Inverse scattering for scattering data with poor regularity or slow decay

Kappeler, T

DOI: https://doi.org/10.1216/JIE-1988-1-1-123

Originally published at:
Kappeler, T (1988). Inverse scattering for scattering data with poor regularity or slow decay. Journal of Integral Equations and Applications, 1(1):123-145.
DOI: https://doi.org/10.1216/JIE-1988-1-1-123

JOURNAL OF INTEGRAL
EQUATIONS AND APPLICATIONS
Volume 1, Number 1, Winter 1988

INVERSE SCATTERING FOR SCATTERING DATA WITH POOR REGULARITY OR SLOW DECAY

TH. KAPPELER

1. Introduction. Motivation to study the inverse scattering problem for scattering data with poor regularity or slow decay is an application which will be given in two subsequent papers [**7, 21**] for the Cauchy problem of the Korteweg-deVries equation (KdV) $u_t - 6uu_x + u_{xxx} = 0$ with irregular initial profile as, e.g., a smooth enough box shaped potential or a steplike a smoothed Heavyside function [**4,5**].

If we consider $u(x)$ as a potential for the Schrödinger equation $-y''(x) + u(x)y(x) = k^2y(x)$ we can associate to u, by a well known procedure [**8,9**], the scattering data of which a part is given by the so called scattering matrix (T_+, R_+, T_-, R_-). To find a solution $u(x,t)$ of the KdV $(t > 0)$ it is enough to study the evolution of the scattering in time and to construct $u(x,t)$ by the inverse problem [**3, 4, 5, 7, 10, 11, 12, 13**]. Often, however, the evolution of the scattering data, especially R_-, does not stay within the set where the inverse problem was known to be solvable [**4, 5**].

Let us briefly outline the organization of the paper. In §2 we discuss the Marchenko equation in $L_2(R_-)$. In §3 we study the inverse scattering problem under weaker decay and regularity properties of R_- and its Fourier transform than in [**8, 9**].

Let us introduce the following notation. Let f be a complex valued function defined on $\mathbf{R}$. By $\tau_x f$ we denote the translated function $\tau_x f(y) := f(x+y)$ (x and y in $\mathbf{R}$). If $h \neq 0$ we denote by $\Delta_h f$ the differential quotient $(\Delta_h f)(x) := \frac{(f(x+h) - f(x))}{h}$.

Let f be in $L_2(\mathbf{R})$. By $\hat{f}$ we denote the Fourier transform $\hat{f}(k) := \int_{-\infty}^{\infty} f(x)e^{2ikx}dx$. By $\tau_x f$ we define the operator on $L_2(\mathbf{R}_-)$ defined by

$$\tau_x f(g)(y) := \int_{-\infty}^{0} \tau_z f(x+z)g(z)dz \quad (g \text{ in } L_2(\mathbf{R}_-)).$$

By $'$ or ∂_x we denote the derivation with respect to x. For a complex