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Josef Zweimüller

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Josef Zweimüller*

University of Zurich and CEPR

Blümlisalpstrasse 10, CH-8006 Zurich

Tel: +41-1-634-3724

Fax: +41-1-634-4907

email: zweim@iew.unizh.ch

Abstract

This paper analyzes the impact of inequality on growth when technical progress is driven by innovations and consumers have hierarchic preferences. Inequality has an impact on growth because it affects the structure and the dynamics of demand. Redistribution from very rich to very poor consumers is beneficial for growth. In general, the growth effect depends on the nature of redistribution. Due to a demand externality of R&D activites multiple equilibria are possible.

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1. Introduction

This paper investigates the impact of income inequality on economic growth when technical progress is driven by innovations and consumers have hierarchic preferences. When consumers have hierarchic preferences the structure of demand is affected by the distribution of income. Poor people concentrate most of their expenditures on basic needs, whereas richer people direct their expenditures to more luxurious goods. The empirical relevance of a hierarchic structure of demand is well documented: it is featured by 'Engel's law' according to which the expenditure share for food decreases with income.

When demand is affected by the income distribution, inequality may be an important determinant of innovations and growth. The empirical importance of the inequality-growth relationship has been emphasized recently. A number of studies have found a robust negative correlation between growth rates and income inequality across countries (Persson and Tabellini (1994), Alesina and Rodrik (1994), Clarke (1995) and the survey by Benabou (1996)).

While recent research has extensively dealt with the question how income inequality affects the long-run growth performance of economies, little attention has been paid to the role of income distribution for the incentives to innovate.[1] This is surprising given the generally accepted view that innovations are an important source of economic development and technical progress. In the standard Schumpeterian growth models income inequality plays no role because consumers have homothetic preferences. By assumption, the level of demand for the various goods - including the innovator's product - does not depend on the income distribution. While the assumption of homothetic preferences has turned out convenient in incorporating monopolistic competition into a general equilibrium framework, it is highly questionable from an empirical point of view. The vast majority of studies of consumer behavior reject the hypothesis of homothetic preferences (see Deaton and Muellbauer (1980)).[2]

A hierarchy of wants implies that goods can be ranked according to their priority in consumption. In this paper, hierarchic preferences are introduced in a stylized way. It is assumed that goods are indivisible and consumers either buy or do not buy a certain good. The utility of a consumer depends then on the number of different goods consumed. The level of market demand for a specific good is affected by the distribution of income as this determines how many consumers can afford it. In a dynamic context, hierarchic preferences imply that inequality determines how the level of demand for a particular good evolves. Today, the good of an innovator may be purchased only by a small group of rich people. But as incomes grow the size of the market grows as less wealthy people also become willing to buy. The novel aspect of this paper is to study how income distribution affects the *time path of demand* for the innovator's good and therefore the reward to an innovation.

As far as the supply side is concerned, the model captures the main features of the standard innovation-driven growth models. In particular, it is assumed that each innovation project increases the stock of public knowledge and thus the productivity level in the whole economy. However, there are differences between the present model and the standard ones. In the present model innovations are process innovations leading to more efficient production of a particular consumption good. This differs from the model of Romer (1987, 1990) where innovators introduce new intermediate inputs which are then used by all final output producers. In the present model, innovators drive less efficient producers (a competitive fringe) out of the market, but are never displaced themselves. This is different from Aghion and Howitt's (1992) framework, where successive innovations take place within the same market and the current innovator disappears with the next innovation.

As mentioned above, inequality affects growth because it affects the time path of demand faced by an innovator. The underlying mechanisms can be illustrated by studying a population with two groups of consumers who differ in income. The analysis can easily be extended to the general case. With two groups, the equilibrium can be characterized by one of three different regimes. Regime (a) corresponds to a situation where initially only the rich buy the good of an innovator, the poor do not purchase until later; in regime (b) both rich and poor consumers immediately buy the innovator's good; and in regime (c) neither the rich nor the poor will buy the innovator's good initially.

The distinction between these three scenarios helps to understand the inequality-growth relationship. On the one hand, income distribution is a determinant of a particular regime. When all incomes are equal, it is evident that only regime (b) or regime (c) are possible, whereas regime (a) requires a sufficient unequal distribution. On the other hand, the impact of changes in income distribution on the growth rate is different across the three regimes. A possible impact of inequality comes from its effect on the time path of demand for a product. In regime (a), inequality is harmful for growth. When income is concentrated among a few the initial market for a new product is small; and with very poor consumers it takes a long time until the size of the market becomes larger. This decreases the profitability of an innovation. In regime (b), growth is independent of inequality. Demand for the good of an innovator is already at its maximum when the new good is introduced. A change in the distribution does not affect demand, so that the incentive to innovate and the growth rate remain constant. Finally, in regime (c) inequality is beneficial for growth. Innovators are better off in getting a small cash flow from the rich in the near future, as opposed to a large cash flow later on.

An interesting aspect of regimes (a) and (c) is the possibility of *multiple equilibria*. Two identical economies can end up in either a high- or in a low-growth regime. Multiplicity is the result of a complementarity between present and future R&D activities. If the expected future innovation rate and therefore the growth rate is high, current innovators can expect that their own markets will grow more rapidly. This creates an incentive to conduct more R&D

today. This complementarity between present and future innovators comes into play when more growth leads to a more favorable time path of demand for an innovator and is supported by the assumption of technological spillovers: more innovative activities lead to a higher growth rate. In other words, a high innovation rate creates a demand externality. This externality is clearly not present in regime (b) where all consumers buy the innovator's good as soon as (s)he enters the market. Since the market is fully developed a higher growth rate does not change the time path of demand for an innovator. In regimes (a) and (c), however, the demand for the innovator's good expands over time and future profits depend on the growth rate. In these two regimes the demand externality is at work and multiplicity can arise. Multiple equilibria are the result of a coordination problem in which expectations determine whether the economy experiences high or low growth.

The role of inequality and hierarchic preferences in the context of economic development has been studied in a number of other papers. The present paper is related to that of Murphy, Shleifer, and Vishny (1989a). Like in my model, they show that the adoption of efficient methods of production requires large markets and excessive concentration of wealth may be an obstacle to economic development. However, Murphy, Shleifer, and Vishny (1989a) focus on a static framework. As a consequence, changes in income distribution matter only if the demand of the marginal firm is affected. This is different from the present model where not only the level but also the time path of demand affects growth. Moreover, the equilibrium in their model is always unique, whereas my model generates multiple equilibria.[3] Finally, their paper emphasizes on the importance of the agricultural sector in generating the necessary demand to promote industrialization. In contrast, the present paper elaborates the idea that growth is driven by industrial R&D.

The importance of a hierarchic structure of demand is also emphasized by Eswaran and Kotwal (1993). If initially workers are too poor to buy manufacturing goods, productivity progress in the manufacturing sector will not trickle down and real wages cannot grow. A more even distribution of wealth as well as openness to international trade are mechanisms to escape underdevelopment. Baland and Ray (1991) consider a situation where a highly unequal distribution of assets generates a high demand for luxury goods. Since basic goods and luxuries are produced by the same resources, unemployment limits the demand for basic goods and allows to cover a high demand for luxuries. Like that of Murphy, Shleifer, Vishny (1989a) these papers stick to a static framework. Income distribution has an impact on the level of income, but no effect on the rate of growth.[4]

Only recently has the literature begun to analyze the impact of income inequality and hierarchic demand on growth. Chou and Talmain (1996) highlight mechanisms also present in the current model. Their consumers have preferences over a standard commodity (‘leisure’) and goods produced in a Grossman and Helpman-type innovation sector. If the demand for ‘leisure’ is not linear in income, inequality affects growth. In the model of Chou and Talmain (1996) new goods are always consumed by all, rich *and* poor consumers. In contrast, I study the potentially important case in which not all consumers can afford an innovator’s good. As a consequence not only the level, but also the time path of demand is affected by the distribution of income. Furthermore, in Chou and Talmain (1996) the equilibrium is always unique, whereas my model generates multiple equilibria. Also Falkinger (1994) studies the impact of income distribution on product development when consumers have a hierarchic structure of demand. He finds that the impact of income inequality on growth depends on the type of technological spillovers (innovations versus learning-by-doing). In this model firms live only for one period whereas in my model innovators live forever. This allows me to study the behavior of innovators who have to consider how demand develops over time.

In the above papers income distribution affects the level of demand, but has no impact on the prices of new goods. The papers by Glass (1996), Li (1996), and Zweimüller and Brunner (1996, 1998) are complementary to the present paper because of their focus on prices. Within a quality ladder framework inequality has an impact on the incentives to innovate by affecting the willingness to pay for quality. The equilibrium price structure among goods of different qualities is affected by the income distribution.

This paper is organized as follows. Section 2 presents the model and section 3 studies the innovation decision in detail. Section 4 studies the general equilibrium and analyzes the relationship between inequality and growth. In these sections the focus will be on regime (a) and on a situation where there are only two groups of consumers, rich and poor. Section 5 first studies the remaining regimes (b) and (c) and then extends the model to a general distribution. Section 6 concludes.

2. The model

2.1 Technology, market structure, and prices

Consider a closed economy. At date *t*, many different consumer goods are supplied with labor as the unique production factor. Goods are produced either by a an efficient modern or by a less efficient traditional technology. The less efficient technology is freely available and requires $a_c(t)$ labor units to produce one unit of output. A market which is served by these low-tech producers has perfect competition. In equilibrium the price is equal to marginal costs. Marginal costs are $w(t)a_c(t)$, where $w(t)$ is the wage rate. Production of each good requires the same labor input, so all goods have the same price. I will normalize this price to unity. The wage rate is then $w(t) = 1/a_c(t)$.

Access to the modern technology requires a fixed R&D labor input $a_r(t)$. Output can then be produced with a unit input $a_m(t)$. The modern technology is more efficient meaning that $a_c(t) > a_m(t)$ for all *t*. If production takes place with the efficient technology, the market is served by a single monopolist.[5] Potential competition comes from the competitive fringe.

To keep this competitors from the market no monopolist ever charges a price larger than 1. It is assumed, that no monopolist ever charges a price below 1.[6] Under these conditions a monopolistic firm earns a profit $\pi(t) = 1 - w(t)a_m(t)$ per unit of output.

2.2 Consumers

Consumers have hierarchic preferences. The hierarchy is captured by an index *j*. A low-*j* good satisfies a basic need, higher-*j* goods satisfy more luxurious wants. Goods are indivisible and consumption is a take-it-or-leave-it decision. A consumer derives utility 1 when consuming a good $j \in [0,1]$ and derives utility $1/j$ when consuming a good $j \in (1,\infty)$.

Given that all prices are unity, consumer *i* will buy good *j* when the utility from consumption is larger than the marginal utility of income, denoted by $\lambda_i(t)$. I will assume that $\lambda_i(t) < 1$ for all *i* and *t*. This means everybody can afford the menu $[0,1]$.[7] Moreover, consumer *i* buys a good $j > 1$ if $\lambda_i(t) \leq 1/j$. The range of consumed goods is then $[0, c_i(t)]$ where $c_i(t)$ satisfies $\lambda_i(t) = 1/c_i(t)$. $c_i(t)$ is not only a measure for the level of consumption but also for the most luxurious good purchased by consumer *i*. The instantaneous utility $u_i(t)$ can now be represented as $u_i(t) = \int_0^1 dj + \int_1^{c_i(t)} \frac{1}{j} dj = 1 + \ln(c_i(t))$. It is assumed that consumers have an infinite horizon. Their objective function is then

$$U_i = \int_0^\infty u_i(t) e^{-\theta t} dt = \int_0^\infty [1 + \ln(c_i(t))] e^{-\theta t} dt, \tag{1}$$

where θ denotes the rate of time preference. Each consumer earns a wage $w(t)$ and owns assets $A_i(t)$ in period *t*. There is a perfect capital market with interest rate *r*. Only steady states will be considered. So r is constant over time. Consumers have perfect foresight and make their choices given the rate of wage growth g which is also constant. The lifetime budget constraint can then be written as

$$\int_0^\infty c_i(t)e^{-rt}dt \le A_{i0} + \int_0^\infty w_0 e^{-(r-g)t}dt\,, \tag{2}$$

where w_0 and A_{i0} are initial wages and assets, respectively. In the steady state also assets and consumption grow at rate g. Moreover, the consumption path is given by the following relations

$$g = r - \theta\,, \tag{3}$$

$$c_i(0) = w_0 + \theta A_{i0}\,. \tag{4}$$

Equation (3) results from the fact that preferences are logarithmic in the number of consumed goods. According to equation (4), the level of consumption is the sum of labor income and a fraction of assets equal to the rate of time preference. As in Bertola (1993), the propensity to consume from labor income (the non-accumulated factor) is unity, whereas the propensity to consume from asset income (the accumulated factor) is lower than one. Current income of household i is $w(t) + rA_i(t)$ and savings are $gA_i(t)$. This means that wealthier households have a higher savings rate.

2.3 The distribution of wealth

To keep things simple, I will first focus on a two-class society with rich (*R*) and poor (*P*) consumers. (Section 5 extends the analysis to more general distributions). Consumers of group R and P have equal preferences and earn the same wage but own different wealth levels. Denoting by *L* the size of the population and by β the group share of the poor, we have βL poor and $(1-\beta)L$ rich consumers. Furthermore, let d_i be the ratio of the value of assets owned by household *i* relative to the average. Poor consumers own less, so $0 \leq d_P < 1$. Rich consumers own more, so $d_R > 1$. The corresponding fractions in aggregate wealth are βd_P for the poor and $(1-\beta)d_R$ for the rich. These two terms must sum up to 1 and one can solve for $d_R = (1-\beta d_P)/(1-\beta) > 1$. Consequently, inequality decreases in d_P and increases in β (holding d_P constant).

Aggregate wealth consists of firm shares. These firms earn a flow profit and the value of a firm k, $v_k(t)$, equals the present value of this flow profit. The value of wealth in the economy $V(t)$ is then the aggregate value of firms, that is the integral of $v_k(t)$ over the interval $[0, n(t)]$ where $n(t)$ is the measure of existing firms. The value of assets of consumer *i* is then

$$A_i(t) = d_i V(t)/L \,. \tag{5}$$

2.4 Technical progress and the resource constraint

Technical progress is the result of innovations. As in most endogenous growth models, it is assumed that researchers of future generations build upon experience of previous innovations. Moreover, it is assumed that also the efficiency of final output production, by both modern firms and the traditional competitive fringe, increases in previous innovations (see also Young

(1993)). In this model firms live forever, so the number of previous innovations equals the number of existing firms $n(t)$. More specifically, I assume

$$a_l(t) = a_l / n(t) \quad \text{for} \quad l = r, c, m, \tag{6}$$

where the a_l's are positive constants.

Total labor supply in the economy is L. The real wage is given by the marginal product in the competitive fringe, fixed at any date *t*. The full employment equilibrium is established by an appropriate allocation of labor across sectors. Part of the work force is employed in the R&D sector to develop new processes, the remaining part is employed in modern or in low-tech firms to produce consumer goods. If $\dot{n}(t)$ measures innovation activities in *t*, the number of R&D employees is $\dot{n}(t)a_r(t)$. Let $Y_m(t)$ denote total production in the monopolistic sector, then $Y_m(t)a_m(t)$ is employment in modern manufacturing. Total output by the competitive fringe is $Y_c(t)$, so $Y_c(t)a_c(t)$ workers are employed in the competitive fringe. The labor market equilibrium condition can then be written as $L = \dot{n}(t)a_r(t) + Y_m(t)a_m(t) + Y_c(t)a_c(t)$.

Above it was shown that the rich consume all goods in the range $[0, c_R(t)]$, whereas the poor buy the menu $[0, c_P(t)]$, with $c_R(t) > c_P(t)$. Suppose that the modern firms supply the menu $[0, n(t)]$. The firm producing good $n(t)$ is the most recent innovator. The following analysis will be made under the assumption that a new innovator sells initially to the rich, but not to the poor (regime (a)). This means that $c_P(t) < n(t) < c_R(t)$. The poor buy only a subset of goods produced in modern manufacturing and will buy no goods produced in the competitive sector. The rich buy all goods produced in modern manufacturing plus other goods. Any additional demand from the rich is satisfied by the competitive fringe. Output in the monopolistic sector is then $Y_m(t) = c_P(t)\beta L + n(t)(1-\beta)L$. Output in the competitive sector is given by $Y_c(t) = [c_R(t) - n(t)](1-\beta)L$. In a steady state, $n(t)$, $c_P(t)$, and $c_R(t)$ grow at the same rate *g*. Using equation (6) and defining $x_i = c_i(t)/n(t)$, *i*=*R*,*P*, the resource balance condition can be written as

$$L = a_r g + a_m\left[(1-\beta) + \beta x_P\right]L + a_c(1-\beta)(x_R - 1)L\,. \tag{7}$$

3. Innovations

3.1 Demand and the value of a firm

The value of a firm depends on its profit flow. The profit margin is constant over time and equal across firms. To see this recall that all prices are equal to 1 and all unit costs can be written as $a_m(t)w(t) = a_m/a_c$ (using (6) and $w(t) = 1/a_c(t)$). Profits per unit of output are then $\pi = (a_c - a_m)/a_c$. The level of demand, however, is different across firms and changes over time. Demand for a monopolistic firm supplying good *j*, $D_j(t)$, is given by

$$D_j(t) = L \qquad \text{if} \quad 0 \le j \le c_P(t), \text{ or}$$

$$D_j(t) = (1-\beta)L \qquad \text{if} \quad c_P(t) < j \le n(t).$$

Goods $j \in [0, c_P(t)]$ are purchased by all consumers. Firms supplying such a good have a fully developed market. All consumers buy today and at all dates in the future, so demand is at its maximum and stays there forever. Thus the value of such a firm is also constant and given by

$$v_j(t) = \pi \int_0^{\infty} D_j(t) e^{-rt} dt = L\pi/r \ .$$

Goods in the interval $(c_P(t), n(t)]$ are too luxurious for poor people. They are only bought by the rich. As incomes grow also the poor will be able to afford that good. The initial period when only rich consumers buy will be short, if the incomes of the poor grow very quickly. And it will also be short, if the current consumption level of the poor is already high. Denote the length of that period by Δt. Obviously, Δt is defined by the equation $c_P(t + \Delta t) = j$. Since $c_P(t)$ grows at rate *g*, I can write $c_P(t) e^{g\Delta t} = j$. Solving for Δt yields $\Delta t = -(1/g)\ln(c_P(t)/j)$. Obviously, Δt is increasing in *g* and decreasing in $c_P(t)$. The value of a firm supplying a good $j \in (c_P(t), n(t)]$ can be written as[8]

$$v_j(t) = \pi \int_0^{\infty} D_j(t) e^{-rt} dt = \frac{L\pi}{r}\left[(1-\beta) + \beta e^{-r\Delta t}\right] = \frac{L\pi}{r}\left[(1-\beta) + \beta [c_P(t)/j]^{r/g}\right].$$

While the level of demand makes a discrete jump when the poor start to buy, the value of a firm increases smoothly over time. The cash flow to be earned from the poor has to be discounted. From the above equation, the discount factors equals $[c_P(t)/j]^{r/g} < 1$. Clearly, discounting depends on the distance between the most luxurious good currently purchased by the poor, $c_P(t)$, and the good under consideration, *j*. When consumption grows this gap decreases and thus the discount factor increases smoothly. Consequently, also the firm value grows smoothly and reaches $L\pi/r$ as soon as the poor can afford good *j*.

3.2 The entry decision

Entering a market is profitable as long as the necessary R&D costs are not larger than the reward to an innovation. The R&D costs are $a_r(t)w(t) = a_r/a_c$ (using (6) and $w(t) = 1/a_c(t)$) and are constant over time. The reward to an innovation equals the present value of the subsequent profit flow. Denote by *n(t)* the good supplied by the most recent innovator. Replacing j by $n(t)$ in the above expression for $v_j(t)$, and using the definition $x_P = c_P(t)/n(t)$, the value of the most recent innovator is equal to

$$v_n(t) = \frac{L\pi}{r}\left[(1-\beta) + \beta x_P^{\,r/g}\right].$$

With free access to the R&D technology the equilibrium is characterized by zero profits, that is by the condition $a_r/a_c \geq v_n(t)$, with equality when g>0. Using (3) and the above expression for $v_n(t)$ the zero-profit equilibrium can be expressed as

$$(g+\theta)(a_r/a_c) \geq L\pi\left[(1-\beta) + \beta x_P^{\,(g+\theta)/g}\right], \quad \text{or} \quad C(g) \geq B(g, x_P) \tag{8}$$

The left hand side of (8) are the *current* costs of innovating $C(g)$: the interest cost of investing in a new firm. In equilibrium these costs must not be smaller than the *current* returns from an innovation $B(g, x_P)$. These returns consist of dividends resulting from sales to the rich $(1-\beta)L\pi$ plus the increase in the firm value, $\beta L\pi(x_P)^{(g+\theta)/g}$.

Figure 1 draws both sides of equation (8) against *g*. $C(g)$ is a straight line with slope a_r/a_c and intercept $\theta(a_r/a_c)$. The innovation costs are increasing in *g* because in equilibrium a higher growth rate goes hand in hand with a higher interest rate (see equation (3)). $B(g, x_P)$ has the shape of a logistic curve. It equals $(1-\beta)L\pi$ for $g = 0$, is convex over the range $g \in [0, -\ln(x_P)\theta/2]$, and concave for $g \geq -\ln(x_P)\theta/2$. For $g \to \infty$, $B(g, x_P)$

approaches $\left(1-\beta+\beta x_P\right)L\pi$. There are two different effects of the growth rate on the returns to an innovation. On the one hand, a rise in *g* means that the profits from the poor accrue earlier since their incomes grow faster. This increases the incentive to innovate. On the other hand, a larger *g* increases the interest rate: future profits have to be discounted at a higher rate. This reduces the reward to an innovation. The former effect always dominates the latter meaning that $B\left(g,x_P\right)$ is increasing in *g*. But for $g \to 0$ and $g \to \infty$ the net effect goes to zero.

The reward to an innovation is not only affected by the growth rate, but also by the current level of consumption of poor people. If the poor have already a high consumption level it takes only a short time until they can afford the innovator's product. This raises the payoff of an innovation. In Figure 1, a higher x_P means that $B\left(g,x_P\right)$ shifts upwards.

Figures 1a,1b

From the non-linear nature of the $B\left(g,x_P\right)$-curve it follows that the equilibrium might not be unique. In Figure 1a the rate of time preference is low and satisfies $\theta\left(a_r/a_c\right)<\left(1-\beta\right)L\pi$. In this case there may either be a unique equilibrium ($x_P > x_P^0$ or $x_P < x_P^1$) or more values of *g* satisfying the no-entry condition ($x_P^0 \geq x_P \geq x_P^1$). Multiple equilibria can arise for $x_P \in \left[x_P^1, x_P^0\right]$.[9] An equilibrium is 'stable' if firms have an incentive to conduct more R&D when *g* falls short of the equilibrium value and vice versa. In terms of Figure 1, in a stable equilibrium the cost-line cuts the benefit-curve from below.[10]

Denote by $\hat{g} \equiv \left(1-\beta\right)L\pi\big/\left(a_r/a_c\right)-\theta$ and $\breve{g} \equiv L\pi\big/\left(a_r/a_c\right)-\theta$ respectively the lower and the upper bound of the range of possible growth rates. Then three regions of *g* can be distinguished: (i) $g \in \left(\hat{g}, g_0\right)$, (ii) $g \in \left[g_0, g_1\right]$, and (iii) $g \in \left(g_1, \breve{g}\right)$. Intersections in the first and in the third interval the stability condition is satisfied. The *C*-line is steeper than the *B*-curve at points of intersection. An increase in x_P will always lead to an increase in *g*. In the second interval, the stability condition is violated. The *C*-line cuts the *B*-curve from above. An increase in x_P leads to a fall in *g*.

In Figure 1b the rate of time preference is larger and satisfies $\theta \geq (1-\beta)\, L\pi / (a_r / a_c)$. There are at most two equilibria with positive values of *g* ($x_P \geq x_P^2$). In that case, also $g = 0$ satisfies the zero-profit condition (benefits are not higher than costs). The interesting intervals are (i) the single point *g=0*, (ii) $g \in (0, g_1]$, and (iii) $g \in (g_1, \breve{g})$. Again intersections in intervals (i) and (iii) are stable, in (ii) unstable.

4. Growth and income distribution

4.1 Equilibrium growth rates

The last section was concerned with the equilibrium innovation rate taking consumption as given (equation (8)). The general equilibrium has to consider in addition which combination of growth and consumption is feasible given the economy's resource constraint (equation (7)). And it has to take into account that consumption choices are optimal given the households' preferences. Denote the average value of all modern firms by $\bar{v} = V(t)/n(t)$ [11] and using equations (4), (5), and $w(t) = 1/a_c(t)$ the optimal level of consumption of consumer *i* can be expressed as

$$x_i = 1/a_c + \theta\, d_i\, \bar{v}/L \qquad (9)$$

Equations (7) - (9) form a system of 4 equations with 4 unknowns: x_P and x_R, g, and $\bar{v}$. I will now discuss the solution to this system. Substituting equation (9) into the resource constraint (7) and using $\pi = (a_c - a_m)/a_c$, it is straightforward to calculate

$$\bar{v} = \frac{L\pi(1-\beta+\beta/a_c) - g\,a_r/a_c}{\theta(1-\beta d_P \pi)}. \tag{10}$$

This is the average firm value consistent with full employment and optimal choices by all consumers.

Now I can determine the equilibrium growth rate, *g**. Solving equation (8) for x_P and setting the resulting expression equal to the value of x_P from (9) using (10) implicitly defines *g**. This yields

$$\left[\frac{(g^*+\theta)(a_r/a_c)-(1-\beta)L\pi}{\beta L\pi}\right]^{\frac{g^*}{g^*+\theta}} = \frac{1}{a_c} + d_P\,\frac{L\pi(1-\beta+\beta/a_c)-g^*a_r/a_c}{L(1-\beta d_P\pi)}. \tag{11}$$

Equation (11) holds $(g^*+\theta)(a_r/a_c) > (1-\beta)L\pi$. If this is not satisfied, *g*=0*. Moreover, there may be more than one value of *g* satisfying (11).

It is convenient to draw both sides of equation (11) in (g, x_P)-space. The left-hand-side satisfies the no-profit condition. This is the "*N*-curve" (Figure 2). Values of x_P equal to the right-hand-side satisfy the resource constraint. This is the "*R*-curve" (Figure 3). The shape of the *N*-curve follows from the discussion in section 3. If the zero-profit equilibrium is unique for all values of x_P the *N*-curve has a positive slope: more consumption by the poor x_P makes innovation more profitable. The result is a higher level of R&D activities, that is a higher g. The R-curve has a negative slope: more consumption by the poor means that more resources are needed for the production of consumer goods. This leaves less resources for R&D.

The intention of this paper is to analyze the impact of inequality on growth. The parameters of particular interest are therefore the distribution parameters β and d_P. Moreover, the rate of time preference θ determines whether there exists a unique general equilibrium or whether multiple equilibria are possible. It is therefore important to know how both the N- and the R-curve react to changes in these three parameters.

Consider first the N-curve (Figure 2). Points on this curve correspond to intersections of the B- and the C-curve in Figures 1a and 1b. The N-curve is independent of the distribution parameter d_P, and shifts downwards with decreasing values of the group share of the poor β. The solid lines correspond to relatively large β, the dashed line is drawn for a smaller β. The shift is negligible for x_P close to unity, but becomes larger for smaller x_P. The role of the rate of time preference θ is less straightforward. For $\theta \to 0$ the N-curve tends towards a constant positive slope.[12] For $\theta \geq L\pi/(a_r/a_c)$, innovation is not profitable and the N-curve coincides with the vertical axis. Figure 2 shows the shape of the N-curve for intermediate values of θ. For relatively low θ, θ_0 in Figure 2, the N-curve is monotonically increasing. For larger θ, θ_1 or θ_2 in Figure 2, there may be multiple equilibrium values of g, given x_P. Denote by $g_0(\theta)$ and $g_1(\theta)$ the lower and the upper bound of the interval of g over which the N-curve has a negative slope. In Figure 2, if $\theta = \theta_1$, this interval is $g \in [g_0(\theta_1), g_1(\theta_1)]$. For all other g, the no-profit curve slopes upward and satisfies the stability condition. From section 3 it is clear that if $\theta < (1-\beta)L\pi/(a_r/a_c)$, $g_0(\theta)$ is positive, whereas if $\theta \geq (1-\beta)L\pi/(a_r/a_c)$, $g_0(\theta) = 0$ (compare Figures 1a and 1b). The latter case occurs if $\theta = \theta_2$ in Figure 2 .

Figure 2

The *R*-curve is a straight line which is independent of θ (Figure 3). If the distribution parameter $d_P = 0$ this line is horizontal with intercept $1/a_c$ on the vertical axis. For $d_P > 0$ the *R*-curve has a negative slope. Increasing d_P means a clockwise rotation around the point *Q* in Figure 3, that is at $x_P = 1/a_c$ and $g_Q = L\pi(1-\beta+\beta/a_c)/(a_r/a_c)$. If the distribution parameter $d_P = 0$, a decrease in the population share of the poor β shifts the point *Q* to the right and the *R*-curve remains otherwise unaffected. If $d_P > 0$, smaller values of β lead to a counter-clockwise rotation around the point *S*, that is at $x_P = 1$ and $g_S = \left[\pi - (1-1/a_c)/d_P\right]L/(a_r/a_c)$. In sum, lower inequality – a higher d_P or a lower β - lead to a higher feasible growth rate *g* for a given level of consumption by the poor x_P.

Figure 3

4.2 The impact of inequality on growth

Now we can analyze the central topic of this paper, namely how income inequality affects the rate of growth. More equality results from a larger d_P or from a lower β (holding d_P constant). By doing comparative statics on these two parameters, the inequality-growth relationship can be established. I will first study the impact of inequality on growth when the general equilibrium is unique. I will then discuss multiplicity.

Figure 4

A unique general equilibrium. Let us first consider the impact of less inequality due to an increase in the distribution parameter d_P. If $d_P = 0$ the *R*-curve in Figure 4 is horizontal and the point E_0 is the unique general equilibrium. Increasing d_P leads to an outward shift of the *R*-curve but leaves the *N*-curve unchanged. In the new equilibrium both the rate of growth *g* and the standard of living of poor people x_P (E_1 in Figure 4) are higher. The reason is a redirection of aggregate consumer demand towards the more efficient monopolistic sector. The most luxurious goods consumed by the rich are produced by the competitive fringe whereas the most luxurious goods consumed by the poor are produced by modern firms. As a result of redistribution the rich reduce and the poor increase their consumption. This leads to a situation where a higher proportion of final output is produced in the monopolistic sector so that production becomes more efficient. This gain in productivity releases resources which can be employed in the R&D sector. In the new equilibrium *g* and x_P are higher whereas x_R is lower; growth and consumption by the poor increase at the expense of consumption by the rich. The fact that growth increases means that absolute consumption by the poor $c_P(t)$ initially jumps to a higher level and has then a steeper path. Consumption by rich people $c_R(t)$ will initially be smaller but the new path will be above the original one after a finite period.

Less inequality can also be the result of a lower group share of the poor β. A reduction in β shifts both the *R*-curve and the *N*-curve outwards. This increases *g* and but has an ambiguous impact on x_P (E_2 in Figure 4). A larger number of rich people makes innovations more profitable because the size of the market in the early stage of production becomes larger. This is why the *N*-curve shifts outwards. At the same time a lower β (holding d_P constant) decreases d_R, leading to a reduction in consumption by the rich. As a result demand for products of the competitive fringe is reduced and aggregate production becomes more efficient. This leaves resources for R&D. This is why the *R*-curve shifts outwards. In sum, in the new equilibrium the growth rate g is higher, the impact on the level of consumption of the poor x_P is ambiguous, and the consumption level of the rich x_R is lower.

Figures 5a, 5b

Multiple equilibria. A unique general equilibrium requires a sufficiently low rate of time preference θ. For a larger θ the *N*-curve becomes non-monotonic and there is the possibility of multiple equilibria. In Figure 5a, $\theta a_r / a_c < (1-\beta) L\pi$ and there are two stable points at $d_P = 0$: F_0 and F_2. (F_1 is unstable). In F_0 the growth rate is low and the size of the market for the competitive fringe is large, so production is relatively inefficient and a small amount of resources is devoted to R&D. This is ‘underdevelopment’. F_2 is a long-run equilibrium where the rich do not excessively consume goods produced by the competitive fringe. More workers are available for research and growth is high. This is ‘prosperity’.

Multiple equilibria arise in this model because innovation activities of present and future innovators are complementary. If current innovators expect high innovation activities of future generations they have an incentive to conduct more R&D. This complementarity is due to technological spillovers. A higher level of R&D activities generates a higher growth rate. And a higher growth rates leads to a more rapid development of an innovator’s market. In other words, a higher expected innovation rate creates a positive demand externality that makes present innovations more profitable. Multiple equilibria are the result of a coordination problem in which the expectation about the future innovation rate determines whether the economy experiences high or low growth. The economy will be trapped in underdevelopment F_0 if agents are pessimistic and expect low growth. The prosperity path F_2 would be feasible but a coordination failure is present. There is no possibility to synchronize expectations. [13] No agent has a reason to expect high growth when all others are pessimistic.

If an economy is trapped in underdevelopment F_0 redistribution can change the situation. Less inequality as a result of an increase in d_P can make the underdevelopment regime infeasible. In terms of Figure 5a an increase in d_P leads to a clockwise rotation of the *R*-curve around point *Q*, and leaves the *N*-curve unaffected. Small changes in d_P will lead to movements along the underdevelopment-trap, but for a $d_P > \tilde{d}_P$ the only feasible and stable equilibrium is the high-growth regime F_2'. As a consequence, redistribution can result in a "Big Push", a discrete jump in the rate of innovation.

Panel b of Figure 5 corresponds to a situation with a relatively high rate of time preference θ, that is when $\theta a_r / a_c \geq (1-\beta)L\pi$. In this scenario, zero growth is a stable equilibrium, F_3 in Figure 5b. This coexists with the high-growth regime F_5 which is also a stable equilibrium. The point F_4 is unstable. Just like before escaping underdevelopment through redistribution is possible by a sufficient increase in d_P, so that $g=0$ is no longer feasible.

The effect of less inequality due to a decrease in the group share of the poor β is straightforward and not shown in Figure 5. For $d_P = 0$, a reduction in β has no effect on x_P, but increases *g*. For $d_P > 0$ the effect of a decreasing β is similar to Figure 5 above. The *N*-curve rotates downwards with decreasing β (see also Figure 2), whereas the *R*-curve rotates upwards (see Figure 3). When an economy is initially trapped in underdevelopment a sufficient increase in β can ultimately lead to a situation where the growth rate jumps from a low to a high growth rate.

To sum up, this section has shown that less inequality is beneficial for growth. More inequality leads to a lower growth rate, because the time path of the market size is more favorable for innovators. Moreover, less inequality may be beneficial for growth by making an underdevelopment trap infeasible.

5. The distribution of wealth

The results derived in the previous section refer to a situation with only two groups of consumers. It was assumed that the products of innovators are initially purchased only by rich people (regime (a)). In this section I will first discuss the conditions under which such an equilibrium exists and discuss possible other regimes with two consumer groups (regimes (a) and (b)). I will then turn to the general case with more types of consumers.

5.1 Existence of the various regimes

Regime (a). If the rich but not the poor buy the innovator's product, $x_P < 1 < x_R$. To see under which circumstances these conditions are met, consider equation (9). To ensure that poor people do not buy the innovator's product $x_P < 1$, two conditions have to be satisfied. First, the wage rate has to be sufficiently low. This means that $1/a_c < 1$. Secondly, the poor must have little wealth, that is d_P has to be small enough. To ensure that the rich buy all *n(t)* goods of monopolistic producers $x_R > 1$, wealth ownership of the rich d_R must be large enough. Since $d_R = (1-\beta d_P)/(1-\beta)$ is increasing in β and goes to infinity as β approaches 1, there exists a β generating a large enough d_R. In sum, low wages and a sufficiently unequal wealth distribution are a necessary and sufficient condition for regime (a).

Regime (b). If the rich and the poor can buy all goods produced by modern firms we have $x_R > x_P > 1$ (regime (b)). In that case the innovation equilibrium condition (8) simplifies to $(g+\theta)a_r/a_c = L\pi$. Using $\pi = (a_c - a_m)/a_c$, the equilibrium growth rate can be expressed as $g^* = (a_c - a_m)L/a_r - \theta$.[14] In that case income distribution does not affect growth. The reason is that demand for an innovator does no longer depend on the distribution of income. Demand is at the maximum when this firm enters the market and stays there forever. Since demand does not change through redistribution, the incentives to innovate remain also unaffected. A similar reasoning holds for the resource constraint. The most luxurious goods purchased by the rich and the poor are produced by the competitive fringe. Redistribution changes the structure of this consumption, but leaves the overall level of production in the competitive sector unchanged. The allocation of resources remains unaffected by the income distribution and growth remains constant.

Regime (b) requires that the poor can afford all *n(t)* goods produced in the monopolistic sector. Using (9) and the fact that in regime (b) all firms have the same value, namely a_r / a_c , the condition $x_P > 1$ can be written as $1/a_c \left(1+\theta d_P \; a_r / L\right) > 1$. This condition is always satisfied when $1/a_c > 1$, that is when the wage rate is large enough. If the wage rate is lower, a high ownership in firm shares d_P, a high rate of time preference θ and/or large set-up costs relative to the market size a_r / L guarantee that the poor will consume all goods of the modern sector. Finally, a situation where rich and poor buy all goods in the modern sector is feasible if production in this sector is efficient enough. When all L households buy all *n(t)* goods labor demand in that sector is $a_m L$. Since labor supply equals L we must have $a_m < 1$ so that additional resources remain available for the competitive sector and the R&D sector.[15]

Regime (c). Regime (c) occurs if neither the poor nor the rich can afford an innovator's product. In that case the competitive sector has no demand and all consumption is satisfied by modern firms. The resource constraint (7) can then be rewritten as

$$L = a_r g + a_m \bar{x} L \qquad (7')$$

where $\bar{x} = \beta x_P + (1-\beta) x_R$ denotes the average consumption level. Moreover, applying the arguments used in section 3.1, it is straightforward to demonstrate that the zero-profit equilibrium condition (8) changes to

$$(g+\theta) a_r / a_c = L\pi\left((1-\beta) x_R{}^{(g+\theta)/g} + \beta x_P{}^{(g+\theta)/g}\right) > L\pi \bar{x}^{(g+\theta)/g} \qquad (8')$$

where the last relation follows from Jensen's inequality.

To understand the inequality growth-relationship in regime (c) it is useful to start from a situation where all consumers are equal. In that case we can draw the *R*-curve and the *N*-curve in (g, $\bar{x}$) space with $\bar{x}$ playing a similar role than x_P in section 4 (Figure 6). Just like before the *R*-curve slopes down and for sufficiently low θ the *N*-curve slopes up. In this case the equilibrium is unique (G_0 in Figure 6). Now consider a spread in the wealth distribution. This leaves the *R*-curve unaffected. The rich increase and the poor decrease their consumption equally. Since all concerned goods are produced in the modern sector, the amount of resources necessary to produce that consumption are the same. The *N*-curve, however, shifts down. The right-hand-side of equation (8') measures the current reward from an innovation and this reward increases when income distribution becomes more dispersed. The reason is that innovators are better off when profits are less 'backloaded'. An innovator prefers a small payoff early in life to a larger payoff later on. This is a result of discounting. Hence in regime (c) inequality is beneficial for growth.

Regime (c) requires that even the rich cannot afford all *n(t)* goods produced in the monopolistic sector. The highest possible average firm value $\bar{v}$ is given by $L\pi / \theta$. This is when the highest possible payoff stream $L\pi$ is realized and the interest rate is at the lowest possible level θ (see equation (3)). Using (9) and substituting $\bar{v} / L = \pi / \theta$ gives the highest possible value of x_R. To ensure that this upper limit for x_R is smaller than unity we must have $(1/a_c)(1+d_R\pi)<1$. This condition holds for a low enough wage rate, $1/a_c<1$, low enough asset holdings by the rich d_R and a limited mark-up of monopolistic firms π.

5.2 More groups of consumers

Now suppose there are many different types $i=(1,...,k,...,K)$, ranked by wealth, so that a higher *i* indexes a type with more assets. Denote by *k* the number of types who are too poor to purchase all $n(t)$ products in the modern sector. The remaining $K-k$ types can afford more than $n(t)$ goods. I will refer to the first *k* types as the group of 'poor' consumers and to the remaining *K*-*k* types as the group of the 'rich'. The aim is to study the impact of *within*-group inequality and *between*-group inequality on growth.

Assume first that $k=1$ but $K-k>2$. We have now more than two types of rich consumers and in addition to one group who does not buy the innovator's product. Which effect has redistribution *within* the rich group? In regime (b) we have seen that redistribution has no impact on the growth rate if all concerned consumers can afford all goods produced by the monopolistic sector. This result carries over to the general case. Such a redistribution leaves the demand for the innovator unchanged and has no impact on the resources necessary to satisfy the same aggregate level of consumption. As a result, redistribution *within the rich* group has no impact on growth.

Now consider the general case when there are many poor types. Denote by β_i is the share of type *i* in the population and by β the group share of the poor, with $\sum_{i=1}^{k}\beta_i=\beta$. The average level consumption among poor households is then $\bar{x}_P = (1/\beta)\sum_{i=1}^{k}\beta_i x_i$.

Moreover, the average poor type owns wealth $\bar{d}_P\,\bar{v}/L = (1/\beta)\sum_{i=1}^{k}\beta_i d_i\,\bar{v}/L$. The *R*-curve remains the same as before (replace x_P and x_R by $\overline{x}_P$ and $\overline{x}_R$ equation (7)). However, the N-curve now changes to

$$(g+\theta)(a_r/a_c) = L\pi\left[(1-\beta)+\sum_{i=1}^{k}\beta_i\,(x_i)^{(g+\theta)/g}\right] \geq L\pi\left[(1-\beta)+\overline{x}_p^{\,(g+\theta)/g}\right]. \qquad (8'')$$

The last relation is again due to Jensen's inequality. What is the impact of redistribution *within* the poor? This is a generalization of regime (c) above and we can apply similar arguments. The resource constraint remains unaffected. Redistribution changes the composition but not aggregate demand by the monopolistic sector. However, a more dispersed income distribution among the poor types is more favorable for innovators. It leads the a time path of demand which is less backloaded. As a result the discounted value of the innovator's profit flow increases. It follows that more inequality *within the poor* group is beneficial for growth.

So far, we have focused on the impact of inequality *within* groups. It remains to discuss the affect of redistribution *between* these groups. This is generalization of regime (a), discussed at length in the previous sections. In that case $\overline{x}_P$, $\overline{d}_P$ and β play exactly the same role as in sections 2 – 4, and the above reasoning can be applied in a straightforward way. This means that more inequality *between groups* is harmful for growth.

Now the growth-maximizing wealth distribution can be characterized. This is the wealth distribution which maximizes the demand of the most recent innovator. Depending on the efficiency of production in the monopolistic sector, there are two possible scenarios. If productivity in the monopolistic sector is very high, the outcome is a wealth distribution such that even the poorest can afford all goods produced in the monopolistic sector.[16] The alternative scenario arises if productivity in the monopolistic sector is low, so that a situation where all consumers buy all goods produced in the monopolistic sector is not feasible. In this case the wealth distribution which maximizes growth is such that only a part of the consumers has enough wealth to consume all goods in the monopolistic sector, but does not consume goods from the competitive fringe. The other part of the consumers has no assets and earns only wage income.

6. Conclusions

When consumers have hierarchic preferences the structure and the dynamics of demand are affected by the distribution of income. Poor people consume more basic goods, whereas rich people direct their expenditure to more luxurious goods. The long-run growth rate depends on the distribution of income because it affects the time path of demand faced by an innovator.

How a change in income inequality affects the long-run growth rate depends on the consumption capacity of the consumer s concerned by the redistribution. First, a redistribution from consumers who can afford the good supplied by the most recent innovator to consumers who cannot afford this good leads to an increase in the growth rate. The reason is that after such a redistribution the market of an innovator grows faster which increases the incentive to innovate. Secondly, if redistribution among consumers who can afford the most recent innovator's product has no effect on the growth rate. This is because the level and the dynamics of demand for an innovator remain unaffected. Finally, redistribution from poor households to even poorer households both of whom cannot afford the most recent innovator's product reduces the growth rate. This is because innovators are increasingly worse off when a given profit flow is shifted towards the future.

The model may generate multiple steady-state equilibria. With a sufficiently unequal distribution and a sufficiently high rate of time preference two identical economies can end up in different growth regimes. In one regime the growth rate is high, and few resources are devoted to the inefficient competitive sector. In another regime, the competitive sector has high demand and R&D activities remain on a low level. Multiple equilibria are the result of a complementarity between present and future R&D activities. If current innovators expect a high future innovation rate they have an incentive to conduct more R&D today. This complementarity is the result of the fact that innovations drive growth and that the economy-wide growth rate has a positive impact on the evolution of an innovator's market.

Figure 1a: Zero Profit Equilibria

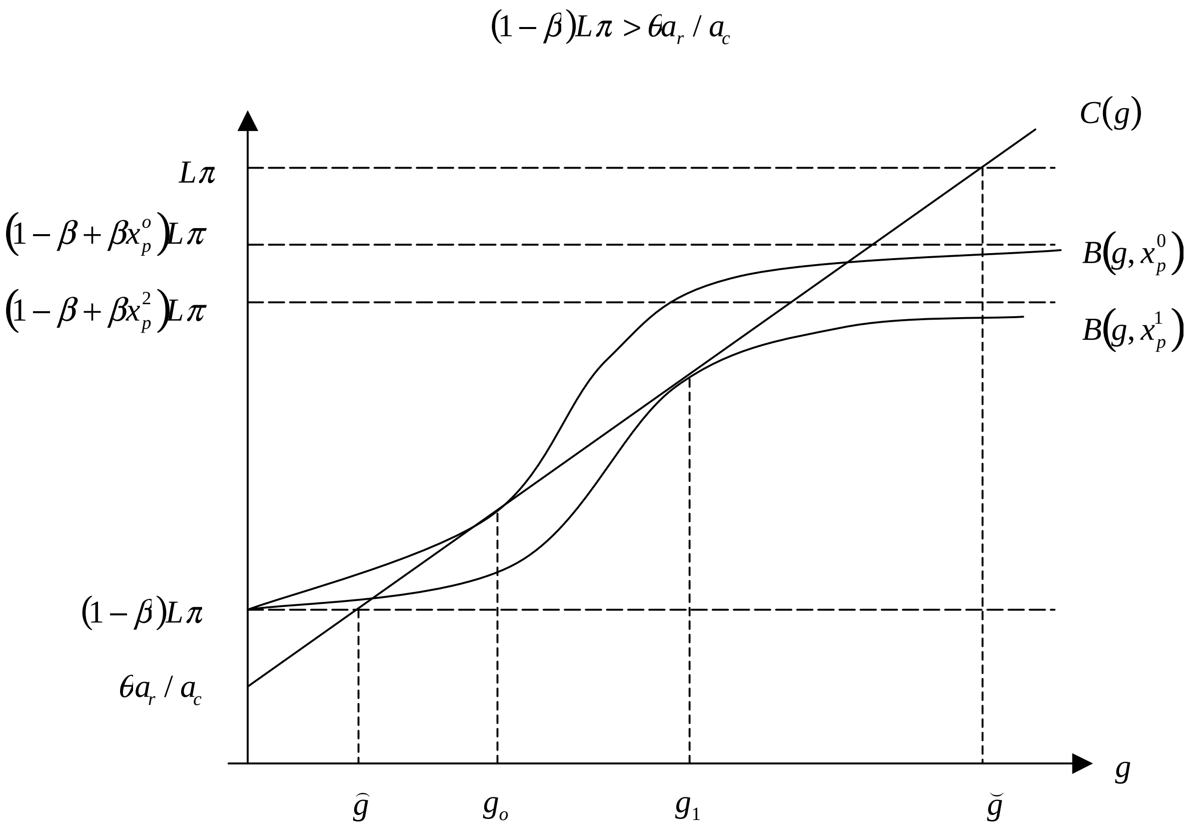


Figure 1b: Zero Profit Equilibria

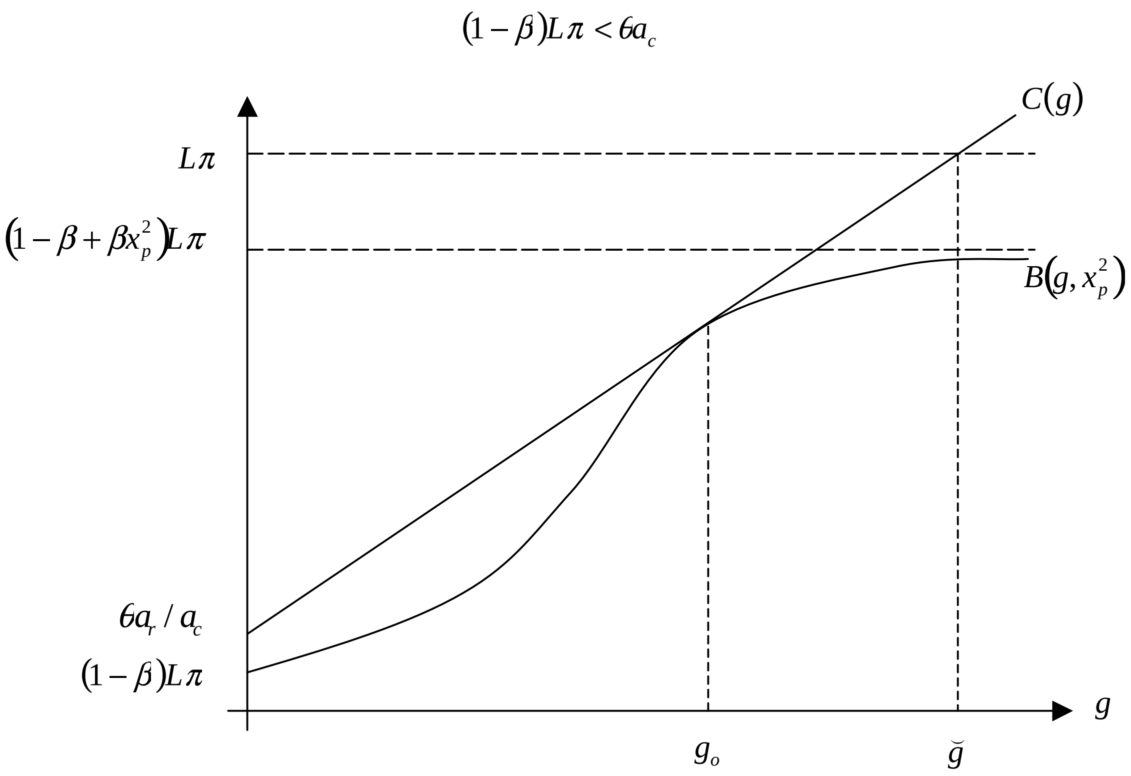


Figure 2: The N-curve

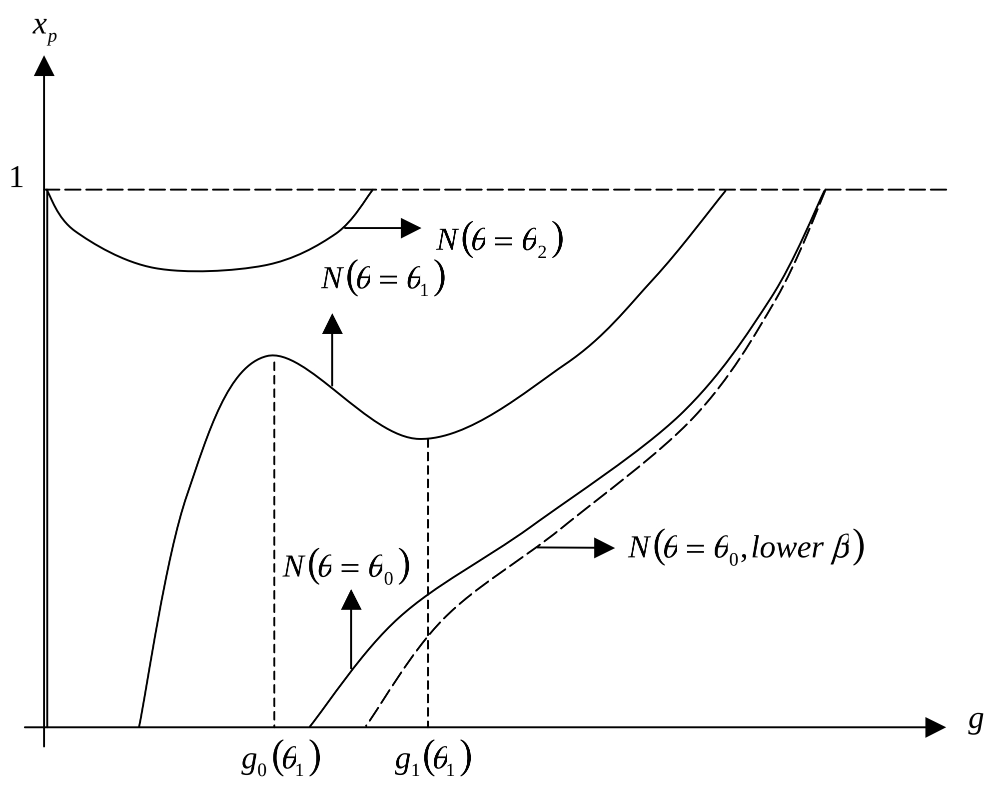


Figure 3: The R-curve

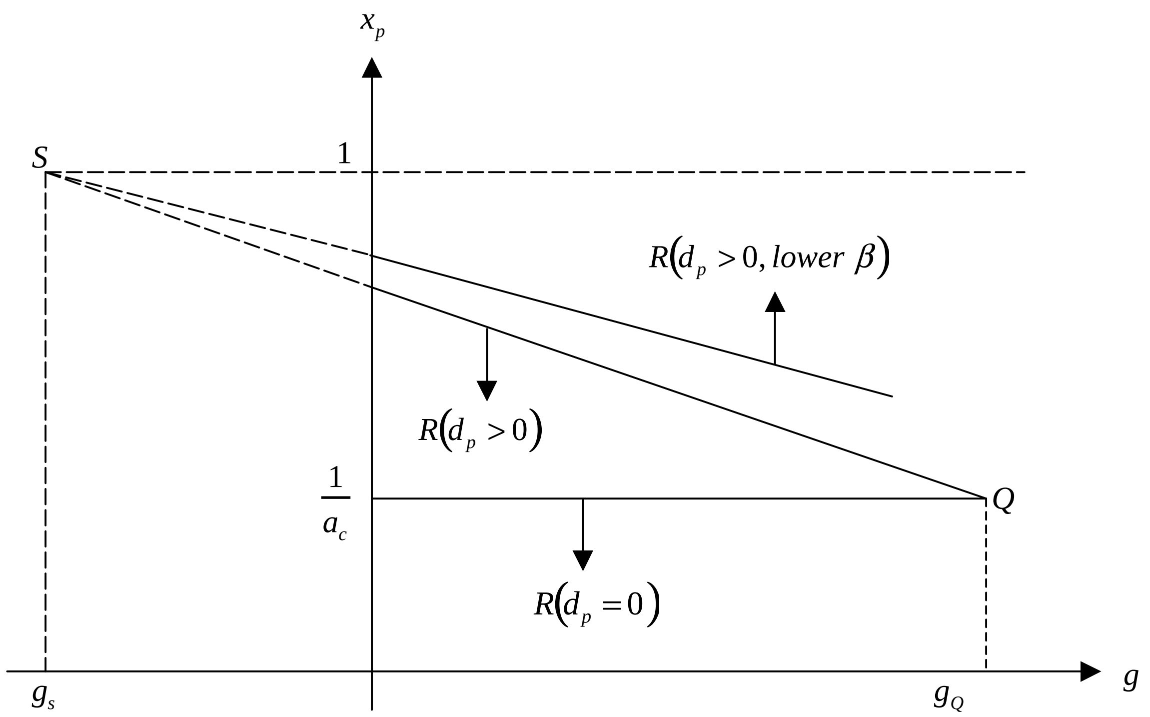


Figure 4: A unique general equilibrium

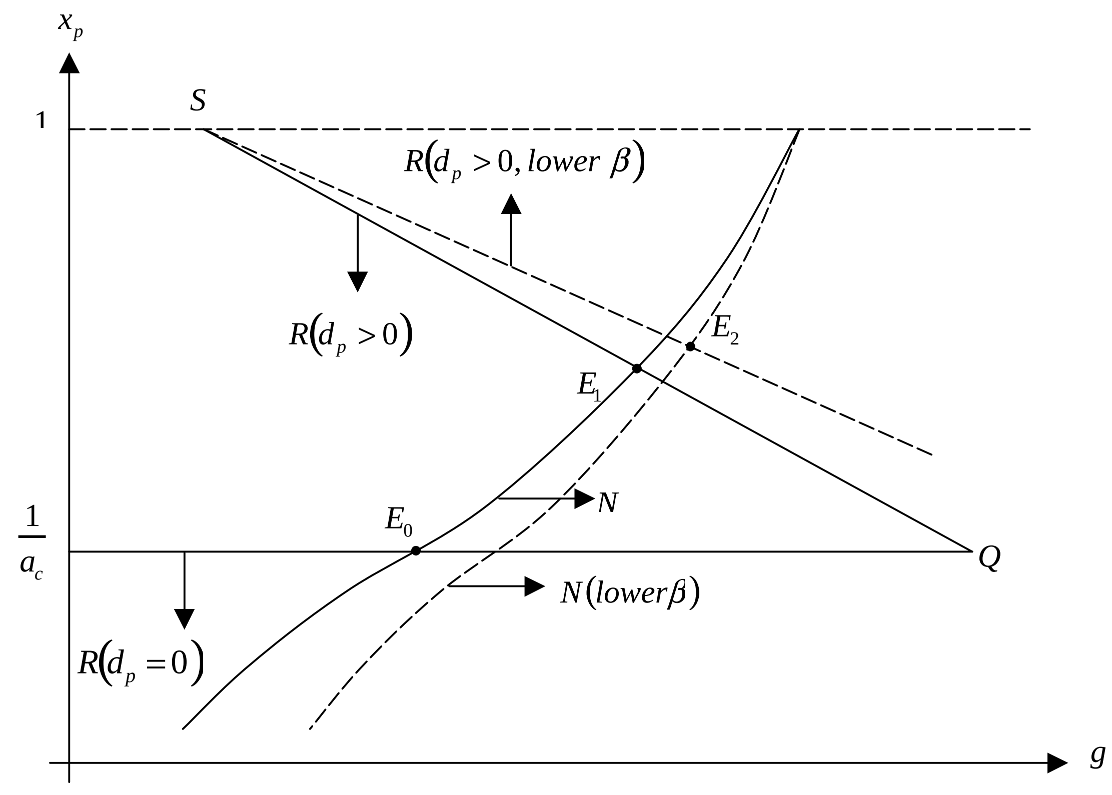


Figure 5a: Multiple steady states

$$\theta a_r / a_c < (1-\beta)L\pi$$

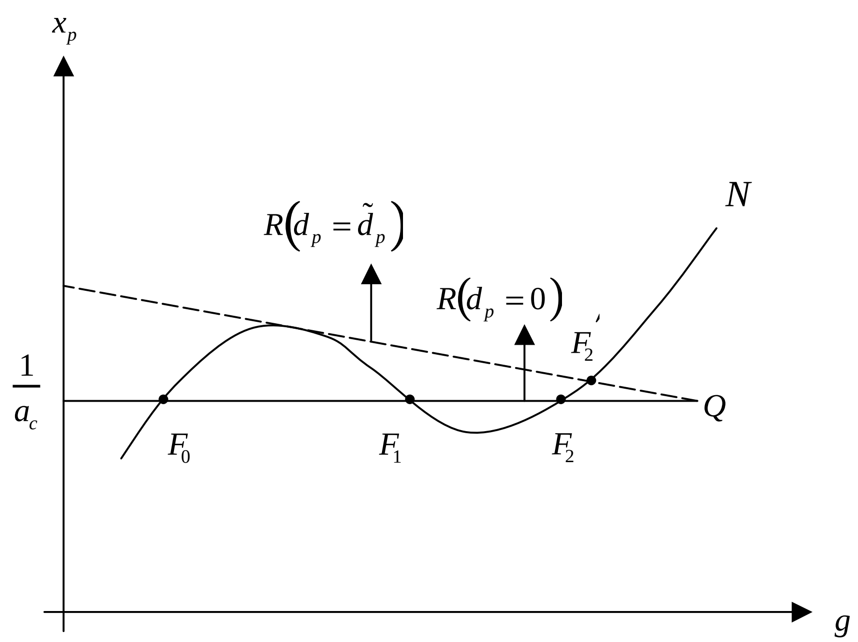


Figure 5b: Multiple steady states

$$\theta a_r / a_c > (1-\beta)L\pi$$

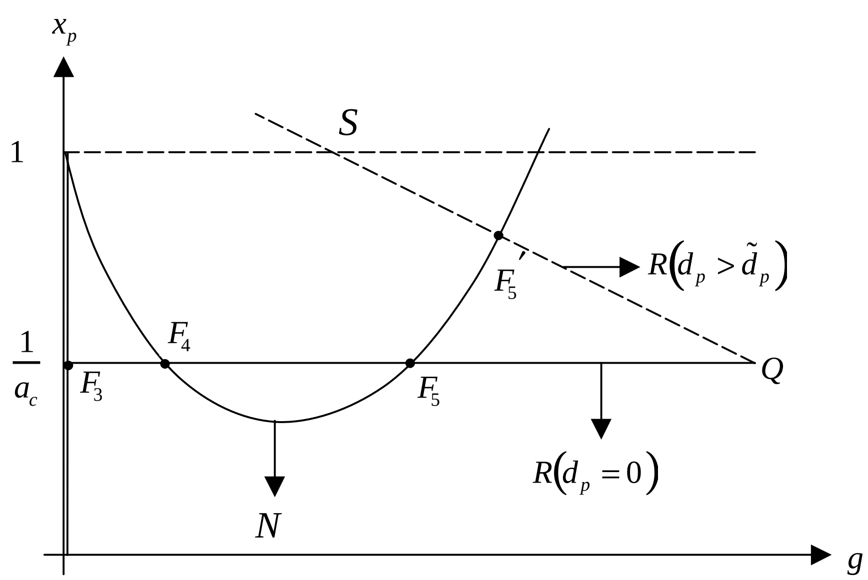
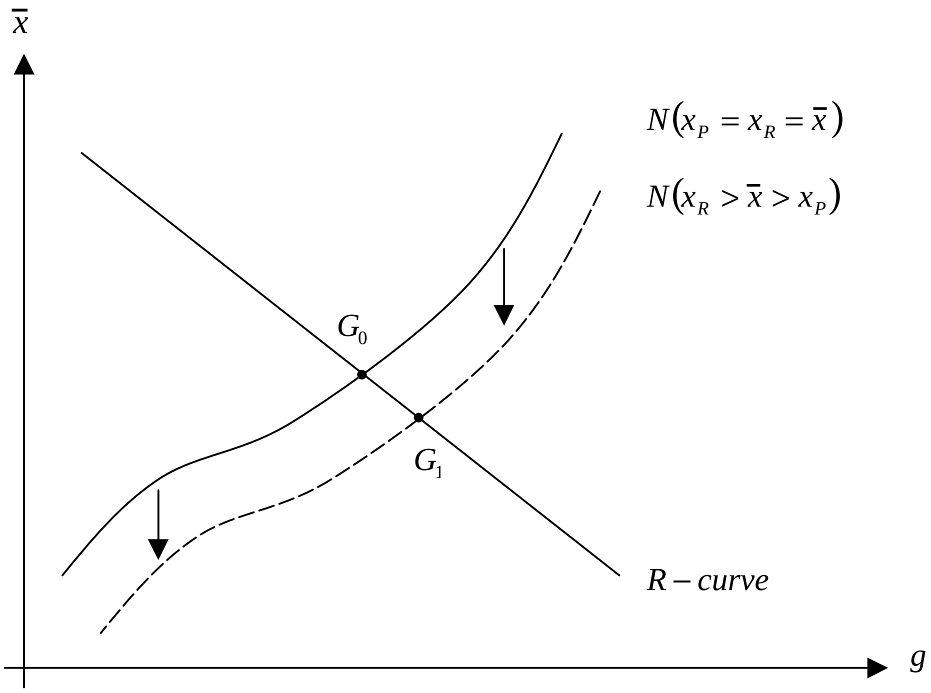


Figure 6: Equlibrium in Regime (c)



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1

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[1] For recent surveys on income distribution and growth see Benabou (1996), Aghion and Howitt (1998, chapters 9 and 10), and Bertola (forthcoming).

[2] See Jackson (1984) for direct evidence in favor of hierarchical structure of demand. Falkinger and Zweimüller (1996) provide similar evidence using aggregate consumption data from the International Comparison Project of the UN.

[3] It is interesting to note that the results in this paper encompass the results of two different papers by Murphy, Shleifer, and Vishny (1989a,b). In their "Big Push" paper, multiple equilibria are essential, but income inequality plays no role. In the paper on market size and income distribution, multiple equilibria cannot arise. In contrast, the conditions for multiple equilibria in my paper include a sufficient degree of inequality. Matsuyama (1993) studies multiple equilibria in a dynamic version of the "Big Push". Contrary to the present paper, multiple equilibria are driven by history, rather than by expectations and inequality-effects are not studied.

[4] See also Bourguignon (1990) for a static model of a dual economy where income distribution affects the equilibrium outcome because the composition of demand varies across income classes.

[5] The absence of imitators of the modern technology could be due to patent protection or some fixed imitation cost with ex-post Bertrand-competition.

[6] This will be an equilibrium outcome if reducing prices below one does not attract many additional consumers, that is if the price elasticity of demand is sufficiently low. The demand curve mirrors the income distribution, so the assumption in the text puts a restriction on the parameters of the distribution. In the special case with two groups there are always some firms with an incentive for price-cutting in the initial phase of the product cycle when additional consumers can be attracted. However, the two-group case serves illustrative purposes and the results carry over to the case of a general distribution in a straightforward way.

[7] This assumption puts a restriction on initial conditions. From the first-order condition to maximizing equation (1) subject to (2) (see below) we must have $\lambda_i(t) = 1/c_i(t)$, so $\lambda_R(t) < \lambda_P(t)$, and $\lambda_i(t)$ will decrease at the same rate as $c_i(t)$ increases. In equilibrium consumption is $c_i(t) = w(t) + \theta A_i(t)$ and the wage rate is $w(t) = n(t)/a_c$ (see equations (4) and (6)). Hence, $\lambda_i(t) < 1$ for all *i* and *t* requires $n(0) > a_c$.

[8] If the firm produces a good $j > c_R(t)$, analogous arguments as above lead to a value $v_j(t) = (L\pi/r)\left[(1-\beta)(c_R(t)/j)^{r/g} + \beta(c_P(t)/j)^{r/g}\right]$.

[9] The possibility of multiple equilibria depends on the curvature of the *B*-function. If $\theta \to 0$, the B-curve becomes a horizontal line, independent of *g*, and the equilibrium will be unique. It is instructive to consider necessary and sufficient conditions for an upward sloping *N*-curve for all *g* and $x_P \geq 1/a_c$. From Figure 1a, a unique zero-profit equilibrium for $x_P = 1/a_c$

implies uniqueness for $x_P > 1/a_c$. A necessary condition for uniqueness when $x_P = 1/a_c$ is $\theta a_r/a_c < (1-\beta)L\pi$. A sufficient condition is $(\tilde{g}+\theta)(a_r/a_c) < (1-\beta)L\pi$ where $\tilde{g}$ is the rate of growth where the $B(1/a_c, g)$-curve has its steepest slope. It is straightforward to verify that $\tilde{g} = \ln(a_c)\theta/2$. The sufficient condition is then $\theta a_r/a_c < \dfrac{(1-\beta)L\pi}{1+\ln(a_c)/2}$.

[10] Differentiating both sides of (8) with respect to g yields the 'stability' condition:

$a_r/a_c > -\ln x_P\left(\theta/g^2\right)\beta L\pi x_P{}^{(g+\theta)/g}$.

[11] Using the definition of $V(t)$ and the expression for the value of a firm $v_j(t)$ derived in section 3.1, it is straightforward to show that $V(t)$ grows at the same rate as $n(t)$ so that $\bar{v}$ is constant over time.

[12] $\theta = 0$ can serve only as a benchmark. In this case the integral in (1) is not defined and $x_R > 1$ does not hold.

[13] For a discussion of path-dependent versus expectation-determined equilibria see Krugman (1991) and Matsuyama (1991).

[14] An alternative way to solve for the equilibrium growth rate uses the resource constraint (7). Substituting (9) into (7) making use of the fact that $\bar{v} = a_r/a_c$ when $x_P > 1$ yields the expression for g^* in the text.

[15] This is a necessary but not a sufficient condition. If inequality is very large demand by the rich for goods produced by competitive firms is high. In that case a_m has to be significantly below unity so that the remaining resources, $(1-a_m)L$, can satisfy the demand for these other goods.

[16] When all consumers buy all *n(t)* goods in the monopolistic sector, the resulting labor demand is $a_m L$. A necessary condition is therefore $a_m < 1$.