# Harmful Signaling in Matching Markets

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## Abstract

Some labor markets have recently developed formal signaling mechanisms, e.g. the signaling for interviews in the job market for new Ph.D. economists. We evaluate the effect of such mechanisms on two-sided matching markets by considering a game of incomplete information between firms and workers. Workers have almost aligned preferences over firms: each worker has "typical" commonly known preferences with probability close to one and "atypical" idiosyncratic preferences with the complementary probability close to zero. Firms have commonly known preferences over workers. We show that the introduction of a signaling mechanism is harmful for this environment. Though signals transmit previously unavailable information, they also facilitate information asymmetry that leads to coordination failures. As a result, the introduction of a signaling mechanism lessens the expected number of matches when signals are informative.

*JEL classification:* C72, C78, D80, J44.

*Key words:* signaling, cheaptalk, matching.

# 1    Introduction

In December 2006, the Ad Hoc Committee of American Economic Association (AEA) implemented signaling as an actual instrument to facilitate match formation in the job market for new Ph.D. economists.[1] This market begins in early Fall each year, when economics departments advertise open faculty positions and graduate students nearing completion of their dissertations apply for these positions according to their preferences. Each student has an opportunity to send two signals to two departments prior to the market. Each signal states only that the student has indicated her interest to a given department. The signals are private, and only the faculty of the chosen department knows the student's signal. Later, departments invite students to interviews and daylong university visits that are followed by job offers to selected candidates. However, each department can interview only a small portion of available students, which creates congestion in the market and introduces a strategic dimension to the interviewing process. A typical department probably does not want to spend time interviewing candidates who are being interviewed by the elite departments.

The Ad Hoc Committee introduced the signals in order to alleviate congestion at the interviewing stage. Roth (2008) suggests that the limited number of signals can credibly transmit information about students' preference, which could help to reduce the coordination failures faced by the market participants and facilitate better match formation (see also "Signaling for Interviews in the Economics Job Market" AEA (2005)[2] for more discussion). Coles et al. (2009) also obtain results that support this intuition. Contrary to these studies, this paper shows that the signals can impede match formation in some environments.

We analyze one-to-one matching market between firms and workers without transfers. Each agent knows its own preferences over agents on the other side of the market, but is uncertain about the preferences of other agents. Workers have almost aligned preferences. Each worker has either "typical" commonly known preferences with a probability close to one or "atypical" preferences taken from some distribution with the complementary probability close to zero. The preferences of workers are ex-ante independently distributed.[3] Firms have some fixed and commonly known preferences over workers.

We concentrate on the analysis of a preference signaling mechanism by assuming that each agent knows its preferences. Therefore, the interviewing stage is not necessary. We

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[1] The Ad Hoc Committee was established in 2005 in order to develop ways to facilitate the job market for new Ph.D. economists. Its members are Alvin E. Roth (chair), John Cawley, Philip Levine, Muriel Niederle, and John Siegfried. The signaling mechanism is implemented via the AEA website: http://www.aeaweb.org/joe/signal/.

[2] The document was created by the Ad Hoc Committee to provide advice to participants in the job market for new Ph.D. economists; http://www.aeaweb.org/joe/signal/signaling.pdf.

[3] We assume that typical workers rank firms according to some public ranking. For example, typical candidates in the job market for new Ph.D. economists rank departments of economics in their field according to the *U.S. News and World Report* ranking.

consider a decentralized matching game with three stages. First, each worker chooses a firm, to which she sends her signal. Each worker sends up to one signal; workers send signals simultaneously. Only firms that receive signals observe them. Second, firms make decisions about job offers by taking into account signals received at the first stage. Each firm can make only one offer. Finally, each worker chooses an offer to accept among the available offers. Each worker can accept at most one offer.

We show that if firms respond to signals in this environment, i.e. treat signals informatively, the introduction of signals decreases the expected number of matches. Prior to the signaling, all firms have almost identical beliefs about worker preferences. However, after the signals are received firms may have disparate beliefs. This disparity in beliefs leads to coordination failure. As a result, the introduction of a signaling mechanism may decrease the total number of matches and the welfare of agents.

Finally, we examine the implications from the introduction of a mechanism with public signals, i.e. signals observed by all firms. Though the expected number of matches increases compared to the game with private signals, public signals still impede match formation for some environments. Public signals do not transmit enough information about worker preferences. This induces some firms to compete for the same workers, which creates mismatches and decreases the expected total number of matches.

In addition to the market of new Ph.D. economists, some versions of the signaling mechanism studied in this paper emerged in other markets. Some online dating websites employ signaling mechanisms, wherein agents send signals to potential partners to indicate special interest. For example, each man of Cupid.com has several "virtual roses" they can attach to their messages. Since there is almost no cost associated with an additional message to a woman, these scarce roses provide men with a means of indicating their genuine interest to particular women. Informal signaling is also an important part of the market for clinical psychologists, described by Roth and Xing (1997). The ability of candidates to convey information about the likelihood to accept an offer is crucial in this market. Program directors for internships in clinical psychology have a tendency to hire applicants who explicitly express their readiness to accept an offer immediately, even if these applicants are of a low quality. Finally, early college admission in the U.S. can be viewed as a form of preference signaling. Many schools require that applicants not send early applications to other schools and view an early application as a signal of a student's enthusiasm for a particular school.

Signaling that we analyze in this paper is a form of costless communication, or cheap talk. There is no penalty attached for lying, and claims do not directly affect payoffs.[4] Therefore, signals can only enlarge the set of equilibria. In contrast to this paper, Crawford and Sobel

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[4]Ration talk is a better name for signals in our setting. Though signals are costless, an agent can send only a limited number of signals.

(1982) show that a cheap talk lead to new equilibria that are Pareto-superior to the one without communication.

Our negative results on expected number of matches in new cheap talk equilibria is in line with Farrell and Gibbons (1989)'s results, though it differs in its intuition.[5] Costless communication in their two-agent bargaining model gives the buyer an opportunity to pretend to have a lower value and the seller an opportunity to pretend to have a higher value (compared to the truthful information transmission in our model). This enhances their bargaining positions at the cost of the risk of no trade. New cheap talk equilibria are characterized by both less trade and a reduction in the expected gains from trade.

In addition, signaling in our paper differs fundamentally from the "job market signaling" first introduced in Spence (1973). First, signals in our model have no direct cost. In addition, signals serve to convey information about worker preferences rather than worker quality.

Some recent works also analyze variants of signaling mechanisms. Coles et al. (2009) is a paper that is most related to ours. They consider the same three stage model of decentralized two-sided matching markets. However, they analyze markets with firm "segments". Workers agree on the ranking of firms across segments, but have idiosyncratic and uniformly distributed preferences within segments. For instance, all workers may agree as to which firms are in the "top five" segment and which are in the "six to ten" segment, etc., but may disagree as to the exact ranking within a segment. Firm preferences over workers are idiosyncratic and uniformly distributed. They show that, on average, introducing a signaling mechanism increases both the expected number of matches as well as the expected welfare of workers for this environment. The welfare of firms, on the other hand, changes ambiguously. Coles et al. (2009) also examine the impact of a signaling mechanism for different environments that vary in market size, the number of signals, positions, and periods of interaction.

Avery and Levin (2009) analyze match formation in the U.S. college admission market. There are two types of early admission programs: *early action* programs, where students may apply early but without any commitment to enroll, and *early decision* programs, where students commit to enroll if accepted. Avery and Levin (2009) analyze early applications as a signal of a student's enthusiasm for a particular school. At the same time, schools benefit more from accepting more enthusiastic students. Their results suggest that selective (or elite) schools benefit from adopting early action policy. At the same time, a lower ranked school, by adopting early decision policy, can attract some highly qualified but cautious students, drawing them away from highly ranked schools.

Lee et al. (2009) run a field experiment with the major Korean online dating website Couple.net to measure potential gains from introducing "virtual roses".[6] They find that

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[5]We are thankful for Lones Smith who drew our attention to this comparison.

[6]This "virtual roses" mechanism is similar to that of Cupid.com described above.

users of both genders are more likely to accept a dating request when a "virtual rose" is attached, and that roses are most helpful in improving acceptance rates for senders of "below average popularity."

Preference signaling can also be useful in centralized matching markets. Abdulkadiroglu et al. (2008) show that the introduction of a signaling technology can improve the ex-ante efficiency of the deferred acceptance algorithm (see Gale and Shapley, 1962) in case of weak preferences. Lee and Schwarz (2007) analyze preferences signaling in centralized matching markets in a three step matching formation process: preference signaling, investments in information acquisition, and the formation of matches based on available information. They show that agents reveal their preference truthfully in an equilibrium under some assumptions on agent utility.

The paper proceeds as follows. Section 2 presents a simple example that illustrates why signals can facilitate coordination failure. Section 3 outlines our general model and introduces some notations. Equilibrium analysis is presented in Section 4. Section 5 analyzes the welfare of agents in the model with and without signals. Section 6 compares these welfare implications with the results in the previous literature and discusses two controversial roles of signals in matching markets. A public signaling mechanism is considered in Section 7. Finally, Section 8 discusses some assumptions of our model and concludes.

# 2    A simple example

Let us consider a market with three firms and three workers. Each firm ranks the workers in the same way $(w_1, w_2, w_3)$, i.e. each firm strictly prefers worker $w_1$ to worker $w_2$ to worker $w_3$. Each worker's preference is either typical $(f_1, f_2, f_3)$ with probability $1-\varepsilon$ or atypical with the complementary probability $\varepsilon$, where $\varepsilon$ is small. The atypical preferences are independently uniformly distributed among all possible preference order lists. All workers are acceptable to all firms and vice versa.

We first examine behavior in the game in the absence of a signaling mechanism. The only possible match in a sequential equilibrium of the game without signals is the assortative match, in which each firm is matched to the corresponding worker. Let now us analyze the game with the signaling mechanism, described above. We consider the following equilibrium strategies of agents.[7] Each worker with typical preferences sends her signal to the corresponding firm, i.e. worker $w_i$ sends her signal to firm $f_i$. Each worker with atypical preferences sends her signal to the best firm worse or equal to the corresponding one (according to typical preferences). Each firm makes its offer to a worker better or equal to the

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[7]See Theorem 1 for the proof that these strategies constitute a sequential equilibrium.

corresponding one, only if it receives a signal from her. Each firm ignores all signals from workers worse than the corresponding one. If a firm receives no signals, it makes an offer to the best worker worse than the corresponding one.
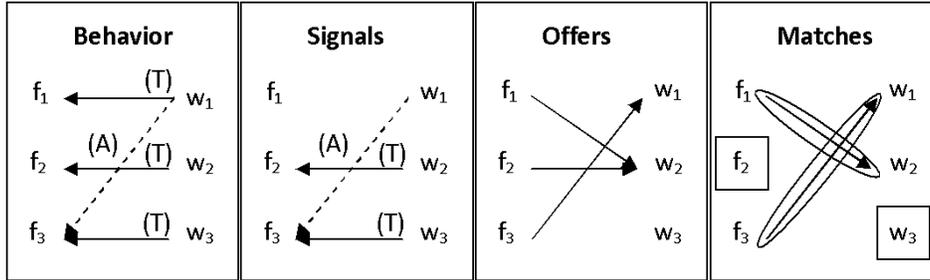


Figure I.

Let us consider the realization of preference profiles when only worker $w_1$ is atypical and firm $f_3$ is her favorite firm. Worker $w_2$ and worker $w_3$ are typical. Figure I illustrates the equilibrium behavior. Worker $w_1$ sends her signal to firm $f_3$. Worker $w_2$ and worker $w_3$ send their signals to firm $f_2$ and firm $f_3$ correspondingly. Firm $f_3$ makes an offer to worker $w_1$, and firm $f_1$ anticipates that worker $w_1$ is atypical and makes an offer to worker $w_2$. Firm $f_2$ also makes its offer to worker $w_2$ and eventually ends up unmatched because worker $w_2$ accepts firm $f_1's$ offer. The coordination failure arises because firm $f_2$ has no information about worker $w_1$'s type and cannot anticipate firm $f_1$'s behavior. Thus, the number of matches for some realization of preferences is smaller than the number of matches when the signals are not allowed. Therefore, the expected number of matches is also smaller.

## 3 Model

We consider a two-sided matching model with $W$ workers and $F$ firms, $W \geq F$. The set of workers and the set of firms are denoted as W and $\mathcal{F}$ correspondingly. Both W and $\mathcal{F}$ include the empty set. Each worker $w$ orders firms according to some strict preference list $\theta_w$. Similarly, each firm $f$ orders workers according to some preference list $\theta_f$. $\Theta_{\mathcal{W}}$ and $\Theta_{\mathcal{F}}$ together comprise the set of all possible workers' and firms' preference lists.

Each agent $a$ has cardinal utility compatible with her/its preference list $\theta_a$.[8] If worker $w$ with preferences $\theta_w$ is matched with firm $f$, she receives cardinal utility $u_w(f, \theta_w)$. Similarly, if firm $f$ with preferences $\theta_f$ is matched with worker $w$, it receives cardinal utility $u_f(w, \theta_f)$. We assume that agent utility depends only on the rank of an agent with which it is matched.

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[8]We employ cardinal utilities compatible with ordinal ranking similar to Bogomolnaia and Moulin (2001).

Specifically, the utility of an agent from being matched with an agent on the *kth* position in her/its preference list equals $u_a(k)$. We assume that agents have the same utility function; i.e. for any agent $a$, $u_a(k) = u(k)$. Our results do not depend on the last assumption; however, this assumption simplifies the exposition.

Additionally, agent's cardinal utility from being unmatched is normalized to zero. We also assume that there is no worker whom firms do not want to hire, and there is no worker who prefers being unemployed to being matched with some firm; i.e. for any $k$, $u(k) > 0$.

Each agent knows only her/its preferences and has some ex-ante common beliefs about the other agents' preferences. We consider an environment where each firm $f$ has some fixed publicly known preference list $\theta_f$. Each worker is one of two types: "typical" or "atypical". A "typical" worker $w$ is denoted as $w(T)$. All workers of typical type have the same commonly known preference list $\theta_0$. An "atypical" worker $w$ is denoted as $w(A)$. The preferences of atypical workers are identically and independently distributed according to some distribution $A(\Theta_{\mathcal{W}})$. Each worker is ex-ante typical with probability $1 - \varepsilon$ and atypical with probability $\varepsilon$, for some $\varepsilon \in (0, 1)$. Our main analysis considers the case when $\varepsilon$ is small.[9] We also assume that the distribution of atypical preferences, $A(\Theta_{\mathcal{W}})$, has a full support, i.e. each firm can be the top firm of an atypical worker with positive probability.[10]

To model the influence of signals on congested markets, we consider a model with three periods:

1. *Agents' preferences are realized. Each worker sends a signal to at most one firm; signals are sent simultaneously. Signals are observed only by firms who have received them.*

2. *Each firm makes an offer to at most one worker; offers are made simultaneously.*[11]

3. *Each worker may accept at most one offer from the set of offers she receives.*

We restrict our analysis to pure strategies.[12] A strategy of worker $w$ is a duple $s_w = (s_w^1, s_w^2)$ that represents her decisions at the first and third stages. A strategy of a worker at the first stage is to choose a firm she sends her signal to, $s_w^1 : \Theta_{\mathcal{W}} \rightarrow \mathcal{F}$. A strategy of a worker on the last stage is to choose an offer among those available to her, $s_w^2 : \Theta_{\mathcal{W}} \times 2^F \rightarrow \mathcal{F}$, where $2^F = \{h : h \subset \mathcal{F}\}$. A strategy of firm $f$ is its decision at the second stage. Firm $f$ chooses the worker to whom it makes an offer based on a set of signals it receives, $s_f : 2^W \rightarrow W$, where

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[9]The exact bound on $\varepsilon$ depends on the parameters of distribution $A(\Theta_{\mathcal{W}})$. However, for each distribution $A(\Theta_{\mathcal{W}})$, one could find an upper bound of $\varepsilon$. We provide a more detailed discussion in Section 8.

[10]Formally, for any $f \in \mathcal{F}$ and any $w \in \mathcal{W}$ $\Pr(f = \max_{\theta_w}(f' : f' \in \mathcal{F})) > 0$.

[11]In practice, some firms should rationally make several offers, anticipating that some workers probably reject their offers. We do not model these strategic decisions.

[12]The analysis of the offer game in which agents can use mixed strategies does not give additional intuition to our main result that signals could impede match formation for some environments. However, this analysis is available upon request.

$2^W = \{h : h \subset \mathcal{W}\}$. The dependence of firm strategy on preferences is omitted, because we assume that each firm has some fixed preferences.

For a given strategy profile of agents $s = (s_w, s_f)$ and realized agents' types $\theta \in (\Theta_{\mathcal{W}})^W \times (\Theta_{\mathcal{F}})^F$ one can determine the final matching and agents' utilities. We denote the utility of agent $a$ given a strategy profile $s$ and a profile of types $\theta$ as $\pi_a(s, \theta)$. The interim expected payoff of worker $w$ with preferences $\theta_w$ from strategy $s_w$ when the other agents follow a strategy profile $s_{-w}$ equals

$$u_w(s_w | s_{-w}, \theta_w) = \sum_{\theta_{-w}} t(\theta_{-w}) \pi_w((s_w, s_{-w}), (\theta_w, \theta_{-w})),$$

where $t(\theta_{-w})$ denotes the joint distribution of all agents except worker $w$ preferences. The interim expected payoff of firm $f$ given a subset of received signals $h \subset \mathcal{W}$, beliefs $\mu_f(\cdot | h)$, and other agents' strategy profile $s_{-f}$ is

$$u_f(s_f | s_{-f}, h) = \sum_{\theta} \mu_f(\theta | h) \pi_f(s_f, s_{-f}, \theta).$$

We employ the concept of sequential equilibrium for multi-stage games with observed actions and incomplete information in order to solve the game (see Fudenberg and Tirole, 1991).

**Definition 1** *A strategy profile $(s_w, s_f)$ and posterior beliefs $\mu_f(\cdot|h)$ for each firm $f$ and each subset of workers $h \subset \mathcal{W}$ is a sequential equilibrium if*

- *for any $w \in \mathcal{W}$, $\theta_w \in \Theta_{\mathcal{W}} : s_w^1(\theta_w) \in \arg\max_{\alpha \in \mathcal{F}} u_w(\alpha|s_{-w}, \theta_w)$,*

- *for any $f \in \mathcal{F}$, $h \subset \mathcal{W} : s_f(h) \in \arg\max_{\beta \in \mathcal{W}} u_f(\beta|s_{-f}, h)$, and*

- *for any $w \in \mathcal{W}$, $\theta_w \in \Theta_{\mathcal{W}}$, $h' \subset \mathcal{F} : s_w^2(h', \theta_w) \in \arg\max_{\gamma \in h'} u_w(\gamma, \theta_w)$,*

*where beliefs are defined using Bayes' rule.*[13]

Now we introduce some notations that will be useful in our further discussion. Though worker strategy is a duple $s_w = (s_w^1, s_w^2)$, we will talk mainly about worker strategies at the first stage. The reason is that each worker has a strictly dominant strategy at the last stage—accept the best offer available—since she knows her preferences and the preferences are strict. To simplify notation, we omit the upper index and write $s_w(\theta_w)$ instead of $s_w^1(\theta_w)$.

For convenience, we name firms according to the typical preference list $\theta_0 = (f_1, ..., f_F)$; i.e. $f_1$ is the best firm, $f_2$ is the second best, etc. Similarly, we name workers in the following way: worker $w_1$ is the best worker among all workers $\mathcal{W}$ according to firm $f_1$'s preferences, $w_1 = \max_{\theta_{f_1}}(w|w \in \mathcal{W})$; worker $w_2$ is the best worker among $\mathcal{W}\backslash\{w_1\}$ according to firm $f_2$'s preferences, $w_2 = \max_{\theta_{f_2}}(w|w \in \mathcal{W}\backslash\{w_1\})$; and so on. Generally, worker $w_i = \max_{\theta_{f_i}}(w|w \in \mathcal{W}\backslash\{w_1, ..., w_{i-1}\})$ if $i \leq F$. The other workers $\mathcal{W}\backslash\{w_1, ..., w_F\}$ are named according to some prespecified order.[14]

We say a subset of workers $h \subset \mathcal{W}$ is *reached for firm $f$ when workers follow strategy profile $s_{\mathcal{W}}$* if ex-ante probability that only workers from set $h$ send their signals to firm $f$ strictly more than zero.

**Definition 2** *A subset of workers $h \subset \mathcal{W}$ is reached for firm $f$ when workers follow strategy profile $s_{\mathcal{W}}$ if*

$$\Pr(h_f = h) = \sum_\theta t(\theta) \prod_{w \in h} I_{s_w(\theta_w)=f} \prod_{w' \notin h} (1 - I_{s_{w'}(\theta_{w'})=f}) > 0,$$

*where $I_{s_w(\theta_w)=f} = \begin{cases} 1 \text{ if } s_w(\theta_w) = f \\ 0 \text{ otherwise} \end{cases}$ and $t(\theta)$ denotes the joint distribution of all agents' preferences.*

We also say that firm $f$ *responds to worker $w$'s signal*, when workers follow strategy profile $s_{\mathcal{W}}$, if her signal changes the strategy of firm $f$ with positive probability.

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[13]Off-equilibrium beliefs are defined by considering the limits of completely mixed strategies.

[14]For instance, if all firms have the same preferences $\theta^*$, workers are named according to this preference list $\theta^* = \{w_1, ..., w_W\}$.

**Definition 3** *Firm $f$ responds to worker $w$'s signal, when workers follow strategy profile $s_\mathcal{W}$, if there exists a subset of workers $h$, $w \notin h$, such that both $h$ and $h \cup w$ are reached for firm $f$, and $s_f(h) \neq s_f(h \cup w)$.*

We proceed with equilibrium analysis in the next section.

# 4    Equilibrium analysis

As a benchmark, we first consider an environment in which workers cannot send signals. Then, the model outlined above is a static game of incomplete information. Therefore, the notion of sequential equilibrium coincides with the notion of Bayesian equilibrium and agents' beliefs are irrelevant. There is a unique equilibrium match in this case.

If signals are not allowed and $\varepsilon$ is small, the only optimal strategy of firm $f_1$ is to make an offer to its best worker $w_1 = \max_{\theta_{f_1}}(w|w \in \mathcal{W})$. The second top firm anticipates that worker $w_1$ is likely to accept firm $f_1$'s offer. Hence, the only optimal strategy of firm $f_2$ is to make an offer to its best worker among $\mathcal{W} \backslash \{w_1\}$, $w_2 = \max_{\theta_{f_2}}(w|w \in \mathcal{W} \backslash \{w_1\})$ and so on. Workers accept the best available offer. Overall, there is the maximum number of matches, $F$ (since $F \leq W$), in the equilibrium when signals are not allowed.

**Proposition 1 (No signaling equilibrium)** *For sufficiently small $\varepsilon$, there is a unique equilibrium when signals are not allowed: firm $f_j$, $j = 1, ..., F$, makes an offer to worker $w_j$; worker $w_i, i = 1, ..., F$, accepts the best available offer.*

We further call the match in our benchmark model as "no signaling" match.

Now, we analyze the set of equilibria in the matching market with signals. Though signals are voluntary in our model, they could still play a negative role and draw away firm offers. In order to eliminate such equilibria, we assume that if firm $f$ makes an offer to worker $w$ when it does not receive her signal, firm $f$ makes an offer to worker $w$ when it receives her signal.[15]

**Assumption PRS (Positive Role of Signals).** *For any firm $f \in \mathcal{F}$ and any worker $w \in \mathcal{W}$ and any $h \subset \mathcal{W}$, $w \notin h$, if $s_f(h) = w$ then $s_f(h \cup w) = w$.*

We further distinguish three types of equilibria in the matching model with signals.

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[15]See Example A1 in Appendix for an example of an equilibrium in which Assumption PRS is violated.

**Definition 4**

- *An equilibrium is "babbling" if no firm responds to any signal.*

- *An equilibrium is "informative", if at least one firm responds to some worker's signal.*

- *An equilibrium is "very informative", if each firm responds to all signals from workers better or equal to its no signaling match.*

The set of the first and second type equilibria, i.e. babbling and informative, exhaust the set of all possible equilibria in our model. The set of equilibria of the last type is a subset of the set of informative equilibria.

A babbling equilibrium always exists in our model because signals are costless. If firms do not respond to signals, signals play no role in equilibria. Hence, the only possible match in a babbling equilibrium is no signaling match.

**Proposition 2** *For sufficiently small $\varepsilon$, the only possible match in a babbling equilibrium is no signaling match.*

If some firms respond to signals, then signals transmit information about workers' preferences in an equilibrium, which changes the overall matching outcome. However, there is a great multiplicity of informative equilibria. One may suggest to use refinements proposed by (Cho and Kreps, 1987) and (Banks and Sobel, 1987).[16] However, these criteria are very powerful in the case of one sender and one receiver. The situation with many senders and receivers is more difficult. Though these criteria significantly reduce the number of equilibria, they do not guarantee uniqueness.

However, it is sufficient to restrict ourselves to the case in which each firm responds to all signals from workers better or equal to its no signaling match, i.e. very informative equilibria, in order to guarantee uniqueness. This equilibrium consists of the following strategies. Worker $w_i$ sends her signal to the best firm among the firms that prefer worker $w_i$ to their no signaling match $\Delta(w_i) = (f_j \in \mathcal{F} : w_i \succeq_{f_j} w_j)$. If firm $f_j$ receives at least one signal from the set of workers $\Delta(f_j) = (w \in \mathcal{W} : w \succeq_{f_j} w_j)$, i.e. workers better or equal to worker $w_j$, it makes its offer to the best such worker; otherwise, it makes an offer to its best worker among $\mathcal{W} \backslash \{w_1, ..., w_j\}$.

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[16] Cho and Kreps (1987) analyze *never a weak responce, intuitive criterion, D1,* and *D2* refinements. Banks and Sobel (1987) analyze *divinity* and *universal divinity* refinements.

**Theorem 1** *For a sufficiently small $\varepsilon$, under Assumption PRS the set of strategies,*

- $s_{w_i}(\theta_{w_i}) = \max_{\theta_{w_i}}(f \in \Delta(w_i))$,

- $s_{f_j}(h) = \begin{cases} \max_{\theta_{f_j}}(w : w \in h) & \text{if } h \cap \Delta(f_j) \neq \varnothing \\ \max_{\theta_{f_j}}(w : w \in \mathcal{W}\backslash\{w_1, ..., w_j\}) & \text{if } h \cap \Delta(f_j) = \varnothing \end{cases}$,

*and the set of firms' beliefs consistent with agents' strategies constitute a unique very informative equilibrium.*[17]

The above theorem is remarkable because it shows that the equilibrium of the model is unique, if we restrict our attention to the case in which firms use signals most extensively. However, we should point out that we do not intend to eliminate all other equilibria. First, the theorem illustrates typical agents' behavior in an informative equilibrium. Workers do not just send signals to the best firms. They send their signals to the best firms that respond to these signals, which is in line with AEA advice to participants in the job market for new Ph.D. economists (see AEA, 2005). Similarly, firms do not respond to all signals. Instead they respond to the signals from workers better than those they could secure in the no signaling equilibrium. Second, our results of welfare comparison do hold for most other sequential equilibria.

# 5   Welfare properties of equilibria

We evaluate the effect of signals on the matching market from an ex-ante perspective. We mainly use the following quantitative characteristics: the expected number of matches, the expected total welfare of firms, and the expected total welfare of workers.

Let us denote the ex-post number of matches for the profile of preferences $\theta \in \Theta_{\mathcal{W}} \times \Theta_{\mathcal{F}}$, when agents follow the profile of strategies $s$ as $m(s, \theta)$. Then, the expected number of matches can be expressed as

$$E[M(s)] = \sum_{\theta} t(\theta) m(s(\theta), \theta),$$

where $t(\theta)$ denotes the joint distribution of all agents' preferences. Similarly, the expected total welfare of workers and firms can be expressed as

$$\begin{aligned} E[W_{\text{firm}}(s)] &= \sum_f \sum_\theta t(\theta) \pi_f(s(\theta), \theta), \text{ and} \\ E[W_{\text{worker}}(s)] &= \sum_w \sum_\theta t(\theta) \pi_w(s(\theta), \theta) \end{aligned}$$

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[17]We should point out that there is a multiplicity of beliefs that could support this equilibrium on off-equilibrium path.

correspondingly.

Proposition 1 shows that the expected number of matches in any no signaling equilibrium is the maximum one. Hence, it is impossible that the expected number of matches in any informative equilibrium exceeds the expected number of matches in any "no signaling" equilibrium. Example 1 and Example 2 demonstrate the case of strict inequality and equality for this welfare criterion.

Example 1 is presented in the introduction and considers the very informative equilibrium with three firms and three workers. To avoid a repetition, we do not discuss it here.

**Example 1 (Fewer expected number of matches)** *There are three firms and three workers .Firms have the same ranking over workers $(w_1, w_2, w_3)$. The typical worker preference list is $\theta_0 = (f_1, f_2, f_3)$. Atypical worker preferences are uniformly distributed. Firm $f_j$, $j = 1, 2, 3$, and worker $w_i$, $i = 1, 2, 3$, equilibrium strategies are*

- $s_{w_i}(\theta_{w_i}) = \max_{\theta_{w_i}}(f \in \Delta(w_i))$,

- $s_{f_j}(h) = \begin{cases} \max_{\theta_{f_j}}(w : w \in h) & \text{if } h \cap \Delta(f_j) \neq \varnothing \\ \max_{\theta_{f_j}}(w : w \in \mathcal{W}\backslash\{w_1, ..., w_j\}) & \text{if } h \cap \Delta(f_j) = \varnothing \end{cases}$ ,

*and the set of firms' beliefs consistent with agents' strategies.*

Example 2 shows that some informative equilibria could have the maximum expected number of matches. Intuitively, it is possible that if some firm $f_j$ secures a better match with some atypical worker $w_i$, firm $f_i$ always makes its offer to firm $f_j$'s no signaling match, worker $w_j$, in an equilibrium. Therefore, firms exchange their matches and it does not decrease the number of matches.

**Example 2 (Equal expected number of matches)** *Let us consider three firms and three workers. All firms have the same preferences $\theta_{f_j} = \{w_1, w_2, w_3\}$. Let us consider the following equilibrium strategies:*

- $s_{w_1}(\theta_{w_1}) = \max_{\theta_{w_1}}(f : f \in \{f_1, f_2\})$ *and* $s_{w_i}(\theta_{w_i}) = f_i$, $i = 2, 3$;

- $s_{f_j}(h) = \begin{cases} \max_{\theta_{f_j}}(w : w \in h) & \text{if } h \cap \Delta(f_j) \neq \varnothing \\ \max_{\theta_{f_j}}(w : w \in \mathcal{W}\backslash\{w_1, ..., w_j\}) & \text{otherwise} \end{cases}$ , *for* $j = 1, 2$;

- $s_{f_3}(h) = \begin{cases} \max_{\theta_{f_3}}(w : w \in h) & \text{if } h \cap \Delta(f_3) \neq \varnothing \\ w_3 & \text{otherwise} \end{cases}$ .

*The set of equilibrium beliefs is such that if firm $f_1$ or $f_2$ receives a signal from worker $w_1$, it believes that it is worker $w_1$'s top firm. If firm $f_3$ receives a signal from worker $w_1$, its belief coincides with her prior, i.e. worker $w_1$ is typical with probability $1 - \varepsilon$ and atypical with probability $\varepsilon$. Similarly, if any firm $f_j$ receives a signal from worker $w_2$ or $w_3$, its belief coincides with its prior. To put it briefly, only firm $f_1$ and firm $f_2$ respond to worker $w_1$'s signal. All other signals are ignored. One may check that the described strategies constitute an informative equilibrium.*

The results about the expected total welfare of firms and the expected total welfare of workers are not so straightforward and depend on the relative magnitudes of $u(k)$. The intuition is that signals in an informative equilibrium play two roles. On the one hand, signals help to secure "better" matches between some atypical workers and firms. On the other hand, the introduction of signals leaves some workers and firms unmatched. Example 3 illustrates that the introduction of signals is beneficial for a matching market according to egalitarian welfare criterion if and only if the decrease in the number of matches is offset by better matches of atypical workers. A similar example can show that the total welfare of firms changes ambiguously.

**Example 3 (Welfare of firms and workers)** *Let us again consider the game of Example 1. Workers' cardinal utilities from being matched to first, second, and third choice are $\delta + \lambda$, $\delta$, and $\delta - \lambda$ correspondingly. The expected total welfare of workers in no signaling match*

$$E[W_{worker}^{nosignals}] = \sum_{i=1}^{3} \left[ (1 - \varepsilon)\, u(i) + \varepsilon \tfrac{1}{3} \sum_{l=1}^{3} u(l) \right] = 3\delta.$$

*One may check that the expected total welfare of workers in very informative equilibrium is[18]*

$$E[W_{worker}^{signals}] = 3\delta + \left( -\tfrac{1}{3}\delta + \tfrac{19}{6}\lambda \right) \varepsilon$$

*Hence, the expected total welfare of workers increases, if and only if the difference in utilities between adjacent firms is large enough, $\lambda > \frac{2}{19}\delta$.*

The theorem below summarizes the results derived above.

**Theorem 2** *For a sufficiently small $\varepsilon$:*

- *the expected number of matches in any informative equilibrium is weakly fewer than in any no signaling equilibrium;*

- *the effect of signals on the expected total welfare of firms and the expected total welfare of workers is ambiguous.*

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[18]Terms of the order of $\varepsilon^2$ and $\varepsilon^3$ are ignored.

We have compared above the properties of any informative and no signaling equilibrium. However, more strict result holds for very informative equilibrium. Under the assumption that there are at least three workers and there exists a worker $w$, such that there are at least three firms that weakly prefer worker $w$ to their no signaling matches, $|\{f_j \in \mathcal{F} : w \succeq_{f_j} w_j\}| \geq 3$, the expected total number of matches is strictly fewer in very informative equilibrium than in the corresponding no signaling match. The intuition for this result is similar to the one for Example 1. If firms respond to signals, some of the realized matches differ from no signaling match. Moreover, if at least three firms respond to some worker's signal the exchange of matches−the situation presented in Example 2−is impossible for each realization of preferences. Therefore, the expected number of matches is smaller than the maximum one in this case.

**Theorem 3** *For sufficiently small $\varepsilon$, if there are at least three workers and for some worker $w$, $|\Delta(w)| \geq 3$, the expected number of matches is strictly smaller in the very informative equilibrium than in the corresponding no signaling equilibrium.*

Theorem 2 proves that the expected total welfare of workers changes ambiguously with the introduction of signals. However, the following proposition shows that signals are harmful to workers only because they deprive them of matches. Workers receive weakly better offers conditional on the fact that they receive any offer.

**Proposition 3** *If a worker receives an offer in any informative equilibrium, this offer is from a firm weakly better than her no signaling match.*

It is easy to see that the above statement is not true for firms, because some firms may have to make offers to workers worse than their no signaling match if she is atypical.

# 6  Role of signals in matching markets

Coles et al. (2009) show that the introduction of signals increases the expected number of matches and the welfare of workers. However, they consider an environment where agents' preferences are block-uniform. Specifically, there exists a partition $\mathcal{F}_1, \ldots, \mathcal{F}_B$ of the firms into *blocks* and

1. For any $b < b'$, where $b, b' \in \{1, \ldots, B\}$, each worker prefers every firm in block $\mathcal{F}_b$ to any firm in block $\mathcal{F}_{b'}$;

2. Each worker's preferences within block $\mathcal{F}_b$ are uniform and uncorrelated, for all $b$;

3. Firm preferences over workers are uniform and uncorrelated.

This paper shows that Coles et al. (2009) results rely on the assumption that preferences are block-uniform. If the preferences of workers are almost aligned and the preferences of firms are fixed and commonly known, the introduction of signals decreases the expected number of matches. The effect of signals on the expected total welfare of agents is ambiguous. Overall, Table I presents the effects from the introduction of the signals for the two different environments: almost complete (this paper) and block-uniform distribution of preferences.

| Preferences | No signals | $Matches$ | $E[W_{\text{worker}}]$ | $E[W_{\text{firm}}]$ |
|---|---|---|---|---|
| Almost complete | 0 | $-$ | $\pm$ | $\pm$ |
| Block-uniform | 0 | $+$ | $+$ | $\pm$ |

Table I. Almost complete VS Block-uniform distribution preferences.

A natural question is why signals influence matching markets in different ways. We argue that the signals play two different roles: transmit information and facilitate information asymmetry. On the one hand, the introduction of signals helps atypical workers to transmit information about their preferences and locate a better match. On the other hand, signals transmit information only to some firms, thus facilitating information asymmetry. This information asymmetry leads to coordination failures that decrease the number of matches.

When there is ex-ante small amount of information about agents' preferences, information transmission plays a more important role in match formation. This happens when agents' preferences are ex-ante block-uniform, as in Coles et al. (2009). However, when there is almost complete information about agents' preferences—as in the model of this paper—the introduction of signals leads to coordination failures. Table II presents the comparison.

| Preferences | Transmit information | Facilitate information asymmetry |
|---|---|---|
| Almost complete | Small | **Large** |
| Block-uniform | **Large** | Small |

Table II. The roles of signals

Overall, the signals play controversial roles in the match formation process. This could make them a less powerful tool than it was previously anticipated.

# 7  Public signals

One could conjecture that should signals be public, they would always benefit match forma-
tion. Public signals introduce no asymmetry of information among firms. Firms have the
same beliefs about the distribution of workers' preferences and the same beliefs about the
strategies other firms use. Therefore, firms should be able to make offers that are more likely
to be accepted. Unfortunately, this intuition is incorrect. This section illustrates that the
expected number of matches in an equilibrium of the offer game with public signals could
be smaller than the expected number of matches in the offer game without signals.

We consider a market with three firms and three workers. The distribution of agents'
preferences is the same as in Section 3. Each worker can send at most one signal and accept
at most one offer. Each firm has only one vacant position and can make at most one offer.
The timing of the game is as follows:

1. Agents' preferences are realized. Each worker sends a signal to at most one firm; signals
   are sent simultaneously. All agents observe what signals each firm receives.

2. Each firm makes an offer to at most one worker; offers are made simultaneously.

3. Each worker chooses an offer to accept from the set of offers she receives.

The only difference from the game we considered previously is that all agents observe the
signals each firm receives. The strategies of workers are the same as in Section 3. However,
a strategy of firm $f$ now depends on the set of signals each firm receives, $s_f : \mathcal{F}^W \to \mathcal{W}$.[19]

As previously, the only equilibrium outcome of the offer game with signals is a full
match. However, the expected number of matches could be smaller than three if we allow
workers to send public signals. Intuitively, public signals do not transmit enough information
about workers' preferences. This could introduce a considerable amount of uncertainty about
workers' preferences. Therefore, some firms can optimally engage in a competitive behavior
for some workers; i.e. firms make their offers to the same worker in an equilibrium. This
produces mismatches.

**Example 4** *There are three firms and three workers. Firms have the same ranking over
workers, which we denote as $(w_1, w_2, w_3)$. The typical worker preference list is $(f_1, f_2, f_3)$.
The atypical worker preferences are uniformly distributed. We assume that all firms have the
same cardinal utility and their utility from being matched to second top worker, i.e. $u(2)$, is
at least twice as great as the cardinal utility from being matched to the third top worker, i.e.
$u(3)$.*

---

[19]Note that we again omit the dependence of strategies on firms' preferences, as we assume that each firm
has fixed commonly known preferences.

*We consider the following strategies of agents in the offer game with public signals. Worker $w_i$ sends her signal to the best firm among the firms that weakly prefer worker $w_i$ to their no signaling match $\Delta(w_i) = (f_j \in \mathcal{F} : w_i \succeq_{f_j} w_j)$:*

$$s_{w_i}(\theta_{w_i}) = \max_{\theta_{w_i}}(f \in \Delta(w_i)).$$

*Firms use the following strategies.*

1. *Firm $f_1$ makes an offer to worker $w_1$, if it receives a signal from her; otherwise, it makes an offer to worker $w_2$.*

2. *Firm $f_2$ makes an offer to worker $w_1$, if it receives a signal from her. Firm $f_2$ makes an offer to worker $w_3$, if either worker $w_1$ sends a signal to firm $f_1$ and worker $w_2$ sends a signal to firm $f_3$ or worker $w_1$ sends a signal to firm $f_3$ and worker $w_2$ sends a signal to firm $f_2$. In all other cases, firm $f_2$ makes an offer to worker $w_2$.*

3. *Firm $f_3$ makes an offer to the best worker from whom it receives a signal. If it receives no signals, it makes an offer to worker $w_3$.*

*Each firm's beliefs on the equilibrium path are consistent with agents' strategies and each firm off-equilibrium beliefs coincide with priors.*

Let us consider the strategies outlined in Example 4. Mismatches happen when both worker $w_1$ and worker $w_2$ are atypical. If at least two atypical workers send their signals to the same firm, only one worker receives an offer from it. Since, signals are public, all other firms infer that the other worker is atypical. This creates a considerable amount of uncertainty about the worker preferences. Since, this worker could be a good one, firms have incentives to compete for her.

Another reason for excessive competition among firms is that signals may not transmit information about workers' top firms. Some workers send their signals to firms that differ from their top ones in an equilibrium, because they want to attract feasible offers. Therefore, several firms could have incentives to make an offer to a given worker. This creates competition among firms, which again lead to mismatches.

Proposition 4 formally proves that the set of strategies in Example 4 constitutes a sequential equilibrium.

**Proposition 4** *The set of strategies in Example 4 constitutes a sequential equilibrium.*
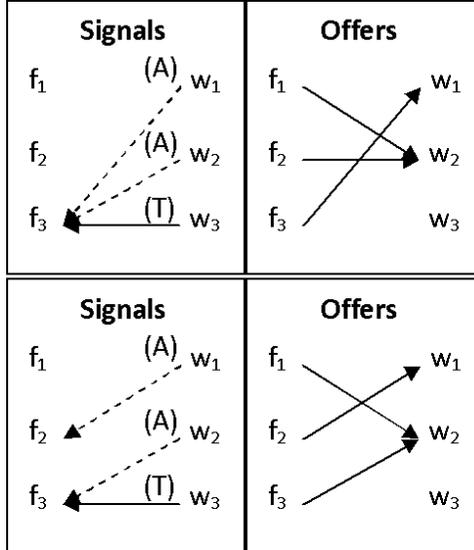
Figure 1: Figure II. Public signals.

The implications of the above example can be summarized by way of two observations. First, public signals do not transmit enough information about workers' preferences. This could introduce uncertainty about workers preferences and induce excessive firm competition for the same workers. This results in mismatches.

In addition, mismatches in the offer game with public signals occur only if there at least two atypical workers, which happens only with probability of the order $\varepsilon^2$. In contrast, mismatches in the offer game with private signals occur with the probability of the order $\varepsilon$. Therefore, mismatches happen less often when signals are public.

# 8 Conclusion

There is a general belief that preference signaling should facilitate match formation (see Crawford and Sobel, 1982; Roth, 2008; AEA, 2005). This belief is also supported by Coles et al. (2009) who show that the introduction of signals increases the expected number of matches and welfare of workers. We show in this paper that this belief can be erroneous for some matching markets. We exemplify an environment in which the introduction of signals harms matching markets: it decreases the expected number of matches and ambiguously affects the welfare of firms and the welfare of workers. Based on this example, we argue that signals play controversial roles in match formation. Though they help to transmit information about participants' preferences, they also facilitate information asymmetry among them. While the former effect reduces coordination failures and facilitates better match formation, the latter effect acts in the opposite direction. Finally, we show that making signals

observable to all agents does not change the welfare applications. We leave here as an open empirical question which effect dominates in real matching markets.

In conclusion, we want to discuss some assumptions of our model. We analyze the introduction of signals to congested decentralized matching markets, as we believe the job market for new Ph.D. economists to be. The fact that we do not analyze any centralized clearinghouse mechanism and stable matches captures the idea of decentralized markets. The fact that we analyze a one-period model captures the idea of congestion. Moreover, several (but finite) periods of interactions between firms and workers would give an opportunity for firms to secure better matches; but signals would still introduce information asymmetry. If each worker sent several signals, these would transmit information to a greater number of firms, but each signal would be less informative. Several vacant positions would make only the preferences of firms more complicated and would not influence the results. Overall, the roles of signals in match formation are robust to these modifications.

The results of this paper are in terms of a sufficiently small $\varepsilon$. However, what we really need is the uniqueness of equilibrium in the benchmark model without signals. The multiplicity of equilibria does not allow a clean comparison between models with and without signals. One could check that there is a unique equilibrium in no signaling model with uniform distribution of atypical preferences if $\varepsilon < \min(\min_i(\frac{u(i)-u(i+1)}{u(i)}), \frac{u(F)}{u(F)+0.5u(1)})$.

We should also highlight that we rely on cardinal utility in our analysis. Our results could not be extended to an ordinal framework, because an Ordinal Bayesian Nash Equilibrium (see d'Aspremont and Peleg, 1988) may not exist in our environment.

# A    Appendix

**Proof of Proposition 1.**

Let us show this statement for each firm sequentially. Each worker $w_i$ has preferences $\theta_0 = (f_1, ..., f_F)$ with probability $1 - \varepsilon$ and with complementary probability some preferences distributed according to $A(\Theta_{\mathcal{W}})$. Let us consider firm $f_1$ which has some preferences $\theta_{f_1}$. If it makes an offer to worker $w_1 = max_{\theta_{f_1}}(w|w \in \mathcal{W})$, its offer will be the best worker $w_1$'s offer with probability at least $1 - \varepsilon$. Hence, its expected utility from making an offer to worker $w_1$ equals at least $(1 - \varepsilon)u(1)$ which is greater than $u(2)$ for sufficiently small $\varepsilon$. Hence, independently on other firms' strategies, firm $f_1$'s optimal strategy is to make an offer to its best worker.

Let us assume that each firm $f_k$, $k < j$, makes its offer to worker $w_k$. Now we consider the decision of firm $f_j$. The expected payoff from making an offer to some worker among $\{w_1, ... w_{j-1}\}$ is less than $\varepsilon u(1)$. In the same time the expected payoff from making offer to some worker among $\mathcal{W} \backslash \{w_1, ... w_{j-1}\}$ is at least $(1 - \varepsilon)u(j)$. Hence, given the strategies of other firms and sufficiently small $\varepsilon$, the optimal strategy of firm $f_j$ is to make an offer to its best worker among $\mathcal{W} \backslash \{w_1, ... w_{j-1}\}$.□

**Proof of Proposition 2.**

The only undominated strategy of a worker at the last stage is to choose the best offer among available ones. Then, under the condition that firm $f$ does not respond to any signal, for any $h \subset \mathcal{W}$ reached in an equilibrium $s_f(h) = const$. Let us assume that there exists a realization of agents' preferences such that firm $f_1$ is matched to some worker $w_i$, $i > 1$, in the equilibrium. Hence, for any $h \subset \mathcal{W}$, reached in the equilibrium, $s_{f_1}(h) = w_i$. Hence, the expected firm 1's payoff equals at most $u(2)$. However, the strategy $s_{f_1}(h) = w_1$ for any $h \subset \mathcal{W}$ is compatible with assumption that firm $f_1$ does not respond to any signals and gives payoff $(1 - \varepsilon)u(1)$ independently of strategies of other firms. Hence, $s_{f_1}(h) = w_i$ cannot be an equilibrium strategy. Similar argument could be applied to any other firm $f_j$, $j = 2, ..., F$.□

**Proof of Theorem 1.**

We prove the theorem by way of several lemmata. In the proof of the lemmata we presume that $\varepsilon$ is sufficiently small. First, we establish that a firm believes about a particular worker is typical with probability more than $1 - \varepsilon$ either when it receives her signal or when it does not receive her signal. Second, we show that firms do not make their offers to a worker better than no signaling match if they do not receive her signal. The third lemma proves that if a firm does receive a signal from a worker better than its no signaling match, it makes its offer to the best such worker. Finally, using the statements of lemmata we show that the set of strategies stated in the theorem constitutes a unique very informative equilibrium.

First two lemmata do not require the assumption that each firm $f_j$, $j = 1, ..., F$, responds to all signals from workers better or equal to worker $w_j$ according to its preferences.

**Lemma A1** *For any worker $w \in \mathcal{W}$, any firm $f \in \mathcal{F}$, and any $h \subset \mathcal{W}$ either $\mu_f(\theta_w = \theta_0|h \cup w) \geq 1 - \varepsilon$ or $\mu_f(\theta_w = \theta_0|h \backslash w) \geq 1 - \varepsilon$. Similarly, either $\mu_f(\theta_w \neq \theta_0|h \cup w) \leq \varepsilon$ or $\mu_f(\theta_w \neq \theta_0|h \backslash w) \leq \varepsilon$.*

**Proof.**

Let us denote as $\alpha_T$ and $\alpha_A$ the probabilities that typical and atypical type of worker $w$ correspondingly send a signal to firm $f$. Then, if worker $w$ sends her signal to firm $f$, $(1 - \varepsilon)\alpha_T + \varepsilon\alpha_A > 0$, we derive its beliefs using Bayes' rule

$$\begin{cases} \mu_f(\theta_w = \theta_0|h \cup w) = \frac{(1-\varepsilon)\alpha_T}{(1-\varepsilon)\alpha_T + \varepsilon\alpha_A} \\ \mu_f(\theta_w = \theta_0|h \backslash w) = \frac{(1-\varepsilon)(1-\alpha_T)}{(1-\varepsilon)(1-\alpha_T) + \varepsilon(1-\alpha_A)} \end{cases}$$

One can verify that

$$\begin{cases} \mu_f(\theta_w = \theta_0|h \cup w) \geq 1 - \varepsilon & \Leftrightarrow & \alpha_T \geq \alpha_A \\ \mu_f(\theta_w = \theta_0|h \backslash w) \geq 1 - \varepsilon & \Leftrightarrow & \alpha_T \leq \alpha_A \end{cases}$$

Hence, either $\mu_f(\theta_w = \theta_0|h \cup w) \geq 1 - \varepsilon$ or $\mu_f(\theta_w = \theta_0|h \backslash w) \geq 1 - \varepsilon$. If worker $w$ never sends her signal to firm $f$, $(1 - \varepsilon)\alpha_T + \varepsilon\alpha_A = 0$, firm $f$'s beliefs are $\mu_f(\theta_w = \theta_0|h \backslash w) = 1 - \varepsilon$ and $\mu_f(\theta_w = \theta_0|h \cup w)$ is arbitrary. The second statement directly follows from the first one. $\square$

**Lemma A2 (Offer to better workers)** *If firm $f_j$ does not receive a signal from worker $w$ strictly better than worker $w_j$, $w \succ_{f_j} w_j$ it does not make an offer to her in an equilibrium.*

**Proof.**

We prove this statement for firms sequentially. Let us first show its validity for $j = 2$. The only worker that could be better than worker $w_2$ for firm $f_2$ is worker $w_1$ by construction. If $w_2 \succ_{f_2} w_1$ we are done. Assume that $w_1 \succ_{f_2} w_2$.

There are two possibilities: either worker $w_1(T)$ sends her signal to firm $f_1$, i.e. $s_{w_1}(\theta_0) = f_1$, or she does not send her signal to firm $f_1$, i.e. $s_{w_1}(\theta_0) \neq f_1$, in an equilibrium.

Assume worker $w_1$ employs strategy $s_{w_1}(\theta_0) = f_1$. If firm $f_2$ does not receive worker $w_1$ signal, firm $f_2$ believes she is atypical with probability less than $\varepsilon$, $\mu_{f_2}(\theta_{w_1} \neq \theta_0 | h \backslash w_1) \leq \varepsilon$ (Lemma A1). According to assumption $F \leq W$, firm $f_2$ can secure a match with some worker $w_i$, $i \geq 2$, with probability at least $1 - \varepsilon$. Hence, firm $f_2$ does not make an offer to worker $w_1$ in an equilibrium.

Worker $w_1$ employs strategy $s_{w_1}(\theta_0) \neq f_1$ in an equilibrium only if firm $f_1$ makes its offer to worker $w_1$ with probability equals to one, and firm $f_2$ has a chance to be matched with worker $w_1$ only if she is atypical. Assume firm $f_2$ makes an offer to worker $w_1$ when it does not receive her signal. If $w_1(T)$ sends her signal to firm $f_2$ in an equilibrium, according to Assumption $PRS$ firm $f_2$ should also make an offer if it receives a signal from $w_1$. However, if it receives a signal from $w_1$, the probability that worker $w_1$ is atypical less than $\varepsilon$ (Lemma A1), which contradicts equilibrium behavior.

Now, we assume that worker $w_1(T)$ does not send her signal to firm $f_2$ in an equilibrium. If firm $f_2$ does not receive worker $w_1$'s signal, firm $f_2$ believes that she is atypical with probability less or equal $\varepsilon$, $\mu_{f_2}(\theta_{w_1} \neq \theta_0 | h \backslash w_1) \leq \varepsilon$ (Lemma A1). Therefore, it is again suboptimal for firm $f_2$ to make an offer to worker $w_1$ if it does not receive a signal from her.

We have shown above that it is suboptimal for firm $f_2$ to make an offer to worker $w_1$ if it does not receive a signal from her. Let us assume that it is suboptimal for any firm $f_j$, $j < k$ to make its offer to a worker $w_t$, $t < j$, if firm $f_j$ does not receive a signal from it and show that the claim for firm $f_k$.

We consider some worker $w_i$, $i < k$. Firm $f_i$ makes its offer to workers $\{w_1, ..., w_{i-1}\}$ with probability less than $\varepsilon(i-1)$. In addition, worker $w_i$ is atypical with probability $\varepsilon$. Hence, firm $f_k$ can secure a match with worker $w_i$ with probability equals at most $i\varepsilon$ if it does not receive a signal from her. For small enough $\varepsilon$ firm $f_k$'s offer to worker $w_i$ is suboptimal. $\square$

Now, we assume that each firm $f_j$, $j = 1, ..., F$, responds to all signals from workers better or equal to worker $w_j$ according to its preferences. The following lemma shows that firm $f_j$ makes its offer to some worker $w$ better or equal to worker $w_j$ if worker $w$'s signal is the best signal firm $f_j$ receives.

**Lemma A3 (Response to signals)** *Assume that $F > W$. Then, for any $h \subset \mathcal{W}$ $s_{f_j}(h) = max_{\theta_{f_j}}(w : w \in h)$ if $h \cap \Delta(f_j) \neq \varnothing$ in very informative equilibrium[20].*

**Proof.**

We prove this statement for firms sequentially. Let us consider firm $f_1$ and worker $w_1$. Assume that worker $w_1$ employs strategy $s_{w_1}(\theta_0) \neq f_1$. Then, firm $f_1$ believes that for any $h \subset \mathcal{W}$ $\mu_{f_1}(\theta_{w_1} = \theta_0|h \backslash w_1) \geq 1 - \varepsilon$. Therefore, for sufficiently small $\varepsilon$, firm $f_1$ always makes its offer to worker $w_1$, which contradicts to our assumption that it responds to worker $w_1$'s signal. Therefore, under the assumption that firm $f_1$ responds to a signal from worker $w_1$, the only possible worker $w_1$'s equilibrium strategy is $s_{w_1}(\theta_0) = f_1$. In this case, for any $h \subset \mathcal{W}$ firm $f_1$'s belief is $\mu_{f_1}(\theta_{w_1} = \theta_0|h \cup w_1) \geq 1 - \varepsilon$. Hence, firm $f_1$'s highest expected payoff when it receives worker $w_1$'s signal is from making an offer to worker $w_1$. Hence, for any $h \subset \mathcal{W}$, firm $f_1$'s strategy $s_{f_1}(h \cup w_1) = w_1$ is optimal.

Assume now that for any $t \leq j < k$, and for any $h \subset \mathcal{W}$, firm $f_j$ employs strategy for $s_{f_j}(h) = max_{\theta_{f_j}}(w : w \in h)$ if $h \cap \Delta(f_j) \neq \varnothing$. We prove below that firm $f_k$'s optimal strategy for any $h \subset \mathcal{W}$ and $s_{f_k}(h) = max(w : w \in h)$ if $h \cap \Delta(f_k) \neq \varnothing$.

There are two possibilities: either $s_{w_k}(\theta_0) \neq f_k$ or $s_{w_k}(\theta_0) = f_k$. For the former case, for any $h \subset \mathcal{W}$ $\mu_{f_k}(\theta_{w_k} = \theta_0|h \backslash w_k) \geq 1 - \varepsilon$. Hence, it is optimal for firm $f_k$ to make an offer to worker $w_k$ when it receives no signals from workers better or equal to worker $w_k$, i.e. for any $h' \subset \mathcal{W}$ such that $h' \cap \Delta(f_k) = \varnothing$, $s_{f_k}(h') = w_k$. Hence, it is also optimal for firm $f_k$ to make an offer to worker $w_k$ when worker $w_k$'s signal is the best signal it receives, i.e. for any $h'' \subset \mathcal{W}$ such that such that $h'' \cap \Delta(f_k) = w_k$, $s_{f_k}(h'') = w_k$. Therefore, firm $f_k$ does not respond to worker $w_k$'s signal. Contradiction.

For the latter case, $s_{w_k}(\theta_0) = f_k$, if firm $f_k$ does not receive a signal from worker $w_k$, it anticipates that she is atypical. Therefore, firm $f_k$ does not make its offer to her. If firm $f_j$ receives signals from any worker $w_i \succeq w_k$ no other firm $f_p$, $p \neq j$ and $p > i$, makes its offer to worker $w_i$ according to Lemma A2. The only offers that compete with firm $f_j$'s offer could be the ones from the set $\{f_p, p < i\}$. However, any firm $f_p$, $p < i$, could make an offer to worker $w_i$ only if worker $w_p$ is atypical, which happens with probability $\varepsilon$. Hence, the interim expected payoff for firm $f_j$ from making its offer to worker $w_i$ equals at least $(1 - (i - 1)\varepsilon)u'$, where $u' = u_{f_j}(w_i, \theta_{f_j})$. Firm $f_j$ expected payoff from making an offer to any other worker from set $\Delta(f_j)$ is smaller than $(1 - (i - 1)\varepsilon)u'$ as this worker either has not sent a signal to firm $f_j$ or has a smaller rank in firm $f_j$' preferences. The expected payoff from making an offer to some worker $\mathcal{W} \backslash \Delta(f_j)$ is smaller either. Therefore, firm $f_j$ optimal strategy is, $s_{f_j}(h) = max_{\theta_{f_j}}(w : w \in h)$ if $h \cap \Delta(f_j) \neq \varnothing$. $\square$

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[20]If $F = W$ the claim is still valid with the same assumption for all firms except firm $f_F$. Firm $f_F$ should respond to a signal from any worker strictly better than the corresponding one.

Now we are ready to prove the theorem. Let us show that the set of strategies, stated in the theorem, constitutes an equilibrium. We first prove that if all agents, except firm $f_l$, follow the strategies, stated in the theorem, firm $f_l$'s strategy is optimal given its belief is consistent with the other agents' strategies. If firm $f_l$ receives a signal from worker $w_t$, $t < l$, firm $f_l$ believes that itself is the best firm among $\Delta(w_t) = \{f_j \in \mathcal{F} : w_t \succeq_{f_j} w_j\}$. Let us assume that worker $w_t$ is the best worker who sends a signal to firm $f_l$. Worker $w_t$ does not accept firm $f_l$'s offer only if she receives an offer from some firm $f_k \in \mathcal{W} \backslash \Delta(w_t)$. However, it happens only if worker $w_k$ is atypical, i.e. with probability less than $\varepsilon$. Hence, firm $f_l$ interim expected payoff from making an offer to worker $w_t$ equals at least $(1 - (n - 1)\varepsilon)u'$, where $u' = u_{f_l}(w_t, \theta_{f_l})$. Firm $f_l$'s offer to a worker better than worker $w_t$ is not optimal according to Lemma A1. Firm $f_l$'s expected payoff from making an offer to some worker $w$, $w_t \succ_{f_l} w$, is also smaller than making an offer to worker $w_t$ for sufficiently small $\varepsilon$. Overall, firm $f_l$'s strategy is optimal.

Let us show that, if all agents, except worker $w_t$, follow the strategies, stated in the theorem, worker $w_t$'s strategy is optimal. Firm $f_t$ does not make an offer to worker $w_t$ when it receives a signal from a better worker. Therefore, if worker $w_t$ is typical, her payoff from sending a signal to firm $f_t$ equals at least $[1 - (l - 1)\varepsilon]u(t)$. If worker $w_t$ does not send her signal to firm $f_t$ it loses her offer and she could get payoff at most $u(t - 1)$, which is less than $[1 - (l - 1)\varepsilon]u(t)$ for sufficiently small $\varepsilon$. There is also no reason for worker $w_t$ to send her signal to a firm better than firm $f_t$, because this firm does not respond to her signal according to its equilibrium strategies. Hence, worker $w_t(T)$'s strategy is optimal. Using similar logic one can show that worker $w_t(A)$'s strategy is also optimal.

Now we show that the above strategies constitute the unique very informative equilibrium. Lemmata A2 and A3 imply that each firm $f_l$, $l = 1, ..., F$, has to follow the following strategies in an equilibrium:

$$\text{for any } h \subset \mathcal{W}, \begin{cases} s_{f_l}(h) \neq w_l \text{ if } h \cap \Delta(f_l) = \varnothing \\ s_{f_l}(h) = w_l \text{ if } h \cap \Delta(f_l) = w_l \end{cases}$$

Straightforwardly, the only worker $w_l(T)$'s optimal strategy is to send her signals to firm $f_l$, $s_{w_l}(\theta_0) = f_l$, otherwise, firm $f_l$'s does not make an offer to student $w_l$.

Let us consider firm $f^* = \max_{\theta_{w_l}} (f' \in \Delta(w_l))$. Firm $f^*$ responds to signals from workers better or equal than no signaling match and its equilibrium beliefs are $\mu_{f^*}(\theta_{w_l} = \theta_0 | h \backslash w_l) \geq 1 - \varepsilon$ and $\mu_{f^*}(\theta_{w_l} \neq \theta_0 | h \cup w_l) = 1$. Therefore, if firm $f^*$ does not receive a signal better than worker $w_l$'s one, it's optimal strategy is to make an offer to worker $w_l$. Taking into account that firm $f^*$ can receive a signal from a better worker with probability less than $(l - 1)\varepsilon$, worker $w_l(A)$'s optimal strategy is to send her signal to firm $f^*$ (for sufficiently small $\varepsilon$). Hence, the strategies, stated in the theorem, constitute the unique equilibrium. $\square$

**Proof of Theorem 3.**

Assumption that $A(\Theta_\mathcal{W})$ has a full support and that the strategies of the very informative equilibrium guarantee that some worker $w_i$ sends her signals to each firm in the set $\Delta(w_i) = |\{f_j \in \mathcal{F} : w_i \succeq_{f_j} w_j\}|$ with positive probability. Then, using logic of Example 1 and Example 2 it is straightforward to show that when there are at least three firms in the set $\Delta(w_i)$ and there are at least three workers the mismatch happens with positive probability. Therefore, the expected number of matches strictly smaller than in the corresponding no signaling equilibrium. $\square$

**Proof of Proposition 3.** The statement directly follows from the strategies of the very informative equilibrium. $\square$

**Example A1 (An equilibrium when assumption PRS does not hold)** *Let us consider two firms and two workers. We assume that all firms have the same preferences over workers $\theta_{f_1} = \theta_{f_2} = \{w_1, w_2\}$. Also we assume that each typical worker has preferences $\theta_0 = (f_1, f_2)$ and each atypical worker has preferences $\theta_A = (f_2, f_1)$ with probability equal to one. Firms prefer worker $w_1$ to worker $w_2$. Agents employ the following strategies:*

- *$s_{w_1}(\theta_0) = f_2$, $s_{w_1}(\theta_A) = f_1$*

- *$s_{w_2}(\theta_0) = f_1$, $s_{w_2}(\theta_A) = f_2$*

- *for any $h \subset \mathcal{W}$ $s_{f_1}(h) = \begin{cases} w_1 \text{ if } w_1 \notin h \\ w_2 \text{ if } w_1 \in h \end{cases}$ , $s_{f_2}(h) = \begin{cases} w_1 \text{ if } w_1 \notin h \\ w_2 \text{ if } w_1 \in h \end{cases}$*

*Agents' believes are:*

- *for any $h \subset \mathcal{W}$ $\mu_{f_j}(\theta_{w_i} : f_j = \max_{\theta_{w_i}}(f \in \mathcal{F})|h\backslash w_i) = 1$ and $\mu_{f_j}(\theta_{w_i} : f_j = \min_{\theta_{w_i}}(f \in \mathcal{F})|h \cup w_i) = 1$*

*It is easy to show that the above strategies and the set of beliefs constitute a sequential equilibrium. One may extend this example for the environment with more firms and workers.*

**Proof of Proposition 4.**

Let us first prove that firms' strategies are optimal. Note that if firm $f$ receives a signal from worker $w_1$ it believes that it is her top firm. Therefore, it is optimal for her to make her an offer. Now, if firm $f_1$ that does not receive a signal from worker $w_1$, firm $f_1$ believes that worker $w_1$ is atypical and will not accept its offer. Then, firm $f_1$ strategy of making an offer to worker $w_2$ is optimal for any signaling pattern, because it believes that her offer will

be accepted at least with probability $\frac{1}{2}$. The worst case is when worker $w_2$ is atypical and sends her signal to firm $f_3$.

Now we consider firm $f_2$ optimal strategy. Let us consider the case when worker $w_1$ sends her signal to firm $f_1$ and worker $w_2$ sends signal to firm $f_3$. Firm $f_2$ believes worker $w_2$ prefers firm $f_3$ to itself. Since, firm $f_3$ makes an offer to worker $w_2$, and firm $f_1$ makes an offer to worker $w_1$, the only worker that could accepts firm $f_2$ offer is worker $w_3$. If worker $w_1$ sends he signal to firm $f_3$ and worker $w_2$ sends her signal to firm $f_2$. In this case firm $f_1$ makes an offer to worker $w_2$, who is typical with probability $(1 - \frac{1}{3}\varepsilon)$. Since, worker $w_1$ most preferred firm is firm $f_3$, the optimal strategy of firm $f_2$ to make an offer to worker $w_3$.

Let us consider the case firm $f_3$ receives signals from all workers. In this case firm $f_3$ makes an offer to worker $w_1$. It is optimal for firm $f_1$ and firm $f_2$ to make an offer to worker $w_2$ because her preferences over these firms could be equally likely. Hence, the payoff from making an offer to worker $w_2$ equal $\frac{1}{2}u(2) > u(3)$. Similar, one could show that in other cases it is optimal for firm $f_2$ to make an offer to worker $w_2$. In a similar way one could show that it is always optimal for firm $f_3$ to make an offer to the best worker it receives a signal from.

Let us now show each worker uses optimal strategy. Worker $w_1$ strategy is optimal, because any firm makes her an offer upon receiving her signal. There is no incentive for worker $w_2$ to make an offer to firm $f_1$ since, all firms upon observing such behavior has believes about workers preferences that coincides with the priors. Therefore, worker $w_2$ optimal strategy is to send her signal to the best firms among $f_1$ and $f_2$. Since firms do not put attention to worker $w_3$ signals, there is no reason for her to deviate from the equilibrium strategy. $\square$

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