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Short-run Risk, Business Cycle, and the Value Premium *

Yunhao He[†] Markus Leippold[‡]

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Abstract

We jointly explain the equity and value premium variations in a model with both short-run (SRR) and long-run (LRR) consumption risk. In our empirical analysis, we find that SRR varies with the business cycle, and it has a substantial predictive power for market excess returns and the value premium—both in-sample and out-of-sample. The LRR component also differs significantly from zero, and value stocks have a larger exposure to both LRR and SRR than growth stocks. To explain these patterns in asset returns, we propose an extended LRR model. The model can be solved using log-linear approximations with economically small errors.

JEL classification: C32, G12, E44

Key Words: Long-run and short-run consumption risk, value premium, business cycle, portfolio selection, stochastic covariance.

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[†]Yunhao He, Swiss Finance Institute (SFI) and University of Zurich, Department of Banking and Finance, Plattenstrasse 14, 8032 Zurich, Switzerland; email: yunhao.he@bf.uzh.ch.

[‡]Markus Leippold, University of Zurich, Department of Banking and Finance, Plattenstrasse 14, 8032 Zurich, Switzerland; email: markus.leippold@bf.uzh.ch.

The value premium refers to the phenomenon that stocks with lower price-to-fundamentals ratios will generate excess returns over those with high ratios. This differential in returns constitutes a puzzle because the return differential cannot be accounted for by CAPM, as documented in Fama and French (1992). A large body of research has tried to reconcile data with theory. A prominent example is the long-run risk (henceforth LRR) model proposed in Bansal and Yaron (2004), which can explain the magnitude of both the equity and value premium.[1] Through a constant leverage parameter, value stocks load more on the LRR component than growth stocks. Therefore, investors require compensation for bearing more LRR, thus generating the value premium.[2] However, the assumption of a constant leverage parameter is a serious drawback of the LRR approach because it fails to explain the variation of equity and value premiums over business cycles. Moreover, the model's design is complicated by the commonly found evidence that the equity premium is pro-cyclical while the value premium is counter-cyclical.[3]

Our paper makes two contributions. The first contribution is to construct non-parametric measures of short-run risk (SRR) to formally study the covariation with the transient consumption risk as indicators of the business cycle. These measures are motivated to capture the priced risk of transient consumption growth, which is overlooked in most LRR literature. The definition of SRRs stems from the specification of consumption and cash flow dynamics in the LRR framework, although they do not depend on the equilibrium solutions. Hence, the empirical SRR measures correspond precisely to the residual consumption risk apart from LRR. We define the SRR in dividends as the short-run covariance with the consumption growth. Similarly, we define the SRR in consumption growth as its short-run variance.

The SRRs fluctuate substantially with business cycles and can even switch sign: the SRR in value stocks appears counter-cyclical, while the SRR in growth stocks seems pro-cyclical. By running predictive regressions of future returns on the estimated SRRs, we find that the SRRs in consumption, growth stocks, and value stocks explain 17.6% of the variations in the future

[1] Other plausible explanations include the over-optimism of extrapolative investors (Bondt and Thaler, 1985), the growth options inherent in growth stocks (Zhang, 2005), cash flow duration (Lettau and Wachter, 2007) and disaster risk exposure (Tsai and Wachter, 2015).

[2] See e.g., Bansal, Dittmar, and Lundblad (2005a), Bansal, Kiku, Shaliastovich, and Yaron (2014), Parker and Julliard (2005), and Hansen, Heaton, and Li (2008).

[3] The counter-cyclicality of the value premium is provided, for example, in Lettau and Ludvigson (2001b), Petkova and Zhang (2005). Meanwhile, Koijen, Lustig, and Nieuwerburgh (2017) also find that during economic downturns, prices and dividend payouts of value stocks plunge, while those of growth stocks are less affected.

one-year market excess returns, and 11.5% of the variations in the future three-year return differentials in value and growth stocks (value-minus-growth returns henceforth). In particular, the SRR in consumption negatively predicts future market returns but positively predicts future value-minus-growth returns, which echoes the evidence that the value premium is counter-cyclical. The regression coefficients on the SRRs are statistically significant. These results are consistent with the interpretation that the predictive power of SRRs stems from the comovements of both SRRs and market returns with the business cycle.

The predictive power of SRR on future returns is not only statistically but also economically significant. Notwithstanding the criticism in Welch and Goyal (2008) that most predictive regressions cannot beat historical average out-of-sample (OOS), the predictive power of SRRs remains strong OOS. Based on predictive regressions, we construct a market-timing strategy that adjusts the positions on the market portfolio and the risk-free asset once every year. This strategy doubles the Sharpe ratio of the market excess returns.

A byproduct of LRR model motivated SRR measures is that we can jointly study SRR and LRR. In a generalized method of moments (GMM) estimation, the null hypothesis of no LRR is rejected at 99.9% significance level. We find the LRR is persistent but large in magnitude relative to consumption growth. The value stocks not only have more exposure to LRR than growth stocks, consistent with Bansal, Dittmar, and Lundblad (2005a) and Parker and Julliard (2005), but they also have larger SRRs than growth stocks.

The second contribution is to extend the LRR model to account for the relationship between SRRs and cyclical variation in the equity and value premium. We model an economy explicitly with a market portfolio and portfolios of growth and value stocks. Guided by Santos and Veronesi (2006, 2010) and Menzly, Santos, and Veronesi (2004), the market portfolio and the cross-section of stocks should be studied jointly to provide a consistent explanation for the stylized facts of asset returns. To account for the time-variation of the SRRs across business cycles, we model stochastic covariances explicitly as state variables.[4] The resulting dynamic covariance structure in the cross-

[4]We model stochastic covariances using a Wishart process. The theoretical foundations of Wishart processes are laid out in Bru (1991) and introduced to finance by Gourieroux and Sufana (2003). Buraschi, Cieslak, and Trojani (2008) subsequently use a Wishart covariance process to study the term structure of interest rates. For derivative pricing, we refer to Gourieroux and Sufana (2004) and Gruber et al. (2015); and for portfolio choice, see Buraschi et al. (2010). More recently, Cieslak and Povala (2016a) exploit the properties of the Wishart process to reflect a

section of assets is necessary to resolve the negative correlation between the market risk premium and SRR in consumption, which would otherwise be positive in a univariate volatility setting.

With the conventional log-linearization approximation whose errors are economically small, our extended LRR model can be solved in quasi-closed form up to Riccati equations and is analytically tractable. This allows us to calibrate the model to match the dynamics in the growth of consumption and dividends, the time-varying SRRs, and asset pricing patterns, such as the equity premium, the value premium, and the price-dividend ratios. Under the calibrations, the model matches market data reasonably well. In particular, the model replicates the predictive power of SRRs on the future market returns and value-minus-growth returns.

We also perform a series of robustness checks on our results. For the empirical studies, the results adopting alternative measures of cash flows by accounting for repurchases remain qualitatively the same. For the model, Pohl, Schmedders, and Wilms (2018) demonstrated that the potential errors induced by the log-linear approximation could be considerable. Consequently, we solve the model via the projection method and find that the errors do not materially affect the model's results. Furthermore, we present empirical results where the SRRs are constructed from the monthly industrial production index instead of monthly consumption. Nonetheless, the industrial production index cannot explain future excess returns. Our regression exercise reveals a fundamental difference in the nature of consumption and industrial production data, rather than measurement errors.

Our paper shares many features with Bansal and Yaron (2004), although with some differences. First, our model is specified in continuous time, but Bansal and Yaron (2004) introduces a discrete-time LRR model. While our parameters are defined similarly to conventional LRR literature, they should be interpreted in the continuous model. Second, Bansal and Yaron (2004) specify the growth rate dynamics of the monthly aggregated consumption, but we specify the growth rate dynamics of the annually aggregated consumption. Thus, in our paper, the growth rates of annually aggregated consumption can be represented by integrals, which is more natural in our continuous-time model. In contrast, in Bansal and Yaron (2004), the growth rates of annually aggregated consumption are approximated by the weighted average of monthly consumption growth rates. Our approach enables

time-varying correlation between short-rate expectations and term premia, which is a difficult feature to achieve with traditional exponential affine models.

us to estimate SRR nonparametrically by realized variances or covariances, which is not done in Bansal and Yaron (2004). However, our model parameters are calibrated to annual variables, while those in Bansal and Yaron (2004) are calibrated to monthly variables.

Bansal et al. (2005b) finds that the conditional volatility of consumption negatively predicts asset valuation ratios, which highlights the role of fluctuating economic uncertainty in asset markets. Consistent with these results, we also find that the SRR in consumption predicts future equity premia with a negative sign. Our paper additionally finds that the SRRs in value and growth stocks relate to asset valuations and that the valuation of the value-minus-growth portfolio is also exposed to the fluctuating economic uncertainty. Furthermore, we formally extend and derive an analytically tractable long-run risk model using log-linear approximations, such that the SRRs correspond precisely to the model as measures of fluctuating economic uncertainty.

Boguth and Kuehn (2013) find that the volatility of the dividend growth of value stocks is more sensitive to the volatility of consumption growth than that of growth stocks, which underscores the importance of transient consumption innovations on the value premium. Our paper differs in several aspects. First, instead of studying the loadings on consumption growth volatility, this paper proposes the time-varying SRR. Second, Boguth and Kuehn (2013) conduct contemporaneous Fama-Macbeth regressions on consumption volatility. While they were able to sort firms according to their exposure to consumption volatility, the value premium variation per se is not studied. Instead, our goal is to explain the variation in the equity and value premiums, for which we perform the predictive regressions using SRRs.

This paper is closely related to Li and Zhang (2017), who jointly study the cross-sectional returns with LRR and SRR components in cash flows. Our paper differs in several areas. First, Li and Zhang (2017) define the SRR component as the regression coefficient of the biannual moving average of consumption growth on the dividend growth, and the LRR component as the covariation between dividend growth and the moving average of consumption growth in the last decades. Meanwhile, our definitions of SRR and LRR is different and consistent with the LRR model. Second, Li and Zhang (2017) attributes the value premium to exposures on LRR, and the momentum returns to exposures on SRR. In contrast, our paper focuses on the implications of SRR to the variation in the market equity premium and value premium. Third, Li and Zhang (2017) simulates a large cross-section of firms and form portfolios on those firms. In contrast, we model growth and value stocks explicitly over time. With the conventional log-linear approximation in lRR literature, our model exploits the Wishart process's affine structure and admits analytically tractable solutions.

The rest of the paper is organized as follows. Section I describes the data source. In Section II, we define the SRR and LRR components within our model framework. We then use the empirical estimates of SRR to run predictive regressions, and we study the SRR and LRR jointly via GMM. In Section III, we introduce an extended LRR model that is flexible enough to account for the salient features of the data. We then proceed to calibrate the model to real data in Section IV. In Section V, we perform some robustness checks. Section VI concludes. All proofs are relegated to Appendix B.

I. Data Source

This section describes the data sources and summarizes the properties of returns, dividends, and price-dividend ratios. All nominal quantities are deflated by CPI-U, published by the U.S. Bureau of Labor Statistics (BLS). We approximate the market portfolio by the value-weighted index from the Center for Research in Security Prices (CRSP). Book-to-market portfolios are constructed in the same way as in Fama and French (1992). We use portfolio returns with and without dividends from the Kenneth R. French data library[5] to construct dividends. We also construct cash payouts adjusting for repurchases in the same way as Bansal et al. (2005a).[6]

The three book-to-market portfolios are the value-weighted stocks with the book-to-market ratio in the lower $0\% - 30\%$ percentiles (growth stocks), middle $30\% - 70\%$ percentiles and upper $70\% - 100\%$ percentiles (value stocks). We construct book-to-market portfolios every end of June by sorting stocks with their book values ratio from the end of the last fiscal year and market values from last calendar year.

We set the end of year price of a portfolio by its price in December of the year. Data on returns

[5] See `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`.

[6] We focus on portfolio dividends as the measure of portfolio cash-flow payout in the majority of this paper. However, in the robustness check section, we study alternative cash-flow measures where share repurchases are considered. To account for repurchases, we adjust monthly equity returns net of stock payouts by $\frac{P_{t+1}}{P_t}\min[\frac{n_{t+1}}{n_t}, 1]$, where n_t is the number of shares after adjusting for splits, stock dividends using the CRSP share adjustment factor.

and dividends are aggregated annually to keep them in line with annually updated macro variables, similar to Bansal and Yaron (2004) and Campbell and Cochrane (1999). The annual real log return is the sum of monthly real log returns. We calculate monthly dividends before seasonal adjustments from the difference of returns with and without dividends in the same way as Beeler and Campbell (2012). We adjust for dividends' seasonality by using the adjusted monthly dividend as the moving average of dividends in the previous 12 months. Dividend growth is calculated as the seasonally adjusted dividend in the current month divided by that in the previous month. To calculate the end-of-year price-dividend ratio, we divide the asset price by the sum of the last 12 months of unadjusted dividends. The nominal 3-month Treasury bill rate data are taken from CRSP Fama risk-free rates. Given that the future inflation is uncertain, we approximate the risk-free rate by the ex-ante real 3-month Treasury rate. Similar to Beeler and Campbell (2012), the ex-ante real 3-month Treasury rate is the fitted value by the regression of the ex-post real rate (deflated 3-month Treasury rate using realized inflation) to the nominal 3-month interest rate and the growth of CPI in the previous year.

II. Short-run and long-run risk

In this section, we formally define SRR and LRR and study their empirical properties.

A. *Identifying short-run risk*

To clarify the definition of SRR in our model, we start with a simplified long-run risk model for consumption and dividend growth dynamics. By denoting aggregated consumption by C_t and by D_t^i the dividend of asset i, we specify

$$\frac{dC_t}{C_t} = (\mu_c + X_t)dt + \sigma_{c,t} dB_t^c, \tag{1}$$

$$\frac{dD_t^i}{D_t^i} = (\mu_i + \phi_i X_t)dt + \sigma_{i,t} dB_t^i, \tag{2}$$

where μ_c and μ_i, are constants, and $\sigma_{c,t}$, and $\sigma_{i,t}$ are possibly time-varying volatilities. The Brownian motions B_t^c and B_t^i may be correlated. X_t is the LRR component in consumption and dividend growth, which is not correlated with B_t^c and B_t^i.[7] The leverage parameter is ϕ_i, which controls the exposure to long-run consumption risk in dividends.

The long-run consumption risk in Equations (1) and (2) has been the focus on the LRR literature. However, the transient consumption risk $\sigma_{c,t} dB_t^c$, which is also a priced risk in equilibrium models, has received relatively little attention. By ignoring the effect of transient consumption risk, we miss an important channel between macroeconomic states and asset prices. We need a measure of exposure to the transient consumption risk, similar to the leverage parameter for the LRR.

To capture the transient consumption risk, we define the short-run risk component in consumption, SRR^c, as the time-t realized integrated variance of consumption growth over one time period, i.e.,

$$\text{SRR}_{t-1,t}^c := \int_{t-1}^t \frac{dC_s}{C_s} \frac{dC_s}{C_s}. \tag{3}$$

Similarly, the short-run risk component in asset or portfolio i, SRR^i, at time t is defined as the realized integrated covariance:

$$\text{SRR}_{t-1,t}^i := \int_{t-1}^t \frac{dC_s}{C_s} \frac{dD_s^i}{D_s^i}. \tag{4}$$

For our analysis, we focus on the SRRs of growth stocks SRR^g, middle BM stocks SRR^{mid}, value stocks SRR^v, and the whole stock market SRR^m.

In our empirical analysis, we assume that one time period corresponds to one year. The above definitions of SRR reflect the (co)variation of the transient components in the consumption and dividend dynamics, as the following equations suggest:

$$\int_{t-1}^t \frac{dC_s}{C_s} \frac{dC_s}{C_s} = \int_{t-1}^t \sigma_{c,s}^2 ds; \tag{5}$$

$$\int_{t-1}^t \frac{dC_s}{C_s} \frac{dD_s^i}{D_s^i} = \int_{t-1}^t \sigma_{c,s} \sigma_{i,s} dB_s^c dB_s^i. \tag{6}$$

There are two advantages to this definition. First, we can estimate the SRRs from realized variances and covariances directly from data. Second, because the LRR component X_t only enters the drift terms of the consumption and dividend dynamics, it does not interfere with the estimation of the SRRs.

[7] We deliberately omit the specification for the dynamics of X_t at this stage because the definition and measurement of SRR do not depend on the dynamics of X_t.

We remark that we impose no structures on the covariation with transient consumption shocks $\sigma_{c,s}\sigma_{i,s}dB_s^c dB_s^i$. Apart from stochastic variances, the covariation of the Brownian motions in consumption and cash flows are also time-varying. Whereas in traditional LRR models, the instantaneous correlation between Brownian shocks is either constant or zero. This deficiency calls for a more advanced model of the stochastic covariance, which is proposed in Section III.

Given the availability of monthly consumption and dividend data, we have ample data to estimate the SRR empirically. However, we face one obstacle—monthly and quarterly aggregated consumption data are seasonally adjusted, whereas dividends are adjusted by taking the yearly moving average. Hence, contemporaneous shocks in dividends and consumption are not reflected in these adjusted time series, which leads to bias in the SRRs estimates. To circumvent this problem, we use as monthly consumption growth the growth rate of the 12-month moving-average of the monthly aggregated consumption, which has some advantages. First, it fits well our model specification in which C_t represents the aggregated consumption in the past year. In particular, under this construction, the sum of the monthly consumption growth rates is equal to the growth rate of the annually aggregated consumption. Second, most of the literature uses seasonally adjusted dividends by taking the 12-month moving average. Seasonalizing consumption in a similar way enables us to calculate the covariance between consumption and dividend growth rates more accurately. We refer the reader to Appendix C for more details about the construction of monthly consumption growth.[8]

Following Equations (3) and (4), we directly estimate SRRs from data:

$$\mathrm{SRR}^c_{t-1,t} = \sum_{k=0}^{k=11}\left(\Delta c_{t+kh,t+(k+1)h} - \frac{1}{12}\sum_{j=0}^{j=11}\Delta c_{t+jh,t+(j+1)h}\right)^2, \tag{7}$$

$$\begin{aligned}\mathrm{SRR}^i_{t-1,t} = \sum_{k=0}^{k=11}&\left(\Delta c_{t+kh,t+(k+1)h} - \frac{1}{12}\sum_{j=0}^{j=11}\Delta c_{t+jh,t+(j+1)h}\right) \\ &\times\left(\Delta d^i_{t+kh,t+(k+1)h} - \frac{1}{12}\sum_{j=0}^{j=11}\Delta d^i_{t+jh,t+(j+1)h}\right),\end{aligned} \tag{8}$$

where $h = 1/12$, $i = v, g$ denote value or growth stocks, and $\Delta c_{t,t+h} = \log(C_{t+h}/C_t)$.

[8]BEA only publishes seasonalized monthly consumption using X13-ARIMA-SEATS. In Appendix C, we also show this algorithm does not affect the 12-month moving average materially.

[Figure 1 about here.]

Figure 1 plots the evolution of SRRs over time. Panel A plots the time-varying SRRs in growth, middle and value stocks, and Panel B the SRRs in consumption. In Panel A, the spikes (troughs) of the SRRs in value (growth) stocks seem to coincide with NBER recorded recessions. This observation is consistent with Koijen et al. (2017). When the macroeconomic activity is low, value stocks strongly align with the business cycle and pay little dividends, but growth stocks perform relatively well. If consumption growth is a measure of economic activity, then the cash flows of value (growth) stocks should move in the same (opposite) direction of the consumption growth rates around recessions. In Panel B, the SRR in consumption shoots up around economic recessions. This observation is in line with the common conception that the consumption volatility is countercyclical.

B. Regression results

To study the link between the SRRs and the business cycle formally, we use SRRs to predict future returns at horizons most pertinent to the business cycle. If SRRs convey bad (good) news about the economy, they should predict negative (positive) future returns at business cycle horizons. Because the value premium is counter-cyclical, we expect the SRRs to have opposite implications for the future value-minus-growth returns.

The predictive regressions are of the following form:

$$r_{t,t+h} = \beta_0 + \beta_c \mathrm{SRR}^c_{t-1,t} + \beta_{cg} \mathrm{SRR}^g_{t-1,t} + \beta_{cMid} \mathrm{SRR}^{mid}_{t-1,t} + \beta_{cv} \mathrm{SRR}^v_{t-1,t} + \boldsymbol{\beta}'_z Z_t + \epsilon_{t,t+h}, \tag{9}$$

where SRR^c, SRR^g, SRR^{mid} and $\mathrm{SRR}^v_{t-1,t}$ are SRRs in consumption, growth, middle, and value stocks. Z_t is a vector of additional optional predictors, which are included to determine the robustness of SRRs in predictive regressions, and $\epsilon_{t,t+h}$ is the residual. The LHS of Equation (9) is either future market excess returns or future value-minus-growth returns.

To test whether the predictive power of SRRs in the regression (9) are already contained in the macroeconomic variables, we include additional macroeconomic variables as predictors. These variables include the approximate log consumption-wealth ratio (*cay*, Lettau and Ludvigson, 2001a), income-consumption ratio (I/C, Santos and Veronesi, 2006) and Cochrane-Piazessi factor (CP, Cochrane and Piazzesi, 2005). We also study the log price-dividend ratio $\log \frac{P}{D}$. Some of the macroeconomic variables are documented to relate to the LRR. If the predictive powers remain significant even with such macroeconomic variables, it suggests that SRRs capture new information not contained in the LRR.

The correlations of the independent variables are shown in Table I. The SRRs correlated with macroeconomic variables, which implies that SRRs contain information in macroeconomic states. The variables CP and $\log \frac{P}{D}$ are strongly correlated, and both are negatively correlated with the income-consumption ratio. These variables capture similar aspects of the business cycle: a positive economic outlook is associated with a large CP factor, a large price-dividend ratio, and a small income-consumption ratio, while for an adverse economic outlook, it is the opposite. The income-consumption ratio is negatively correlated with *cay* because both are related to the representative agent's consumption-wealth ratio. The *cay* is pro-cyclical, which is in line with the argument in Lettau and Ludvigson (2001a) that the representative agent consumes a larger share of her total wealth in anticipation of good portfolio returns. SRR^c is positively correlated with the income-consumption ratio and is negatively correlated with *cay* and the price-dividend ratio, which indicates that SRR^c is negatively correlated with the consumption-wealth ratio. The SRR in value stocks is negatively correlated with that in growth stocks, consistent with the observations that these two variables move in opposite directions during recessions. However, the business cycle risks captured by SRRs are not adequately represented in those macroeconomic variables, as suggested by the next section's regression results.

[Table I about here.]

B.1. In-sample predictability

Figure 2 shows the adjusted R^2 of the predictive regressions at different horizons using Equation (9). For future market excess returns, forecasting regressions using past consumption, growth and value SRRs has the best predictive power at the four-quarter horizon, where the R^2 is 17.6%. Beyond the business cycle horizons, predictability wears off. This supports our hypothesis that

SRRs capture business cycle risks, which do not persist over the long term. For the value-minus-growth returns, predictability increases over time, where R^2 continues to rise to 11.5% at 12-quarter horizon using consumption, growth, and value SRRs.

[Figure 2 about here.]

[Table II about here.]

[Table III about here.]

To test the significance of SRRs in the presence of other macroeconomic variables as predictors, we focus on the four-quarter horizon for the market excess returns and 12-quarter horizon for the value-minus-growth returns. The horizons are chosen to maximize predictability. Table II and Table III report the regression results using SRRs and macroeconomic variables as predictors.[9]

A large consumption and value stock SRR, or a small growth stock SRR, usually accompanies an adverse economic outlook in the future one-year, and the market excess return drops. The SRR of the middle bucket does not seem to have significant predictive power. The adjusted R^2s using SRRs are even more significant than those of in-sample constructed *cay* in both samples. If we use both the SRRs and *cay* in prediction, then R^2 would increase more, and *cay* would remain significant. This result suggests that SRRs are not redundant, even when given predictors such as *cay*. Other predictors—including $\log(\frac{P}{D})$, CP factor, and income-consumption ratio—manifest little forecasting power of the future market excess returns in the one-year horizon. Consistent with the negative sign of the income-consumption ratio, which is first documented in Santos and Veronesi (2006), SRR^m is positively associated with the future market excess returns.

To supplement this study, we also ran predictive regressions using univariate SRRs. Univariate regressions of the future market excess returns on SRRs gave coefficients similar to those in the multivariate regressions. This observation indicates that SRRs carry orthogonal information for

[9] We have further tested those predictive regressions using IVX estimation (Kostakis et al., 2014). The predictive regression on market excess returns is significant at the 99% level, and the regression on value-minus-growth returns is significant at the 90% level.

the equity risk premia. The invariance of regression coefficients in univariate and multivariate regressions provides additional confidence in our regression results' robustness. The finding that SRRs in consumption negatively correlate with future market excess returns seems counter-intuitive because conventional wisdom would suggest that the market risk premium is positively correlated with market return volatility. However, consumption volatility is not perfectly correlated with all of the covariance matrix components of the cross-section of assets. Indeed, the correlation between the SRR in growth stocks and the SRR in consumption is almost zero. In contrast, the correlation between the SRR in value stocks and the SRR in consumption is nontrivial. The regression results suggest that the information in the cross-section of SRRs plays a vital role beyond the univariate market volatility.

For the future value-minus-growth excess returns, the SRR in consumption is significant. The SRRs in value and growth stocks do not explain the variations in the value-minus-growth excess returns. The negative coefficient sign suggests the value premium is counter-cyclical, which is, for example, consistent with Zhang (2005). Indeed, the quarterly aggregated market excess returns and value-minus-growth returns have a negative correlation of -16.15%. The value-minus-growth returns tend to rise post-crisis and are typically associated with a spike in the SRRs in consumption. This observation is in line with Avramov et al. (2013), such that the value premium is mainly derived from survived distressed firms that are valued lower and bounce back harder post-crisis. Because recoveries last longer than recessions, the adjusted R^2 in forecasting the future value-minus-growth excess returns increases over time.

B.2. OOS predictability and market timing strategy

Welch and Goyal (2008) argue that most predictive regressions cannot beat the historical average in forecasting the OOS market excess returns. In contrast, we find that our predictive regressions using SRRs have out-of-sample explanatory power. This out-of-sample predictability gives rise to an out-of-sample market-timing strategy, which leads to an economically significant improvement in portfolio performance for mean-variance investors.

We consider the out-of-sample R^2, Sharpe ratio and the cumulated excess returns corresponding to five kinds of predictive regressions—four univariate regressions using SRRs in consumption, growth, middle and value stocks, respectively; and a multivariate regression using all SRRs given above except middle stocks—to forecast the future one-year market excess returns. At the start of each year, we estimate the regression coefficients using data from the previous 35 years.[10] We then compare our forecasts and the historical average to calculate out-of-sample R^2:

$$R^2_{OS} = 1 - \frac{\sum_{t=1}^{T} \left(r^e_{t,t+1} - \hat{r}^e_{t,t+1} \right)^2}{\sum_{t=1}^{T} \left(r^e_{t,t+1} - \bar{r}^e_t \right)^2} \tag{10}$$

where $\bar{r}^e_t$, measured at beginning of year t, is the historical average of the annual market excess returns in the past 35 years.

For each predictive regression, we construct a market-timing strategy. At the beginning of each year, the regression gives an out-of-sample estimate of the market excess return of the following year, and we set the estimate as the weight to adjust the position in the market equity premium.[11] To make returns in such zero net position strategies comparable to the market excess returns, we ex post scale weights of these strategies such that their returns have ex post the same volatility as the market excess returns.

Table IV reports the out-of-sample R^2s of predictive regressions and the Sharpe ratios of corresponding market-timing strategies. Note that scaling positions of a strategy ex-post do not affect the Sharpe ratio. We report the results in Panel A for sample 1959-2017 using more recent data. Sharpe ratios are calculated from monthly returns. We also list the annualized Sharpe ratios for the ease of reference. To test the OOS returns' significances in the market-timing strategies, we adopt the OOS F-test in (McCracken, 2007).[12] To gauge the performance of the SRR predictors relatively, we compare its OOS R^2 to those documented in Campbell and Thompson (2008). To make our results comparable to the analysis in Campbell and Thompson (2008), we additionally restrict our sample to 1959-2005 and present the results in Panel B. We recall that Campbell and Thompson (2008) report OOS R^2 for the subperiods 1956-1980 and 1980-2005. For convenience, we added the results from their Table 3 for dividend-price ratio (D/P), earnings-price ratio (E/P),

[10]We estimate the regression coefficients from a quarterly sample. To avoid using future information, we exclude observations after the beginning of last year.

[11]Given CRRA utility and constant variance in market excess returns, and supposing that the investor chooses from a risk-free asset and a market portfolio to invest, her position in the market portfolio is proportional to its estimated return.

[12]In particular, we choose the asymptotic ratio of out-of-sample and in-sample points π to be 0.4. The significance levels are taken from Table 5.

smooth earnings-price ratio (Smooth E/P), and dividend-price ratio growth (D/P Growth), since those turned out to have the best OOS predictive power.

[Table IV about here.]

The out-of-sample R^2s of predictive regressions using individual and all SRRs except those in middle BM stocks are positive in the 1959-2017 sample, which suggests that SRRs can robustly forecast the future one-year market excess returns. The OOS F-statistics are all significant at 95%-level except $\text{SRR}^{mid}_{t-1,t}$ (Mid). Compared with the Sharpe ratio of the market excess returns, the Sharpe ratio of any market-timing strategy based on consumption SRR is improved by about 60%, and the Sharpe ratio of the strategy using all SRRs is even doubled. The predictive power of SRRs is more pronounced in the 1959-2005 sample, where the R^2 for using SRR in consumption alone is more than 20%. Moreover, the F-statistics are significant at 99% for using SRRs in consumption, value, or a combination of consumption, value, and growth. As a comparison, the best performing predictor in Campbell and Thompson (2008) for the first subperiod, the dividend-price ratio, has an out-of-sample R^2s of 9.46% (in their unconstrained case) and 6.88% in the second subperiod (in the case of fixed coefficients).

[Figure 3 about here.]

Figure 3 plots the cumulative returns using different investment strategies starting at the end of 1995. A few comments are in order. First, our market-timing strategies have predicted the 2000 dot-com bubble and the 2008 financial crisis. Although we do not report the weights on market excess returns in the strategies, we find that the market-timing strategies based on SRRs actually short the market following the dot-com bubble. Second, Market-timing strategies adjust positions in the market, and the risk-free asset only once every year. Unlike many portfolio selection strategies, where the bid-ask spread could eat up a significant part of the profits, this market-timing strategy is almost free from such costs. Moreover, the coefficients on rolling predictive regressions always have the same sign as their in-sample counterparts. In summary, the predictive power remains strong, even in out-of-sample regressions, which leads to a consistent improvement in the asset allocation strategies for a mean-variance investor.

C. Estimating the LRR

In what follows, we study the properties of the LRR and the SRR jointly via GMM.[13] The results only depend on consumption and dividend growth rates and are thus independent of any assumption on the transient consumption component. Hence, the results in this section apply also to the continuous-time version of Bansal and Yaron (2004) model. Unlike the SRRs, the LRR component does not admit a time series of empirical estimates. However, with the help of SRR estimates, we can study the LRR component's properties in consumption growth dynamics via GMM.

Although X_t is highly persistent and changes little over shorter periods, X_t could potentially vary a lot from year to year. From Equation (1), we obtain the unconditional variance of the annually aggregated consumption growth as,

$$\begin{aligned}
\operatorname{Var}\left(\int_{t-1}^{t} \frac{dC_s}{C_s}\right) &= \operatorname{Var}\left(\int_{t-1}^{t} X_s ds\right) + \mathbb{E}\left(\left(\int_{t-1}^{t} \sigma_{c,s} dB_s^c\right)^2\right) + \mathbb{E}\left(\left(\int_{t-1}^{t} \sigma_{c,s} dB_s^c\right)\left(\int_{t-1}^{t} X_s ds\right)\right) \\
&= \operatorname{Var}\left(\int_{t-1}^{t} X_s ds\right) + \mathbb{E}\left(\int_{t-1}^{t} \sigma_{c,s}^2 ds\right) \qquad (11) \\
&= \operatorname{Var}\left(\int_{t-1}^{t} X_s ds\right) + \mathbb{E}(\mathrm{SRR}_{t-1,t}^c), \qquad (12)
\end{aligned}$$

where the cross-term between X_t and B_s^c is zero because they are uncorrelated. Thus, the variance of the annually aggregated consumption growth is the sum of the variance of integrated LRR and the expectation of SRR. Similarly, the contribution of LRR to the covariance between dividend and consumption growth is controlled by ϕ_i, as illustrated in Equation (13)

$$\operatorname{Cov}\left(\int_{t-1}^{t} \frac{dC_s}{C_s}, \int_{t-1}^{t} \frac{dD_s^i}{D_s^i}\right) = \phi_i \operatorname{Var}\left(\int_{t-1}^{t} X_s ds\right) + \mathbb{E}(\mathrm{SRR}_{t-1,t}^i) \qquad (13)$$

Motivated by Equations (12) and (13), we can study the variance of integrated LRR by GMM. At the end of each year, we can estimate the annually aggregated growth of consumption and

[13] The LRR literature has developed methods to identify the unobservable state variables, the LRR component, and the stochastic volatility. In Bansal et al. (2016), the two-state variables LRR and stochastic volatility are backed out from observed risk-free rate and market price-dividend ratio. However, we construct SRRs empirically directly from cash flows without pricing data and additional assumptions on instantaneous cash flows. Recently, Schorfheide et al. (2018) decomposes consumption dynamics by using a Bayesian approach. While the Bayesian approach identifies the persistent LRR and stochastic volatility directly from cash flows for Bansal and Yaron (2004) model, there is no non-trivial extension to time-varying covariance structure. Taking those into consideration, we choose the GMM to decompose consumption, and dividend cash flows like in Bansal et al. (2016), but additionally includes SRRs from realized covariance estimators.

dividends $\Delta c_{t-1,t} := \int_{t-1}^{t} \frac{dC_s}{C_s} ds$, $\Delta d_{t-1,t}^{i} := \int_{t-1}^{t} \frac{dD_s^i}{D_s^i} ds$, and SRRs as the realized variances or covariances. For notational brevity, we denote the variance of the annually integrated LRR and the mean of SRRs in consumption by

$$\sigma_X^2 := \mathrm{Var}\left(\int_{t-1}^{t} X_s ds\right), \quad \mu_{\mathrm{SRR}^i} := \mathbb{E}(\mathrm{SRR}_{t-1,t}^i) \tag{14}$$

Hence, we can formulate the GMM according to the following moment conditions:

$$\mathbb{E}(\Delta c_{t-1,t}) = \mu_c, \qquad \mathbb{E}(\Delta d_{t-1,t}^i) = \mu_i, \tag{15}$$

$$\mathbb{E}(\Delta c_{t-1,t}^2) = \mu_c^2 + \sigma_X^2 + \mu_{\mathrm{SRR}^c}, \qquad \mathbb{E}(\Delta d_{t-1,t}^i \Delta c_{t-1,t}) = \mu_c \mu_i + \phi_i \sigma_X^2 + \mu_{\mathrm{SRR}^i}, \tag{16}$$

$$\mathbb{E}(\mathrm{SRR}_{t-1,t}^c) = \mu_{\mathrm{SRR}^c}, \qquad \mathbb{E}(\mathrm{SRR}_{t-1,t}^i) = \mu_{\mathrm{SRR}^i}. \tag{17}$$

where $i = v, g, mid, m$ represents the value, growth, mid, or market portfolio.

[Table V about here.]

The GMM estimation results are reported in Table V. We test all of the parameters with the null hypothesis that it equals zero against the alternative that it is larger than zero. The σ_X^2 is significantly different from zero at the 90.0%-level, which suggests the existence of a nontrivial LRR component. Consistent with the previous literature, such as Bansal et al. (2005a), the value stocks load more LRR than the growth stocks. The leverage parameter of value stocks ϕ_v is significantly larger than zero at 99%-level, but the leverage parameter of growth stocks ϕ_g is not significantly larger than 0. Apart from LRR, our paper also identifies that value stocks have higher SRR than the growth stocks, with the mean of SRR in value stocks significantly larger than zero at the 99%-level.

[Table VI about here.]

To further study the properties of the LRR, we summarize the statistics of the SRRs and consumption growth in Panel A of Table VI.[14] The means of SRRs are smaller than the covariances between the annually aggregated consumption and cash flows growth rates, which confirms the existence of LRR in consumption and dividends of book-to-market portfolios. The first-order

[14] The middle portfolio is excluded since it is not economically interesting.

autocorrelation of the monthly consumption growth rates over the whole sample is 92.4%. The high persistence could be explained by the persistent LRR component. The autocorrelation of the annually aggregated consumption growth rates is 47.9%, which is smaller than the autocorrelation for the monthly growth rates because the LRR X_t varies more over a longer period. Given the almost constant LRR component within a year, within year autocorrelation of monthly aggregated growth rates almost removes the LRR part as the mean and only considers the transient component of the consumption growth. Therefore, within-year first-order autocorrelation of monthly aggregated growth rates should be smaller. Indeed, the first-order autocorrelations of the monthly aggregated consumption growth rates calculated within each year are small and volatile, with the mean 35.5% and the standard deviation 26.5%.

To verify that our approach does not falsely detect a persistent and large LRR component, we simulate consumption growth processes under different assumptions regarding the LRR component. Our results are summarized in Panel B of Table VI. Under a persistent and large LRR component, the simulated consumption growth process has similar patterns in the statistics: the variance of the annually aggregated growth rates is larger than the sample mean of the SRRs, and the GMM estimation rejects the LRR component at zero. However, if the persistence of LRR component is zero or the variance of LRR component is zero such that there are only transient shocks, then the SRRs are almost as large as the variance of annually aggregated consumption growth, and the GMM estimation cannot reject the variance of persistent LRR at zero. Further details about the simulation exercise can be found in Appendix D.

III. Theoretical model

In this section, we introduce the model, and we derive solutions to generate the patterns in asset prices and SRRs.

A. *Model setup*

We formulate our economy in continuous time, and we equip our representative agent with the recursive utility function, as defined in Duffie and Epstein (1992). We depart from the previous literature on LRR models in how we incorporate fluctuating economic uncertainty.

To model the covariance structure of the transient shocks in consumption and dividend growth, we impose a matrix-valued Wishart process given by

$$d\Sigma_t = \left(kQQ' + M\Sigma_t + \Sigma_t M'\right) dt + \sqrt{\Sigma_t} dB_t^\sigma Q + Q' d\left(B_t^\sigma\right)' \sqrt{\Sigma_t}, \tag{18}$$

where $B_t^\sigma \in \mathbb{R}^{n\times n}$ is a matrix of independent Brownian motions. The constant matrices $M \in \mathbb{R}^{n\times n}$ and $Q \in \mathbb{R}^{n\times n}$ control the mean reversion and volatility of the Wishart process.[15]

To maintain parsimony, we fix the long-term mean for Σ_t to kQQ' and we set the scalar $k = n + 1$.[16]

As in Bansal and Yaron (2004), we let both dividend and consumption growth be characterized by a persistent LRR component X_t, which follows a mean-reverting process with stochastic volatility,

$$dX_t = -\alpha X_t dt + \underbrace{\sqrt{\delta_x' \Sigma_t \delta_x}}_{\in\mathbb{R}} dB_t^X, \tag{19}$$

where α controls the speed of mean-reversion, $\delta_x \in \mathbb{R}^n$ is a constant vector, and $B_t^X \in \mathbb{R}$ is a Brownian motion. Note that the volatility of the transient shock $\sqrt{\delta_x' \Sigma_t \delta_x}$ is univariate, despite being a function of the stochastic matrix $\Sigma_t \in \mathbb{R}^{n\times n}$.

Our economy models n portfolios jointly. Each portfolio pays out dividends D_t^i, $i = 1, \ldots, n$ with the following dynamics,

$$\frac{dD_t^i}{D_t^i} = (\mu_i + \phi_i X_t)dt + \underbrace{\delta_i' \sqrt{\Sigma_t} dB_t}_{\in\mathbb{R}} + \sigma_i dB_t^i, \tag{20}$$

where $B_t \in \mathbb{R}^n$ is a vector of Brownian motions shared by all firms, B_t^i is univariate Brownian motion for firm i, σ_i is the volatility of firm-specific shock, μ_i measures the mean of firm i's dividend growth process, $\delta_i \in \mathbb{R}^n$ is a constant vector, and ϕ_i measures its loading on the LRR component X_t.

To generate a time-varying correlation between consumption growth and dividend growth, we link the stochastic covariance matrix to the consumption process C_t. We assume that C_t has the

[15] To guarantee stationarity, we assume M to be negative definite. For $Q \in \mathbb{R}^{n\times n}$, we impose symmetry and positive definiteness to reduce the number of parameters in our estimation. These restrictions on Q are without loss of generality.

[16] This is a sufficient condition for Σ_t to stay positive definite, see Mayerhofer et al. (2011).

following dynamics:

$$\frac{dC_t}{C_t} = (\mu_c + X_t)dt + \underbrace{\delta_c' \sqrt{\Sigma_t} dB_t}_{\in \mathbb{R}} + \sigma_{c,t} dB_t^c, \tag{21}$$

where $B_t^c \in \mathbb{R}$ is a Brownian motion independent of B_t, μ_c is the mean consumption growth rate, and the constant vector $\delta_c \in \mathbb{R}^n$ together with Σ_t controls the loading on the transient component of consumption growth. Our representative agent may not generate income solely from dividends, but may also generate income from other sources, such as labor. Hence, we add an additional source of risk in the consumption growth dynamics, $\sigma_{c,t} dB_t^c$, which is not spanned by asset markets. We specify

$$\sigma_{c,t}^2 = \bar{\sigma}_c^2 + \mathrm{Tr}\left(\chi_c \Sigma_t\right), \tag{22}$$

where $\mathrm{Tr}(\cdot)$ denotes the trace of a square matrix. We assume that the Brownian motions B_t, B_t^X, B_t^i and B_t^c are mutually independent.

Our specifications in Equations (20) and (21) are consistent with the LRR framework in Equations (1) and (2). However, we extend the LRR model with a Wishart process, which models the multivariate stochastic volatility structure for consumption and dividend growth rates. In particular, we have

$$\mathrm{Var}_t\left(\frac{dC_t}{C_t}\right) = \mathrm{Tr}(\delta_c \delta_c' \Sigma_t)dt + \sigma_{c,t}^2 dt = \mathrm{Tr}\left((\delta_c \delta_c' + \chi_c)\Sigma_t\right) dt + \bar{\sigma}_c^2 dt, \tag{23}$$

$$\mathrm{Cov}_t\left(\frac{dC_t}{C_t}, \frac{dD_t^i}{D_t^i}\right) = \mathrm{Tr}(\delta_i \delta_c' \Sigma_t)dt, \quad i = 1, \ldots n. \tag{24}$$

The vectors δ_c and δ_i determine how much the variances and covariances load on the different elements of the matrix Σ_t. Furthermore, we can construct the theoretical counterparts of Equations (3) and (4) to accommodate the time-varying SRR in closed-form:

$$\mathrm{SRR}^i = \int_{t-1}^{t} \underbrace{\delta_c' \Sigma_s \delta_i}_{\in \mathbb{R}} ds, \tag{25}$$

$$\mathrm{SRR}^c = \int_{t-1}^{t} \left(\mathrm{Tr}\left((\delta_c \delta_c' + \chi_c)\Sigma_t\right) + \bar{\sigma}_c^2\right) ds. \tag{26}$$

The SRR in asset dividends only captures the common shocks between consumption and dividends through δ_c and δ_i. Nonetheless, the SRR in consumption includes an additional component from $\sigma_{c,t}$ as defined in Equation 22. In models with univariate dividend and consumption growth variances, a larger consumption growth variance accompanies larger expected returns. In our model, dividend growth has a stochastic covariance structure. The loadings χ_c allow the consumption volatility to load flexibly on the components in the cash flow covariance matrix Σ_t, which is crucial in replicating the negative relationship between SRR in consumption and future asset returns.

B. *Model solutions*

We follow Duffie and Epstein (1992) and assume that the representative agent has recursive preferences. The results in this section are subject to a log-linear approximation conventional in LRR literature.

The value function J satisfies

$$J_t = \max_{C_s} \mathbb{E}_t \left[\int_t^T f(C_s, J_s) ds \right], \tag{27}$$

where

$$f(C_t, J_t) = \begin{cases} \beta\theta J_t \left[\left(\frac{C_t}{((1-\gamma)J_t)^{1/(1-\gamma)}} \right)^{1-\frac{1}{\psi}} - 1 \right] & \text{if } \psi \neq 1,\ \theta = \frac{1-\gamma}{1-1/\psi}, \\ \beta(1-\gamma) J_t \log \left(\frac{C_t}{((1-\gamma)J_t)^{1/(1-\gamma)}} \right) & \text{if } \psi = 1. \end{cases} \tag{28}$$

where γ denotes the risk aversion coefficient and ψ the intertemporal elasticity of substitution. We assume that the representative agent prefers early resolution of risk, such that $\gamma > 1$ and $\psi > 1$.[17] To solve the model, we make use of the log-linear approximation as in Campbell and Shiller (1988). Thus, we obtain a quasi-closed-form solution up to generalized continuous-time algebraic Riccati equations (CARE).[18]

PROPOSITION 1: *The value function is given by*

$$J(W_t, X_t, \Sigma_t) = \exp\left(A_0 + A_1 X_t + \mathrm{Tr}(A_2 \Sigma_t)\right) \frac{W_t^{1-\gamma}}{1-\gamma}, \tag{29}$$

and the consumption-wealth ratio is given by

$$\frac{C_t}{W_t} = \beta^{\psi} \exp(A_{0a} + A_{1a} X_t + \mathrm{Tr}(A_{2a} \Sigma_t)), \tag{30}$$

[17] We discuss the case $\psi = 1$ in Appendix B.

[18] The potential errors introduced by the log-linear approximation have come under scrutiny in a recent paper by Pohl et al. (2018). We perform some robustness checks in Section V and we find that, for our setup, the errors induced by log-linear approximations are negligible.

where $A_{ka} = \frac{1-\psi}{1-\gamma} A_k$, *for* $k = 0, 1, 2$. *Furthermore,*

$$\begin{aligned} A_0 &= \frac{1}{g_1 \psi}\left(\theta(g_1 - g_1 \log g_1 + g_1 \psi \log \beta) - \beta\theta + \psi \operatorname{Tr}(A_2 \Omega\Omega') + (1-\gamma)\mu_c - \frac{(1-\gamma)\gamma}{2}\bar{\sigma}_c^2\right) \quad (31) \\ A_1 &= \frac{1-\gamma}{(g_1+\alpha)\psi}, \end{aligned}$$

where $g_1 = \exp(\mathbb{E}(c_t - w_t))$, $c_t := \log C_t$, $\theta = \frac{1-\gamma}{1-1/\psi}$ *and* $w_t := \log W_t$. *The term* g_1 *and the constant positive semidefinite symmetric matrix* A_2 *need to be solved by generalized CARE.*

Some comments are in order here. First, the generalized CARE admits a positive semidefinite solution with reasonable computational efficiency. Hence, although some numerical calculations are required, the model is still highly tractable. Second, with $\gamma > 1$ and $\psi > 1$, we have $A_{1a} < 0$. Therefore, following the standard LRR model's interpretations, the representative agent reacts to positive news in long-term consumption growth X_t by consuming less out of her wealth portfolio, thereby smoothing consumption. Consequently, the substitution effect dominates the income effect. Third, A_2 is positive semidefinite. Therefore, the consumption-wealth ratio increases when an overall increase in variance occurs, similar to Bansal and Yaron (2004). However, because in our model, each element of the stochastic covariance matrix could affect the consumption-wealth ratio through A_2, elements of the covariance matrix have mixed effects on the consumption-wealth ratio. Finally, the persistent component X_t on the consumption-wealth ratio A_1 increases with the persistence of X_t, which is inversely related to the mean reversion coefficient α.

PROPOSITION 2: *The state price deflator follows the dynamics*

$$\frac{d\pi_t}{\pi_t} = -r_f dt - \Lambda dB_t - \Lambda^c dB_t^c - \Lambda^X dB_t^X - \operatorname{Tr}(\Lambda^\sigma dB_t^\sigma), \quad (32)$$

with

$$\Lambda = \gamma \delta_c' \sqrt{\Sigma_t}, \quad \Lambda^c = \gamma \sigma_{c,t}, \quad \Lambda^X = -\frac{1-\psi\gamma}{1-\gamma} A_1 \sqrt{\delta_x' \Sigma_t \delta_x}, \quad \Lambda^\sigma = -2\frac{1-\psi\gamma}{1-\gamma} Q A_2 \sqrt{\Sigma_t}. \quad (33)$$

Furthermore, the risk-free interest rate is given as

$$r_f = r_0 + r_x X_t + \operatorname{Tr}(r_\Sigma \Sigma_t), \quad (34)$$

where the expressions for the coefficients r_0, r_x, *and* r_Σ *are given in equations* (B.26) *to* (B.28).

From Equation (33), we can identify four components for the market price of risk in our model.

The first two components, Λ and Λ^c, are the market prices of risk on transient consumption shocks, where Λ^c arises from the additional source of risk that is not spanned by the asset market. These two components are proportional to the risk aversion coefficient γ, and they do not depend on the intertemporal elasticity of substitution ψ. The third component, Λ^X, is the market price of risk for exposure to innovations in LRR. The fourth component, Λ^σ, represents the market price of risk for innovations in the Wishart covariance process.

Our specification of the market price of risk extends the previous LRR models in that we account not only for the variance risk as in Zhou and Zhu (2015) but we also account for the covariance risk. The off-diagonal elements of the covariance matrix are needed to match the time-varying returns in the assets' cross-section.

Our LRR model generates a risk-free interest rate in Equation (34) as an affine function of the LRR component X_t and elements of Σ_t. This specification of the risk-free rate is similar to the term structure models in Buraschi et al. (2008) and Cieslak and Povala (2016b). However, our focus is on the dynamics of the cross-section of equity returns instead of the risk-free rate term structure.[19]

PROPOSITION 3: *The dividend-price ratio for asset i has the following form*

$$\frac{D_t^i}{P_t} = \exp\left(A_{0i} + A_{1i}X_t + \mathrm{Tr}(A_{2i}\Sigma_t)\right), \tag{35}$$

A_{0i} and A_{1i} are given by

$$A_{0i} = \frac{1}{g_{1i}}\left(-\mu_i + \mathrm{Tr}(A_{2i}\Omega\Omega') - g_{0i} + r_0\right), \tag{36}$$

$$A_{1i} = -\frac{\phi_i - \frac{1}{\psi}}{g_{1i} + \alpha}, \tag{37}$$

where $g_{1i} = \exp(\mathbb{E}(d_t^i - p_t^i))$, $d_t^i = \log D_t^i$, $p_t^i = \log P_t^i$, and A_{2i} is a symmetric positive semidefinite matrix of coefficients. The expressions for g_{1i} and A_{2i} need to be solved by generalized CARE. The equity risk premium for asset i is

$$-\frac{d\pi_t}{\pi_t}\frac{dP_t^i}{P_t^i}/dt = \gamma\delta_c'\Sigma_t\delta_i + \psi\left(1 - \frac{1}{\theta}\right)A_1A_{1i}\delta_x'\Sigma_t\delta_x + 4\psi\left(1 - \frac{1}{\theta}\right)\mathrm{Tr}(QA_{2i}\Sigma_tA_2Q). \tag{38}$$

[19] Note that if $\psi\gamma = 1$, then the utility function reduces to CRRA form. Under CRRA, uncertainty in the future utility arising from uncertainty in the consumption growth process is no longer priced, so Λ^X and Λ_σ are zero. Furthermore, we would obtain $r_\Sigma = 0$, which shuts down the major channel of variation in risk-free interest rate because X_t moves only slowly.

A_{1i} and A_{2i} play similar roles to A_1 and A_2. If $\phi_i > 1/\psi$, then we have $A_{1i} < 0$ and an increase in the LRR component X_t drives up the valuation of asset i. In other words, the substitution effect dominates the income effect. Because A_{2i} is positive semidefinite, an increase in overall volatility in Σ_t drives down the valuation of asset i.

The equity risk premium in Equation (38) comprises three parts. The first part is determined by the covariance of dividend growth and consumption growth, scaled by the risk aversion coefficient. A higher covariance implies a higher risk premium. The second part is the contribution of the variation in the LRR component. Higher LRR volatility or intertemporal elasticity of substitution leads to a larger equity premium. As Bansal and Yaron (2004) show, sufficiently high persistence in the LRR component dynamics helps generate a large equity premium. Under our assumption that the representative agent prefers early resolution of risk, $\gamma > 1$ and $\psi > 1$. Hence, a high persistence (a low value for α), leads to the large product $A_1 A_{1i}$. Furthermore, because Σ_t is positive definite, $\delta_x' \Sigma_t \delta_x$ is positive. Therefore, the risk premium part arising from long-run risk is always positive. The third part arises from the exposure to the innovations in transient consumption shocks, which captures the compensation for the SRR. In models without stochastic covariation, the correlation between shocks is constant, and the only variation in this part stems from the stochastic volatility, which lacks the flexibility to model the compensation from the SRR. In contrast, the Wishart process enables SRR to manifest its importance in the risk premium.

To avoid over-parametrization, we impose additional restrictions on the model. We assume that there are three portfolios in the economy: the market portfolio, the portfolio of growth stocks, and the portfolio of value stocks. We further assume that the stochastic covariance matrix is a 2×2 Wishart process, which has three free components because any covariance matrix is symmetric. Under these restrictions, the risk premium for asset i in Equation (38) is the linear combination of the three components in the Wishart process. Recall from Equations (23) and (24) that the instantaneous variance of consumption growth and the instantaneous covariance between consumption growth and dividend growth are linear combinations of the components in the Wishart process Σ_t and a constant. Hence, if $\left(\frac{dC_t}{C_t}\right)^2$, $\left(\frac{dC_t}{C_t}\frac{dD_t^g}{D_t^g}\right)$ and $\left(\frac{dC_t}{C_t}\frac{dD_t^v}{D_t^v}\right)$ are linearly independent, then any risk premium can be written as their combinations and a constant part. Therefore,

$$\begin{aligned} E_t(r^i_{e,t}) &= \gamma\delta_c'\Sigma_s\delta_i + \psi\left(1-\frac{1}{\theta}\right)A_1A_{1i}\delta_x'\Sigma_s\delta_x + 4\psi\left(1-\frac{1}{\theta}\right)\mathrm{Tr}(QA_{2i}\Sigma_sA_2Q) \\ &= \beta_0 + \beta_c\left(\frac{dC_t}{C_t}\right)^2 + \beta_{cg}\left(\frac{dC_t}{C_t}\frac{dD^g_t}{D^g_t}\right) + \beta_{cv}\left(\frac{dC_t}{C_t}\frac{dD^v_t}{D^v_t}\right) \end{aligned} \tag{39}$$

for some β_0, β_c, β_{cg}, β_{cv} chosen to match the four dimensions in the 2×2 Wishart process and the constant. Hence, the model implies that the equity risk premia of an asset can be explained by the instantaneous variance of the consumption growth and the instantaneous covariance between the growth of consumption and the dividends in growth stocks and value stocks.

Equation (39) shows that the representative agent takes into account the time-varying covariance in the cash flows of assets, which leads to the time-varying equity risk premia. Given that SRRs are realized variances and covariances, the model could replicate the relation between SRRs and asset returns.

PROPOSITION 4: *The model-implied regression coefficients $\beta_0, \beta_c, \beta_{cg}, \beta cv$ can be derived in closed-form for predictive regressions of future returns on SRRs*

$$r_{t,t+\frac{Q}{4}} = \beta_0 + \beta_c SRR^c_{t-1,t} + \beta_{cg} SRR^g_{t-1,t} + \beta_{cv} SRR^v_{t-1,t} + \epsilon_{t,t+\frac{Q}{4}}, \tag{40}$$

where Q denotes quarters, $SRR^c_{t-1,t}$ and $SRR^g_{t-1,t}$ ($SRR^v_{t-1,t}$) are defined in Equations (3),(4), *$\epsilon_{t,t+\frac{Q}{4}}$ is the residual. We refer to Section B.4 for further details.*

IV. Quantitative model results

In this section, we aim to calibrate the parameters to match moments in the sample from 1959–2017,[20] and we will study the quantitative results implied by our model. More details about the calibrations and the derivations of model-implied moments are given in Appendix B.6.[21]

To reflect the dynamics of cash flows, we match the unconditional mean, the first-order autocorrelation, and the volatility of growth rates of consumption and dividends of value stocks, growth stocks, and the market portfolio. To ensure that our model captures asset returns patterns, we

[20] Monthly aggregated consumption data is not available before 1959.

[21] We remark that we only use the value and growth portfolios for calibration. As a robustness check and for completeness, we also report the results when we also include the middle portfolio in the calibration exercise. These results are summarized in Appendix F.

also match the unconditional mean, the volatility, and the first-order autocorrelation of the risk-free rate, the aggregated market equity premia, and equity premia of value and growth stocks, as well as their price-dividend ratios. In particular, to verify the additional pricing channel of SRR, we match the theoretical moments of SRRs with sample moments of SRRs. Moreover, we match regression coefficients in predictive regressions of future returns on SRRs. The model is calibrated by matching these quantities jointly.

For a Wishart process of dimension n, Q has $n(n+1)/2$ free parameters while M has n^2 parameters. To reduce the number of parameters, we set $n = 2$. To avoid over-identification and further reduce the number of parameters, w.l.o.g. we specify[22]

$$\delta_m = (\delta_1^m, \delta_2^m)', \; \delta_g = (1, 0)', \; \delta_v = (0, 1)', \tag{41}$$

where δ_m, δ_g, δ_v correspond to the market, growth and value portfolio, respectively. We restrict M to be lower triangular, which further reduces the number of parameters. Table VII shows our baseline calibration.[23]

[Table VII about here.]

A. *Consumption and cash-flow growth*

First, we study the dynamics of the growth rates of consumption and dividends under the joint calibration. In Panel A of Table VIII, we report the mean, standard deviation, and first-order autocorrelation of the growth rates of the annually aggregated consumption and dividends. Most model-implied first moments lie within one standard deviation from their sample counterparts. The growth rates' volatilities implied by the model are also close to the sample's realized volatilities. Our model also generates realistic first-order autocorrelations for dividend growth rates, where the model-implied values lie within one standard deviation from the sample counterparts. This model replicates the pattern that the first-order autocorrelation of consumption growth is larger than

[22]This restriction is without loss of generality. For a Wishart process Σ_t with mean reversion M and scale of shocks Q, $L\Sigma_t L'$ is still a Wishart process with LML and QL' replacing M and Q. If δ_i are arbitrary, set $L' = (\delta_g, \delta_v)$ and we transform Wishart process so that our restriction in Equation 41 holds without changing any model implications.

[23]R package *DEoptim* (Mullen et al., 2011) is used to estimate the parameter values.

those of dividend growth, although the model-implied $AC1(\Delta c)$ seems larger than the empirical value.

[Table VIII about here.]

Our baseline calibration has $\alpha = 0.087$, which translates into a monthly persistence of the consumption growth rate at 92.3%. The persistence in the monthly consumption growth is comparable to the persistence at 97.8% in the BY model. The Wishart covariance matrix is mean reverting and is controlled by the mean reversion matrix M. M has eigenvalues $-0.163, -0.0875$, so the Wishart covariance matrix has a monthly persistence between 98.7% and 99.3%. While the half-life notion is not immediately applicable to the mean reversion matrix M, the half-lives implied from the eigenvalues of M are between 2 and 7 years. The Wishart covariance matrix's variation cycle has a similar length as a business cycle, which substantiates our claim that SRR is sensitive to business cycle risks. We match both the correlations of the annually aggregated growth rates and SRRs, see Panel B of Table VIII. The results in this section differ from Section II.C in that parameters are calibrated jointly to asset returns and SRRs and correlations.

The leverage parameter ϕ^v of value stocks is estimated to be 8.17, which is much higher than that of growth stocks $\phi^g = 4.68$. Growth stocks have less exposure to the LRR. The differential exposure to the LRR affects the correlations with the growth rates of the annually aggregated consumption. In our sample, the correlation between the growth rates of the annually aggregated consumption and dividends of value stocks $\mathrm{Corr}(\Delta c, \Delta d^v)$ is 0.588, while that between consumption and growth stocks $\mathrm{Corr}(\Delta c, \Delta d^g)$ is 0.323. The higher loadings replicate the higher correlation with value stocks on the LRR in the dividend growth rate dynamics. Our model generates SRRs similar to their empirical levels. We find that most variations consumption growth comes from LRR, whereas SRRs account for most variations in dividend growth. Although the volatility of the annually aggregated consumption's growth rates is 0.865%, the mean of SRR is only about $\sqrt{0.0283}\% \approx 0.168\%$.

B. Asset Returns

In this part, we study the model-implied asset returns, particularly the channels through which the agent's preferences and consumption risks determine asset prices. In Table IX, we report the model-implied and sample moments of asset returns. The model replicates the risk-free rate dynamics closely, matching its mean, volatility, and first-order autocorrelation. The model generates realistic equity risk premium levels averaging 5.461%, compared with 4.803% in data. The model further replicates excess returns in growth and value stocks, hence generating a significant value premium. The model captures the low persistence in stock returns. The model further generates realistic levels of volatilities. The volatilities of annualized excess returns in market, growth, and value portfolios in the model match their empirical values closely. Moreover, the model produces realistic price-dividend ratios, where growth stocks have a much higher valuation ratio (price-dividend ratio) than value stocks.

[Table IX about here.]

Table X decomposes the risk premia in the market portfolio, growth, and value stocks. The risk premia arise from different channels. According to Proposition 3, the risk premia can be decomposed into the compensation for the instantaneous covariance between the growth rates of consumption and dividends, the LRR, and the SRR. From Table X, both the SRR and the LRR matter for asset returns. However, the SRR accounts for most of the risk premia.

[Table X about here.]

A few comments are in order here. First, in the baseline calibration, we get $\gamma = 2.4899$. Hence, the risk aversion lies in a reasonable range between one and ten documented in Mehra and Prescott (1985). Moreover, the risk aversion γ is smaller than in Bansal and Yaron (2004). The EIS ψ is $\psi = 1.0325 > \frac{1}{\gamma}$, so that the representative agent has a preference for the early resolving of risk. The EIS is also smaller than in Bansal and Yaron (2004). Consequently, the representative agent requires less compensation for the LRR. For the model to generate realistic levels of the risk premia in the cross-section, the compensation for the SRR must be sufficiently large. Second, although the leverage parameter of value stocks ϕ^v is larger than growth stocks ϕ^g, the difference in LRR alone is not sufficient to account for the value premium. Therefore, SRRs in value and growth stocks contribute a significant proportion to the observed value premium.

C. Predictability

In Section II.B, we demonstrated that SRR could predict the future market excess returns and value-minus-growth returns. The predictability could be partially explained by the business cycle: SRR in consumption is counter-cyclical, and SRR in growth (value) stocks is pro-cyclical (counter-cyclical). Our model replicates the link between the SRR and asset returns. In the data, the predictability for the market excess returns peaks at the four-quarter horizon—also, the predictability for the value-minus-growth returns increases in horizons of up to 12 quarters.

Our model focuses on those horizons where SRR has the most predictability. In addition, we incorporate the correlation structure between the cyclical SRRs documented in Section II.A. The loading $\delta_c\delta_c' + \chi_c$ of transient consumption volatility on the components corresponding to growth stocks are negative, while those on the components corresponding to value stocks are positive. These loadings mimic the small correlation between SRR in consumption and growth stocks and the large correlation between SRR in consumption and value stocks. While a univariate volatility structure cannot generate a negative correlation between market risk premium and volatility, our model with a dynamic covariance structure resolves the negative correlation. [24] The details for the derivations of the model-implied regression coefficients are given in Appendix B.4.

[Table XI about here.]

Table XI reports the estimated coefficients of the predictive regressions. Our model implies a negative coefficient using the SRR in consumption to predict the future market excess returns, which lies within one standard error from the sample estimate at the four-quarter horizon. The SRR in growth stocks has a positive model-implied coefficient in predicting the market excess returns, albeit smaller than the sample counterpart. The model-implied regression coefficient of the SRR in growth stocks is less than one standard error away from sample estimates, where the coefficients are positive both in-sample and in-model. The model-implied coefficients of the SRR in value stocks are slightly outside the one-standard-deviation interval but have the same negative sign as in the

[24] Although the negative values on χ_c raise the potential concern that $\sigma_{c,t}$ could turn negative, the probability for $\sigma_{c,t}$ to reach 0 in our calibration is around 0.00002. In a simulation of 2,400,000 months, $\sigma_{c,t}$ only turned negative in 118 months.

data. Overall, our model does a good job replicating the forecasting patterns of SRRs for the future market excess returns.

We have shown that to predict the future value-minus-growth returns, the SRR in consumption is the only significant predictor among SRRs. Hence, we focus only on the SRR in consumption for the forecasts. We find that the model-implied coefficient is less than one standard error away from the sample estimate at the 12-quarter horizon.

V. Robustness

In this section, we check our results' robustness to an alternative measurement of cash flows and log-linear approximation. We also empirically investigate the results replacing consumption by the industrial production index.

A. Dividends adjusted for repurchases

As a first robustness check, we adjust dividends to account for equity repurchases by the method proposed in Bansal et al. (2005a). Details about the adjustment method can be found in Appendix I.

[Figure 4 about here.]

[Table XII about here.]

First, we study the properties of SRRs measured with dividends adjusted for repurchases. From Figure 4, SRRs in adjusted dividends also fluctuate with the business cycle. We run the regression (9) with adjusted cash flows to confirm this, see Table XII. Similar to the case using cash dividends, the SRRs in consumption and value stocks negatively predict the future equity premia at the four-quarter horizon, and the predictability declines as the horizon expands. The adjusted R^2 at the four-quarter horizon is 10.5%, which remains reasonably large. The SRR in consumption positively predicts the future value premia, and the predictive power scales up with the horizon. Meanwhile, SRRs in growth and value stocks are less significant in predicting future value premia. In summary, in line with the case of cash dividends, SRRs fluctuate with the business cycle and carry similar signals in predicting future returns.

B. Errors in log-linearization

In this part, we quantify the impact of approximation via the log-linearization. Pohl, Schmedders, and Wilms (2018) finds that log-linearization ignores the higher-order effects in long-run risk models. However, those higher-order effects can lead to "a strong impact on key financial statistics."

To solve the equilibrium in the LRR model, we adapt the projection method proposed in Pohl, Schmedders, and Wilms (2018). More technical details can be found in Appendix E. Pohl, Schmedders, and Wilms (2018) suggests that the projection method is sufficiently accurate to solve the general equilibrium numerically and requires less computational cost than other methods such as Tauchen and Hussey (1991).

[Table XIII about here.]

Table XIII lists the simulated sample moments for the log-linearized solutions and the projection method based solutions, under the baseline calibration of parameters. Because Table XIII is based on simulation, the results obtained through log-linearization could differ from theoretical moments. Compared with the more accurate solution by the projection method, the log-linearized solution provides moments with economically negligible errors, while providing better tractability by admitting quasi-closed-form solutions.

Pohl, Schmedders, and Wilms (2018) study several models in the LRR framework. They find that the log-linearization approximation induces large errors, especially when the risk aversion γ and the EIS ϕ are large, or the LRR component X_t is highly persistent. Although the LRR component remains persistent in our study, our model implies a calibration with a small risk aversion γ around three and an EIS close to one, which are both smaller than in models studied in Pohl, Schmedders, and Wilms (2018). Therefore, solving our model with log-linearization approximation induces smaller errors than those analyzed in Pohl, Schmedders, and Wilms (2018), and the approximated solution suffices for our analysis.

C. Results using the industrial production index

In this part, we look into the short-run industrial product index risk (henceforth SRIR), which are defined similarly to SRR but with the production index replacing the role of consumption. We estimate SRIRs empirically, and we then run the predictive regressions of future returns on SRIRs. Figure 5 resembles Figure 1 to plot SRIRs. Table XIV shows the results regarding SRIR in a similar manner to Table XII.

[Figure 5 about here.]

[Table XIV about here.]

The industrial production index growth in Panel B of Figure 5 appears to be less volatile than consumption growth most of the time, except during the Great Recession and the Oil Crisis. In Panel A of Figure 5, we see that SRIRs in cash flows are stable except around the Great Recession. Other than these periods, variation in industrial production is disconnected from economic outlook. The variation in industrial production index growth is less informative about the business cycle than the variation in consumption growth.

In Table XIV, predictive regressions SRIRs explain little variation in future market excess returns and value-minus-growth returns. The discrepancy in predictive power between consumption and industrial production is unlikely to be merely due to measurement errors in monthly consumption data. It is also unlikely that the pure noises in monthly consumption predict future returns. A more plausible explanation for the discrepancy involves investigating the economic differences in industrial production index and consumption composition. We leave this aspect to further studies.

VI. Conclusions

This paper studies the relationship between SRRs in consumption, Book-to-Market portfolios, the business cycle, and asset returns. The SRRs vary with the business cycle. The SRR in growth stocks predicts the future equity premia negatively, while the SRRs in growth stocks and consumption predict the future equity premia positively. For the future one-year (three-year) horizon excess market (value-minus-growth) returns, the adjusted R^2 of forecasting regression is 17.6% (11.5%). This predictability remains robust in out-of-sample regressions.

To capture the cyclical variations in SRR and asset returns, we propose an LRR model where a Wishart process models the stochastic covariance process. The model reproduces the growth dynamics in consumption and dividends, the cross-sectional asset pricing moments (particularly the value premium), and the predictive regressions' coefficients. Both SRR and LRR components contribute to the equity and value risk premia.

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Appendix A. Wishart process

In what follows, we summarize some essential properties of the Wishart process for solving our model. More details can be found, for example, in Gourieroux et al. (2009).

A.1. Moments and autocovariances

In this section, we will give the first two moments of the Wishart process without detailed derivations. For $l \in \mathbb{R}^+$, denote $A_l := \exp(lM)$ and $\Xi_l := \int_0^l \exp(sM)QQ'[\exp(sM)]'ds$. To calculate matrix exponential, we use the formula in Van Loan (1978). Gourieroux et al. (2009) gives

$$\mathbb{E}_t(\Sigma_{t+l}) = A_l \Sigma_t (A_l)' + K\Xi_l \tag{A.1}$$

Let $\Sigma(\infty) = \mathbb{E}(\Sigma_t)$ denote the expectation of the Wishart process in the stationary distribution. Let $\Xi(\infty) = \Sigma(\infty)/K$ be the counterpart of Ξ in the stationary distribution, then $\mathbb{E}(\Sigma_t) = \Sigma(\infty) = K\Xi(\infty)$. $\Sigma(\infty)$ is solved by

$$\Sigma(\infty) = A_l \Sigma(\infty) A_l' + K\Xi_l \tag{A.2}$$

Let h_1, h_2 be $n \times n$ constant symmetric matrices, $l > 0$, the second moments of the Wishart process are given as

$$\begin{aligned}
\mathrm{Cov}\left(\mathrm{Tr}(h_1\Sigma_t), \mathrm{Tr}(h_2\Sigma_t)\right) &= 2K\,\mathrm{Tr}\left(\Xi(\infty)h_1\Xi(\infty)h_2\right) \\
\mathrm{Cov}\left[\mathrm{Tr}(h_1\Sigma_{t+l}), \mathrm{Tr}(h_2\Sigma_t)\right] &= \mathrm{Cov}\left[\mathbb{E}_t\left(\mathrm{Tr}(h_1\Sigma_{t+l})\right), \mathrm{Tr}(h_2\Sigma_t)\right] \\
&= \mathrm{Cov}\left[\mathrm{Tr}\left((\exp(lM))'h_1\exp(lM)\Sigma_t\right), \mathrm{Tr}(h_2\Sigma_t)\right] \\
&= 2K\,\mathrm{Tr}\left(\Xi(\infty)(\exp(lM))'h_1\exp(lM)\Xi(\infty)h_2\right)
\end{aligned} \tag{A.3}$$

To solve the model, we also need calculate the second moments of the integrated Wishart process

$$\mathrm{Var}\left(\mathrm{Tr}\left(h_1\int_t^{t+1}\Sigma_s ds\right)\right), \mathrm{Cov}\left(\mathrm{Tr}\left(h_1\int_t^{t+1}\Sigma_s ds\right), \mathrm{Tr}\left(h_2\int_{t-1}^{t}\Sigma_u du\right)\right)$$

Equation (A.3) reduces this problem to the calculation of matrix exponential integrals, for which there exists closed-form solutions (see Van Loan (1978)).

A.2. *Quadratic variation of matrix SDE*

Here, we study the quadratic variation of traces of matrix stochastic processes. Denote the quadratic variation by $\langle , \rangle$.

Lemma A.1: *Assume W_t is a $n \times n$ matrix Brownian Motion, and $A_t, \bar{A}_t$ are predictable $n \times n$ matrix processes. Then, $\langle \mathrm{Tr}(\int A_. dW_.), \mathrm{Tr}(\int \bar{A}_. dW_.) \rangle_t = \mathrm{Tr}(A_t \bar{A}_t')$.*

Proof.

$$\begin{aligned}
\langle \mathrm{Tr}(\int A_. dW_.), \mathrm{Tr}(\int \bar{A}_. dW_.) \rangle_t &= \langle \sum_{i,j=1,\ldots,n} \int A_t^{ij} dW_t^{ji}, \sum_{l,m=1,\ldots,n} \int \bar{A}_t^{lm} dW_t^{ml} \rangle_t \\
&= \sum_{i,j=1,\ldots,n} A_t^{ij} \bar{A}_t^{ji} \\
&= \mathrm{Tr}(A_t \bar{A}_t')
\end{aligned}$$

□

Given that we work with Wishart process, the following corollary comes in handy.

Corollary A.2: *Assume Σ_t is the Wishart process given in* (18)*. Let $A_t, \bar{A}_t$ be predictable $n \times n$ symmetric matrix processes. Then $\langle \mathrm{Tr}(\int A_. d\Sigma_.), \mathrm{Tr}(\int \bar{A}_. d\Sigma_.) \rangle_t = \mathrm{Tr}(4QA_t\Sigma_t\bar{A}_tQ)$.*

Appendix B. Details of the model solutions

B.1. *Proof of proposition 1*

Given the affine structure of the underlying problem, we guess the following exponential affine form for the value function:

$$J(W_t, X_t, \Sigma_t) = \exp(A_0 + A_1 X_t + \mathrm{Tr}(A_2 \Sigma_t)) \frac{W_t^{1-\gamma}}{1-\gamma}. \tag{B.1}$$

Because Σ_t is symmetric, w.l.o.g. we can assume A_2 to be a symmetric matrix. From the optimization problem in (27), we obtain the Hamilton-Jacobi-Bellman (HJB) equation as:

$$\max_C \{ f(C, J) + \mathcal{A}^c J \} = 0, \tag{B.2}$$

where $\mathcal{A}^c$ is the infinitesimal generator associated with state variables (W_t, X_t, Σ_t). The first order condition of the HJB equation for consumption C_t is

$$\frac{1-\gamma}{W} J = J_W = f_C = \beta(1-\gamma)\frac{C^{-1/\psi} J}{((1-\gamma)J)^{\frac{1-1/\psi}{1-\gamma}}}. \tag{B.3}$$

For notational convenience, we define

$$G_t := A_0 + A_1 X_t + \mathrm{Tr}(A_2 \Sigma_t). \tag{B.4}$$

Then, from the first-order condition (B.3), we obtain the consumption-wealth ratio $\frac{C_t}{W_t}$ as:

$$\frac{C_t}{W_t} = \beta^{\psi} \exp\left(\frac{1-\psi}{1-\gamma} G_t\right) = \beta^{\psi} \exp\left(A_{0a} + A_{1a} X_t + \mathrm{Tr}(A_{2a}\Sigma_t)\right). \tag{B.5}$$

where $A_{ka} = \frac{1-\psi}{1-\gamma} A_k$, for $k = 0, 1, 2$. The consumption-wealth ratio is an exponential affine function of the state variables. Note that if $\psi = 1$, consumption-wealth ratio $\frac{C_t}{W_t}$ is constant and equal to β. By substituting (B.5) back into (B.1), we get

$$J_t(C_t, X_t, \Sigma_t) = \exp(\psi G_t)\beta^{-\psi(1-\gamma)} \frac{C_t^{1-\gamma}}{1-\gamma} \tag{B.6}$$

Then, for the case $\psi \neq 1$, the HJB equation can be rewritten as:

$$\begin{aligned} 0 &= \beta\theta\left[\beta^{\psi-1}\exp\left(\frac{1-\psi}{1-\gamma}G_t\right) - 1\right] + \psi\frac{\mathbb{E}_t[dG_t]}{dt} + \frac{\psi^2}{2}\frac{(dG_t)^2}{dt} \\ &\quad + (1-\gamma)\frac{\mathbb{E}_t[dC_t]}{C_t dt} + (1-\gamma)(-\gamma)\frac{(dC_t)^2}{2C_t^2 dt} + (1-\gamma)\psi\frac{dC_t dG_t}{C_t dt} \qquad \text{(B.7)} \\ &= \theta\left(\frac{C_t}{W_t} - \beta\right) + \psi\left(-\alpha A_1 X_t + \mathrm{Tr}[A_2(\Omega\Omega' + M\Sigma_t + \Sigma_t M')]\right) \\ &\quad + \frac{\psi^2}{2}\left(A_1^2 \delta_x' \Sigma_t \delta_x + \mathrm{Tr}(4QA_2\Sigma_t A_2 Q)\right) \\ &\quad + (1-\gamma)(\mu_c + X_t) + \frac{(1-\gamma)(-\gamma)}{2}(\delta_c'\Sigma_t\delta_c + \mathrm{Tr}(\chi_c\Sigma_t) + \bar{\sigma}_c^2) \qquad \text{(B.8)} \end{aligned}$$

and, for $\psi = 1$,

$$\begin{aligned} 0 &= \beta(1-\gamma)\left(\log(\beta) - \frac{G_t}{1-\gamma}\right) + \psi\left(-\alpha A_1 X_t + \mathrm{Tr}[A_2(\Omega\Omega' + M\Sigma_t + \Sigma_t M')]\right) \\ &\quad + \frac{1}{2}\left(A_1^2\delta_x'\Sigma_t\delta_x + \mathrm{Tr}(4QA_2\Sigma_t A_2 Q)\right) \\ &\quad + (1-\gamma)(\mu_c + X_t) + \frac{(1-\gamma)(-\gamma)}{2}\left(\delta_c'\Sigma_t\delta_c + \mathrm{Tr}(\chi_c\Sigma_t) + \bar{\sigma}_c^2\right). \end{aligned} \tag{B.9}$$

To obtain the coefficients of the representation in (B.1) for the case when $\phi \neq 1$, we adopt the standard log-linear approximation of Campbell and Shiller (1988). Defining $c_t := \log C_t$ and $w_t :=$

$\log W_t$, we approximate the consumption-wealth ratio as

$$\frac{C_t}{W_t} = \exp(c_t - w_t) \approx g_1 - g_1 \log g_1 + g_1(c_t - w_t), \tag{B.10}$$

where $g_1 = \exp(\mathbb{E}(c_t - w_t))$. Because C_t/W_t depends on A_0, A_1, A_2, which in turn depend on g_1, it is not possible to give an analytical expression of g_1. Hence, g_1 must be calculated numerically. We refer to Appendix B.5 for details. Then, the log-linearized HJB equation is

$$\begin{aligned} 0 \;=\; & \theta\left[g_1 - g_1 \log g_1 + g_1\left(\psi \log \beta + A_{0a} + A_{1a} X_t + \operatorname{Tr}(A_{2a}\Sigma_t)\right) - \beta\right] \\ & + \psi\left(-\alpha A_1 X_t + \operatorname{Tr}\left[A_2(\Omega\Omega' + M\Sigma_t + \Sigma_t M')\right]\right) + (1-\gamma)(\mu_c + X_t) \\ & + \frac{\psi^2}{2}(A_1^2 \delta_x' \Sigma_t \delta_x + 4QA_2\Sigma_t A_2 Q) - \frac{(1-\gamma)\gamma}{2}\left(\delta_c'\Sigma_t\delta_c + \operatorname{Tr}(\chi_c\Sigma_t) + \bar{\sigma}_c^2\right). \end{aligned} \tag{B.11}$$

For the case of $\psi = 1$, no approximation is needed since the log-linearization is exact. The resulting HJB equation is

$$\begin{aligned} 0 \;=\; & \beta(1-\gamma)\log(\beta) - \beta\left(A_0 + A_1 X_t + \operatorname{Tr}(A_2\Sigma_t)\right) \\ & - \alpha A_1 X_t + \operatorname{Tr}[A_2(\Omega\Omega' + M\Sigma_t + \Sigma_t M')] + (1-\gamma)(\mu_c + X_t) \\ & + \frac{1}{2}\left(A_1^2\delta_x'\Sigma_t\delta_x + 4QA_2\Sigma_t A_2 Q\right) - \frac{(1-\gamma)\gamma}{2}\left(\delta_c'\Sigma_t\delta_c + \operatorname{Tr}(\chi_c\Sigma_t) + \bar{\sigma}_c^2\right). \end{aligned} \tag{B.12}$$

Now we solve for A_0, A_1, and A_2. Irrespective of the value of ψ, A_1 satisfies

$$-g_1\psi A_1 - \alpha\psi A_1 + (1-\gamma) = 0, \tag{B.13}$$

If $\psi = 1$, then $g_1 = \beta$. For $\psi > 1$, A_0 satisfies

$$\theta(g_1 - g_1\log g_1 + g_1\psi\log\beta) - \beta\theta - g_1\psi A_0 + \psi\operatorname{Tr}(A_2\Omega\Omega') + (1-\gamma)\mu_c + \frac{(1-\gamma)(-\gamma)}{2}\bar{\sigma}_c^2 = 0. \tag{B.14}$$

For $\psi = 1$, we have

$$\beta(1-\gamma)\log(\beta) - \beta A_0 + \operatorname{Tr}(A_2\Omega\Omega') + (1-\gamma)\mu_c + \frac{(1-\gamma)(-\gamma)}{2}\bar{\sigma}_c^2 = 0. \tag{B.15}$$

To obtain A_2, we first note that the terms involving Σ_t in the HJB equation (B.11) should sum up to zero:

$$\operatorname{Tr}\left[\left(-g_1\psi A_2 + \psi(M'A_2 + A_2M) + \frac{\psi^2}{2}(A_1^2\delta_x\delta_x' + 4A_2QQA_2) + \frac{(1-\gamma)(-\gamma)}{2}(\delta_c\delta_c' + \chi_c)\right)\Sigma_t\right] = 0$$

If we denote the matrix left-multiplying Σ_t inside the trace operator by L, then L must satisfy $L + L' = 0$ because Σ_t is symmetric. L does not have to be a zero matrix. Thus,

$$A_2 g_1 \psi - \psi(A_2 M + M' A_2) = \frac{\psi^2}{2}(A_1^2 \delta_x \delta_x' + 4A_2 QQA_2) + \frac{(1-\gamma)(-\gamma)}{2}(\delta_c \delta_c' + \chi_c) \tag{B.16}$$

We then solve for a symmetric A_2 numerically from (B.16). This equation has the form of a generalized continuous time algebraic Riccati equation, which have a positive semidefinite solution under certain assumptions.[25] In particular, the generalized continuous time algebraic Riccati equation for X is of the form

$$A'XE + E'XA - (E'XB + S)R^{-1}(B'XE + S') + V = 0, \tag{B.17}$$

where A, Q and E are square matrices of the same dimension. Furthermore, Q and R are symmetric matrices. Hence, in our case,

$$\begin{aligned}
BR^{-1}B' &= 2\psi^2 QQ, \\
SR^{-1}S' - V &= \frac{\psi^2}{2} A_1^2 \delta_x \delta_x' - \frac{(1-\gamma)\gamma}{2}(\delta_c \delta_c' + \chi_c), \\
A - BS' &= \frac{g_1 \psi}{2} I - \psi M.
\end{aligned}$$

B.2. Proof of proposition 2

To derive the state price deflator, we take partial derivatives of $f(C, J)$ and use identities (B.5) and (B.6) to obtain:

$$\begin{aligned}
f_J(C_t, J_t) &= \begin{cases} (\theta - 1)\frac{C_t}{W_t} - \beta\theta & \text{if } \psi \neq 1, \\ \beta(1-\gamma)\left[\log\beta - \frac{G_t}{1-\gamma}\right] - \beta & \text{if } \psi = 1, \end{cases} \\
f_C(C_t, J_t) &= \frac{\beta^{\psi\gamma} \exp\left(\frac{1-\psi\gamma}{1-\gamma} G_t\right) C_t^{-\gamma}}{1-\gamma}.
\end{aligned}$$

The expression for SDF under recursive utility is, according to Duffie and Epstein (1992),

$$\pi_t = \exp\left[\int_0^t f_J(C_s, J_s) ds\right] f_C(C_t, J_t), \tag{B.18}$$

[25] See Kawamoto et al. (1999) and Bittanti et al. (2012).

By plugging in expressions of $f_J(C_t, J_t)$ and $f_C(C_t, J_t)$, we obtain the dynamics of π_t,

$$\begin{aligned}\frac{d\pi_t}{\pi_t} &= f_J(C_t, J_t)dt + \frac{df_C(C_t, J_t)}{f_C(C_t, J_t)} && \text{(B.19)}\\ &= -\left(r_f dt + \mathrm{Tr}(\Lambda^\sigma dB_t^\sigma) + \Lambda dB_t + \Lambda^X dB_t^X + \Lambda^c dB_t^c\right). && \text{(B.20)}\end{aligned}$$

where Λ^σ, Λ, Λ^X and Λ^c are the prices of risk, which we can identify as

$$\begin{aligned}\Lambda^\sigma &= -2\left(\frac{1-\psi\gamma}{1-\gamma}\right) QA_2\sqrt{\Sigma_t} && \text{(B.21)}\\ \Lambda &= \gamma\delta_c'\sqrt{\Sigma_t} && \text{(B.22)}\\ \Lambda^X &= -\left(\frac{1-\psi\gamma}{1-\gamma}\right) A_1\sqrt{\delta_x'\Sigma_t\delta_x}, && \text{(B.23)}\\ \Lambda^c &= \gamma\sigma_c. && \text{(B.24)}\end{aligned}$$

We can read off risk-free interest rate directly from SDF. The risk-free interest rate can be decomposed in the following:

$$r_f = r_0 + r_x X_t + \mathrm{Tr}(r_\Sigma \Sigma_t), \qquad \text{(B.25)}$$

where by matching constants and coefficients on X_t and Σ_t, we obtain

$$\begin{aligned}r_0 &= \begin{cases} -(\theta-1)\left(g_1 - g_1\log(g_1) + g_1\psi\log(\beta) - \frac{g_1\psi}{\theta}A_0\right) & \\ +\beta\theta - \frac{1-\psi\gamma}{1-\gamma}\mathrm{Tr}(A_2KQQ) + \gamma\mu_c - \gamma(\gamma+1)\frac{\sigma_c^2}{2} & \text{if } \psi > 1 \\ -\beta(1-\gamma)\left(\log(\beta) - A_0/(1-\gamma)\right) + \beta & \\ -\mathrm{Tr}(A_2KQQ) + \gamma\mu_c - \gamma(\gamma+1)\bar{\sigma}_c^2/2 & \text{if } \psi = 1\end{cases} && \text{(B.26)}\\ r_\Sigma &= g_1\left(\frac{1-\psi\gamma}{1-\gamma}\right)A_2 - \left(\frac{1-\psi\gamma}{1-\gamma}\right)(A_2M + M'A_2) \\ &\quad -\frac{1}{2}\left(\frac{1-\psi\gamma}{1-\gamma}\right)^2 (A_1^2\delta_x\delta_x' + 4A_2QQA_2) - \frac{\gamma(\gamma+1)}{2}(\delta_c\delta_c' + \chi_c) && \text{(B.27)}\\ r_x &= \frac{1}{\psi}. && \text{(B.28)}\end{aligned}$$

B.3. Proof of proposition 3

Assume that the dividend-price ratio has the following exponential affine form,

$$\frac{D_t^i}{P_t} = \exp(A_{0i} + A_{1i}X_t + \mathrm{Tr}(A_{2i}\Sigma_t)), \qquad \text{(B.29)}$$

where A_{2i} is a symmetric matrix. The instantaneous return of asset i is

$$\begin{aligned}\frac{dP_t^i}{P_t^i} &= (\mu_i + \phi_i X_t)dt + \delta_i' \sqrt{\Sigma_t} dB_t + \sigma_i dB_t^i \\ &\quad - \left[A_{1i}(-\alpha X_t dt + \sqrt{\delta_x' \Sigma_t \delta_x} dB_t^X)\right] - \mathrm{Tr}\left[A_{2i}(\Omega\Omega' + M\Sigma_t + \Sigma_t M')dt + 2QA_{2i}\sqrt{\Sigma_t} dB_t^\sigma\right] \\ &\quad + \frac{1}{2} A_{1i}^2 \delta_x' \Sigma_t \delta_x dt + 2\,\mathrm{Tr}(QA_{2i}\Sigma_t A_{2i} Q)dt. \end{aligned} \tag{B.30}$$

We perform a log-linear approximation as in Campbell and Shiller (1988). Defining $d_t^i = \log D_t^i$ and $p_t^i = \log P_t^i$, then

$$\frac{D_t^i}{P_t^i} \approx g_{0i} + g_{1i}\left(A_{0i} + A_{1i}X_t + \mathrm{Tr}(A_{2i}\Sigma_t)\right), \tag{B.31}$$

where $g_{1i} = \exp(\mathbb{E}(d_t^i - p_t^i))$ and $g_{0i} = g_{1i} - g_{1i}\log g_{1i}$. Because $d(\pi_t P_t) + \pi_t D_t dt$ must have zero drift,

$$\mathbb{E}_t\left[\frac{dP_t}{P_t}\right] + \frac{D_t}{P_t}dt = r_f dt - \frac{d\pi_t}{\pi_t}\frac{dP_t}{P_t}, \tag{B.32}$$

where $-\frac{d\pi_t}{\pi_t}\frac{dP_t}{P_t}$ is the risk premium of the asset. The formula for the risk premium is the following:

$$-\frac{d\pi_t}{\pi_t}\frac{dP_t}{P_t}/dt = \gamma\delta_c'\Sigma_t\delta_i + \psi(1-\frac{1}{\theta})A_1 A_{1i}\delta_x'\Sigma_t\delta_x + 4\psi(1-\frac{1}{\theta})\,\mathrm{Tr}(QA_{2i}\Sigma_t A_2 Q) \tag{B.33}$$

where we used Proposition A.1 to calculate the quadratic variation of Wishart diffusions. By comparing coefficients in Equations (B.30) and (B.32), we find that A_{0i} must satisfy

$$\mu_i - \mathrm{Tr}(A_{2i}\Omega\Omega') + g_{0i} + g_{1i}A_{0i} = r_0. \tag{B.34}$$

Similarly, for A_{1i}:

$$\phi_i + \alpha A_{1i} + g_{1i}A_{1i} = r_x, \tag{B.35}$$

and for A_{2i}:

$$\begin{aligned} &\mathrm{Tr}\left[(-2A_{2i}M + \frac{A_{1i}^2}{2}\delta_x\delta_x' + 2A_{2i}QQA_{2i} + g_{1i}A_{2i})\Sigma_t\right] \\ &= \mathrm{Tr}\left[\left(r_\Sigma + \gamma\delta_i\delta_c' + 4\left(\frac{1-\psi\gamma}{1-\gamma}\right)A_2QQA_{2i} + \left(\frac{1-\psi\gamma}{1-\gamma}\right)A_1A_{1i}\delta_x\delta_x'\right)\Sigma_t\right]. \end{aligned} \tag{B.36}$$

Hence, A_{2i} is a solution to:

$$\begin{aligned} &-(A_{2i}M + M'A_{2i}) + \frac{A_{1i}^2}{2}\delta_x\delta_x' + 2A_{2i}QQA_{2i} + g_{1i}A_{2i} \\ &= r_\Sigma + \gamma\frac{\delta_d\delta_c' + \delta_c\delta_d'}{2} + \left(\frac{1-\psi\gamma}{1-\gamma}\right)\left(2A_2QQA_{2i} + 2A_{2i}QQA_2 + A_1A_{1i}\delta_x\delta_x'\right). \end{aligned} \tag{B.37}$$

To obtain A_{2i} numerically, we can again cast it into the form of a generalized continuous time algebraic Riccati equation (B.17). In this case,

$$\begin{aligned} SR^{-1}S' - V &= \frac{A_{1i}^2}{2}\delta_x\delta_x' - r_\Sigma - \gamma\frac{\delta_d\delta_c' + \delta_c\delta_d'}{2} - \left(\frac{1-\psi\gamma}{1-\gamma}\right)A_1A_{1i}\delta_x\delta_x', \\ A - BS' &= -\frac{g_{1i}}{2}I + M + 2\left(\frac{1-\psi\gamma}{1-\gamma}\right)QQA_2, \\ B &= \sqrt{2}Q. \end{aligned}$$

Given proper technical conditions, a positive semidefinite solution A_{2i} exists.

B.4. Proof of proposition 4

In what follows, we derive model-implied regression coefficients, in which we regress future asset return on SRRs. We want to study the regressions of the form

$$r^e_{t,t+\frac{Q}{4}} = \beta_0 + \beta_1 SRR^c_{t-1,t} + \beta_2 SRR^g_{t-1,t} + \beta_3 SRR^v_{t-1,t} + \epsilon_{t,t+\frac{Q}{4}} \tag{B.38}$$

where

$$\begin{aligned} SRR^c_{t-1,t} &= \int_{t-1}^t \delta_c'\Sigma_s\delta_c ds + \int_{t-1}^t \mathrm{Tr}(\chi_c\Sigma_t) + \bar{\sigma}_c^2 \\ SRR^i_{t-1,t} &= \int_{t-1}^t \delta_c'\Sigma_s\delta_i ds \end{aligned}$$

Thus, we derive the model-implied coefficients for the following regression:

$$r^e_{t,t+\frac{Q}{4}} = \beta_0 + \mathrm{Tr}\left(\int_{t-1}^t h_1\Sigma_s ds\right)\beta_1 + \mathrm{Tr}\left(\int_{t-1}^t h_2\Sigma_s ds\right)\beta_2 + \cdots + \mathrm{Tr}\left(\int_{t-1}^t h_m\Sigma_s ds\right)\beta_m + \epsilon_{t,t+\frac{Q}{4}}$$

where r^e is excess stock return over risk-free rate in the period from t to $t+\frac{Q}{4}$, $h_i \in \mathbb{R}^{3\times 3}$ for $i = 1, 2, ..m$. Denote by $\boldsymbol{\beta} := (\beta_0, \beta_1, \cdots, \beta_m)$ the vector of model-implied regression coefficients.

For convenience, we denote the right-hand-side independent variables by

$$\mathrm{RHSVAR} := \left(1, \mathrm{Tr}\left(\int_{t-1}^t h_1\Sigma_s ds\right), \cdots, \mathrm{Tr}\left(\int_{t-1}^t h_m\Sigma_s ds\right)\right)'$$

Regression coefficients are therefore given by

$$\boldsymbol{\beta} = E(\mathrm{RHSVAR}\times\mathrm{RHSVAR}')^{-1}E\left(\mathrm{RHSVAR}\times r^e_{t,t+\frac{Q}{4}}\right), \tag{B.39}$$

where $E(\mathrm{RHSVAR}\times\mathrm{RHSVAR}')$ is

$$\begin{bmatrix} 1 & \mathbb{E}[\mathrm{Tr}(\int_{t-1}^{t} h_1\Sigma_s ds)] & \cdots & \mathbb{E}[\int_{t-1}^{t} h_m\Sigma_s ds)] \\ \mathbb{E}[\mathrm{Tr}(\int_{t-1}^{t} h_1\Sigma_s ds)] & \mathbb{E}[\mathrm{Tr}(\int_{t-1}^{t} h_1\Sigma_s ds)\,\mathrm{Tr}(\int_{t-1}^{t} h_1\Sigma_s ds)] & \cdots & \mathbb{E}[\mathrm{Tr}(\int_{t-1}^{t} h_1\Sigma_s ds)\,\mathrm{Tr}(\int_{t-1}^{t} h_m\Sigma_s ds)] \\ \vdots & \vdots & \ddots & \vdots \\ \mathbb{E}[\mathrm{Tr}(\int_{t-1}^{t} h_m\Sigma_s ds)] & \mathbb{E}[\mathrm{Tr}(\int_{t-1}^{t} h_m\Sigma_s ds)\,\mathrm{Tr}(\int_{t-1}^{t} h_1\Sigma_s ds)] & \cdots & \mathbb{E}[\mathrm{Tr}(\int_{t-1}^{t} h_m\Sigma_s ds)\,\mathrm{Tr}(\int_{t-1}^{t} h_m\Sigma_s ds)] \end{bmatrix}$$

Because the expression of $r^e_{t,t+\frac{Q}{4}}$ is given by Equations (B.48)-(B.51) and Brownian motions in $r^e_{t,t+\frac{Q}{4}}$ are uncorrelated with Σ_t, we can calculate $\mathbb{E}\left(\mathrm{RHSVAR} \times r^e_{t,t+\frac{Q}{4}}\right)$ similarly. Then we use Equation (A.3) and techniques in Van Loan (1978) to calculate model-implied $\boldsymbol{\beta}$.

B.5. Numerically solving g_1 and g_{1i}

Here we develop the algorithm to calculate g_1 and g_{1i} numerically. Recall Equation (30),

$$g_1 = \mathbb{E}\left(\frac{C_t}{W_t}\right) = \beta^{\psi}\exp(A_{0a})\mathbb{E}\left[\exp\left(A_{1a}X_t + \mathrm{Tr}(A_{2a}\Sigma_t)\right)\right] \tag{B.40}$$

Then,

$$\begin{aligned} g_1 &= \beta^{\psi}\exp(A_{0a})\mathbb{E}\left[\mathbb{E}[\exp(A_{1a}X_t + \mathrm{Tr}(A_{2a}\Sigma_t))|\Sigma_{0\le s\le t}]\right] && \text{(B.41)} \\ &= \beta^{\psi}\exp(A_{0a})\mathbb{E}\left[\exp\left(A_{1a}e^{-\alpha t}X_0 + \frac{1}{2}A_{1a}^2 e^{-2\alpha t}\int_0^t e^{2\alpha s}\delta_x'\Sigma_s\delta_x ds + \mathrm{Tr}(A_{2a}\Sigma_t)\right)\right] && \text{(B.42)} \end{aligned}$$

Note that this equation holds for $\forall t > 0$. Obviously $e^{-\alpha t}X_0$ converges to zero in probability as $t \to \infty$, so

$$g_1 = \lim_{t\to\infty}\beta^{\psi}\exp(A_{0a})\mathbb{E}\left[\frac{1}{2}A_{1a}^2 e^{-2\alpha t}\int_0^t e^{2\alpha s}\delta_x'\Sigma_s\delta_x ds + \mathrm{Tr}(A_{2a}\Sigma_t)\right]. \tag{B.43}$$

Because $\delta_x'\Sigma_t\delta_x$ has the long term mean $\delta_x'\Sigma(\infty)\delta_x$, we approximate g_1 by

$$\begin{aligned} g_1 &\approx \lim_{t\to\infty}\beta^{\psi}\exp(A_{0a})\mathbb{E}\left[\frac{1}{2}A_{1a}^2 e^{-2\alpha t}\int_0^t e^{2\alpha s}\delta_x'\Sigma(\infty)\delta_x ds + \mathrm{Tr}(A_{2a}\Sigma_t)\right] \\ &= \beta^{\psi}\exp\left(A_{0a} + \frac{1}{4\alpha}A_{1a}^2\delta_x'\Sigma(\infty)\delta_x\right)\mathbb{E}\left[\exp\left(\mathrm{Tr}(A_{2a}\Sigma_t)\right)\right]. \end{aligned}$$

Laplace transform of $W(K, 0, \Xi(\infty))$ is given in Gourieroux et al. (2009):

$$\mathbb{E}[\exp(\Gamma\Sigma_t)] = \det(I_n - 2\Xi(\infty)\Gamma)^{-K/2}$$

Thus, we numerically solve for g_1 from

$$g_1 = \beta^{\psi}\exp\left(A_{0a} + \frac{1}{4\alpha}A_{1a}^2\delta_x'\Sigma(\infty)\delta_x\right)\det(I_n - 2\Xi(\infty)A_{2a})^{-K/2} \tag{B.44}$$

Similarly for asset i, its stationary mean of dividend-price ratio g_{1i} is solved from

$$g_{1i} = \exp\left(A_{0i} + \frac{1}{4\alpha}A_{1i}^2 \delta_x' \Sigma(\infty)\delta_x\right) \det(I_n - 2\Xi(\infty)A_{2i})^{-K/2} \tag{B.45}$$

B.6. Theoretical moments

This section gives the analytical expressions for moments used in GMM estimation. There are 36 moments in total. Under our assumptions, M is negative definite and lower triangular, $Q = qI_n$ and $\delta_x = \eta\delta_c$, the expressions of the following moments can be further simplified. The parameters are:

$$\alpha, \sigma_X, \mu_c, \delta_c, \mu_i, \delta_i, \phi_i, m, q, \beta, \psi, \gamma,$$

where $i = m, 1, 2, 3$ represents market portfolio and three Fama-French portfolios respectively. There are 26 parameters to be estimated. To make the estimations easier, we restrict μ_i to be the sample mean of the corresponding mean of cash flow growth, which leaves us with 21 parameters to match 31 moments. To estimate the over-identified system, we introduce the weight matrix W, which we specify as a diagonal matrix that adjusts for the magnitudes of moments.

Bansal et al. (2016) argues that the decision interval for the long-run risk model should be a month. To reflect the more frequent decision making (i.e., more than once per year), we model the agent to make decisions dynamically and continuously. Because observations are only available yearly in aggregate, we calculate theoretical moments at yearly aggregations. $\Delta c_{t,t+1} := \int_t^{t+1} \frac{dC_t}{C_t}$, and $\Delta d^i_{t,t+1}$, $r^i_{e,t,t+1}$, $r_{f,t,t+1}$ are similarly defined. Therefore,

$$
\begin{aligned}
\Delta c_{t,t+1} &= \mu_c + \int_t^{t+1} X_s ds + \int_t^{t+1} \delta_c' \sqrt{\Sigma_s} dB_s + \int_t^{t+1} \sigma_{c,t} dB_s && \text{(B.46)} \\
\Delta d^i_{t,t+1} &= \mu_i + \phi_i \int_t^{t+1} X_s ds + \int_t^{t+1} \delta_i' \sqrt{\Sigma_s} dB_s + \int_t^{t+1} \sigma_i dB^i_s && \text{(B.47)} \\
r^i_{e,t,t+1} &= \gamma \delta_c' \int_t^{t+1} \Sigma_s ds \delta_i + \left(\frac{1-\psi\gamma}{1-\gamma}\right) A_1 A_{1i} \delta_x' \int_t^{t+1} \Sigma_s ds \delta_x && \text{(B.48)} \\
&\quad +4 \left(\frac{1-\psi\gamma}{1-\gamma}\right) \mathrm{Tr}\left(Q A_{2i} \int_t^{t+1} \Sigma_s ds A_2 Q\right) - A_{1i} \int_t^{t+1} \sqrt{\delta_x' \Sigma_s \delta_x} dB^X_s && \text{(B.49)} \\
&\quad - \mathrm{Tr}\left(A_{2i} \int_t^{t+1} \sqrt{\Sigma_s} dB^\sigma_s Q + Q' \left(\int_t^{t+1} \sqrt{\Sigma_s} dB^\sigma_s\right)' A_{2i}'\right) && \text{(B.50)} \\
&\quad +\delta_i' \int_t^{t+1} \sqrt{\Sigma_s} dB_s + \int_t^{t+1} \sigma_i dB^i_s && \text{(B.51)} \\
r_{f,t,t+1} &= r_0 + r_x \int_t^{t+1} X_s ds + \mathrm{Tr}\left(r_\Sigma \int_t^{t+1} \Sigma_s ds\right) && \text{(B.52)}
\end{aligned}
$$

We plug in Equations (A.2) and (A.3) to calculate moments of Wishart process, expressions for the model implied moments are shown in Table XV.

[Table XV about here.]

Appendix C. Monthly consumption growth

We define consumption as the sum of nondurable goods and services, where the data are from the U.S. Bureau of Economic Analysis (BEA). Per capita annual consumption data range from 1927 to 2017 in real terms.[26] The monthly consumption data are available from January 1959 to December 2017, as the national aggregate and in nominal dollar amounts.[27] We construct the monthly per capita consumption in real terms to make the consumption data at monthly and yearly frequency consistent. We divide aggregate nominal monthly consumption by population and the personal consumption deflator to get personal consumption in real terms. Given that the population is only measured quarterly, we linearly interpolate the quarterly population to get a monthly estimate

[26]Table 2.4.5, Personal Consumption Expenditures by Type of Product, where per capita annual consumption denoted in chained dollar (U.S. dollar fixed in 2009) is available.

[27]Table 2.8.5, Personal Consumption Expenditures by Major Type of Product, Monthly. BEA provides chained dollar monthly consumption only from 1999, so we choose this table with a longer time series of nominal consumption data.

of population level. We also linearly interpolate quarterly personal consumption deflator to get a monthly personal consumption deflator, where the quarterly personal consumption deflator is the ratio of nondurable consumption plus services in nominal terms divided by those in chained dollars. To compare consumption and dividend growth rates and estimate their covariance, one should be cautious about the different constructions of seasonal adjusted consumption and dividends. While monthly personal consumption data from BEA are seasonally adjusted by removing the seasonal component,[28] dividends are seasonal adjusted simply by calculating the yearly moving average. Consequently, any macroeconomic shock has an immediate impact on consumption growth but affects dividend growth data only after several quarters. Therefore, to make consumption and dividends comparable, we calculate seasonally adjusted consumption as its moving average in the last 12 months. We let one unit of time interval corresponding to one year, and we set $h = 1/12$. We denote the personal real consumption before seasonal adjustment between time t and time $t + h$ by $C_{t,t+h}$, the contemporary seasonal component by $S_{t,t+h}$. Hence, the seasonal adjusted consumption, corresponding to the data from BEA, is given by $C^{SA}_{t,t+h} = C_{t,t+h} - S_{t,t+h}$. We then calculate the moving average of consumption $C^{MA}_{t,t+h}$ between time t and $t + h$:

$$
\begin{aligned}
C^{MA}_{t,t+h} &= \frac{1}{12}\sum_{i=0}^{11} C^{SA}_{t-ih,t-ih+h} \\
&= \frac{1}{12}\sum_{i=0}^{11}(C_{t-ih,t-ih+h} - S_{t-ih,t-ih+h}) \\
&\approx \frac{1}{12}\sum_{i=0}^{11} C_{t-ih,t-ih+h}.
\end{aligned}
$$

The approximation in the last step holds as long as the seasonal components derived from X13-ARIMA-SEATS within a year sum up to a small value close to 0. In principle, we would prefer to calculate the moving average of consumption directly from an unadjusted time series of monthly consumption data instead of using C^{SA}, but no such data is available as of now. We confirm insensitivity to seasonalization by X13-ARIMA-SEATS in Panel A of Figure 6, where we plot the annual consumption growth directly calculated from the ratio of consecutive annually aggregated consumption, and that calculated from summing up the monthly changes in the 12-month moving average

[28]See, `https://www.bea.gov/faq/index.cfm?faq_id=123`, where X13-ARIMA-SEATS is implemented.

of the monthly aggregated consumption. For the years where both data are available, the two series are almost identical with a high correlation at about 99.6%.[29] Panel B of Figure 6 plots the fluctuations of (rescaled) monthly consumption growth around their annual mean. Yearly aggregated consumption $C_{t,t+1}$ is the sum of the monthly consumption within the year $\sum_{i=1}^{12} C^{MA}_{t+(i-1)h,t+ih}$. Then, we construct the monthly consumption growth in a way similar to how we construct the growth of dividend moving average. Log monthly consumption growth $\Delta c_{t,t+h}$ is

$$\Delta c_{t,t+h} = \log \frac{C^{MA}_{t,t+h}}{C^{MA}_{t-h,t}} = \log \frac{C_{t-11h,t-10h} + \cdots + C_{t,t+h}}{C_{t-12h,t-11h} + \cdots + C_{t-h,t}} \tag{C.1}$$

This monthly consumption growth is different from, for example, Bansal et al. (2016), in which the monthly consumption growth is $\log \frac{C_{t,t+h}}{C_{t-h,t}}$. In their setup, the annually aggregated consumption growth is not the sum of the monthly consumption growth rates within the year. In contrast, our construction of monthly consumption growth reflects the monthly changes in the annual consumption, which sum up to the annual consumption growth:

$$\begin{aligned}\sum_{k=1}^{12} \Delta c_{t+(k-1)h,t+kh} &= \sum_{k=1}^{12} \log \frac{C_{t+(k-1)h-11h,t+(k-1)h-10h} + \cdots + C_{t+(k-1)h,t+(k-1)h+h}}{C_{t+(k-1)h-12h,t+(k-1)h-11h} + \cdots + C_{t+(k-1)h-h,t+(k-1)h}} \\ &= \Delta c_{t,t+1}\end{aligned}$$

[Figure 6 about here.]

Appendix D. Simulation details in Table VI

The consumption growth is generated from the following specification:

$$\log \frac{C_t}{C_{t-1}} = \mu_c + x_t + \sigma_c \epsilon_t \tag{D.1}$$

$$x_t = \rho x_{t-1} + \sigma_x e_t \tag{D.2}$$

$$\epsilon_t,\, e_t \sim N(0,1), \tag{D.3}$$

[29]To further confirm that the moving average is not sensitive to whether data is seasonalized, we seasonalize dividends and perform moving average in the same manner to calculate dividend growth. Indeed, dividend growth with seasonalization are highly correlated with dividend growth from raw data, with correlations above 97%. Seasonalization before moving average does not alter the key results in this paper, and therefore, these results are not included in this paper due to the limitations of space.

where the unit of time is one month. We simulate 1,000 years of monthly consumption growth rates and use the last 900 years for estimation to minimize the effect of the choice of initial value of long-run risk x_0. We refer to the results in Panel B of Table VI. To study the effect of long-run risk, we study different calibration setups: baseline parameters ($\rho = 0.975, \sigma_x = 0.0237, \sigma_c = 0.032, \mu_c = 0.16$), and three alternatives with $\rho = 0$, $\sigma_x = 0$ and $\mu_c = 0$ respectively.

The baseline parameters are chosen in a way to match the persistence of long-run risk in Bansal and Yaron (2004) and consumption growth dynamics. In the baseline case, we are able to generate similar short- and long-run consumption growth rates variance to real data, and the long-run risk component accounts for most of the variance annually aggregated consumption growth. Moreover, the within-year monthly autocorrelation is much smaller than the monthly autocorrelation estimated using the whole sample, which confirms that the persistent long-run consumption risk does not affect the within-year autocorrelation of the monthly aggregated consumption growth rates as much as in the autocorrelation of the annually aggregated consumption growth rates. For comparison, we shut down the LRR channel in other calibrations, and we find that without LRR, the dynamics of consumption growth behave distinctly from real consumption data. In the second column, we let the $\rho = 0$ so that x_t is just another source of transitory risk. In this case, the variance of annually aggregated consumption is smaller due to zero persistence in x_t, and the variance of monthly growth rates is almost identical to the variance of annual growth rates. All measures of autocorrelation are close to zero. In the third column we let $\sigma_x = 0$ so that $x_t = 0$ throughout. Like the second column, the variance of monthly growth rates accounts for almost all of the annually aggregated consumption growth rates. All autocorrelation measures are close to zero. The fourth column studies the sensitivity to the mean of consumption growth, and we find the mean of consumption growth rates has no impact on the variance or autocorrelation measures.

Appendix E. Details of projection method

The projection method in this paper roughly follows the steps described in Pohl, Schmedders, and Wilms (2018). Because our model is in continuous-time, the HJB equation (B.7) is the counterpart of the wealth-Euler equation (A13) in Pohl, Schmedders, and Wilms (2018). Instead of solving the HJB Equation by a log-linear approximated consumption-wealth ratio like Equation (B.4), the projection method aims to find the real solution G_t to the HJB equation as a higher-order polynomial of state variables. Since the state variables in our model are the long-run risk component x_t and three covariance components in Σ_t, we assume G_t is a polynomial of those state variables of degree five. The polynomials are products of univariate Chebyshev polynomials of state variables. The degree of those polynomials is guided by the choice in Pohl, Schmedders, and Wilms (2018).

We use the collocation method to solve for G_t. I.e., determine the polynomial coefficients in G_t by solving it on a grid points of the state variables, as described in Equation (A11) in Pohl, Schmedders, and Wilms (2018). For this polynomial of four variables of degree five, we solve G_t on $5 \times 5 \times 5 \times 5 = 625$ points, where each state variable has a grid of five points. We set the ranges of the grid of state variables a bit larger than would have been realized in Monte-Carlo simulations.

We also used log-linear approximation to solve the log price-dividend ratios of individual portfolios (Equation (B.29)). Similar to the wealth-consumption ratio, we solve the real log dividend-price ratio as the sum of products of univariate Chebyshev polynomials on state variables. The polynomial coefficients are determined by solving Equation (B.32) on $5 \times 5 \times 5 \times 5 = 625$ points, where each state variable has a grid of five points. The ranges of grid points are slightly larger than those that would have been realized in Monte-Carlo simulations.

Appendix F. Quantitative model results with three BM portfolios

In our analysis, we find that the middle book-to-market portfolio does not carry any useful financial information. Therefore, we have excluded it in our calibration exercise. Nevertheless, for completeness, we also report the results of the calibration using all three book-to-market portfolios. To keep the model in line with our previous analysis and to reduce the model complexity, we stick to a two-dimensional Wishart process for modeling the SRRs. Note that the SRR of the middle portfolio is controlled by $\delta_{mid} = (\delta_1^{mid}, \delta_2^{mid})'$.

The calibration results are reported in Tables XVI to XIX. The persistence of the monthly consumption growth remains large with $\alpha = 0.082$. Value stocks have larger exposure to LRR than growth stocks, with $\phi^v = 4.4065$, $\phi^g = 0.3391$. The risk aversion parameter $\gamma = 2.2466$ and the EIS $\psi = 1.0139$ are similar to those in the base calibration. The decomposition of risk premiums similarly shows that the SRRs play an essential role in explaining the market risk premium and the value premium. As expected, the middle portfolio exhibits exposures to the LRR and SRR which lie between the exposures of the growth and value portfolios. Although the parameter values differ from those in the base calibration with two portfolios, the calibration using all three book-to-market portfolios conveys similar economic information and intuition.

[Table XVI about here.]

[Table XVII about here.]

[Table XVIII about here.]

[Table XIX about here.]

Tables

Table I. Correlation Matrix

	C	G	Mid	V	M	I/C	$\log\frac{P}{D}$	cay	CP
C	1	0.030	0.040	0.165	0.182	0.430	−0.238	−0.275	0.019
G	0.030	1	0.030	−0.195	0.394	0.034	−0.134	0.098	0.064
Mid	0.040	0.030	1	−0.097	0.626	−0.133	0.058	0.230	0.118
V	0.165	−0.195	−0.097	1	0.142	0.088	0.079	−0.184	−0.030
M	0.182	0.394	0.626	0.142	1	0.062	0.016	−0.045	−0.006
I/C	0.430	0.034	−0.133	0.088	0.062	1	−0.696	−0.504	0.028
$\log\frac{P}{D}$	−0.238	−0.134	0.058	0.079	0.016	−0.696	1	0.036	−0.106
cay	−0.275	0.098	0.230	−0.184	−0.045	−0.504	0.036	1	0.320
CP	0.019	0.064	0.118	−0.030	−0.006	0.028	−0.106	0.320	1

This table presents the correlation of independent variables in predictive regressions. The independent variables include $\mathrm{SRR}^{c}_{t-1,t}$ (C), $\mathrm{SRR}^{g}_{t-1,t}$ (G), $\mathrm{SRR}^{mid}_{t-1,t}$ (Mid), $\mathrm{SRR}^{v}_{t-1,t}$ (V), $\mathrm{SRR}^{m}_{t-1,t}$ (M), the logarithm of price-dividend ratio ($\log\frac{P}{D}$), the ratio of income over consumption (I/C) in Santos and Veronesi (2006), the cay in Lettau and Ludvigson (2001a) and the CP factor in Cochrane and Piazzesi (2005).

Table II. Forecast 4-Quarter Market Excess Returns, 1959–2017

	Future Real Market Return											
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
C	-193.189^{***}	-188.191^{***}	-202.475^{***}									-215.915^{***}
	(40.476)	(44.948)	(56.081)									(44.508)
G	8.682^{*}	8.752^{*}		10.246^{**}								7.407
	(5.203)	(5.119)		(4.934)								(5.396)
Mid	8.197^{*}				8.538							7.071
	(4.746)				(6.124)							(4.903)
V	-3.897^{***}	-4.280^{***}				-6.265^{**}						-3.542^{***}
	(1.337)	(1.608)				(3.067)						(1.170)
M							13.712					
							(10.083)					
I/C								-386.914				$1,278.115$
								(685.024)				$(1,015.940)$
$\log \frac{P}{D}$									-2.372			2.409
									(5.291)			(7.865)
cay										245.627^{***}		190.423
										(95.262)		(122.470)
CP											169.363	60.884
											(111.884)	(85.792)
Constant	10.273^{***}	10.238^{***}	10.869^{***}	3.673^{*}	4.515^{**}	5.465^{***}	4.051^{*}	384.567	13.136	4.965^{***}	3.018	$-1,253.095$
	(2.407)	(2.442)	(2.223)	(2.163)	(1.889)	(1.672)	(2.070)	(673.000)	(18.342)	(1.748)	(2.490)	$(1,019.787)$
Adjusted R^2	0.196	0.176	0.121	0.025	0.021	0.064	0.012	0.002	-0.002	0.074	0.016	0.231

Note: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

This table presents the regression results forecasting the future 4-quarter equity premia from 1959 Q1 to 2017 Q4, performed on quarterly data. The independent variables include $\mathrm{SRR}^{c}_{t-1,t}$ (C), $\mathrm{SRR}^{g}_{t-1,t}$ (G), $\mathrm{SRR}^{mid}_{t-1,t}$ (Mid), $\mathrm{SRR}^{v}_{t-1,t}$ (V), $\mathrm{SRR}^{m}_{t-1,t}$ (M), the logarithm of price-dividend ratio ($\log \frac{P}{D}$), the ratio of income over consumption (I/C) and *cay*. Newey-West standard errors with lag 8 are shown in parentheses, based on which the significance level is determined. SRRs are represented in squared percentage and returns in percentage terms.

Table III. Forecast 12-Quarter Value-Minus-Growth Returns, 1959–2017

	Future Real Market Return											
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
C	81.522***	79.010***	81.677***									46.610**
	(19.798)	(23.668)	(23.986)									(18.418)
G	0.440	0.401		0.322								0.800
	(1.922)	(2.038)		(2.117)								(1.818)
Mid	−4.044*				−3.825*							−3.490
	(2.312)				(2.223)							(2.677)
V	0.437	0.632				1.159						0.103
	(0.687)	(0.780)				(0.893)						(0.818)
M							−4.609					
							(3.985)					
I/C								902.488**				1,531.829***
								(357.931)				(505.669)
$\log \frac{P}{D}$									−2.438			7.960*
									(3.686)			(4.726)
cay										−105.818**		74.278
										(53.768)		(69.359)
CP											40.296	32.983
											(54.860)	(40.397)
Constant	1.028	1.043	1.082	3.626***	3.696***	3.499***	3.844***	−882.744**	12.420	3.610***	3.273**	−1,531.281***
	(1.484)	(1.474)	(1.419)	(1.284)	(1.208)	(1.205)	(1.287)	(351.814)	(12.989)	(1.162)	(1.539)	(509.569)
Adjusted R^2	0.146	0.115	0.119	-0.004	0.027	0.010	0.007	0.192	0.013	0.077	0.002	0.297

Note: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

This table presents the regression results forecasting the future 12-quarter value-minus-growth returns from 1959 Q1 to 2017 Q4. The independent variables include $SRR^{c}_{t-1,t}$ (C), $SRR^{g}_{t-1,t}$ (G), $SRR^{mid}_{t-1,t}$ (Mid), $SRR^{v}_{t-1,t}$ (V), $SRR^{m}_{t-1,t}$ (M), the logarithm of the price-dividend ratio ($\log \frac{P}{D}$), the ratio of income over consumption (I/C) and *cay*. Newey-West standard errors with lag 8 are shown in parentheses, based on which significance level is determined. The variance and covariances are represented in squared percentage and returns in percentage terms.

Table IV. Summaries of Out-of-Sample Regressions and Strategies

	Market	C	G	Mid	V	All
Panel A: Results of SRR in 1959 - 2017 Sample						
Annualized SR	0.372	0.572	0.431	0.194	0.517	0.663
Monthly SR	0.107	0.165	0.124	0.056	0.149	0.192
R^2_{OS}		0.021	0.025	-0.174	0.024	0.033
OOS F-stat		1.748**	2.084**	-14.652	2.055**	2.733**
Panel B: Results of SRR in 1959 - 2005 Sample						
Annualized SR	0.343	0.693	0.270	0.147	0.527	0.734
Monthly SR	0.099	0.200	0.078	0.042	0.152	0.212
R^2_{OS}		0.242	0.011	-0.081	0.103	0.280
OOS F-stat		8.955***	0.411	-3.015	3.815***	10.354***
Panel C: R^2_{OS}s in Campbell & Thompson (2008)						
Predictor	1956-1980	1980-2005		Predictor	1956-1980	1980-2005
D/P	0.095	-0.162		E/P	0.051	-0.061
Smooth E/P	0.049	-0.089		D/P Growth	0.018	0.019

This table summarizes the out-of-sample regression results. Panel A reports the Sharpe Ratios, the annualized out-of-sample R^2_{OS}, and out-of-sample F-statistics (McCracken, 2007) for excess returns of market-timing strategies derive from predictive regressions in 1959-2017 sample. The predictors are resp., $\text{SRR}^c_{t-1,t}$ (C), $\text{SRR}^g_{t-1,t}$ (G), $\text{SRR}^{mid}_{t-1,t}$ (Mid), $\text{SRR}^v_{t-1,t}$ (V), and all the above SRRs except $\text{SRR}^{mid}_{t-1,t}$ (All). Panel B reports the same results for the 1959-2005 sample for comparisons to (Campbell and Thompson, 2008). Panel C reports the out-of-sample R^2s in 1956-1980 and 1980-2005 samples documented in Campbell and Thompson (2008), Table 3.

Table V. Joint GMM Estimation of SRR and LRR

	Estimate	Std. Error	t-value	p-value
μ_c	0.8631	0.2877	2.9998	0.0014
μ_m	2.0189	1.2509	1.6140	0.0533
μ_g	2.3026	1.4013	1.6431	0.0502
μ_{mid}	2.1204	1.4740	1.4385	0.0751
μ_v	3.8946	2.5325	1.5378	0.0620
σ_X^2	1.4175	0.9067	1.5634	0.0590
ϕ_m	1.4532	1.6807	0.8646	0.1936
ϕ_g	0.1630	1.9564	0.0833	0.4668
ϕ_{mid}	1.6471	2.4789	0.6645	0.2532
ϕ_v	4.9598	3.0059	1.6500	0.0495
μ_{SRR^c}	0.0571	0.0054	10.6210	0.0000
μ_{SRR^m}	0.0216	0.0221	0.9736	0.1651
μ_{SRR^g}	0.0487	0.0479	1.0187	0.1542
$\mu_{SRR^{mid}}$	0.0363	0.0686	0.5283	0.2986
μ_{SRR_v}	0.2888	0.1047	2.7584	0.0029

This table summarizes the GMM estimation results for moment conditions (15), (16) and (17). For each parameter, the p-value is calculated from the one-sided test of the parameter equal to 0 against larger than 0. The covariance matrix is estimated by Newey-West estimator. Growth rates are in percent and SRRs are in square percentage.

Table VI. Statistics of the SRRs and Growth Rates

Panel A: Statistics of SRRs and Growth Rates, 1959–2017				
	C	M	G	V
Mean of $\text{SRR}_{t-1,t}$	0.028	0.027	0.054	0.250
SE of $\text{SRR}_{t-1,t}$	(0.030)	(0.162)	(0.323)	(0.715)
$\widehat{\text{Cov}}(\cdot, \Delta c_{t-1,t})$	1.488	2.113	0.261	7.478
Mean of $\text{ACF}_{t-1,t}(1/12)$	0.355	−0.261	−0.163	−0.243
SE of $\text{ACF}_{t-1,t}(1/12)$	(0.265)	(0.282)	(0.259)	(0.283)
Full Sample ACF(1)	0.479	0.286	0.078	0.304
Full Sample ACF(1/12)	0.924	0.039	−0.056	0.082
Panel B: Statistics of SRRs and Growth Rates, Simulations				
	Baseline	$\rho = 0$	$\sigma_x = 0$	$\mu_c = 0$
Mean of $\text{SRR}^c_{t-1,t}$	0.023	0.017	0.011	0.023
SE of $\text{SRR}^c_{t-1,t}$	(0.014)	(0.007)	(0.005)	(0.014)
$\widehat{\text{Var}}(\Delta c_{t-1,t})$	1.673	0.020	0.012	1.673
Mean of $\text{ACF}_{t-1,t}(1/12)$	0.188	−0.086	−0.076	0.188
SE of $\text{ACF}_{t-1,t}(1/12)$	(0.290)	(0.252)	(0.243)	(0.290)
Full Sample ACF(1)	0.836	−0.002	−0.032	0.836
Full Sample ACF(1/12)	0.905	0.003	0.004	0.905

Panel A summarizes the statistics of the SRRs, consumption growth (C), and the cash-flow growth of the market (M), growth (G), and value portfolios (V). The first row reports the averages of SRRs. In the second row, we list in parentheses the standard errors of the SRRs. The third row reports the unconditional variance of the annually aggregated consumption growth and the covariance between the annually aggregated consumption growth and cash-flow growth. The fourth row reports the average yearly observations of the first-order autocorrelations within the year of monthly aggregated growth rates, and the fifth row shows their standard errors in parentheses. The sixth row reports the first-order autocorrelations of the annual consumption growth and cash-flow growth. We report the first-order autocorrelation of the monthly consumption growth and cash-flow growth in the seventh row, both calculated using the full sample. Panel B lists the same statistics for simulated consumption growth under different dynamics specifications. We simulate monthly consumption growth for 1000 years and use the last 900 years to calculate statistics. The first column displays the baseline calibration ($\rho = 0.975, \sigma_x = 0.0237, \sigma_c = 0.032, \mu_c = 0.16$), the remaining columns correspond to cases where $\rho = 0$, $\sigma_x = 0$ and $\mu_c = 0$. SRRs, variances, and covariances are expressed in squared percentage terms.

Table VII. Baseline Calibrations

μ_c	0.0090	δ_1^m	0.2970	δ_1^x	0.1853	ϕ^m	5.5759	γ	2.4899
μ_m	0.0282	δ_2^m	0.1472	δ_2^x	0.2001	ϕ^g	4.6816		
μ_g	0.0320	δ_1^c	0.0904	σ^m	0.0808	ϕ^v	8.1676		
μ_v	0.0205	δ_2^c	0.0181	σ^g	0.0998	β	0.0210		
α	0.0869	$\bar{\sigma}^c$	0.0025	σ^v	0.0786	ψ	1.0325		

$$Q = \begin{bmatrix} 0.0029 & -0.0006 \\ -0.0006 & 0.0004 \end{bmatrix}, \; M = \begin{bmatrix} -0.1625 & -0.0000 \\ 0.1989 & -0.0875 \end{bmatrix}$$

$$\chi_c = \begin{bmatrix} -168.5532 & -8.2767 \\ -8.2767 & -0.5919 \end{bmatrix} \times 10^{-4}$$

This table reports the choice of values in the baseline calibration. All the matrices are of dimension 2×2. $\delta_m = (\delta_1^m, \delta_2^m)'$, $\delta_g = (1, 0)'$, $\delta_v = (0, 1)'$.

Table VIII. Consumption and Cash-Flow Growth

Panel A: Means, Standard Deviations and Autocorrelations							
	Model	Data	SE		Model	Data	SE
$\mathbb{E}(\Delta c)$	0.8989	0.8652	(0.3029)	$\sigma(\Delta d^g)$	10.6117	9.3667	(1.6733)
$\mathbb{E}(\Delta d^m)$	2.8205	2.0102	(0.9796)	$\sigma(\Delta d^v)$	11.9888	15.3426	(3.3309)
$\mathbb{E}(\Delta d^g)$	3.1957	2.2951	(1.3532)	$AC1(\Delta c)$	0.8593	0.4763	(0.1357)
$\mathbb{E}(\Delta d^v)$	2.0487	3.8589	(1.8820)	$AC1(\Delta d^m)$	0.1969	0.2847	(0.0846)
$\sigma(\Delta c)$	0.7806	1.2091	(0.1271)	$AC1(\Delta d^g)$	0.1019	0.0761	(0.1738)
$\sigma(\Delta d^m)$	8.0935	6.7134	(1.3045)	$AC1(\Delta d^v)$	0.3501	0.3050	(0.1495)
Panel B: The Correlations of Annual Growth and SRRs							
	Model	Data	SE		Model	Data	SE
$\text{Corr}(\Delta c, \Delta d^m)$	0.4402	0.2559	(0.1310)	$\mathbb{E}(\text{SRR}^m)$	0.0324	0.0272	(0.0205)
$\text{Corr}(\Delta c, \Delta d^g)$	0.3233	0.0226	(0.1046)	$\mathbb{E}(\text{SRR}^g)$	0.0817	0.0544	(0.0466)
$\text{Corr}(\Delta c, \Delta d^v)$	0.5880	0.3961	(0.0933)	$\mathbb{E}(\text{SRR}^v)$	0.2552	0.2501	(0.1185)
$\mathbb{E}(\text{SRR}^c)$	0.0547	0.0283	(0.0086)				

This table reports the model-implied (Model) and sample (Data) moments of the variables of interest, as well as their corresponding standard deviation (SE). In Panel A, we summarize the mean ($\mathbb{E}(\cdot)$), standard deviation ($\sigma(\cdot)$), and first-order autocorrelation ($AC1(\cdot)$) of the growth rates of the annually aggregated consumption and cash flows in value and growth stocks. In Panel B, we summarize the correlations of the growth rates in the annually aggregated consumption and dividends, the means of SRRs, and the means of short-run covariances between dividend growth rates. We consider the cash flows in the market portfolio (m), growth stocks (g,) and value stocks (v). Standard deviations are constructed by the delta method with Newey-West standard errors at eight lags. The growth rates are in percent. SRRs, variances, and covariances are in square percentage terms.

Table IX. Asset Returns

	Model	Data	SE		Model	Data	SE
$\mathbb{E}(r_f)$	0.6766	0.9931	(0.4603)	$\mathbb{E}(r_e^g)$	4.9702	4.5216	(2.3462)
$AC1(r_f)$	0.7592	0.8406	(0.0828)	$AC1(r_e^g)$	0.0355	-0.0420	(0.1555)
$\sigma(r_f)$	1.8632	1.6664	(0.2443)	$\sigma(r_e^g)$	15.6401	17.7419	(2.8189)
$\mathbb{E}(r_e^m)$	5.4609	4.8030	(2.2349)	$\mathbb{E}(P^g/D^g)$	53.7977	54.5673	(15.4136)
$AC1(r_e^m)$	0.0411	-0.0661	(0.1548)	$\mathbb{E}(r_e^v)$	7.3582	7.9921	(2.8214)
$\sigma(r_e^m)$	16.0553	16.8272	(2.7533)	$AC1(r_e^v)$	0.0555	-0.1216	(0.1610)
$\mathbb{E}(P^m/D^m)$	39.1322	39.9934	(11.3543)	$\sigma(r_e^v)$	19.8531	18.0551	(2.6285)
				$\mathbb{E}(P^v/D^v)$	20.3240	35.3418	(10.3722)

This table reports the model-implied (Model) and sample (Data) moments of the asset price dynamics, including the means ($E(r)$), the standard deviations ($\sigma(r)$) and the first-order autocorrelations ($AC1(r)$) of the annually aggregated returns and the mean of price-dividend ratios ($E(P/D)$). The in-sample standard deviations (SE) are also reported. Standard deviations are constructed by the delta method with Newey-West standard errors at eight lags. The assets under consideration are risk-less asset (f), market portfolio (m), growth stocks (g) and value stocks (v). Numbers are in percent.

Table X. The Decomposition of the Risk Premium

	$\gamma\,\mathrm{Cov}_t(\Delta c, \Delta d)$	LRR	SRR
Market	0.0008	0.4752	4.9849
Growth	0.0020	0.4082	4.5600
Value	0.0014	0.6134	6.7434

This table reports the decomposition of the risk premium in the market portfolio, and growth and value stocks. The risk premium can be attributed to three sources: the risk aversion times the instantaneous covariance between the growth rates of consumption and dividends ($\gamma\,\mathrm{Cov}_t(\Delta c, \Delta d)$), the LRR and the SRR. Returns are in percent.

Table XI. Predictive Regression Coefficients

	Market Excess Returns			V-G Returns
	C	G	V	C
Model	-216.417	11.597	-3.213	167.466
Data	-202.475	10.246	-6.265	81.677
SE	56.081	4.934	3.067	23.986

This table reports the coefficients of the predictive regressions in model (Model) and data (Data), and standard errors of estimates in data (SE). In the first three columns, we predict 4-quarter horizon future market-excess returns using SRR in consumption (C), dividends of growth stocks (G), and value stocks (V). In the last column, the SRR in consumption (C) is used to predict 12-quarter horizon value-minus-growth returns.

Table XII. Forecast Future Excess Returns with SRRs (Adjusted for Repurchases)

	Market Excess Returns			Value-Minus-Growth Returns		
	4Q	8Q	12Q	4Q	8Q	12Q
C	-194.006^{***}	-93.434^{**}	-37.034	79.491^{***}	86.510^{***}	81.961^{***}
	(55.536)	(41.937)	(31.696)	(26.781)	(25.305)	(21.195)
G	0.180	0.340	0.477^{**}	0.405^{**}	-0.134	-0.265^{*}
	(0.453)	(0.316)	(0.207)	(0.206)	(0.205)	(0.143)
V	-1.844^{*}	-1.547^{***}	0.009	0.654	0.903^{**}	0.501^{*}
	(1.003)	(0.590)	(0.450)	(0.631)	(0.383)	(0.302)
Constant	10.959^{***}	7.760^{***}	5.457^{***}	0.770	0.688	0.985
	(2.248)	(2.145)	(2.072)	(1.598)	(1.405)	(1.228)
Adjusted R^2	0.133	0.090	0.010	0.057	0.133	0.161

Note: *p<0.1; **p<0.05; ***p<0.01

This table reports the results of the predictive regressions. The independent variables include $\text{SRR}^c_{t-1,t}$ (C), $\text{SRR}^g_{t-1,t}$ (G), $\text{SRR}^v_{t-1,t}$ (V). The first (last) three columns report results from regression forecasting the future 4, 8, and 12 quarters market excess returns (value-minus-growth returns). Newey-West (NW) standard errors with lag 8 are shown in parentheses, based on which the significance level is determined. The variances and covariances are represented in square percentage terms, and returns are represented in percent.

Table XIII. Annualized Moments and Errors

	Log-Lin	Real	Error		Log-Lin	Real	Error
$\mathbb{E}(r_f)$	1.1353	1.1456	0.90%	$\mathbb{E}(r_e^g)$	4.7788	4.7474	0.66%
$\sigma(r_f)$	1.4091	1.3984	0.77%	$\sigma(r_e^g)$	7.5050	7.5004	0.06%
$\mathbb{E}(r_e^m)$	4.8350	4.8216	0.28%	$\mathbb{E}(p^g - d^g)$	3.9680	3.9671	0.02%
$\sigma(r_e^m)$	10.2310	10.2492	0.18%	$\mathbb{E}(r_e^v)$	7.1432	7.1916	0.67%
$\mathbb{E}(p^m - d^m)$	3.6494	3.6487	0.02%	$\sigma(r_e^v)$	20.2725	20.3363	0.31%
				$\mathbb{E}(p^v - d^v)$	3.3703	3.3701	0.01%

This table shows the means and standard deviations of the annualized risk-free rate, the market excess returns, and the excess returns of growth and value stocks. The table also reports the means of logarithms of the price-dividend ratios. The relative errors in moments are determined through dividing the absolute difference between the projection method (Real) and the log-linearization (Log-Lin) moments by Real. The model moments are calculated via Monte-Carlo method by taking the average of a simulated sample of 2,400,000 monthly data.

Table XIV. Forecast Future Returns with SRIRs

	Market Excess Returns			Value-Minus-Growth Returns		
	4Q	8Q	12Q	4Q	8Q	12Q
Ind	1.097	0.702	0.671	2.407	2.102	2.009**
	(1.412)	(1.328)	(1.112)	(1.703)	(1.283)	(0.812)
G	−1.127	0.348	0.638*	−0.495	−0.431	−0.559
	(1.158)	(0.606)	(0.329)	(0.845)	(0.717)	(0.698)
V	−0.657**	−0.099	−0.029	0.347**	0.455***	0.303***
	(0.307)	(0.174)	(0.123)	(0.155)	(0.109)	(0.115)
Constant	4.171*	4.059*	3.995**	2.790*	3.072**	3.150**
	(2.438)	(2.112)	(1.741)	(1.679)	(1.481)	(1.316)
Adjusted R^2	0.013	−0.004	0.006	0.015	0.052	0.061

Note: *p<0.1; **p<0.05; ***p<0.01

This table reports the results of predictive regressions. Independent variables include $\widehat{\text{SRIR}}^c_{t-1,t}$ (Ind), $\widehat{\text{SRIR}}^g_{t-1,t}$ (G), $\widehat{\text{SRIR}}^v_{t-1,t}$ (V). The first (last) three columns report results from regression forecasting future 4, 8, and 12 quarters market excess returns (value-minus-growth returns). Newey-West (NW) standard errors with eight lag are shown in parentheses, based on which significance level is determined. SRIRs are represented in square percentage terms, and returns are represented in percent.

Table XV. Theoretical Moments

$$
\begin{aligned}
\mathbb{E}[\Delta c_t] &= \mu_c \\
\sigma(\Delta c_t) &= \sqrt{\delta_c'\Sigma(\infty)\delta_c + \mathrm{Tr}\left(\chi_c\Sigma(\infty)\right) + \bar{\sigma}_c^2 + \frac{\alpha-1+e^{-\alpha}}{\alpha^3}\delta_x'\Sigma(\infty)\delta_x} \\
AC1(\Delta c_t) &= \frac{\frac{1-2e^{-\alpha}+e^{-2\alpha}}{2\alpha^3}\delta_x'\Sigma(\infty)\delta_x}{\frac{\alpha-1+e^{-\alpha}}{\alpha^3}\delta_x'\Sigma(\infty)\delta_x + \delta_c'\Sigma(\infty)\delta_c + \mathrm{Tr}\left(\chi_c\Sigma(\infty)\right) + \bar{\sigma}_c^2} \\
\mathbb{E}[\Delta d_t^i] &= \mu_i \\
\sigma(\Delta d_t^i) &= \sqrt{\delta_i'\Sigma(\infty)\delta_i + \frac{\alpha-1+e^{-\alpha}}{\alpha^3}\delta_x'\Sigma(\infty)\delta_x\phi_i^2 + \sigma_i^2} \\
AC1(\Delta d_t^i) &= \frac{\frac{1-2e^{-\alpha}+e^{-2\alpha}}{2\alpha^3}\delta_x'\Sigma(\infty)\delta_x\phi_i^2}{\delta_i'\Sigma(\infty)\delta_i + \frac{\alpha-1+e^{-\alpha}}{\alpha^3}\delta_x'\Sigma(\infty)\delta_x\phi_i^2 + \sigma_i^2} \\
\mathrm{Cov}(\Delta d_t^i, \Delta c_t) &= \frac{\alpha-1+e^{-\alpha}}{\alpha^3}\phi_i\delta_x'\Sigma(\infty)\delta_x + \delta_i'\Sigma(\infty)\delta_c \\
\mathrm{Cov}(\Delta d_t^i, \Delta d_t^j) &= \frac{\alpha-1+e^{-\alpha}}{\alpha^3}\phi_i\phi_j\delta_x'\Sigma(\infty)\delta_x + \delta_i'\Sigma(\infty)\delta_j \\
\mathbb{E}[r_f] &= r_0 + \mathrm{Tr}(r_\Sigma\Sigma(\infty)) \\
\sigma(r_f) &= \sqrt{\frac{\alpha-1+e^{-\alpha}}{\alpha^3}r_x^2\delta_x'\Sigma(\infty)\delta_x + \mathrm{Var}\left(\mathrm{Tr}\left(r_\Sigma\int_t^{t+1}\Sigma_s ds\right)\right)} \\
AC1(r_f) &= \frac{\frac{1-2e^{-\alpha}+e^{-2\alpha}}{2\alpha^3}r_x^2\delta_x'\Sigma(\infty)\delta_x + \mathrm{Cov}(\mathrm{Tr}(r_\Sigma\int_t^{t+1}\Sigma_s ds), \mathrm{Tr}(r_\Sigma\int_{t+1}^{t+2}\Sigma_u du))}{\frac{\alpha-1+e^{-\alpha}}{\alpha^3}r_x^2\delta_x'\Sigma(\infty)\delta_x + \mathrm{Var}(\mathrm{Tr}(r_\Sigma\int_t^{t+1}\Sigma_s ds))} \\
\mathbb{E}[r_e^i] &= \gamma\delta_c'\Sigma(\infty)\delta_i + \left(\frac{1-\psi\gamma}{1-\gamma}\right)A_1A_{1i}\delta_x'\Sigma(\infty)\delta_x + 4\left(\frac{1-\psi\gamma}{1-\gamma}\right)\mathrm{Tr}(QA_{2i}\Sigma(\infty)A_2Q) \\
\sigma(r_e^i) &= \sqrt{A_{1i}^2\delta_x'\Sigma(\infty)\delta_x + \delta_i'\Sigma(\infty)\delta_i + 4\,\mathrm{Tr}(QA_{2i}\Sigma(\infty)A_{2i}Q) + \mathrm{Var}\left(\mathrm{Tr}\left(\lambda_e^i\int_t^{t+1}\Sigma_s ds\right)\right) + \sigma_i^2} \\
AC1(r_e^i) &= \frac{\mathrm{Cov}\left(\mathrm{Tr}(\lambda_e^i\int_t^{t+1}\Sigma_s ds), \mathrm{Tr}(\lambda_e^i\int_{t-1}^{t}\Sigma_u du)\right)}{\sigma(r_e^i)^2}
\end{aligned}
$$

This table shows the expressions of model-implied moments. For notational simplicity, we denote the coefficient on Σ_t for excess returns of portfolio i by

$$\lambda_e^i := \gamma\delta_i\delta_c' + \left(\frac{1-\psi\gamma}{1-\gamma}\right)A_1A_{1i}\delta_x\delta_x' + 4\left(\frac{1-\psi\gamma}{1-\gamma}\right)A_2QQA_{2i}$$

Table XVI. Three BM Portfolios Calibrations

μ_c	0.011733	δ_1^m	0.257017	$\bar{\sigma}^c$	0.001361	σ^v	0.093024	ψ	1.013851
μ_m	0.030143	δ_2^m	0.682196	δ_1^x	0.246294	ϕ^m	3.88298	γ	2.246633
μ_{mid}	0.029882	δ_1^{mid}	0.132705	δ_2^x	0.365603	ϕ^g	0.339146		
μ_g	0.020635	δ_2^{mid}	0.092662	σ^m	0.093194	ϕ^{mid}	4.334124		
μ_v	0.029515	δ_1^c	0.016963	σ^g	0.147981	ϕ^v	4.406522		
α	0.082343	δ_2^c	0.043852	σ^{mid}	0.074732	β	0.021125		

$$Q = \begin{bmatrix} 0.0201 & -0.0062 \\ -0.0062 & 0.0029 \end{bmatrix}, \ M = \begin{bmatrix} -0.4546 & -0.0000 \\ 0.0560 & -0.1018 \end{bmatrix}$$

$$\chi_c = \begin{bmatrix} -372.0080 & -221.1621 \\ -221.1621 & -227.3284 \end{bmatrix} \times 10^{-4}$$

This table reports the choice of values in the calibration using all three book-to-market portfolios. All the matrices are of dimensional 2×2. $\delta_m = (\delta_1^m, \delta_2^m)'$, $\delta_g = (1, 0)'$, $\Delta_{Mid} = ($, $\delta_v = (0, 1)'$.

Table XVII. Consumption and Cash-Flow Growth of Three BM Portfolios

Panel A: Means, Standard Deviations and Autocorrelations							
	Model	Data	SE		Model	Data	SE
$\mathbb{E}(\Delta c)$	1.1733	0.8652	(0.3029)	$\sigma(\Delta d^{mid})$	9.0337	7.6470	(1.5664)
$\mathbb{E}(\Delta d^m)$	3.0143	2.0102	(0.9796)	$\sigma(\Delta d^v)$	10.7972	15.3426	(3.3309)
$\mathbb{E}(\Delta d^g)$	2.9882	2.2951	(1.3532)	$AC1(\Delta c)$	0.9354	0.4763	(0.1357)
$\mathbb{E}(\Delta d^{mid})$	2.0635	2.1010	(1.0559)	$AC1(\Delta d^m)$	0.1806	0.2847	(0.0846)
$\mathbb{E}(\Delta d^v)$	2.9515	3.8589	(1.8820)	$AC1(\Delta d^g)$	0.0006	0.0761	(0.1738)
$\sigma(\Delta c)$	1.1752	1.2091	(0.1271)	$AC1(\Delta d^{mid})$	0.2974	0.4228	(0.0771)
$\sigma(\Delta d^m)$	10.3846	6.7134	(1.3045)	$AC1(\Delta d^v)$	0.2152	0.3050	(0.1495)
$\sigma(\Delta d^g)$	15.2888	9.3667	(1.6733)				
Panel B: The Correlations of Annual Growth and SRRs							
	Model	Data	SE		Model	Data	SE
$\text{Corr}(\Delta c, \Delta d^m)$	0.4363	0.2559	(0.1310)	$\mathbb{E}(\text{SRR}^m)$	0.0267	0.0272	(0.0205)
$\text{Corr}(\Delta c, \Delta d^g)$	0.0242	0.0226	(0.1046)	$\mathbb{E}(\text{SRR}^g)$	-0.0275	0.0544	(0.0466)
$\text{Corr}(\Delta c, \Delta d^{mid})$	0.5570	0.2587	(0.1683)	$\mathbb{E}(\text{SRR}^{mid})$	0.0009	-0.0105	(0.0492)
$\text{Corr}(\Delta c, \Delta d^v)$	0.4777	0.3961	(0.0933)	$\mathbb{E}(\text{SRR}^v)$	0.2394	0.2501	(0.1185)
$\mathbb{E}(\text{SRR}^c)$	0.0168	0.0283	(0.0086)				

This table reports the model-implied (Model) and sample (Data) moments of the variables of interest, as well as their corresponding standard deviation (SE) in sample. In Panel A, we summarize the mean ($\mathbb{E}(\cdot)$), standard deviation ($\sigma(\cdot)$) and first-order autocorrelation ($AC1(\cdot)$) of the growth rates of the annually aggregated consumption and cash flows in value and growth stocks. In Panel B, we summarize the correlations of the growth rates in the annually aggregated consumption and dividends, the means of SRRs, and the means of short-run covariances between dividend growth rates. We consider the cash flows in the market portfolio (m), growth stocks (g), middle stocks (Mid) and value stocks (v). Standard deviations are constructed by the delta method with NW errors at eight lags. The growth rates are in percentage points. SRRs, variances, and covariances in square percentage terms.

Table XVIII. Asset Returns of Three BM Portfolios

	Model	Data	SE		Model	Data	SE
$\mathbb{E}(r_f)$	1.1108	0.9931	(0.4603)	$\sigma(r_e^g)$	15.9970	17.7419	(2.8189)
$AC1(r_f)$	0.6795	0.8406	(0.0828)	$\mathbb{E}(P^g/D^g)$	40.7345	54.5673	(15.4136)
$\sigma(r_f)$	1.8421	1.6664	(0.2443)	$\mathbb{E}(r_e^{mid})$	5.4156	5.7247	(2.2927)
$\mathbb{E}(r_e^m)$	5.2068	4.8030	(2.2349)	$AC1(r_e^{mid})$	0.0261	-0.0586	(0.1492)
$AC1(r_e^m)$	0.0228	-0.0661	(0.1548)	$\sigma(r_e^{mid})$	15.9774	15.7099	(2.6285)
$\sigma(r_e^m)$	16.4399	16.8272	(2.7533)	$\mathbb{E}(P^{mid}/D^{mid})$	26.7539	33.7448	(9.5053)
$\mathbb{E}(P^m/D^m)$	38.7657	39.9934	(11.3543)	$\mathbb{E}(r_e^v)$	7.6516	7.9921	(2.8214)
$\mathbb{E}(r_e^g)$	4.3950	4.5216	(2.3462)	$AC1(r_e^v)$	0.0221	-0.1216	(0.1610)
$AC1(r_e^g)$	0.0171	-0.0420	(0.1555)	$\sigma(r_e^v)$	18.1427	18.0551	(2.6285)
				$\mathbb{E}(P^v/D^v)$	34.7359	35.3418	(10.3722)

This table reports the model-implied (Model) and sample (Data) moments of the asset price dynamics, including the means ($E(r)$), the standard deviations ($\sigma(r)$) and the first-order autocorrelations ($AC1(r)$) of the annually aggregated returns and the mean of price-dividend ratios ($E(P/D)$). The in-sample standard deviations (SE) are also reported. Standard deviations are constructed by the delta method with NW errors at eight lags. The assets under consideration are risk-less asset (f), market portfolio (m), growth stocks (g) and value stocks (v). Numbers are reported in percentage points.

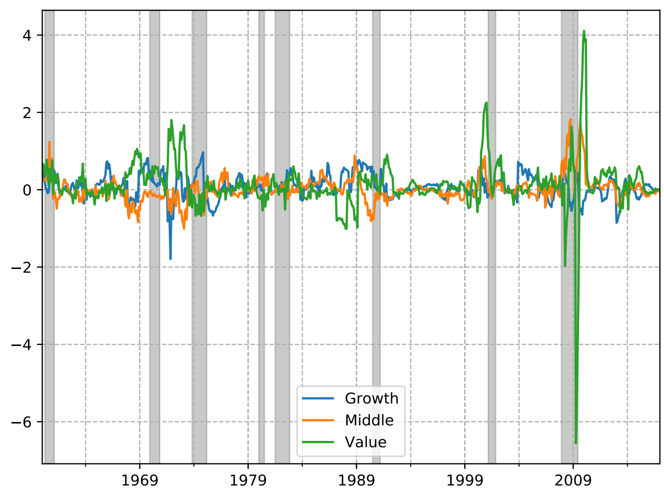
Table XIX. The Decomposition of the Risk Premium of Three BM Portfolios

	$\gamma\,\mathrm{Cov}_t(\Delta c, \Delta d)$	LRR	SRR
Market	0.0006	0.6252	4.5810
Growth	-0.0006	-0.1413	4.5370
Middle	0.0000	0.6526	4.7629
Value	0.0011	0.7183	6.9322

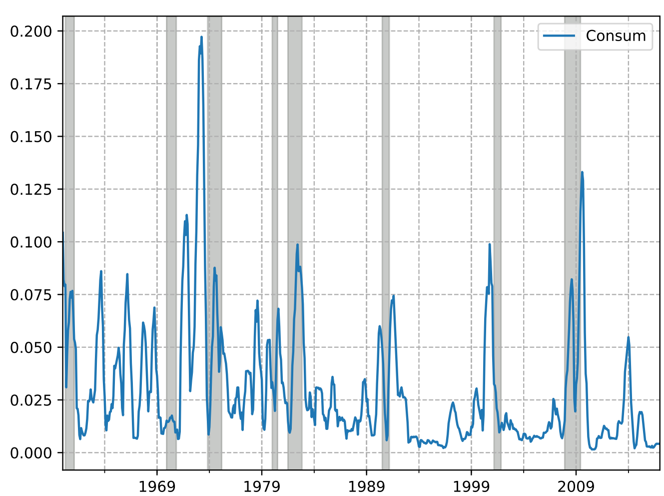
This table reports the decomposition of the risk premium in the market portfolio and growth, middle, and value stocks. The risk premium can be attributed to three sources: the risk aversion times the instantaneous covariance between the growth rates of consumption and dividends ($\gamma\,\mathrm{Cov}_t(\Delta c, \Delta d)$), the LRR, and the SRR. Numbers are reported in percentage points.

Figures

Figure 1. Short-run consumption risks (SRRs)



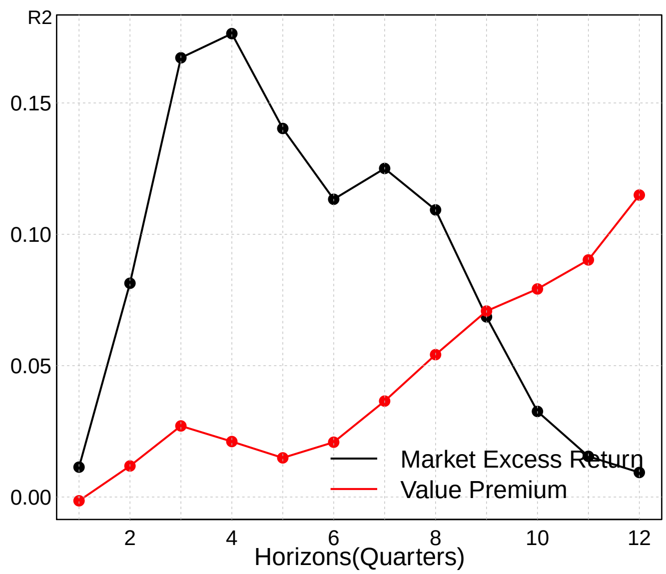
Panel A



Panel B

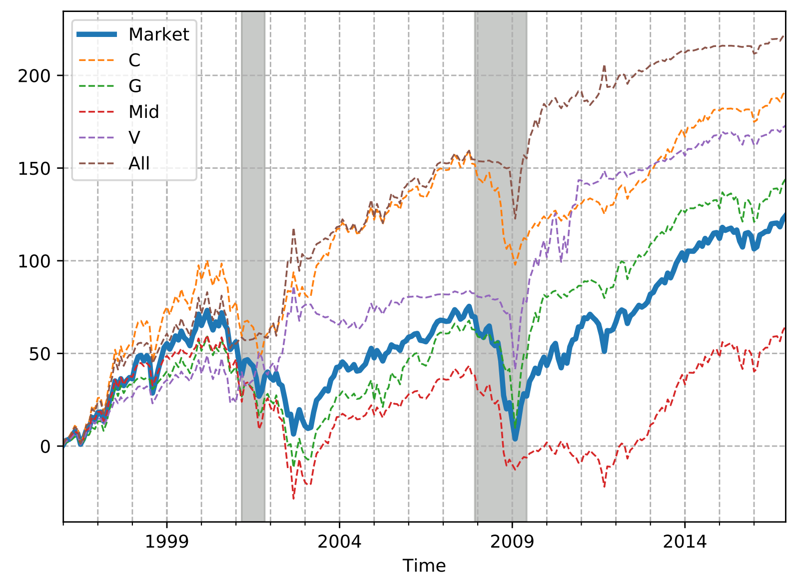
In Panel A, we plot the monthly observations of SRRs in value, middle, and growth stocks. In Panel B, we plot the monthly observations of SRRs in consumption. The shaded areas are NBER recorded recession periods. To construct value, growth and middle portfolios, we use the upper and lower 30 percentiles, and the middle 40 percentiles of the value-weighted portfolios formed on book-to-market of individual stocks. Our sample ranges from 1959 to 2017. In k-th month in year t, we calculate the SRRs according to Equations (7) and (8). SRRs are represented in squared percentage terms.

Figure 2. Adjusted R^2 of Predictive Regressions Over Different Horizons



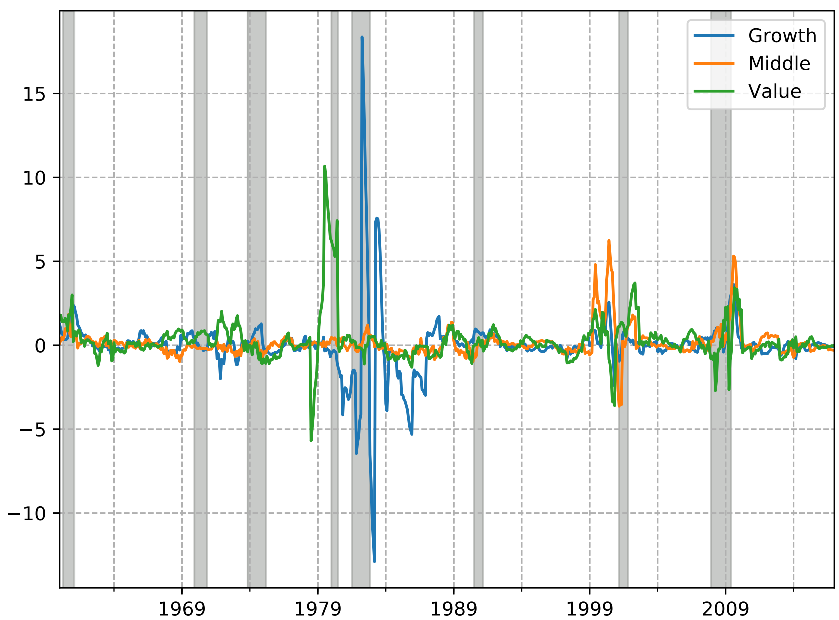
This figure plots the adjusted R^2s (in percent) of the predictive regression of future returns on SRRs specified by Equation (9). The dependent variables are the future market excess returns or the value-minus-growth returns at future Q-quarter horizons. The independent variables are the SRRs in consumption, value, and growth stocks. The black line shows the adjusted R^2 for predicting the future market excess returns, and the red line the adjusted R^2 for predicting the future value-minus-growth returns.

Figure 3. Cumulative Excess Returns



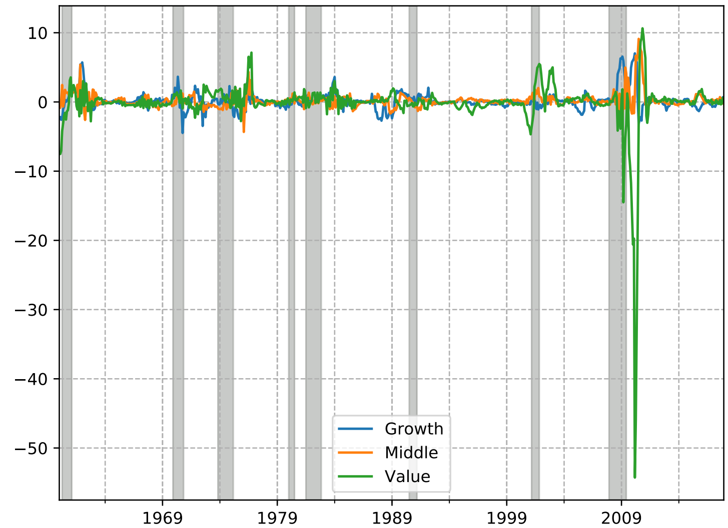
We plot cumulative excess returns using different investment strategies starting at the end of 1995. In each such strategy, we multiply the position in the market excess returns by the predictions from the out-of-sample regressions. As predictors, we use $\text{SRR}^{c}_{t-1,t}$ (C), $\text{SRR}^{g}_{t-1,t}$ (G), $\text{SRR}^{mid}_{t-1,t}$ (Mid), $\text{SRR}^{v}_{t-1,t}$ (V), and all the above SRRs except $\text{SRR}^{mid}_{t-1,t}$ (All). As a benchmark, we plot the cumulative market excess returns. We rescale the weights on the market excess returns ex post to ensure that all of the portfolios have the same volatility in returns. The out-of-sample predictive regressions estimate coefficients on a rolling basis from the past 35 years of data, and the weight on the market excess returns is updated at the beginning of each year.

Figure 4. SRRs in Value, Middle and Growth Stocks (Adjusted for Repurchases)

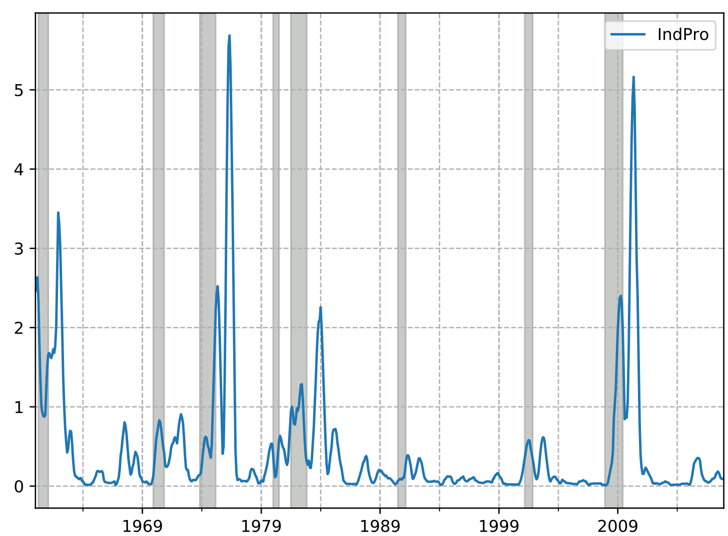


This figure displays the monthly observations of SRRs (adjusted for repurchases) in value, middle, and growth stocks. The shaded areas are NBER recorded recession periods. For the portfolios of value and growth stocks, we use the upper and lower 30 percentiles, and the middle 40 percentiles of the value-weighted portfolios formed on book-to-market. Our sample ranges from 1959–2017.

Figure 5. SRIRs



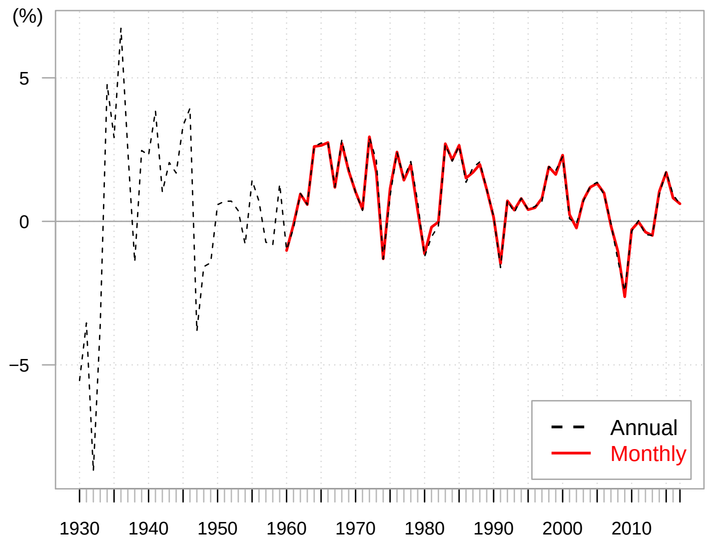
Panel A



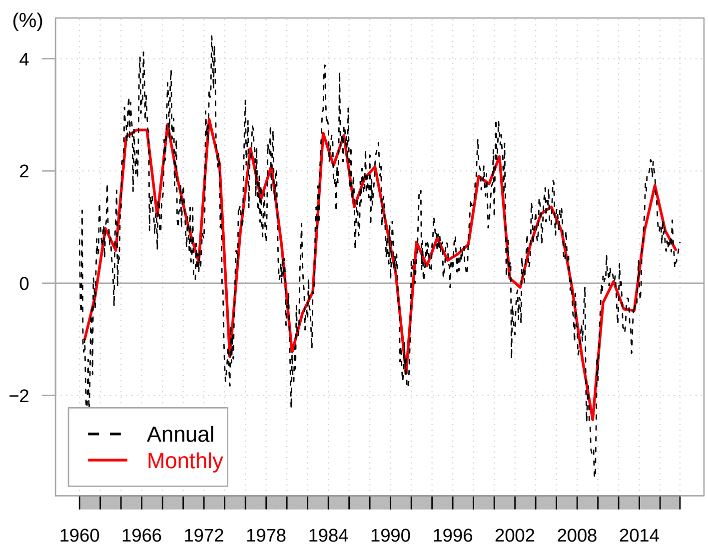
Panel B

In Panel A, we plot the monthly observations of SRIRs in the value, middle, and growth portfolios. In Panel B, we plot the monthly observations of SRIRs in the industrial production index growth. The shaded areas correspond to the NBER recorded recession periods. For the value, growth, and middle portfolios, we use the upper and lower 30 percentiles, and the middle 40 percentiles of the value-weighted portfolios formed on book-to-market. Our sample ranges from 1959–2017. We estimate the SRIRs similar to Equations (7) and (8), with consumption replaced by the industrial production index. SRIRs are represented in square percentage terms.

Figure 6. Annual Consumption Growth Constructed from Annual and Monthly Aggregated Consumptions



Panel A



Panel B

Panel A plots the log annual growth rates of annually aggregated consumption calculated as $\log \frac{C_{t,t+1}}{C_{t-1,t}}$ (dashed line) and calculated from summing up the log monthly growth rates within a year of annually aggregated consumption $\sum_{k=1}^{12} \log \frac{C_{t+(k-1)/12-1,t+(k-1)/12}}{C_{t+k/12-1,t+k/12}}$ (solid line). Panel B plots the annualized ($\times 12$) log monthly growth rate of annually aggregated consumption calculated as $\log \frac{C_{t+(k-1)/12-1,t+(k-1)/12}}{C_{t+k/12-1,t+k/12}}$ (dash line) together with the log annual growth rates of annually aggregated consumption $\log \frac{C_{t,t+1}}{C_{t-1,t}}$ (solid line). All growth rates are in percent.