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Overbidding in fixed rate tenders: The role of exposure risk[◊]

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Abstract

The fixed rate tender is one of the main procedures used by central banks in the implementation of their monetary policies. While academic research has largely dismissed the procedure owing to its tendency to encourage overbidding, central banks such as the ECB and the Bank of England have continued using it. We investigate this apparent conflict by considering an auction-theoretic setting with private information about declining marginal valuations. Since overbidding entails *exposure risk*, an equilibrium may exist even if bids are costless and the intended volume is pre-announced. In fact, the allotment quota may be strictly below one with certainty. Also with adaptive expectations, overbidding need not escalate. However, the resulting allocation is typically inefficient. Empirical proxies of exposure risk are significant in both euro and sterling operations. Our findings have implications, in particular, for the potential reintroduction of pro rata allotment in the main refinancing operations of the Eurosystem.

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1. Introduction

The fixed rate tender is one of the main procedures used by central banks in the implementation of their monetary policies. In a routine application of this procedure, a central bank first announces its intention to provide a given amount of interbank liquidity against payment of the interest and delivery of eligible collateral. Market participants then submit "bids" that specify the amount of liquidity they wish to obtain. Finally, the central bank determines how much liquidity is allotted to each of the bidders. If all bids can be allotted in full, allotments correspond in size to individual bids. Typically, however, there is excess demand and bids must be allotted on a pro rata basis.

It is probably fair to say that academic research has largely dismissed the fixed rate tender because of its tendency to encourage strategic *overbidding*. Indeed, the procedure cannot work in a complete-information setting when there is excess demand. For example, consider $n = 10$ bidders with a demand of $\theta_i = 0.2$ each when the amount of interbank liquidity to be provided is 1. If all bids are for the amounts that are genuinely required, each bidder obtains only $q_i = 0.1$ after pro rata allotment. Therefore, bidders have an incentive to overbid. But bidders that expect competing bidders to overbid have an incentive to inflate their own bids even more. Thus, expectations of overbidding are spiraling, and there is no equilibrium. Moreover, in reality, the fixed rate tender has not always performed to the satisfaction of all market participants. In particular, during a well-documented episode at the start of Stage III of Economic and Monetary Union in Europe, levels of aggregate bids escalated out of all proportion to the amounts of liquidity to be provided. As a consequence, the procedure was abandoned in June 2000.[1]

Notwithstanding this experience, central banks such as the European Central Bank (ECB) and the Bank of England (BoE) have continued using the fixed rate tender. In the euro area, the fixed rate tender with pro rata allotment has been employed to absorb liquidity in fine-tuning operations, to distribute funds provided through the US dollar term auction facility, and to allot EUR/CHF foreign exchange swaps. In the United Kingdom, the procedure has been used in the weekly open market operations and in fine-tuning operations. Interestingly, the mechanism has tended to work quite smoothly in all these cases. Indeed, as Table 1 shows, the overbidding episode in the euro area, with a mean log bid-to-cover ratio of 2.92, is a rather exceptional case.[2]

-Table 1 about here-

To investigate the apparent conflict between theory and practice, and to better understand the determinants of the smooth operation of the procedure, we study the possibility of an equilibrium in a somewhat richer theoretical

[1] See, e.g., Ayuso and Repullo (2001), Breitung and Nautz (2001), Ehrhart (2001), and Nautz and Oechssler (2003, 2006).

[2] The *bid-to-cover ratio* is the ratio of total bids to the "intended" volume of the operation. All logs are with respect to the natural base.

framework. Specifically, we assume that bidders have private information, possibly correlated, about their own marginal valuations functions. In this setting, overbidding entails *exposure risk*, i.e., the risk of receiving an overly large allotment. We show that the presence of exposure risk has important consequences for bidding behavior.[3]

For example, in a liquidity-providing operation, a bidder that excessively inflates its bid may receive a very large allotment if aggregate competing bids are significantly below the level it expected. That bidder not only receives excess liquidity but is also obliged to deliver the corresponding amount of collateral. However, holding excess liquidity is costly, e.g., in terms of interest rate spreads. Moreover, the obligation to deliver collateral may entail high fees for securities lending if there is a shortage of eligible collateral after the tender.[4]

Because overbidding entails exposure risk, we can show that an equilibrium is sustainable even if bids are costless and the intended volume of the operation is pre-announced. Intuitively, if there is a possibility of exposure risk, incentives to overbid remain limited provided that competitors are likewise expected not to overbid to an excessive extent. Using this observation,

[3] A related *exposure problem* arises in the theory of multi-unit auctions. There, a bidder with *increasing* marginal valuations may end up making losses when competition is unexpectedly strong. See, e.g., Krishna and Rosenthal (1996).

[4] Nyborg and Strebulaev (2001) stress the possibility that bidders fear the scarcity of *liquidity* in the after-market. In the present example, exposure risk is essentially the mirror image of that effect, capturing a scarcity of *collateral* in the after-market.

the existence of an equilibrium follows from a standard existence theorem for Bayesian games. We also discuss extensions and welfare implications. Specifically, we consider an example of an equilibrium in which rationing occurs with certainty. Then, in a dynamic extension with adaptive expectations and myopic optimization, moderate overbidding factors are shown to return quickly to the equilibrium level. Finally, we prove that the fixed rate tender typically yields inefficient allocations even if there is no failure to coordinate.

To test the empirical importance of exposure risk for bidding behavior in fixed rate tenders, we ran regressions of bid amounts in the weekly euro operations conducted between January 1999 and June 2000, and in the weekly sterling operations conducted between May 2006 and October 2008. Proxies of exposure risk prove to be significant for both cases, confirming our main hypothesis. However, exposure risks are less pronounced in the euro area than in the UK. Our interpretation of these findings is that, apparently, there was a mismatch between the collateral framework of the Eurosystem and its tender procedures during the considered period, while there was no such mismatch in the case of the BoE. We use our findings to comment on the potential reintroduction of fixed rate tenders with pro rata allotment in the main refinancing operations of the Eurosystem.

The rest of the paper is structured as follows. In Section 2, we survey some recent uses of the fixed rate tender. Section 3 introduces the theoretical framework. The existence theorem is stated and proved in Section 4. Section 5 contains an example which illustrates the possibility of rationing with probability one. A dynamic extension is considered in Section 6. Section 7 deals with the efficiency of the equilibrium allocation. The empirical test is conducted in Section 8. Section 9 reviews some related literature. Section 10 concludes. The Appendix contains technical proofs.

2. Recent uses of the fixed rate tender

This section surveys recent uses of the fixed rate tender in the euro area and in the UK. We focus on the fixed rate tender with pro rata allotment, in which potential bidders are informed in advance about the intended (maximum) volume of the operation or have at least a good idea of what it will be. Operations in which participants are notified in advance that all bids will be allotted in full as used, e.g., by the Eurosystem from October 2008 in its main and longer-term refinancing operations are not consistent with this definition, neither are operations in which the central bank provides little information prior to the operation regarding how it intends to allot liquidity, as conducted, for example, by the Swiss National Bank.[5]

Altogether, we find nine recent cases of the use of the fixed rate tender with pro rata allotment (cf. Table 1). These cases originate from the

[5] See, e.g., Jordan and Kugler (2004).

following six data series:

(1) Euro fine-tuning operations. Between May 2003 and October 2008, the ECB conducted 32 of its fine-tuning operations as fixed rate tenders with pro rata allotment. These operations were liquidity-absorbing. Figure 1 shows the development of the allotment quota.[6] An occurrence of excessive overbidding appears in the figure as a very short bar. As can be seen, the performance of the pro rata allotment scheme was overall quite satisfactory in these operations.

-Figure 1 about here-

(2) Term auction facility. Between December 2007 and October 2008, the Eurosystem allotted US dollar liquidity to its counterparties through 21 fixed rate tenders. Collateral had to be ECB eligible. The US dollars were provided by the Federal Reserve by means of a temporary swap line established in connection with the Term Auction Facility. Two features are remarkable. First, the interest rate was determined as the marginal rate of the simultaneous Federal Reserve auction. Second, there was a maximum bid limit, equal to 10 percent of the pre-announced amount of liquidity to be provided. Table 2 lists some key data for these operations. We note a clearly discernible downward trend in the allotment quota.

-Table 2 about here-

(3) Foreign exchange swaps. Between October 2008 and January 2010, the

[6] The *allotment quota* is the ratio of the total allotment to total bids.

ECB conducted 70 liquidity-absorbing EUR/CHF foreign exchange swap operations in co-operation with the Swiss National Bank. The announcements included the price (i.e., swap points) as well as the intended volume of the respective operation. Figure 2 compares intended volumes for the one-week swaps with total bids. The intended volumes were €20 billion before February 2009, and €25 billion thereafter. The bid-to-cover ratio initially rose, with a tendency to destabilize, but recovered later without changes to the intended volumes.

-Figure 2 about here-

(4) The overbidding episode. From January 1999 to June 2000, the Eurosystem conducted 76 main refinancing operations as fixed rate tenders. These weekly tenders offered credit for two weeks. There was no explicit announcement prior to the tenders regarding the total amount of liquidity to be provided. Instead, the decision about the total allotment was based, without binding commitment, upon a benchmark allotment, which is the amount of interbank liquidity that allows counterparties, on aggregate, to smoothly fulfill their reserve requirements.[7] In fact, on average during the episode, the ECB tended to allot amounts somewhat above the benchmark allotment. Counterparties were able to estimate the benchmark allotment reasonably well from information on the liquidity situation of the overnight

[7] The formula for the calculation of the benchmark allotment can be found, e.g., in the Annex of European Central Bank (2002). For a theoretical rationale of the benchmark allotment, see Ewerhart et al. (2009).

market. The performance of the main refinancing operations with pro rata allotment was unsatisfactory owing to a rising bid-to-cover ratio.[8]

(5) Sterling fine-tuning operations. During the period June 2006 to March 2009, the BoE conducted 45 fine-tuning operations as fixed rate tenders. Somewhat less than half of those operations were liquidity-providing, a small majority were liquidity-absorbing. These operations performed well.

(6) Sterling weekly open market operations. Between May 2006 and February 2009, the BoE conducted altogether 149 regular open market operations as fixed rate tenders, all of which had a maturity of one week. The size of the operations was part of the scheduled announcements. The operations were liquidity-providing until early October 2008, after which they were liquidity-absorbing and conducted via the sale of Bank of England sterling bills. A maximum bid of 40 percent of the total amount on offer applied. In all the operations, there was significant overbidding, yet the bid-to-cover ratio always remained within reasonable bounds.

The examples show that an escalation of bids in a series of fixed rate tenders with pro rata allotment is by no means inevitable. In fact, excessive overbidding occurred in only one of the nine series. Before continuing the discussion

[8] The liquidity policy of the ECB did not stop the overbidding. Bindseil (2005) argues that the central bank cannot maintain excess liquidity until the end of the reserve maintenance period. An alternative explanation, suggested by our findings, is the absence of exposure risk.

of the evidence, we examine a theoretical model of the fixed rate tender.

3. The model

The central bank intends to distribute one unit of a perfectly divisible good at price $p_0 \geq 0$. There are $n \geq 2$ bidders. Each bidder $i = 1, ..., n$ is asked to submit a bid $b_i \geq 0$. The allotment to bidder i is then

$$\widehat{q}_i(b_i, b_{-i}) = \begin{cases} b_i & \text{if} \quad b_i + b_{-i} \leq 1 \\ \dfrac{b_i}{b_i + b_{-i}} & \text{if} \quad b_i + b_{-i} > 1, \end{cases} \tag{1}$$

where $b_{-i} = \sum_{j \neq i} b_j$ denotes the aggregate competing bids of bidders $j \neq i$.

Bidder i's *marginal valuation* $v_i(q_i, \theta_i)$ at quantity $q_i \geq 0$ depends on a type parameter θ_i, drawn from an interval $[\underline{\theta}_i; \overline{\theta}_i]$, where $0 \leq \underline{\theta}_i < \overline{\theta}_i$. Only bidder i observes θ_i. The joint distribution of types is given by a density $f(\theta_1, ..., \theta_n)$. Thus, types may be correlated. We assume that $v_i(q_i, \theta_i)$ is continuously differentiable in (q_i, θ_i) with $\partial v_i / \partial q_i < 0$, as illustrated in Figure 3.[9] Type θ_i's *demand* at p_0 is defined as the maximum quantity q_i such that $v_i(q_i, \theta_i) \geq p_0$. In fact, to simplify the exposition, we assume that type θ_i has a demand of just θ_i.[10] We also assume that an individual bidder's demand is bounded away from total supply, i.e., $\overline{\theta}_i < 1$ for all i.

-Figure 3 about here-

[9] Marginal valuations may be declining, e.g., owing to collateral requirements. See Ewerhart et al. (2010).

[10] In fact, two bidders with the same demand at p_0 might possess different marginal valuation functions. Our results do not depend on this assumption.

Payoffs for bidder i are given by

$$\Pi_i(b_i, b_{-i}; \theta_i) = \int_0^{\widehat{q}_i(b_i, b_{-i})} v_i(q_i, \theta_i) dq_i - p_0 \widehat{q}_i(b_i, b_{-i}). \tag{2}$$

In Figure 3, $\Pi_i(b_i, b_{-i}; \theta_i)$ corresponds to the signed area bordered by the vertical axis, the horizontal line at p_0, the marginal valuation curve, and a vertical line at the allotment to bidder i.

A *(pure) strategy* for bidder i is a mapping $\beta_i : [\underline{\theta}_i; \overline{\theta}_i] \rightarrow \mathbb{R}_+$ that assigns a bid $b_i = \beta_i(\theta_i) \geq 0$ to each type θ_i. For a given profile $\{\beta_j(.)\}_{j \neq i}$ of strategies for bidders $j \neq i$, *expected payoffs* for a bidder i of type θ_i resulting from a bid b_i are given by

$$\overline{\Pi}_i(b_i, \theta_i) = \int_0^{\infty} \Pi_i(b_i, b_{-i}; \theta_i) dG_{\theta_i}(b_{-i}), \tag{3}$$

where $G_{\theta_i}(.)$ denotes the distribution function of $b_{-i} = \sum_{j \neq i} \beta_j(\theta_j)$, conditional on θ_i.

The fixed rate tender is modeled here as a game of incomplete information (cf. Harsanyi, 1967-68). In line with the auction-theoretic literature pioneered by Vickrey (1961), we search for a Bayesian Nash equilibrium for this game. In equilibrium, each bidder is deemed to correctly anticipate the strategies (i.e., the bid functions) chosen by the other bidders. Via the strategies, the uncertainty about types translates into uncertainty about the size of aggregate competing bids. Thereby, any bid determines a probability distribution for the allotment to the individual bidder. Each bidder then chooses its bid so as to maximize the expected payoffs from this uncertain outcome.

We now discuss the optimal bid in the fixed rate tender. Bidding below demand is obviously strictly dominated. Indeed, the only case where demand reduction is (weakly) optimal is when an individual bidder's demand exceeds total supply and aggregate competing bids are nil. This, however, is not possible under our assumptions. Further, note that a bidder with vanishing demand optimally bids zero because a strictly positive bid always leads to a strictly positive allotment, which would be suboptimal. The trade-off for a bidder i with type $\theta_i > 0$ is as follows. When bidding $b_i \geq \theta_i$, there are three cases, depending on aggregate competing bids. First, if $b_{-i} \leq 1 - b_i$, then there is no rationing, and the allotment to bidder i weakly exceeds its demand. Of course, this case is not possible if b_i exceeds unity. Second, if $1 - b_i < b_{-i} \leq b^0_{-i} \equiv b_i \frac{1-\theta_i}{\theta_i}$, then there will be rationing, yet the allotment to bidder i nevertheless weakly exceeds its demand. Finally, if $b_{-i} > b^0_{-i}$, then the allotment to bidder i after rationing falls short of its demand.

The first-order condition reflects these cases and balances the likelihood of gains from receiving marginal units within demand against the likelihood of losses from receiving marginal units exceeding demand. For example, for a well-behaved distribution of b_{-i} that allows a density $g_{\theta_i}(.)$, the necessary first-order condition for an interior maximum is[11]

$$
\begin{aligned}
&\{p_0 - v_i(b_i,\theta_i)\}G_{\theta_i}(1-b_i) \\
&+\int_{\{1-b_i\}^+}^{b^0_{-i}} \{p_0 - v_i(\frac{b_i}{b_i+b_{-i}},\theta_i)\}\frac{b_{-i}}{(b_i+b_{-i})^2} g_{\theta_i}(b_{-i})db_{-i} \\
&= \int_{b^0_{-i}}^{\infty} \{v_i(\frac{b_i}{b_i+b_{-i}},\theta_i) - p_0\}\frac{b_{-i}}{(b_i+b_{-i})^2} g_{\theta_i}(b_{-i})db_{-i}. \qquad (4)
\end{aligned}
$$

Figure 4 illustrates the expressions appearing in the first-order condition (4), such as the marginal valuation and the derivative of the allotment rule, both of which are considered here as functions of b_{-i}.[12]

-Figure 4 about here-

4. Equilibrium bidding

This section deals with the existence of an equilibrium in the fixed rate tender with a pre-announced intended volume. Intuitively, the crucial condition for an equilibrium to exist is to exclude the possibility of mutually reinforcing expectations of overbidding. To achieve this, bidders should find it in their interests to overbid only moderately when their competitors do the same. Thus, in equilibrium, the likelihood and potential detriment of a low demand realization should be important in view of the extent and potential detriment of expected excess demand.

Formally, choose parameters m and M, once and for all, such that $m > 0$ and $\max\{\overline{\theta}_1, ..., \overline{\theta}_n\} < M < 1$. Consider a bidder i of type θ_i. Let

[11] As usual, $\{x\}^+ = x$ if $x \geq 0$, and $\{x\}^+ = 0$ if $x < 0$.

[12] Of course, the derivative of the allotment rule is well-defined for $b_{-i} \neq 1 - b_i$ only.

$\pi(i,\theta_i)$ =pr$\{\theta_i + \sum_{j\neq i}\theta_j \leq M$ and $\sum_{j\neq i}\theta_j \geq m|\theta_i\}$ denote that bidder's assessment of the probability that total demand is bounded by M, while aggregate competing demand weakly exceeds m.[13] Further, let $\varepsilon(i,\theta_i) = E[\{\theta_i + \sum_{j\neq i}\theta_j - 1\}^+|\theta_i]$ denote the excess demand expected by that bidder. Write ε and π, respectively, for the supremum of $\varepsilon(i,\theta_i)$ and the infimum of $\pi(i,\theta_i)$, taken across all bidders and type realizations. Finally, let $s = \min\{|\frac{\partial v_i}{\partial q_i}| : i = 1,...,n; q_i \leq \theta_i\}$ and $S = \max\{|\frac{\partial v_i}{\partial q_i}| : i = 1,...,n; \theta_i \leq q_i \leq \frac{\theta_i}{M}\}$, respectively, denote the minimum slope, in absolute terms, of the marginal valuation curve for allotments below θ_i, and the corresponding maximum slope for allotments between θ_i and $\frac{\theta_i}{M}$, as illustrated in Figure 3.[14]

The following lemma is the key result driving the existence theorem.

Lemma 1. *Let α satisfy $\frac{1}{M} \equiv \alpha_* \leq \alpha < \alpha^* \equiv m\frac{1-M}{M^3}\frac{\pi s}{\varepsilon S}$. Assume that each bidder $j \neq i$ follows some strategy $\beta_j(.)$ such that $\theta_j \leq \beta_j(\theta_j) \leq \alpha\theta_j$ for all θ_j. Then bidder i's best response $\beta_i(.)$ exists and satisfies $\theta_i \leq \beta_i(\theta_i) \leq \alpha\theta_i$.*

The proof is in the Appendix. The lemma finds conditions under which expectations of moderate overbidding induce likewise moderate overbidding. Moderate means here using a factor of no more than α, where α is taken

[13] To understand the intuitive role of the second constraint in the definition of $\pi(i,\theta_i)$, note that $\widehat{q}_i$ may be quite inelastic in b_i when b_{-i} is small. E.g., for $b_i = 2$ and $b_{-i} = 0.1$, a decrease of b_i by 0.1 lowers $\widehat{q}_i$ by merely 0.02. Therefore, the risk of low demand with b_{-i} too small may indeed matter little as a limiting factor to overbidding.

[14] Note that the parameters s and S are well-defined and strictly positive. Indeed, $\partial v_i/\partial q_i$ is continuous on compact areas defined by either $q_i \leq \theta_i$ or $\theta_i \leq q_i \leq \theta_i/M$.

from the interval $[\alpha_*; \alpha^*)$. Note that the upper bound α^* increases in π and s, and decreases in ε and S. Thus, consistent with intuition, the range of non-escalating overbidding factors widens as exposure risk becomes more important in view of expected excess demand.

If $\alpha_* < \alpha^*$, then Lemma 1 implies existence of a mixed Nash equilibrium in the fixed rate tender. A *mixed strategy* for bidder i is here a probability distribution μ_i on $\mathbb{R}_+ \times [\underline{\theta}_i; \overline{\theta}_i]$ such that the marginal distribution of μ_i on $[\underline{\theta}_i; \overline{\theta}_i]$ coincides with bidder i's type distribution. We arrive at the main theoretical result of this paper, a proof of which may be found in the Appendix.

Theorem 1. *Assume $\varepsilon S < m\frac{1-M}{M^2}\pi s$. Then the fixed rate tender with pro rata allotment allows a mixed Nash equilibrium. In this equilibrium, bidders overbid by a factor of at most $1/M$, and the allotment quota never drops below $M/(\sum_{i=1}^n \overline{\theta}_i)$.*

Thus, provided that expected excess demand is low in view of exposure risk, the fixed rate tender with pro rata allotment allows an equilibrium even if bids are costless and the intended volume is pre-announced.[15]

5. Rationing with probability one

The following example illustrates the possibility of a Bayesian equilib-

[15] As an illustration, let types be uniformly distributed, with all $\overline{\theta}_i$ exceeding $1/n$ by a small amount only. Then an equilibrium exists.

rium. It also shows that the allotment quota may stay strictly below one with certainty.

Example 1. There are two bidders, with independent types. Bidder i's type θ_i is drawn from the interval $[\frac{1}{\lambda+1}; \frac{\lambda}{\lambda+1}]$ according to the density

$$f_i(\theta_i) = \frac{2(\lambda+1)}{\lambda-1}(1-\theta_i), \tag{5}$$

where $\lambda > 1$. This bidder's marginal valuation for an allotment q_i is given by

$$v_i(q_{i,}\theta_i) = p_0 + \frac{\theta_i - q_i}{q_i^2(1-q_i)}. \tag{6}$$

There is a pure Nash equilibrium in which type θ_i bids $\beta_i(\theta_i) = \lambda\theta_i/(1-\theta_i)$. Moreover, for any combination of bidder types, the allotment quota is strictly smaller than one.[16] $\square$

In the example above, rationing occurs with probability one. Nevertheless, an equilibrium exists because marginal valuations fall quickly, and the probability of high types, that overbid to a greater degree, is comparatively low.

Notably, an equilibrium in which rationing occurs with certainty differs structurally from an equilibrium in which with positive probability all bids are fulfilled. In the former, there is a coordination problem with respect to the

[16] To verify these claims, assume that $\beta_j(\theta_j) = \lambda\theta_j/(1-\theta_j)$ for all θ_j, where $j \neq i$. Then, bidder j's bids are distributed on the interval $[1; \lambda^2]$, so that any positive bid by bidder i engenders pro rata allotment. Hence, when bidding b_i against type θ_j, bidder i obtains the allotment $\widehat{q}_i = b_i/(b_i + \beta_j(\theta_j))$. Bidder i's problem is strictly concave, and leads to the necessary and sufficient first-order condition $\int \frac{\theta_i - \widehat{q}_i}{\widehat{q}_i} f_j(\theta_j) d\theta_j = 0$. Replacing $\widehat{q}_i$ by the explicit expression shows then that $\beta_i(\theta_i)$ is indeed optimal.

extent of overbidding. That is, the equilibrium strategies of all bidders may be scaled up by an arbitrary factor without affecting the resulting allotments. In an equilibrium in which with positive probability all bids are fulfilled, however, re-scaling of all bids does not in general lead to a new equilibrium. This is because the allotment changes in the case of bids where the bidder is uncertain as to whether they lead to rationing or not.

The potential indeterminacy of the equilibrium has implications in a repeated setting where expectations about bidding may depend on the outcome of earlier operations. The prevalent view in the literature is that the development of myopic and adaptive expectations is a consequence of the perceived non-existence of the equilibrium. A weakness of that interpretation, however, is that participants in a bidding race would be required to adhere to the hypothesis of, say, an unchanged allotment quota despite the recurrent empirical rejection of that hypothesis. In an alternative interpretation, suggested by Example 1, expectations in an overbidding episode are in fact rational, only the coordination along the linear trend is adaptive.

6. Dynamics

This section studies a dynamic extension of the model introduced in Section 3. Time $\tau = \tau_0, \tau_0 + 1, ...$ is discrete and tenders are organized sequentially. At the beginning of each period τ, a new type vector $(\theta_1(\tau), ..., \theta_n(\tau))$

is drawn according to the density $f(\theta_1, ...\theta_n)$. Thus, types are independent across periods, but possibly correlated within periods. In contrast to our assumptions made so far, bidders' expectations are not necessarily aligned. Instead, we assume *adaptive expectations*, meaning that bidders expect in period $\tau + 1$ that competitors will overbid to the same degree as in period τ. Moreover, bidders myopically choose their bids $b_i(\tau)$ so as to maximize expected payoffs in period τ.

Theorem 2. *Under the assumption of Theorem 1, assume that all bidders have exaggerated their demands in period $\tau \geq \tau_0$ by a factor of no more than $\alpha(\tau)$, where $\alpha(\tau) \in [\alpha_*; \alpha^*)$. Then, with adaptive expectations and myopic optimization, bidders exaggerate their demands in period $\tau + 1$ by a factor of no more than $\alpha(\tau + 1)$, where $\alpha(\tau + 1) < \alpha(\tau)$. In fact, starting from any initial value $\alpha(\tau_0) \in [\alpha_*; \alpha^*)$, the sequence $\{\alpha(\tau)\}_{\tau=\tau_0,...}$ declines exponentially until it falls and stays below α_*.*

Proof. By assumption, bidder i expects that aggregate competing bids $b_{-i}(\tau+1) = \sum_{j\neq i} b_j(\tau+1)$ satisfy $b_{-i}(\tau+1) \leq \alpha(\tau) \sum_{j\neq i} \theta_j(\tau+1)$ for all type profiles $(\theta_1(\tau+1), ..., \theta_{i-1}(\tau+1), \theta_{i+1}(\tau+1), ..., \theta_n(\tau+1))$. We claim that the optimal bid for bidder i in period $\tau+1$ satisfies $b_i(\tau+1) \leq \alpha(\tau+1)\theta_i(\tau+1)$, where $\alpha(\tau + 1) = (1 - \delta)\alpha(\tau)$ for some $\delta > 0$. Clearly, $b_i(\tau + 1) = 0$ if $\theta_i(\tau + 1) = 0$. Consider therefore the case $\theta_i(\tau + 1) > 0$. To establish a contradiction, assume that $b_i(\tau+1) > \alpha(\tau+1)\theta_i(\tau+1)$. We compare the LHS and RHS of bidder i's first-order condition, as in the proof of Lemma 1. For this, note that

$$b_{-i}(\tau+1) - b^0_{-i}(\tau+1) \equiv b_{-i}(\tau+1) - \frac{1-\theta_i(\tau+1)}{\theta_i(\tau+1)} b_i(\tau+1) \tag{7}$$

$$< \alpha(\tau) \sum_{j \neq i} \theta_j(\tau+1) - \{1-\theta_i(\tau+1)\}\alpha(\tau+1) \tag{8}$$

$$\leq \alpha(\tau)\{\theta_i(\tau+1) + \sum_{j \neq i} \theta_j(\tau+1) - 1 + \delta\}. \tag{9}$$

Hence, in analogy to inequality (23),

$$\int_{b^0_{-i}}^{\infty} (b_{-i}(\tau+1) - b^0_{-i}(\tau+1)) dG_{\theta_i}(b_{-i}) \leq \alpha(\tau)(\varepsilon+\delta). \tag{10}$$

For a sufficiently small δ such that $\frac{1}{M} \leq \alpha(\tau) < m \frac{1-M}{M^3} \frac{\pi s}{(\varepsilon+\delta)S}$, we obtain the desired contradiction. Finally, note that δ can be chosen independently of τ because $\alpha(\tau)$ is decreasing in the relevant domain. □

Thus, myopic best responses to adaptive expectations may lead to a steady decline in overbidding factors to the level predicted for equilibrium strategies. The range of overbidding factors over which this type of convergence is obtainable clearly depends on the relative importance of expected excess demand and exposure risk, as discussed before. Nevertheless, Theorem 2 can be seen as an "optimistic" counterpart to Nautz and Oechssler's (2003) divergence prediction.[17]

[17] On the other hand, Theorem 2 makes no prediction for the case that overbidding factors already exceed α^*. This leaves room for the interesting possibility that a bid race might continue *solely* as a result of misaligned expectations (cf. Ehrhart, 2001).

7. Welfare

The fixed rate tender commonly results in an inefficient allocation whenever bidders find it difficult to coordinate their expectations. As we show in this section, however, the situation is not much better when bidders have equilibrium expectations. Intuitively, when there is a possibility of excess demand, the allotment for some bidder must fall short of demand with strictly positive probability. Since that bidder makes an optimal choice in equilibrium, the allotment must likewise exceed demand with strictly positive probability. In an efficient allocation, however, the allotment never exceeds a bidder's demand.

Formally, let

$$W(q;\theta) = \sum_{i=1}^{n} \{ \int_0^{q_i} v_i(\widetilde{q}_i, \theta_i) d\widetilde{q}_i - p_0 q_i \} \tag{11}$$

denote the welfare associated with an *ex-post allocation* $q = (q_1, ..., q_n)$ and a type vector $\theta = (\theta_1, ..., \theta_n)$. Note that no welfare is associated with any fraction of the good potentially left with the central bank. An ex-post allocation q is *feasible* if $q_i \geq 0$ for all bidders i and $\sum_{i=1}^{n} q_i \leq 1$. Given θ, an ex-post allocation q is *efficient* if it maximizes $W(.;\theta)$ under the feasibility constraint.

Theorem 3. *Assume that* $\sum_{i=1}^{n} \theta_i > 1$ *with strictly positive probability. Then, in any equilibrium of the fixed rate tender, pure or mixed, the ex-post* *allocation will be inefficient with strictly positive probability.*

Proof. Consider first a pure Nash equilibrium $\{\beta_i^*(.)\}_{i=1,...,n}$. Denote by $q^*(\theta) = (q_1^*(\theta), ..., q_n^*(\theta))$ the ex-post allocation resulting from that equilibrium as a function of $\theta = (\theta_1, ..., \theta_n)$. To provoke a contradiction, assume that $q^*(\theta)$ is efficient with probability one. Since individual allotments exceeding demand are inefficient, $v_i(q_i^*(\theta), \theta_i) \geq p_0$ with probability one, for all i. Furthermore, since there is a possibility of excess demand, there must be some bidder i such that $v_i(q_i^*(\theta), \theta_i) > p_0$ with strictly positive probability. However, from the optimality of $\beta_i^*(\theta_i)$,

$$\int \{v_i(\widehat{q}_i(\beta_i^*(\theta_i), b_{-i}), \theta_i) - p_0\} \frac{\partial_+ \widehat{q}_i}{\partial b_i}(\beta_i^*(\theta_i), b_{-i}) dG_{\theta_i}(b_{-i}) \leq 0, \qquad (12)$$

where $\frac{\partial_+ \widehat{q}_i}{\partial b_i} > 0$ denotes the right derivative of $\widehat{q}_i(b_i, b_{-i})$ with respect to b_i. Integrating (12) over θ_i yields

$$\int \{v_i(q_i^*(\theta), \theta_i) - p_0\} \frac{\partial_+ \widehat{q}_i}{\partial b_i} f(\theta) d\theta_1 ... d\theta_n \leq 0, \qquad (13)$$

which is the desired contradiction. Thus, any pure Nash equilibrium is inefficient. In a mixed equilibrium $\mu^* = (\mu_1^*, ..., \mu_n^*)$, the ex-post allocation $q^*(b)$ depends directly on $b = (b_1, ..., b_n)$. As above, there is then a bidder i such that $v_i(q_i^*(b), \theta_i) \geq p_0$ with probability one and $v_i(q_i^*(b), \theta_i) > p_0$ with strictly positive probability. Denote by $\mu_i^{(\alpha)}$ the mixed strategy that increases all bids in μ_i^* uniformly by a factor $\alpha > 1$. Optimality of μ_i^* implies

$$\frac{\partial_+}{\partial\alpha}\bigg|_{\alpha=1} \int \Pi_i d(\mu_i^{(\alpha)}, \mu_{-i}^*) = \int \{v_i(q_i^*(b), \theta_i) - p_0\} \frac{\partial_+ \widehat{q}_i}{\partial b_i} b_i d\mu^*(b, \theta) \leq 0. \quad (14)$$

Since in equilibrium, $b_i > 0$ with probability one, the assertion follows as above. $\square$

Thus, when there is a positive probability that aggregate demand exceeds supply, then the allocation implemented through the fixed rate tender will be inefficient even with equilibrium expectations.

8. Empirical test

To assess the practical importance of exposure risk for bidding behavior in fixed rate tenders, we use data on liquidity-providing operations conducted by the Eurosystem in the period January 1999 to June 2000, and by the BoE in the period May 2006 to October 2008.

The test is based on the following reduced-form model of bidding behavior. The endogenous variable is the log of total bids b_t submitted in tender t, where $t = 1, 2, 3,...$ counts the weekly tenders in chronological order.

$$\begin{aligned} \ln b_t &= \gamma_0 + \gamma_1 t + \gamma_2 \ln a_t + \gamma_3 \ln \widehat{a}_{t-1} + \gamma_4 \ln b_{t-1} + \gamma_5 \ln b_{t-2} \\ &\quad + \gamma_6 (\rho - r)_t + \gamma_7 (\rho - \sigma)_t + \eta_t \end{aligned} \quad (15)$$

In this specification, a_t and $\widehat{a}_{t-1}$ are the *intended volume* and the *previous allotment*, respectively.[18] To take account of the two-week maturity in the euro area, we include two lagged variables of bids, b_{t-1} and b_{t-2}. The interest rate applied by the central bank in the tender is denoted by r. Arbitrage possibilities are proxied by the spread between the unsecured rate and the tender rate, $(\rho - r)_t$, where the unsecured rate ρ corresponds to the two-week EONIA swap rate for the euro area and to the one-week LIBOR for the UK, respectively.[19] The hypothesis of exposure risk is operationalized by the spread between unsecured and secured rates, $(\rho - \sigma)_t$, where the secured rate σ is the ECB's market repo rate for the euro area and the one-week general collateral rate for the UK, respectively.[20]

The time series for the BoE was split into operations that settled on or before August 2, 2007, and operations that settled on or after August 9, 2007. We refer to the former as the *pre-crisis sample*, and to the latter as the *crisis sample*. The model has been estimated by ordinary least squares for the three samples, i.e., ECB, BoE pre-crisis, and BoE crisis. Table 3 summarizes the results of the regressions.

-Table 3 about here-

The left-hand column shows the regression for the Eurosystem operations.

[18] As a proxy for the intended volume in the euro area, we use the benchmark allotment at the announcement date of the tender. Cf. Section 2.

[19] EONIA = euro overnight index average; LIBOR = London interbank offered rate.

[20] For the euro area, we use the one-week repo spread, for which the longest time series is available (settlement not before April 21, 1999). However, the results are almost identical for the two-week repo.

Consistent with findings in prior contributions, arbitrage possibilities had a strong influence on bids in the euro area. Indeed, the coefficient of the spread $(\rho - r)_t$ is highly significant and positive. The time trend is also significant and positive.[21] The new element here is the regressor for exposure risk. Its coefficient is highly significant and negative, as predicted by the theoretical analysis.

The regression results for the BoE during the pre-crisis period are shown in the middle column. While the coefficient for arbitrage is positive, it is not significant. Moreover, the trend is weakly significant, but negative. In sum, this suggests that neither the arbitrage hypothesis nor the rationing hypothesis had a strong role in the pre-crisis sterling auctions. There is, however, a significant and strongly negative effect of exposure risk. In economic terms, an increase in $(\rho - \sigma)_t$ of, say, 5 basis points would lower total bids by 23.0 percent in the BoE pre-crisis sample, compared with 4.4 percent in the Eurosystem sample.

Finally, in the right-hand column, the table shows the coefficients for the BoE during the crisis. The intended volume is significant, potentially because bidders became more responsive to information after August 2007. More importantly for our present analysis, the coefficient for exposure risk

[21] The usual interpretation of the time trend is that it captures the adaptive response to rationing, see Nautz and Oechssler (2006).

is again highly significant and negative. The somewhat smaller coefficient for exposure risk, compared with the pre-crisis period, might reflect the fact that the relative scarcity of cash and high-quality collateral became more balanced during the crisis. That interpretation would also be consistent with the significant arbitrage coefficient. In sum, we find that exposure risk has been a significant determinant of bidding behavior in all three samples.

The results of the regressions are consistent with anecdotal evidence on central bank collateral in euro and sterling markets. In the euro area, eligibility criteria for collateral have traditionally been broad. In fact, the first mention of a scarcity of collateral came at a time when banks in some euro area countries felt at a disadvantage during the peak of the overbidding episode.[22] The situation was very different in sterling markets. Indeed, the largest fraction of collateral used in sterling operations was gilts (i.e., UK government bonds), and these were, typically, not owned outright by the bidders, i.e., by banks and building societies, but were borrowed through securities lending transactions from the ultimate holders of gilt securities, namely pension funds and insurance companies. Therefore, when the bid-to-cover ratio happened to be unexpectedly low, bidders with insufficient collateral were indeed forced to go in search of more.[23]

[22] Cf. Deutsche Bundesbank (2000).

[23] In fact, there was a squeeze in the gilt market on July 31, 2006. Cf. Bank of England (2007).

In the remainder of this section, we discuss informally the performance of the fixed rate tender in the other six cases (cf. Table 1). Consider first the three samples in which operations withdrew liquidity from the market, i.e., the fine-tuning euro operations as well as the liquidity-absorbing weekly and fine-tuning sterling operations. The exposure risk in these operations differed from the exposure risk in the liquidity-providing operations discussed above. If a bidder receives an excessive allotment in a liquidity-absorbing operation, it has an unexpected outflow of liquidity. To compensate, the bidder must then either borrow in the market or have recourse to the central bank's lending facility.[24] Typically, both options will be costly in terms of the interest rate spread. More critically, credit limits may be exhausted in the market, and the bidder may also sustain a reputational damage from seeking additional credit. Finally, it has been suggested that not only in the US but also in Europe, there may be a stigma attached to the use of the central bank facility. Therefore, even if protected by anonymity, the bidder might still be exposed to rumors and insinuations. We conclude that receiving an oversized allotment in a liquidity-absorbing operation is potentially even more harmful than receiving an oversized allotment in a liquidity-providing operation. In particular, this would explain the quite satisfactory performance of the fixed

[24] Further alternatives such as selling assets on short notice are possible, but may not always be desirable.

rate tender in these three examples, all of which have very low mean log bid-to-cover ratios.

Next, we turn to the BoE's liquidity-providing fine-tuning operations. On the one hand, operations in that sample tend to exhibit a somewhat higher mean log bid-to-cover ratio than their liquidity-absorbing counterparts, which is consistent with the factors discussed above. On the other hand, their better performance compared with the BoE's weekly liquidity-providing tenders might be related to the fact that reserve account holders in the UK had some flexibility with regard to the fulfillment of their reserve requirements, which should have lowered demand in the fine-tuning operations.

Regarding foreign exchange swap operations, Figure 2 shows that in contrast to the other examples, the liquidity policy for these operations was biased towards excess supply, which explains the very low mean log bid-to-cover ratio of -0.73. During the short period with excess demand, however, the fixed rate tender generated a rather special situation. Specifically, if a bidder received an oversized allotment, it would have a large Swiss franc liquidity inflow as well as a large euro liquidity outflow. The exposure risk associated with the liquidity outflow might have contributed to the relatively swift decline in total bids after the peak of the overbidding period.

Finally, the US dollar tender was similar to an ordinary liquidity-providing

Eurosystem operation in terms of exposure risk. That is to say, exposure risk was probably not very pronounced owing to the relatively broad definition of eligible collateral. This view is consistent with the downward trend in the allotment quota. Indeed, our results suggest that even lower quotas might have been possible if these operations had continued in an unchanged market environment.

9. Related literature

Our main theoretical result is a robust condition for equilibrium existence when the intended volume of the operation is pre-announced and there are no costs of bidding. To the best of our knowledge, this result is new in the literature. Ayuso and Repullo (2003) explain the overbidding observed in the Eurosystem over the period January 1999 to June 2000 as a consequence of an asymmetric objective function for the central bank. In contrast to the present paper, however, Ayuso and Repullo have a cost function that depends on the size of the bid. Nyborg and Strebulaev (2001) consider equilibria in fixed rate tenders with subsequent short squeezes in liquidity, assuming that bids are constrained by the amount of collateral held by the bidders. Nautz and Oechssler (2003) document the overbidding phenomenon. They show that the rationing game does not allow an equilibrium to exist in a complete-information setting, and explain the development of aggregate bids during the overbidding period as being driven by a myopic best-reply process. Ehrhart (2001) draws similar conclusions from an experimental study. Bindseil (2005) provides a survey of the experience of modern central banks with fixed rate tenders, stressing in particular the case of the Eurosystem. He also analyzes the macro behavior of a banking system facing a cost of bidding that depends on the aggregate bid.[25] Ehrhart (2002) extends the non-existence result in various ways, allowing in particular for uncertainty about supply and repeated interaction. Välimäki (2003) assumes a two-part penalty consisting of a punitive interest rate charged on the share of collateral that is not delivered and a fixed fine for non-compliance. He then studies the decision of an individual bank to bid optimally against a given probability distribution of aggregate bids submitted by the other banks. Catalão-Lopes (2010) compares the fixed rate tender with the uniform-price tender, stressing the non-existence result and collateral constraints on bids. Thus, it appears that the case considered in this paper is not covered by the existing literature.[26]

10. Conclusion

Central banks are fond of the fixed rate tender because it conveys monetary policy signals to the market usually with very little noise. Pro rata

[25] See also Bindseil (2004).

[26] This statement includes related work on market disequilibrium (e.g., Bénassy, 1977) and supply chain management (e.g., Lee et al., 1997).

allotment is applied in these tenders when the total of bids received exceeds the total amount that the central bank intends to provide. In the present paper, we argue that exposure risk is a critical determinant of bidding behavior in fixed rate tenders. Owing to exposure risk, an equilibrium exists even when bids are costless and the intended volume of the operation is preannounced. In this equilibrium, the extent of overbidding is limited, and there is a bound below which the allotment quota never falls. We also find conditions under which temporarily elevated overbidding factors will, with adaptive expectations, decline to the levels predicted for the equilibrium. In particular, this suggests a rationale for the continued use of the fixed rate tender by central banks such as the ECB and the BoE.[27]

By clarifying the role of exposure risk for bidding behavior in fixed rate tenders, our analysis allows the overbidding phenomenon in the Eurosystem operations to be put into a somewhat broader context. The comparison with the sterling market strongly suggests that an absence of exposure risk, owing to the market-friendly eligibility criteria for central bank collateral, was a critical factor in the escalation of bids in the euro area. In particular, our findings suggest that a potential reintroduction of the fixed rate tender with pro rata allotment in the main refinancing operations of the Eurosystem

[27] Having an equilibrium is also a prerequisite for further theoretical work such as comparing the fixed rate and variable rate tenders in terms of efficiency. Exploring that issue, however, would go beyond the scope of the present analysis.

would presuppose a substantial review of the existing collateral framework.

Appendix: Proofs

Proof of Lemma 1. Recall from the discussion of the optimal bid that $\beta_i(0) = 0$, and $\beta_i(\theta_i) \geq \theta_i$ for all θ_i. In particular, we may assume in the sequel that $\theta_i > 0$ and $b_i > 0$. The first-order condition associated with a marginal decrease in b_i is

$$\int_0^{\infty} \{v_i(\widehat{q}_i(b_i, b_{-i}), \theta_i) - p_0\} \frac{\partial_- \widehat{q}_i}{\partial b_i}(b_i, b_{-i}) dG_{\theta_i}(b_{-i}) \geq 0, \tag{16}$$

where

$$\frac{\partial_- \widehat{q}_i}{\partial b_i}(b_i, b_{-i}) = \begin{cases} 1 & \text{if} \quad b_i + b_{-i} \leq 1 \\ \dfrac{b_{-i}}{(b_i + b_{-i})^2} & \text{if} \quad b_i + b_{-i} > 1 \end{cases} \tag{17}$$

denotes the left derivative of the allotment rule (1). Since $v_i - p_0$ vanishes at $b_{-i} = b^0_{-i}$, we may safely ignore a potential mass point of $G_{\theta_i}(.)$ at $b_{-i} = b^0_{-i}$ and rewrite (16) as

$$\begin{aligned} &\int_0^{b^0_{-i}} \{p_0 - v_i(\widehat{q}_i(b_i, b_{-i}), \theta_i)\} \frac{\partial_- \widehat{q}_i}{\partial b_i}(b_i, b_{-i}) dG_{\theta_i}(b_{-i}) \\ &\leq \int_{b^0_{-i}}^{\infty} \{v_i(\frac{b_i}{b_i + b_{-i}}, \theta_i) - p_0\} \frac{b_{-i}}{(b_i + b_{-i})^2} dG_{\theta_i}(b_{-i}). \end{aligned} \tag{18}$$

We claim that under the assumptions of the lemma, for $b_i > \alpha\theta_i$, the left-hand side (LHS) of the first-order condition (18) exceeds the right-hand side (RHS). But then, lowering b_i marginally would raise expected profits. In fact, the continuous function $\overline{\Pi}_i(., \theta_i)$ must then be strictly declining for $b_i > \alpha\theta_i$. Thus, an optimal bid exists and satisfies $\beta_i(\theta_i) \leq \alpha\theta_i$.

RHS. By the definition of S, we have $v_i(\widehat{q}_i, \theta_i) - p_0 \leq S(\theta_i - \widehat{q}_i)$ for all $b_{-i} \geq b^0_{-i}$. Since $\widehat{q}_i(b_i, .)$ is convex for $b_{-i} \geq b^0_{-i}$,

$$\theta_i - \widehat{q}_i(b_i, b_{-i}) \leq (b_{-i} - b^0_{-i}) \left.\frac{\partial}{\partial b_{-i}}\right|_{b_{-i}=b^0_{-i}} \{\theta_i - \widehat{q}_i(b_i, b_{-i})\} \tag{19}$$

$$= (b_{-i} - b^0_{-i})\frac{\theta_i^2}{b_i}. \tag{20}$$

Using $b_{-i} \leq \alpha\sum_{j\neq i}\theta_j$ and $b_i > \alpha\theta_i$, one finds $b_{-i} - b^0_{-i} \leq \alpha(\theta_i + \sum_{j\neq i}\theta_j - 1)$. Moreover, for $b_{-i} \geq b^0_{-i}$,

$$\frac{b_{-i}}{(b_i + b_{-i})^2} \leq \frac{1}{b_i + b_{-i}} \leq \frac{1}{b_i + b^0_{-i}} = \frac{\theta_i}{b_i}. \tag{21}$$

Thus,

$$\text{RHS} \leq \frac{\theta_i^3 S}{b_i^2} \int_{b^0_{-i}}^{\infty} (b_{-i} - b^0_{-i}) dG_{\theta_i}(b_{-i}) \tag{22}$$

$$\leq \frac{\alpha\theta_i^3 S}{b_i^2} \int \{\theta_i + \textstyle\sum_{j\neq i}\theta_j - 1\}^+ dF_{\theta_i}(\theta_{-i}), \tag{23}$$

where $F_{\theta_i}(.)$ is the conditional distribution of $\theta_{-i} = (\theta_1, ..., \theta_{i-1}, \theta_{i+1}, ..., \theta_n)$ given θ_i.

LHS. Using $b_i > \alpha\theta_i \geq \theta_i/M$, it is straightforward to verify that the allotment to bidder i exceeds θ_i/M if and only if $b_{-i} < b^M_{-i} \equiv b_i(M - \theta_i)/\theta_i$. Note also that $b^M_{-i} > 1 - b_i$. As marginal valuations are declining, and because

$\widehat{q}_i(b_i, .)$ is weakly decreasing, one obtains for $b_{-i} \leq b^M_{-i}$ that

$$
\begin{aligned}
p_0 - v_i(\widehat{q}_i(b_i, b_{-i}), \theta_i) &\geq p_0 - v_i(\widehat{q}_i(b_i, b^M_{-i}), \theta_i) && (24)\\
&= v_i(\theta_i, \theta_i) - v_i(\frac{\theta_i}{M}, \theta_i) && (25)\\
&\geq \frac{1-M}{M}\theta_i s. && (26)
\end{aligned}
$$

Moreover, for $1 - b_i < b_{-i} \leq b^M_{-i}$,

$$\frac{\partial_- \widehat{q}_i}{\partial b_i}(b_i, b_{-i}) = \frac{b_{-i}}{(b_i + b_{-i})^2} = (\frac{b_i}{b_i + b_{-i}})^2 \frac{b_{-i}}{b_i^2} \geq (\frac{\theta_i}{b_i M})^2 b_{-i}. \tag{27}$$

Also for $b_{-i} \leq 1 - b_i$,

$$\frac{\partial_- \widehat{q}_i}{\partial b_i}(b_i, b_{-i}) = 1 \geq \frac{b_{-i}}{(\alpha M)^2} \geq (\frac{\theta_i}{b_i M})^2 b_{-i}, \tag{28}$$

because $\alpha \geq 1/M$. Hence,

$$\text{LHS} \geq \frac{(1-M)\theta_i^3 s}{M^3 b_i^2} \int_0^{b^M_{-i}} b_{-i} dG_{\theta_i}(b_{-i}). \tag{29}$$

As above, one shows that $b_{-i} - b^M_{-i} \leq \alpha(\theta_i + \sum_{j \neq i} \theta_j - M)$. Thus, if $\theta_i + \sum_{j \neq i} \theta_j \leq M$, then $b_{-i} \leq b^M_{-i}$. Therefore,

$$
\begin{aligned}
\text{LHS} &\geq \frac{(1-M)\theta_i^3 s}{M^3 b_i^2} \int_{\theta_i + \sum_{j \neq i} \theta_j \leq M} \sum_{j \neq i} \theta_j dF_{\theta_i}(\theta_{-i}) && (30)\\
&\geq \frac{(1-M)\theta_i^3 s}{M^3 b_i^2} \cdot m \cdot \pi(i, \theta_i). && (31)
\end{aligned}
$$

This implies that, indeed, LHS>RHS for all $b_i > \alpha\theta_i$. $\square$

Proof of Theorem 1. Consider the *restricted game* in which each bidder $i = 1, ..., n$ chooses an overbidding factor $\alpha_i \in [1; \alpha_*]$, and bidder i's payoffs are given by $U_i(\alpha_1, ..., \alpha_n; \theta_1, ..., \theta_n) = \Pi_i(\alpha_i\theta_i, \sum_{j\neq i} \alpha_j\theta_j; \theta_i)$. Glicksberg's theorem implies existence of an equilibrium in mixed strategies in the restricted game provided that (R1) payoffs are equicontinuous, and (R2) information is absolutely continuous. For details, see Milgrom and Weber (1985), and note that our definition of a mixed strategy corresponds to their notion of a distributional strategy. Condition (R1) holds if the family of functions $\{U_i(.; \theta_1, ..., \theta_n)|\theta_i \in [\underline{\theta}_i; \overline{\theta}_i] \text{ for all } i\}$ is equicontinuous. Since payoffs are continuous in actions, it suffices to show that all functions $\alpha_k \mapsto U_i(\alpha_1, ..., \alpha_n; \theta_1, ..., \theta_n)$, keeping the other entries fixed, are piecewise differentiable with a uniformly bounded derivative. There are several cases. Consider first $k \neq i$ and $\theta_k > 0$. Then $\frac{\partial U_i}{\partial \alpha_k} = \{\frac{\partial v_i}{\partial q_i} - p_0\}\frac{\partial \widehat{q}_i}{\partial b_{-i}}\theta_k$ provided that $\alpha_k \neq \frac{1}{\theta_k}(1 - \sum_{j\neq k} \alpha_j\theta_j)$. It is not hard to check that $|\frac{\partial \widehat{q}_i}{\partial b_{-i}}| \leq 1$. Moreover, $\frac{\partial v_i}{\partial q_i}$ is continuous on the compact set $[0; 1] \times [\underline{\theta}_i; \overline{\theta}_i]$. Hence, $|\frac{\partial U_i}{\partial \alpha_k}|$ is uniformly bounded for all $k \neq i$ and $\theta_k > 0$. For $k = i$ and $\theta_i > 0$, the argument is analogous. Finally, if $\theta_k = 0$, then $\frac{\partial U_i}{\partial \alpha_k} \equiv 0$. This proves (R1). Condition (R2) holds because the joint distribution of types has a density. Thus, there is an equilibrium $(\nu_1^*, ..., \nu_n^*)$ in mixed strategies in the restricted game. Consider now the mapping $\varphi_i : [1; \alpha_*] \times [\underline{\theta}_i; \overline{\theta}_i] \rightarrow \mathbb{R}_+ \times [\underline{\theta}_i; \overline{\theta}_i]$ defined by $\varphi_i(\alpha_i, \theta_i) = (\alpha_i\theta_i, \theta_i)$. Given that φ_i preserves the type, the pushforward measure of ν_i^* with respect to φ_i, denoted by $\mu_i^* = \nu_i^* \circ \varphi_i^{-1}$, is a mixed strategy in the fixed rate tender. Moreover, $\int \Pi_i d(\mu_1, ..., \mu_n) = \int U_i d(\nu_1, ..., \nu_n)$

if $(\nu_1, ..., \nu_n)$ is a profile of mixed strategies in the restricted game, and $(\mu_1, ..., \mu_n)$ is the corresponding profile of pushforward strategies. We claim that $(\mu_1^*, ..., \mu_n^*)$ is an equilibrium in the fixed rate tender. Let μ_i be an arbitrary deviation by bidder i. Consider the mixed strategy $\widetilde{\mu}_i = \mu_i \circ \phi_i^{-1}$, where

$$\phi_i(b_i, \theta_i) = \begin{cases} (\theta_i, \theta_i) & \text{if} \quad b_i < \theta_i \\ (b_i, \theta_i) & \text{if} \quad \theta_i \leq b_i \leq \alpha_* \theta_i \\ (\alpha_* \theta_i, \theta_i) & \text{if} \quad b_i > \alpha_* \theta_i \end{cases} \tag{32}$$

maps $\mathbb{R}_+ \times [\underline{\theta}_i; \overline{\theta}_i]$ into itself. Intuitively, $\widetilde{\mu}_i$ raises all bids $b_i < \theta_i$ to θ_i, and lowers all bids $b_i > \alpha_* \theta_i$ to $\alpha_* \theta_i$. Using Lemma 1, one finds that

$$\int \Pi_i d(\widetilde{\mu}_i, \mu_{-i}^*) = \int \overline{\Pi}_i(\phi_i(b_i, \theta_i)) d\mu_i(b_i, \theta_i) \tag{33}$$

$$\geq \int \overline{\Pi}_i(b_i, \theta_i) d\mu_i(b_i, \theta_i) = \int \Pi_i d(\mu_i, \mu_{-i}^*), \tag{34}$$

where $\mu_{-i}^* = (\mu_1^*, ..., \mu_{i-1}^*, \mu_{i+1}^*, ..., \mu_n^*)$. Consider, finally, the mapping $\chi_i : \mathbb{R}_+ \times [\underline{\theta}_i; \overline{\theta}_i] \to [1; \alpha_*] \times [\underline{\theta}_i; \overline{\theta}_i]$ defined by $\chi_i(b_i, \theta_i) = (b_i/\theta_i, \theta_i)$ if $\theta_i > 0$, and by $\chi_i(b_i, \theta_i) = (1, 0)$ if $\theta_i = 0$. Note that $\widetilde{\nu}_i = \widetilde{\mu}_i \circ \chi_i^{-1}$ satisfies

$$\widetilde{\nu}_i \circ \varphi_i^{-1} = \widetilde{\mu}_i \circ \chi_i^{-1} \circ \varphi_i^{-1} = \widetilde{\mu}_i \circ (\varphi_i \circ \chi_i)^{-1} = \widetilde{\mu}_i. \tag{35}$$

Hence, using that $(\nu_1^*, ..., \nu_n^*)$ is an equilibrium,

$$\int \Pi_i d(\mu_i^*, \mu_{-i}^*) = \textstyle\int U_i d(\nu_i^*, \nu_{-i}^*) \geq \textstyle\int U_i d(\widetilde{\nu}_i, \nu_{-i}^*) = \displaystyle\int \Pi_i d(\widetilde{\mu}_i, \mu_{-i}^*), \tag{36}$$

where $\nu_{-i}^* = (\nu_1^*, ..., \nu_{i-1}^*, \nu_{i+1}^*, ..., \nu_n^*)$. Thus, $\int \Pi_i d(\mu_i^*, \mu_{-i}^*) \geq \int \Pi_i d(\mu_i, \mu_{-i}^*)$, and there is indeed no profitable deviation. $\square$

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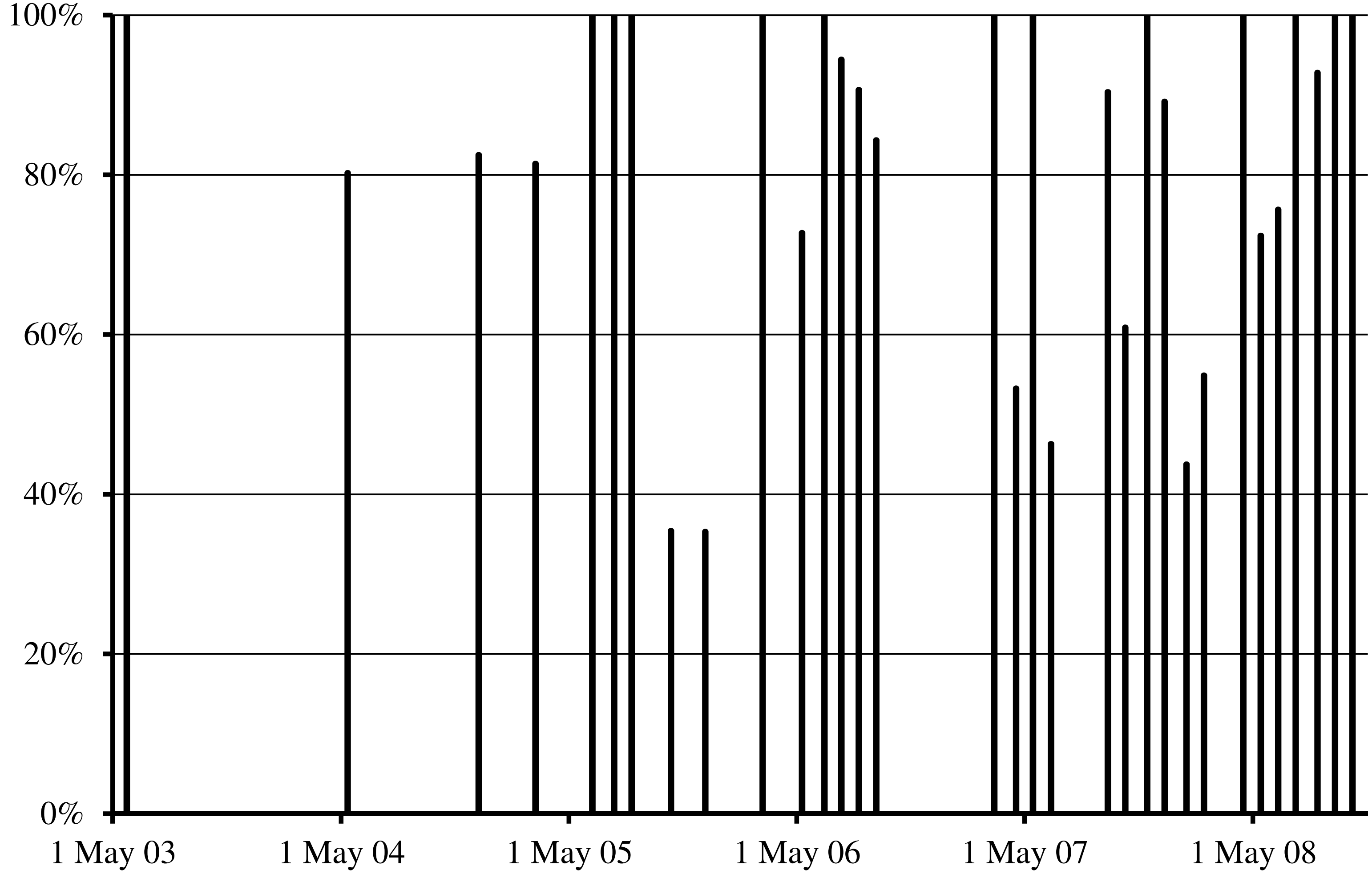

Figure 1: Allotment quotas in ECB fine-tuning operations

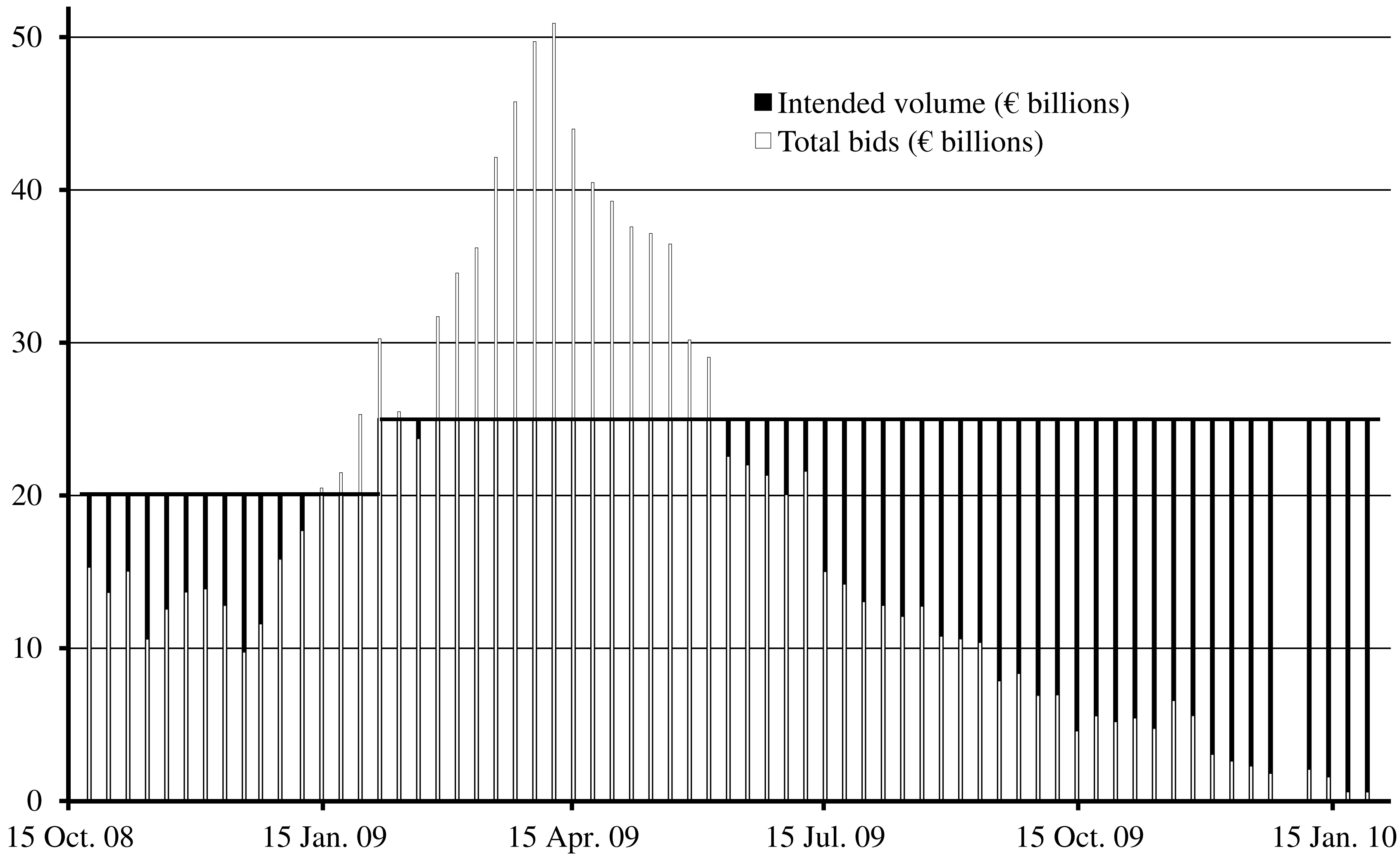

Figure 2: Intended volume and total bids in the weekly
EUR/CHF foreign exchange swap operations

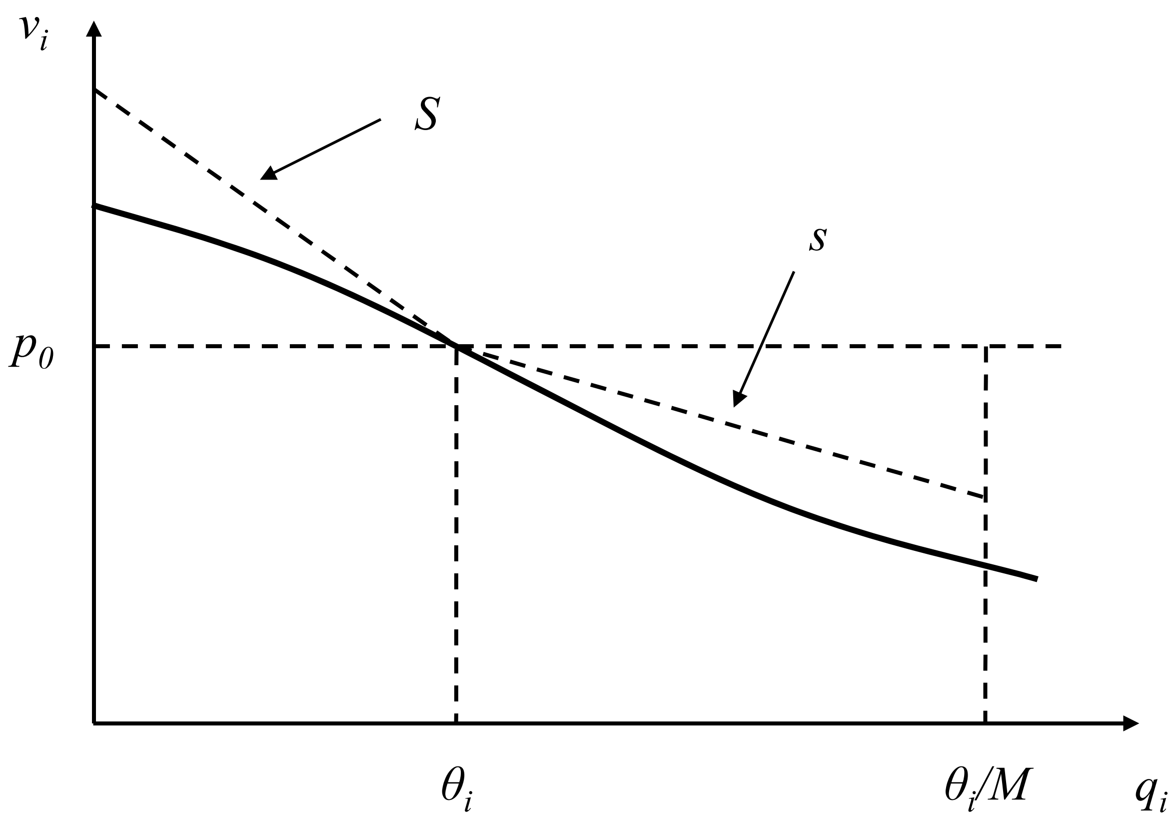

Figure 3: Marginal valuation function

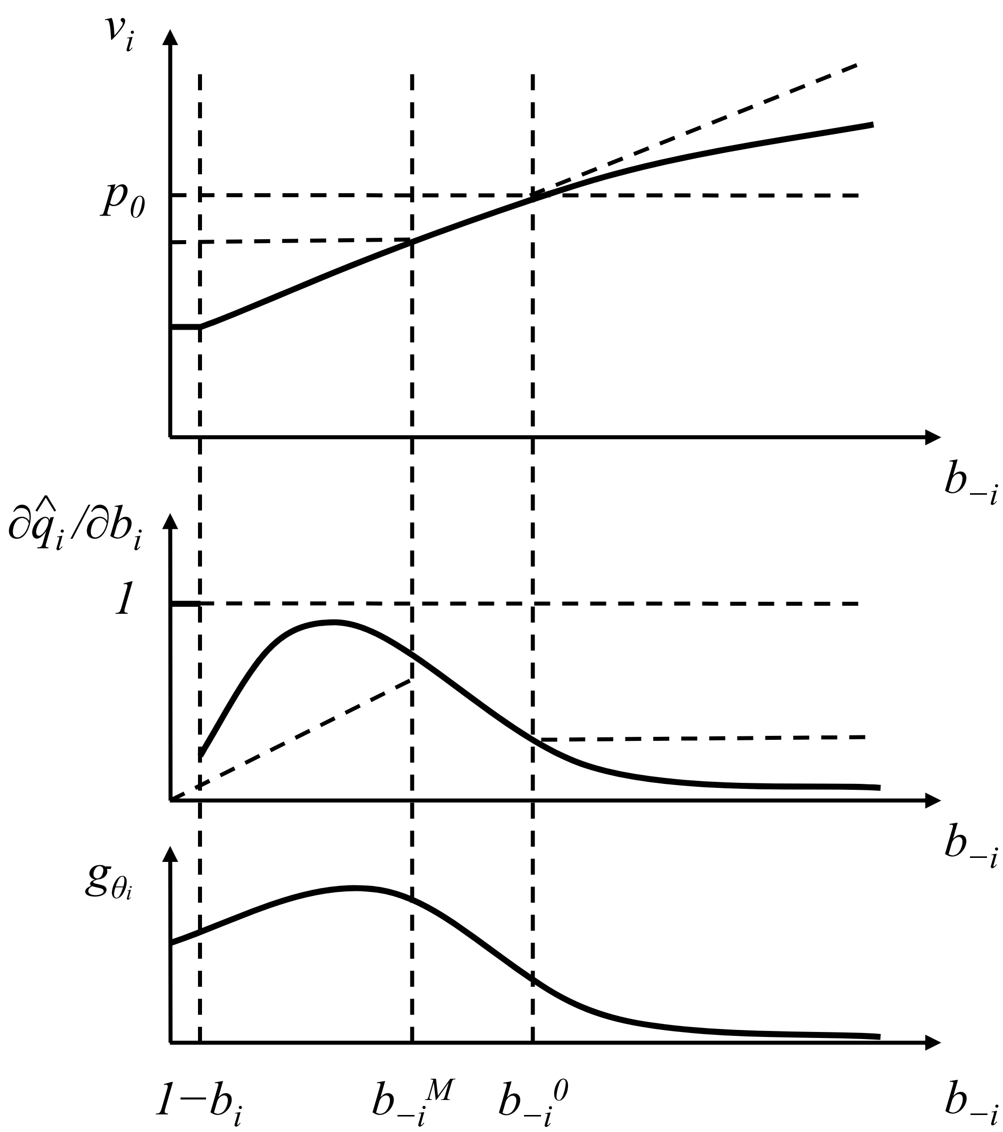

Figure 4: First-order condition

Table 1
Recent uses of the fixed rate tender

Series	Period of usage	Mean log bid-to-cover ratio	Number of operations conducted
Euro main refinancing operations	Jan. 99 – June 00	2.92	76
Sterling weekly, liquidity-providing operations, crisis	Aug. 07 – Oct. 08	1.50	62
US dollar Term Auction Facility	Dec. 07 – Oct. 08	1.01	21
Sterling weekly, liquidity-providing operations, pre-crisis	May 06 – Aug. 07	0.94	64
Sterling fine-tuning, liquidity-providing operations	Aug. 06 – Feb. 09	0.37	22
Euro fine-tuning operations	May 03 – Oct. 08	-0.03	32
Sterling weekly, liquidity-absorbing operations	Oct. 08 – Feb. 09	-0.14	23
Sterling fine-tuning, liquidity-absorbing operations	June 06 – Mar. 09	-0.37	17
EUR/CHF foreign exchange swap operations	Oct. 08 – Jan. 10	-0.73	70

Notes. The table lists recent uses of the fixed rate tender in the euro area and the UK. Shown are the period of usage, the mean of the log bid-to-cover ratio, and the number of operations conducted. The term pre-crisis (crisis) refers to operations with settlement on or before August 2, 2007 (on or after August 9, 2007). The nine series are listed in order of the mean log bid-to-cover ratio, starting with the highest value. Note that, owing to the log transform applied to the bid-to-cover ratio, a negative value of the mean indicates excess supply.

Table 2
Euro area US dollar operations under the Term Auction Facility conducted as fixed rate tenders

Settlement date	Duration (days)	Intended volume (€ billions)	Maximum bid (€ billions)	Number of bidders	Total bids (€ billions)	Allotment quota
20 Dec. 2007	28	10	1.0	39	22.080	0.45
27 Dec. 2007	35	10	1.0	27	14.115	0.71
17 Jan. 2008	28	10	1.0	22	14.790	0.68
31 Jan. 2008	28	10	1.0	19	12.400	0.81
27 Mar. 2008	28	15	1.5	34	31.237	0.48
10 Apr. 2008	28	15	1.5	32	30.760	0.49
24 Apr. 2008	28	15	1.5	33	30.128	0.50
8 May 2008	28	25	2.5	31	39.530	0.63
22 May 2008	28	25	2.5	54	58.876	0.42
5 June 2008	28	25	2.5	50	64.855	0.39
19 June 2008	28	25	2.5	56	78.460	0.32
3 July 2008	28	25	2.5	57	84.830	0.29
17 July 2008	28	25	2.5	59	90.075	0.28
31 July 2008	28	25	2.5	63	101.683	0.25
14 Aug. 2008	84	10	1.0	57	38.522	0.26
14 Aug. 2008	28	20	2.0	66	91.100	0.22
28 Aug. 2008	28	20	2.0	69	89.249	0.22
11 Sep. 2008	84	10	1.0	40	31.720	0.32
11 Sep. 2008	28	10	1.0	53	43.340	0.23
25 Sep. 2008	28	25	2.5	71	110.100	0.23
9 Oct. 2008	85	20	2.0	70	88.650	0.23

Notes. The table lists data related to the US dollar operations conducted by the Eurosystem within the framework of the Term Auction Facility. Shown are the settlement date, the duration, the intended volume, the maximum bid, the number of bidders, total bids, and the allotment quota.

Table 3
Bid functions for fixed rate tenders

	ECB	Bank of England	
		Pre-crisis	Crisis
	Apr. 1999 – June 2000	May 2006 – Aug. 2007	Aug. 2007 – Oct. 2008
trend × 10^2	0.785** (0.306)	-0.407* (0.225)	0.096 (0.159)
a_t	0.031 (0.179)	0.918 (1.346)	0.908*** (0.071)
$\hat{a}_{t-1}$	-0.194 (0.187)	0.074 (1.271)	-0.216* (0.120)
b_{t-1}	0.336*** (0.072)	0.575*** (0.127)	0.318*** (0.111)
b_{t-2}	0.019 (0.069)	0.107 (0.120)	-0.047 (0.064)
$(\rho\text{-}r)_t$	4.024*** (0.376)	0.688 (0.811)	2.085*** (0.540)
$(\rho\text{-}\sigma)_t$	-0.890*** (0.268)	-5.317*** (1.481)	-2.956*** (0.515)
constant	10.30*** (3.53)	-6.202 (8.402)	1.360*** (0.374)
R-squared	0.902	0.692	0.954
Observations	61	62	60

Notes. The table shows the estimated bid (b_t) functions for the fixed rate tenders of the ECB (left-hand column) and the Bank of England (middle and right-hand columns), compare Eq. (15) in the text. Confidence levels are *** for p<0.01, ** for p<0.05, and * for p<0.1. Standard errors are noted in parentheses.