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Wittgenstein and Frege on Assertion

Abstract: In the *Philosophical Investigations*, Wittgenstein famously criticizes Frege's conception of assertion. "Frege's opinion that every assertion contains an assumption", says Wittgenstein, rests on the possibility of parsing every assertoric sentence into two components: one expressing the assumption that is put forward for assertion, the other expressing that it is asserted. But this possibility does not entail that the "assertion consists of two acts, entertaining and asserting" – any more than the possibility of rendering assertions as pairs of questions and affirmative answers entails that they consist of questions. Frege scholars protest that such criticism is inappropriate, not only because Frege doesn't speak about assumptions, but also – and crucially – because Wittgenstein fails to address the logical nature of assertion as reflected in Frege's use of the judgment stroke. They seem to read Wittgenstein's argument in the light of a remark in the *Tractatus* saying that the judgment stroke is "logically meaningless" because it simply indicates that the author holds the propositions marked with this sign to be true. In this paper, I argue that Wittgenstein's criticism of Frege is not that the latter's conception of judgment and assertion contains a corrupting psychological element. Rather, the criticism is that for Frege judgment and assertion are composed of two separate acts, i.e. an act of *referring* to a truth value and an act of *determining* which of the two it is. Through a detailed examination of the "black-spot analogy" in the *Tractatus*, I want to show that Wittgenstein presents a serious objection to Frege's conception of judgment and assertion.

1 Introduction

The notion of *judgment* is essential to Frege's conception of logic. He considers judgment to be a "logically primitive activity" (Frege 1979: 15) and introduces a special symbol for it in order to make judgments recognizable in logical derivations. Indeed, the "judgment stroke" represents such an important discovery for Frege that he wishes to have cited it in first place when responding to the question "*What may I regard as the Result of my Work?*" (Frege 1979: 184).

For all his admiration for Frege's work, Wittgenstein doesn't share Frege's enthusiasm, and both in early and late periods he expresses reservations about Frege's conception of judgment. Ignoring certain subtleties such as that Frege

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distinguishes between judgment and assertion, or that he only calls the vertical part of the complex symbol “judgment stroke”, Wittgenstein disapproves of the logical significance of assertion and its representation in logical symbolism. He says that “assertion is merely psychological” (Wittgenstein 1913: 95) and that “Frege’s ‘judgment-stroke’ ‘|—’ is logically quite meaningless” because it simply indicates that the author holds the propositions marked with this sign to be true (TLP 4.442).¹

Wittgenstein’s negative verdict seems to be repeated in §22 of *Philosophical Investigations*, where he discusses “Frege’s opinion that every assertion contains an assumption”. The assertion sign turns out to be “superfluous” if its function is to indicate assertion, since the specification of what is asserted characteristically takes the form of an assertion already. Wittgenstein grants that the “assertion sign” can be used to distinguish assertions from questions, fictions or assumptions, but it is a mistake to think of assertion as composed of two separate acts, one of which is represented by the judgment stroke.

Dummett made no bones about his take on these considerations: “The confused objection of *Philosophical Investigations*, §22, is not to the point” (Dummett 1991: 247). And many Frege scholars seem to share this negative assessment. For one thing, Wittgenstein still seems to slide over the complexity of the assertion sign as well as the fact that for Frege it is not “assumptions” but *thoughts* that are put forward as true in judgments and assertions.² For another, it has been objected that Wittgenstein’s criticism misfires because it completely ignores the *normative* dimension judgment. Frege’s notion of judgment is essentially normative, since the judgments he is dealing with are made on the grounds of other judgments in accordance with logical laws as the “guiding principles for thought” (Frege 1964: xv). Against this background, the judgment stroke is anything but superfluous, since far from indicating what anyone holds to be true, it indicates what everyone *should* acknowledge as true.³

Can Wittgenstein’s objection be defused by highlighting the link between Frege’s notion of judgment and norms for logical inference? Does Wittgenstein

1 Wittgenstein’s hostility echoes in a letter from Philip Jourdain, who asks Frege for permission to publish some passages from *Grundgesetze* in *The Monist*, assuring him that Wittgenstein has agreed to check the translation: “Also, will you tell me, [...] whether you now regard assertion (|—) as merely psychological” (Frege 1980: 78).

2 This has given rise to the suspicion that Wittgenstein targets Russell’s Frege, rather than Gottlob Frege, since Russell portrays Frege as thinking that judgments consist of assumptions (Russell 1903 §477); see Anscombe 1959: 105.

3 Among the interpreters who try to dissolve the tension between the logical role of Frege’s judgment stroke and the threat of psychologism in valuable ways are Smith 2000, Greimann 2000, Taschek 2008, Textor 2010, Pedriali 2017 and van der Schaar 2018.

really complain about a corrupting psychological element in Frege's conception of logic? The fact that he doesn't sound particularly hostile when he claims that assertion is merely psychological suggests that the core of his objection doesn't concern psychologism at all. Admittedly, one easily gets this impression when reading §22 of the *Investigations* in the light of Wittgenstein's negative remark in the *Tractatus*. However, what Wittgenstein, early and late, reject is Frege's idea that "the assertion consists of two acts, entertaining and asserting" (PI §22). Defending Frege against this charge is more difficult than showing that the judgment stroke is not logically superfluous. In order to avoid barking up the wrong tree, therefore, I suggest getting a proper understanding of Wittgenstein's criticism by reading his objection in the *Tractatus* in the light of §22 of the *Investigations*. Before that, however, we need to define clearly the proper target of Wittgenstein's criticism, viz. Frege's view of assertion.

2 A Problem for Frege's Conception of Assertion

For Frege, the distinction between thought, judgment and assertion is crucial. He famously distinguishes between "the grasp of a thought – thinking, the acknowledgement of the truth of a thought – the act of judgment, the manifestation of this judgment – assertion" (Frege 1918a: 62). The difference between the content of a judgment and the acknowledgment of its truth is logically relevant and thus has to be expressed in *Begriffsschrift*, in which "everything that is necessary for a valid inference is fully expressed" (Frege 1879 §3). Frege uses a horizontal line to express that a content is "judgeable" (— A), and he draws a vertical line at the left end of the horizontal line to express the recognition of its truth (⊢— A). In order to paraphrase the difference in natural language, Frege suggests reading the former as "the circumstance that A" and the latter as "the circumstance that A is a fact". The complex symbol "⊢—" is something like "the common predicate for all judgments" (Frege 1879 §3).⁴

In order to explain the difference between content and judgment, Frege makes essential use of *nominalizations*. For example, the content of the judgment that Archimedes was killed at the capture of Syracuse is expressed as "the violent death of Archimedes at the capture of Syracuse", or some other nominalization that goes

⁴ The comparison must be taken with a pinch of salt, since Frege dispenses with the grammatical distinction between subject and predicate, distinguishing instead between function and argument. Taken at his word, the "single predicate for all judgments, namely 'is a fact'" (Frege 1879 §3) would turn out to be logically irrelevant.

with "... is a fact", and makes it clear that we are not yet dealing with a judgment. However, this early conception of judgment is exposed to two objections. First, the distinction between assertables and unassertables seems to be *ad hoc*; e.g., nominalizations such as "the death of Cesar" can be asserted, but nouns such as "house", "the number 2" as well as propositions involving vague concepts can't, as they don't express assertable contents. As we will see later, this problem does not arise on Frege's mature conception of the content of a judgment.

Second, and more importantly, Frege faces a grammatical *dilemma*: If the content of a judgment is to be expressed by the use of a nominalization, then the content is nothing that can strictly speaking be judged. Grammatically, nominalizations introduced with "the circumstance that ..." function like names or other noun-phrases whose function is referential rather than expressive. If, on the other hand, the content of a judgment is not nominalized but paraphrased with a sentence in the indicative mood, then the content is assertoric and there is no point in adding the judgment stroke. Therefore, the content of a judgment is either nominalized and hence unfit for being the subject of a possible judgment, or it is in the indicative mood and thereby steals the judgment stroke's thunder. To be clear, the cause of this predicament resides in Frege's proposal on how to paraphrase the difference between content and judgment. Frege's development of his semantic theory of sentences and names in the early 1890s should allow him to cope with this problem too.

According to Wittgenstein, however, the dilemma just outlined is not merely a problem of the paraphrase, but rather a problem of what is so paraphrased. Immediately after his misleading remark about Frege's opinion that every assertion contains an assumption, he seems to allude to an argument that runs parallel to Frege's dilemma:

But "that such-and-such is the case" is not a sentence in our language – it is not yet a move in the language game. And if I write, not "It is asserted that ...", but "It is asserted: such-and-such is the case", the words "It is asserted" simply become superfluous. (PI §22)

Although Wittgenstein is not strictly targeting Frege's nominalizing device "the circumstance that ...", the structure of his argument corresponds to what I called the grammatical dilemma above. If the content of a judgment or of an assertion is to be isolatable, such as an assumption that we agree or disagree with, then the content needs to be expressed in a sentence that is either true or false. But neither "that *p*" nor "the circumstance that *p*" meets this requirement, as with these phrases one makes no move in the language game; i.e., they can be used as parts of utterances, but one doesn't perform a speech act when expressing

them in isolation. Remedying this deficiency means understanding the content assertorically, which ipso facto makes the judgment stroke redundant.⁵

According to Baker and Hacker, this predicament already shows that Frege's conception of judgment and assertion is incoherent:

Any attempt to represent Frege's claim that every assertion contains an assumption by transformations permissible in language is thus subject to contradictory demands. For the linguistic expression of the contained assumption must both be, and not be, a sentence. (Baker/Hacker 2005: 80)

However, such general verdict might be premature, as it is essentially Frege's early conception of a judgment that causes all the trouble, and so the question naturally arises whether Frege's mature conception avoids the dilemma.

3 Does Semantics Come to the Rescue?

With the discovery of the sense-reference distinction and the prior extension of the notion of a *function*, Frege modifies his *Begriffsschrift* in a way that also affects his conception of judgment and assertion. The “horizontal” now represents a truth function, the value of which is the True if the argument is true, and the False in all other cases (see Frege 1891: 21; Frege 1964 §5). Thanks to this function, Frege no longer needs to stipulate that the content of a judgment is assertable: “— x ” already represents something that can be judged for any meaningful instance of x . Consequently, “ \vdash — 2” expresses a judgment just as “ \vdash — $3 > 2$ ” does.

As is well known, a function whose value is always a truth value is for Frege a *concept*. Since the horizontal stands for such a function, the question arises as to *which* concept it represents? There is no agreement among Frege scholars on this point. Some have suggested reading “— x ” in the sense of “ x is identical to the True”.⁶ The merits of this reading are obvious: as a relational concept under which nothing but the True falls, the concept represented by the horizontal applies equally to assertables and unassertables. However, the proposal faces the problem

5 Just to be clear, Frege can't possibly agree to paraphrasing “ \vdash — p ” with “it is asserted: p ”, as one is entitled to apply the judgment stroke only to *true* propositions. Regardless of whether truth is a norm for assertions, contrary to “ \vdash — p ”, “it is asserted: p ” does not entail the truth of p (see Künne 2009b: 337). Yet, as for the distinction between content and judgment, the tension is real, since Frege makes demands that are difficult to reconcile. The plausibility of Frege's distinction depends crucially on what is “contained” in judgment and assertion.

6 See Walker 1965: 132 and Noonan 2001: 150 for explicit statements of this suggestion; for a critical but nevertheless approving discussion of the proposal see Greimann 2000: 232.

that “ $3 > 2$ is identical to the True” is both ungrammatical and assertoric. In terms of grammar, it won’t help to opt for a metalinguistic alternative, such as “‘ $3 > 2$ ’ is identical to the True”, since the expression “ $3 > 2$ ” in this reading does not refer to the True but to itself, and hence the value of the horizontal would be the False, not the True. Moreover, the assertoric mode of the metalinguistic alternative preempts the role of the judgment stroke.⁷

In order to overcome this second problem, David Bell has suggested reading “— x ” as corresponding to the complex noun phrase “ x ’s being identical to the True” (Bell 1979: 23). Thus nominalized, the horizontal can be equally applied to names and sentences without assertoric import, but in the latter case the resulting expression still sounds grammatically odd. For how are we to understand the judgment that 3 is greater than 2, or that Caesar is dead? According to Bell’s proposal, “Caesar’s death’s being identical to the True is a fact” expresses a true judgment stating identity between Caesar’s death and the True. But since truth values are abstract objects, Caesar’s death should also be an abstract object, and perhaps even more controversially, it should be the *same* abstract object as the True. To stay within Frege’s framework, one can say that “Caesar’s death” *refers* to the True, just as any other true sentence does (via its sense), but reference and identity are not one and the same.

Apart from this difficulty, Bell’s proposal departs from Frege’s language use. In a letter to Husserl, he explicitly says how the horizontal of the modified *Begriffsschrift* is to be read: “Instead of speaking of a ‘circumstance’, one should speak of a ‘truth-value’” (Frege 1980: 64). Unfortunately, the key formulation is lost in the English translation, since Frege gives precise instructions on *how* to speak of a truth value: “Wahrheitswert davon, dass” (Frege 1976: 98). The definite description “the truth value of (that) x ” is applicable to assertables as well as unassertables without assertoric import and without infringing on grammar. Hence, Frege’s mature semantic theory seems to provide the resources necessary to avoid the quandaries related to his early conception of judgment and assertion.

Wittgenstein’s charge, however, is not completely settled, as will be shown in the next section. But before examining whether Frege’s semantic conception of judgment and assertion is coherent, let me just highlight an immediate consequence of his modified account. If the content of a judgment is *referential*, insofar as it is referring to a truth value, then a judgment is by its very nature something linguistic, since the reference relation essentially holds between a linguistic object and something else. Bluntly put, therefore, a judgment for Frege is always

⁷ Heck/Lycan 1979 conclude from this that it is impossible to determine which concept is represented by the horizontal.

about a name's reference to the True. This may be acceptable for the judgments made within a framework such as the *Grundgesetze*, but how does this referential conception work for judgments that are not put into writing?⁸

4 Wittgenstein's Criticism in the *Tractatus*

On Frege's mature conception, assertion essentially involves reference to a truth value. When introducing the horizontal and the judgment stroke in *Grundgesetze*, he explicitly states that the part of a "proposition of Begriffsschrift" (*Begriffsschriftsatz*) that determines the judgment's content "simply designates a truth-value, without saying which of the two it is" (Frege 1964 §5). He continues to say that "we therefore need a special sign to be able to assert something as true". Thus, it falls to the judgment stroke to say which truth value is denoted by the rest of a *Begriffsschriftsatz*. As I read him, Wittgenstein opposes this division of labor both in the *Investigations* and in the *Tractatus* (as well as in *Notes on Logic*). For a proper understanding of his objection, however, one has to look at the earlier of these writings too, since in *Investigations* he only presents the diagnostics of the mistake:

Of course, one has the right to use an assertion sign in contrast with a question mark [...] It is a mistake only if one thinks that the assertion consists of two acts, entertaining and asserting (assigning a truth-value, or something of the kind), and that in performing these acts we follow the sentence sign by sign roughly as we sing from sheet music. (PI §22)

Wittgenstein seems to grant the use of an assertion sign (⊢—) for the purpose of contrasting assertions with other speech acts such as questions and demands. However, the mistake to which he wants to draw our attention distinctively in connection with Frege is to think that this contrastive sign would represent the performance of *two separate* acts. Since Fregean assertion and judgment essentially involve the act of referring to a truth value, he seems to be guilty of making this mistake.

Wittgenstein does not, however, say in this passage *why* the two-stage model of assertion is mistaken. The model could be rejected simply on the intuitive ground

⁸ It has been argued that "judging that *p* is attempting to refer to the True, by thinking that *p*" (Heck/May 2007: 19). It seems to me that one can maintain the Fregean spirit of this general proposal only if one is prepared to accept the controversial claim that thought presupposes language, since Fregean reference is essentially a relation between linguistic signs and their denotation.

that in making a judgment or an assertion one is not doing *two* things in a row, as one reads note after note when singing from a score.⁹ But Wittgenstein has a stronger objection to Frege's model of judgment and assertion, although it occurs elsewhere and Wittgenstein obviously feels no need to repeat it. The decisive argument can be found in the *Tractatus*, where Wittgenstein criticizes Frege's notion of truth with a comparison:

Imagine a black spot on white paper: you can describe the spot by saying for each point on the sheet, whether it is black or white. To the fact that a point is black there corresponds a positive fact, and to the facts that a point is white (not black), a negative fact. If I designate a point on the sheet (a truth-value according to Frege), then this corresponds to the supposition that is put forward for judgment, etc. etc. (TLP 4.063; see also NL B10)

The thought is that a random black stain on white paper representing a totality of facts can be completely described by, for example, indicating whether each spot is black or white by means of Cartesian coordinates. In this analogy, each point on the sheet corresponds to a Fregean truth value, and pointing to a particular spot corresponds to a Fregean supposition (*Annahme*). Just as one can point to the color of, say, J9, so one can refer to the truth value of *p*. Moreover, just as the reference to J9 is a substantial component of the judgment that this particular spot is black, so reference to the truth value of *p* is a substantial component (on Frege's terms) of the judgment that *p* is true. So according to this comparison, “— *p*” refers to a truth value without telling whether it is the True or the False, just as “the color of *x*” refers to a color without telling whether it is black or white.¹⁰

However, Wittgenstein thinks that the analogy breaks down because referring to a truth value is not relevantly similar to pointing at color stains:

But in order to be able to say that a point is black or white, I must first know when a point is called black, and when white: in order to be able to say, “*p* is true (or false)”, I must have

9 The two-stage model of assertion is not a strawman's position and surfaces in many of Frege's characterizations. He sometimes describes judgment in terms of “taking steps” (Frege 1892: 34), “advances from a thought to a truth-value” (Frege 1892: 35), or making a “choice between opposite thoughts” (Frege 1979: 198). The literal interpretation of these characterizations is critically discussed in Stepanians 1998.

10 Note that “the truth value of *p* is the True” is an identity statement with definite descriptions on both sides; accordingly, the corresponding sentence in Wittgenstein's analogy would have to be “the color of spot *x* is the color black”. Otherwise, the comparison would not make sense, since truth values are *objects* and colors are *properties*. Whether it is plausible to use “the color of *x*” as a referring device that parallels the horizontal's reference to a truth value seems to be more problematic and will be discussed below.

determined in what circumstances I call “*p*” true, and in so doing I determine the sense of the proposition.

Now the point where the simile breaks down is this: we can indicate a point on the paper even if we do not know what black and white are, but if a proposition has no sense, nothing corresponds to it, since it does not designate a thing (a truth-value) which might have properties called “false” or “true”. The verb of a proposition is not “is true” or “is false”, as Frege thought: rather, that which “is true” must already contain the verb. (TLP 4.063; see also NL B10)

This passage is rich and notoriously difficult to understand, partly because it contains some elements that do not fit the Fregean picture at all.¹¹ Although Frege’s notion of truth is the declared target of the whole section, it remains unclear, for example, whether Wittgenstein’s argument builds upon Fregean or Tractarian *sense*. Furthermore, according to the conclusion of Wittgenstein’s argument, Frege allegedly took “is true” (and “is false”) to be the verb of a proposition, thus imposing a *predicational* conception of judgment and assertion that Frege couldn’t possibly accept.¹² Regardless of these incongruities, I am going to suggest a reading of this section according to which Wittgenstein raises a serious objection to the two-stage model of judgment and assertion. In my interpretation, the middle section of 4.063 is a straightforward continuation of the analogy because it makes explicit the similarity between the statement that a particular thought is true and the statement that a particular color patch is black; i.e., one has to know the conditions of application for expressions such as “true” and “black”.¹³

Having emphasized the similarity between the color case and the semantic case at the level of judgment and assertion, Wittgenstein goes on to explain why the analogy breaks down. One can point to a particular spot on the paper – either

11 Unfortunately, many commentators end up rephrasing this passage instead of elucidating it; notable exceptions are Proops 1997: 129ff., Ricketts 2002: 239ff., and Potter 2009: 89ff.

12 For the difference between Fregean and Tractarian sense, see Künne 2009a: 45ff. and Hacker 2001: 206f.; for the second supposition, see Proops 1997: 131. I will give reasons below why the predicational conception of judgment is not acceptable for Frege.

13 In this respect, I deviate from Proops, who argues that this paragraph “is not a continuation of the analogy”, but “presents Wittgenstein’s own views about what it is to have a grasp of the notion of truth” (Proops 1997: 131). According to Proops, “to sustain the analogy, truth and falsity would have to be applicable to truth-values, not propositions” (Proops 1997: 143). Yet, this is not a result of sustaining the analogy but a result of misconceiving judgment and assertion as *predicating* truth. In my reading, *at the level of judgment and assertion* – and that is what the middle section is about – the color case is similar to the semantic case, regardless of how truth attaches to thought. Frege can confidently accept what Wittgenstein says in the middle section: one cannot *judge* a proposition to be true without determining its sense. It is *at the level of thought* that the analogy breaks down, because designating a truth value is in relevant respects not like pointing to a color patch.

ostensively or by using Cartesian coordinates – without knowing the application conditions for such expressions as “black” and “white”. But one cannot designate a truth value by the use of a sentence without knowing the application conditions for such expressions as “true” and “false”. The pointing device in the semantic case may be a sentence or a definite description such as “the truth value of x ”; either way, one cannot make use of the device without a prior understanding of what the device is supposed to refer to. This is in stark contrast to the use of a pointing device in the color case, because one can make use of a finger or of coordinates without knowing anything of the colors of the point thus indicated. Wittgenstein explains the dissimilarity by alluding to *some* notion of *sense*: the target in the semantic case (truth) surfaces in the requirements for semantic pointing, insofar as the use of a sentence, for whatever purpose, cannot be detached from grasping the thought expressed, and grasping the thought is grasping the truth conditions of the sentence. As I read him, Wittgenstein is not saying that grasping the truth conditions of a sentence is knowing *whether* the sentence is true, for this would obviously forestall the point of judgment and assertion. He seems to make the more subtle observation that by referring to a truth value with a sentence one has to make use of the notion of *truth* as it occurs in judgment and assertion, because one has to know that the sentence is either true or false. This is where the semantic case differs from the color case, as pointing to a specific spot of a stain can be done without knowing that it is either black or white.¹⁴

5 Drawing the Right Conclusion

In this last section, I want to discuss the conclusion to be drawn from this argument. On the one hand, as Künne (Künne 2009a: 57) and others have pointed out, Wittgenstein’s official conclusion is indeed bewildering, since Frege never said that “is true” is the verb of the proposition. On the other hand, if the analogy only

¹⁴ Thus, the argument does not necessarily presuppose Wittgenstein’s notion of *sense*, as it seems to be equally valid for logical tautologies, which characteristically lack Tractarian sense. When grasping the truth conditions of, say, “ $(c \rightarrow (b \rightarrow a)) \rightarrow ((c \rightarrow b) \rightarrow (c \rightarrow a))$ ”, one is computing a large number of conditionals (including a case differentiation) regarding the truth and falsity of the whole sentence with respect to the truth or falsity of its parts. For these computations one has to make tentative use of the notion of *truth* as it occurs in judgment and assertion. However, the argument does presuppose that by simply writing down a well-formed formula preceded by the horizontal one is not referring to a truth value. But this assumption is compatible with Frege’s demand that it must be possible to express a thought without acknowledging its truth, for by merely writing down a sentence one has not yet grasped the thought it expresses.

stresses the fact that judgment and assertion involve grasping truth conditions, then it is hard to see why this is an objection to Frege. In short, Wittgenstein's criticism is either unjustified or inconsequential, or so it seems.

Let me begin with the first half of the lesson that Wittgenstein officially wants us to draw: "The verb of a proposition is not 'is true' or 'is false', as Frege thought[.]" This conclusion will strike Fregeans as puzzling, since Frege almost always insists that judgment and assertion are *not* predications of truth (the only exception is in *Begriffsschrift* §3; see my footnote 4 above). He famously observes that "the thought that 5 is a prime number is true" contains the same thought as "5 is a prime number", and that the relation of the thought to the True may therefore not be compared with that of subject to predicate (Frege 1892: 34). So he would oppose at least the first part of Wittgenstein's conclusion by pointing to the *redundancy* of the truth predicate. Moreover, Frege not only thinks that the truth predicate so used contributes nothing to a thought; he also offers a compelling argument against the mistaken conception of judgment and assertion as *predications* of truth:

By combining subject and predicate, one reaches only a thought, never passes from a sense to its *Bedeutung*, never from a thought to its truth-value. One moves at the same level but never advances from one level to the next. (Frege 1892: 35)

This rules out the predicational view of judgment and assertion that Wittgenstein allegedly ascribes to Frege. Judgment and assertion cannot consist in predicating "is true" of a thought, for the result of combining thought and truth in terms of predication yields just another, and more complex, thought. As an account of judgment and assertion, the predicational view rather amounts to an *infinite regress* at the level of thought than of judgments and assertions, which are at another level to stay with Frege's picture (cf. Textor 2010: 637f.).

Wittgenstein's misrepresentation of Frege's views harbors the danger of concealing the second part of the lesson to be drawn from the analogy: "that which 'is true' must already contain the verb". The claim is not that the truth predicate is the verb of the proposition, but rather that that to which such a predicate applies, whether redundant or not, must already contain *some* verb. If this is the conclusion that follows from the analogy, then it must stand independently of the failed prelude. Whatever "is true" contributes, it cannot make its contribution to something that doesn't already contain a verb and thereby is assertable. Regarding the dilemma that arises from paraphrasing Frege's judgment stroke (see section 2 above), Wittgenstein seems to be willing to accept one of the alternatives, namely that the content of an assertion is assertoric because of the verb. However, Frege cannot agree to this conclusion, as it confuses predicating with judging (cf. Frege 1979: 185). Therefore, if it follows from the analogy that the content of the

assertion is assertoric, Wittgenstein seems to have a point which seriously threatens Frege's conception of judgment and assertion, and which is valid regardless of Wittgenstein's unfortunate portrayal.

To reiterate, Frege wants to drive a wedge between merely grasping a thought on the one hand and acknowledging its truth on the other. What is sometimes characterized informally as temporally distinct acts (cf. Frege 1918b: 151; Frege 1979: 7, 138) is formally represented by symbols depicting the logical relation between them. The representation of an overt speech act of assertion and of its silent counterpart, a judgment, incorporates the representation of an act whose performance is logically independent of the first type of act. Just as " $\text{— } p$ " is a graphical component of " $\text{⊢ } p$ " that has an independent meaning, so the act of grasping a thought, of referring to a truth value, is a component of judgment and assertion that can be performed independently of these latter acts. It is precisely this 'logical anatomy' that Wittgenstein's analogy addresses, since it questions the logical autonomy of truth value reference in Frege's two-stage model of judgment and assertion. As a separate act one should be able to perform it without performing the other.

For Frege, judgment and assertion are composed of two separate acts, represented by the horizontal and the judgment strokes. This makes it comparable to the two-stage process of pointing to a particular spot on a piece of paper and telling what color it is. But whereas the color has no bearing on the autonomous act of pointing at the spot, the act of designating a truth value doesn't have this autonomy. For one can only refer to a truth value by means of a proposition that is either true or false, that is, by the use of a vehicle containing a verb. According to Wittgenstein then, the two-stage model of judgment and assertion is mistaken because it conceives of reference to a truth value as a separate act on which judgment and assertion have no bearing. If truth value designation is a component of making a judgment, but cannot be described independently from judgment and assertion, then Frege's attempt to drive a wedge between designating a truth value ($\text{— } p$) on the one hand, and the judgment that the thing so designated is identical with the True ($\text{⊢ } p$), will not succeed. Therefore, it is a mistake to think of judgment and assertion as containing a separate act of reference to a truth value.

By way of conclusion, I shall briefly respond to an objection that has been raised by one of Frege's most insightful scholars. In his discussion of Wittgenstein's analogy, Wolfgang Kühne (Kühne 2009a: 55ff.) suggests reading *Begriffsschriftsätze* as pairings of sentence questions and affirmative answers. The idea is taken from Frege's remarks about questions as a form of words that can be used to express a truth without asserting it (Frege 1918a: 62, Frege 1918b: 143 – 147), and it is launched against Hacker's negative verdict that there is no such form of words (Hacker 2001: 211). Kühne realizes that for grammatical reasons we cannot simply paraphrase the judgment that the Earth moves as "Is the Earth moves identical

with the true? Yes”, and that some kind of nominalization is needed. His proposal is to parse the judgment as follows: “Is the truth-value of the thought that the Earth moves identical with the True? Yes.” I consider it to the merit of this proposal that the paraphrase for the judgment stroke (Yes) applies to something containing a verb (moves) without rendering the content of the judgment assertoric, as it is embedded in the wordy nominalization “the truth-value of the thought that ...”.

However, I see no way of reconciling this proposal with Frege’s function-theoretic interpretation of the horizontal, according to which simple nouns can also be used as arguments (see section 3). Does it make sense to ask whether the truth value of the thought that 2 is identical with the True, if there is no such thing as the thought that 2? The hesitation at this point could be an indication that Wittgenstein is not wrong in claiming that assertables should contain a verb – the grammatical dilemma is hard to overcome. Apart from technical sophistry, Künne seems to be glossing over Frege’s remark that “a judgement is often preceded by questions” (Frege 1976: 7), because according to Künne’s own proposal, Frege should rather have made the general claim that judgments are *always* preceded by questions. However, if my interpretation of Wittgenstein’s criticism is conclusive, then Künne’s proposal seems to be grist to Wittgenstein’s mill. Not only is Künne’s analysis of judgments as pairings of questions and affirmative answers a clear manifestation of the two-stage model in terms of two separate speech acts; it also demonstrates how the speech act of asking a question already draws on the notion of truth.

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