h(\xi_1^*(c_1)) \qquad (c_1 \in C_1^*, c_2 \in C_2). \tag{107}$$

Now, by assumption, some types of player 1 remain inactive in the original contest. Since, by Lemma A.2, ξ_1^* is monotone declining, this clearly implies $\xi_1^*(\bar{c}_1) = 0$. Consequently, the marginal productivity at the zero bid level $h'(0) = \lim_{\varepsilon \searrow 0} \frac{h(\varepsilon)}{\varepsilon}$ is finite. Moreover, type $\bar{c}_1$'s marginal payoff at the zero bid level is weakly negative, i.e.,

$$E_{c_2}\left[\frac{\gamma_1 h'(0)}{\gamma_2 h(\xi_2^*(c_2))}\right] \leq \bar{c}_1. \tag{108}$$

Plugging (107) into (108), we see that

$$\frac{h'(0)}{h(\xi_1^*(c_1))} \leq \bar{c}_1 \qquad (c_1 \in C_1^*). \tag{109}$$

Moreover, Assumption 1 implies

$$\frac{\gamma_2}{\gamma_1} = \gamma < \underbrace{\frac{\pi_1 + 2\pi_2 - 2}{2 - \pi_1}}_{\leq 1} \cdot \underbrace{\widehat{\sigma}(\sigma, \underline{\rho})}_{\leq \sigma} \leq \sigma = \frac{\underline{c}_2}{\bar{c}_1}. \tag{110}$$

Multiplying inequality (109) by $(\gamma/q_1^*) > 0$, exploiting (110), and taking expectations over all

$c_1 \in C_1^*$, we get

$$\sum_{c_1 \in C_1^*} \frac{q_1(c_1)}{q_1^*} \frac{\gamma_2 h'(0)}{\gamma_1 h(\xi_1^*(c_1))} < \frac{\underline{c}_2}{q_1^*}. \tag{111}$$

Thus, in the modified contest, the marginal expected payoff of type $(\underline{c}_2/q_1^*)$ at the zero bid level is strictly negative. But this is impossible given that she is active and her expected payoff against $\xi_1^*|_{C_1^*}$ is strictly concave. The contradiction shows that, indeed, all types of player 1 are active in the original contest. □

Proof of Theorem 1. We start by showing that self-disclosure by all types of both players constitutes a perfect Bayesian equilibrium. To this end, we specify off-equilibrium beliefs $\mu_1^0 \in \Delta(C_1)$ and $\mu_2^0 \in \Delta(C_2)$ as follows. The underdog expects a favorite that does not disclose her private information to be of type $c_1 = \overline{c}_1$ with probability one. Thus, $\mu_1^0(c_1) = 1$ if $c_1 = \overline{c}_1$, and $\mu_1^0(c_1) = 0$ otherwise. Similarly, the favorite expects an underdog that does not disclose her private information to be of type $c_2 = \underline{c}_2$ with probability one. Thus, $\mu_2^0(c_2) = 1$ if $c_2 = \underline{c}_2$, and $\mu_2^0(c_2) = 0$ otherwise. To check the equilibrium property, consider first an arbitrary type $c_1 \in C_1$ of the favorite. If c_1 complies with equilibrium self-disclosure, and is matched with some type $c_2 \in C_2$ of the underdog, then c_1 receives a complete-information equilibrium payoff of

$$\begin{aligned} \Pi_1^\circ(c_1, c_2) &= \Pi_1(x_1^\circ(c_1, c_2), x_2^\circ(c_1, c_2); c_1) && (112) \\ &= \Pi_1(\widetilde{\beta}_1(x_2^\circ(c_1, c_2); c_1), x_2^\circ(c_1, c_2); c_1). && (113) \end{aligned}$$

If, however, c_1 chooses to not disclose then, given the off-equilibrium beliefs specified above, an underdog of type c_2 expects the favorite to be of the worst-case type $\overline{c}_1$ and, having revealed her own type c_2, chooses an effort of $x_2^\circ(\overline{c}_1, c_2)$. Responding optimally to type c_2's bid, the deviating favorite of type c_1 chooses an effort of $\widetilde{\beta}_1(x_2^\circ(\overline{c}_1, c_2); c_1)$ at the contest stage, and consequently receives a payoff of

$$\Pi_1^{\text{dev}}(c_1, c_2) = \Pi_1(\widetilde{\beta}_1(x_2^\circ(\overline{c}_1, c_2); c_1), x_2^\circ(\overline{c}_1, c_2); c_1). \tag{114}$$

A straightforward application of Monaco and Sabarwal (2016, Th. 3) shows that, given Assumption 1, $x_2^\circ(c_1, c_2) \leq x_2^\circ(\overline{c}_1, c_2)$.[67] We claim that $\Pi_1^\circ(c_1, c_2) \geq \Pi_1^{\text{dev}}(c_1, c_2)$. Indeed, if $x_2^\circ(c_1, c_2) <$

[67] For a self-contained argument, it suffices to replicate earlier arguments. Indeed, suppose that $x_2^\circ(c_1, c_2) > x_2^\circ(\overline{c}_1, c_2)$. Clearly, all equilibrium efforts are positive under complete information. Therefore, using Lemma 2(ii), player 1's domain condition holds at $(x_2^\circ(c_1, c_2); c_1)$, so that, by Lemma 3(i), $x_1^\circ(c_1, c_2) > x_1^\circ(\overline{c}_1, c_2)$. Moreover, using Lemma 2(ii) another time, player 2's domain condition is seen to hold at $(x_1^\circ(\overline{c}_1, c_2); c_2)$, so that by Lemma

$x_2^\circ(\overline{c}_1, c_2)$ then, by Lemma A.4(ii), $\Pi_1^\circ(c_1, c_2) > \Pi_1^{\text{dev}}(c_1, c_2)$. Moreover, if $x_2^\circ(c_1, c_2) = x_2^\circ(\overline{c}_1, c_2)$ then $\Pi_1^\circ(c_1, c_2) = \Pi_1^{\text{dev}}(c_1, c_2)$, which proves the claim. Taking expectations over all $c_2 \in C_2$ yields

$$E_{c_2}\left[\Pi_1^\circ(c_1, c_2)\right] \geq E_{c_2}\left[\Pi_1^{\text{dev}}(c_1, c_2)\right] \qquad (c_1 \in C_1). \tag{115}$$

Hence, a deviation is not profitable for any type $c_1 \in C_1$. On the other hand, if any type of the underdog deviates, and the favorite interprets this as a tactic of the strongest type of the underdog, then one can show in complete analogy that the equilibrium condition holds.[68] It follows that self-disclosure by all types of both players is indeed a perfect Bayesian equilibrium.

Next, suppose there is a perfect Bayesian equilibrium in which not all private information is revealed. Then, for at least one player $i \in \{1, 2\}$, the set of types concealing their signal, $C_i \backslash S_i$, has at least two elements. By suitably redefining C_1 and C_2, we may assume without loss of generality that all types conceal their types. Suppose first that $K_2 \geq 2$. Then, Proposition 1 implies that the weakest type of the underdog has a strict incentive to unilaterally deviate at the revelation stage, in conflict to the equilibrium assumption. Suppose next that $K_2 = 1$. Then, since there is incomplete information, $K_1 \geq 2$. But, again, this cannot be part of a perfect Bayesian equilibrium by Proposition 2. Thus, either way, we obtain a contradiction, and the claim follows. This proves the theorem. □

Best-response monotonicity. We will say that *player 1's domain condition* holds at $(\xi_2; c_1) \in X_2^* \times C_1$ if (i) $\widetilde{\beta}_1(\xi_2; c_1) > 0$, and (ii) $p_1(\widetilde{\beta}_1(\xi_2; c_1), \xi_2(c_2)) > \frac{1}{2}$ for any $c_2 \in C_2$. Thus, player 1's domain condition at $(\xi_2; c_1)$ requires that type c_1's best-response bid against ξ_2 is interior, and wins with a probability strictly exceeding one half against any of player 2's types. Similarly, we will say that *player 2's domain condition* holds at $(\widehat{\xi}_1, c_2) \in X_1^* \times C_2$ if (i) $\widetilde{\beta}_2(\widehat{\xi}_1; c_2) > 0$, and (ii) $p_2(\widehat{\xi}_1(c_1), \widetilde{\beta}_2(\widehat{\xi}_1; c_2)) < \frac{1}{2}$ for any $c_1 \in C_1$. Thus, player 2's domain condition at $(\widehat{\xi}_1; c_1)$ requires that type c_2's best-response bid against $\widehat{\xi}_1$ is interior, and wins with a probability strictly below one half against any of player 1's types. Using these definitions, we obtain the following useful result.

Lemma B.1 (Strict monotonicity of best-response mappings)

(i) Let $\xi_2, \widehat{\xi}_2 \in X_2^*$ with $\xi_2 \succ \widehat{\xi}_2$, and let $c_1 \in C_1$. If player 1's domain condition holds at $(\xi_2; c_1)$,

3(ii), $x_2^\circ(c_1, c_2) < x_2^\circ(\overline{c}_1, c_2)$, which yields the desired contradiction.

[68] The details are omitted.

then $\widetilde{\beta}_1(\xi_2; c_1) > \widetilde{\beta}_1(\widehat{\xi}_2; c_1)$. *In particular, if player 1's domain condition holds at* $(\xi_2; c_1)$ *for every* $c_1 \in C_1$, *then* $\beta_1(\xi_2) \succ \beta_1(\widehat{\xi}_2)$.

(ii) Let $\xi_1, \widehat{\xi}_1 \in X_1^*$ *with* $\xi_1 \succ \widehat{\xi}_1$, *and let* $c_2 \in C_2$. *If player 2's domain condition holds at* $(\widehat{\xi}_1; c_2)$, *then* $\widetilde{\beta}_2(\xi_1; c_2) < \widetilde{\beta}_2(\widehat{\xi}_1; c_2)$. *In particular, if player 2's domain condition holds at* $(\widehat{\xi}_1; c_2)$ *for every* $c_2 \in C_2$, *then* $\beta_2(\xi_1) \prec \beta_2(\widehat{\xi}_1)$.

Proof. (i) Let $\xi_2, \widehat{\xi}_2 \in X_2^*$ with $\xi_2 \succ \widehat{\xi}_2$, and $c_1 \in C_1$. By assumption, player 1's domain condition holds at $(\xi_2; c_1)$. We wish to show that $x_1 \equiv \widetilde{\beta}_1(\xi_2; c_1) > \widetilde{\beta}_1(\widehat{\xi}_2; c_1) \equiv \widehat{x}_1$. To provoke a contradiction, suppose that $\widehat{x}_1 \geq x_1$. From the domain condition, we have $x_1 > 0$. Therefore, both x_1 and $\widehat{x}_1$ are positive, so that the corresponding first-order conditions imply

$$E_{c_2}\left[\frac{\gamma_1 h'(x_1)\gamma_2 h(\xi_2(c_2))}{(\gamma_1 h(x_1) + \gamma_2 h(\xi_2(c_2)))^2}\right] = E_{c_2}\left[\frac{\gamma_1 h'(\widehat{x}_1)\gamma_2 h(\widehat{\xi}_2(c_2))}{(\gamma_1 h(\widehat{x}_1) + \gamma_2 h(\widehat{\xi}_2(c_2)))^2}\right] = c_1. \tag{116}$$

Fix some $c_2 \in C_2$ for the moment. Letting $x = \gamma_1 h(\widetilde{\beta}_1(\xi_2; c_1))$ and $y = \gamma_2 h(\xi_2(c_2))$, the domain condition implies $x > y$. Clearly, the mapping $y \mapsto y/(x+y)^2$ is strictly increasing over the interval $[0, x]$. Therefore, noting that $\xi_2 \succ \widehat{\xi}_2$ implies $y \geq \widehat{y} \equiv \gamma_2 h(\widehat{\xi}_2(c_2))$, we see that

$$\frac{\gamma_1 h'(x_1)\gamma_2 h(\xi_2(c_2))}{(\gamma_1 h(x_1) + \gamma_2 h(\xi_2(c_2)))^2} \geq \frac{\gamma_1 h'(x_1)\gamma_2 h(\widehat{\xi}_2(c_2))}{(\gamma_1 h(x_1) + \gamma_2 h(\widehat{\xi}_2(c_2)))^2} \qquad (c_2 \in C_2), \tag{117}$$

with strict inequality for at least one $c_2 \in C_2$. Moreover, from $\widehat{x}_1 \geq x_1$,

$$\frac{\gamma_1 h'(x_1)\gamma_2 h(\widehat{\xi}_2(c_2))}{(\gamma_1 h(x_1) + \gamma_2 h(\widehat{\xi}_2(c_2)))^2} \geq \frac{\gamma_1 h'(\widehat{x}_1)\gamma_2 h(\widehat{\xi}_2(c_2))}{(\gamma_1 h(\widehat{x}_1) + \gamma_2 h(\widehat{\xi}_2(c_2)))^2} \qquad (c_2 \in C_2). \tag{118}$$

Combining (117) and (118), and subsequently taking expectations , we arrive at

$$E_{c_2}\left[\frac{\gamma_1 h'(x_1)\gamma_2 h(\xi_2(c_2))}{(\gamma_1 h(x_1) + \gamma_2 h(\xi_2(c_2)))^2}\right] > E_{c_2}\left[\frac{\gamma_1 h'(\widehat{x}_1)\gamma_2 h(\widehat{\xi}_2(c_2))}{(\gamma_1 h(\widehat{x}_1) + \gamma_2 h(\widehat{\xi}_2(c_2)))^2}\right], \tag{119}$$

in conflict with (116). The contradiction shows that $x_1 > \widehat{x}_1$, as claimed. Moreover, if player 1's domain condition holds for any $c_1 \in C_1$, then $\widetilde{\beta}_1(\xi_2; c_1) > \widetilde{\beta}_1(\widehat{\xi}_2; c_1)$ for any $c_1 \in C_1$, which indeed implies $\beta_1(\xi_2) \succ \beta_1(\widehat{\xi}_2)$.

(ii) The proof is similar. Let $\xi_1, \widehat{\xi}_1 \in X_1^*$ with $\xi_1 \succ \widehat{\xi}_1$, and $c_2 \in C_2$. By assumption, player 2's domain condition holds at $(\widehat{\xi}_1; c_2)$. Suppose that $x_2 \equiv \widetilde{\beta}_2(\xi_1; c_2) \geq \widetilde{\beta}_2(\widehat{\xi}_1; c_2) \equiv \widehat{x}_2$. Then, from the domain condition, $\widehat{x}_2 > 0$. Hence,

$$E_{c_1}\left[\frac{\gamma_2 h'(\widehat{x}_2)\gamma_1 h(\widehat{\xi}_1(c_1))}{\left(\gamma_1 h(\widehat{\xi}_1(c_1)) + \gamma_2 h(\widehat{x}_2)\right)^2}\right] = E_{c_1}\left[\frac{\gamma_2 h'(x_2)\gamma_1 h(\xi_1(c_1))}{(\gamma_1 h(\xi_1(c_1)) + \gamma_2 h(x_2))^2}\right] = c_2. \qquad (120)$$

Fix some $c_1 \in C_1$, and let $\widehat{x} = \gamma_2 h(\widetilde{\beta}_2(\widehat{\xi}_1; c_2))$ and $\widehat{y} = \gamma_1 h(\widehat{\xi}_1(c_1))$. By the domain condition, $\widehat{x} < \widehat{y}$. Moreover, the mapping $\widehat{y} \mapsto \widehat{y}/(\widehat{x} + \widehat{y})^2$ is strictly declining for $\widehat{y} \geq \widehat{x}$. Hence, given that $\widehat{\xi}_1 \prec \xi_1$ implies $\widehat{y} \leq y \equiv \gamma_1 h(\xi_1(c_1))$, we see that

$$\frac{\gamma_2 h'(\widehat{x}_2)\gamma_1 h(\widehat{\xi}_1(c_1))}{\left(\gamma_1 h(\widehat{\xi}_1(c_1)) + \gamma_2 h(\widehat{x}_2)\right)^2} \geq \frac{\gamma_2 h'(\widehat{x}_2)\gamma_1 h(\xi_1(c_1))}{(\gamma_1 h(\xi_1(c_1)) + \gamma_2 h(\widehat{x}_2))^2} \qquad (c_1 \in C_1), \qquad (121)$$

with strict inequality for some $c_1 \in C_1$. Moreover, from $\widehat{x}_2 \leq x_2$,

$$\frac{\gamma_2 h'(\widehat{x}_2)\gamma_1 h(\xi_1(c_1))}{(\gamma_1 h(\xi_1(c_2)) + \gamma_2 h(\widehat{x}_2))^2} \geq \frac{\gamma_2 h'(x_2)\gamma_1 h(\xi_1(c_1))}{(\gamma_2 h(\xi_1(c_1)) + \gamma_2 h(x_1))^2} \qquad (c_1 \in C_1). \qquad (122)$$

Combining (121) and (122), and taking expectations, we arrive at

$$E_{c_1}\left[\frac{\gamma_2 h'(\widehat{x}_2)\gamma_1 h(\widehat{\xi}_1(c_1))}{\left(\gamma_1 h(\widehat{\xi}_1(c_1)) + \gamma_2 h(\widehat{x}_2)\right)^2}\right] > E_{c_1}\left[\frac{\gamma_2 h'(x_2)\gamma_1 h(\xi_1(c_1))}{(\gamma_1 h(\xi_1(c_1)) + \gamma_2 h(x_2))^2}\right], \qquad (123)$$

in contradiction to (120). It follows that, indeed, $\widehat{x}_2 > x_2$. In particular, provided that player 2's domain condition holds for any $c_2 \in C_2$, it follows that $\beta_2(\xi_1) \prec \beta_2(\widehat{\xi}_1)$. This concludes the proof. □

This lemma shows that the domain conditions are sufficient to ensure that a type's best-response bid and a player's best-response bid schedule, respectively, move in a strictly monotone way to changes in the opponent's bid schedule. For example, in the case of player 1, the best-response bid of type c_1 will strictly rise in response to an increase of player 2's bid schedule. If player 1's domain condition holds at all of her types, then we get a strict order relation even between the best-response bid schedules. Similar comparative statics properties hold for player 2, whose best-response mapping is, however, strictly declining under the assumptions of Lemma B.1. In sum, the contest with two-sided incomplete information exhibits, subject to domain conditions, comparative statics properties analogous to those of the complete-information contest.

Proof of Proposition 1. The conclusions of Proposition 1 are immediate if $\xi_2^*(\overline{c}_2) = 0$. Suppose that $\xi_2^*(\overline{c}_2) > 0$. Since, by Lemma A.2, the equilibrium bid schedule ξ_2^* is weakly declining, actually all types of player 2 are active in ξ_2^*. Using Lemma A.2 another time, one sees that ξ_2^* is even strictly declining. These observations will be tacitly used below. We now prove the three assertions made in the statement of the proposition.

(i) First, it is shown that self-disclosure induces the weakest type of the underdog to strictly raise her bid, i.e., $\xi_2^*(\overline{c}_2) < x_2^\#$. To provoke a contradiction, suppose that $\xi_2^*(\overline{c}_2) \geq x_2^\#$. Then, because ξ_2^* is strictly declining and there are at least two possible type realizations for player 2, we get $\xi_2^* \succ \psi_2(x_2^\#)$. We claim that player 1's domain condition holds at $(\xi_2^*; c_1)$, for any $c_1 \in C_1$. To see this, take some $c_1 \in C_1$. Then, from Lemma 2(i), $\widetilde{\beta}_1(\xi_2^*; c_1) = \xi_1^*(c_1) > 0$. Further, since all types of player 2 are active in ξ_2^*, Lemma 2(ii) implies that $p_1(\xi_1^*(c_1), \xi_2^*(c_2)) > \frac{1}{2}$ for any $c_2 \in C_2$, which proves the claim. We may, therefore, apply Lemma B.1(i) so as to obtain

$$\xi_1^* = \beta_1(\xi_2^*) \succ \beta_1(\psi_2(x_2^\#)) = \xi_1^\#. \tag{124}$$

Next, it is claimed that player 2's domain condition holds at $(\xi_1^\#; \overline{c}_2)$. Since $(\xi_1^\#(.), x_2^\#)$ is an equilibrium in the contest with one-sided incomplete information, we have $x_2^\# > 0$, i.e., player 2 is active with probability one. Applying Lemma 2(ii) shows, therefore, that $p_2(\xi_1^\#(c_1), x_2^\#) < \frac{1}{2}$ holds true for any $c_1 \in C_1$. Since $\widetilde{\beta}_2(\xi_1^\#; \overline{c}_2) = x_2^\#$, this means that $p_2(\xi_1^\#(c_1), \widetilde{\beta}_2(\xi_1^\#; \overline{c}_2)) < \frac{1}{2}$, for any $c_1 \in C_1$. I.e., player 2's domain condition at $(\xi_1^\#; \overline{c}_2)$ is indeed satisfied. Therefore, using relationship (124) and Lemma B.1(ii), we see that

$$\xi_2^*(\overline{c}_2) = \widetilde{\beta}_2(\xi_1^*; \overline{c}_2) < \widetilde{\beta}_2(\xi_1^\#; \overline{c}_2) = x_2^\#, \tag{125}$$

in contradiction to $\xi_2^*(\overline{c}_2) \geq x_2^\#$. Thus, $\xi_2^*(\overline{c}_2) < x_2^\#$, as claimed.

(ii) Next, it is shown that, after disclosure, the probability of winning for the weakest type of the underdog rises strictly, i.e.,

$$p_2^\# = E_{c_1}[p_2(\xi_1^\#(c_1), x_2^\#)] > E_{c_1}[p_2(\xi_1^*(c_1), \xi_2^*(\overline{c}_2))] = p_2^*. \tag{126}$$

In fact, we will prove the somewhat stronger statement

$$p_2(\xi_1^{\#}(c_1), x_2^{\#}) > p_2(\xi_1^*(c_1), \xi_2^*(\overline{c}_2)) \qquad (c_1 \in C_1). \tag{127}$$

Take some type $c_1 \in C_1$. It is claimed first that $\widetilde{\beta}_1(\psi_2(\xi_2^*(\overline{c}_2)); c_1) > 0$, as shown in the left diagram of Figure 4. Indeed, because player 2 is always active in ξ_2^*, the mapping $x_1 \mapsto E_{c_2}[\Pi_1(x_1, \xi_2^*(c_2); c_1)]$ is strictly concave on $\mathbb{R}_+$, and vanishes at $x_1 = 0$. Therefore, the optimality of $\xi_1^*(c_1) > 0$ implies $E_{c_2}[\Pi_1(\xi_1^*(c_1), \xi_2^*(c_2); c_1)] > 0$. But the flat bid schedule $\psi_2(\xi_2^*(\overline{c}_2))$ is everywhere weakly lower than ξ_2^*. Therefore, $E_{c_2}[\Pi_1(\xi_1^*(c_1), \psi_2(\xi_2^*(\overline{c}_2)); c_1)] > 0$, i.e., type c_1 is able to realize a positive payoff against the flat bid schedule $\psi_2(\xi_2^*(\overline{c}_2))$. Since $\xi_2^*(\overline{c}_2) > 0$, it follows that type c_1's best-response bid against $\psi_2(\xi_2^*(\overline{c}_2))$ is positive, as claimed. Next, from the previous step, we know that $x_2^{\#} > \xi_2^*(\overline{c}_2)$. Invoking Lemma A.4(i), and noting that $\xi_1^{\#} = \beta_1(\psi_2(x_2^{\#}))$, it follows that

$$p_2(\xi_1^{\#}(c_1), x_2^{\#}) > p_2(\widetilde{\beta}_1(\psi_2(\xi_2^*(\overline{c}_2)); c_1), \xi_2^*(\overline{c}_2)) \qquad (c_1 \in C_1). \tag{128}$$

Next, comparing the strictly declining equilibrium bid schedule $\xi_2^* = \beta_2(\xi_1^*)$ with the flat bid schedule $\psi_2(\xi_2^*(\overline{c}_2))$, and recalling that there are at least two types, we obtain $\xi_2^* \succ \psi_2(\xi_2^*(\overline{c}_2))$. Moreover, as seen above, all types of player 2 are active. Hence, by Lemma 2(ii), $p_1(\xi_1^*(c_1), \xi_2^*(c_2)) > \frac{1}{2}$ for any $c_1 \in C_1$ and any $c_2 \in C_2$, so that via $\widetilde{\beta}_1(\xi_2^*; c_1) = \xi_1^*(c_1)$, player 1's domain condition is seen to hold at $(\xi_2^*; c_1)$, for any $c_1 \in C_1$. Therefore, by Lemma B.1(i), $\xi_1^* = \beta_1(\xi_2^*) \succ \beta_1(\psi_2(\xi_2^*(\overline{c}_2)))$, as illustrated in Figure 4.[69] In particular,

$$\xi_1^*(c_1) \geq \widetilde{\beta}_1(\psi_2(\xi_2^*(\overline{c}_2)); c_1) \qquad (c_1 \in C_1). \tag{129}$$

Therefore,

$$p_2(\widetilde{\beta}_1(\psi_2(\xi_2^*(\overline{c}_2)); c_1), \xi_2^*(\overline{c}_2)) \geq p_2(\xi_1^*(c_1), \xi_2^*(\overline{c}_2)) \qquad (c_1 \in C_1). \tag{130}$$

Combining (128) and (130) yields (127). In particular, this proves $p_2^{\#} > p_2^*$, as claimed.

[69] The figure shows an example where $x_2^{\#} < \xi_2^*(\underline{c}_2)$. In general, we may also have that $x_2^{\#} \geq \xi_2^*(\underline{c}_2)$.

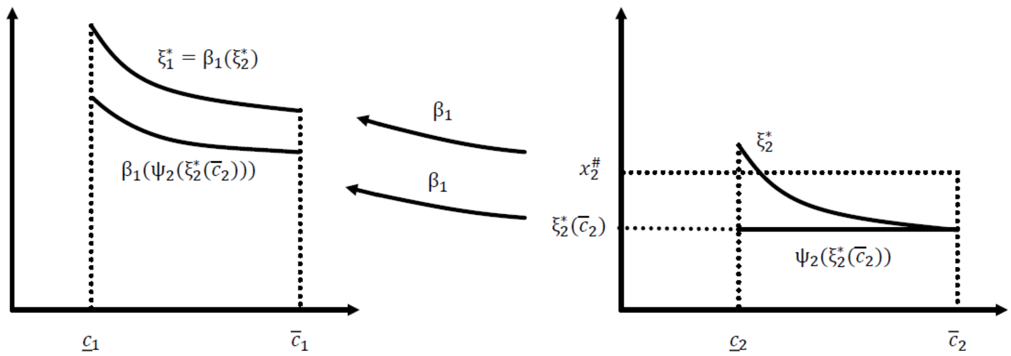

Figure 4. Proof of Proposition 1(ii).

(iii) Finally, we show that the weakest type of the underdog has a strict incentive to disclose her type. Clearly, the equilibrium effort $x_2^{\#}$ is positive. One can check that type $\overline{c}_2$'s first-order condition is equivalent to

$$E_{c_1}\left[p_2(\xi_1^{\#}(c_1),x_2^{\#})-\left(p_2(\xi_1^{\#}(c_1),x_2^{\#})\right)^2\right]=\overline{c}_2\Phi(x_2^{\#}). \tag{131}$$

Exploiting (131), we obtain for type $\overline{c}_2$'s expected payoff from self-disclosure,

$$\Pi_2^{\#}=E_{c_1}\left[\left(p_2(\xi_1^{\#}(c_1),x_2^{\#})\right)^2\right]+\overline{c}_2\left(\Phi(x_2^{\#})-x_2^{\#}\right). \tag{132}$$

In a completely analogous fashion, we can convince ourselves that concealment grants type $\overline{c}_2$ a payoff of

$$\Pi_2^{*}(\overline{c}_2)=E_{c_1}\left[\left(p_2(\xi_1^{*}(c_1),\xi_2^{*}(\overline{c}_2))\right)^2\right]+\overline{c}_2\left(\Phi(\xi_2^{*}(\overline{c}_2))-\xi_2^{*}(\overline{c}_2)\right). \tag{133}$$

Now, from (127), we see that

$$E_{c_1}\left[\left(p_2(\xi_1^{\#}(c_1),x_2^{\#})\right)^2\right]>E_{c_1}\left[\left(p_2(\xi_1^{*}(c_1),\xi_2^{*}(\overline{c}_2))\right)^2\right]. \tag{134}$$

Moreover, from Lemma A.1, $\Phi'\geq 1$, so that the mapping $x_2\mapsto\Phi(x_2)-x_2$ is monotone increasing in x_2. But, as shown above, $\xi_2^{*}(\overline{c}_2)<x_2^{\#}$. It follows that the weakest type of the underdog has indeed a strict incentive to reveal her type. This proves the final claim, and concludes the proof of the proposition. □

Proof of Proposition 2. Since x_1° and x_2° are equilibrium efforts under complete information, we have $x_1^\circ > 0$ and $x_2^\circ > 0$. Similarly, one notes that $x_2^{\#} > 0$. Moreover, by Lemma 2(i), all types of player 1 are active in $\xi_1^{\#}$, so that by Lemma A.2, the bid schedule $\xi_1^{\#}$ is strictly declining. We now prove the four assertions made in the statement of Proposition 2.

(i) It is claimed that $x_2^\circ < x_2^{\#}$. To provoke a contradiction, suppose that $x_2^\circ \geq x_2^{\#}$. Lemma 2(ii) implies $p_1(x_1^\circ, x_2^\circ) > \frac{1}{2}$, so that in view of $x_1^\circ = \widetilde{\beta}_1(x_2^\circ; \underline{c}_1)$, player 1's domain condition holds at $(x_2^\circ; \underline{c}_1)$. Hence, by Lemma B.1(i), if even $x_2^\circ > x_2^{\#}$, then

$$x_1^\circ = \widetilde{\beta}_1(x_2^\circ; \underline{c}_1) > \widetilde{\beta}_1(x_2^{\#}; \underline{c}_1) = \xi_1^{\#}(\underline{c}_1). \tag{135}$$

If, however, $x_2^\circ = x_2^{\#}$, then it is immediate that $x_1^\circ = \xi_1^{\#}(\underline{c}_1)$. Thus, either way, we arrive at $x_1^\circ \geq \xi_1^{\#}(\underline{c}_1)$, so that $\psi_1(x_1^\circ) \succeq \psi_1(\xi_1^{\#}(\underline{c}_1))$. Moreover, given that player 1 has at least two types, and that $\xi_1^{\#}$ is strictly declining, $\psi_1(\xi_1^{\#}(\underline{c}_1)) \succ \xi_1^{\#}$. Hence, $\psi_1(x_1^\circ) \succ \xi_1^{\#}$. Lemma 2(ii) implies that $p_2(\xi_1^{\#}(c_1), x_2^{\#}) < \frac{1}{2}$ for any $c_1 \in C_1$. Thus, recalling that $x_2^{\#} = \widetilde{\beta}_2(\xi_1^{\#}; c_2^{\#})$, player 2's domain condition holds at $(\xi_1^{\#}; c_2^{\#})$. Therefore, using Lemma B.1(ii), we arrive at

$$x_2^{\#} = \widetilde{\beta}_2(\xi_1^{\#}; c_2^{\#}) > \widetilde{\beta}_2(\psi_1(x_1^\circ); c_2^{\#}) = x_2^\circ, \tag{136}$$

a contradiction. It follows that $x_2^\circ < x_2^{\#}$, as claimed.

(ii) Next, it is shown that $x_1^\circ < \xi_1^{\#}(\underline{c}_1)$. From the previous step, we know that $x_2^{\#} > x_2^\circ$. Via Lemma 2(ii), we see that $p_1(\xi_1^{\#}(\underline{c}_1), x_2^{\#}) > \frac{1}{2}$. Thus, the domain condition for player 1 holds at $(x_2^{\#}; \underline{c}_1)$. Lemma B.1(i) implies, therefore, that

$$\xi_1^{\#}(\underline{c}_1) = \widetilde{\beta}_1(x_2^{\#}; \underline{c}_1) > \widetilde{\beta}_1(x_2^\circ; \underline{c}_1) = x_1^\circ. \tag{137}$$

Thus, the effort of the strongest type of the favorite will indeed be strictly lower after self-disclosure.

(iii) Given part (i) above, we have $x_2^\circ < x_2^{\#}$. Recalling that $x_1^\circ > 0$, Lemma A.4(i) implies $p_2(x_1^\circ, x_2^\circ) < p_2(\xi_1^{\#}(\underline{c}_1), x_2^{\#})$, so that $p_1(x_1^\circ, x_2^\circ) > p_1(\xi_1^{\#}(\underline{c}_1), x_2^{\#})$. Thus, type $\underline{c}_1$ indeed wins with a strictly higher probability after self-disclosure.

(iv) The claim that $\Pi_1^\circ > \Pi_1^{\#}$ follows now directly from Lemma A.4(ii). This completes the proof. □

Details on Example 1. The following numerical example shows that monotone comparative statics results availabe for games of strategic heterogeneity do not apply to Example 1. Let $\overline{\beta}_1(\xi_2) = \beta_1(\psi_2(\xi_2(\overline{c}_2)))$ denote player 1's best-response bid schedule against $\psi_2(\xi_2(\overline{c}_2))$, where $\xi_2 \in X_2^*$. Monaco and Sabarwal (2016, Th. 5) required that $\overline{\beta}_1(\widehat{\xi}_2) \preceq \xi_1^*$, where $\widehat{\xi}_2 = \beta_2(\widehat{\xi}_1)$ and $\widehat{\xi}_1 = \overline{\beta}_1(\xi_2^*)$. A computation shows that $\widehat{\xi}_1(\underline{c}_1) = 0.1016$, $\widehat{\xi}_1(\overline{c}_1) = 0.0715$, and $\widehat{\xi}_2(\overline{c}_2) = 0.0194$. As a result, $\overline{\beta}_1(\widehat{\xi}_2)(\underline{c}_1) = 0.4208 > 0.1592 = \xi_1^*(\underline{c}_1)$ and $\overline{\beta}_1(\widehat{\xi}_2)(\overline{c}_1) = 0.2919 > 0.1042 = \xi_1^*(\overline{c}_1)$. It follows that $\overline{\beta}_1(\widehat{\xi}_2) \succ \xi_1^*$, in conflict with the required condition. Thus, it is indeed not feasible to apply existing methods to obtain Proposition 1.

Proof of Theorem 2. Without loss of generality, we may assume that the equilibrium is interior. Then, the vector $\{c_2^k \Phi(x_2^k)\}_{k=1}^{K_2}$ is strictly hump-shaped as a consequence of the first-order condition

$$p_2^k(1-p_2^k) = c_2^k \Phi(x_2^k) \qquad (k \in \{1, \ldots, K_2\}),$$

and the strict declining monotonicity of the bid schedule $\{x_2^k\}_{k=1}^{K_2}$. Now, the "all-or-nothing" disclosure by a type $\widehat{c}_2 \in C_2$ may be seen as a continuous accumulation of identical pieces of evidence δ_2 with $\delta_2(\widehat{c}_2) = 1$ and $\delta_2(c_2) = \frac{1}{K_2 - 1}$ for any $c_2 \in C_2 \backslash \{\widehat{c}_2\}$. Suppose first that $K_2 \geq 3$. Then, there exists an extremal type $\widehat{c}_2 \in \{\underline{c}_2, \overline{c}_2\}$ such that, for the just defined marginal piece of evidence, the condition in Lemma 3 is fulfilled. Specifically, $\widehat{c}_2 = \underline{c}_2$ if $c_1^\# > \sqrt{\underline{c}_2 \overline{c}_2}$, and $\widehat{c}_2 = \overline{c}_2$ if $c_1^\# \leq \sqrt{\underline{c}_2 \overline{c}_2}$. Clearly, this condition remains valid on the entire path. Hence, there is one extremal type that strictly prefers to disclose. Suppose next that $K_2 = 2$. Then the same argument goes through provided that $c_1^\# \neq \sqrt{\underline{c}_2 \overline{c}_2}$. This proves the claim. □

Proof of Proposition 3. Consider an ex-ante symmetric, unbiased lottery contest specified by $c_1^1 = c_2^1 \equiv \underline{c} < \overline{c} \equiv c_1^2 = c_2^2$ and $q_1(\underline{c}) = q_2(\underline{c}) = q_1(\overline{c}) = q_2(\overline{c}) = \frac{1}{2}$.[70] Assumption 1 does not hold. However, for $\overline{c}/\underline{c}$ sufficiently large, all types concealing their private information is a perfect Bayesian equilibrium. To understand why, note that the efficient type of player 1 has expected payoffs from not disclosing of

$$E_{c_2}[\Pi_1(\xi_1^*(\underline{c}), \xi_2^\#(c_2)); \underline{c}] = \frac{1}{8} + \frac{\overline{c}^2}{2(\underline{c} + \overline{c})^2}. \tag{138}$$

This is strictly increasing in $\overline{c}/\underline{c}$, reflecting a mitigating effect of uncertainty. Suppose that the

[70] This case is isomorphic to Malueg and Yates (2004).

efficient type $\underline{c}$ of player 1 reveals her private information. If $\overline{c}/\underline{c} \geq 9$, this marginalizes the inefficient type of player 2, i.e, $\xi_2^{\#}(\underline{c}, \overline{c}) = 0$. As a result, two equally efficient opponents meet with probability one half, so that player 1's expected payoff is

$$E_{c_2}[\Pi_1(x_1^{\#}(\underline{c}), \xi_2^{\#}(c_2; \underline{c})); \underline{c}] = \frac{5}{9}. \tag{139}$$

Therefore, not disclosing is optimal for the efficient type if $\overline{c}/\underline{c} \geq \frac{6\sqrt{31}-31}{5} \approx 12.8$.

For the inefficient type of player 1, we compare the expected payoff from not disclosing,

$$E_{c_2}[\Pi_1(\xi_1^*(\overline{c}), \xi_2^*(c_2)); \overline{c}] = \frac{1}{8} + \frac{\underline{c}^2}{2(\underline{c} + \overline{c})^2}, \tag{140}$$

with the expected payoff in the contest with one-sided private information,

$$E_{c_2}[\Pi_1(x_1^{\#}(\underline{c}), \xi_2^{\#}(c_2; \underline{c})); \underline{c}] = \frac{(\sqrt{\underline{c}} + \sqrt{\overline{c}})^2(\underline{c} + \overline{c})}{2(\underline{c} + 3\overline{c})^2}, \tag{141}$$

which is always strictly lower. □

Proof of Proposition 4. (i) Let $c_1^{\#} \in C_1$ denote the public type of the favorite. For the unbiased lottery contest, an interior equilibrium may be easily derived from the corresponding first-order conditions (Hurley and Shogren, 1998a; Epstein and Mealem, 2013; Zhang and Zhou, 2016). In our set-up, this yields equilibrium bids

$$x_1^{\#} = \left(\frac{E\left[\sqrt{c_2}\right]}{c_1^{\#} + E\left[c_2\right]} \right)^2, \text{ and} \tag{142}$$

$$\xi_2^{\#}(c_2) = \sqrt{\frac{x_1^{\#}}{c_2}} - x_1^{\#} \qquad (c_2 \in C_2), \tag{143}$$

where we dropped, for convenience, the subscript c_2 from the expectation operator. Using these expressions, total expected costs under mandatory concealment are easily derived as

$$\begin{aligned}
\mathbf{C}^{\mathrm{MC}} &= c_1^{\#}x_1^{\#} + E[c_2\xi_2^{\#}(c_2)] && (144)\\
&= (c_1^{\#} - E[c_2])x_1^{\#} + E[\sqrt{c_2}]\sqrt{x_1^{\#}} && (145)\\
&= \frac{(c_1^{\#} - E[c_2])E[\sqrt{c_2}]^2}{(c_1^{\#} + E[c_2])^2} + \frac{E[\sqrt{c_2}]^2}{c_1^{\#} + E[c_2]} && (146)\\
&= \frac{2c_1^{\#}E[\sqrt{c_2}]^2}{(c_1^{\#} + E[c_2])^2}. && (147)
\end{aligned}$$

Note that this formula entails, in particular, the complete-information case where c_2 is public as well. Therefore, being an expectation over such complete-information scenarios, total expected costs under full revelation amount to

$$\mathbf{C}^{\mathrm{FR}} = E\left[\frac{2c_1^{\#}c_2}{(c_1^{\#} + c_2)^2}\right]. \tag{148}$$

To compare the two expressions, we apply Lemma A.5 with $Y = \sqrt{c_2/c_1^{\#}}$ and $g(x,y) = g_1(x,y) \equiv \frac{2x^2}{(1+y)^2}$. The Hessian of the mapping g_1 is given by

$$H_{g_1}(x,y) = \begin{pmatrix} \frac{4}{(1+y)^2} & -\frac{8x}{(1+y)^3} \\ -\frac{8x}{(1+y)^3} & \frac{12x^2}{(1+y)^4} \end{pmatrix}. \tag{149}$$

It suffices to show that, for any $x > 1$, $y \geq x^2$, $d_x > 0$, $d_y > 0$ such that $\frac{d_y}{d_x} > \frac{y-1}{x-1}$, the quadratic form

$$\begin{aligned}
(d_x\ d_y)\,(H_{g_1}(x,y))\begin{pmatrix} d_x \\ d_y \end{pmatrix} &= \frac{4}{(1+y)^2}{d_x}^2 - \frac{16x}{(1+y)^3}d_xd_y + \frac{12x^2}{(1+y)^4}{d_y}^2 && (150)\\
&= \frac{4{d_x}^2}{(1+y)^2}\left(1 - \frac{x}{1+y}\frac{d_y}{d_x}\right)\left(1 - \frac{3x}{1+y}\frac{d_y}{d_x}\right) && (151)
\end{aligned}$$

attains a positive value. To see this, one checks that

$$\frac{x}{y+1}\cdot\frac{d_y}{d_x} > \underbrace{\frac{x}{y+1}\cdot\frac{y-1}{x-1}}_{\text{increasing in } y} \geq \frac{x}{x^2+1}\cdot\frac{x^2-1}{x-1} = \frac{x^2+x}{x^2+1} > 1. \tag{152}$$

Clearly then, the right-hand side of (151) is positive. This proves the claim. It follows that

$$\mathbf{C}^{\mathrm{FR}} = E\left[\frac{2(c_2/c_1^{\#})}{(1+(c_2/c_1^{\#}))^2}\right] > \frac{2E\left[\sqrt{c_2/c_1^{\#}}\right]^2}{(1+E[c_2/c_1^{\#}])^2} = \mathbf{C}^{\mathrm{MC}}, \tag{153}$$

i.e., total expected costs are indeed strictly higher under full revelation than under mandatory concealment. In particular, given that, by Eq. (35), expected costs in the lottery contest are the same across contestants, and given that the favorite's type is public, the favorite exerts a higher effort under full revelation than under mandatory concealment.

(ii) From (142) and (143), player 1's probability of winning is easily determined as

$$p_1^{\mathrm{MC}} = E\left[\frac{x_1^{\#}}{x_1^{\#}+\xi_2^{\#}(c_2)}\right] = E\left[\sqrt{x_1^{\#}c_2}\right] = \frac{E\left[\sqrt{c_2}\right]^2}{c_1^{\#}+E\left[c_2\right]}, \tag{154}$$

under mandatory concealment, and by

$$p_1^{\mathrm{FR}} = E\left[\frac{c_2}{c_1^{\#}+c_2}\right] \tag{155}$$

under full revelation. Again, we apply Lemma A.5 for $Y = \sqrt{c_2/c_1^{\#}}$, using this time the mapping $g(x,y) = g_2(x,y) \equiv \frac{x^2}{1+y}$. The corresponding Hessian reads

$$H_{g_2}(x,y) = \begin{pmatrix} \dfrac{2}{1+y} & -\dfrac{2x}{(1+y)^2} \\ -\dfrac{2x}{(1+y)^2} & \dfrac{2x^2}{(1+y)^3} \end{pmatrix}. \tag{156}$$

Suppose that $x > 1$, $y \geq x^2$, $d_x > 0$, and $d_y > 0$. Then, clearly,

$$(d_x \ d_y)\left(H_{g_2}(x,y)\right)\begin{pmatrix} d_x \\ d_y \end{pmatrix} = \frac{2{d_x}^2}{1+y}\left(1 - \frac{x}{1+y}\frac{d_y}{d_x}\right)^2 \geq 0. \tag{157}$$

Moreover, from relationship (152), inequality (157) is even strict, which implies strict convexity of g_2 along the relevant linear path segment. Thus, we have

$$p_1^{\text{FR}} = E\left[\frac{(c_2/c_1^{\#})}{1+(c_2/c_1^{\#})}\right] > \frac{E\left[\sqrt{c_2/c_1^{\#}}\right]^2}{1+E[c_2/c_1^{\#}]} = p_1^{\text{MC}}, \tag{158}$$

and, consequently, also $p_2^{\text{FR}} < p_2^{\text{MC}}$.

(iii) Since expected costs are equal across players in the lottery contest, ex-ante expected payoffs for the underdog are given by $\Pi_2^{\text{FR}} = p_2^{\text{FR}} - \frac{\mathbf{C}^{\text{FR}}}{2}$ under full revelation, and by $\Pi_2^{\text{MC}} = p_2^{\text{MC}} - \frac{\mathbf{C}^{\text{MC}}}{2}$ under mandatory concealment. As seen above, $p_2^{\text{FR}} < p_2^{\text{MC}}$ and $\mathbf{C}^{\text{FR}} > \mathbf{C}^{\text{MC}}$. Hence, $\Pi_2^{\text{FR}} < \Pi_2^{\text{MC}}$, as claimed. $\square$

Proof of Proposition 5. The underdog's expected profits under mandatory concealment (i.e., the underdog "closes her eyes") and under full revelation (i.e., the underdog "opens her eyes"), respectively, are easily derived as

$$\mathbf{\Pi}_2^{\text{MC}} = \frac{E[\sqrt{c_1}]^2 E[c_1]}{(E[c_1]+c_2^{\#})^2}, \tag{159}$$

$$\mathbf{\Pi}_1^{\text{FR}} = E\left[\frac{c_1^2}{(c_1+c_2^{\#})^2}\right]. \tag{160}$$

To compare the respective expressions for $\mathbf{\Pi}^{\text{MC}}$ and $\mathbf{\Pi}^{\text{FR}}$, we apply Lemma A.6, in which the support of the random variable Y is assumed to be $(0,1)$, with $Y = \sqrt{c_1/c_2^{\#}}$ and $g(x,y) = g_3(x,y) \equiv \frac{x^2 y}{(1+y)^2}$. The Hessian of the mapping g_3 is given by

$$H_{g_3}(x,y) = \begin{pmatrix} \frac{2y}{(y+1)^2} & \frac{2x(1-y)}{(y+1)^3} \\ \frac{2x(1-y)}{(y+1)^3} & -\frac{2x^2(2-y)}{(y+1)^4} \end{pmatrix}. \tag{161}$$

It suffices to show that, for any $x, y \in (0,1)$, $y \geq x^2$, $d_x > 0$, $d_y > 0$ such that $\frac{d_y}{d_x} < \frac{1-y}{1-x}$, the quadratic form

$$(d_x \ d_y)\,(H_{g_3}(x,y))\begin{pmatrix} d_x \\ d_y \end{pmatrix} = \frac{2y}{(1+y)^2}{d_x}^2 + \frac{4x(1-y)}{(y+1)^3}d_x d_y - \frac{2x^2(2-y)}{(y+1)^4}{d_y}^2 \tag{162}$$

$$= \frac{2{d_x}^2}{(1+y)^2}\left(1 - \frac{x}{1+y}\frac{d_y}{d_x}\right)\left(y + \frac{x(2-y)}{1+y}\frac{d_y}{d_x}\right). \tag{163}$$

attains a positive value. To see this, one checks that

$$\frac{x}{y+1}\cdot\frac{d_y}{d_x} < \underbrace{\frac{x}{y+1}\cdot\frac{1-y}{1-x}}_{\text{decreasing in } y} \leq \frac{3x}{x^2+1}\cdot\frac{1-x^2}{1-x} = \frac{x^2+x}{x^2+1} < 1. \tag{164}$$

This proves the claim. □

Proof of Lemma 3. We use the shorthand notation $x_2^k \equiv \xi_2^{\#}(c_2^k)$ and $p_2^k \equiv p_2(x_1^{\#}, x_2^k)$ for $k \in \{1, \ldots, K_2\}$. We have the first-order conditions

$$\sum_{k=1}^{K_2} q_2^k p_2^k (1-p_2^k) = c_1^{\#}\Phi(x_1^{\#}), \tag{165}$$

$$p_2^k(1-p_2^k) = c_2^k \Phi(x_2^k) \qquad (k \in \{1, \ldots, K_2\}). \tag{166}$$

Total differentiation of (165) yields

$$\sum_{k=1}^{K_2} q_2^k (1-2p_2^k) dp_2^k + \left\{\sum_{k=1}^{K_2} \delta_2^k p_2^k (1-p_2^k)\right\} d\varepsilon = c_1^{\#}\Phi'(x_1^{\#}) dx_1^{\#}. \tag{167}$$

Moreover, for $k \in \{1, \ldots, K_2\}$, using (166),

$$dp_2^k = \frac{\partial p_2^k}{\partial x_1^{\#}} dx_1^{\#} + \frac{\partial p_2^k}{\partial x_2^k} dx_2^k \tag{168}$$

$$= -\frac{p_2^k(1-p_2^k)}{\Phi(x_1^{\#})} dx_1^{\#} + \frac{p_2^k(1-p_2^k)}{\Phi(x_2^k)} dx_2^k \tag{169}$$

$$= c_2^k \left\{ -\frac{\Phi(x_2^k)}{\Phi(x_1^{\#})} dx_1^{\#} + dx_2^k \right\}. \tag{170}$$

From Lemma A.1(iv),

$$dx_2^k = \frac{\Phi(x_2^k)}{\Phi(x_1^\#)} \frac{2p_2^k - 1}{\Phi'(x_2^k) - 1 + 2p_2^k} dx_1^\#, \tag{171}$$

so that

$$dp_2^k = -c_2^k \frac{\Phi(x_2^k)}{\Phi(x_1^\#)} \frac{\Phi'(x_2^k)}{\Phi'(x_2^k) - 1 + 2p_2^k} dx_1^\# \qquad (k \in \{1, \ldots, K_2\}). \tag{172}$$

Using (172) to eliminate dp_2^k in (167), we obtain

$$\begin{aligned}
&\left\{\sum_{k=1}^{K_2} \delta_2^k c_2^k \Phi(x_2^k)\right\} d\varepsilon \\
= \;& \frac{dx_1^\#}{\Phi(x_1^\#)} \left\{ c_1^\# \Phi(x_1^\#)\Phi'(x_1^\#) - \sum_{k=1}^{K_2} q_2^k c_2^k \Phi(x_2^k) \frac{(2p_2^k - 1)\Phi'(x_2^k)}{\Phi'(x_2^k) + 2p_2^k - 1} \right\} &(173)\\
= \;& \frac{dx_1^\#}{\Phi(x_1^\#)} \sum_{k=1}^{K_2} q_2^k c_2^k \Phi(x_2^k) \left\{ \Phi'(x_1^\#) - \frac{(2p_2^k - 1)\Phi'(x_2^k)}{\Phi'(x_2^k) + 2p_2^k - 1} \right\}. &(174)
\end{aligned}$$

We claim that

$$\Phi'(x_1^\#) - \frac{(2p_2^k - 1)\Phi'(x_2^k)}{\Phi'(x_2^k) + 2p_2^k - 1} > 0 \qquad (k \in \{1, \ldots, K_2\}). \tag{175}$$

Indeed, this is obvious for $p_2^k \le \frac{1}{2}$ since $\Phi' \ge 1$ by Lemma A.1(ii). On the other hand, if $p_2^k > \frac{1}{2}$, then $2p_2^k - 1 > 0$, and hence,

$$(2p_2^k - 1) \cdot \frac{\Phi'(x_2^k)}{\Phi'(x_2^k) + 2p_2^k - 1} < 2p_2^k - 1 \le 1 \le \Phi'(x_1^\#). \tag{176}$$

Hence, the claim (175) is indeed true. Therefore,

$$\frac{dx_1^\#}{d\varepsilon} = \{\text{sth. positive}\}^{-1} \cdot \left\{ \sum_{k=1}^{K_2} \delta_2^k \varphi^k \right\}, \tag{177}$$

where $\varphi^k = c_2^k \Phi(x_2^k)$, for $k \in \{1, \ldots, K_2\}$. The hump-shape of the sequence $(\varphi^1, \ldots, \varphi^{K_2})$ follows immediately from (166) and Lemma A.2. This proves the lemma. □

Proof of Proposition 6. As seen above, before the shift,

$$x_1^{\#,\text{ before}} = \left(\frac{E[\sqrt{c_2}]}{c_1^\# + E[c_2]} \right)^2. \tag{178}$$

It suffices to prove the claim for a FOSD shift in the type distribution of player 2 that makes $\overline{c}_2$ more likely by a probability $\varepsilon > 0$, and another type $\widehat{c}_2 < \overline{c}_2$ less likely by the same probability. Then, after the shift, we get

$$x_1^{\#,\text{ after}} = \left(\frac{E[\sqrt{c_2}] + \varepsilon(\sqrt{\overline{c}_2} - \sqrt{\widehat{c}_2})}{c_1^{\#} + E[c_2] + \varepsilon(\overline{c}_2 - \widehat{c}_2)} \right)^2, \tag{179}$$

Let $\widehat{\varepsilon} = \varepsilon(\sqrt{\overline{c}_2} - \sqrt{\widehat{c}_2})$. Then,

$$x_1^{\#,\text{ after}} = \left(\frac{E[\sqrt{c_2}] + \widehat{\varepsilon}}{c_1^{\#} + E[c_2] + \widehat{\varepsilon}(\sqrt{\overline{c}_2} + \sqrt{\widehat{c}_2})} \right)^2. \tag{180}$$

It follows that $x_1^{\#,\text{ after}} < x_1^{\#,\text{ before}}$ holds if and only if

$$\frac{1}{\sqrt{\overline{c}_2} + \sqrt{\widehat{c}_2}} < \frac{E[\sqrt{c_2}]}{c_1^{\#} + E[c_2]} \tag{181}$$

$$\Leftrightarrow \quad c_1^{\#} + E[c_2] < E[\sqrt{c_2}](\sqrt{\overline{c}_2} + \sqrt{\widehat{c}_2}). \tag{182}$$

This is equivalent to

$$c_1^{\#} + E[c_2] < \underbrace{E[\sqrt{c_2 \overline{c}_2}]}_{>E[c_2]} + \underbrace{E[\sqrt{c_2 \widehat{c}_2}]}_{\geq c_1^{\#}}, \tag{183}$$

which holds true. □

The following lemma is used in the proof of Proposition 7.

Lemma B.2 *An active type $c_2 \in C_2$ prefers a marginal change $d\xi_1$ in player 1's bid schedule if and only if*

$$E_{c_1}\left[\frac{p_2(\xi_1^*(c_1), \xi_2^*(c_2))(1 - p_2(\xi_1^*(c_1), \xi_2^*(c_2)))}{\xi_1^*(c_1)} d\xi_1(c_1) \right] < 0 \tag{184}$$

Proof. Type c_2's expected payoff is given as

$$E_{c_1}[\Pi_2] = \max_{x_2 \geq 0} E[p_2(\xi_1^*(c_1), x_2) - c_2 x_2]. \tag{185}$$

The claim follows now by the envelope theorem. □

Proof of Proposition 7. First-order conditions read

$$q_2^1\frac{\mathbf{p}_{11}(1-\mathbf{p}_{11})}{x_1^1}+q_2^2\frac{\mathbf{p}_{12}(1-\mathbf{p}_{12})}{x_1^1} = \underline{c}_1 \tag{186}$$

$$q_2^1\frac{\mathbf{p}_{21}(1-\mathbf{p}_{21})}{x_1^2}+q_2^2\frac{\mathbf{p}_{22}(1-\mathbf{p}_{22})}{x_1^2} = \overline{c}_1 \tag{187}$$

$$q_1^1\frac{\mathbf{p}_{11}(1-\mathbf{p}_{11})}{x_2^1}+q_1^2\frac{\mathbf{p}_{21}(1-\mathbf{p}_{21})}{x_2^1} = \underline{c}_2 \tag{188}$$

$$q_1^1\frac{\mathbf{p}_{12}(1-\mathbf{p}_{12})}{x_2^2}+q_1^2\frac{\mathbf{p}_{22}(1-\mathbf{p}_{22})}{x_2^2} = \overline{c}_2, \tag{189}$$

where we use the shorthand notation

$$\mathbf{p}_{k_1k_2} = p_1(\xi_1^*(c_1^{k_1}),\xi_2^*(c_2^{k_2})) = 1-p_2(\xi_1^*(c_1^{k_1}),\xi_2^*(c_2^{k_2})). \tag{190}$$

We consider now a marginal increase of q_2^2 by ε, combined with a simultaneous reduction of q_2^1 by ε (so that the resulting probabilities for player 2's types continue to sum up to one). Intuitively, this corresponds to a marginal counterpart of the scenario studied in Proposition 1. Taking the total differential with respect to the variables $x_1^1=\xi_1^*(c_1^1)$, $x_1^2=\xi_1^*(c_1^2)$, $x_2^1=\xi_2^*(c_2^1)$, $x_2^2=\xi_2^*(c_2^2)$, and ε, we obtain the system

$$\begin{pmatrix} \mathbf{d}_1^1 & 0 & -\mathbf{a}_{11} & -\mathbf{a}_{12} \\ 0 & \mathbf{d}_1^2 & -\mathbf{a}_{21} & -\mathbf{a}_{22} \\ \mathbf{a}_{11} & \mathbf{a}_{21} & \mathbf{d}_2^1 & 0 \\ \mathbf{a}_{12} & \mathbf{a}_{22} & 0 & \mathbf{d}_2^2 \end{pmatrix} \begin{pmatrix} q_1^1 d\ln x_1^1 \\ q_1^2 d\ln x_1^2 \\ q_2^1 d\ln x_2^1 \\ q_2^2 d\ln x_2^2 \end{pmatrix} = \begin{pmatrix} \mathbf{p}_{11}(1-\mathbf{p}_{11})-\mathbf{p}_{12}(1-\mathbf{p}_{12}) \\ -\mathbf{p}_{21}(1-\mathbf{p}_{21})+\mathbf{p}_{22}(1-\mathbf{p}_{22}) \\ 0 \\ 0 \end{pmatrix} d\varepsilon, \tag{191}$$

where the diagonal entries of the matrix in (191) are given by

$$\mathbf{d}_1^1 = -\frac{2}{q_1^1}\left(q_2^1(\mathbf{p}_{11})^2(1-\mathbf{p}_{11})+q_2^2(\mathbf{p}_{12})^2(1-\mathbf{p}_{12})\right), \tag{192}$$

$$\mathbf{d}_1^2 = -\frac{2}{q_1^2}\left(q_2^1(\mathbf{p}_{21})^2(1-\mathbf{p}_{21})+q_2^2(\mathbf{p}_{22})^2(1-\mathbf{p}_{22})\right), \tag{193}$$

$$\mathbf{d}_2^1 = -\frac{2}{q_2^1}\left(q_1^1\mathbf{p}_{11}(1-\mathbf{p}_{11})^2+q_1^2\mathbf{p}_{21}(1-\mathbf{p}_{21})^2\right), \tag{194}$$

$$\mathbf{d}_2^2 = -\frac{2}{q_2^2}\left(q_1^1\mathbf{p}_{12}(1-\mathbf{p}_{12})^2+q_1^2\mathbf{p}_{22}(1-\mathbf{p}_{22})^2\right), \tag{195}$$

while the off-diagonal entries are given by

$$\mathbf{a}_{k_1k_2} = (1 - 2\mathbf{p}_{k_1k_2})\mathbf{p}_{k_1k_1}(1 - \mathbf{p}_{k_1k_2}) \qquad (k_1, k_2 \in \{1, 2\}). \tag{196}$$

Inverting the matrix in (191) yields

$$\begin{pmatrix} \mathbf{d}_1^1 & 0 & -\mathbf{a}_{11} & -\mathbf{a}_{12} \\ 0 & \mathbf{d}_1^2 & -\mathbf{a}_{21} & -\mathbf{a}_{22} \\ \mathbf{a}_{11} & \mathbf{a}_{21} & \mathbf{d}_2^1 & 0 \\ \mathbf{a}_{12} & \mathbf{a}_{22} & 0 & \mathbf{d}_2^2 \end{pmatrix}^{-1} \tag{197}$$

$$= \frac{1}{\Delta} \begin{pmatrix} (\mathbf{a}_{21})^2\mathbf{d}_2^2 + (\mathbf{a}_{22})^2\mathbf{d}_2^1 + \mathbf{d}_1^2\mathbf{d}_2^1\mathbf{d}_2^2 < 0 & -\mathbf{a}_{11}\mathbf{a}_{21}\mathbf{d}_2^2 - \mathbf{a}_{12}\mathbf{a}_{22}\mathbf{d}_2^1 & * & * \\ -\mathbf{a}_{11}\mathbf{a}_{21}\mathbf{d}_2^2 - \mathbf{a}_{12}\mathbf{a}_{22}\mathbf{d}_2^1 & (\mathbf{a}_{11})^2\mathbf{d}_2^2 + (\mathbf{a}_{12})^2\mathbf{d}_2^1 + \mathbf{d}_1^1\mathbf{d}_2^1\mathbf{d}_2^2 < 0 & * & * \\ -\mathbf{a}_{22}(\mathbf{a}_{11}\mathbf{a}_{22} - \mathbf{a}_{12}\mathbf{a}_{21}) - \mathbf{a}_{11}\mathbf{d}_1^2\mathbf{d}_2^2 & \mathbf{a}_{12}(\mathbf{a}_{11}\mathbf{a}_{22} - \mathbf{a}_{12}\mathbf{a}_{21}) - \mathbf{a}_{21}\mathbf{d}_1^1\mathbf{d}_2^2 & * & * \\ \mathbf{a}_{21}(\mathbf{a}_{11}\mathbf{a}_{22} - \mathbf{a}_{12}\mathbf{a}_{21}) - \mathbf{a}_{12}\mathbf{d}_1^2\mathbf{d}_2^1 & -\mathbf{a}_{11}(\mathbf{a}_{11}\mathbf{a}_{22} - \mathbf{a}_{12}\mathbf{a}_{21}) - \mathbf{a}_{22}\mathbf{d}_1^1\mathbf{d}_2^1 & * & * \end{pmatrix},$$

with

$$\Delta = \det \begin{pmatrix} \mathbf{a}_{11} & \mathbf{a}_{21} \\ \mathbf{a}_{12} & \mathbf{a}_{22} \end{pmatrix}^2 + \mathbf{d}_1^1 \left[(\mathbf{a}_{22})^2\mathbf{d}_2^1 + (\mathbf{a}_{21})^2\mathbf{d}_2^2\right] + \mathbf{d}_1^2 \left[(\mathbf{a}_{12})^2\mathbf{d}_2^1 + (\mathbf{a}_{11})^2\mathbf{d}_2^2\right] + \mathbf{d}_1^1\mathbf{d}_2^1\mathbf{d}_1^2\mathbf{d}_2^2 > 0. \tag{198}$$

These equations allow to disentangle the implications of a marginal shift in player 2's type distribution. First, one notes that, while the repective signs of $\mathbf{d}_1^1$, $\mathbf{d}_1^2$, $\mathbf{d}_2^1$, and $\mathbf{d}_2^2$ are all negative, the sign of $\mathbf{a}_{k_1k_2}$ is positive (negative) if $\mathbf{p}_{k_1k_1} < \frac{1}{2}$ (if $\mathbf{p}_{k_1k_1} > \frac{1}{2}$) or equivalently, if $x_1^{k_1} < x_2^{k_2}$ (if $x_1^{k_1} > x_2^{k_2}$). Next, one notes that, as a consequency of the monotonicity of bid schedules (Lemma A.3),

$$\left\{\mathbf{p}_{k_11} > \frac{1}{2}\right\} \quad \Rightarrow \quad \mathbf{p}_{k_11}(1 - \mathbf{p}_{k_11}) - \mathbf{p}_{k_12}(1 - \mathbf{p}_{k_12}) > 0 \tag{199}$$

$$\left\{\mathbf{p}_{k_12} < \frac{1}{2}\right\} \quad \Rightarrow \quad \mathbf{p}_{k_11}(1 - \mathbf{p}_{k_11}) - \mathbf{p}_{k_12}(1 - \mathbf{p}_{k_12}) < 0. \tag{200}$$

Using these facts, we can now prove the proposition. Claims (i) and (ii) are immediate from the analysis above. As for claim (iii), using Lemma B.2, the expected payoffs of player 2's weak type c_2^2 rises strictly if

$$q_1^1 \frac{\mathbf{p}_{11}(1-\mathbf{p}_{11})}{x_1^1} dx_1^1 + q_1^2 \frac{\mathbf{p}_{12}(1-\mathbf{p}_{12})}{x_1^2} dx_1^2 < 0, \tag{201}$$

or equivalently, if

$$\mathbf{p}_{11}(1-\mathbf{p}_{11}) \left(q_1^1 d \ln x_1^1\right) + \mathbf{p}_{12}(1-\mathbf{p}_{12}) \left(q_1^2 d \ln x_1^2\right) < 0. \tag{202}$$

Using the results from above, this is seen to be equivalent to

$$\begin{aligned} 0 \quad > \quad & \mathbf{p}_{11}(1-\mathbf{p}_{11}) \left\{ \begin{array}{c} ((\mathbf{a}_{21})^2 \mathbf{d}_2^2 + (\mathbf{a}_{22})^2 \mathbf{d}_2^1 + \mathbf{d}_1^2 \mathbf{d}_2^1 \mathbf{d}_2^2)(\mathbf{p}_{11}(1-\mathbf{p}_{11}) - \mathbf{p}_{12}(1-\mathbf{p}_{12})) \\ +(\mathbf{a}_{11}\mathbf{a}_{21}\mathbf{d}_2^2 + \mathbf{a}_{12}\mathbf{a}_{22}\mathbf{d}_2^1)(-\mathbf{p}_{21}(1-\mathbf{p}_{21}) + \mathbf{p}_{22}(1-\mathbf{p}_{22})) \end{array} \right\} \\ & +\mathbf{p}_{12}(1-\mathbf{p}_{12}) \left\{ \begin{array}{c} -(\mathbf{a}_{11}\mathbf{a}_{21}\mathbf{d}_2^2 + \mathbf{a}_{12}\mathbf{a}_{22}\mathbf{d}_2^1)(\mathbf{p}_{11}(1-\mathbf{p}_{11}) - \mathbf{p}_{12}(1-\mathbf{p}_{12})) \\ +((\mathbf{a}_{11})^2 \mathbf{d}_2^2 + (\mathbf{a}_{12})^2 \mathbf{d}_2^1 + \mathbf{d}_1^1 \mathbf{d}_2^1 \mathbf{d}_2^2)(-\mathbf{p}_{21}(1-\mathbf{p}_{21}) + \mathbf{p}_{22}(1-\mathbf{p}_{22})) \end{array} \right\}. \end{aligned} \tag{203}$$

This proves the last claim, and hence, the proposition. □

Proof of Proposition 8. To provoke a contradiction, suppose there are two messages m_1 and m_2, both used in equilibrium with positive probability, so that the equilibrium effort of the uninformed player is strictly lower for message m_1 than for m_2. In straightforward extension of Lemma A.4(ii), all types of the informed player share the same strict preference for lowering the uninformed player's bid—provided they are active against the lower bid. Therefore, any type active after sending m_1 strictly prefers sending m_1 over sending m_2. Moreover, only types anticipating marginalization after sending m_1 can rationally send m_2 (because they bid zero regardless of the message they sent). But then, the uninformed player would have a strict incentive to lower her effort for message m_2 weakly below her equilibrium effort for m_1. Hence, we arrive at the desired contradiction. □

Proof of Proposition 9. (i) In a fully separating equilibrium, the uninformed player 1 chooses the bid

$$x_1^\circ = \frac{c_2}{\left(c_1^\# + c_2\right)^2} \tag{204}$$

when receiving type c_2's message. Recalling that $c_1^{\#}$ is constant, the right-hand side of Eq. (204) is positive and strictly hump-shaped in c_2. However, by Proposition 8, player 1 necessarily chooses the same bid in response to all types $c_2 \in C_2$. Therefore, there can be at most two types in C_2. (ii) In the lottery contest with complete information, the best response of player 1 against a given effort level x_2 of player 2 is $x_1 = \max\{0, \sqrt{x_2/c_1} - x_2\}$. Plugging this into the payoff function yields $\Pi_1 = (\max\{0, 1 - \sqrt{c_1 x_2}\})^2$. Suppose that player 1's type c_1 claims being $\widehat{c}_1$. As type c_1 is active under truth-telling, she will consider such deviation only if she stays active, but from the profit formula, this will be the case for lower-cost types. Then, player 2 chooses $x_2 = \widehat{c}_1/(\widehat{c}_1 + c_2)^2$. Hence, type c_1's profit from claiming being type $\widehat{c}_1$ is

$$E_{c_2}[\Pi_1] = E_{c_2}\left[\left(\underbrace{1 - \frac{\sqrt{c_1 \widehat{c}_1}}{\widehat{c}_1 + c_2}}_{>0 \text{ by activity}}\right)^2\right]. \tag{205}$$

Suppose now that $\overline{c}_1 \leq \underline{c}_2$. Then, the right-hand side of Eq. (205) is strictly declining in $\widehat{c}_1$. Hence, truth-telling cannot be an equilibrium—as, e.g., player 1's weakest type $\overline{c}_1$ would have an incentive to claim to be stronger. (iii) Let $\underline{q}$ denote the probability of the efficient type of player 1. Under the given condition, the incentive compatibility condition for the efficient type (205) reads

$$\underline{q}\underbrace{\left(1 - \frac{\underline{c}}{\underline{c} + \underline{c}}\right)^2}_{=\frac{1}{4}} + (1 - \underline{q})\left(1 - \frac{\underline{c}}{\underline{c} + \overline{c}}\right)^2 \geq \underline{q}\left(1 - \frac{\sqrt{\underline{c}\overline{c}}}{\overline{c} + \underline{c}}\right)^2 + (1 - \underline{q})\left(1 - \frac{\sqrt{\underline{c}\overline{c}}}{\overline{c} + \overline{c}}\right), \tag{206}$$

while the incentive compatibility condition for the inefficient type reads

$$\underline{q}\left(1 - \frac{\overline{c}}{\underline{c} + \overline{c}}\right)^2 + (1 - \underline{q})\underbrace{\left(1 - \frac{\overline{c}}{\overline{c} + \overline{c}}\right)^2}_{=\frac{1}{4}} \geq \underline{q}\left(\max\left\{0, 1 - \frac{\sqrt{\underline{c}\overline{c}}}{\underline{c} + \underline{c}}\right\}\right)^2 + (1 - \underline{q})\left(1 - \frac{\sqrt{\underline{c}\overline{c}}}{\underline{c} + \overline{c}}\right). \tag{207}$$

Note that

$$\left(1-\frac{\sqrt{\underline{c}\overline{c}}}{\overline{c}+\underline{c}}\right)^2 > \frac{1}{4}, \tag{208}$$

$$\left(1-\frac{\overline{c}}{\underline{c}+\overline{c}}\right)^2 > \left(1-\frac{\sqrt{\underline{c}\overline{c}}}{\underline{c}+\underline{c}}\right)^2. \tag{209}$$

Therefore, rewriting yields

$$\frac{\left(1-\frac{\underline{c}}{\underline{c}+\overline{c}}\right)^2-\left(1-\frac{\sqrt{\underline{c}\overline{c}}}{\overline{c}+\overline{c}}\right)}{\left(1-\frac{\sqrt{\underline{c}\overline{c}}}{\underline{c}+\overline{c}}\right)^2-\frac{1}{4}} \geq \frac{\underline{q}}{1-\underline{q}} \geq \frac{\left(1-\frac{\sqrt{\underline{c}\overline{c}}}{\underline{c}+\overline{c}}\right)-\frac{1}{4}}{\left(1-\frac{\overline{c}}{\underline{c}+\overline{c}}\right)^2-\left(\max\left\{0,1-\frac{\sqrt{\underline{c}\overline{c}}}{\underline{c}+\underline{c}}\right\}\right)^2} \tag{210}$$

as a necessary condition for truth-telling. For convenience, we write $x=\overline{c}/\underline{c}$. Then, we get

$$\frac{\left(1-\frac{1}{1+x}\right)^2-\left(1-\frac{1}{2\sqrt{x}}\right)}{\left(1-\frac{\sqrt{x}}{1+x}\right)^2-\frac{1}{4}} \geq \frac{\left(1-\frac{\sqrt{x}}{1+x}\right)-\frac{1}{4}}{\left(1-\frac{x}{1+x}\right)^2-\left(\max\left\{0,1-\frac{\sqrt{x}}{2}\right\}\right)^2}. \tag{211}$$

Multiplying out leads to

$$\begin{aligned} &((1-\frac{1}{1+x})^2-(1-\frac{1}{2\sqrt{x}})^2)((1-\frac{x}{1+x})^2-(1-\frac{\sqrt{x}}{2})^2)-((1-\frac{\sqrt{x}}{1+x})^2-\frac{1}{4})^2 \\ =\ & -\frac{1}{4}\frac{\left(\sqrt{x}+1\right)^2\left(\sqrt{x}-1\right)^6}{\sqrt{x}\left(x+1\right)^3} \qquad (212) \\ <\ & 0 \qquad (213) \end{aligned}$$

for the case without self-marginalization (i.e., for $x<4$), and to

$$\begin{aligned} &((1-\frac{1}{1+x})^2-(1-\frac{1}{2\sqrt{x}})^2)(1-\frac{x}{1+x})^2-((1-\frac{\sqrt{x}}{1+x})^2-\frac{1}{4})^2 \\ =\ & -\frac{1}{16}\left(9x+46x^2+9x^3-8\sqrt{x}-38x^{\frac{3}{2}}-30x^{\frac{5}{2}}+4\right)\frac{\left(\sqrt{x}-1\right)^2}{\left(\sqrt{x}\right)^2\left(x+1\right)^3} \qquad (214) \end{aligned}$$

Inspection shows that the polynomial

$$P=9y^2+46y^4+9y^6-8y-38y^3-30y^5+4 \tag{215}$$

has two real zeros in the interval $[0,2]$ and four complex zeros. Therefore, for $x \geq 4$, the right-hand side of (214) is negative. Thus, regardless of whether the inefficient type self-marginalizes herself or not, truth-telling cannot be simultaneously optimal for both types. (iv) Let $i,j \in \{1,2\}$ such that $j \neq i$. With marginal variation of the reported type $\widehat{c_i}$ vis-a-vis the true type c_i, self-marginalization does not occur. Hence, incentive compatibility implies

$$\begin{aligned} 0 &= \frac{\partial}{\partial \widehat{c_i}} E_{c_j}\left[\left(1-\frac{\sqrt{c_i\widehat{c_i}}}{\widehat{c_i}+c_j}\right)^2\right]\Bigg|_{\widehat{c_i}=c_i} && (216)\\ &= E_{c_j}\left[\frac{\partial}{\partial \widehat{c_i}}\left(1-\frac{\sqrt{c_i\widehat{c_i}}}{\widehat{c_i}+c_j}\right)^2\Bigg|_{\widehat{c_i}=c_i}\right] && (217)\\ &= E_{c_j}\left[\frac{c_j(c_i-c_j)}{(c_i+c_j)^3}\right]. && (218)\end{aligned}$$

Without loss of generality, the least efficient type in the contest is $\overline{c}_i$. Then, letting $c_i = \overline{c}_i$ shows that

$$E_{c_j}\left[\frac{c_j(c_i-c_j)}{(c_i+c_j)^3}\right] < 0, \tag{219}$$

a contradiction. □

Proof of Lemma 4. Take a posterior belief $\mu_2 \in \Delta(C_2)$. Then, the weakest type $\overline{c}_2 \in C_2$ is active if and only if

$$\overline{c}_2 < \left(\frac{c_1^{\#} + E[\widetilde{c}_2|\,\mu_2]}{E[\sqrt{\widetilde{c}_2}|\,\mu_2]}\right)^2, \tag{220}$$

or equivalently, if

$$m\sqrt{\underline{c}_2}(\underline{c}_2 - \underline{c}_2) < c_1^{\#}, \tag{221}$$

where $m = \mu_2(\underline{c}_2)$. Letting $m = 1$ yields the condition in the statement of the lemma. □

Proof of Proposition 10. The sender (player 2) solves the problem

$$\max_{\tau_2 \text{ s.t. } (12)} \tau_2^{\mathrm{A}} E_{c_2}\left[\Pi_2^{\#}(c_2|\mu_2^{\mathrm{A}})\right] + \tau_2^{\mathrm{B}} E_{c_2}\left[\Pi_2^{\#}(c_2|\mu_2^{\mathrm{B}})\right]. \tag{222}$$

More explicitly, this becomes

$$\max_{\substack{\tau_2^{\mathrm{A}} \equiv 1-\tau_2^{\mathrm{B}} \in [0,1], \\ \mu_2^{\mathrm{A}}(\underline{c}_2) \equiv 1-\mu_2^{\mathrm{A}}(\overline{c}_2) \in [0,1], \\ \mu_2^{\mathrm{B}}(\underline{c}_2) \equiv 1-\mu_2^{\mathrm{B}}(\overline{c}_2) \in [0,1], \\ \text{s.t. } (12)}} \quad \tau_2^{\mathrm{A}} E_{c_2} \left[\Pi_2^{\#}(c_2|\mu_2^{\mathrm{A}}) \right] + \tau_2^{\mathrm{B}} E_{c_2} \left[\Pi_2^{\#}(c_2|\mu_2^{\mathrm{B}}) \right]. \tag{223}$$

We shart with the case where $c_1^{\#} \geq \sqrt{\underline{c}_2 \overline{c}_2}$. By Lemma 4, both types of player 2 are active. Therefore, the question if player 2 benefits from persuasion (or not) is linked to the strict convexity (or weak concavity) of the function

$$\widehat{\Pi}_2(\mu_2) = q_2(\underline{c}_2) \left(1 - \frac{\sqrt{\underline{c}_2} E[\sqrt{\widetilde{c}_2}|\mu_2]}{c_1^{\#} + E[\widetilde{c}_2|\mu_2]} \right)^2 + q_2(\overline{c}_2) \left(1 - \frac{\sqrt{\overline{c}_2} E[\sqrt{\widetilde{c}_2}|\mu_2]}{c_1^{\#} + E[\widetilde{c}_2|\mu_2]} \right)^2, \tag{224}$$

where the posterior μ_2 is given as

$$\mu_2 \equiv (\mu_2(\underline{c}_2), \mu_2(\overline{c}_2)) \in \Delta(C_2) = \{(m, 1-m) : 0 \leq m \leq 1\}. \tag{225}$$

Let $c_2 \in C_2$. Based on the computation of the first derivative,

$$\begin{aligned} & \frac{\partial}{\partial m} \left(1 - \frac{\sqrt{c_2} E[\sqrt{\widetilde{c}_2}|\mu_2]}{c_1^{\#} + E[\widetilde{c}_2|\mu_2]} \right)^2 \\ = \; & \frac{\partial}{\partial m} \left(1 - \frac{\sqrt{c_2}(m\sqrt{\underline{c}_2} + (1-m)\sqrt{\overline{c}_2})}{c_1^{\#} + m\underline{c}_2 + (1-m)\overline{c}_2} \right)^2 & (226) \\ = \; & 2 \cdot \left(1 - \frac{\sqrt{c_2}(m\sqrt{\underline{c}_2} + (1-m)\sqrt{\overline{c}_2})}{c_1^{\#} + m\underline{c}_2 + (1-m)\overline{c}_2} \right) \cdot \frac{\sqrt{c_2}(\sqrt{\overline{c}_2} - \sqrt{\underline{c}_2})(c_1^{\#} - \sqrt{\underline{c}_2 \overline{c}_2})}{(c_1^{\#} + m\underline{c}_2 + (1-m)\overline{c}_2)^2}, & (227) \end{aligned}$$

we see that the second derivative is given by

$$
\begin{aligned}
& \frac{\partial^2}{\partial m^2}\left(1-\frac{\sqrt{c_2}E[\sqrt{\widetilde{c}_2}|\mu_2]}{c_1^\# + E[\widetilde{c}_2|\mu_2]}\right)^2 \\
= \quad & 2\cdot\left(\frac{\sqrt{c_2}(\sqrt{\overline{c}_2}-\sqrt{\underline{c}_2})(c_1^\# - \sqrt{\underline{c}_2\overline{c}_2})}{(c_1^\# + m\underline{c}_2+(1-m)\overline{c}_2)^2}\right)^2 \qquad (228) \\
& +4\cdot\underbrace{\left(1-\frac{\sqrt{c_2}E[\sqrt{\widetilde{c}_2}|\mu_2]}{c_1^\# + E[\widetilde{c}_2|\mu_2]}\right)}_{>0 \text{ by activity}}\cdot\frac{\sqrt{c_2}(\sqrt{\overline{c}_2}-\sqrt{\underline{c}_2})(c_1^\# - \sqrt{\underline{c}_2\overline{c}_2})(\overline{c}_2-\underline{c}_2)}{(c_1^\# + m\underline{c}_2+(1-m)\overline{c}_2)^3}.
\end{aligned}
$$

Clearly, the right-hand side of Eq. (228) is positive (zero) if $c_1^\# > \sqrt{\underline{c}_2\overline{c}_2}$ (if $c_1^\# > \sqrt{\underline{c}_2\overline{c}_2}$) regardless of $c_2 \in C_2$ and $m \in [0,1]$, which proves parts (i) and (ii). Suppose next that $\sqrt{\underline{c}_2\overline{c}_2} > c_1^\#$. Then, combining (224) and (228), we get

$$
\frac{\partial^2\widehat{\Pi}_2(\mu_2)}{\partial m^2} = \frac{2(\sqrt{\overline{c}_2}-\sqrt{\underline{c}_2})^2(c_1^\# - \sqrt{\underline{c}_2\overline{c}_2})}{(c_1^\# + E[\widetilde{c}_2|\mu_2])^4}\cdot\left\{\begin{array}{c} E[c_2](c_1^\# - \sqrt{\underline{c}_2\overline{c}_2}) \\ +2(c_1^\# + E[\widetilde{c}_2|\mu_2])E[\sqrt{c_2}](\sqrt{\overline{c}_2}+\sqrt{\underline{c}_2}) \\ -2E[c_2]E[\sqrt{\widetilde{c}_2}|\mu_2](\sqrt{\overline{c}_2}+\sqrt{\underline{c}_2}) \end{array}\right\}. \qquad (229)
$$

Exploiting that

$$
\begin{aligned}
E[\sqrt{c_2}](\sqrt{\overline{c}_2}+\sqrt{\underline{c}_2}) &= E[c_2]+\sqrt{\underline{c}_2\overline{c}_2}\ , \qquad (230) \\
E[\sqrt{\widetilde{c}_2}|\mu_2](\sqrt{\overline{c}_2}+\sqrt{\underline{c}_2}) &= E[\widetilde{c}_2|\mu_2]+\sqrt{\underline{c}_2\overline{c}_2}\ , \qquad (231)
\end{aligned}
$$

we see that

$$
\frac{\partial^2\widehat{\Pi}_2(\mu_2)}{\partial m^2} = \frac{2(\sqrt{\overline{c}_2}-\sqrt{\underline{c}_2})^2(c_1^\# - \sqrt{\underline{c}_2\overline{c}_2})}{(c_1^\# + m\underline{c}_2+(1-m)\overline{c}_2)^4}\cdot\left\{\begin{array}{c} (3E[c_2]+2\sqrt{\underline{c}_2\overline{c}_2})\,c_1^\# \\ -(3E[c_2]-2(m\underline{c}_2+(1-m)\overline{c}_2))\sqrt{\underline{c}_2\overline{c}_2} \end{array}\right\}. \qquad (232)
$$

As the expression in the curly brackets is linear in m, we certainly find a unique cut-off level $m^* \in \mathbb{R}$ such that, if replaced for m in (232), renders this term equal to zero. Moreover, $\widehat{\Pi}_2(\mu_2)$ is strictly concave for $m \leq m^*$, and strictly convex for $m \geq m^*$. There are now three cases. Suppose first that $m^* \geq 1$. Then, $\widehat{\Pi}_2(\mu_2)$ is globally strictly concave regardless of $q_2(\underline{c}_2)$, so that full concealment is optimal. In this case, we may set $\chi^* = 1$. Next, suppose that $m^* \in (0,1)$. Then, taking the convex closure of $\widehat{\Pi}_2(\mu_2)$ over the interval $[0,1]$, we find a "tangential" point at some $\chi^* \in [0, m^*)$, as illustrated conceptually in Figure 5 in the case where $m^* \in (0,1)$ and $\chi^* > 0$. (For $\chi^* = 0$, the slope of $\widehat{\Pi}_2(\mu_2)$ at $m = 0$ and the slope of the upper boundary of the convex closure may differ). If $q_2(\underline{c}_2) \leq \chi^*$, then full concealment remains optimal. If $q_2(\underline{c}_2) > \chi^*$, however, player 2's signal randomizes, in response to her type and the randomizing commitment device, between the two signals causing Bayesian posteriors μ_2^{A} with $\mu_2^{\mathrm{A}}(\underline{c}_2) = \chi^*$ and μ_2^{B} with $\mu_2^{\mathrm{B}}(\underline{c}_2) = 1$.[71] Suppose, finally, that $m^* \leq 0$. Then, $\widehat{\Pi}_2(\mu_2)$ is globally strictly convex regardless of $q_2(\underline{c}_2)$, so that full disclosure is optimal. In this case, we may set $\chi^* = 0$. This proves the claim. □

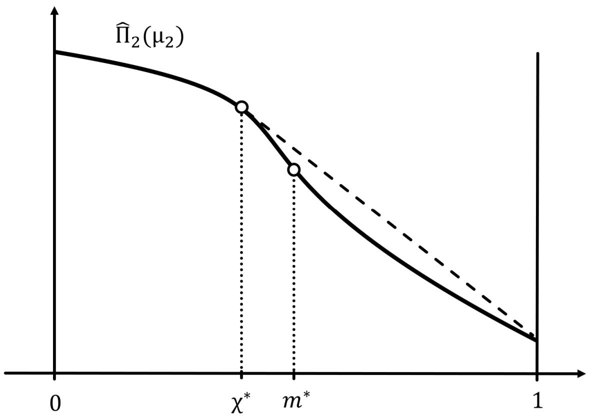

Figure 5. Proof of Proposition 10.

Proof of Proposition 11. (i) This result follows immediately from Zhang and Zhou (2016, Prop. 3) by replacing valuations by reciprocals of marginal costs. The details are omitted.

(ii) In an unbiased lottery contest with incomplete information about marginal costs, the ex-ante expected expenses are identical across players. Moreover, the prize is always assigned to one player. Therefore, maximizing total expected payoffs is equivalent to *minimizing* player 1's expenses, $c_1^{\#} x_1^{\#}$. However, as shown in the proof of Lemma 3, all types $c_2 \in C_2$ prefer a strictly lower bid $x_1^{\#}$ over any higher bid. The claim follows.

[71] An example for this case is $c_1^{\#} = 1$, $\underline{c}_2 = 5$, $\overline{c}_2 = 6$, $q_2(\underline{c}_2) = 0.75$, and $q_2(\overline{c}_2) = 0.25$. Then, $m^* = 0.56$ and $\chi^* = 0.32$. The reader is cautioned, however, that the effects are numerically small.

(iii) In an unbiased lottery contest, we have

$$\begin{aligned} & \frac{1}{4}\left\{1 - E_{c_2}\left[\left(p_1(x_1^{\#}, \xi_2^{\#}(c_2)) - p_2(x_1^{\#}, \xi_2^{\#}(c_2))\right)^2\right]\right\} \\ = & \ E_{c_2}\left[p_1(x_1^{\#}, \xi_2^{\#}(c_2))(1 - p_1(x_1^{\#}, \xi_2^{\#}(c_2)))\right] \qquad (233) \\ = & \ x_1^{\#} c_1^{\#}. \qquad (234) \end{aligned}$$

Hence, minimizing the expected quadratic distance between players' winning probabilities is equivalent to *maximizing* player 1's expenses. But, by the arguments just explained, this is equivalent to the problem considered under part (i). The proposition follows. □

Proof of Proposition 12. We first show that all information must be disclosed in any perfect Bayesian equilibrium. The key point to note is that, even if the underdog's conditional belief $\mu_1(\cdot|\, c_2) \in \Delta(C_1)$ is weakly decreasing in c_2 in the FOSD sense, the underdog's bid schedule remains weakly decreasing globally, as well as strictly decreasing in the interior. Indeed, this follows immediately by combining Lemma A.2 and Lemma B.1, where the underdog's domain condition holds as a consequence of Assumption 1. Therefore, the proof of Proposition 1, which exploits only the monotonicity properties of the bid schedules and the monotonicity properties of the best-response mappings applies without change to this more general setting. Thus, the underdog side unravels. For the favorite, correlation now does not matter anymore, i.e., Proposition 2 applies as before. This proves the claim. Next, we show that self-disclosure by all types of both players is indeed a perfect Bayesian equilibrium. Even in the presence of arbitrary correlation, this is the case provided we keep the specification of off-equilibrium beliefs used in the proof of Theorem 1. The reason is that, under this specification, type-specific payoffs resulting from self-disclosure or unilateral concealment are expected values of complete-information payoffs. Therefore, the correlation does not affect the interim payoff ranking for the contest stage, and the argument proceeds as before. □

Proof of Proposition 13. We repeatedly apply Benoît and Dubra (2006, Thm. 1), for which we refer the reader to the original paper. In a first step, we note that type c_2's expected payoff from disclosure,

$$\begin{aligned} & u_2(c_2, c_2, S_1, S_2) \\ \equiv & \int_{S_1} \Pi_2(x_1^\circ, x_2^\circ; c_2) dF_1(c_1) + \int_{C_1 \backslash S_1} \Pi_2(\xi_1^\#(c_1), x_2^\#; c_2) dF_1(c_1) \quad (235) \\ = & \int_{S_1} \left(\frac{c_1}{c_1 + c_2} \right)^2 dF_1(c_1) + \mathrm{pr}\{C_1 \backslash S_1\} \frac{E[\sqrt{c_1} | \, c_1 \in C_1 \backslash S_1]^2 E[c_1 | \, c_1 \in C_1 \backslash S_1]}{((E[c_1 | \, c_1 \in C_1 \backslash S_1] + c_2)^2}, \quad (236) \end{aligned}$$

is continuous in c_2 as a consequence of Lebesgue's theorem of dominated convergence. Similarly, type c_2's expected payoff from concealment,

$$\begin{aligned} & u_2(c_2, \emptyset, S_1, S_2) \\ \equiv & \int_{S_1} \Pi_2(x_1^\#, \xi_2^\#(c_2); c_2) dF_1(c_1) + \int_{C_1 \backslash S_1} \Pi_2(\xi_1^*(c_1), \xi_2^*(c_2); c_2) dF_1(c_1) \quad (237) \\ = & \ \mathrm{pr}\{S_1\} \left(1 - \frac{\sqrt{c_2} E[\sqrt{\widetilde{c}_2} | \, \widetilde{c}_2 \in C_2 \backslash S_2]}{c_1 + E[\widetilde{c}_2 | \, \widetilde{c}_2 \in C_2 \backslash S_2]} \right)^2 \quad (238) \\ & + \int_{C_1 \backslash S_1} \left(\frac{\xi_1^*(c_1)}{\xi_1^*(c_1) + \xi_2^*(c_2)} - c_2 \xi_2^*(c_2) \right) dF_1(c_1), \end{aligned}$$

is well-defined as a consequence of the existence and uniqueness results in Ewerhart (2014, Thm. 3.4 & 4.2). Moreover, $u_2(c_2, \emptyset, S_1, S_2)$ is continuous in c_2, because the equilibrium strategy ξ_1^* in the second term does not depend on c_2. By a straightforward generalization of Proposition 1, for any non-degenerate conditional distribution $F_2(\cdot | \, \emptyset, S_2)$, the lowest type $\widehat{c}_2$ in the support of $F_2(\cdot | \, \emptyset, S_2)$ has the property that $u_2(\widehat{c}_2, \widehat{c}_2, S_1, S_2) > u_2(\widehat{c}_2, \emptyset, S_1, S_2)$. By Benoît and Dubra (2006, Thm. 1), the underdog's signal is almost surely known in any perfect Bayesian equilibrium. Next, we consider the decision of the favorite under the assumption that the underdog's type is revealed with probability one. Continuity of the expected payoff functions $u_1(c_1, c_1, S_1, C_2)$ and $u_1(c_1, \emptyset, S_1, C_2)$, defined in analogy to (235) and (237), may be checked as above. Then, by a straightforward generalization of Proposition 2, almost surely across c_2, for any non-degenerate conditional distribution $F_1(\cdot | \, \emptyset, S_1)$, the highest type $\widehat{c}_1$ in the support of $F_1(\cdot | \, \emptyset, S_1)$ has, typewise across $c_2 \in C_2$, but hence also globally the property that $u_1(\widehat{c}_1, \widehat{c}_1, S_1, C_2) > u_1(\widehat{c}_1, \emptyset, S_1, C_2)$. Applying Benoît and Dubra (2006, Thm. 1) again, we see that also the favorite's type is necessarily almost surely known in any perfect Bayesian equilibrium. This concludes the proof and proves the proposition. □

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