University of Zurich[UZH]

Department of Business Administration

UZH Business Working Paper Series

(ISSN 2296-0422)

Working Paper No. 315

The Effects of Introducing Advertising in Pay TV: A Model of Asymmetric Competition between Pay TV and Free TV

Helmut M. Dietl, Markus Lang and Panlang Lin

31 January 2012

University of Zurich, Plattenstrasse 14, CH-8053 Zurich,
http://www.business.uzh.ch/forschung/wps.html

UZH Business Working Paper Series

Contact Details

Helmut M. Dietl

University of Zurich

helmut.dietl@business.uzh.ch

Markus Lang

University of Zurich

helmut.dietl@business.uzh.ch

Panlang Lin

University of Zurich

panlang.lin@business.uzh.ch

The Effects of Introducing Advertising in Pay TV: A Model of Asymmetric Competition between Pay TV and Free TV*

Helmut Dietl, Markus Lang, Panlang Lin†

January 31, 2012

Abstract

This paper develops a theoretical model of asymmetric competition between a pay TV and a free TV broadcaster. Our model shows that the pay TV broadcaster has incentives to place advertising on its channel if the marginal return on advertising exceeds the viewers' disutility from advertising. In this case, however, the pay TV advertising level is always below the corresponding level on free TV. The pay TV advertising level can increase with a higher viewer disutility from advertising but the pay TV channel will never attract a larger viewership than the free TV channel. Furthermore, we show that introducing advertising on pay TV induces a decrease of the subscription fees on this channel and a decrease in the advertising level of the free TV channel. Moreover, pay TV viewer demand can increase if the pay TV broadcaster places advertising on its channel. If the viewer disutility of advertising is sufficiently large, aggregate broadcaster profits increase through the introduction of advertising in pay TV, while aggregate consumer surplus always increases.

Keywords: Asymmetric competition, advertising, television broadcasting, media

JEL Classification: D40, L10

*We would like to thank Andreas Hefti for helpful comments and suggestions. Markus Lang is grateful for the financial support which was provided by the Ecoscientia Foundation and the Foundation for the Advancement of Young Scientists (FAN) of the Zürcher Universitätsverein (ZUNIV).

†All the authors are from the Department of Business Administration, University of Zurich, Plattenstrasse 14, 8032 Zurich, Switzerland. Phone: +41 44 634 53 11, Fax: +41 44 634 53 29. E-mails: helmut.dietl@business.uzh.ch, markus.lang@business.uzh.ch, panlang.lin@business.uzh.ch. Corresponding author: Markus Lang.

1 Introduction

The television broadcasting industry is of crucial economic and social importance in the US. Nearly every household owns a television set and the average American adult spends more than five hours a day viewing television (TVB, 2011a). Television is also a very important media for placing advertisements. In 2010, advertisers spent about $71 billion (total advertising expenditures were $131 billion) to promote their products and services on television (TVB, 2011b). There are two generic business models in the television broadcasting industry: commercial free-to-air television and pay TV. Traditionally, the television broadcasting industry has been dominated by free TV. In this business model, viewers are fully subsidized because broadcasters want to attract as many viewers as possible in order to maximize advertising revenues. Basically, a TV channel is more attractive for advertisers, the larger its viewership. On the other hand, viewers typically dislike advertising as it is perceived as a nuisance. This trade off suggests that there is an optimal amount of advertising.

Due to technological progress (i.e. encryption techniques and digital decoding) pay TV has emerged as a competing business model. Originally, pay TV broadcasters did not air advertisements, but directly charged viewers for TV access. More recently, however, some pay TV channels (in the US mostly cable subscription television networks such as TNT, TBS, ESPN) started to air commercials in addition to charging subscription fees. What are the economic effects of placing advertising on pay TV channels? How will free TV broadcasters react? Will market shares of pay TV increase or decrease? Whose profits will increase and whose will decrease? And, most importantly, will consumers benefit or not?

To answer these and related questions, we develop a theoretical duopoly model of asymmetric competition between a pay TV and a free TV broadcaster by utilizing the Hotelling approach. We analyze the effects of placing pay TV advertising on broadcaster market strategies (i.e., choice of subscription fee and advertising level), viewer demands, broadcaster profits and consumer surplus. So far, the literature has concentrated on the symmetric competition between either pay TV or free TV broadcasters. Because the reality is often characterized by the coexistence of both pay TV and free TV, more research that analyzes the competition between both business models is needed. Research on symmetric competition neglects the important aspect of consumer migration from one business model to the other. This paper tries to fill this gap in the literature.

Our model shows that the pay TV broadcaster has incentives to place advertising on its channel if the marginal return from advertising exceeds the viewers' disutility from advertising. In this case, however, the pay TV advertising level is always below the corresponding level on free TV. Surprisingly, the pay TV advertising level can increase with a higher viewer disutility from advertising but the pay TV channel will never attract a larger viewership than the free TV channel. Furthermore, we show that introducing advertising in pay TV induces a decrease of the subscription fees on this channel and a decrease in the advertising level of the free TV channel. Moreover, pay TV viewer demand can increase if pay TV places advertising on its channel. Surprisingly, if the viewer disutility of advertising is sufficiently large, aggregate broadcaster profits increase through the introduction of advertising in pay TV, while on aggregate, consumers always benefit from advertising on pay TV.

With increasing popularity and coverage, one can observe that various revenues models are chosen by the pay TV broadcasters. Some pay TV channels choose to charge a higher subscription fee, but air little or no advertising, while other pay TV channels combine both viewer and advertiser incomes. This paper provides a conceptual framework for how to choose and implement an optimal pay TV strategy when a free TV competitor preexists in the market. It can also serve as a guide for pay TV broadcasters considering a possible pay TV strategy change. Furthermore, our paper derives implications for policy makers and regulatory authorities. For example, we find additional pay TV advertising is not necessarily socially undesirable due to the strategic market reactions.

The remainder of this paper is organized as follows: Section 2 briefly reviews the related literature. In Section 3, we introduce the analytical framework. Section 4 presents the equilibrium analysis of the model. In Section 5, we examine the effects of introducing pay TV advertising on relevant equilibrium outcomes. Section 6 concludes and provides implications for future research.

2 Related Literature

The literature on the economics of media markets is flourishing and has experienced a significant growth in recent years.[1] The focus of the extant literature is on the choice of optimal market strategies by media platforms (i.e., content differentiation, advertising level and viewer charge) and welfare implications in models with symmetric competition between either free TV or pay TV. Some papers also consider two separate scenarios, in which competition takes place between pay TV and free TV platforms, respectively, and they compare the two independent scenarios. The key distinction of our paper from previous research is that we explicitly model the direct competition between pay TV and free TV, rather than treating them independently. Thus, our paper offers insights about a scenario, in which pay TV and free TV exist at the same time and compete for the same viewership. Although in reality pay TV and free TV often coexist, the literature has neglected to model this aspect of asymmetric competition.

For example, in a model of symmetric competition, Anderson and Coate (2005) ana-

[1] See Anderson and Gabszewicz (2006), who provide a comprehensive overview to the literature.

lyze the nature of market failure in the broadcasting industry. Commercial broadcasters provide advertising levels and programming amount that can be above or below socially optimal levels, depending on how strongly viewers dislike advertising. With the ability to price programming, broadcasters can internalize the nuisance of advertisements by substituting prices for advertising at the margin. However, this is not necessarily socially desirable because pricing may also result in some viewers being inefficiently excluded.

In a similar context, Peitz and Valletti (2008) also introduce endogenous content provision by competing TV broadcasters into their model.[2] They assume that pay TV broadcasters generate revenues from both advertisers and viewers, whereas free TV broadcasters are solely funded by advertising. Their model shows that under free TV the advertising level is higher than under pay TV when viewers strongly dislike advertising. Under free TV, broadcasters tend to provide less differentiated content while they always maximally differentiate their content under pay TV. However, they do not model asymmetric competition because they consider two separate regimes with two competing pay TV channels and two competing free TV channels, respectively.

Rather than horizontal program diversity, Armstrong (2005) studies the endogenous choice of vertical program quality by competing broadcasters in a model of symmetric competition. He also considers two separate funding regimes, in which either two pay TV channels or two free TV channels compete against each other and finds that the program quality level in the pay TV regime is higher than under the free TV regime. Our model builds on Armstrong (2005), but we model asymmetric competition and focus on the effects of introducing advertising on pay TV.

Moreover, Armstrong (2006) analyzes the competition between media firms in the broader concept of a two-sided market.[3] Inter alia, he analyzes a competitive bottleneck in a two-sided media market, in which media platforms compete for viewers but not for advertisers.[4] If the consumers join only one specific media platform, advertisers have to place advertising on all competing platforms to reach all consumers. The model shows that the equilibrium price for the access to media platforms (i.e., magazines) depends on how advertising charges are levied. The equilibrium reader price and platforms' profit is lower (higher) if platforms charge advertisers on a lump-sum basis than under a per-reader basis in the case that readers like (dislike) advertising.

All papers mentioned above have considered models of duopoly competition in Hotelling

[2]For further work on the endogenous choice of content diversity between media platforms, see also Gabszewicz et al. (2001), Gabszewicz et al. (2002), Gal-Or and Dukes (2003), and Gabszewicz et al. (2004).

[3]Two-sided markets are markets in which the agents of two distinct market sides interact via a platform and exert indirect network externalities upon each other. General research on two-sided markets include Caillaud and Jullien (2003), Rochet and Tirole (2003), Wright (2004), Armstrong (2006), Hagiu (2006), Belleflamme and Toulemonde (2009) and Weyl (2010).

[4]This assumption is also made in most other related models on media markets. An exception is Reisinger (2011), who analyzed a two-sided market model in which media platforms compete for both advertisers and users.

fashion. Crampes et al. (2009) extend these analyses by considering the number of active media platforms as endogenous. They develop a Salop-Vickrey style model of media competition with free entry by assuming that media platforms are financed from advertising receipts and consumer subscriptions or only from advertising receipts, in the case of free media platforms.[5] Their model suggests that there is an excessive level of entry and an insufficient level of advertising under constant or increasing returns to scale in the audience size. Similarly, Kind et al. (2009) allows the number of media platforms to vary and investigates how media platforms raise revenue in a model with quadratic consumer preferences. They find that media firms' scope for raising advertising revenues is constrained by the number of media firms. Moreover, a low level of horizontal differentiation between two media firms, or rather their content, restricts the scope for raising revenues from direct consumer payment.

However, neither of these papers on media competition have analyzed the asymmetric competition between pay TV and free TV. To the best of our knowledge, the only paper that analyzes asymmetric competition is Lin (2011), who develops a duopoly model of asymmetric competition between pay TV and free TV. His model focuses on the endogenous choice of program quality made by television broadcasters. The paper shows that the broadcasters vertically differentiate their channel programs if viewers strongly or weakly dislike advertising. Depending on the degree of horizontal differentiation, pay TV offers higher or lower quality programming than free TV. However, contrary to our model, Lin (2011) excludes the possibility that pay TV can introduce advertising.

3 Analytical Framework

We consider a TV viewing market that is served by one pay TV broadcaster and one free TV broadcaster. TV viewers, who are of mass one, are uniformly distributed along the unit interval. The two competing channels are situated at the extremes of the interval with the pay TV channel located at $x = 0$ and the free TV channel located at $x = 1$. We consider the Hotelling model with linear transport cost of $t > 0$ per unit of length. Hence, the two channel programs are horizontally differentiated from the perspective of viewers and the parameter t can be interpreted as the differentiation parameter. A lower value of t means that the channels or their program are perceived as closer substitutes by the viewers.

The indirect utility of a viewer, located at point $x \in [0, 1]$ when choosing the pay TV

[5] Choi (2006) has presented a related Salop-Vickrey model with free entry.

channel denoted by p and the free TV channel denoted by f, respectively, is defined as

$$u_p = v - s_p - \gamma a_p - tx, \tag{1}$$
$$u_f = v - \gamma a_f - t(1-x), \tag{2}$$

where $v > 0$ represents the viewers' intrinsic value from watching TV. The pay TV broadcaster charges viewers a subscription fee $s_p > 0$ for the access to the channel programming.[6] We assume that both channels can place advertising, where the amount of advertising placed on the pay TV and free TV channel is given by a_p and a_f, respectively. The parameter $\gamma > 0$ describes the extent to which viewers dislike advertising because each advertisement produces a perceived nuisance cost of γ by the viewers.[7] Moreover, we make the assumption of full viewer market coverage, i.e., the viewers' intrinsic value from watching TV is sufficiently large such that all viewers will watch one program.

We further assume that viewers are single-homing, i.e., they choose only one channel to watch. The marginal viewer, who is indifferent between watching pay TV and free TV, can be identified at the location $\overline{x} = \frac{1}{2}[1 + \frac{1}{t}(\gamma(a_f - a_p) - s_p)]$. All viewers to the left of $\overline{x}$, i.e., $x \in [0, \overline{x})$, decide to watch pay TV programming and all viewers to the right of $\overline{x}$, i.e., $x \in (\overline{x}, 1]$, consume free TV programming. As a result, the viewer demand functions for the pay TV and free TV channels, respectively, can be derived as

$$n_p = \frac{1}{2}\left[1 + \frac{1}{t}(\gamma(a_f - a_p) - s_p)\right], \tag{3}$$
$$n_f = 1 - n_p = \frac{1}{2}\left[1 + \frac{1}{t}(\gamma(a_p - a_f) + s_p)\right]. \tag{4}$$

We exclude the possibility of demand tipping at the viewer market which implies that each broadcaster captures a positive market share.

We assume that placing advertising on its channel produces costs for each broadcaster given by $c(a)$ where $c(a) \in C^2$ is a strictly convex cost function with $c'(a) > 0$ and $c''(a) > 0$ for $a > 0$ and $c'(0) = c''(0) = 0$.[8] These costs can be interpreted either as transaction costs when broadcasters negotiate, serve, and deal with the advertisers, or as direct costs for the operative/technical realization of advertising implementation.

[6] Note that we assume a positive subscription fee. Otherwise, the model would collapse to one with symmetric competition between two free TV broadcasters.

[7] Kaiser and Song (2009) use data from the German magazine market to empirically assess the extent to which readers (dis-)like advertising. They show that the readers' attitude toward advertising depends on the nature of advertisements. For example, readers in many magazine segments, such as women's magazines, business and politics magazines, and car magazines, in which advertisements are relatively more informative, appreciate advertising. However, advertising is perceived as a nuisance to readers of adult magazines, a segment, in which advertisements are particularly uninformative. Moreover, Rysman (2004) found in the market for Yellow Pages that consumers value advertising.

[8] For the assumption that media firms incur costs for placing advertisements, see also Blair and Romano (1993) and Armstrong (2006).

Alternatively, these costs can be interpreted as broadcasters' opportunity costs for given-up programming slots/time. For simplicity, we assume that there are no other costs.

The profit functions of the broadcaster are then given by

$$\pi_p = (s_p + ka_p)\, n_p - c\,(a_p)\,, \quad (5)$$
$$\pi_f = ka_f n_f - c\,(a_f)\,, \quad (6)$$

where $k > 0$ is a parameter that captures how much revenue a broadcaster can generate per ad a. The parameter k can be interpreted as a measure for the effectiveness of advertising. For tractability, we assume a linear specification of advertising revenue, i.e., a broadcaster generates revenues of kan if its channel programming attracts n viewers and a advertising are placed on the channel.

Moreover, the consumers (viewers) of the pay TV and free TV channel derive the following surpluses:

$$CS_p = \int_0^{n_p} (v - s_p - \gamma a_p - tz)\, dz = (v - s_p - \gamma a_p)\, n_p - \frac{t}{2} n_p^2, \quad (7)$$
$$CS_f = \int_{n_p}^{1} (v - \gamma a_f - t(1-z))\, dz = (1 - n_p)\left(v - \gamma a_f - \frac{t}{2}(1 - n_p)\right). \quad (8)$$

Aggregate consumer (viewer) surplus is then given by the sum of pay TV and free TV consumer surpluses, i.e., $CS = CS_p + CS_f$.

4 Analysis

The pay TV broadcaster has two strategic variables at its disposal: the subscription fee s_p and the level of advertising a_p. For the free TV broadcaster, the level of advertising a_f is the only choice variable. Therefore, the broadcasters solve the following maximization problems:

$$\max_{(s_p, a_p) \geq 0} \left\{\pi_p = [s_p + ka_p]\, n_p(s_p, a_p) - c\,(a_p)\right\}, \quad (9)$$
$$\max_{a_f > 0} \left\{\pi_f = ka_f n_f(a_f) - c\,(a_f)\right\}. \quad (10)$$

The first-order conditions for the pay TV broadcaster are given by

$$\frac{\partial \pi_p}{\partial s_p} = \underbrace{n_p^*(s_p^*, a_p^*)}_{>0} + \underbrace{(s_p^* + ka_p^*)\frac{\partial n_p^*}{\partial s_p}}_{<0} = 0, \quad (11)$$
$$\frac{\partial \pi_p}{\partial a_p} = \underbrace{kn_p^*(s_p^*, a_p^*)}_{>0} + \underbrace{(s_p^* + ka_p^*)\frac{\partial n_p^*}{\partial a_p}}_{<0} - c'\left(a_p^*\right) = 0, \quad (12)$$

where $c'(a) \equiv \frac{\partial c(a)}{\partial a}$. To ensure that the second-order conditions for a maximum are satisfied, the differentiation parameter t has to be sufficiently large, i.e.,[9]

$$t > t_{SOC} \equiv \frac{(k-\gamma)^2}{4c''\left(a_p^*\right)},$$

where $c''(a) \equiv \frac{\partial^2 c(a)}{\partial a^2}$. From the first-order conditions (11) and (12), we observe that increasing either the subscription fee or the advertising level triggers a positive revenue effect and a negative demand effect because pay TV viewer demand decreases with s_p and a_p, respectively. Additionally, increasing the advertising level induces higher costs (cost effect).

For the free TV broadcaster, the first-order condition is given by[10]

$$\frac{\partial \pi_f}{\partial a_f} = kn_f^*(a_f^*) + ka_f^* \frac{\partial n_f^*}{\partial a_f} - c'(a_f^*) = 0. \tag{13}$$

and has a similar interpretation as above. By solving this system of equations, we can establish the following lemma:

Lemma 1 *(i) For the pay TV channel, the subscription fee s_p^* and advertising level a_p^* in equilibrium are implicitly given by*

$$s_p^* = \frac{1}{2}(\gamma a_f^* - (k+\gamma)a_p^* + t) \text{ and } c'(a_p^*) = \frac{k-\gamma}{4t}(\gamma a_f^* + (k-\gamma)a_p^* + t), \tag{14}$$

and for the free TV channel the equilibrium advertising level a_f^* is implicitly given by

$$c'(a_f^*) = \frac{k}{4t}(-3\gamma a_f^* - (k-\gamma)a_p^* + 3t). \tag{15}$$

(ii) The equilibrium (s_p^*, a_p^*, a_f^*) exists and is unique if the differentiation parameter t is sufficiently large with $t > t_{eq}$.

Proof. See Appendix A.1. ■

Regarding part (i), we assume that $t > t_s \equiv ka_p^* + \gamma(a_p^* - a_f^*)$ to ensure the pay TV broadcaster sets a positive subscription fee. Hence, if the channel programs are not sufficiently differentiated, the viewers are not willing to pay an additional subscription fee for watching pay TV containing advertising because they can enjoy similar programs on free TV. As a result our model would collapse to one with symmetric competition between two free TV broadcasters. Moreover, to ensure the free TV sets a positive advertising level, the differentiation parameter t has to be sufficiently large, with $t > t_{a_f} \equiv \gamma a_f^* + \frac{k-\gamma}{3} a_p^*$.

[9]For a derivation of the second-order conditions, see the proof of Lemma 1 in Appendix A.1.

[10]The second-order condition is satisfied because $\frac{\partial^2 \pi_f}{\partial a_f^2} = -\frac{k\gamma}{t} - \frac{\partial^2 c(a_f^*)}{\partial a_f^2} < 0$.

According to part (ii), a necessary assumption to ensure the existence and uniqueness of the equilibrium is also a sufficiently large differentiation parameter with $t > t_{eq}$. This assumption guarantees that the advertising level, which pay TV would have to set such that the free TV broadcaster has no incentives to place advertising on its channel (i.e., free TV is driven out of the market) is larger than the pay TV advertising level if free TV refrains from advertising. As shown in the proof of Lemma 1, we need this technical property to show that the reaction functions have at least one intersection point, which ensures the existence of an equilibrium. The uniqueness then follows from the strictly monotonous reaction functions. In the case of a quadratic cost function $c(a) = 1/2a^2$, the threshold is given by $t_{eq} = \frac{(k-\gamma)^2}{3} > \frac{(k-\gamma)^2}{4} = t_{SOC}$. Hence, a fulfillment of the second-order condition for pay TV implies the existence and uniqueness of the equilibrium. However, this is not necessarily true for a general cost function $c(a)$ because the threshold t_{eq}, which is only given implicitely, depends on $c''(a)$. Therefore, in the subsequent analysis, we assume that the differentiation parameter is sufficiently large with $t > t^* \equiv \max\{t_{SOC}, t_{eq}, t_s, t_{a_f}\}$.

With the subscription fee s_p^*, the equilibrium demands of the viewers for the pay TV and free TV channels, respectively, are given by

$$n_p^* = \frac{1}{2}\left[1 + \frac{1}{2t}(\gamma(a_f^* - a_p^*) + ka_p^* - t)\right], \tag{16}$$

$$n_f^* = \frac{1}{2}\left[1 + \frac{1}{2t}(\gamma(a_p^* - a_f^*) - ka_p^* + t)\right]. \tag{17}$$

Note that $t > t^*$ also ensures that n_p^* and n_f^* are both positive.

In a further step, we establish the following proposition:

Proposition 1 *(i) For $\gamma = k$, the pay TV broadcaster will not place advertising on its channel, while for $\gamma \in [0, k)$, the pay TV broadcaster places advertising on its channel and its level of advertising is always below the corresponding level on the free TV channel.*

(ii) The level of advertising a_f^* on the free TV channel always decreases with a higher viewer disutility γ from advertising. For the pay TV channel, the level of advertising a_p^* increases with a higher viewer disutility if this disutility is sufficiently small. Formally, $\frac{\partial a_f^*}{\partial \gamma} < 0$ and $\frac{\partial a_p^*}{\partial \gamma} > 0 \Leftrightarrow \gamma < \widehat{\gamma} \equiv \frac{k\left(a_f^* - 2a_p^*\right) - t}{2\left(a_f^* - a_p^*\right)}$.

(iii) The pay TV channel never attracts a larger viewership than the free TV channel independent of the viewer disutility from advertising, i.e., $n_f^* \geq n_p^*$.

Proof. See Appendix A.2. ■

Part (i) of the proposition indicates that a pay TV broadcaster has incentives to place advertising on its channel in addition to a positive subscription fee, if the marginal return on advertising exceeds the viewers' disutility from advertising, i.e., $k > \gamma$. In other words, for a given level of k, placing advertising is beneficial for the pay TV broadcaster's profits whenever the viewers' disutility from advertising is not too strong. Otherwise, i.e., if $\gamma = k$, the pay TV broadcaster is better off in terms of profits when relying only on subscription fees.[11] It implies that there is no scope for pay TV to place advertising when the viewers' disutility from advertising is too strong ($\gamma = k$). Otherwise, viewers will simply migrate away from pay TV where they originally intend to avoid annoying advertising by paying a subscription fee. If the viewers' disutility from advertising is below the marginal return on advertising, then the pay TV broadcaster is able to choose an optimal income mix from both subscription and advertising revenues. However, in this case, the pay TV channel will always set a lower advertising level than the free TV channel.

Our result for the case $\gamma < k$ also rejects the claim of so-called "profit neutrality." It has been often demonstrated that there is a full pass-through of advertising revenues into lower subscription fees.[12] It implies that advertising revenues do not affect the profits of two media platforms in equilibrium; however, this finding is only valid for the case of symmetric competition between two pay TV channels both using advertising. We have shown, under certain circumstances, profit neutrality does not apply to the pay TV broadcaster under asymmetric competition with a free TV channel because advertising revenues are not neutral, but rather increase overall profit.

Regarding the comparative statics results in part (ii), we derive that the advertising level a_f^* of the free TV channel always decreases with a higher viewer disutility from advertising γ. To observe the intuition behind this result, we rearrange the first-order condition (13) of the free TV broadcaster and write

$$c'(a_f^*) = kn_f^* + ka_f^* \frac{\partial n_f^*}{\partial a_f}. \tag{18}$$

Recall that a higher free TV advertising level triggers a positive effect kn_f^* through higher advertising revenues and a negative effect $ka_f^* \frac{\partial n_f^*}{\partial a_f}$ through a lower viewer demand on profits. We derive that a higher γ diminishes the positive effect and strengthens the negative effect, which lowers the right-hand side (rhs) of (18).[13] As a result, the free TV broadcaster lowers its advertising level a_f^* to ensure a decrease of the left-hand side (lhs).

Regarding the behavior of the pay TV broadcaster with respect to a higher disutility parameter γ, we rearrange its first-order condition (12) and write

$$c'\left(a_p^*\right) = kn_p^* + (s_p^* + ka_p^*) \frac{\partial n_p^*}{\partial a_p}. \tag{19}$$

[11] The case $\gamma > k$ is not feasible as shown in Appendix A.2.

[12] See, for example, Peitz and Valletti (2008).

[13] Formally, with $\tau_1 \equiv kn_f$ and $\tau_2 \equiv ka_f \frac{\partial n_f}{\partial a_f}$, we derive $\frac{\partial \tau_1}{\partial \gamma} = \frac{k}{4t}(a_p - a_f) < 0$ and $\frac{\partial \tau_2}{\partial \gamma} = -ka_f \frac{1}{4t} < 0$. Hence, the positive effect is diminished and the strength of the negative effect increases.

Similarly to above, a higher free TV advertising level triggers a positive effect kn_p^* through higher advertising revenues and a negative effect $(s_p^* + ka_p^*)\frac{\partial n_p^*}{\partial a_p}$ through a lower viewer demand on profits. Contrary to above, a higher viewer disutility from advertising strenghens both the positive and the negative effect. Hence, the effect of a higher γ on the rhs of (19) is ambiguous and depends on the level of γ. First, suppose that the threshold $\widehat{\gamma}$ is positive, which implies that the differentiation parameter t is sufficiently small, i.e., $t < \widehat{t} \equiv k(a_f^* - 2a_p^*)$. If γ is sufficiently small with $\gamma < \widehat{\gamma}$, then the rhs increases with a higher viewer disutility from advertising such that the pay TV broadcaster increases a_p to ensure an increase of the lhs. If $\gamma > \widehat{\gamma}$, the reverse is true and the advertising level in pay TV decreases with a higher γ. However, if the differentiation parameter t is sufficiently large, i.e., $t > \widehat{t}$, then the threshold $\widehat{\gamma}$ is negative, and hence, a higher viewer disutility from advertising induces a decrease in the advertising level in pay TV independent of γ.

Part (iii) of the proposition indicates that the free TV channel never attracts a lower viewership than the pay TV channel even though viewers might derive a comparatively large disutility from advertising and the advertising level in free TV is always higher than in pay TV. To observe the intuition behind this result, consider the case where viewers do not perceive advertisement as a nuisance, i.e., $\gamma = 0$. In this case, it is not surprising that the free TV channel attracts more viewers than the pay TV channel because viewers are charged via the subscription fee for pay TV. Increasing the disutility parameter γ has the following effects on pay TV viewer demand. Total differentiation of $n_p^* = \frac{1}{2}[1 + \frac{1}{t}(\gamma(a_f^* - a_p^*) - s_p^*)]$ yields

$$\frac{dn_p^*}{d\gamma} = \frac{1}{2t}[\underbrace{(a_f^* - a_p^*)}_{>0} + \gamma\underbrace{\frac{\partial(a_f^* - a_p^*)}{\partial\gamma}}_{\gtreqless 0} - \underbrace{\frac{\partial s_p^*}{\partial\gamma}}_{>0}].$$

Given that the free TV channel has a higher level of advertising than the pay TV channel, a higher γ pronounces this difference and therefore has a direct positive effect $a_f^* - a_p^*$ on viewer demand. However, it is ambiguous whether the difference in advertising levels $a_f^* - a_p^*$ shrinks or expends such that the sign of the second effect is undetermined. Finally, a higher γ induces an increase in the subscription fee s_p^*, which has a negative effect on pay TV viewer demand. Overall, it is ambiguous whether pay TV viewer demand follows a U-shaped or an inverted U-shaped pattern in the disutility parameter γ. However, we know if the differentiation parameter t is sufficiently small, i.e., $t < \widehat{t}$,[14] then pay TV and free TV viewer demand follows an inverted U-shaped and U-shaped pattern in γ, respectively, because a higher viewer disutility from advertising induces a decrease in the pay TV advertising level. Formally, $\partial n_p^*/\partial\gamma\big|_{\gamma=0} = [(a_f^* - a_p^*) + k(\partial a_p^*/\partial\gamma)]/(4t) > 0$ if $\partial a_p^*/\partial\gamma > 0$. Finally, we derive that the pay TV channel attracts the same viewership as

[14]Note that $t < \widehat{t}$ implies $\widehat{\gamma} > 0$.

the free TV channel if the disutility parameter γ is given by $\gamma = \frac{t-ka_p^*}{a_f^*-a_p^*}$.

By substituting (n_p^*, n_f^*, s_p^*) into the profit functions (5) and (6), we derive the equilibrium profits of the pay TV and free TV broadcaster, respectively, as

$$\begin{aligned}\pi_p^* &= \frac{1}{8t}(\gamma a_f^* + (k-\gamma)a_p^* + t)^2 - c(a_p^*),\\ \pi_f^* &= \frac{ka_f^*}{4t}(-\gamma a_f^* - (k-\gamma)a_p^* + 3t) - c(a_f^*).\end{aligned}$$

5 The Effects of Placing Advertisements in Pay TV

In this section, we investigate the effects of marginally increasing the pay TV advertising level on the equilibrium outcomes. With this approach, we can deduce the consequences if a pay TV broadcaster switches from a strategy that solely relies on subscription fees to a strategy where, in addition to subscription fees, the pay TV broadcaster places advertising on its channel.[15] To ensure that the pay TV broadcaster has incentives to place advertising on its channel, we assume that $\gamma < k$.

First, we analyze the effects of introducing pay TV advertising on the free TV advertising level, the pay TV subscription fee and viewer demands. We establish the following proposition.

Proposition 2 *Introducing advertising on the pay TV channel has the following effects:*

(i) The subscription fee on the pay TV channel decreases and the free TV channel reduces its advertising level.

(ii) Viewer demand for pay TV decreases and viewer demand for free TV increases if the viewers' disutility from advertising is sufficiently large with $\gamma > \gamma_n \equiv \frac{k}{1-da_f^*/da_p}$. *The reverse holds true for* $\gamma < \gamma_n$.

Proof. Below in the text. ■

Part (i). The subscription fee on the pay TV channel decreases if the pay TV broadcaster decides to place advertising on its channel. To prove this finding, we compute the total derivative of s_p^* with respect to a_p as

$$\frac{ds_p^*}{da_p} = \frac{\partial s_p^*}{\partial a_p} + \frac{\partial s_p^*}{\partial a_f}\frac{da_f^*}{da_p} = \frac{1}{2}\left[-(\gamma+k) + \gamma\frac{da_f^*}{da_p}\right] < 0.$$

Both the direct effect, as well as, the indirect effect of a higher a_p are negative, yielding a lower subscription fee. To compensate the viewers for watching advertising, the pay TV broadcaster lowers its subscription fee. In more common terms of the two-sided market

[15]Formally, the effect of introducing advertising on pay TV is given by $\frac{\partial}{\partial a_p}\Big|_{a_p=0}$.

literature, additional advertising revenues will be passed through to the viewers in the form of a lower subscription fee.[16]

To prove the claim that the free TV channel always reduces its own advertising level when the pay TV channel places advertising, we rearrange the optimality condition (15) of the free TV channel and we derive[17]

$$\frac{da_f^*}{da_p} = -\frac{k(k-\gamma)}{3k\gamma + 4tc''(a_f^*)} < 0.$$

Intuitively, one might think that when a pay TV channel introduces advertising, the free TV channel faces less fierce competition and hence, tends to increase its own advertising level. However, as we have shown, there is also an accompanying effect from pay TV, lowering its subscription fee. The latter effect dominates, and hence, the free TV channel reduces its own advertising level.

Part (ii) shows that the demand effect of introducing advertising in pay TV depends on the level of viewers' disutility from advertising. If the disutility is sufficiently small (i.e., $\gamma < \gamma_n$) then viewer demand decreases for free TV and increases for pay TV. If the disutility is sufficiently large (i.e., $\gamma > \gamma_n$) then the opposite holds true. Formally, the effect of a higher pay TV advertising level on the pay TV viewer demand (16) is given by

$$\frac{dn_p^*}{da_p} = \frac{\partial n_p^*}{\partial a_p} + \frac{\partial n_p^*}{\partial a_f}\frac{da_f^*}{da_p} = \frac{1}{4t}\left[k + \gamma\left(\frac{da_f^*}{da_p} - 1\right)\right] \gtreqless 0.$$

On one hand, a higher advertising level a_p^* has a positive (direct) effect on the viewer demand n_p^* because the lower subscription fee overcompensates for a higher advertising level. On the other hand, the indirect effect via a lower advertising level a_f^* on the free TV channel is negative. The overall effect on n_p^* depends on the extent γ to which viewers dislike advertising, i.e.,

$$\frac{dn_p^*}{da_p} < 0 \Leftrightarrow \gamma > \gamma_n \equiv \frac{k}{1 - da_f^*/da_p},$$

with $\gamma_n < k$ because $da_f^*/da_p < 0$. Hence, if the viewers' disutility of advertising is sufficiently small, i.e. $\gamma < \gamma_n$, introducing advertising on the pay TV channel induces a higher demand on this channel but a lower demand on the free TV channel because $n_f^* = 1 - n_p^*$. If, however, the viewers' disutility of advertising is sufficiently large, i.e. $\gamma > \gamma_n$, then introducing advertising on the pay TV channel induces a lower demand on this channel but a higher demand on the free TV channel. The intuition is as follows: when viewers strongly dislike advertising, they will value a reduction in the level of

[16] Using data from the German magazine market, Kaiser and Wright (2006) find results which are consistent with this result. That is, advertisers value readers more than readers value advertisements, and thus, magazines subsidize cover prices and make their profits from advertisers.

[17] See part (ii) in the proof of Lemma 1.

advertising on free TV more than a decrease in the subscription fee for pay TV. As a result, free TV becomes relatively more attractive for the viewers and more viewers will choose free TV over pay TV.

In the next proposition, we analyze the effects of placing pay TV advertising on broadcaster profits and consumer surplus.

Proposition 3 *Introducing advertising on the pay TV channel has the following effects:*

(i) Aggregate broadcaster profits increase if the viewers' disutility from advertising is sufficiently large with $\gamma > \gamma_{\Pi}$.

(ii) Consumer surplus of pay TV viewers increases if the viewers' disutility from advertising is sufficiently large with $\gamma > \gamma_{CS_p}$*, while the viewers' disutility from advertising must be sufficiently small with* $\gamma < \gamma_{CSf}$ *to ensure an increase in consumer surplus of free TV viewers. However, aggregate consumer (viewer) surplus always increases independent of the viewers' disutility from advertising.*

Proof. Below in the text. ■

Regarding the effects on pay TV broadcaster profits in part (i), we know that the profit-maximizing advertising level for the pay TV channel is positive, i.e., $a_p^* > 0$ if the viewers' disutility from advertising is smaller than the marginal return of advertising, i.e., $\gamma < k$. Hence, it is clear that profits of the pay TV channel must increase when switching from a strategy that solely relies on subscription fees to a strategy consisting of both subscription and advertising income, i.e., $\pi_p^*(s_p^*, 0) < \pi_p^*(s_p^*, a_p^*)$. Otherwise, the pay TV channel would opt not to place any advertising and choose $a_p^* = 0$. Hence, for $\gamma < k$, additional advertising revenues will overcompensate the loss in subscription revenues and additional costs. Formally, we derive

$$\frac{d\pi_p^*}{da_p} = (\underbrace{\frac{ds_p^*}{da_p}}_{<0} + k)n_p^* + (s_p^* + ka_p^*)\underbrace{\frac{dn_p^*}{da_p}}_{\gtreqless 0} - \underbrace{\frac{dc}{da_p}}_{>0} > 0.$$

However, two questions remain whether and under which conditions the free TV broadcaster can benefit from the pay TV's strategy change. The effect of introducing pay TV advertising on the free TV broadcaster profit is given by

$$\frac{d\pi_f^*}{da_p} = k\underbrace{\frac{da_f^*}{da_p}}_{<0} n_f^* + ka_f^* \underbrace{\frac{dn_f^*}{da_p}}_{>0 \Leftrightarrow \gamma > \gamma_n} - \underbrace{\frac{\partial c}{\partial a_f}\frac{da_f^*}{da_p}}_{<0}.$$

We know that the introduction of pay TV advertising leads to a reduction of the advertising level on the free TV channel. This reduction triggers a negative effect through lower advertising revenues and a positive effect through lower advertising costs. Moreover, the reduction of the advertising level on the free TV channel affects viewer demand.

The demand effect can be positive or negative dependent on the viewers' disutility from advertising. We find that even in the case of a negative demand effect, i.e., $\gamma < \gamma_n$, the profit of the free TV channel may increase through the introduction of advertising on the pay TV because

$$\frac{d\pi_f^*}{da_p} > 0 \Leftrightarrow \gamma > \gamma_{\pi_f} \equiv \frac{k + \frac{4t}{ka_f^*}(\frac{\partial c}{\partial a_f}\frac{da_f^*}{da_p})}{1 - da_f^*/da_p}.$$

We derive $\gamma_{\pi_f} < \gamma_n$ because $\frac{4t}{ka_f^*}(\frac{\partial c}{\partial a_f}\frac{da_f^*}{da_p}) < 0$. Hence, for $\gamma \in (\gamma_{\pi_f}, \gamma_n)$ the negative effect on profits through lower advertising revenues and lower viewership are overcompensated for by lower advertising costs. Due to the profit-increasing cost effect, the critical threshold γ_{π_f} above which profits of the free TV channel increases through the introduction of advertising on the pay TV is smaller than the threshold for increasing viewer demand γ_n. However, when γ is sufficiently small ($\gamma < \gamma_{\pi_f}$), the positive cost effect cannot catch up the negative revenue effect caused by a lower advertising level and a shrinking viewership such that the overall free TV broadcaster's profit decreases after pay TV implements advertising.

Regarding aggregate broadcaster profits $\Pi^* = \pi_p^* + \pi_f^*$, we derive

$$\frac{d\Pi^*}{da_p} = \underbrace{\frac{d\pi_p^*}{da_p}}_{>0} + \underbrace{\frac{d\pi_f^*}{da_p}}_{>0 \Leftrightarrow \gamma > \gamma_{\pi_f}}.$$

Given that the profit of the pay TV channel increases if this channel introduces advertising, it is clear that aggregate profits increase whenever the profit of the free TV channel increases, i.e., $\gamma > \gamma_{\pi_f}$. However, due to the positive impact of pay TV profits, it suffices that $\gamma > \gamma_\Pi$ with $\gamma_\Pi < \gamma_{\pi_f}$ to ensure an increase in aggregate profits, i.e., $\frac{d\Pi^*}{da_p^*} > 0 \Leftrightarrow \gamma > \gamma_\Pi$.

Regarding part (ii), we derive that an introduction of advertising in pay TV has the following effect on the pay TV consumer surplus:

$$\frac{dCS_p^*}{da_p} = \underbrace{(-\frac{ds_p^*}{da_p} - \gamma)}_{>0} n_p^* + u_p^* \underbrace{\frac{dn_p^*}{da_p}}_{>0 \Leftrightarrow \gamma < \gamma_n},$$

where $u_p^* = v - s_p^* - \gamma a_p^* - tn_p^* > 0$. Although the advertising level a_p increases, pay TV consumers benefit from a higher advertising level because the subscription fee s_p^* decreases stronger than their disutility from advertising γ increases, i.e., $\left|\frac{ds_p^*}{da_p}\right| > \gamma$.[18] If, in addition, a higher pay TV advertising level induces more viewer demand for pay TV, which is the case for $\gamma < \gamma_n$, then the effect of a higher a_p on pay TV consumer surplus

[18] Note that $\left|\frac{ds_p^*}{da_p}\right| = \left|\frac{1}{2}(-(\gamma + k) + \gamma\frac{da_f^*}{da_p})\right| = \frac{1}{2}(\gamma+k) + \frac{\gamma}{2}\left|\frac{da_f^*}{da_p}\right| > \gamma \Leftrightarrow \frac{k}{2} + \frac{\gamma}{2}\left|\frac{\partial a_f^*}{\partial a_p}\right| > \frac{\gamma}{2}$ because $k > \gamma$.

is clearly positive. However, a higher pay TV advertising level triggers a negative effect on the consumer surplus through a lower viewer demand for pay TV if $\gamma > \gamma_n$. We can conclude that $\gamma < \gamma_{CS_p}$ with $\gamma_n < \gamma_{CS_p}$ suffices to ensure an increase in pay TV consumer surplus, i.e., $\frac{dCS_p^*}{da_p} > 0 \Leftrightarrow \gamma < \gamma_{CS_p}$.

Similarly, for the free TV consumer surplus, we derive

$$\frac{dCS_f^*}{da_p} = \underbrace{\gamma \left(n_p^* - 1\right) \frac{\partial a_f^*}{\partial a_p}}_{>0} - u_f^* \underbrace{\frac{dn_p^*}{da_p}}_{<0 \Leftrightarrow \gamma > \gamma_n},$$

where $u_f^* = v - \gamma a_f^* - t\left(1 - n_p^*\right) > 0$. Pay TV advertising exerts a positive effect on the free TV viewer surplus through a lower free TV advertising level. If, in addition, free TV viewer demand increases, i.e., $\gamma > \gamma_n$, then the overall effect of introducing pay TV advertising on the free TV viewer surplus is positive. However, even if the free TV viewer demand decreases through a higher pay TV advertising level, i.e., $\gamma < \gamma_n$, the lower free TV advertising level can overcompensate for the lower viewer demand. Hence, $\frac{dCS_f^*}{da_p} > 0 \Leftrightarrow \gamma > \gamma_{CS_f}$, where $\gamma_n > \gamma_{CS_f}$.

In aggregate, we derive

$$\frac{dCS^*}{da_p} = (-\frac{ds_p^*}{da_p} - \gamma)n_p^* + \gamma \left(n_p^* - 1\right) \frac{da_f^*}{da_p} + \underbrace{(u_p^* - u_f^*)}_{=0} \frac{dn_p^*}{da_p} > 0.$$

Introducing pay TV advertising has an unambiguously positive effect on the overall consumer surplus of both channels. Additional benefit derived by the viewers from a lower subscription fee and a lower free TV advertising level exceeds the disutility from additional pay TV advertising. Because of the Hotelling model specification, the demand effects are neutral.

Finally, we derive the welfare implications of introducing advertising in pay TV. We assume that social welfare is given as an unweighted sum of aggregate profits and consumer surplus[19]

$$W = (\pi_p + \pi_f) + (CS_p + CS_f).$$

We find that social welfare increases with the introduction of advertising in pay TV if the viewers' disutility from advertising is sufficiently large, i.e., $\frac{dW^*}{da_p} > 0 \Leftrightarrow \gamma > \gamma_W$, where $\gamma_W < \gamma_\Pi$. Given that aggregate consumer surplus always increases, it is clear that social welfare increases if aggregate profits increase, which is the case if $\gamma > \gamma_\Pi$. However, a possible decrease in profits can be overcompensated for by an increase in consumer surplus such that $\gamma_W < \gamma_\Pi$.

[19]Because our model does not explicitly model the advertising market, our welfare function does not capture the advertisers' surplus.

6 Conclusion

This paper develops a model of asymmetric competition between a pay TV broadcaster and a free TV broadcaster to analyze the effects of introducing advertising in pay TV. Our model shows that the pay TV broadcaster will place advertising on its channel if the viewers' disutility of watching advertising is not too strong. Introducing advertising on the pay TV channel leads to a lower subscription fee because additional advertising revenues will be passed through to the viewers. Moreover, as a strategic reaction, the free TV channel reduces its own advertising level. The corresponding viewer demand effects are ambiguous. If the viewers derive a sufficiently low disutility from advertising, then placing advertising in pay TV induces a higher demand on this channel but a lower demand on the free TV channel. In this case, the viewers value a decrease of the subscription fee in pay TV more than a reduction of the advertising level in free TV.

Because the demand effects are neutral in our Hotelling model, the overall consumer surplus on both channels increases through the introduction of pay TV advertising. Consumers of pay TV benefit from the introduction of advertising on their channel if their disutility derived from watching advertising is sufficiently low, while the reverse holds true for the consumers of free TV. On aggregate, the consumers benefit more from a lower subscription fee and a lower free TV advertising level than they suffer from additional pay TV advertising. Moreover, if the marginal return from advertising exceeds the viewers' disutility from advertising then introducing advertising in pay TV increases profits of this channel. Regarding the free TV profits, we find that placing advertising in pay TV has a positive effect on the free TV broadcaster's profit if the viewers' disutility from advertising is sufficiently strong. In this case, the positive viewer demand and cost effect overcompensate for the negative revenue effect through a lower advertising level.

Because the model in this paper is a stylized one, we have made some simplifying assumptions. For example, viewers are supposed to join either pay TV or free TV. It is reasonable that free TV viewers without paying the subscription fee are excluded from the pay TV programming such that they are "forced" to single-home. By contrast, pay TV viewers still have the opportunity to watch free TV programming and thus they have the possibility to multi-home.[20] However, the single-homing assumption of viewers is widely adopted in the existing literature when it comes to capture the competition for viewer market share between media firms.[21] One can justify the assumption by pointing out that viewers cannot physically watch two broadcasts at the same time (at least without decreasing the entertainment value) and thus have to make a choice for watching only

[20] Note that pay TV packages can also include free TV channels.

[21] Rasch (2007) introduced a model of the so-called partial multi-homing behavior of agents in two-sided markets, assuming that some agents single-home, while the rest multi-homes. Anderson et al. (2010) explore how equilibrium prices behave if media consumers single-purchase (i.e., they buy either Time or Newsweek) or if some consumers multi-purchase (i.e., they buy both).

one channel. Furthermore, we have assumed that the broadcaster's marginal return on advertising and the viewers' disutility parameter is symmetric between channels.

An interesting avenue for further research would be to allow for heterogeneity in the effectiveness of advertising and the disutility parameter, respectively. Moreover, it would be interesting to extend our framework and to explicitly model the advertiser's market. This extension would allow deriving social welfare that includes the advertiser's surplus. In our model, social welfare comprises only aggregate broadcasting profit and viewer surplus. Finally, further research regarding the viewers' disutility from advertising is needed.[22] For example, it would be potentially fruitful to empirically assess the value of the disutility parameter.

[22]As also suggested by Peitz and Valletti (2008).

A Appendix

A.1 Proof of Lemma 1

First, we derive the second-order conditions. By noting that $\frac{\partial n_p}{\partial s_p} = -\frac{1}{2t}$, $\frac{\partial^2 n_p}{\partial s_p^2} = 0$, $\frac{\partial n_p}{\partial a_p} = -\frac{\gamma}{2t}$, $\frac{\partial^2 n_p}{\partial a_p^2} = 0$, $\frac{\partial c}{\partial a_p} > 0$ and $\frac{\partial^2 c}{\partial a_p^2} > 0$, the second-order conditions for a maximum require

$$
\begin{aligned}
\text{(a)}\ \frac{\partial^2 \pi_p}{\partial (s_p)^2} &= -\frac{1}{t} < 0 \text{ and } \frac{\partial^2 \pi_p}{\partial a_p^2} = -\frac{k\gamma}{t} - \frac{\partial^2 c}{\partial a_p^2} < 0, \\
\text{(b)}\ \frac{\partial^2 \pi_p}{\partial s_p^2}\frac{\partial^2 \pi_p}{\partial a_p^2} - \frac{\partial^2 \pi_p}{\partial s_p \partial a_p}\frac{\partial^2 \pi_p}{\partial a_p \partial s_p} &= \frac{1}{t}\left(\frac{k\gamma}{t} + \frac{\partial^2 c}{\partial a_p^2}\right) - \left(-\frac{k+\gamma}{2t}\right)^2 > 0.
\end{aligned}
$$

It is easy to see that (a) is satisfied by definition. To ensure that (b) is satisfied, we must assume that $t > \frac{(k-\gamma)^2}{4c''(a_p^*)}$, where $c''(a_p) = \frac{\partial^2 c}{\partial a_p^2}$.

Second, we derive the optimality conditions in part (i). By solving $\frac{\partial \pi_p}{\partial s_p} = 0$ for s_p with $\frac{\partial n_p}{\partial s_p} = -\frac{1}{2t}$, we obtain $s_p^* = \frac{1}{2}(\gamma a_f^* - (k+\gamma)a_p^* + t)$. Plugging s_p^* into $\frac{\partial \pi_p}{\partial a_p} = kn_p^*(s_p^*, a_p^*) + (s_p^* + ka_p^*)\frac{\partial n_p^*}{\partial a_p} - c'\left(a_p^*\right) = 0$ and $\frac{\partial \pi_f}{\partial a_f} = kn_f^*(a_f^*) + ka_f^*\frac{\partial n_f^*}{\partial a_f} - c'(a_f^*) = 0$ and rearranging these equations produces the optimality conditions for pay TV and free TV.

Part (ii). To prove the existence and uniqueness of the equilibrium (s_p^*, a_p^*, a_f^*), we proceed as follows. First, we show that the reaction function $R_f(a_p)$ of free TV is a monotonous decreasing function in a_p and the reaction function $R_p(a_f)$ of pay TV is a monotonous increasing function in a_f. We rearrange the optimality conditions of pay TV and free TV and we define

$$
\begin{aligned}
F_p(a_p^*, a_f^*) &= \frac{k-\gamma}{4t}(\gamma a_f^* + (k-\gamma)a_p^* + t) - c'(a_p^*), \\
F_f(a_p^*, a_f^*) &= \frac{k}{4t}(-3\gamma a_f^* - (k-\gamma)a_p^* + 3t) - c'(a_f^*).
\end{aligned}
$$

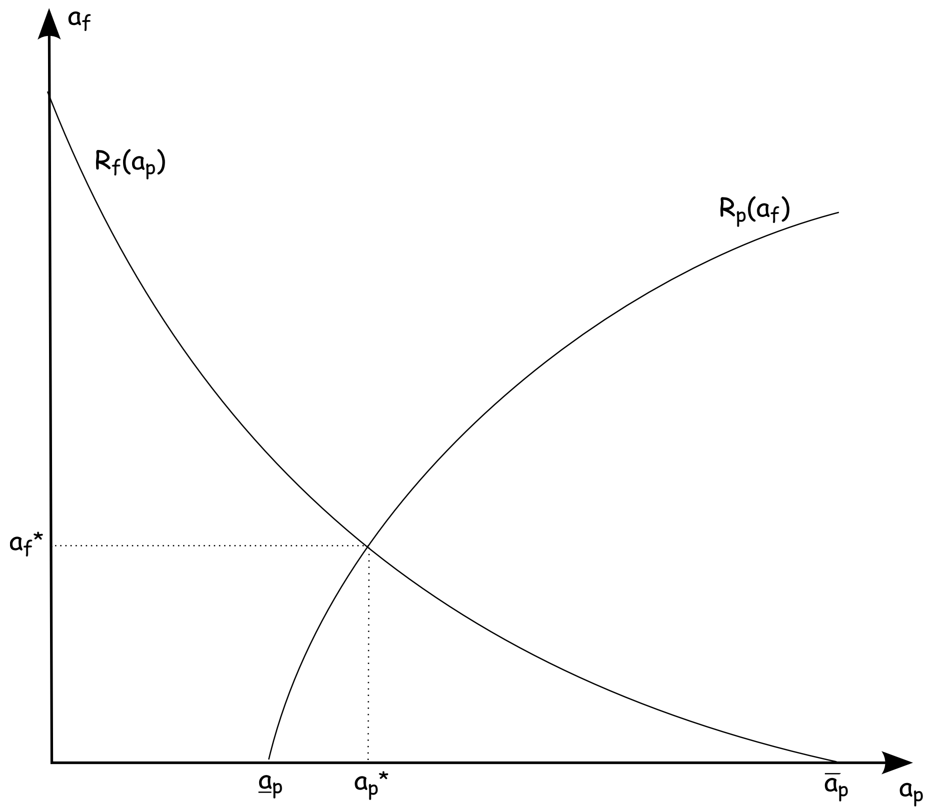
With the implicit function theorem, we derive

$$
\begin{aligned}
\frac{da_p^*}{da_f} &= -\frac{\partial F_p/\partial a_f}{\partial F_p/\partial a_p} = -\frac{\gamma(k-\gamma)}{(k-\gamma)^2 - 4tc''(a_p^*)} \overset{t>t_{SOC}}{>} 0, \\
\frac{da_f^*}{da_p} &= -\frac{\partial F_f/\partial a_p}{\partial F_f/\partial a_f} = -\frac{k(k-\gamma)}{3k\gamma + 4tc''(a_f^*)} < 0.
\end{aligned}
$$

Hence, pay TV reacts with a higher advertising level to an increase in the free TV advertising level, while the opposite holds true for the reaction of free TV. We illustrate the reaction functions in Figure 1.

We define $\overline{a}_p$ as the advertising level, which pay TV would have to set such that the free TV broadcaster has no incentives to places advertising on its channel, i.e., $R_f(\overline{a}_p) = 0$.

Figure 1: Reaction Functions



Moreover, $\underline{a}_p$ is the pay TV advertising level if free TV refrains from advertising, i.e., $\underline{a}_p = R_p(0)$. Now, we have to show that $\overline{a}_p > \underline{a}_p$. This guarantees that the monotonous reaction function have exactly one intersection point, which characterizes the unique equilibrium.

Because $c'(a_f^*) = \frac{k}{4t}(-3\gamma a_f^* - (k-\gamma)a_p^* + 3t)$ is the optimality condition for free TV, we derive that if $a_p = \frac{3t}{k-\gamma}$ then $a_f = 0$. Hence, $\overline{a}_p(t) = \frac{3t}{k-\gamma}$ and thus $\frac{\partial \overline{a}_p}{\partial t} > 0$. Moreover, $\underline{a}_p$ is implicitely characterized by the pay TV's optimality condition $c'(\underline{a}_p) = \frac{k-\gamma}{4t}((k-\gamma)\underline{a}_p + t)$. With the implicit function theorem, we derive

$$\frac{\partial \underline{a}_p}{\partial t} = \frac{\underline{a}_p(k-\gamma)^2}{t\left[(k-\gamma)^2 - 4tc''(\underline{a}_p)\right]} < 0,$$

for t sufficiently large. Because $\overline{a}_p(t)$ is continuous and a monotonously increasing function in t and $\underline{a}_p(t)$ is continuous and a monotonously decreasing function in t, there exists a value t_{eq} such that $\overline{a}_p(t) > \underline{a}_p(t)$ for all $t > t_{eq}$. Hence, we have shown that for a sufficiently large differentiation parameter t_{eq}, an unique equilibrium (a_p^*, a_f^*) exists. Hence, the equilibrium subscription fee s_p^* also exists and is unique.

For example, for a quadratic cost function $c(a) = 1/2a^2$, the advertising level $\underline{a}_p(t)$ is given by

$$\underline{a}_p(t) = \frac{t(k-\gamma)}{4t-(k-\gamma)^2},$$

and thus

$$\overline{a}_p(t) > \underline{a}_p(t) \Leftrightarrow t > t_{eq} \equiv \frac{(k-\gamma)^2}{3}.$$

A.2 Proof of Proposition 1

To prove part (i), we differentiate three cases:

1. Suppose that $\gamma = k$. In this case, we derive $c'(a_p^*) = 0$ and hence $a_p^* = 0$. The subscription fee is then given by $s_p^* = \frac{1}{2}(ka_f^* + t)$ and a_f^* is implicitly defined by $c'(a_f^*) = \frac{3k}{4t}(-ka_f^* + t)$. To ensure that the free TV broadcaster sets a positive advertising level, it must hold $t > ka_f^*$ and hence $a_f^* > a_p^*$ for $\gamma = k$.

2. Suppose that $\gamma = 0$. In this case, the subscription fee is given by $s_p^* = \frac{1}{2}(-ka_p^* + t)$. To ensure a positive fee, we assume that $t > ka_p^*$. The advertising level (a_p^*, a_f^*) are then implicitly defined by $c'(a_p^*) = \frac{k}{4t}(ka_p^* + t)$ and $c'(a_f^*) = \frac{k}{4t}(-ka_p^* + 3t)$. Since $t > ka_p^*$, the free TV broadcaster sets a positive advertising level because $-ka_p^* + 3t > 0$. Next, we derive

$$c'(a_p^*) - c'(a_f^*) = \frac{k(ka_p^* - t)}{2t} < 0,$$

   because $t > ka_p^*$. It follows that $a_f^* > a_p^*$ for $\gamma = 0$.

3. Suppose that $\gamma \in (0, k)$. We can show that a_p^* and a_f^* will never coincide in the interval $\gamma \in (0, k)$. Suppose that $a_p^* = a_f^* = a^*$ and hence $c(a_p^*) = c(a_f^*)$ such that $\frac{k-\gamma}{4t}(ka^* + t) = \frac{k}{4t}\left[-(2\gamma + k)a^* + 3t\right]$. This equality is satisfied if and only if $\gamma = -2k$, which is not in the interval of feasible γ.

From 1.-3., it follows that $a_f^* > a_p^*$ for $\gamma \in (0, k]$.

Moreover, we can rule out the case $\gamma > k$. To show this claim, we provide a proof by contradiction. We know that $c'(a_p^*) = \frac{k-\gamma}{4t}(\gamma a_f^* + (k-\gamma)a_p^* + t) \geq 0$. Now, suppose that $\gamma > k$. Hence, it must be the case that $t + \gamma a_f^* \leq (\gamma - k)a_p^*$. However, a positive subscription fee $s_p^* > 0$ implies $t + \gamma a_f^* > (\gamma + k)a_p^*$. That is, $t + \gamma a_f^* \in ((\gamma + k)a_p^*, (\gamma - k)a_p^*]$, which cannot be satisfied in equilibrium. Therefore, our assumption was wrong and it must hold $\gamma \leq k$.

To prove the comparative statics result in part (ii), we rearrange (15) and (14) and define $F_p(\gamma, a_p^*) = \frac{k-\gamma}{4t}(\gamma a_f^* + (k-\gamma)a_p^* + t) - c'(a_p^*) = 0$ and $F_f(\gamma, a_f^*) = \frac{k}{4t}(-3\gamma a_f^* - (k - \gamma)a_p^* + 3t) - c'(a_f^*) = 0$. With the implicit function theorem, we derive

$$\frac{\partial a_f^*}{\partial \gamma} = -\frac{\partial F_f/\partial \gamma}{\partial F_f/\partial a_f} = -\frac{(3a_f^* - a_p^*)k}{3k\gamma + 4tc''(a_f^*)} < 0.$$

Hence, the advertising level on the free TV channel is always decreasing in γ. Regarding the advertising level on pay TV, we derive

$$\frac{\partial a_p^*}{\partial \gamma} = -\frac{\partial F_p/\partial \gamma}{\partial F_p/\partial a_p} = \frac{(a_f^* - a_p^*)(2\gamma - k) + t + ka_p^*}{(k-\gamma)^2 - 4tc''(a_p^*)}.$$

The second-order conditions require $4tc''(a_p^*) > (k-\gamma)^2$, which implies that the denominator is negative. Moreover, it is always the case that $a_f^* > a_p^*$. As a result, we derive that for $\gamma < \widehat{\gamma}$, the numerator will be negative, and hence,

$$\frac{\partial a_p^*}{\partial \gamma} > 0 \Leftrightarrow \gamma < \widehat{\gamma} \equiv \frac{k\left(a_f^* - 2a_p^*\right) - t}{2\left(a_f^* - a_p^*\right)}.$$

Part (iii). To show that $n_f^* \geq n_p^*$ $\forall \gamma \in [0, k]$, we differentiate three cases:

1. Suppose that $\gamma = k$. In this case, we derive $n_f^* - n_p^* = \frac{1}{2t}(t - ka_f^*) > 0$ because $t > ka_f^*$.

2. Suppose that $\gamma = 0$. In this case, we derive $n_f^* - n_p^* = \frac{1}{2t}(t - ka_p^*) > 0$ because $t > ka_p^*$.

3. Suppose that $\gamma \in (0, k)$. To show that $n_f^* \geq n_p^*$ $\forall \gamma \in (0, k)$, we provide a proof by contradiction. Suppose that n_f^* can fall below n_p^* in the interval $\gamma \in (0, k)$. Note that $n_f^*(\gamma)$ and $n_p^*(\gamma)$ are both continuous functions in γ. Hence, it must be the case that n_f^* and n_p^* intersect twice in the interval $\gamma \in (0, k)$ because $n_f^*(0) > n_p^*(0)$ and $n_f^*(k) > n_p^*(k)$. However, only one point of intersection exists because

$$n_f^* = n_p^* = \frac{1}{2} \Leftrightarrow \gamma = \frac{t - ka_p^*}{a_f^* - a_p^*}.$$

   It follows that the assumption was wrong and $n_f^* \geq n_p^*$ $\forall \gamma \in (0, k)$.

This completes the proof of the proposition.

References

Anderson, S. and Coate, S. (2005), 'Market Provision of Broadcasting: A Welfare Analysis', *Review of Economic Studies* **72**, 947–972.

Anderson, S., Foros, Ø. and Kind, H. (2010), 'Hotelling Competition with Multi-purchasing: Time Magazine, Newsweek, or both?', *CESifo Working Paper Industrial Organisation 3096* .

Anderson, S. and Gabszewicz, J. (2006), 'The Media and Advertising: a Tale of Two-sided Markets', *Handbook on the Economics of Art and Culture* **1**, 567–614.

Armstrong, M. (2005), 'Public Service Broadcasting', *Fiscal Studies* **26**(3), 281–299.

Armstrong, M. (2006), 'Competition in Two-Sided Markets', *RAND Journal of Economics* **37**(3), 668–691.

Belleflamme, P. and Toulemonde, E. (2009), 'Negative Intra-Side Externalities in Two-Sided Markets', *International Economic Review* **50**(1), 245–272.

Blair, R. and Romano, R. (1993), 'Pricing Decisions of the Newspaper Monopolist', *Southern Economic Journal* **59**(4), 721–732.

Caillaud, B. and Jullien, B. (2003), 'Chicken & Egg: Competition among Intermediation Service Providers', *RAND Journal of Economics* **34**(2), 309–328.

Choi, J. (2006), 'Broadcast Competition and Advertising with Free Entry: Subscription vs. Free-to-Air', *Information Economics and Policy* **18**(2), 181–196.

Crampes, C., Haritchabalet, C. and Jullien, B. (2009), 'Advertising, Competition and Entry in Media Industries', *Journal of Industrial Economics* **57**(1), 7–31.

Gabszewicz, J., Laussel, D. and Sonnac, N. (2001), 'Press Advertising and the Ascent of the "Pensee Unique"', *European Economic Review* **45**(4-6), 641–651.

Gabszewicz, J., Laussel, D. and Sonnac, N. (2002), 'Press Advertising and the Political Differentiation of Newspapers', *Journal of Public Economic Theory* **4**(3), 317–334.

Gabszewicz, J., Laussel, D. and Sonnac, N. (2004), 'Programming and Advertising Competition in the Broadcasting Industry', *Journal of Economics & Management Strategy* **13**(4), 657–669.

Gal-Or, E. and Dukes, A. (2003), 'Minimum Differentiation in Commercial Media Markets', *Journal of Economics & Management Strategy* **12**(3), 291–325.

Hagiu, A. (2006), 'Pricing and Commitment by Two-Sided Platforms', *RAND Journal of Economics* **37**(3), 720–737.

Kaiser, U. and Song, M. (2009), 'Do Media Consumers Really Dislike Advertising? An Empirical Assessment of the Role of Advertising in Print Media Markets', *International Journal of Industrial Organization* **27**(2), 292–301.

Kaiser, U. and Wright, J. (2006), 'Price Structure in Two-Sided Markets: Evidence from the Magazine Industry', *International Journal of Industrial Organization* **24**(1), 1–28.

Kind, H., Nilssen, T. and Sorgard, L. (2009), 'Business Models for Media Firms: Does Competition Matter for How They Raise Revenue?', *Marketing Science* **28**(6), 1112–1128.

Lin, P. (2011), 'Market Provision of Program Quality in the Television Broadcasting Industry', *The BE Journal of Economic Analysis & Policy (Contributions)* **11**(1), 1–17.

Peitz, M. and Valletti, T. (2008), 'Content and Advertising in the Media: Pay-tv versus Free-to-air', *International Journal of Industrial Organization* **26**(4), 949–965.

Rasch, A. (2007), 'Platform Competition with Partial Multihoming under Differentiation: a Note', *Economics Bulletin* **12**(7), 1–8.

Reisinger, M. (2011), 'Platform Competition for Advertisers and Users in Media Markets', *International Journal of Industrial Organization* .

Rochet, J. and Tirole, J. (2003), 'Platform Competition in Two-Sided Markets', *Journal of the European Economic Association* **1**(4), 990–1029.

Rysman, M. (2004), 'Competition between Networks: A Study of the Market for Yellow Pages', *Review of Economic Studies* **71**(2), 483–512.

TVB (2011*a*), 'Ad Spending: First Half 2011 vs. 2010 Television', <http://www.tvb.org/trends/4705>, Last accessed on January, 25, 2012.

TVB (2011*b*), 'TV Basics', <http://www.tvb.org/media/file/TVBasics.pdf>, Last accessed on January, 25, 2012.

Weyl, E. G. (2010), 'A Price Theory of Multi-Sided Platforms', *American Economic Review* **100**(4), 1642–1672.

Wright, J. (2004), 'One-Sided Logic in Two-Sided Markets', *Review of Network Economics* **3**(1), 44–64.