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Cosmological model discrimination from weak lensing data

Pires, S ; Starck, J L ; Amara, A ; Réfrégier, A ; Teyssier, R

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Cosmological model discrimination from weak lensing data

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Abstract. Weak gravitational lensing provides a unique way of mapping directly the dark matter in the Universe. The majority of lensing analyses use the two-point statistics of the cosmic shear field to constrain the cosmological model, a method that is affected by degeneracies, such as that between σ_8 and Ω_m which are respectively the rms of the mass fluctuations on a scale of $8Mpc/h$ and the matter density parameter, both at $z = 0$. However, the two-point statistics only measure the Gaussian properties of the field, and the weak lensing field is non-Gaussian. It has been shown that the estimation of non-Gaussian statistics for weak lensing data can improve the constraints on cosmological parameters. In this paper, we systematically compare a wide range of non-Gaussian estimators to determine which one provides tighter constraints on the cosmological parameters. These statistical methods include skewness, kurtosis, and the higher criticism test, in several sparse representations such as wavelet and curvelet; as well as the bispectrum, peak counting, and a newly introduced statistic called wavelet peak counting (WPC). Comparisons based on sparse representations indicate that the wavelet transform is the most sensitive to non-Gaussian cosmological structures. It also appears that the most helpful statistic for non-Gaussian characterization in weak lensing mass maps is the WPC. Finally, we show that the σ_8 - Ω_m degeneracy could be even better broken if the WPC estimation is performed on weak lensing mass maps filtered by the wavelet method, MRLens.

Keywords: Cosmology: Weak Lensing, Methods : Statistics, Data Analysis

INTRODUCTION

Measurements of the image distortion of background galaxies caused by large-scale structures provide a direct way to study the statistical properties of the growth of structures in the Universe. Weak lensing measures the mass and can thus be directly compared to theoretical models of structure formation. Most lensing studies use the two-point statistics of the cosmic shear field because of its potential to constrain the power spectrum of density fluctuations in the late Universe [1, 2, 3]. Two-point statistics measure the Gaussian properties of the field. This represents a limited amount of information since it is well known that the low redshift Universe is highly non-Gaussian on small scales. Gravitational clustering is, indeed, a non linear process and on small scales, in particular, the mass distribution is highly non-Gaussian. Consequently, using only two-point statistics to place constraints on the cosmological model provides limited insight. Stronger constraints can be obtained using 3D weak lensing maps [4, 5, 6]. An alternative procedure is to consider higher-order statistics enabling a characterization of the non-Gaussian nature of the signal [see e.g., 7, 8, 9].

In this paper, we systematically compare a range of non-Gaussian statistics. For this purpose, we focus on the degeneracy between σ_8 and Ω_m which are respectively, the amplitude of the matter power spectrum and the matter density parameter. We attempt to determine the most effective method for breaking this degeneracy that exists when only the two-point correlation is considered. A wide range of statistical methods are systematically applied to a set of simulated data to characterize the non-Gaussianity present in the mass maps due to the growth of structures. Their ability to discriminate between different possible cosmological models are compared. For the analysis of CMB data, it has been proposed that statistics such as wavelet kurtosis or wavelet higher criticism can be used to detect clusters and curvelet kurtosis or curvelet higher criticism to detect anisotropic features such cosmic strings. Since weak lensing data may contain filamentary structures, we also considered statistical approaches based on sparse representations.

In sect. 2, we review the major statistical methods used in the literature to constrain cosmological parameters from weak lensing data. Section 3 is dedicated to the description of our analysis and we describe the different statistics that we studied along with the different multi-scale transforms investigated. We also present a new statistic that we call wavelet peak counting (WPC). In sect. 4, we present our results and finally, sects. 5, we summarize our conclusions.

WEAK LENSING STATISTICS AND COSMOLOGICAL MODELS CONSTRAINTS

The most common method for constraining cosmological parameters in weak lensing studies uses two-point statistics of the shear field calculated either in real or Fourier space. In general, there is an advantage to using Fourier space statistics such as the power spectrum because the modes are independent. The power spectrum $P_\kappa(l)$ of the 2D convergence is defined as a function of the modes l by

$$\langle \hat{\kappa}(\vec{l}) \hat{\kappa}(\vec{l}') \rangle = (2\pi)^2 \delta(\vec{l} - \vec{l}') P_\kappa(l), \quad (1)$$

where hat symbols denote Fourier transforms, δ is the delta function, and the brackets denote an average over l . $P_\kappa(l)$ depends only on $l = |\vec{l}|$ and κ is the convergence (i.e., which is proportional to the projected mass distribution). This power spectrum $P_\kappa(l)$ can be expressed in terms of the 3D matter power spectrum $P(k, \chi)$ of the mass fluctuations $\delta\rho/\rho$ and of cosmological parameters as

$$P_\kappa(l) = \frac{9}{16} \left(\frac{H_0}{c} \right)^2 (\Omega_m)^2 \int d\chi \left[\frac{g(\chi)}{a r(\chi)} \right]^2 P\left(\frac{l}{r}, \chi\right), \quad (2)$$

where a is the expansion parameter, H_0 is the Hubble constant, and Ω_m is the matter density parameter. The correlation properties are expressed more conveniently in Fourier space, but for surveys with complicated geometry because of the removal of bright stars and artifacts, the missing data need proper handling [a review of the different existing methods can be found in 10]. Real space statistics are easier to estimate, but require more computational time. The following two-point statistics can be related to the underlying 3D matter power spectrum by means of the 2D convergence power spectrum $P_\kappa(l)$:

- The shear variance $\langle \gamma^2 \rangle$, which is an example of real space two-point statistic is defined as the variance in the average shear $\bar{\gamma}$ evaluated in circular patches of varying radius θ_s . The shear variance $\langle \gamma^2 \rangle$ is related to the power spectrum $P_\kappa(l)$ of the 2D convergence by

$$\langle \gamma^2 \rangle = \int \frac{dl}{2\pi} l P_\kappa(l) \frac{J_1^2(l\theta_s)}{(l\theta_s)^2}, \quad (3)$$

where J_n is a Bessel function of order n . This shear variance has been used in many weak lensing analysis to constrain cosmological parameters [1, 11].

- The shear two-point correlation function, which is currently the most widely used statistic because it is easy to implement and can be estimated even in cases of complex geometry. The shear two-point correlation functions can be related to the 2D convergence power spectrum by

$$\xi_+(\theta) = \xi_{1,1}(\theta) + \xi_{2,2}(\theta) = \int_0^\infty \frac{dl}{2\pi} l P_\kappa(l) J_0(l\theta). \quad (4)$$

These two-point correlation functions are the most popular statistical tools and have been used in the most recent weak lensing analyses [11, 12].

Two-point cosmic shear measurements yield interesting constraints on the amplitude of the matter power spectrum σ_8 on which the lensing strongly depends. However, when deriving joint constraints, a degeneracy exists between σ_8 and Ω_m [see e.g., 1, 13]. This degeneracy between σ_8 and Ω_m is typical of cosmic shear measurements that use only two-point statistics. Two-point statistics are inadequate for characterizing non-gaussian features. Non-Gaussianity produced by the non-linear evolution of the Universe is of great importance for the understanding of the underlying physics, and it may help us to differentiate between cosmological models. In the standard structure formation model, initial random fluctuations are amplified by non-linear gravitational instability to produce a final distribution of mass that is highly non-Gaussian. The weak lensing field is thus highly non-Gaussian. On small scales, we observe structures such as galaxies and clusters of galaxies, and on larger scales, some filament structures. Detecting these non-Gaussian features in weak lensing mass maps can be very useful for constraining the cosmological model parameters.

- The three-point correlation function $\xi_{i,j,k}$ is the lowest-order statistics that can be used to detect non-Gaussianity, and is defined to be

$$\xi_{i,j,k}(\theta) = \langle \kappa(\vec{\theta}_1) \kappa(\vec{\theta}_2) \kappa(\vec{\theta}_3) \rangle. \quad (5)$$

In Fourier space, it is called bispectrum and depends only on the distances $|\vec{l}_1|$, $|\vec{l}_2|$, and $|\vec{l}_3|$, i.e.,

$$B(|\vec{l}_1|, |\vec{l}_2|, |\vec{l}_3|) \propto \langle \hat{\kappa}(|\vec{l}_1|) \hat{\kappa}(|\vec{l}_2|) \hat{\kappa}^*(|\vec{l}_3|) \rangle. \quad (6)$$

It has been shown that tighter constraints can be obtained with the three-point correlation function [4, 14].

- A simpler quantity than the three-point correlation function is provided by measuring the third-order moment (skewness) of the smoothed convergence κ [4] or of the aperture mass M_{ap} [7].
- Another approach to searching for non-Gaussianity is to perform a statistical analysis directly on non-Gaussian structures such as clusters. Galaxy clusters are the largest virialized cosmological structures in the Universe. They provide a unique way to focus on non-Gaussianity present on small scales. One interesting statistic is *peak counting*, which searches the number of peaks detected on the 2D convergence corresponding to the cluster abundance [see e.g., 8].

The methods to search for non-Gaussianity in the weak lensing literature concentrate mainly on the higher-order correlation function. For the analysis of the CMB, the skewness and kurtosis of the wavelet coefficients are also standard tools to check the CMB Gaussianity [15], and it was shown that curvelets [16] were useful in detecting anisotropic feature such as cosmic strings in the CMB [17]. In the following, we perform a comparison between most existing methods to find the most suitable higher order statistic for constraining cosmological parameters from weak lensing data.

COSMOLOGICAL MODEL DISCRIMINATION FRAMEWORK

To explore the effectiveness of a non-Gaussian measure, we use a battery of N-body simulations. By choosing five models whose two-point correlation statistics are degenerate, we attempt to determine which statistics are able to distinguish between these models.

Simulations of weak lensing mass maps

We obtained realistic realizations of convergence mass maps from N-body cosmological simulations using the RAMSES code [18]. The cosmological models were assumed to be in concordance with the Λ CDM model. We limited the model parameters to within a realistic range choosing five models along the (σ_8, Ω_m) -degeneracy (see Fig. (1)). For each of our 5 models, we run 21 N-body simulations, each containing 256^3 particles. In the N-body simulations that are commonly used in cosmology, the dark matter distribution is represented by discrete massive particles. The simplest way of treating these particles is to map their positions onto a pixelized grid. In the case of multiple sheet weak lensing, we do this by taking slices through the 3D simulations. These slices are then projected into 2D mass sheets. Using the previous 3D N-body simulations, we derived 100 realizations of the five models.

FIGURE 1. The 5 cosmological models along the (σ_8, Ω_m) -degeneracy : model 1 ($\sigma_8 = 1$ and $\Omega_m = 0.23$), model 2 ($\sigma_8 = 0.9$ and $\Omega_m = 0.3$), model 3 ($\sigma_8 = 0.8$ and $\Omega_m = 0.36$), model 4 ($\sigma_8 = 0.7$ and $\Omega_m = 0.47$) and model 5 ($\sigma_8 = 0.6$ and $\Omega_m = 0.64$)

In practice, the observed shear γ_i is obtained by averaging over a finite number of galaxies and is therefore noisy. The noise arises in both the measurement errors and from the intrinsic ellipticities dispersion of galaxies.

Characterization of the discrimination

In identifying the most suitable statistic for breaking the (σ_8, Ω_m) -degeneracy, we are interested in comparing different statistics estimated in different representations using the set of simulated data described in the previous section. For each statistic, we need to characterize, the discrimination obtained between each couple of models. The optimal statistic will be the one that maximizes the discrimination for all model pairs. To find the most suitable statistic, we need to quantitatively characterize for each statistic the discrimination between two different models m_1 and m_2 . One way to proceed is to define a discrimination efficiency that expresses the ability of a statistic to discriminate in percentage. We then need to define, for each individual statistic, two different thresholds :

- a threshold $\mathcal{T}_1$: if the estimation of a given statistic in a map κ_i is below $\mathcal{T}_1$, the map κ_i is classified to belong to the model m_1 , and not if it is above;
- a threshold $\mathcal{T}_2$: if the estimation of a given statistic in a map κ_i is above $\mathcal{T}_2$, the map κ_i is classified to belong to the model m_2 , and not if it is below.

We have used a statistical tool called FDR (false discovery rate) introduced by [19] to set these two thresholds ($\mathcal{T}_1$ and $\mathcal{T}_2$) correctly. This FDR method is a competitive tool that sets a threshold in an adaptive way without any assumption, given a false discovery rate (α). The false discovery rate is given by the proportion of false detections over the total number of detections. The threshold being estimated, we can derive a discrimination efficiency for each statistic. The discrimination efficiency measures the ability of a statistic to differentiate a model and another by calculating the ratio of detections (true or false) to the total number of samples. It corresponds basically to the part of the distribution that does not overlap. The more the distributions overlap, the lower the discrimination efficiency will be and the less the models will be easily separated with this statistic.

A set of statistical tools

The first objective of this study is to compare different statistics to identify the one that places tighter constraints on the cosmological parameters. By adopting two-point statistics that contain all the information about a Gaussian signal leading to the (σ_8, Ω_m) -degeneracy, we have opted for statistics currently used to detect non-Gaussianity to probe the non-linear process of gravitational clustering. The statistics that we selected are the following :

- Skewness (S_κ):
The skewness is the third-order moment of the convergence κ and is a measure of the asymmetry of a distribution. The convergence skewness is related primarily to rare and massive dark-matter halos. The distribution will be more or less skewed positively depending on the abundance of rare and massive halos.
- Kurtosis (K_κ):
The kurtosis is the fourth-order moment of the convergence κ and is a measure of the peakedness of a distribution. A high kurtosis distribution has a sharper “peak” and flatter “tails”, while a low kurtosis distribution has a more rounded peak with wider “shoulders”.
- Bispectrum (B_κ):
There have been many theoretical studies of the three-point correlation function to constrain the cosmological parameters. However, the direct computation of three-point correlation function would take too long for our large maps. We therefore used its Fourier analog: the bispectrum. And we consider the equilateral configuration.
- Higher Criticism (HC):
The HC statistic was developed by [9]. It measures non-Gaussianity by identifying the maximum deviation after comparing the sorted p -values of a distribution with the sorted p -values of a normal distribution. A large HC value implies non-Gaussianity. We consider two different forms of HC [9]: HC^* and HC^+ .
- Peak counting (P_c):
We now investigate the possibility of using peak counting to differentiate between cosmological models. By peak counting (or cluster count), we mean the number of halos that we can detect per unit area of sky (identified as peak above a mass threshold in mass maps). This cluster count enables us to constrain the matter power spectrum normalization σ_8 for a given Ω_m and the formalism exist to predict the peak count to a given cosmological model [8].
- Wavelet peak counting (WPC):
We introduce a new statistic that we call wavelet peak counting (WPC). This statistic consists of estimating a

cluster count per scale of a wavelet transform i.e., it is an approximate cluster count that depends on the size of the clusters. In the following, we show how WPC is superior to peak counting in characterizing the non-linear structure formation process.

All statistics are estimated in terms of the 2D convergence κ map. In practice, the complex geometry of surveys gives the convergence κ , which is in turn inferred from the shear field γ_i of the same complex geometry. However, it becomes possible with the inpainting method developed in [10] to reconstruct a full convergence κ map. The solution proposed enables judiciously masked regions to be filled-in so as to reduce the impact of missing data on the estimation of both the power spectrum and higher-order statistical measures, while requiring $O(N \log N)$ operations.

Representations

The second objective of the present study was to compare different transforms and find the sparsest representation of weak lensing data that makes the discrimination easier. Some studies of CMB data used multiscale methods to detect non-Gaussianity [20]. Weak lensing maps exhibit both isotropic and anisotropic features. These kind of features can be represented more successfully by some basis functions. A transform is, indeed, optimal for detecting structures which have the same shape of its basis elements. We thus tested different representations :

- The Fourier transform
- The anisotropic bi-orthogonal wavelet transform which we expect to be optimal for detecting mildly anisotropic features.
- The isotropic "à trous" wavelet transform which is well adapted to the detection of isotropic features such as the clumpy structures (clusters) in the weak lensing data.
- The ridgelet transform developed to process images including ridge elements which therefore provides a good representation of perfectly straight edges.
- The curvelet transform which allows us to approximate curved singularities with few coefficients and then provides a good representation of curved structures.

We note that the estimation of a given statistic at each resolution level of a representation can be considered to be individual statistic in itself.

ANALYSIS AND RESULTS

Treatment of the noise

As explained previously, the weak lensing mass maps are measured from the distortions of a finite number of background galaxies and therefore suffer from shot noise. Furthermore, each galaxy provides only a noisy estimator of the distortion field. We added the expected level of Gaussian noise to simulations of weak lensing mass maps to obtain simulated noisy mass maps corresponding to space observations.

- No filtering :
We started by applying the different transformations directly to the noisy data and, for each representation estimating the statistics described in the previous section, with the exception of cluster count and WPC, which required filtering.
We expect that the noise will make the third and fourth order statistics tend to zero. The more noisy the data are, the more the distribution will indeed look like a Gaussian.
- Gaussian filtering :
As a second step, we performed Gaussian filtering obtained by convolving the noisy mass maps κ_n with a Gaussian window G of standard deviation σ_G :

$$\kappa_G = G * \kappa_n. \quad (7)$$

On the left, Fig. (2) shows the original map without noise, and on the right, the result obtained by Gaussian filtering of the noisy mass map. The quality of the filtering depends strongly on the value of σ_G . For the

FIGURE 2. Left, noiseless simulated mass map, and right, filtered mass map by convolution with a Gaussian kernel. The field is $1.975^\circ \times 1.975^\circ$.

simulations, an optimal value is about 0.9 arcmin. We then computed all our statistics for this new set of filtered mass maps.

- MRLens filtering :

We finally used a method of non-linear filtering based on the wavelet representation, i.e., the MRLens filtering proposed by [21]. The MRLens¹ filtering is based on Bayesian methods that incorporate prior knowledge into data analysis. Choosing the prior is one of the most critical aspects of Bayesian analysis. Here a multiscale entropy prior is used.

In [21], it was shown that this method outperforms several standard techniques such as Gaussian filtering, Wiener filtering, and MEM filtering). On the left, Fig. (3) shows the original map without noise, and on the right, the result of FDR filtering of the noisy mass map. The visual aspect indicates that many clusters are reconstructed and the intensity of the peaks are well recovered.

FIGURE 3. Left, noiseless simulated mass map, and right, filtered mass map by the FDR multiscale entropy filtering. The field is $1.975^\circ \times 1.975^\circ$.

As before, all the statistics were estimated for these MRLens filtered mass maps. By essentially reconstructing the clusters, we anticipate that this MRLens method will improve statistical methods such as peak counting or WPC more than statistics that focus on the background.

¹ The MRLens software is available at the following address : <http://www-irfu.cea.fr/Ast/878.html>.

Results

For each statistic described in the previous section, we can derive a discrimination efficiency. A mean discrimination efficiency for each individual statistic can be estimated by averaging the discrimination efficiency across all pairs of models. For statistics estimated in multiscale representations, the mean discrimination efficiency is calculated for each scale and we adopt the most reliable statistic. These values are given in Tables (1), (2), and (3) for respectively, (i) unfiltered mass maps, (ii) Gaussian filtered mass maps and (iii) MRLens mass maps.

Reliable differentiation between the five models is achievable if the mean discrimination efficiency is about 100%. If between 80% and 100%, the discrimination is possible except for adjacent models. At a level of below 40% no discrimination is possible even for distant models.

Differentiation between the noisy mass maps

The mean discrimination efficiency obtained for unfiltered mass maps is displayed in Table (1). The peak counting and WPC differ from the others. They cannot be computed on unfiltered maps because the clusters cannot be extracted from noisy mass maps. Another point is that the bispectrum by definition can only be estimated in the Fourier domain.

Without filtering the results are poor and no discrimination can be achieved in direct space because the non-Gaussian signal is hidden by the Gaussian noise. The different transforms appear to be unable to bring out efficiently the non-Gaussianity characteristics except for the isotropic wavelet transform which performs rather well, whatever the statistic. This is probably because it is an optimal transform for detecting clusters. Indeed, the clusters are a direct probe of non-Gaussianity and by concentrating on the cluster information, the isotropic wavelet transform seems to be a better representation of non-Gaussianity.

TABLE 1. Mean discrimination efficiencies (in percent) achieved on noisy mass maps with a false discovery rate of $\alpha = 0.05$.

Basis/statistics	S	K	HC_n^*	HC_n^+	B_l	P_c	WPC
Direct space	21.0	1.6	1.8	1.9	x	x	x
Fourier space	0.7	10.7	3.0	21.3	5.75	x	x
Isotropic wavelet transform	40.5	29.3	24.9	34.0	x	x	x
Bi-orthogonal wavelet transform	7.3	10.8	4.1	2.9	x	x	x
Ridgelet transform (block = 16)	6.2	3.7	7.2	5.6	x	x	x
Ridgelet transform (block = 32)	8.3	15.2	9.5	20.4	x	x	x
Fast curvelet transform	1.1	4.2	2.6	4.2	x	x	x

The skewness in the wavelet transform representation appears to be the most successful statistic in unfiltered mass maps. Fig. (4) shows the discrimination efficiency obtained for the skewness on the second scale of an isotropic wavelet transform which is the scale for which optimal differentiation between models is achieved. We can see that the 5 distributions overlap, the differentiation is only achieved between the farthest models, which is quite poor. Some groups have already used the skewness aperture mass statistic to try to break the σ_8 - Ω_m degeneracy, [see e.g., 7]. This processing consists of convolving the noisy signal with Gaussian windows of different scales and is quite similar to an isotropic wavelet transform. They showed that by combining second and third-order statistics, the degeneracy can be diminished but not broken.

Discrimination in Gaussian filtered mass maps

To increase the signal-to-noise ratio, we applied a Gaussian filter to the noisy simulated mass maps. Table (2) shows the results. After Gaussian filtering, the noise is removed but the structures are over-smoothed. Except for the direct space where the results are clearly improved by the noise removal, the results after Gaussian filtering are relatively unchanged. Some statistics are a bit improved by the noise removal, while others become worse. In contrast, the peak counting and WPC that can now be estimated for these filtered mass maps, perform well. Fig. (5) shows the discrimination efficiency obtained with peak counting estimated on Gaussian-filtered mass maps. We can see that except for adjacent models, discrimination is now possible. The distributions barely overlap for models that are not

FIGURE 4. Distribution of the skewness calculated from the second scale of an isotropic wavelet transform on the simulated realizations of the 5 models. No discrimination is possible except between the farthest models (i.e., between models 1 and 5)

TABLE 2. Mean discrimination efficiencies (in percent) achieved on Gaussian-filtered mass maps with a false discovery rate of $\alpha = 0.05$.

Basis/statistics	S	K	HC_n^*	HC_n^+	B_l	P_c	WPC
Direct space	39.2	34.3	28.8	40.2	x	79.3	x
Fourier space	2.2	2.2	3.8	2.9	6.75	x	x
Isotropic wavelet transform	31.9	29.5	24.	38.4	x	x	79.2
Bi-orthogonal wavelet transform	12.2	10.5	5.1	6.8	x	x	x
Ridgelet transform (block = 16)	7.4	20.2	22.7	35.3	x	x	x
Ridgelet transform (block = 32)	2.5	7.3	4.	9.1	x	x	x
Fast curvelet transform	1.3	4.7	4.8	8.3	x	x	x

adjacent. The ability of the weak-lensing cluster count (peak counting) to discriminate between the five different models chosen along the degeneracy, can be explained. If we assume that the dark matter lies at $z \simeq 0.5$, which would correspond to the maximum lensing efficiency for background galaxies at $z = 1$, and assuming a constant dark energy model, the number density of massive clusters in the weak lensing mass maps is sensitive to both the amplitude of the mass fluctuations σ_8 and the matter density parameter Ω_m , both at $z = 0$. If instead of σ_8 , we consider $\sigma_8^{z \simeq 0.5}$, which represents the amplitude of the fluctuations of the projected weak lensing mass map, the $\sigma_8^{z \simeq 0.5}$ is now a constant for the five models because the five corresponding weak lensing power spectrum are indistinguishable. This leaves the Ω_m parameter which controls the structure formation. A low value of Ω_m ensures that the structures form earlier and a high Ω_m ensures that the structures form later. Then, for a low value of Ω_m , the abundance of massive clusters at $z \simeq 0.5$ become more significant than for a high Ω_m . The cluster count can then be used to differentiate between cosmological models. The massive cluster abundance has already been used to probe Ω_m .

FIGURE 5. Distribution of the peak counting estimated directly on the simulated realizations of the 5 models. The discrimination is possible except between adjacent models (that is to say between model 1 and model 2, model 2 and model 3, model 3 and model 4, model 4 and model 5).

Table (3) shows the results obtained when the MRLens filtering scheme is applied to the noisy simulated mass maps.

TABLE 3. Mean discrimination efficiencies (in percent) achieved on MRLens filtered mass maps with a false discovery rate of $\alpha = 0.05$.

Basis/statistics	S	K	HC_n^*	HC_n^+	B_l	P_c	WPC
Direct space	65.2	54.5	72.9	8.4	x	93.2	x
Fourier space	3.2	3.4	30.0	38.4	9.95	x	x
Isotropic wavelet transform	81.7	68.1	74.8	74.1	x	x	97.3
Bi-orthogonal wavelet transform	52.2	67.3	29.7	65.0	x	x	x
Ridgelet transform (block = 16)	45.1	54.5	77.1	37.2	x	x	x
Ridgelet transform (block = 32)	61.5	52.6	70.3	69.0	x	x	x
Fast curvelet transform	25.	65.5	68.8	63.3	x	x	x

After MRLens filtering, the sensitivity of all transforms is greatly improved. However, the isotropic wavelet transform remains the most efficient transform certainly helped by the MRLens filtering that uses this transform. The most effective statistic remains peak counting, which is also helped by the MRLens filtering that essentially reconstructs the clusters. However, the other statistics also achieve good results in these MRLens filtered mass maps compared to those with Gaussian filtered mass maps. This is probably because the MRLens filtering, by favoring clusters reconstruction, helps all statistics that search for non-Gaussianity. The best result is obtained with WPC on the third scale of the isotropic wavelet transform as shown in Figure (6). The 5 distributions barely overlap. This statistic allows us to discriminate between different models even for adjacent models for which the discrimination is challenging. A comparison of these results with those obtained for noisy mass maps by the skewness in a wavelet

FIGURE 6. Distribution of the wavelet peak counting estimated at the third scale of an isotropic wavelet transform on the simulated realizations of the 5 models. We obtain a good discrimination even for adjacent models.

representation, shows that the accuracy of the constraints on σ_8 and Ω_m is improved greatly by using WPC estimated for MRLens filtered mass maps.

CONCLUSION

By using only two-point statistics to constrain the cosmological model, various degeneracies between cosmological parameters are possible, such as the $(\sigma_8-\Omega_m)$ -degeneracy. In this paper, we have considered a range of non-Gaussian statistics to attempt to place tighter constraints on cosmological parameters. For this purpose, we have run N-body simulations of 5 models selected along the $(\sigma_8-\Omega_m)$ -degeneracy, and we have examined different non-Gaussian statistical tools in different representations, comparing their abilities to differentiate between models. Using non-Gaussian statistics, we have searched for non-Gaussian signal for small scales caused by gravitational clustering.

The main conclusions of our analysis are the following:

1. The isotropic wavelet transform has been found to be the best representation of the non-Gaussian structures in weak lensing data because it can differentiate between the models the most reliably.

2. We have shown that a wavelet transform denoising method such as the MRLens filtering, which reconstructs essentially the non-Gaussian structures (the clusters), helps the statistics to characterize the non-Gaussianity more reliably.
3. We have introduced a new statistic called wavelet peak counting (WPC), which consists of estimating a cluster count per scale of an isotropic wavelet transform.
4. WPC has been found to be the best statistic compared to the others that we have tested (skewness, kurtosis, bispectrum, HC, P_c), and we have shown that this statistic estimated on MRLens filtered maps provide a strong discrimination between the 5 selected models.

In this paper, we have compared systematically a wide range of non-Gaussian statistics proposed in the literature, and WPC has been found to be the best statistic. By comparing models selected along the $(\sigma_8\text{--}\Omega_m)$ -degeneracy, this study shows that the power spectrum (or the two-point correlation function) and WPC should therefore be used simultaneously. However, these two statistics probe the same field, therefore the correlations introduced by the combined measurement need to be taken into account [see 22]. This will be investigated in the future. Finally, while peak counting and WPC provide a lot of information, further statistics such as the cluster count per mass and the spatial cluster correlation will provide further constraints. Future work will be needed to fully exploit these approaches.

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