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ASYMPTOTIC DEPTH OF TWISTED HIGHER DIRECT IMAGE SHEAVES

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ABSTRACT. Let $\pi : X \rightarrow X_0$ be a projective morphism of schemes, such that X_0 is Noetherian and essentially of finite type over a field K . Let $i \in \mathbb{N}_0$, let \mathcal{F} be a coherent sheaf of \mathcal{O}_X -modules and let \mathcal{L} be an ample invertible sheaf over X . Let $Z_0 \subseteq X_0$ be a closed set. We show that the depth of the higher direct image sheaf $\mathcal{R}^i \pi_*(\mathcal{L}^n \otimes_{\mathcal{O}_X} \mathcal{F})$ along Z_0 ultimately becomes constant as n tends to $-\infty$, provided X_0 has dimension ≤ 2 . There are various examples which show that the mentioned asymptotic stability may fail if $\dim(X_0) \geq 3$. To prove our stability result, we show that for a finitely generated graded module M over a homogeneous Noetherian ring $R = \bigoplus_{n \geq 0} R_n$ for which R_0 is essentially of finite type over a field and an ideal $\mathfrak{a}_0 \subseteq R_0$, the \mathfrak{a}_0 -depth of the n -th graded component $H_{R_+}^i(M)_n$ of the i -th local cohomology module of M with respect to $R_+ := \bigoplus_{k > 0} R_k$ ultimately becomes constant in codimension ≤ 2 as n tends to $-\infty$.

1. INTRODUCTION

Let $\pi : X \rightarrow X_0$ be a projective morphism of schemes, such that X_0 is Noetherian and essentially of finite type over a field. Let \mathcal{F} be a coherent sheaf of \mathcal{O}_X -modules and let \mathcal{L} be an ample invertible sheaf of \mathcal{O}_X -modules. Let $i \in \mathbb{N}_0$. In [2, Theorem 5.5] we did show:

(1.1) *As $n \rightarrow -\infty$, the set*

$$\begin{aligned} & \text{Ass}_{X_0}(\mathcal{R}^i \pi_*(\mathcal{L}^n \otimes_{\mathcal{O}_X} \mathcal{F}))^{\leq 2} \\ & := \{x_0 \in \text{Ass}_{X_0}(\mathcal{R}^i \pi_*(\mathcal{L}^n \otimes_{\mathcal{O}_X} \mathcal{F})) \mid \dim(\mathcal{O}_{X_0, x_0}) \leq 2\} \end{aligned}$$

ultimately becomes constant.

Our aim is to prove a corresponding but stronger stability result for the depths in codimension ≤ 2 of the sheaves $\mathcal{R}^i \pi_*(\mathcal{L}^n \otimes_{\mathcal{O}_X} \mathcal{F})$ along a closed subset $Z_0 \subseteq X_0$.

To make this precise we introduce the following notion: If $Z_0 \subseteq X_0$ is a closed set, \mathcal{G} is a coherent sheaf of \mathcal{O}_{X_0} -modules and $t \in \mathbb{N}_0$, we define the *depth of \mathcal{G} along Z_0* and the *depth in codimension $\leq t$ of \mathcal{G} along Z_0* respectively by

$$(1.2) \text{ depth}(Z_0, \mathcal{G}) := \inf\{\text{depth}(\mathcal{G}_{x_0}) \mid x_0 \in Z_0\};$$

$$(1.3) \text{ depth}(Z_0, \mathcal{G})^{\leq t} := \inf\{\text{depth}(\mathcal{G}_{x_0}) \mid x_0 \in Z_0, \dim(\mathcal{O}_{X_0, x_0}) \leq t\}.$$

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We use the convention that $\inf(\emptyset) := \infty$, so that

$$\text{depth}(Z_0, \mathcal{G})^{\leq t} = \infty \text{ if } \text{codim}(Z_0, X_0) > t.$$

Our main result now says (cf. Theorem 3.5)

(1.4) *As $n \rightarrow -\infty$, the number*

$$\text{depth}(Z_0, \mathcal{R}^i \pi_* (\mathcal{L}^n \otimes_{\mathcal{O}_X} \mathcal{F}))^{\leq 2}$$

ultimately becomes constant.

As an immediate consequence we obtain (cf. Corollary 3.6)

(1.5) *Let $\dim(X_0) \leq 2$. Then, as $n \rightarrow -\infty$, the number*

$$\text{depth}(Z_0, \mathcal{R}^i \pi_* (\mathcal{L}^n \otimes_{\mathcal{O}_X} \mathcal{F}))$$

ultimately becomes constant.

The examples constructed by Chardin-Cutkosky-Herzog-Srinivasan [5] illustrate that the conclusion of (1.5) need not hold if $\dim(X_0) \geq 3$.

The basic tool to prove our main result is a corresponding stability result for the depths of graded components of certain local cohomology modules. We shall establish this result in the next section.

2. DEPTH AND LOCAL COHOMOLOGY

By \mathbb{N}_0 we denote the set of non-negative integers, and by \mathbb{N} we denote the set of positive integers.

Notation and Conventions 2.1. (A) Throughout this paper let $R = R_0 \oplus R_1 \oplus \dots$ be a *standard graded ring*. So, R is \mathbb{N}_0 -graded, R_0 is Noetherian and there are finitely many elements $a_1, \dots, a_k \in R_1$ such that $R = R_0[a_1, \dots, a_k]$. By R_+ we denote the *irrelevant ideal* of R ; thus $R_+ = R_1 \oplus R_2 \oplus \dots$.

(B) If $i \in \mathbb{N}_0$ and M is a graded R -module, we write $H_{R_+}^i(M)$ for the i -th *local cohomology module of M with respect to R_+* , and we always furnish this module with its natural grading.

For $n \in \mathbb{Z}$ we denote by $H_{R_+}^i(M)_n$ the n -th *graded component of $H_{R_+}^i(M)$* . Keep in mind that $H_{R_+}^i(M)_n$ is a finitely generated R_0 -module for all $n \in \mathbb{Z}$ and vanishes for all $n \gg 0$, provided that the graded R -module M is finitely generated.

(C) Now, fix an ideal $\mathfrak{a}_0 \subseteq R_0$. We write $\text{Var}(\mathfrak{a}_0)$ for the *variety* $\{\mathfrak{p}_0 \in \text{Spec}(R_0) \mid \mathfrak{a}_0 \subseteq \mathfrak{p}_0\}$ of \mathfrak{a}_0 . Keep in mind that for a finitely generated R_0 -module T we always have

$$\text{depth}(\mathfrak{a}_0, T) = \inf\{\text{depth}(T_{\mathfrak{p}_0}) \mid \mathfrak{p}_0 \in \text{Var}(\mathfrak{a}_0)\}.$$

Now, for any finitely generated R_0 -module T and any $t \in \mathbb{N}_0$ we define the *depth in codimension $\leq t$ of T with respect to \mathfrak{a}_0* by

$$\text{depth}(\mathfrak{a}_0, T)^{\leq t} = \inf\{\text{depth}(T_{\mathfrak{p}_0}) \mid \mathfrak{p}_0 \in \text{Var}(\mathfrak{a}_0), \text{height}(\mathfrak{p}_0) \leq t\}.$$

Again we use the convention that $\inf(\emptyset) = \infty$, so that

$$\text{depth}(\mathfrak{a}_0, T)^{\leq t} \in \{0, 1, \dots, t, \infty\}$$

with

$$\text{depth}(\mathfrak{a}_0, T)^{\leq t} = \infty \iff \forall \mathfrak{p}_0 \in \text{Var}(\mathfrak{a}_0) \cap \text{Supp}(T) : \text{height}(\mathfrak{p}_0) > t.$$

(D) We say that a graded R -module $U = \bigoplus_{n \in \mathbb{Z}} U_n$ is *tame* if

$$U_n = 0 \text{ for all } n \ll 0 \text{ or } U_n \neq 0 \text{ for all } n \ll 0.$$

(E) Let $(\mathcal{S}_n)_{n \in \mathbb{Z}}$ be a family of numbers or sets. We say that \mathcal{S}_n is *asymptotically stable* for $n \rightarrow -\infty$ if there is some $n_0 \in \mathbb{Z}$ such that $\mathcal{S}_n = \mathcal{S}_{n_0}$ for all $n \leq n_0$.

Lemma 2.2. *Assume that (R_0, \mathfrak{m}_0) is local and of dimension ≤ 2 . Let $i \in \mathbb{N}_0$ and let M be a finitely generated graded R -module such that $\text{Ass}_{R_0}(H_{R_+}^i(M)_n)$ is asymptotically stable for $n \rightarrow -\infty$.*

Then $\text{depth}_{R_0}(H_{R_+}^i(M)_n)$ is asymptotically stable for $n \rightarrow -\infty$.

Proof. By the asymptotic stability of $\text{Ass}_{R_0}(H_{R_+}^i(M)_n)$ for $n \rightarrow -\infty$ we have

$$\begin{aligned} \text{depth}_{R_0}(H_{R_+}^i(M)_n) = 0 & \text{ for all } n \ll 0, \text{ or else} \\ \text{depth}_{R_0}(H_{R_+}^i(M)_n) > 0 & \text{ for all } n \ll 0. \end{aligned}$$

In the first case we are done. So, assume that we are in the second case.

According to [3, Proposition 5.10] the graded R -module $H_{\mathfrak{m}_0 R}^1(H_{R_+}^i(M))$ is Artinian and hence tame. Therefore either

$$\begin{aligned} H_{\mathfrak{m}_0}^1(H_{R_+}^i(M)_n) = 0 & \text{ for all } n \ll 0 \text{ or else} \\ H_{\mathfrak{m}_0}^1(H_{R_+}^i(M)_n) \neq 0 & \text{ for all } n \ll 0. \end{aligned}$$

As $\dim(R_0) \leq 2$ we thus respectively have either

$$\begin{aligned} \text{depth}_{R_0}(H_{R_+}^i(M)_n) = 2 & \text{ for all } n \ll 0 \text{ or} \\ \text{depth}_{R_0}(H_{R_+}^i(M)_n) = 1 & \text{ for all } n \ll 0. \quad \square \end{aligned}$$

Proposition 2.3. *Let $\mathfrak{a}_0 \subseteq R_0$ be an ideal, let $i \in \mathbb{N}_0$ and let M be a finitely generated graded R -module such that $\text{Ass}_{R_0}(H_{R_+}^i(M)_n)^{\leq 2}$ is asymptotically stable for $n \rightarrow -\infty$.*

Then $\text{depth}(\mathfrak{a}_0, H_{R_+}^i(M)_n)^{\leq 2}$ is asymptotically stable for $n \rightarrow -\infty$.

Proof. Let $t := \liminf_{n \rightarrow -\infty} \text{depth}(\mathfrak{a}_0, H_{R_+}^i(M)_n)^{\leq 2}$. Observe that $t \in \{0, 1, 2, \infty\}$ (cf. 2.1 (C)). We have to show that $t(n) := \text{depth}(\mathfrak{a}_0, H_{R_+}^i(M)_n)^{\leq 2} = t$ for all $n \ll 0$.

If $t = \infty$, this is clear. So, let $t \in \{0, 1, 2\}$. We set

$$V := \{\mathfrak{p}_0 \in \text{Var}(\mathfrak{a}_0) \mid \text{height}(\mathfrak{p}_0) \leq 2\}.$$

By our hypothesis the set $V \cap \text{Ass}_{R_0}(H_{R_+}^i(M)_n)$ takes a constant value U for all $n \ll 0$. If $U \neq \emptyset$, we have $\text{depth}(\mathfrak{a}_0, H_{R_+}^i(M)_n)^{\leq 2} = 0$ for all $n \ll 0$. If $U = \emptyset$, we have $\text{depth}(\mathfrak{a}_0, H_{R_+}^i(M)_n)^{\leq 2} > 0$ for all $n \ll 0$. This gives our claim if $t = 0$.

So, assume that $t \in \{1, 2\}$. Then $\text{depth}(\mathfrak{a}_0, H_{R_+}^i(M)_n)^{\leq 2} \geq 1$ for all $n \ll 0$.

By our hypothesis, the set $V \cap \text{Supp}_{R_0}(H_{R_+}^i(M)_n) = V \cap \overline{\text{Ass}_{R_0}(H_{R_+}^i(M)_n)}$ takes a constant value W for all $n \ll 0$. If $\text{height}(\mathfrak{p}_0) \leq 1$ for some $\mathfrak{p}_0 \in W$, we have $\text{depth}(\mathfrak{a}_0, H_{R_+}^i(M)_n) \leq \text{height}(\mathfrak{p}_0) \leq 1$ for all $n \ll 0$, so that $\text{depth}(\mathfrak{a}_0, H_{R_+}^i(M)_n) = 1$ for all $n \ll 0$. Thus, our claim follows in this case.

Therefore, we may assume that $\text{height}(\mathfrak{p}_0) = 2$ for all $\mathfrak{p}_0 \in W$. As W is closed in V and V is closed under generalization, the set W must be finite. As $t < \infty$ we must have $\text{depth}(\mathfrak{a}_0, H_{R_+}^i(M)_n)^{\leq 2} < \infty$ for infinitely many $n < 0$. Therefore $W \neq \emptyset$ (cf. 2.1 (C)).

Now by Lemma 2.2 and the Flat Base-Change Property of local cohomology (cf. [4, Theorem 13.1.8]) we get that $\text{depth}_{(R_0)_{\mathfrak{p}_0}}((H_{R_+}^i(M)_n)_{\mathfrak{p}_0})$ is asymptotically stable for $n \rightarrow -\infty$ for all $\mathfrak{p}_0 \in W$. As W is finite it follows that $\text{depth}(\mathfrak{a}_0, H_{R_+}^i(M)_n) = \min\{\text{depth}_{(R_0)_{\mathfrak{p}_0}}((H_{R_+}^i(M)_n)_{\mathfrak{p}_0}) \mid \mathfrak{p}_0 \in W\}$ is asymptotically stable for $n \rightarrow -\infty$. \square

Corollary 2.4. *Assume that R_0 is essentially of finite type over a field. Let $i \in \mathbb{N}_0$, let $\mathfrak{a}_0 \subseteq R_0$ be an ideal and let M be a finitely generated graded R -module.*

Then $\text{depth}(\mathfrak{a}_0, H_{R_+}^i(M)_n)^{\leq 2}$ is asymptotically stable for $n \rightarrow -\infty$.

Proof. According to [2, Proposition 3.5] the set $\text{Ass}_{R_0}(H_{R_+}^i(M)_n)^{\leq 2}$ is asymptotically stable for $n \rightarrow -\infty$. Now, we may conclude by Proposition 2.3. \square

Corollary 2.5. *Assume that $\dim(R_0) \leq 2$ and R_0 is essentially of finite type over a field. Let $i \in \mathbb{N}_0$, let $\mathfrak{a}_0 \subseteq R_0$ be an ideal and let M be a finitely generated graded R -module.*

Then $\text{depth}(\mathfrak{a}_0, H_{R_+}^i(M)_n)$ is asymptotically stable for $n \rightarrow -\infty$.

This result actually is shown in [1].

Remark 2.6. (A) According to [5], there is a normal homogeneous domain $R = R_0 \oplus R_1 \oplus \dots$, such that (R_0, \mathfrak{m}_0) is local of dimension 3, essentially of finite type over \mathbb{C} and such that $H_{R_+}^2(R)$ is not tame. In particular $\text{depth}(\mathfrak{m}_0, H_{R_+}^2(M)_n)$ is not asymptotically stable for $n \rightarrow -\infty$. So in codimensions ≥ 3 the depth of $H_{R_+}^i(M)_n$ need not be asymptotically stable.

(B) Let $R = R_0 \oplus R_1 \oplus \dots$ be as in 2.1 (A), let $\mathfrak{a}_0 \subseteq R_0$ be an ideal and let M be a finitely generated graded R -module. Let

$$c := \sup\{i \in \mathbb{N}_0 \mid H_{R_+}^i(M) \neq 0\}$$

be the *cohomological dimension of M with respect to R_+* and let

$$f := \inf\{i \in \mathbb{N} \mid H_{R_+}^i(M) \text{ not finitely generated}\}$$

be the *cohomological finiteness dimension of M with respect to R_+* .

If $f < \infty$, then clearly $f \leq c$. Moreover, it is well known that $\text{Ass}_{R_0}(H_{R_+}^c(M)_n)$ need not be asymptotically stable for $n \rightarrow -\infty$ (cf. [8] for example) and that $\text{Ass}_{R_0}(H_{R_+}^f(M)_n)$ is asymptotically stable for $n \rightarrow -\infty$. In [7] it is shown:

If $f = c$, then $\text{depth}(\mathfrak{a}_0, H_{R_+}^c(M)_n)$ is asymptotically stable for $n \rightarrow -\infty$.

3. DEPTH AND HIGHER DIRECT IMAGES

For the unexplained terminology of this section we refer to [6].

Notation and Conventions 3.1. (A) For the rest of this paper let X_0 denote a Noetherian scheme, let $\pi : X \rightarrow X_0$ denote a projective scheme over X_0 with very ample sheaf $\mathcal{O}_X(1)$ and let $Z_0 \subseteq X_0$ be a closed set.

(B) If \mathcal{G} is a coherent sheaf of \mathcal{O}_{X_0} -modules and $t \in \mathbb{N}_0$ we always use the notation introduced in (1.2) and (1.3).

(C) If X_0 is affine and $I(Z_0) \subseteq \mathcal{O}(X_0)$ is the vanishing ideal of Z_0 , we write

$$\begin{aligned} \text{depth}(Z_0, T) &:= \text{depth}(I(Z_0), T) = \text{depth}(Z_0, \widetilde{T}) \text{ and} \\ \text{depth}(Z_0, T)^{\leq t} &:= \text{depth}(I(Z_0), T)^{\leq t} = \text{depth}(Z_0, \widetilde{T})^{\leq t} \end{aligned}$$

for each finitely generated $\mathcal{O}(X_0)$ -module T .

Proposition 3.2. *Assume that X_0 is affine and essentially of finite type over a field. Let \mathcal{F} be a coherent sheaf of \mathcal{O}_X -modules and let $i \in \mathbb{N}_0$.*

Then $\text{depth}(Z_0, H^i(X, \mathcal{F}(n)))^{\leq 2}$ is asymptotically stable for $n \rightarrow -\infty$.

Proof. Let $R_0 := \mathcal{O}(X_0)$, $\mathfrak{a}_0 := I(Z_0) \subseteq R_0$. Then, there is a homogeneous Noetherian R_0 -algebra $R = R_0 \oplus R_1 \oplus \dots$ with $X = \text{Proj}(R)$ and $\mathcal{O}_X(1) = R(1)^\sim$. Moreover there is a finitely generated graded R -module M such that $\mathcal{F} = \widetilde{M}$. Now, for each $n \in \mathbb{Z}$ the Serre-Grothendieck Correspondence gives rise to a short exact sequence of R_0 -modules

$$0 \rightarrow H_{R_+}^0(M)_n \rightarrow M_n \rightarrow H^0(X, \mathcal{F}(n)) \rightarrow H_{R_+}^1(M)_n \rightarrow 0$$

and to isomorphisms of R_0 -modules

$$H^j(X, \mathcal{F}(n)) \cong H_{R_+}^{j+1}(M)_n \text{ for all } j > 0.$$

Therefore, our claim follows by Corollary 2.4. □

Proposition 3.3. *Assume that X_0 is affine and essentially of finite type over a field. Let \mathcal{F} be a coherent sheaf of \mathcal{O}_X -modules, let \mathcal{L} be an ample invertible sheaf of \mathcal{O}_X -modules and let $i \in \mathbb{N}_0$.*

Then, $\text{depth}(Z_0, H^i(X, \mathcal{L}^n \otimes_{\mathcal{O}_X} \mathcal{F}))^{\leq 2}$ is asymptotically stable for $n \rightarrow -\infty$.

Proof. This follows from Proposition 3.2 by essentially the same arguments as used in the proof of [2, Theorem 5.3]. □

Corollary 3.4. *Let $X_0, \mathcal{L}, \mathcal{F}$ and i be as in Proposition 3.3. Assume in addition that $\dim(X_0) \leq 2$.*

Then, $\text{depth}(Z_0, H^i(X, \mathcal{L}^n \otimes_{\mathcal{O}_X} \mathcal{F}))$ is asymptotically stable for $n \rightarrow -\infty$.

Theorem 3.5. *Let X_0 be essentially of finite type over a field. Let \mathcal{L} be an ample invertible sheaf of \mathcal{O}_X -modules and let \mathcal{F} be a coherent sheaf of \mathcal{O}_X -modules. Let $i \in \mathbb{N}_0$.*

Then $\text{depth}(Z_0, \mathcal{R}^i \pi_(\mathcal{L}^n \otimes_{\mathcal{O}_X} \mathcal{F}))^{\leq 2}$ is asymptotically stable for $n \rightarrow -\infty$.*

Proof. We may assume that X_0 is affine. Now, we can conclude by Proposition 3.3 as $\mathcal{R}^i \pi_*(\mathcal{L}^n \otimes_{\mathcal{O}_X} \mathcal{F}) \cong H^i(X, \mathcal{L}^n \otimes_{\mathcal{O}_X} \mathcal{F})^\sim$. □

Corollary 3.6. *Let $X_0, \mathcal{L}, \mathcal{F}$ and i be as in Theorem 3.5. Assume in addition that $\dim(X_0) \leq 2$.*

Then $\text{depth}(Z_0, \mathcal{R}^i \pi_(\mathcal{L}^n \otimes_{\mathcal{O}_X} \mathcal{F}))$ is asymptotically stable for $n \rightarrow -\infty$.*

Remark 3.7. The observations made in Remark 2.6 (A) show that the conclusions of Corollaries 3.4 and 3.6 need not hold if $\dim(X_0) \geq 3$.

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