Institute for Empirical Research in Economics
University of Zurich

Working Paper Series
ISSN 1424-0459

Working Paper No. 501

Multimarket Contact Effect on Collusion through Diversification

Hwa Ryung Lee

September 2010

Multimarket Contact Effect on Collusion through Diversification

Hwa Ryung Lee

Institute for Empirical Economics
University of Zurich
This version: *September 2010*

Abstract

This study establishes the potential positive relationship between multimarket contact (MMC) and sustainable collusive profits under demand fluctuations. In particular, I focus on the correlation structure between demand shocks over multiple markets and show how it can lead to a positive link between collusive profit and MMC. Simple theoretical models show that, regardless of whether demand shocks are observable or not, MMC may improve collusive profits through diversification of demand shocks over overlapping markets. If firms meet in multiple markets and link those markets in the sense that deviation in any market will trigger simultaneous retaliations in every market, then a cheating firm will optimally deviate in every market. Demand fluctuation that a firm is facing in its markets in total will be reduced as the number of markets increases, unless demand shocks are perfectly and positively correlated between the markets. The reduction of demand fluctuations can boost collusion (1) by reducing the temptation to deviate in the period of high demand when demand shocks are observable and (2) by reducing the frequency of costly punishment on the equilibrium path when demand shock is unobservable. The conclusion in the case of observable demand shock provides us with a new testable implication that price competition will be muted by MMC in periods of high demand.

1 Introduction

When demand is fluctuating, so is the sustainability of collusion. When demand shock is observable, Rotemberg and Saloner (1986) pointed out that a firm is more tempted to deviate from collusion in the period of high demand because the immediate gain from deviation increases while the expected future loss from it remains the same. When demand shock is unobservable, on the other hand, Green and Porter (1984) argued that firms may enter into a non-collusive punishment phase when they observe low profit, even though it is caused by low demand shock rather than by secrete cheating of some firms, due to the lack of monitoring ability to distinguish deviation from low demand shock. These results suggest that demand fluctuations have a negative relationship with the level of sustainable collusive profits.

When firms meet with each other in more than a single market, that is, when firms have multimarket contacts (MMC), it may make a difference in a competitive environment and lead to a new implication on collusion as compared to a single market setting. This study investigates how MMC can affect the sustainability of collusive outcome under demand fluctuations. In particular, we propose a possible mechanism in which MMC boosts the sustainable collusive profits when firms face stochastic demand shocks using the model of repeated games. The short conclusion is that, regardless of whether demand shocks are observable or not, multimarket contacts may improve collusive profits through diversification of demand shocks across the markets, when firms link the overlapping markets in the sense that a deviation in a market will trigger retaliations in all overlapping markets. Less demand fluctuations from diversification may facilitate collusion (1) by reducing the temptation to deviate in the period of high demand when aggregate demand shock is observable and (2) by reducing the frequency of costly punishment on the equilibrium path when aggregate demand shock is unobservable.

Collusion will break down when the expected gain from deviation is higher than that from collusion. This relationship is clear to see when demand shock is observable as a firm can tell when the best time to deviate is: a cheating firm will deviate when the realized demand is at a peak. If rival firms are meeting with each other in multiple markets and they engage in linking strategy, in which a deviation in a single market triggers simultaneous punishments in all overlapping markets, then the best opportunity to deviate would be those times when demand shock is the highest in every overlapping market. Note that, as the number of overlapping markets gets larger, the demand fluctuation of all overlapping markets combined will get smaller unless the demand shocks are perfectly and positively correlated. In other words, a common high demand shock in every market is unlikely. It is likely that some overlapping markets experience a negative demand shock. In this sense, MMC may lead to a higher sustainable collusive profit as firms can take advantage of diversification of demand shocks by linking the overlapping markets.

When demand shock is unobservable, monitoring is imperfect. The knowledge of correlation strucutre between demand shocks can be useful for firms to detect cheating. Although individual market outcomes may not be informative, the profile of outcomes across overlapping markets could be informative. Note that, if firms are meeting in multiple markets and a firm decides to deviate, the firm will optimally deviate in every market. This is because the markets are linked to the extent that deviation in any market triggers simultaneous retaliations in all the markets. Then, cheating will affect all overlapping markets and firms can have a better sense of whether some firms have deviated or not by looking at the profile of profits across overlapping markets. Better monitoring can lead to higher collusive profits.

The rest of the paper proceeds as follows. In Section 2, we introduce related literatures and highlight my contributions. In Section 3, theoretical models will be described to show the potential positive impact of multimarket contacts on collusion under stochastic demand shocks and the possible extension of the model will be discussed. Finally, Section 4 concludes.

2 Literature Review

Bernheim and Whinston (1990) show that MMC may facilitate collusion by pooling incentive constraints and transferring the slacks in the constraints between markets. They emphasize the asymmetry between markets or rival firms as a source of positive MMC effect on collusion. Adding stochastic demand shocks to their story provides us with another implication on the link between MMC and collusion through diversification effect.

Diversification is often regarded to have two different but related economic effects. On the one hand, a firm may operate in multiple markets in which the tasks are unrelated or products are heterogeneous. In this case, diversified tasks or products may have a positive impact on a firm's performance through the economies of scope. On the other hand, diversification can indicate that market outcomes such as profits and returns are unrelated or negatively correlated between the markets. In this sense, the role of diversification is the reduction of risk or fluctuation of firm performance, similarly with that investors reduce the risk of investment for the same expected return by diversifying a portfolio in finance. The role of diversification as a mean to enjoy the economy of scope has been raised in several studies on collusion and MMC. The reduction of risk by diversification, however, has not been emphasized in pervious works on the topic and this role will be the focus of this study.

The link between diversification and collusion through MMC was noted by Hughes and Oughton (1993). However, their work is limited in the sense that diversification induces a higher collusive profit simply because it extends the chances for firms to meet with each other and thereby increases firms' mutual recognition of interdependence. In other words, when firms are diversified in terms of product lines or operations, it is more likely for them to meet with each other, which in turn will lead the firms to know each other better and not to compete hard against each other. Their work does not address the direct role of diversification in collusion. Rather, it argues that diversified firms tend to have more MMC and hence higher collusive profits. This study shows that firms with more MMC tend to be more diversified and diversification has a direct (positive) effect on the sustainable collusive profits by reducing demand fluctuations.

Marketing literatures take a different approach on the link between diversification and MMC. Similarly with Hughes and Oughton, they note that "diversification and multimarket contact are complementary activities because the former provides the opportunity for the latter" (Li and Greenwood, 2004). In addition, since diversification usually involves the economy of scope, it can lead to a higher profit. Basically, they argue that diversification tends to lead to MMC and hence the effect of MMC on profits will include the benefit from diversification which arises from the economy of scope, although it is not specific to collusion, resulting in the pattern of higher profits under MMC. As in Hughes and Oughton, however, this argument does not present the direct effect of diversification on collusion with MMC.

When it comes to the markets with stochastic demand shocks, diversification can offer an additional channel in which MMC may affect collusive outcome. In particular, the reduction of demand fluctuations through diversification of demand shocks across overlapping markets, combined with linking strategy (which involves simultaneous retaliations in multiple markets), may have a direct effect on collusive profits. This idea of linking the effect of diversification on the expected collusive profits under stochastic demand shock is new.

Now, let's think about how demand fluctuation can affect collusion. The link between stochastic demand shock and collusion can be found in Rotemberg and Saloner (1986) and Green and Porter (1984). The both works have the same implication that demand fluctuations can undermine collusion. However, the situations in which the fluctuations undermine collusion are different due to different assumptions on the characteristics of demand shock.

Rotemberg and Saloner assume an "observable" demand shock and conclude that firms are more tempted to deviate from collusion when demand shock is positive, implying more competition in the period of high demand. This is because the immediate gains from deviation increases while the future profits lost during punishment phase remains the same. In contrast, Green and Porter assume an "unobservable" demand shock and conclude that a price war is more likely to occur when demand is low. Note that demand fluctuations are not observed directly by firms in their setting. Thus, a low profit can occur either due to negative demand shock or from secrete cheating by some firms. As a result, firms trigger a price war when demand is lower than a certain level, even when no one has actually deviated, in the equilibrium path of collusion.

Applying the two works to a MMC setting provides us with a new perspective on the effect of MMC on collusion under demand fluctuations. If rival firms link the overlapping markets in the sense a deviation in any overlapping market will trigger simultaneous retaliations in every market, a cheating firm will optimally deviate in every market. Note that, unless demand shocks are perfectly and positively correlated between markets, the average demand fluctuations will be reduced as the number of markets increases. The reduction of demand fluctuations from MMC implies that (1) the best opportunity to deviate, i.e. high demand shock in every market, will come less often when demand shock is observable while (2) the probability of low demand shock in at least one market, i.e. the likelihood of triggering a price war even without cheating, rises, which is basically same as the "risk of contagion" noted by Thomas and Willig (2006). In this sense, MMC and diversification from linking the overlapping markets may facilitate collusion when demand shock is observable but not when it is unobservable.

Thomas and Willig focused on the strategy that links the risky front where demand shock is unobservable to the safe front where there is no variation in demand. Linking strategy may rather reduce the players' payoffs because it permits negative demand shocks to spread from the risky front to the safe front. Even though collusion used to be sustainable only in the safe front and linking might enable firms to collude in the risky front as well, the reduced (or sacrificed) profit in the safe front might exceed the profit from the collusion in the risky front.

If we consider the strategy that links the risky front to another risky front, instead of the safe front, so that a low profit in any front triggers a price war in both fronts, then the linkage will always make collusion less attractive because a price war is more likely to be triggered unless the shocks are perfectly and positively correlated. In this sense, diversification can amplify the "risk of contagion" from linking strategy. The more markets firms are meeting, the more likely is that demand shock is negative in at least one market, which results in a higher frequency of a price war and a lower expected collusive profit. Thus, diversification may hurt collusion by raising the type 1 error, given the number of market contacts.

The risk of contagion, however, can be overstated given the trigger strategy that does not take into account the number of overlapping markets and the correlation structure of demand shocks between the markets. So far, we have implicitly assumed that a trigger event is the same regardless of the number of market contacts or the degree of diversification; a firm will enter into the punishment phase if the realized profit is lower than a certain level in any of the overlapping markets. However, firms will take advantage of their knowledge of the correlation structure of demand shocks in order to set an optimal trigger strategy.

Matsushima (2001) argued that extensive MMC enhances monitoring ability and firms adjust their trigger strategies and thereby improves the expected collusive profits. In particular, he assumed independent and identical demand shocks across markets. Then firms can adjust trigger events such as the accumulation of low profits in more than a certain number of markets. This trigger strategy will make monitoring perfect since the number of markets with a low demand will not exceed a certain level as the number of market contacts increases by the law of large numbers.

Diversification may amplify the benefit from better specification of trigger events because the knowledge of the correlation structure of demand shocks between markets can provide a more scope that firms can use in setting trigger events, even without an infinitely many number of market contacts. That is, even with only two market contacts, if firms know how the demand shocks are correlated between the two markets, they can set a better trigger event based on the joint probability of realized market outcomes derived from the correlation structure. Although a single market outcome may not have any information about other firms' actions, the distribution of outcomes across the overlapping markets may be informative.

For example, firms can optimally adjust trigger events so that they enter into the punishment phase if the profile of profits across the markets becomes much more likely when cheating has occurred than when other firms have been cooperative. Under this trigger strategy, a cheating firm cannot optimally deviate in every market because it will increase the probability of getting caught significantly when markets are diversified. That is, the new trigger strategy reduces the number of markets that a cheating firm can profitably deviate and so the immediate gain from deviation, which will curb the temptation to deviate. In this way, firms can actually benefit from reduced demand fluctuations because the frequency of trigger events will decrease when the markets are diversified, as in the case of observable demand shock. Therefore, firms can better distinguish cheating from negative demand shocks, implying the lower the probability of rejecting the idea that all firms are cooperative (Type I error[1]) and higher expected collusive profits. Moreover, this improvement in monitoring is even larger when markets are diversified and the correlation structure is known than demand shocks are independent as in Matsushima, given the number of market

[1] There are two types of errors that can be made when testing the statistical significance of estimates. When a null hypothesis is erroneously rejected, it is called a Type I error. When a null hypothesis is erroneously accepted, it is called a Type II error. Here, a null hypothesis is that all firms were cooperative while an alternative hypothesis is that some firms deviated from collusion. When the null hypothesis is rejected, a price war is triggered.

contacts.

Note that there are two forces of diversification that affect the Type I error in opposite directions. One is the increased probability of low demand at least in one market given a trigger event, which raises the Type I error, and the other is the better specification of a trigger event, which lowers the Type I error. It is noteworthy that diversification under linking strategy when demand shock is unobservable amplifies both the risk of contagion and the benefits from better specification of trigger events. However, if firms know how demand shocks are correlated between the markets, they can reduce the risk of contagion by setting optimal trigger events and hence benefit from diversification. Thus, MMC may facilitate collusion through diversification if firms know the correlation structure of demand shocks between the markets.

In sum, regardless of whether demand shocks are observable of not, MMC may improve collusive profits through diversification. In particular, diversification aids collusion (1) by creating asymmetry between markets when demand shock is observable and (2) by providing informational advantage in monitoring when demand shock is unobservable.

3 Model

In this section, we will develop simple theoretical models using repeated games. The analyses follow the traditional game theoretical analysis, and the possible extension of the model will be discussed.

First of all, let me define the observability of demand shocks. Either if firms make a price decision after they know the realized demand or if firms can predict demand and make a decision based on the prediction, demand shocks are regarded as "observable" by the firms in the market. In contrast, if demand shock is not directly observed by firms neither before nor after their price decision, it is thought of as "unobservable" in the market.

The observability of demand shock matters when firms are coordinating their actions. Under observable demand shock, the temptation to deviate in the period of high demand is most likely to make a binding constraint for collusion. Under unobservable demand shock, imperfect monitoring raised by firms' inability to distinguish cheating from negative demand shock is the obstacle in collusion. In the following sections, whether and how diversification may alleviate these problems will be studied in each case. The basic models follow the traditional game theoretical analysis but the possible extension of the model will be discussed later in the section.

3.1 Observable Demand Shock

In this section, we will present a basic model that suggests that MMC may mute price competition and improve sustainable collusive profits especially in the period of high demand, through diversification of demand shocks between overlapping markets. That is, the more the overlapping markets are diversified in demand shocks, the higher the sustainable profits are for firms participating in collusion.

Key intuition is that good chances to cheat will come less often when firms are linking the diversified overlapping markets. In particular, when markets are linked strategically in the sense that a deviation in any market will trigger the same massive punishment in multiple markets, a firm will deviate in every market once it decides to cheat. Then it would be the best for the firm to deviate when demand is high in every market. If the linked markets are diversified, however, when demand is high in some markets, demand will be low in other markets, meaning that the immediate gain from deviation is reduced.

The intuition has the same flavor as Bernheim and Whinston's theory. Here, the source of asymmetry comes from statistically different realization of demand shocks between markets. When markets are diversified in terms of demand shocks, there will be slack in incentive constraints in some markets in general and so firms can transfer the slacks to the other markets where the incentive constraint is binding, although the markets in which firms have slack in the incentive constraint will be different from time to time depending on the realized demand shocks. So, the source of asymmetry is diversification of demand shocks across the markets.

First of all, let us assume the followings:

(A1) There exist two identical firms competing in two duopoly markets M_1 and M_2, without product differentiation. The markets open simultaneously and repeatedly.

(A2) ε_i is a random demand shock in market i. Demand shock is realized to be either "high" (when $\varepsilon = \varepsilon_H$) or "low" (when $\varepsilon = \varepsilon_L$) with equal probability (=0.5) in each market. The demand shocks are independently and identically distributed over time but may be correlated between the markets. So, in each period, the distribution of random demand shock in the two markets is

$$\varepsilon = \begin{matrix} \varepsilon_H \text{ with prob. } = 0.5 \\ \varepsilon_L \text{ with prob. } = 0.5 \end{matrix}$$

(A3) $\Pi^M(\varepsilon)$ is defined as a firm's payoff from joint profit maximization (as if the two firms are maximizing one monopoly profit) when demand shock ε is realized. For notational simplicity, $\Pi_L^M \equiv \Pi^M(\varepsilon_L)$, $\Pi_H^M \equiv \Pi^M(\varepsilon_H)$. Assume $0 < \Pi_L^M < \Pi_H^M$. $\Pi^S(\varepsilon)$ is the highest sustainable collusive profit given demand shock ε.

(A4) Decision variable is price and firms decide their own price based on the observation of demand shock in each period.

(A5) $\delta \in (0, 1)$ is a discount factor common to the two firms.

(A6) Firm's payoff can be any value from zero to infinity.

Let us consider the case where the firms employ a grim trigger strategy in which they revert to the Nash Bertrand competition (meaning zero profits) forever once any firm defects. When it comes to observable demand shocks, punishment will not be realized in the equilibrium path of collusion. So, the severer is the punishment, the higher is the sustainable collusive profits. In this sense, the Nash Bertrand competition is the optimal choice for firms in the punishment phase. Under the grim trigger strategy, the loss from deviation is the present value of the expected future collusive profits ($= \frac{\delta}{1-\delta} E[\Pi^S(\varepsilon)]$).

We will begin with the benchmark case in which firms do not link the markets. That is, a defect in one market will lead to punishment only in that specific market and will not affect the other market. Then the collusive profit when a linking strategy is employed by the firms will be explored and compared to the benchmark result, under different correlation structures of demand shocks between the markets.

[**Benchmark Case**] Assume a separating strategy; firms maximize profit in each market separately.

If taken separately, the two markets can be viewed as identical. So, looking at one market is sufficient. In a single market, joint profit maximization is sustainable if

$$\Pi_i^M \leq \frac{\delta}{1-\delta} E[\Pi^S(\varepsilon)]$$

where $i = H$ or L. The LHS is the immediate gain from deviation and the RHS is the future loss from deviation (i.e. the present value of the expected sustainable profits in the future).

If the joint profit maximizing profit is sustainable regardless of the realization of demand shock, i.e.

$$\Pi^S(\varepsilon) = \begin{matrix} \Pi_H^M & \text{if } \varepsilon = \varepsilon_H \\ \Pi_L^M & \text{if } \varepsilon = \varepsilon_L \end{matrix}$$

then the loss from deviation will be

$$\frac{\delta}{1-\delta} E[\Pi^S(\varepsilon)] = \frac{\delta}{1-\delta} \cdot \frac{\Pi^S(\varepsilon_H) + \Pi^S(\varepsilon_L)}{2} = \frac{\delta}{1-\delta} \cdot \frac{\Pi_H^M + \Pi_L^M}{2}.$$

That is, firms can maximize the joint profit in any state of demand if

$$(\Pi_L^M <)\ \Pi_H^M \leq \frac{\delta}{1-\delta} E[\Pi^S(\varepsilon)] = \frac{\delta}{1-\delta} \cdot \frac{\Pi_H^M + \Pi_L^M}{2}$$

$$\longleftrightarrow \lambda \equiv \frac{\delta}{1-\delta} \geq \frac{2\Pi_H^M}{\Pi_H^M + \Pi_L^M} \tag{1}$$

On the other hand, for a lower discount factor δ with which (1) does not hold, the joint profit-maximizing profit may be sustainable only when demand is low because the immediate gain from deviation is larger in the period of high demand ($\Pi_L^M < \Pi_H^M$). Then the firms have to settle for lower profit than Π_H^M in the period of high demand. In particular, the highest sustainable profit in the period of high demand ($\equiv \Pi_{BM}^S$) must satisfy the following condition:

$$\Pi_{BM}^S = \frac{\delta}{1-\delta} E[\Pi^S(\varepsilon)] = \frac{\delta}{1-\delta} \cdot \frac{\Pi^S(\varepsilon_H) + \Pi^S(\varepsilon_L)}{2} = \frac{\delta}{1-\delta} \cdot \frac{\Pi_{BM}^S + \Pi_L^M}{2}$$

$$\rightarrow \Pi_{BM}^S = \frac{\lambda}{2-\lambda} \Pi_L^M$$

provided $\lambda < 2$ (otherwise, (1) holds, meaning that Π_H^M is sustainable, which is contradictory). In addition,

$$\Pi_L^M \leq \frac{\delta}{1-\delta} E[\Pi^S(\varepsilon)] = \Pi_{BM}^S < \Pi_H^M$$

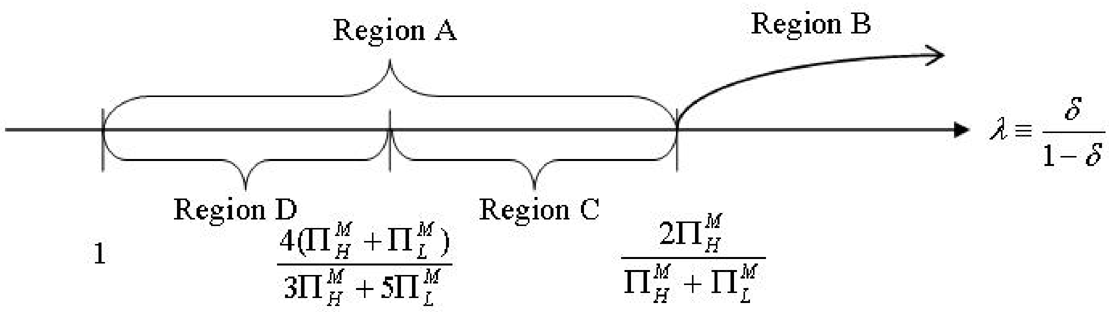

Figure 1: Regions of Sustainable Collusive Profits under Observable Demand Shock

This implies that firms can sustain joint profit maximization in the period of low demand shock $(= \Pi_L^M)$, but they can sustain only as high as Π_{BM}^S in the period of high demand shock. Then

$$\Pi_L^M \leq \frac{\lambda}{2-\lambda}\Pi_L^M < \Pi_H^M$$

$$\longleftrightarrow 1 \leq \lambda \equiv \frac{\delta}{1-\delta} < \frac{2\Pi_H^M}{\Pi_H^M + \Pi_L^M}$$

Therefore, for δ such that $1 \leq \frac{\delta}{1-\delta} < \frac{2\Pi_H^M}{\Pi_H^M+\Pi_L^M}$, the highest sustainable profit is

$$\Pi^S(\varepsilon) = \begin{cases} \Pi_{BM}^S = \frac{\lambda}{2-\lambda}\Pi_L^M & \text{if } \varepsilon = \varepsilon_H \\ \Pi_L^M & \text{if } \varepsilon = \varepsilon_L \end{cases}$$

Figure 1 summarizes the results. Moving from the left to the right on the horizontal line, λ increases. Note that λ is defined as $\frac{\delta}{1-\delta}$ and it is increasing in δ. Joint profit maximization is sustainable regardless of the realization of demand shock as long as λ is in Region B of Figure 1. On the other hand, $\Pi_{BM}^S(< \Pi_H^M)$ is the highest sustainable level of profit in the period of high demand if λ is in Region A of Figure 1.

It is noteworthy that the higher is Π_H^M than Π_L^M (i.e. the higher ε_H is than ε_L), the larger is Region A. This implies that sustainable collusive profit will be reduced as the degree of demand fluctuation increases.

Now, let us turn to the cases where firms are linking the two markets in various situations in terms of the correlation of demand shocks between the markets. In particular, we will compare the cases where the demand shocks are perfectly and negatively correlated, perfectly and positively correlated, and independent of each other, to the benchmark case in order to see the effect of linking the markets under demand fluctuations.

[Case 1] Assume ε_1 and ε_2 are perfectly and *negatively* correlated.

When firms are linking the two markets, they consider the profile of realized demand shocks in the both markets, i.e. $(\varepsilon_1, \varepsilon_2)$. The probability density function of $(\varepsilon_1, \varepsilon_2)$, $f(\varepsilon_1, \varepsilon_2)$ is defined as follows:

$$f(\varepsilon_1, \varepsilon_2) = \begin{cases} 0.5 & \text{if } (\varepsilon_1, \varepsilon_2) = (\varepsilon_H, \varepsilon_L) \text{ or } (\varepsilon_L, \varepsilon_H) \\ 0 & \text{otherwise} \end{cases}$$

Since we assume the markets are identical except for the realization of demand shock, the incentive constraint not to deviate is the same in the two cases $(\varepsilon_H, \varepsilon_L)$ or $(\varepsilon_L, \varepsilon_H)$. Note that the immediate gain from deviation is the same in both cases. In addition, only these two cases take place with a positive probability. Thus, what we need to consider is one of the two cases. Demand fluctuations disappear as the total demand shock $(= \varepsilon_H + \varepsilon_L)$ does not vary over time.

By linking the markets in the sense a cheating in any market triggers punishment in every overlapping market, firms are pooling the incentive constraints not to deviate. So, the incentive constraint for collusion in the both markets when $(\varepsilon_H, \varepsilon_L)$ (or $(\varepsilon_L, \varepsilon_H)$) is

$$\Pi_H^M + \Pi_L^M \leq \frac{\delta}{1-\delta} E[\Pi^S(\varepsilon)] = \frac{\delta}{1-\delta} (\Pi_H^M + \Pi_L^M)$$

$$\longleftrightarrow \lambda \equiv \frac{\delta}{1-\delta} \geq 1$$

Note that joint profit maximization is now possible for both Region A and Region B in the Figure 1. Therefore, when $\frac{\delta}{1-\delta}$ is in Region A, the joint-profit maximizing profit is sustainable even in the period of high demand with linking strategy, which cannot be sustained with separating strategy. This shows the possibility that MMC mute price competition and improve the expected collusive profits in the period of high demand.

[CASE 2] Assume ε_1 and ε_2 are perfectly and *positively* correlated.
The probability density function of $(\varepsilon_1, \varepsilon_2)$, $f(\varepsilon_1, \varepsilon_2)$ is defined as follows:

$$f(\varepsilon_1, \varepsilon_2) = \begin{cases} 0.5 & \text{if } (\varepsilon_1, \varepsilon_2) = (\varepsilon_H, \varepsilon_H) \text{ or } (\varepsilon_L, \varepsilon_L) \\ 0 & \text{otherwise} \end{cases}$$

Firms can sustain joint profit maximization in any state of demand if

$$\begin{aligned} (2\Pi_L^M \quad < \quad)\ 2\Pi_H^M &\leq \frac{\delta}{1-\delta} E[\Pi^S(\varepsilon)] = \frac{\delta}{1-\delta} \cdot \frac{2\Pi_H^M + 2\Pi_L^M}{2} \\ \longleftrightarrow \quad \lambda \equiv \frac{\delta}{1-\delta} &\geq \frac{2\Pi_H^M}{\Pi_H^M + \Pi_L^M} \end{aligned}$$

This condition is exactly the same as in the benchmark case. Also as in the benchmark case, for a lower discount factor that does not satisfy (1), it can be the case that firms are able to sustain joint profit maximization only in the period of low demand in the both markets. In this case, the highest sustainable profit in the period of high demand in the both markets is lower than the joint profit maximizing profit. The highest sustainable profit in the two markets combined ($\equiv \Pi^S_{Total}(\varepsilon_1, \varepsilon_2)$) must satisfy

$$\begin{aligned}\Pi^S_{Total}(\varepsilon_H, \varepsilon_H) &= \frac{\delta}{1-\delta} E[\Pi^S(\varepsilon_1, \varepsilon_2)] = \frac{\delta}{1-\delta} \cdot \frac{\Pi^S(\varepsilon_H, \varepsilon_H) + \Pi^S(\varepsilon_L, \varepsilon_L)}{2} \\ &= \frac{\delta}{1-\delta} \cdot \frac{\Pi^S_{Total}(\varepsilon_H, \varepsilon_H) + 2\Pi^M_L}{2}\end{aligned}$$

$$\rightarrow \Pi^S_{Total}(\varepsilon_H, \varepsilon_H) = \frac{2\lambda}{2-\lambda} \Pi^M_L$$

when demand is high (i.e. $(\varepsilon_1, \varepsilon_2) = (\varepsilon_H, \varepsilon_H)$) in both markets. Note that $\Pi^S_{Total}(\varepsilon_H, \varepsilon_H) = 2\Pi^S_{BM}$, meaning that the incentive constraint is the same as in benchmark case, again.

In sum, when demand shocks are perfectly and positively correlated between markets, another market is nothing but a replication of the same market and linking these markets is irrelevant to collusive profits.

[CASE 3] Assume ε_1 and ε_2 are *independent* of each other.

The probability density function of $(\varepsilon_1, \varepsilon_2)$, $f(\varepsilon_1, \varepsilon_2)$ is defined as follows:

$$f(\varepsilon_1, \varepsilon_2) = \begin{matrix} 0.25 & \text{if } (\varepsilon_1, \varepsilon_2) = (\varepsilon_H, \varepsilon_H), (\varepsilon_H, \varepsilon_L), (\varepsilon_L, \varepsilon_H), \text{ or } (\varepsilon_L, \varepsilon_L) \\ 0 & \text{otherwise} \end{matrix}$$

As in case 2, joint profit maximization is always possible regardless of the realization of demand shocks if

$$\begin{aligned}(2\Pi^M_L &< \Pi^M_H + \Pi^M_L <)\ 2\Pi^M_H \leq \frac{\delta}{1-\delta} \cdot \frac{2\Pi^M_L + 2\Pi^M_H + (\Pi^M_L + \Pi^M_H) + (\Pi^M_H + \Pi^M_L)}{4} \\ &= \frac{\Pi^M_H + \Pi^M_L}{2}\end{aligned}$$

This is the same condition as in the benchmark case (and Case 2).

For a lower discount factor which does not satisfy (1), firms may not be able to sustain joint profit maximization when demand is high in the both markets. When demand is high in both markets, the highest sustainable profit ($= \Pi^S_{Total}(\varepsilon_H, \varepsilon_H)$) satisfies

$$\begin{aligned}\Pi^S_{Total}(\varepsilon_H,\varepsilon_H) &= \frac{\delta}{1-\delta}E[\Pi^S(\varepsilon_1,\varepsilon_2)] \\ &= \frac{\delta}{1-\delta}\cdot\frac{2\Pi^M_L+\Pi^S_{Total}(\varepsilon_H,\varepsilon_H)+(\Pi^M_L+\Pi^M_H)+(\Pi^M_H+\Pi^M_L)}{4} \\ &= \frac{\delta}{1-\delta}\cdot\frac{2\Pi^M_H+\Pi^S_{Total}(\varepsilon_H,\varepsilon_H)+4\Pi^M_L}{4}\end{aligned}$$

$$\rightarrow \Pi^S_{Total}(\varepsilon_H,\varepsilon_H)=\frac{2\lambda}{4-\lambda}(\Pi^M_H+2\Pi^M_L)$$

and

$$\Pi^M_H+\Pi^M_L\le\Pi^S_{Total}(\varepsilon_H,\varepsilon_H)\le 2\Pi^M_H$$

$$\longleftrightarrow \frac{4(\Pi^M_H+\Pi^M_L)}{3\Pi^M_H+5\Pi^M_L}\le\lambda\le\frac{2\Pi^M_H}{\Pi^M_H+\Pi^M_L}$$

That is, for δ such that $\frac{4(\Pi^M_H+\Pi^M_L)}{3\Pi^M_H+5\Pi^M_L}\le\frac{\delta}{1-\delta}\le\frac{2\Pi^M_H}{\Pi^M_H+\Pi^M_L}$, joint profit maximization is not sustainable when demand is high in both markets. In this case, the highest sustainable profit is

$$\Pi^S_{Total}(\varepsilon_1,\varepsilon_2)=\begin{cases}\frac{2\lambda}{4-\lambda}(\Pi^M_H+2\Pi^M_L) & \text{if } (\varepsilon_1,\varepsilon_2)=(\varepsilon_H,\varepsilon_H)\\ \Pi^M_H+\Pi^M_L & \text{if } (\varepsilon_1,\varepsilon_2)=(\varepsilon_H,\varepsilon_L)\text{ or }(\varepsilon_L,\varepsilon_H)\\ 2\Pi^M_L & \text{if } (\varepsilon_1,\varepsilon_2)=(\varepsilon_L,\varepsilon_L)\end{cases}$$

For an even lower discount factor, firms may not be able to sustain joint profit maximization when demand is high at least in one of the two markets. When demand is high in at least one of the markets, the highest sustainable profit $(\Pi^S_{Total}(\varepsilon_H,\cdot))$[2] satisfies

$$\begin{aligned}\Pi^S_{Total}(\varepsilon_H,\cdot) &= \frac{\delta}{1-\delta}E[\Pi^S(\varepsilon_1,\varepsilon_2)] \\ &= \frac{\delta}{1-\delta}\cdot\frac{2\Pi^M_L+\Pi^S_{Total}(\varepsilon_H,\varepsilon_L)+\Pi^S_{Total}(\varepsilon_H,\varepsilon_L)+\Pi^S_{Total}(\varepsilon_H,\varepsilon_L)}{4} \\ &= \frac{\delta}{1-\delta}\cdot\frac{2\Pi^M_L+3\Pi^S_{Total}(\varepsilon_H,\varepsilon_L)}{4}\end{aligned}$$

$$\rightarrow \Pi^S_{Total}(\varepsilon_H,\cdot)=\frac{2\lambda}{4-3\lambda}\Pi^M_L$$

[2] In this case, $\Pi^S_{Total}(\varepsilon_H,\varepsilon_H)=\Pi^S_{Total}(\varepsilon_H,\varepsilon_L)=\Pi^S_{Total}(\varepsilon_L,\varepsilon_H)$. Thus, the highest sustainable profit can be expressed as either $\Pi^S_{Total}(\varepsilon_H,\cdot)$ or as $\Pi^S_{Total}(\cdot,\varepsilon_H)$. Here, without loss of generality, we will use $\Pi^S_{Total}(\varepsilon_H,\cdot)$.

and

$$2\Pi_L^M \leq \Pi_{Total}^S(\varepsilon_H, \cdot) \leq \Pi_H^M + \Pi_L^M$$

$$\longleftrightarrow \frac{4(\Pi_H^M + \Pi_L^M)}{3\Pi_H^M + 5\Pi_L^M} \leq \lambda \leq \frac{2\Pi_H^M}{\Pi_H^M + \Pi_L^M}$$

That is, for δ such that $1 \leq \frac{\delta}{1-\delta} \leq \frac{4(\Pi_H^M+\Pi_L^M)}{3\Pi_H^M+5\Pi_L^M}$, joint profit maximization is not sustainable when demand is high in at least one market. In this case, the highest sustainable profit is

$$\Pi_{Total}^S(\varepsilon_1, \varepsilon_2) = \begin{cases} \frac{2\lambda}{4-3\lambda}\Pi_L^M & \text{if } (\varepsilon_1, \varepsilon_2) = (\varepsilon_H, \varepsilon_H) \\ 2\Pi_L^M & \text{if } (\varepsilon_1, \varepsilon_2) = (\varepsilon_H, \varepsilon_L) \text{ or } (\varepsilon_L, \varepsilon_H) \end{cases}$$

In sum, for a low discount factor with which joint profit maximization is not sustainable if the overlapping markets are taken separately, i.e. $\lambda = \frac{\delta}{1-\delta} \leq \frac{2\Pi_H^M}{\Pi_H^M+\Pi_L^M}$, firms can improve their expected profits by linking the two markets. In particular, when a discount factor, δ is in Region C of Figure 1 (i.e. $\frac{4(\Pi_H^M+\Pi_L^M)}{3\Pi_H^M+5\Pi_L^M} \leq \frac{\delta}{1-\delta} \leq \frac{2\Pi_H^M}{\Pi_H^M+\Pi_L^M}$), a high demand shock in an overlapping market will trigger a price war in the market under separating strategy while firms will be able to sustain joint profit maximization in the market under linking strategy. Although joint profit maximization is not sustainable even with linking strategy if demand shocks are high in both markets, collusion does not break up under a single market high demand shock in this case. When a discount factor is lower so that it falls into Region D of Figure 1 (i.e. $1 \leq \frac{\delta}{1-\delta} \leq \frac{4(\Pi_H^M+\Pi_L^M)}{3\Pi_H^M+5\Pi_L^M}$), on the other hand, joint profit maximization is not sustainable when demand shock is high at least in one market. However, firms can sustain a higher collusive profit under high demand shock when they link the markets. That is, the highest sustainable profit is still higher when linking the markets than taking them separately as $\Pi_{Total}^S(\varepsilon_1, \cdot) = \frac{2\lambda}{4-3\lambda}\Pi_L^M > \frac{2\lambda}{2-\lambda}\Pi_L^M = 2\Pi_{BM}^S$.[3] This implies that, if MMC leads to diversification, then diversification reduces the probability of the realization of high demand in the both markets, which is the best opportunity to deviate, and thereby improves the expected collusive profits.

Table 2.1. summarizes the results. Separating strategy implies that firms take each market separately. Linking strategy, on the other hand, means that firms pool the incentive constraints in all overlapping markets. Thus, the difference between separating and linking strategies represent the effect of MMC on collusion. When a discount factor is high enough (Region B), then firms can maximize joint profits in any state of demand regardless of MMC. When a discount factor is lower (Region A), then MMC make differences in the period of high demand unless demand shocks are perfectly and positively correlated between the markets (Case 2: $\rho = 1$). In particular, when demand shocks are perfectly and negatively correlated between the markets (Case 1: $\rho = -1$), joint profit maximization is sustainable with linking strategy. When demand shocks are uncorrelated (Case 3: $\rho = 1$), we can see that linking strategy leads to higher sustainable profits in the period of high demand. That is, incentive to deviate in those periods is reduced and collusion becomes easier to sustain.

[3] This equation holds because $\lambda = \frac{\delta}{1-\delta} \geq 1$.

Table 2.1. Highest Sustainable Collusive Profits

Correl. Structure	Realized Shocks $(\varepsilon_1, \varepsilon_2)$	Range of Discount Factor: Region A, Separating	Region A, Linking, Region D	Region A, Linking, Region C	Region B, Separating	Region B, Linking
Case 1 $(\rho = -1)$	$(\varepsilon_H, \varepsilon_L)$	$\frac{\lambda}{2-\lambda}\Pi_L^M + \Pi_L^M$	$\Pi_H^M + \Pi_L^M$		$\Pi_H^M + \Pi_L^M$	$\Pi_H^M + \Pi_L^M$
Case 3 $(\rho = 0)$	$(\varepsilon_H, \varepsilon_H)$	$\frac{2\lambda}{2-\lambda}\Pi_L^M$	$\frac{2\lambda}{4-\lambda}(\Pi_H^M + 2\Pi_L^M)$		$2\Pi_H^M$	$2\Pi_H^M$
	$(\varepsilon_H, \varepsilon_L)$	$\frac{\lambda}{2-\lambda}\Pi_L^M + \Pi_L^M$	$\frac{2\lambda}{4-3\lambda}\Pi_L^M$	$\Pi_H^M + \Pi_L^M$	$\Pi_H^M + \Pi_L^M$	$\Pi_H^M + \Pi_L^M$
	$(\varepsilon_L, \varepsilon_L)$	$2\Pi_L^M$	$2\Pi_L^M$		$2\Pi_L^M$	$2\Pi_L^M$
Case 2 $(\rho = 1)$	$(\varepsilon_H, \varepsilon_H)$	$\frac{2\lambda}{2-\lambda}\Pi_L^M$	$\frac{2\lambda}{2-\lambda}\Pi_L^M$		$2\Pi_H^M$	$2\Pi_H^M$
	$(\varepsilon_L, \varepsilon_L)$	$2\Pi_L^M$	$2\Pi_L^M$		$2\Pi_L^M$	$2\Pi_L^M$

Region A: $1 \leq \frac{\delta}{1-\delta} \leq \frac{2\Pi_H^M}{\Pi_H^M + \Pi_L^M}$, Region B $\left(\frac{\delta}{1-\delta} \geq \frac{2\Pi_H^M}{\Pi_H^M + \Pi_L^M}\right)$

Region C: $\frac{4(\Pi_H^M + \Pi_L^M)}{3\Pi_H^M + 5\Pi_L^M} \leq \frac{\delta}{1-\delta} \leq \frac{2\Pi_H^M}{\Pi_H^M + \Pi_L^M}$, Region D: $1 \leq \frac{\delta}{1-\delta} \leq \frac{4(\Pi_H^M + \Pi_L^M)}{3\Pi_H^M + 5\Pi_L^M}$

ρ is the correlation of demand shocks between the two overlapping markets M_1 and M_2.

Since the two overlapping markets M_1 and M_2 are symmetric,

$(\varepsilon_H, \varepsilon_L)$ and $(\varepsilon_L, \varepsilon_H)$ are essentially the same case.

In conclusion, unless the demand shocks are perfectly and positively correlated, the expected profit of collusion in each market in the period of high demand is higher as compared to the benchmark case where markets are not linked strategically. Notice that, when markets are perfectly and positively correlated, it is merely a replication of the same market where the irrelevance result noted by Bernheim and Whinston applies. In the real world, the probability of perfect positive correlation of demand shocks between markets is practically zero. So, if firms are operating in several overlapping markets in which demand shocks are imperfectly correlated, they may be able to sustain joint profit maximization in a market where demand is high by linking the market to the other overlapping markets where demand is low.

3.2 Unobservable Demand Shock

The assumptions are similar with those in the case of observable demand shocks, except that (1) firms cannot observe demand shocks directly and observe only their own profit which is the result of realized demand shocks and firms' actions combined, and (2) when some firms cheated, innocent firms will have the same low profit as in the period of low demand, that is, monitoring is imperfect. These differences, unobservability and imperfect monitoring change the implication of MMC and diversification in collusion.

The key ingredients of the model that affect the result are the risk of contagion[4] and the specification

[4] The term "risk of contagion" is used in Thomas and Willig (2006).

of a trigger event. These two factors do not show up in the analysis when demand shock is observable because there is no probability that low demand is misconstrued as cheating and moreover a firm can detect cheating for sure if it has occurred. When demand shock is unobservable, punishment is triggered by not only deviation but also low demand in order to reduce the incentive to deviate and sustain collusion. Given a fixed trigger event, this costly punishment will take place more often under more market contacts. However, the trigger event is chosen from some optimization problem, meaning that the number of markets and the joint distribution of demand shocks in the markets become arguments for the trigger level. Then, more markets do not necessarily induce higher probability of mistakes and diversification may improve a firm's ability to infer a rival firm's action and lead to better specification of trigger events. Let me begin the model with the following assumptions:

(A1) There exist two identical firms competing in two duopoly markets M_1 and M_2. The markets open simultaneously and repeatedly.

(A2) ε_i is a random demand shock in market i. Demand shock is realized to be either "high" (when $\varepsilon = \varepsilon_H$) or "low" (when $\varepsilon = \varepsilon_L$) with equal probability (=0.5) in each market. The demand shocks are independently and identically distributed over time but may be correlated between the markets. So, in each period, the distribution of random demand shock in the two markets is

$$\varepsilon = \begin{matrix} \varepsilon_H \text{ with prob. } = 0.5 \\ \varepsilon_L \text{ with prob. } = 0.5 \end{matrix}$$

(A3) The payoff matrix for firms when demand is high, i.e. $\varepsilon = \varepsilon_H$, depends on the combination of actions of the two firms as follows:

		Firm 2	
		Cooperate	Defect
Firm 1	Cooperate	(Π_H, Π_H)	$(\Pi_L, \Pi_H + k)$
	Defect	$(\Pi_H + k, \Pi_L)$	$(\Pi_H - m, \Pi_H - m)$

where $0 < k \leq \Pi_H$ and $0 < m < \Pi_H - \Pi_L$. Meanwhile, when demand shock is low, the payoff for a firm does not depend on its action. That is, firms cannot distinguish cheating from low demand shock. In particular, regardless of a firm's action, the profit is Π_L when demand shock is low. This assumption implies imperfect monitoring.

(A4) Firms set their prices without knowledge of the state of demand in each period.

(A5) $\delta \in (0, 1)$ is a discount factor common to the two firms.

First of all, let us consider a trigger strategy that firms enter into a punishment phase forever from the next period when they observe in any market. Since there is possibility that firms trigger a price war erroneously due to imperfect monitoring, the punishment phase should be finite. Instead, we assume a low discount factor.

[**Benchmark Case**] Assume a separating strategy; firms maximize profit in each market separately. If taken separately, firms will be able to sustain collusion in each market if

$$\frac{\Pi_H + \Pi_L}{2} + \delta \frac{V_C + V_P}{2} \geq \frac{(\Pi_H + k) + \Pi_L}{2} + \delta V_P \tag{2}$$

where V_C is the value in a collusion phase and V_P is the value in a punishment phase. In each side, the first term is the expected profit today and the second term is the present value of the play starting tomorrow. The left hand side (LHS) is the total expected profit when being cooperative in this period. The right hand side (RHS) is the total expected profit when cheating in this period. Firms will experience either high demand shock or low demand shock with the same probability, the expected profit. So, the expected profit for the current period will be $\frac{\Pi_H+\Pi_L}{2}$ when being cooperative and $\frac{(\Pi_H+k)+\Pi_L}{2}$ when cheating. In the next period, firms will enter into a punishment phase with probability .5 even when they have been cooperative because low demand shock will be regarded as the sign of secret cheating. Meanwhile, collusion will break down for certain if cheating has occurred. Thus, the present value of the play starting tomorrow is $\delta\frac{V_C+V_P}{2}$ when being cooperative and δV_P when cheating.

If the value of collusion is stable over time, the LHS is the value of collusion ($= V_C$) and the RHS is the value of deviation. In addition, the optimal punishment will satisfy equation (2) with an exact equality. Therefore, $V_C = \frac{\Pi_H+\Pi_L-k}{2(1-\delta)}$ (and $V_P = V_C - \frac{1}{\delta}k$).

Now, turn to the multiple market cases where firms trigger a price war based on their own profits in the two markets together.

[**Case 1**] Assume ε_1 and ε_2 are perfectly and *negatively* correlated.

First of all, if firms enter into a punishment phase when they observe low profit in any of the two markets, firms will always become to be in a punishment phase because they will observe low profit in at least one market. So, firms' incentive constraint not to deviate is

$$\Pi_H + \Pi_L + \delta V_C \geq (\Pi_H + k) + \Pi_L + \delta V_P$$

which is not sustainable as long as $k > 0$. That is, collusion is impossible because low demand in one market hurts collusion not only in that market but also the other market where demand is high. This illustrates the "risk of contagion" noted by Thomas and Willig (2006). Because the negative demand shock in one market spreads its effect to the other market with positive demand shock, linking the markets can be even worse than separating the markets.

However, if firms know the correlation structure of demand shocks between the markets, they can adjust a trigger strategy optimally based on the information. In particular, consider the case where a firm triggers punishment if the firm observes a pair of its own profits in the two markets, (Π_1, Π_2) s.t. $(\Pi_1, \Pi_2) = Arg \max \Pr\{(\Pi_1, \Pi_2)|D\} - \Pr\{(\Pi_1, \Pi_2)|C\}$, where D and C stand for "other firm deviated" and "other firm was cooperative", respectively, when I was cooperative. Intuitively, it will be optimal for a firm to punish the other firm if the firm's profits in the two markets are much more likely to be realized when the other firm cheated than when the other firm was cooperative. So, a firm can specify a better trigger event than a simple strategy that triggers punishment when it observed low profit in any of the two markets, based on the increase in probability of a pair of its own profits when the other firm cheated.

When the demand shocks are perfectly and negatively correlated, the pair of profits in the two markets for an innocent firm is either (Π_H, Π_L) or (Π_L, Π_H) with equal probability if the other firm has also been cooperative, but it is (Π_L, Π_L) for certain if the other firm has deviated in the both markets. So, for each possible pair of profits of an innocent firm in the two markets, the change in probability is as follows:

$$\Pr\{(\Pi_1, \Pi_2)|D\} - \Pr\{(\Pi_1, \Pi_2)|C\} = \begin{matrix} 0 - .5 = -.5 & \text{if } (\Pi_1, \Pi_2) = (\Pi_H, \Pi_L) \text{ or } (\Pi_L, \Pi_H) \\ 0 - 0 = 0 & \text{if } (\Pi_1, \Pi_2) = (\Pi_H, \Pi_H) \\ 1 - 0 = 1 & \text{if } (\Pi_1, \Pi_2) = (\Pi_L, \Pi_L) \end{matrix}$$

Based on the change in probability of (Π_1, Π_2) when some firm deviated as compared to when all firms cooperated, a firm will adjust a trigger event so that it starts retaliations in every market if it observed low profit in every market because it can happen only when the other firm deviated in the both markets. In this case, the incentive constraint not to deviate becomes

$$\Pi_H + \Pi_L + \delta V_C \geq (\Pi_H + k) + \Pi_L + \delta V_P$$

Note that punishment no longer occurs in the equilibrium path of collusion (so V_P does not appear in the LHS). So the optimal choice of punishment will be as harsh as possible. That is, $V_P = 0$ (perfect competition). The value of collusion is now $V_C = \frac{\Pi_H + \Pi_L}{1-\delta}$, and the collusion is sustainable if $\frac{\delta}{1-\delta}(\Pi_H + \Pi_L) \geq k$. The value of collusion is larger than the value of collusion when markets are taken separately $(= \frac{\Pi_H + \Pi_L - k}{1-\delta})$.

However, knowing that it will be punished only when low profits are realized in the both markets, a cheating firm might decide to deviate in only one of the markets in order to reduce the probability of getting caught although it will reduce the immediate gains from deviation. Without loss of generality, let's assume that a firm will cheat in M_1 if it decides to deviate. Then, an innocent firm will get (Π_L, Π_H) if demand happens to be low in M_1 (and high in M_2) and (Π_L, Π_L) if demand happens to be low in M_2 (and high in M_1), both with probability 0.5. So, for each possible pair of profits of an innocent firm in the two markets, the change in probability is now

$$\Pr\{(\Pi_1, \Pi_2)|D\} - \Pr\{(\Pi_1, \Pi_2)|C\} = \begin{matrix} .5 - .25 = .25 & \text{if } (\Pi_1, \Pi_2) = (\Pi_L, \Pi_H) \\ 0 - .25 = -.25 & \text{if } (\Pi_1, \Pi_2) = (\Pi_H, \Pi_L) \\ 0 - 0 = 0 & \text{if } (\Pi_1, \Pi_2) = (\Pi_H, \Pi_H) \\ .5 - 0 = .5 & \text{if } (\Pi_1, \Pi_2) = (\Pi_L, \Pi_L) \end{matrix}$$

Again, the biggest change in probability happens for the cases where low profit is realized in the both markets. So, a trigger strategy is the same as before and the incentive constraint not to deviate becomes

$$\Pi_H + \Pi_L + \delta V_C \geq \frac{(\Pi_L + \Pi_H) + ((\Pi_H + k) + \Pi_L)}{2} + \delta \frac{V_P + V_C}{2}$$

Note that punishment does not take place on the equilibrium path but, at the same time, cheating may not be caught by an innocent firm. The optimal punishment and the value of collusion are the same as before, i.e. $V_P = 0, V_C = \frac{\Pi_H+\Pi_L}{1-\delta}$. In addition, the condition for sustainable collusion is the same as well, i.e. $\frac{\delta}{1-\delta}(\Pi_H + \Pi_L) \geq k$. Thus, the optimal trigger event will be that low profit is realized in every market and, if a firm observes this trigger event, it will enter into a punishment phase where firms are in perfect competition.

Therefore, for any deviation strategy, firms can improve their collusive profits by adjusting a trigger event based on how much the distribution of realized profits is more likely when there has been cheating compared to when there has not. Moreover, if the overlapping markets are perfectly diversified, that is, perfectly and negatively correlated in terms of demand shocks, then the knowledge of correlation structure leads to perfect monitoring and so the temptation of firms to deviate is reduced and so is the frequency of price wars.

[**Case 2**] Assume ε_1 and ε_2 are perfectly and *positively* correlated.

If firms enter into a punishment phase when they observe low profit in any of the two markets, firms can sustain collusion if

$$\frac{2\Pi_H + 2\Pi_L}{2} + \delta\frac{V_P + V_C}{2} \geq \frac{2(\Pi_H + k) + 2\Pi_L}{2} + \delta V_P$$

Then, with the optimal punishment, $V_C = \frac{\Pi_H+\Pi_L-k}{1-\delta}$ (and $V_P = V_C - \frac{2}{\delta}k$), which is exactly the same as in the benchmark case (note that V_C in the benchmark case is for a single market and so should be doubled for comparison). That is, the value of collusion remains the same with and without linking the markets because there is no risk of contagion.

Moreover, the knowledge of correlation structure does not affect collusion either because one more market is only a replication of the same market, meaning no additional information that firms can take advantage of. Therefore, MMC is irrelevant in this case.

[**Case 3**] Assume ε_1 and ε_2 are *independent* of each other.

If firms trigger retaliations in the both markets if it observes low profit in at least one of the markets and a cheating firm optimally deviates in the both markets, collusion is sustainable if

$$\begin{aligned} & \frac{2\Pi_H + 2\Pi_L + (\Pi_H + \Pi_L) + (\Pi_L + \Pi_H)}{4} + \delta\frac{V_C + 3V_P}{4} \\ \geq \ & \frac{2(\Pi_H + k) + 2\Pi_L + ((\Pi_H + k) + \Pi_L) + (\Pi_L + (\Pi_H + k))}{4} + \delta V_P \end{aligned}$$

Then, with the optimal punishment, $V_C = \frac{\Pi_H+\Pi_L-k}{1-\delta/2}$ (and $V_P = \frac{1}{3}(V_C - \frac{4}{\delta}k)$), which is less that the value of collusion in the benchmark case ($= \frac{\Pi_H+\Pi_L-k}{1-\delta}$). The reduction in the value of collusion is because of the risk of contagion which increases the type I error that a firm erroneously accuses low profit of cheating and enters into a punishment phase.

However, if firms know how demand shocks are correlated between the markets, they can adjust a trigger strategy optimally. First of all, for each possible pair of profits of an innocent firm in the two markets, the change in probability is

$$
\begin{aligned}
& \Pr\{(\Pi_1,\Pi_2)|D\} - \Pr\{(\Pi_1,\Pi_2)|C\} \\
= & \begin{cases} 0-.25=-.25 & \text{if } (\Pi_1,\Pi_2)=(\Pi_L,\Pi_H),\ (\Pi_H,\Pi_L), \text{ or } (\Pi_H,\Pi_H) \\ 1-.25=.75 & \text{if } (\Pi_1,\Pi_2)=(\Pi_L,\Pi_L) \end{cases}
\end{aligned}
$$

So, the biggest change in probability occurs for the case where profit is low in every market, i.e. (Π_L,Π_L). Based on this, consider the trigger strategy that a firm will retaliate in the both markets if it observed low profits in the both markets. Under this strategy, collusion is sustainable if

$$
\begin{aligned}
& \frac{2\Pi_H+2\Pi_L+(\Pi_H+\Pi_L)+(\Pi_L+\Pi_H)}{4}+\delta\frac{3V_C+V_P}{4} \\
\geq & \frac{2(\Pi_H+k)+2\Pi_L+((\Pi_H+k)+\Pi_L)+(\Pi_L+(\Pi_H+k))}{4}+\delta V_P
\end{aligned}
$$

With the optimal punishment that satisfies the incentive constraint with an exact equality, the value of collusion is $V_C=\frac{\Pi_H+\Pi_L-k/3}{1-\delta}$ (and $V_P=V_C-\frac{4}{3\delta}k$). We can see that the value of collusion is larger with this trigger strategy than with a simple trigger strategy that triggers punishment when a firm observes low profit in at least one market because the risk of contagion becomes large when the latter strategy is chosen. The value of collusion is actually even larger than in the benchmark case where the markets are taken separately $(=\frac{\Pi_H+\Pi_L-k}{1-\delta})$.

However, a cheating firm might want to deviate in one market at random rather than in the both markets because it will reduce the probability of getting caught. Then, for each possible pair of profits of an innocent firm in the two markets, the change in probability is

$$
\Pr\{(\Pi_1,\Pi_2)|D\} - \Pr\{(\Pi_1,\Pi_2)|C\} = \begin{cases} .25-.25=0 & \text{if } (\Pi_1,\Pi_2)=(\Pi_H,\Pi_L) \text{ or } (\Pi_L,\Pi_H) \\ 0-.25=-.25 & \text{if } (\Pi_1,\Pi_2)=(\Pi_H,\Pi_H) \\ .5-.25=.25 & \text{if } (\Pi_1,\Pi_2)=(\Pi_L,\Pi_L) \end{cases}
$$

The probability changes the most for (Π_L,Π_L). Again, the optimal trigger event would be (Π_L,Π_L), i.e. entering into a punishment phase if low profit is realized in the both market. In this case, the incentive constraint not to deviate becomes

$$
\begin{aligned}
& \Pi_H+\Pi_L+\delta\frac{3V_C+V_P}{4} \\
\geq & \frac{((\Pi_H+k)+\Pi_H)+2\Pi_L+((\Pi_H+k)+\Pi_L)+(\Pi_L+\Pi_H)}{2}+\delta\frac{V_P+V_C}{2}
\end{aligned}
$$

In this case, with the optimal punishment, $V_C=\frac{\Pi_H+\Pi_L-k/2}{1-\delta}$ (and $V_P=V_C-\frac{2}{\delta}k$). Note that firms cannot sustain collusion with the optimal punishment level when a firm expects that a cheating firm will deviate in every market ($V_P = \frac{\Pi_H+\Pi_L-k/3}{1-\delta} - \frac{4}{3\delta}k$) because a firm will be tempted to deviate in only one market, with less immediate gain but less probability of getting caught. Therefore, the optimal trigger event is that a firm observes low profit in the both markets and, once punishment is triggered, the highest value that the firms can get in a punishment phase will be $V_P = \frac{\Pi_H+\Pi_L-k/2}{1-\delta} - \frac{2}{\delta}k$. Still, the value of collusion, $\frac{\Pi_H+\Pi_L-k/2}{1-\delta}$ is higher than in the benchmark case.

Table 2.2. Value of Collusion (V_c) under Unobservable Demand Shocks

	Separating	Linking		
Trigger Strategy	Benchmark	Case 1 ($\rho = -1$)	Case 3 ($\rho = -1$)	Case 2 ($\rho = -1$)
Simple	$\frac{\Pi_H^M+\Pi_L^M-k}{1-\delta}$	$\frac{\Pi_H^M+\Pi_L^M-k}{1-\delta}$	$\frac{\Pi_H^M+\Pi_L^M-k}{1-\delta/2}$	No level of V_c sustainable
Optimal		$\frac{\Pi_H^M+\Pi_L^M-k}{1-\delta}$	$\frac{\Pi_H^M+\Pi_L^M-k/2}{1-\delta}$	$\frac{\Pi_H^M+\Pi_L^M}{1-\delta}$

Optimal strategy refers to the strategy in which firms.

Table 2.2 summarizes the results in this section. A separating strategy means that a trigger strategy and punishment is determined in a market separately. In contrast, a linking strategy means that a trigger strategy and punishment is determined, based on the outcomes in every overlapping market. On the other hand, there could be two types of strategies when linking markets; a simple strategy that a firm triggers punishment if a firm observes low profit in at least one market and an optimal strategy that a firm enters into punishment phase if a firm observes the market outcome that is much more likely when other firms cheated than when other firms were collusive (in this basic model, the optimal strategy is to trigger punishment if a firm observes low profit in the both markets).

The first row of the value of collusion in Table 2.2 shows that the risk of contagion becomes more serious as the overlapping markets get more diversified (is the largest for perfectly and positively correlated demand shocks, followed by independent shocks, and the least for perfectly and negatively correlated shocks). However, once we assume that firms know the correlation structure of demand shocks between the overlapping markets, then the simple trigger strategy is not optimal and they can specify a better trigger strategy that incorporates the information. That is, the knowledge of the correlation structure of demand shocks between the overlapping markets can improve monitoring, which not only offsets the risk of contagion but also may increase the value of collusion. In particular, under better monitoring, firms may not profitably deviate in every market and thus the gains from deviation decreases while the future loss from deviation remains the same. This will curb the temptation to deviate and facilitate collusion. Moreover, the informational advantage from linking markets becomes larger as the overlapping markets get more diversified.

In conclusion, even in the markets where demand shocks are unobservable, MMC may facilitate collusion if the markets are diversified and firms are aware of how demand shocks are correlated between the markets. This is because, although the reduction in demand fluctuations from diversification makes collusion even harder, if firms know the correlation structure of demands shocks between the markets they are meeting, they optimally adjust a trigger strategy that prevents the risk of contagion and furthermore improves the value of collusion. That is, the informational advantage of observing the distribution of outcomes of the diversified may exceed the risk of contagion of linking the diversified markets. So, we can conclude that MMC may help collusion through diversification even when demand shock is unobservable.

In addition, the conditional probability is even more precise than the joint probability when in comes to inferring a rival firm's action in the market with unobservable demand shocks. In this sense, there is an advantage for a firm in diversifying to the industry where a rival firm is operating and demand shock is observable if it is currently meeting with the rival firm in the industry with unobservable demand shock.

4 Extension

So far, I have focused on the incentive not to deviate in the period of high demand given that firms can coordinate their actions if they want. However, explicit communication of price is mostly illegal and coordination of actions itself might be hard to begin with. In some industries, firms might find coordination rather easy even without explicit discussion of pricing. For example, firms might have been operating in the same market for a long time and managers and practitioners know each other, even personally. Or, the advanced internet technology might enhance the communication between firms through the third party that posts some information that signals firms' actions. In these industries, the incentive to deviate is the concern for firms participating in collusion and the theory above applies.

However, when it comes to what will actually happen after cheating, the easiness of coordinating actions might rather undermine collusion because of possible renegotiation. First of all, the Folk Theorem asserts that basically any collusive outcome can be implemented in infinitely repeated game as long as it is feasible and individually rational, i.e. subgame-perfect, which is sometime called as the "embarrassment of riches." As a mean to narrow down a set of subgame perfect equilibria, credibility of equilibrium has been presented by Farrell and Maskin (1989), which is called as "(Weak) Renegotiation-Proofness." Basically, if not only a cheating firm but also an innocent firm can be better off by restarting collusion rather than implementing punishments, they will have an incentive to renegotiate. Especially, the only credible equilibrium in the symmetric Bertrand price competition without capacity constraints is the Bertrand Nash Equilibrium, meaning perfect competition and no collusion. Therefore, if renegotiation is possible after deviation, collusion may be simply impossible even with a high discount factor when firms are competing with price and homogeneous products.

Recall that, when demand shock is observable, firms are more tempted to deviate in the period of high demand and MMC may be able to alleviate this problem through diversification and thereby improves collusive profits. However, if a discount factor is high enough, the incentive constraint is not binding and any collusive outcome can be implemented even without MMC. In contrast, if renegotiation is possible, any collusive outcome might be unsustainable in the first place.

In order to account for the two problems regarding a high discount factor and renegotiation, I modify the general game theoretic approach by introducing a firm's belief in whether there is a firm that has a finite time horizon and how short the time horizon would be. I further assume that firms adjust their beliefs based on current market outcomes when deciding whether to continue the collusive behavior in the next period.

If a firm has a finite time horizon and other firms are not aware of it, the firm will deviate at the end. Firms can have finite time horizon due to various reasons, e.g. managers may serve only a finite term, or the wage structure is based on short-term performances, or firms may face cash constraints in any moment. Short-term oriented managers will put more weight on today and the future will become more and more negligible. Also, financially constrained firms are likely to trigger a price war regardless of whether cheating has taken place, as noted by Busse (2002) that airlines under financial distress are more likely to lead a price war.

I assume that firms rationally expect that other firms might have finite time horizon and end up with deviation in the next period. In particular, firms have a prior belief that there is a firm that will deviate in the next period because of a finite time horizon with probability $\alpha \in [0, 1]$. For example, if there are N identical firms in a market, in a traditional game theoretical approach, the incentive constraint for a firm not to deviate is

$$N\Pi < \frac{\delta}{1-\delta}\Pi \ (= \Pi + \delta\Pi + \delta^2\Pi + \cdots)$$

where Π is a payoff to a participating firm from collusion and δ is a discount factor. If we incorporate α into this model, the incentive constraint now becomes

$$N\Pi < \frac{(1-\alpha)\delta}{1-\delta}\Pi \ (= (1-\alpha)\Pi + \delta(1-\alpha)\Pi + \delta^2(1-\alpha)\Pi + \cdots)$$

because a firm might end up being cheated by other firm with finite time horizon with probability α in every period in the future. In other words, since firms believe that collusion can continue only with probability $1 - \alpha$, the expected payoff for a participating firm in each period is now only $(1 - \alpha)\Pi$. If $\alpha = 0$, it goes back to the traditional game theoretic model. On the other hand, positive has the same effect as the decrease in the discount factor in the incentive constraints for firms not to deviate. In other words, positive α discounts the future profit that will be lost if a firm defects and thus makes collusion harder. As a result, the possible set of collusion equilibria shrinks.

In addition, assume that the belief is a function of market outcome; a firm has a prior belief $\alpha = \alpha_0 \in (0, 1)$ until any one deviates (if demand shock is observable) or it observes a certain profit level that triggers punishment (if demand shock is unobservable), and, once those trigger events take place, the belief α is adjusted to one for good because the firm now knows that some firms do have finite time horizon. Notice that, once the belief is adjusted after the trigger events, any level of collusion is no longer sustainable. As I assume this adjustment of the belief based on market outcome, there becomes no dynamic inconsistency of incentives before and after cheating and thus renegotiation will not take place.

Now, if I incorporate the belief α, which is a function of whether or not trigger events have taken place, to the basic models with unobservable/observable demand shocks, then the collusion will become harder to sustain and the equilibrium is renegotiation-free. However, the key conclusion will remain the same that MMC may facilitate collusion and, if demand shock is observable, the effect will be more significant especially in the period of high demand.

5 Conclusion

In this study, I explored how collusive outcome is affected by MMC and diversification when the competing firms face stochastic demand shocks. I consider two kinds of demand shocks depending on their "observability."

First of all, when demand shock is observable, Rotemberg and Saloner (1986) pointed out that firms are more tempted to deviate from collusion in the period of high demand (because the immediate gain from deviation increases while the expected future loss from it remains the same). In this case, unless the demand shocks are perfectly and positively correlated across overlapping markets, the incentive to deviate in the period of high demand will decrease. Given that overlapping markets are strategically linked in the sense that a deviation in a single market will trigger retaliations in all markets, a firm will optimally deviate in every market once it decides to cheat, and then the best opportunity to deviate is when demand is high in every overlapping market. If the linked markets are diversified, however, when demand is high in some markets, demand will be not-so-high in other markets, meaning that the immediate gain from deviation is reduced and so is the temptation to deviate. (That is, the probability that demand is high in every market will decrease with the number of overlapping markets.) In this sense, MMC and diversification of demand shocks by linking the markets will facilitate collusion by reducing the temptation to deviate in the period of high demand.

Now, let's turn to the case of unobservable demand shocks. In this case, the implication of MMC and diversification may be different as monitoring is imperfect. The negative link between imperfect monitoring and collusion has been noted by Green and Porter (1984). With unobservable demand shock, detection of cheating is not perfect as, when a firm observes a profit below a certain level, it cannot tell negative demand shock from secrete cheating by other firms. So, a price war is triggered not only by cheating but also by low demand. This price war is costly but necessary to sustain collusion. In this case, MMC facilitate collusion by improving monitoring ability and by reducing the frequency of costly punishment on the equilibrium path. We need to note that there can be two opposite effects of MMC on collusion. First, in the sense that low demand in a local market may falsely trigger a price war in all overlapping markets, MMC may have negative impact on expected collusive profits. However, MMC may improve firms' monitoring ability as firms now can use the information on the joint distribution of market outcomes across overlapping markets, in addition to individual market outcome, in order to infer other firms' actions. That is, firms will optimally adjust trigger events so that they will enter into the punishment phase if the profile of profits across the markets becomes much more likely when cheating has occurred than when other firms have been cooperative. One of the optimal trigger events can come from the Likelihood Ratio test in the Maximum Likelihood Estimation. (Although a single market outcome may not have any information about other firms' actions, the joint distribution of outcomes across the overlapping markets may be informative.). Using this trigger strategy, I showed that MMC can improve collusive profits if firms optimally adjust punishment trigger event based on the information about the joint distribution of demand shocks.

The previous empirical works on the topic have examined either the decrease in rivalry associated with MMC on average or the effect of heterogeneity in markets or firms on the link between MMC and competition. This study provides a new testable implication on the topic, which is about a dynamic relationship between MMC and price competition. When demand shock is observable, the theory predicts that price competition will be muted by MMC in the period of high demand. The test of this idea will be an interesting future research.

6 References

Abreu, Dilip.1986. "Extremal equilibria of oligopolistic supergames", *Journal of Economic Theory*, 39: 191-225.

_______. 1988. "On the theory of infinitely repeated games with discounting", *Econometrica*, 56: 383–396.

_______, David Pearce and Ennio Stacchetti. 1990. "Toward a Theory of Discounted Repeated Games with Imperfect Monitoring", *Econometrica*, 58: 1041–1063.

_______, _______ and _______. 1993. "Renegotiation and Symmetry in Repeated Games", *Journal of Economic Theory*, 60: 217-240.

Baum, Joel A. C. 1999. "Chance, Imitative, and Strategic Antecedents to Multimarket Contact", *Academy of Management Journa*l, 42: 171-193.

_______ and Helaine J. Korn. 1996. "Competitive Dynamics of Interfirm Rivalry: Linking Structural Conditions of Competition to Patterns of Market Entry and Exit", *Academy of Management Journal*, 39: 255-291.

Bernheim, Douglas and Michael D. Whinston. 1990. "Multimarket Contact and Collusive Behavior", *RAND Journal of Economics*, 21: 1-26.

Borenstein, Severin and Andrea Shepard. 1996. "Dynamic Pricing in Retail Gasoline Markets", *Rand Journal of Economics*, 27(3): 429-451.

Busse, Meghan R. 2000. "Multimarket Contact and Price Coordination in the Cellular Telephone Industry", *Journal of Economics & Management Strategy*, 9: 287–320.

Evans, William N and Ioannis Kessides. 1995. "Living by the 'Golden rule': Multimarket Contact in the U.S. Airline Industry", *Quarterly Journal of Economics*, 109 (2), 341-366.

Farrell, Joseph. 2000. "Renegotiation in Repeated Oligopoly Interaction", *Working Paper*, Department of Economics, University of California at Berkeley.

_______ and Eric Maskin. 1989. "Renegotiation in Repeated Games", *Games and Economic Behavior*, 1(4): 327-360.

Feuerstein, Switgard. 2005. "Collusion in Industrial Economics - A Survey", *Journal of Industry, Competition and Trade*, 5(3/4): 163–198.

Fournier, Gary M. and Thomas W. Zuehlke. 2008. "Multimarket Contact and the Antitrust Settlement in Airline Tariff Publishing Co (1994)", *Working Paper*, Department of Economics, Florida State University.

Fudenberg, Drew, David Levine and Eric Maskin. 1994. "The Folk Theorem with Imperfect Public Information", *Econometrica*, 62(5): 997–1039.

Green, Edward J. and Robert H. Porter. 1984. "Noncooperative Collusion under Imperfect Price Information", *Econometrica,* 52(1): 87-100.

Heggestad, Arnold A. and Stephen A. Rhoades. 1978, "Multimarket Interdependence and Local Market Competition in Banking", *Review of Economics and Statistics,* 60: 523-532.

Hughes, Kirsty and Christine Oughton. 1993. "Diversification, Multi-market Contact and Profitability", *Economica,* 60: 203-224.

Jans, Ivette and David I. Rosenbaum. 1996. "Multimarket contact and pricing: Evidence from the U.S. cement industry", *International Journal of Industrial Organization,* 15(3): 391-412.

Jayachandran, Satish, Javier Gimeno and P. Rajan Varadarajan. 1999. "The Theory of Multimarket Competition: A Synthesis and Implications for Marketing Strategy", *Journal of Marketing*, 63: 49-66.

Kwoka, John E., Jr. and Lawrence J. White. 2004. *The Antitrust Revolution: Economics, Competition, and Policy*, Oxford University Press.

Levenstein, Margaret C. and Valerie Y. Suslow. 2006, "What Determines Cartel Success?", *Journal of Economic Literature*, 44(1): 43-95.

Li, Stan Xiao and Royston Greenwood. 2004. "The Effect of Within-Industry Diversification on Firm Performance: Synergy Creation, Multi-Market Contact and Market Structuration", *Strategic Management Journal*, 25(12): 1131–1153.

Matsushima, Hitoshi. 2001., "Multimarket Contact, Imperfect Monitoring, and Implicit Collusion", *Journal of Economic Theory*, 98(1): 158-178.

Prince, Jeffrey and Daniel H. Simon. 2006. "Multimarket Contact and On-Time Performance in the US Airline Industry", *Working Paper*, Cornell University.

Rhoades, Stephen A. and Arnold A. Heggestad. 1985. "Multimarket Interdependence and Performance in Banking: Two Tests", *Antitrust Bulletin*, 30: 975-995.

Rotemberg, Julio J. and Garth Saloner (1986), "A Supergame-Theoretic Model of Business Cycles and Price Wars during Booms", *American Economic Review*, 76(3): 390-407.

Singal, Vijay. 1998. "Airline Mergers and Multimarket Contact", *Managerial and Decision Economics*, 17(6): 559-574.

Sorenson, Timothy L. 2007. "Credible Collusion in Multimarket Oligopoly", *Managerial and Decision Economics*, 28(2): 115–128.

Spagnolo, Giancarlo. 1999. "On Interdependent Supergames: Multimarket Contact, Concavity, and Collusion", *Journal of Economic Theory*, 89: 127–139.

Thomas, Charles J. and Robert D. Willig. 2006. "The Risk of Contagion from Multimarket Contact", *International Journal of Industrial Organization*, 24(6): 1157-1184.