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Jacob K. Goeree, Theo Offerman and Randolph Sloof

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Abstract

Multi-unit ascending auctions allow for equilibria in which bidders strategically reduce their demand and split the market at low prices. At the same time, they allow for preemptive bidding by incumbent bidders in a coordinated attempt to exclude entrants from the market. We consider an environment where both demand reduction and preemptive bidding are supported as equilibrium phenomena of the ascending auction. In a series of experiments, we compare its performance to that of the discriminatory auction. Strategic demand reduction is quite prevalent in the ascending auction even when entry imposes a (large) negative externality on incumbents. As a result, the ascending auction performs worse than the discriminatory auction both in terms of revenue and efficiency, while entrants.chances are similar across the two formats.

JEL Code: D44, D45, C91

Keywords: Multi-license auctions, demand reduction, external effects, preemption

*Goeree: Institute for Empirical Research in Economics, University of Zürich, Blümlisalpstrasse 10, CH-8006, Zürich, Switzerland, and Division of the Humanities and Social Sciences, California Institute of Technology, Mail code 228-77, Pasadena, CA 91125, USA. Offerman and Sloof: Department of Economics, University of Amsterdam, Roetersstraat 11, 1018 WB Amsterdam, The Netherlands. We gratefully acknowledge .nancial support from the National Science Foundation (SBR-0098400), the Alfred P. Sloan Foundation, the Dutch Royal Academy of Sciences (KNAW) and the Dutch National Science Foundation (NWO-VICI 453.03.606). We are grateful for the helpful comments of three anonymous referees and the editor. We thank CREED programmer Jos Theelen for programming the experiment.

1 Introduction

Following the successful US spectrum auctions in 1994, highly valuable public assets around the world are now assigned by auction. Gas stations, airport slots, phone numbers, and telecommunication frequencies have been put up for bid, mostly using some variant of the open ascending auction. However, despite their popularity, open ascending auctions are known to be vulnerable to collusion, sometimes allowing bidders to split the market at low prices.[1] Before the 2001 Austrian UMTS auction, for example, the largest incumbent, Telekom Austria, announced it "... would be satisfied with just two out of the twelve blocks for offer and if the [five] other bidders behaved similarly, it should be possible to get the frequencies on sensible terms ... but that it would bid on a third block if one of its rivals did..." Other bidders understood the hint and bidding stopped after a couple of rounds at low prices with each bidder obtaining just two blocks (Klemperer 2004, p. 136). Several papers have demonstrated that this type of demand reduction can be supported in an equilibrium of the sealed-bid uniform-price auctions, see, e.g., Noussair (1995), Engelbrecht-Wiggans and Kahn (1998), and Ausubel and Cramton (1998). As Ausubel and Cramton (1998) note, such a result can usually be adapted to apply for the open ascending auction.[2]

Incentives for demand reduction are likely affected when incumbents compete with possible entrants for a fixed number of market licenses. Entrants typically impose a negative externality on incumbents in the ensuing product market and in an attempt to keep the entrants out, incumbents may engage in "predatory bidding" and drive up license prices beyond their economic values. Open ascending auctions are particularly conducive to predatory bidding as they allow incumbents to coordinate their attempts to keep an entrant out. One example of successful preemptive bidding occurred in the US C&F spectrum auction. After round 14, only

[1] For instance, in his analysis of bidding behavior in the FCC's AB-block auction, Weber (1997) finds evidence that the large bidders dropped out of some markets at prices far below market expectations. See also Cramton and Schwartz (2000).

[2] Demand reduction has also been observed in the laboratory. Alsemgeest, Noussair, and Olson (1998), for instance, compare the open ascending auction to a sealed-bid auction where the price for each unit equals the lowest accepted bid. They find evidence for demand reduction in the open ascending auction, which generally yields lower revenues than the sealed-bid format. Kagel and Levin (2001) consider environments without strategic uncertainty where a single human bidder competes against robot bidders in a sealed-bid uniform-price auction, an open ascending auction, and the Vickrey/Ausubel auction. Demand reduction occurs in both the sealed-bid uniform-price and the open ascending auction, but the level of demand reduction is more pronounced in the latter. Kagel and Levin (2005) obtain a similar finding for a setting where the single human bidder faces a tradeoff between bidding above value due to synergies and demand reduction. Engelmann and Grimm (2009) compare five auction formats: the sealed-bid uniform-price, the open ascending, the discriminatory, the Vickrey, and the Ausubel auction. They observe more demand reduction in the open ascending auction than in the sealed-bid uniform-price auction. Finally, List and Lucking-Reiley (2000) conduct a field experiment with sports cards and find evidence for demand reduction in the sealed-bid uniform-price auction, although revenues do not differ from those of a Vickrey auction because bidders bid too high on the first unit. Pooling the results of these different studies suggests that demand reduction is more pronounced in open ascending auctions than in sealed-bid uniform-price auctions.

Verizon, Cingular, and AT&T were competing for the three available licenses in New York. At this point prices were \$782 million but Verizon continued to bid for two licenses until Cingular dropped out, resulting in license prices in excess of \$2 billion (Cramton 2002).[3]

When both incumbent and entrant bidders are present, a revenue-maximizing seller thus has to gauge the likelihood of demand reduction versus preemptive bidding in the open ascending auction.[4] The drawing power of either equilibrium is essentially an empirical issue, which is complicated by the existence of "cheap preemptive equilibria." In such an equilibrium, incumbents first try to keep entrants out of the market but when preemption turns out to be rather costly they reduce demand and split the market at an intermediate price level. In the German UMTS auction, for instance, Deutsche Telekom, one of the incumbents, continued pushing up the price when the market could be split among the six active bidders but it later ended the auction before any of its competitors had conceded, paying an extra \$2 billion for the two blocks it could have acquired before. This sequence of events surprised many and some even interpreted it as evidence of irrational behavior by Deutsche Telekom.[5] Ewerhart and Moldovanu (2001) show, however, that preemption followed by demand reduction can be rationalized as equilibrium behavior.[6]

This paper presents the first experimental study of the impact of negative externalities on outcomes of the open ascending auction. In this auction, the auctioneer steadily raises the price for the goods for sale and the bidders decide at what prices they want to reduce demand. A bidder's decision to reduce demand is irrevocable and observed by the other bidders. The auction stops when demand equals supply. For brevity we will simply refer to this auction as the ascending auction. In the environment we consider, two incumbents compete with one entrant for six identical licenses. We assume bidders have flat demands for the licenses offered, i.e. a bidder's independent private value applies to each license bought. Every bidder can buy at most three licenses and if the entrant acquires one or more licenses, both incumbents incur a negative externality (even when an incumbent buys no license). Hence, the only way to avoid the negative externality is for the incumbents to each buy three licenses and keep the entrant out of the market. We consider three different regimes: no externality, a weak externality, and a strong externality.

We prove there exists a continuum of "cheap preemptive equilibria" in all three regimes – ranging from a "pure" demand reduction equilibrium to a "pure" preemptive equilibrium. We

[3] See Jehiel, Moldovanu, and Stacchetti (1996, 1999), Jehiel and Moldovanu (2000) and Das Varma (2002, 2003) for a theoretical analysis of bidding behavior in the presence of externalities.

[4] A similar evaluation has to be made by a seller interested in maximizing efficiency or entry.

[5] Klemperer (2004, p. 159, footnote 27) compares Deutsche Telekom's behavior to that of someone who waits in a queue for a long time but then quits in frustration before it is his turn.

[6] Klemperer (2004, p. 202) criticizes the assumption made by Ewerhart and Moldovanu (2001) that there is only a single strong bidder; their model cannot explain why initially both Deutsche Telekom and Mannesman pushed up the price but then stopped doing so.

thus extend the analysis of Ewerhart and Moldovanu (2001) beyond the particular setting of the German UMTS auction and show that preemptive bidding followed by demand reduction can occur more generally, e.g. with multiple symmetric incumbent bidders. Our experimental setting is conducive to any of these equilibria, all of which prescribe identical strategies to players of the same type. Intuitively, one might expect that demand reduction becomes less focal when the negative externality becomes stronger. Our experiment provides a controlled way to evaluate this conjecture empirically.

The equilibrium-selection issue present in ascending auctions translates into large uncertainty about revenues. This may explain why some high-stakes license auctions employ a simpler sealed-bid format where high bidders pay their own bids. In some instances, such discriminatory auctions have performed remarkably well. For example, in the Brazilian auction for wireless telephone services, a consortium including BellSouth and Splice do Brazil submitted the winning bid of \$2.45 billion for the license covering Sao Paulo. This bid was 60% higher than the second highest bid, i.e. about \$1 billion was left on the table (Milgrom 2004, p. 17). In another instance, Spain's biggest bank BSCH won the Sao Paulo state bank Banespa for a bid of \$3.6 billion. To the embarassment of the managers of the Spanish bank, the second highest bid was only \$1.1 billion, leaving \$2.5 billion on the table (Klemperer 2004, p. 136). While these examples are somewhat extreme, they show that the discriminatory auction may outperform the ascending auction in terms of revenues.

Our experiments also compare the ascending auction to the discriminatory auction.[7] Discriminatory auctions do not support demand reduction in equilibrium. Moreover, preemptive bidding plays less of a role in that entrants have better chances than in the preemptive equilibrium of the ascending auction. A comparison of the performance of the two formats thus hinges on the type of equilibrium selected in the ascending auction. Our experimental results indicate that demand reduction occurs even when incumbents' incentives are to keep the entrant out of the market. In particular, while the presence of a negative externality makes strategic demand reduction less focal in the ascending auctions, it is always more prevalent than preemptive bidding. Because demand reduction is so wide-spread, the ascending auction is outperformed by the discriminatory auction in terms of revenue and efficiency while entry levels are similar.

The remainder of the paper is organized as follows. Section 2 details the auction formats, the experimental design, and procedures. Section 3 presents the theoretical analysis for our setup. In Section 4 we discuss the experimental results. Section 5 concludes.

[7] We chose to focus attention to the two auction formats that are most often observed in practice. The optimal format is unknown for our setting, see Jehiel, Moldovanu, and Stacchetti (1999).

2 Experimental design

The experiment was computerized. Subjects started with on-screen instructions. They also received a summary of the instructions on paper (see Appendix A). The actual experiment consisted of two parts. Part 1 started with a practice period followed by 15 periods. Part 2 consisted of 1 period only. Subjects received instructions for the second part only after the first part was completed. They earned points in each (real) period of the experiment, which were exchanged at the end of the experiment at a rate of 1 euro for 80 points. Table 1 summarizes the details of the 6 treatments. Each subject participated in one treatment only.

The setup of our experiment is roughly consistent with the actual German UMTS license auction, where 12 blocks of spectrum were put up for sale from which bidders could create licenses of either 2 or 3 blocks, so that the market would be split between 6 or 4 firms, respectively. Basically, we consider a scaled down version of the German UMTS auction. In our experiment, 6 identical goods were sold to 3 bidders. Each bidder could buy at most 3 goods so that the market could be split between 2 or 3 bidders. Restrictions on the number of licenses are common in licence auctions, because they help to prevent the creation of an excess of market power.

Subjects were assigned to the same group of 3 bidders throughout the whole first part. In each period, 6 identical goods were sold to the 3 subjects of a group. Subjects submitted bids in accordance with the auction rules of the treatment and at the end of each period the goods were assigned to the winning bidders.

At the start of each period, each subject received one integer private value from the $U[0, 100]$ distribution, which was valid for each of the 6 goods being offered for sale. Subjects derived a constant marginal payoff equal to their private value for each good bought. This feature of the design is roughly consistent with the practice of license auctions. Klemperer (2002) concludes from the bidding in the UK UMTS auctions that most bidders value large licenses consisting of 3 blocks more than small licenses of 2 blocks. We chose constant marginal costs to keep the design and the instructions simple and we leave it to future work to investigate other interesting cases, for instance the one where marginal costs decrease. Subjects were only informed about their own private value and private value draws were independent across subjects and periods. All these rules were common knowledge. We kept the private value draws constant across treatments. Thus, differences between treatments cannot be attributed to differences in draws.

We used three levels of the negative external effect: $x = 0$, $x = 50$ and $x = 100$. The external effect was kept constant within a treatment. Each subject was assigned a fixed role that she kept during the whole experiment. For the treatments with $x = 0$ subjects were assigned to 'symmetric' roles of Types A, B and C. In the treatments with $x > 0$, Types A and B represented the incumbents and Type X represented the entrant. Each bidder received

Table 1: Experimental design

treatment	auction	external effect	# groups	size	composition	# licenses	restriction	private values
asc0	ascending	$x = 0$	8	3	3 sym	6	3	$U[0, 100]$
asc50	ascending	$x = 50$	8	3	2 inc, 1 entr	6	3	$U[0, 100]$
asc100	ascending	$x = 100$	8	3	2 inc, 1 entr	6	3	$U[0, 100]$
disc0	discriminatory	$x = 0$	8	3	3 sym	6	3	$U[0, 100]$
disc50	discriminatory	$x = 50$	8	3	2 inc, 1 entr	6	3	$U[0, 100]$
disc100	discriminatory	$x = 100$	8	3	2 inc, 1 entr	6	3	$U[0, 100]$

a profit on purchases equal to the number of goods bought times the bidder's private value minus the sum of the prices paid for the goods. Type X's profits were entirely determined by the profit margins on the goods bought. Types A and B knew that if Type X would buy 1, 2 or 3 goods, an amount of 50 (100) points would be subtracted from their profits on purchases when $x = 50$ ($x = 100$). The negative external effect was also inflicted upon an incumbent when she did not buy any good herself. So there was no escape from an external effect once it occurred. The only possibility to prevent the negative external effect was to keep the entrant completely out of the market.

Our modeling of the external effect was based on the idea that in most license auctions winners engage in a form of Bertrand competition in the aftermarket. With pure Bertrand competition, theory predicts that the price-level is not dependent on the number of competitors. This prediction seems at odds with real-life observations and it is indeed rejected in an experimental study by Dufwenberg and Gneezy (2000). They find that in groups of 3 or 4 competitors, winning price bids converged rather rapidly to the marginal cost. With 2 competitors prices remained much higher than the marginal costs. Dufwenberg and Gneezy offer an explanation based on bounded rationality. If with some small probability any firm prices differently from the Bertrand prediction, then the predictions will depend on the number of competitors. Thus, with 3 competitors in the aftermarket, lower prices and profits will result compared to the case where the entrant is successfully kept out of the market by the incumbents. With Bertrand competition, a newcomer may drive down consumer prices to the competitive level independent of the number of licenses it acquires. Consistent with this possibility, we chose the external effect to be independent of the number of licenses acquired by the entrant in our reduced form model. An alternative justification of a negative external effect of the newcomer on the incumbent is provided by Spulber (1995), who shows that an increase in competition should result in lower prices in an extended Bertrand model where competitors possess private information about marginal costs. Abbink and Brandts (2005) provide experimental support for the main prediction of that model.

Notice that subjects in the treatments with $x > 0$ could easily lose money in some periods because of the external effect. Therefore, we provided subjects with a starting capital that they did not have to pay back after the experiment. The starting capital in treatments $x = 0$, $x = 50$ and $x = 100$ equalled 200, 750 and 1500 points, respectively, for incumbents as well as for entrants. Subjects knew that if they finished the experiment with a negative balance they would go home without any money. It never happened that a subject's balance actually became negative. The case of $x = 100$ represents a substantive negative external effect because incumbents in the discriminatory auction would actually make an average loss of 9.9 points per period if they played according to the Nash equilibrium.

Part 2 lasted for just a single period. To cover potential losses in this part, at the beginning of part 2 subjects received an additional bonus of 500 (1000) points in the treatments with $x = 50$ ($x = 100$). The only difference between a period of part 1 and the period of part 2 was that the payoff in part 2 was automatically multiplied by 10. Subjects thus played for much more money in the period of part 2. For statistical reasons we kept the group compositions the same as in part 1. We did not inform subjects about this aspect and none of them asked about it. The auction format and the level of the external effect were kept constant across parts.

In the ascending auctions, bidders first simultaneously submitted their 'initially demanded quantity'. This initial demand represented the number of goods on which they wanted to start bidding. It had to be an integer number from the set $\{0, 1, 2, 3\}$. When the sum of initial demands within a group was less than or equal to 6, the period ended immediately and all bidders received their requested goods at a price of zero.[8] In case the sum of initial demands was greater than 6, a thermometer (or clock) started rising point-by-point from 0 points onwards at a pace of 1 point per second. The thermometer's 'temperature' showed the price level that all active bidders were prepared to pay for the number of goods that they still demanded. Bidders could reduce their demand at any price level. The thermometer continued to rise until a price level was reached where total demand was equal to 6. This was the price that each bidder had to pay for all the goods assigned to her in accordance with her quantity demanded.

When the thermometer started rising, each bidder was and remained completely informed about each of the demanded quantities of the other two bidders. In case one of the bidders decreased her demand, the thermometer halted for four seconds to give the bidders the possibility to process the information. Bidders were not able to increase their demand within a period. They only had the possibility to reduce demand by 1, 2 or 3 units at a time, as long as this would not decrease total demand below 6. So like in actual license auctions, bidders could not increase their activity during the auction and they could also not withdraw bids that belonged to the provisionally winning bids. The computer kept track of how much a bidder could reduce her demanded quantity. At the end of a period bidders were informed about their own earnings but not about the earnings of others.

In the discriminatory auctions, bidders simultaneously submitted three (integer) bids. They also had the possibility to bid on fewer than 3 goods.[9] All the bids in a group were ordered and the 6 goods were assigned to the 6 highest bids.[10] The bidder who submitted a highest bid

[8]In accordance with the usual practice of license auctions, we did not use reserve prices in either auction format. In the recent 3G auctions, most countries refrained from setting a reserve price (Netherlands) or they set very low reserve prices (e.g., Germany, Austria, Switzerland, Italy). As Klemperer (2003) notes, "[But] serious reserve prices are often unpopular with politicians and bureaucrats who –even if they have the information to set them sensibly– are often reluctant to run even a tiny risk of not selling the objects, which outcome they fear would be seen as a 'failure'."

[9]In all treatments, we had a lower bound of 0 on subjects' bids. In the treatments with $x = 0$, $x = 50$ and $x = 100$ the upper limit of subjects' bids was equal to respectively 100, 125 and 150. The upper limit was never reached in the experiments.

[10]Notice that in both auctions there was a possibility that some goods remained unsold in a period. In the

bought the good at a price equal to the amount bid. In case of tied bids the bids were ordered on the basis of a lottery. At the end of a period bidders were informed about the number of goods that they bought, the price that they paid, whether the entrant entered (if applicable) and their own earnings but they were not informed about the earnings of others.

In total we recruited 144 subjects from the student population of the University of Amsterdam. The subjects were equally divided over the 6 treatments, so we obtained per treatment data on 8 independent groups of 3 subjects each. The experiment lasted for about one and a half hours. Subjects earned on average 30.80 euros with a minimum of 5.10 euros and a maximum of 80.30 euros. Subjects in the role of entrant earned with an average of 42.95 euros more than subjects in the role of incumbent with an average of 26.60 euros.

3 Equilibrium predictions

In section 3.1 we discuss, for the environment described above, the demand-reduction and (cheap) preemptive equilibrium outcomes for the ascending auction. In section 3.2 we discuss the preemptive equilibrium for the discriminatory auction. For ease of exposition we consider bidder values that are uniformly distributed on [0,1] rather than on [0,100], i.e. we choose dollar units rather than pennies so that values and bids are scaled by 1/100. Consequently, the external effect used in the experiment is $x = 0$, $x = \frac{1}{2}$, and $x = 1$.

3.1 Ascending auction

Because of its dynamic nature, the relevant solution concept for the ascending auction is the perfect Bayesian equilibrium (PBE), which specifies a strategy profile and a set of beliefs such that strategies are optimal given beliefs and beliefs are "consistent," i.e. they follow from Bayes' rule where applicable. As usual this leaves some room for the specification of off-the-equilibrium-path beliefs, but once a deviation occurs, subsequent play again has to follow the logic of sequential rationality dictated by PBE. Below we show that there are multiple belief/strategy systems that form a PBE of the ascending auction, some that involve demand reduction and others that involve preemptive behavior and some that involve both.

First, consider for the symmetric case with no externality ($x = 0$) the "competitive" strategy profile in which each bidder demands three units until the price reaches the bidder's value at which point the bidder reduces demand to zero units. It is easy to derive bidders' on-the-equilibrium-path beliefs: at price p, a bidder's beliefs about a rival's value is that it is uniformly distributed on $[p, 1]$. To show that the competitive strategy profile can be part of

ascending auctions this happened when the sum of the initially demanded quantities was less than 6. In the discriminatory auctions this would occur when in total fewer than 6 bids were submitted.

a PBE, we have to consider deviations from equilibrium play and specify corresponding off-the-equilibrium-path beliefs to show that such deviations are not profitable. Suppose that at price level p, bidder 3 reduces demand from three units to a single unit (we deal with the case in which demand is reduced to two units below) and let $f_i(v_3|p)$ denote the beliefs of bidders $i = 1, 2$ about bidder 3's value once this deviation occurs. Depending on bidder i's belief, it may be optimal for bidder i to continue bidding or to reduce demand to two units in which case the auction stops. In the latter case bidder 3's deviation can be profitable,[11] which would destroy the competitive PBE. In other words, for the competitive strategy profile to be part of a PBE, bidders 1 and 2 have to believe it is optimal for them to continue bidding after bidder 3 deviates by reducing demand to a single unit at price p. This occurs when bidders 1 and 2 think that the deviation indicates that bidder 3 is relatively "weak:"

$$f_i(v_3|p) = \frac{\lambda e^{-\lambda(v_3-p)}}{1-e^{-\lambda(1-p)}}, \tag{1}$$

for $p \leq v_3 \leq 1$ and $\lambda > 0$, i.e. bidders 1 and 2 believe that bidder 3's value is (exponentially) skewed towards lower values on $[p, 1]$.

We next show that given these off-the-equilibrium-path beliefs, bidders 1 and 2 will demand three units until the price is (arbitrarily close to) their values, *as they would have if no deviation had occurred.* Suppose not and bidder i reduces demand to two units at some price level p' where $p \leq p' < v_i$. The marginal gain of reducing demand a little later, say at price $p' + \epsilon$, is that one of the other two bidders reduces demand by one unit between p' and $p' + \epsilon$ and an additional unit is won. The marginal cost is that bidder i has to pay ϵ more for the two units. Hence, the net gain is

$$\epsilon \cdot \Big\{(v_i - p')\big(f_i(p'|p') + \frac{1}{1-p'}\big) - 2\Big\},$$

where the first (second) term in parentheses occurs when bidder 3 (bidder $j \neq i, 3$) reduces demand by one unit between p' and $p' + \epsilon$. Since $f_i(p'|p') > \lambda$, the marginal gain of reducing demand a little later is strictly positive for all $p' < v_i - 2/\lambda$. Hence, for large λ, it is optimal for bidder i to keep demanding three units until the price level is (arbitrarily close) to her value.

To summarize, given the off-the-equilibrium-path beliefs in (1), bidders 1 and 2 continue demanding three units until the price reaches their values. In other words, their behavior is unaffected by bidder 3's deviation, which is therefore not profitable: if, after deviating, bidder 3 wins one unit at some price less than her value then she could have won three units at that price had she not deviated. For the same reason, the off-the-equilibrium-path beliefs in (1) ensure that it is not profitable for bidder 3 to reduce demand to two units (instead of one unit)

[11] Note that in the competitive PBE, the probability that a bidder with value v_3 wins three units is $1-(1-v_3)^2$, i.e. the chance that the bidder's value is not the lowest, and a simple envelope argument establishes that her equilibrium payoffs are $3v_3^2 - v_3^3$. The payoff from deviating is v_3 if bidder 3 reduces demand to 1 unit at $p = 0$ and another bidder reduces demand to two units in response. For small v_3, the deviation is therefore profitable.

as such a deviation makes it even less attractive for bidder $i = 1, 2$ to reduce demand early (i.e. before the price reaches their values). We thus have:

Lemma 1. *With no externality ($x = 0$), the competitive strategy profile in which each bidder demands three licenses until the price level reaches her value (plus a consistent belief system) is a perfect-Bayesian equilibrium of the ascending auction.*

As is typical for dynamic games, the competitive PBE is not the unique equilibrium of the ascending auction. We next consider the possibility of a demand reduction equilibrium, i.e. a symmetric strategy profile where all three bidders behave competitively up to some price level p and then all bidders reduce their demand to two units. To verify that this is a PBE, suppose that if bidder 3 deviates and continues demanding three units when the price reaches p then the beliefs of bidders $i = 1, 2$ are again given by (1) so that they bid competitively after bidder 3's deviation. We next verify that bidder 3's deviation is not profitable. Bidder 3's payoff of deviating is $\max_b(\pi_3(b|v_3, p))$, where $\pi_3(b|v_3, p)$ denotes bidder 3's payoff when she is willing to keep bidding on three licenses until the price level reaches $b \geq p$ after which she "stops," i.e. reduces her demand to two licenses, and the auction ends.[12]

$$\begin{aligned}\pi_3(b|v_3, p) &= 3\int_p^b \int_p^b (v_3 - \min(v_1, v_2))dv_1 dv_2 \\ &+ 3\int_p^b \int_b^1 (v_3 - v_1)dv_1 dv_2 + 3\int_b^1 \int_p^b (v_3 - v_2)dv_1 dv_2 \\ &+ 2\int_b^1 \int_b^1 (v_3 - b)dv_1 dv_2. \end{aligned} \tag{2}$$

Here the top line corresponds to the case where bidder 3 has the highest-value, in the middle line bidder 3 has the middle value, and in the bottom line bidder 3 has the lowest value. It is readily verified that

$$\pi_3'(b|v_3, p) = -2(1 - b)(1 - v_3) \leq 0, \tag{3}$$

so bidder 3's payoff is maximized by choosing $b = p$, i.e. reducing demand to two licenses at price p. We thus have:

Lemma 2. *With no externality ($x = 0$), the strategy profile in which each bidder bids competitively until the price level reaches $0 \leq p \leq 1$ and then reduces demand to two units (plus a consistent belief system) is a perfect-Bayesian equilibrium of the ascending auction.*

Note that for $p = 1$, the PBE of Lemma 2 reproduces that of Lemma 1.

[12] Recall that if the other bidders each demand two licenses and bidder 1 initially demands three licenses, she cannot lower her demand below two licenses since total demand cannot fall below the total supply of six licenses.

Now suppose the entrant imposes a negative externality ($x > 0$) on the incumbents when she wins a license. Also in this case there exists a continuum of PBEs parameterized by a price level $0 \leq p \leq 1$ at which all bidders reduce demand to two units. The intuition is that the demand reduction part of the PBEs described in Lemma 2, i.e. the part that applies once the price reaches p, can be sustained when $x > 0$ since no incumbent bidder can avoid the negative externality once the other two bidders demand only two units. (Once the price level reaches p, the negative externality becomes a sunk cost that does not affect an incumbent's optimization problem.) However, when $x > 0$, it is not necessarily optimal for an incumbent bidder to demand three units until the price reaches her value and then reduce demand to zero units. Since incumbents profit from excluding the entrant from the market, it can be optimal for them to continue demanding three units at even higher prices.

We next derive the set of *cheap preemptive* PBEs, where an incumbent bidder with value v demands three units up to a price level $B_I(v)$ that may exceed her value and all bidders reduce their demand to two units if the price level reaches p. The differential equation that determines $B_I(v)$ can be derived from a simple marginal argument. Suppose an incumbent who has value v acts as if her value were $v + \epsilon$. Such a deviation alters the outcome of the auction only if the bidder turns from a loser into one of the winners. This requires that either (i) the other incumbent has a value between v and $v + \epsilon$ and the entrant has a value higher than $B_I(v)$, or (ii) the entrant has a value between $B_I(v)$ and $B_I(v + \epsilon)$ and the other incumbent has a value higher than v. The former case happens with probability $\epsilon(1 - B_I(v))$ and the bidder's net gain of deviating from v to $v + \epsilon$ in this case would be $3v - 3B_I(v)$. The latter case happens with probability $\epsilon B_I'(v)(1 - v)$ and the net gain would be $3v - 3B_I(v) + x$. In equilibrium, the total net gain should be zero:

$$(1 - B_I(v))(3v - 3B_I(v)) + B_I'(v)(1 - v)(3v - 3B_I(v) + x) \;=\; 0. \tag{4}$$

The incumbent's optimal bid function is uniquely determined by this first-order condition and the boundary condition $B_I(p) = p$.[13]

To solve the first-order condition in (4) it will prove useful to consider a related differential equation

$$zG'(z) = 2(1 + G(z) - z^2 G(z)^2), \tag{5}$$

with general solution

$$G_{\alpha_{p,x}}(z) \;=\; \frac{1}{z}\,\frac{I_1(2z) + \alpha_{p,x} K_1(2z)}{I_2(2z) - \alpha_{p,x} K_2(2z)} \tag{6}$$

for $z \geq 0$. Here I_n (K_n) is the n^{th} modified Bessel function of the first (second) kind and $\alpha_{p,x}$ is a constant chosen such that the boundary condition $B_I(p) = p$ is satisfied. In the proof of

[13] Suppose instead that $B_I(v) = p$ where $v < p$. As the price level approaches p, an incumbent of type v would be better off reducing demand to two units slightly before p and incur the negative externality, rather than waiting to reduce demand to two units at p since then she also incurs the negative externality plus $2(v - p) < 0$.

Proposition 1 below we show that this boundary condition is met if $\alpha_{p,x} = \alpha\left(\frac{1-p}{x/3}\right)$ where we define

$$\alpha(z) \;=\; \frac{I_2(2z) - (1-1/z)I_1(2z)}{K_2(2z) + (1-1/z)K_1(2z)} \tag{7}$$

Lemma 3. *$G_{\alpha_{p,x}}(\cdot)$ and $\alpha(\cdot)$ satisfy the following properties:*

(i) *$\alpha(z)$ is strictly increasing in z with $\alpha(0) = 0$.*

(ii) *$G_{\alpha_{p,x}}(z)$ has an asymptote at $z = z^*$ where z^* solves $I_2(2z^*)/K_2(2z^*) = \alpha_{p,x}$.*

(iii) *The inverse $G^{(-1)}_{\alpha_{p,x}}(z)$ is well defined for $z \in \mathcal{D}$ where*

$$\mathcal{D} \;=\; \begin{cases} [\frac{x/3}{1-x/3}, \frac{x/3}{1-x/3-p}] & \text{if} \quad p < 1 - x/3 \\ [\frac{x/3}{1-x/3}, \infty) \cup (-\infty, \frac{x/3}{1-x/3-p}] & \text{if} \quad p > 1 - x/3 \end{cases}$$

and $G^{(-1)}_{\alpha_{p,x}}(z)$ for $z \in \mathcal{D}$ is minimized at $z = \frac{x/3}{1-x/3-p}$ with $G^{(-1)}_{\alpha_{p,x}}(\frac{x/3}{1-x/3-p}) = \frac{1-p}{x/3}$.

(iv) *For $z \in Int(\mathcal{D})$ we have*

$$\frac{1}{z}\Big(\frac{1}{z} + 1\Big) \;<\; G^{(-1)}_{\alpha_{p,x}}(z)^2 \;<\; \Big(\frac{1}{z} + 1\Big)^2 \tag{8}$$

Proof. Properties (i) and (ii) can be verified by plotting the right side of (7) and the ratio of modified Bessel functions $I_2(2z)/K_2(2z)$. To establish (iii) note that if $G_{\alpha_{p,x}}(z) \geq 0$ then (6) implies that $G_{\alpha_{p,x}}(z) \geq G_0(z) \equiv I_1(2z)/(zI_2(2z))$ and using standard properties of the modified Bessel functions we have

$$\frac{1}{G_0(z)}\Big(\frac{1}{G_0(z)} + 1\Big) \;=\; \Big(\frac{I_0(2z)I_2(2z)}{I_1(2z)I_1(2z)}\Big)\, z^2 \;<\; z^2$$

Since $G_{\alpha_{p,x}}(z) \geq G_0(z)$ we conclude that

$$\frac{1}{G_{\alpha_{p,x}}(z)}\Big(\frac{1}{G_{\alpha_{p,x}}(z)} + 1\Big) \;<\; z^2 \tag{9}$$

when $G_{\alpha_{p,x}}(z) \geq 0$. Moreover, inequality (9) is (trivially) satisfied when $G_{\alpha_{p,x}}(z) \leq -1$ since then the left side is non-positive. Combined with (5) inequality (9) implies that $G_{\alpha_{p,x}}(z)$ is strictly decreasing, and, hence, invertible when $G_{\alpha_{p,x}}(z) \leq -1$ or $G_{\alpha_p}(z) \geq 0$. This proves that $G^{(-1)}_{\alpha_{p,x}}(z)$ is well defined for $z \in \mathcal{D}$ since $\mathcal{D} \subseteq [0,\infty) \cup (-\infty,-1]$ for all $0 \leq p \leq 1$ and $x > 0$. If $p < 1 - x/3$ then $G^{(-1)}_{\alpha_{p,x}}(z)$ is strictly decreasing on $[\frac{x/3}{1-x/3}, \frac{x/3}{1-x/3-p}]$ and is thus minimized at $z = \frac{x/3}{1-x/3-p}$. If $p > 1 - x/3$ then $G^{(-1)}_{\alpha_{p,x}}(z)$ declines on $[\frac{x/3}{1-x/3}, \infty)$ and it declines on $(-\infty, \frac{x/3}{1-x/3-p}]$ with $G^{(-1)}_{\alpha_{p,x}}(\pm\infty) = z^*$, so the minimum is again attained at $z = \frac{x/3}{1-x/3-p}$. A direct computation verifies that $G_{\alpha_{p,x}}(\frac{1-p}{x/3}) = \frac{x/3}{1-x/3-p}$, or, equivalently $G^{(-1)}_{\alpha_{p,x}}(\frac{x/3}{1-x/3-p}) = \frac{1-p}{x/3}$.

Finally, the left inequality of (iv) follows from (9) and the fact that $G^{(-1)}_{\alpha_{p,x}}(z)$ is well defined for $z \in \mathcal{D}$. To show the right inequality of (iv), note that $G^{(-1)}_{\alpha_{p,x}}(z) < 1/z + 1$ is equivalent to $z < 1/G_{\alpha_{p,x}}(z) + 1$, which using (6) can be written as

$$1 - \frac{1}{z} \;<\; \frac{I_2(2z) - \alpha_{p,x} K_2(2z)}{I_1(2z) + \alpha_{p,x} K_1(2z)}$$

Using $\alpha_{p,x} = \alpha\big(\frac{1-p}{x/3}\big)$ and the definition of $\alpha(\cdot)$ in (7) this can be rewritten as $\alpha(z) > \alpha\big(\frac{1-p}{x/3}\big)$. Since $\alpha(\cdot)$ is increasing, the right inequality in (iv) follows if $z > \big(\frac{1-p}{x/3}\big)$ for all z such that $G_{\alpha_{p,x}}(z) \in \text{Int}(\mathcal{D})$. Since $\frac{1-p}{x/3} = G^{(-1)}_{\alpha_{p,x}}\big(\frac{x/3}{1-x/3-p}\big)$ this is true if $G^{(-1)}_{\alpha_{p,x}}(z) > G^{(-1)}_{\alpha_{p,x}}\big(\frac{x/3}{1-x/3-p}\big)$ for all $z \in \text{Int}(\mathcal{D})$, which holds since $G^{(-1)}_{\alpha_{p,x}}(z)$ is minimized at $z = \frac{x/3}{1-x/3-p}$. ■

We are now in the position to characterize the cheap preemptive PBEs for the ascending auction:

Proposition 1. *For $0 \le p \le 1$ and $x > 0$, the following strategy profile (plus a consistent belief system) is a cheap preemptive perfect-Bayesian equilibrium of the ascending auction:*

(i) *an entrant with value v demands three units until the price reaches the lower of v, at which point she demands zero units, and p, at which point she demands two units,*

(ii) *an incumbent with value v demands three units until the price reaches the lower of $B_I(v)$, at which point she demands zero units, and p, at which point she demands two units, where*

$$B_I(v) \;=\; 1 - \frac{(x/3)^2}{1-v}\, G^{(-1)}_{\alpha_{p,x}}\Big(\frac{x/3}{1-x/3-v}\Big)^2 \tag{10}$$

is strictly increasing for $0 \le v < p$ with $B_I(p) = p$.

(iii) *The incumbent's bid function (10) satisfies $v < B_I(v) < v + x/3$ for $0 \le v < p$.*

(iv) *When $x \to 0$, the incumbent's bid function (10) limits to $B_I(v) = v$ and the cheap preemptive PBE reproduces that of Lemma 2.*

The limit cases of $p = 0$ and $p = 1$ will be referred to as the "demand reduction PBE" and the "preemptive PBE" respectively.

Proof. Note that for $0 \le v \le p$ and $0 \le p \le 1$, the ratio $(x/3)/(1 - x/3 - v)$ lies in the set $(-\infty, -1] \cup [0, \infty)$ on which $G^{(-1)}_{\alpha_{p,x}}$ is well defined (see proof of Lemma 3). We first verify the necessary (first-order and boundary) conditions. Differentiating (10) with respect to v yields

$$B_I'(v) \;=\; -\frac{(x/3)^2}{(1-v)^2} G^{(-1)}_{\alpha_{p,x}}\Big(\frac{x/3}{1-x/3-v}\Big)^2 - \frac{(x/3)^2}{(1-v)}\frac{x/3}{(1-x/3-v)^2}\frac{2G^{(-1)}_{\alpha_{p,x}}\big(\frac{(x/3)}{1-x/3-v}\big)}{G'_{\alpha_{p,x}}\big(G^{(-1)}_{\alpha_{p,x}}\big(\frac{x/3}{1-x/3-v}\big)\big)}$$

Using (10) we can rewrite this as

$$B_I'(v) \;=\; -\frac{1-B_I(v)}{1-v}\Big(1+\frac{(1-v)(x/3)}{(1-x/3-v)^2}\frac{2}{G^{(-1)}_{\alpha_{p,x}}\big(\frac{x/3}{1-x/3-v}\big)G'_{\alpha_{p,x}}\big(G^{(-1)}_{\alpha_{p,x}}(\frac{x/3}{1-x/3-v})\big)}\Big)$$

which can be rewritten using (5)

$$B_I'(v) \;=\; -\frac{1-B_I(v)}{1-v}\Big(1+\frac{(1-v)(x/3)}{(1-x/3-v)^2}\frac{1}{1+\frac{(x/3)}{(1-x/3-v)}-\frac{(x/3)^2}{(1-x/3-v)^2}G^{(-1)}_{\alpha_{p,x}}\big(\frac{x/3}{1-x/3-v}\big)^2}\Big)$$

which, using (10), further simplifies to

$$B_I'(v) \;=\; -\frac{1-B_I(v)}{1-v}\Big(1+\frac{(1-v)(x/3)}{(1-x/3-v)^2}\frac{1}{1+\frac{(x/3)}{(1-x/3-v)}-\frac{(1-v)(1-B_I(v))}{(1-x/3-v)^2}}\Big)$$

and, after rearranging terms

$$B_I'(v) \;=\; \frac{(1-B_I(v))(B_I(v)-v)}{(1-v)(v-B_I(v)+x/3)}$$

which is the first-order condition in (4). To verify the boundary condition, note that $B_I(p)=p$ can be rewritten as

$$G^{(-1)}_{\alpha_{p,x}}\Big(\frac{x/3}{1-x/3-p}\Big) \;=\; \frac{1-p}{x/3}$$

or, equivalently,

$$\frac{x/3}{1-x/3-p} \;=\; G_{\alpha_{p,x}}\Big(\frac{1-p}{x/3}\Big)$$

which yields a linear equation in $\alpha_{p,x}$, see (6), that can readily be solved to yield $\alpha_{p,x}=\alpha\big(\frac{1-p}{x/3}\big)$, with $\alpha(\cdot)$ defined in (7).

To show that $B_I(v)<v+x/3$, use the left inequality of (8) in property (iv) of Lemma 3

$$B_I(v) \;<\; 1-\frac{(x/3)^2}{1-v}\,\frac{1}{\frac{x/3}{1-x/3-v}}\Big(\frac{1}{\frac{x/3}{1-x/3-v}}+1\Big) \;=\; v+x/3$$

Similarly, the right inequality in (8) implies

$$B_I(v) \;>\; 1-\frac{(x/3)^2}{1-v}\Big(\frac{1}{\frac{x/3}{1-x/3-v}}+1\Big)^2 \;=\; v$$

Note that $v<B_I(v)<v+x/3$ for $0\le v<p$ implies that $B_I'(v)>0$ for all $0\le v<p$, see the first-order condition (4).

Finally, property (iv) follows from the fact that $G^{(-1)}_{\alpha_{p,x}}(z)\sim 1/z$ for z small. Hence, as x tends to zero, $B_I(v)$ limits to

$$\lim_{x\to 0} B_I(v) \;=\; \lim_{x\to 0} 1-\frac{(x/3)^2}{1-v}\Big(\frac{1-x/3-v}{x/3}\Big)^2 \;=\; 1-(1-v) \;=\; v$$

We next have to show that no bidder wants to deviate from the proposed PBE. For the demand reduction part, i.e. the part corresponding to when the price reaches p, this simply follows from the logic preceding Lemma 2 because at price level p the negative externality cannot be avoided (and becomes a sunk cost). For price levels less than p we have to define appropriate off-the-equilibrium-path beliefs if one of the bidders deviates from the proposed PBE and show that the resulting optimal choices (given these beliefs) render the deviation unprofitable.

Suppose the entrant deviates by reducing demand to one unit at price $p' < p$ and incumbents' beliefs about the entrant's value are given by (1) for very high λ, i.e. the incumbents believe the price is very close to the entrant's value and the entrant is about to reduce demand to zero units. We show that given these beliefs the entrant's deviation will cause the incumbents to bid *more aggressively*: an incumbent with value v will now demand three units up to a price level of $v + x$. Suppose not and an incumbent bidder reduces demand to two units at some price level p'' where $p' \leq p'' < v + x$. The marginal gain of reducing demand a little later, say at price $p'' + \epsilon$, is that the entrant drops out in between p'' and $p'' + \epsilon$ in which case an additional unit is won and the externality is avoided.[14] The marginal cost is that the incumbent has to pay ϵ more for the two units. Hence, the net gain is

$$\epsilon \cdot \Big\{(v + x - p'')f_i(p''|p'') - 2\Big\},$$

which is strictly positive for all $p'' < v + x - 2/\lambda$. So an incumbent will not reduce demand before $v + x > B_I(v)$ after the entrant's deviation, and, hence, the entrant's deviation is not profitable. (The case where the entrant reduces demand to two units at $p' < p$ can be treated similarly.)

Finally, suppose an incumbent with value v deviates by reducing demand to one or two units at price $p' < B_I(v)$. In this case, the other incumbent cannot avoid the negative externality. If others' beliefs about the deviating incumbent's value are again given by (1) they will not reduce demand until their values are reached. Given this, the deviating incumbent will not reduce demand further until her value, v, is reached. Moreover, if the deviating incumbent's out-of-equilibrium beliefs about others' values are that they are no less than her own, then the gain from her deviation is the same as when she would have reduced demand to zero units, which cannot be profitable because the optimal point at which to reduce demand to zero units is given by (10). ■

The incumbents' bid functions for the three levels of x employed in the experiment are given in Figure 1 for $p = 0.5$, $p = 0.75$, and $p = 0.99$. Note that incumbents bids exceed their true values.

[14]Note that we only consider the event that the entrant reduces her demand to zero units, not the other incumbent reducing her demand to two units, since this event is much more likely for λ very high.

[Figure 1 about here]

Proposition 1 characterizes a continuum of PBE, ranging from the "demand reduction PBE" ($p = 0$) to the "preemptive PBE" ($p = 1$). As usual, by considering different out-of-equilibrium beliefs, other equilibria become possible. In particular, asymmetric equilibria in which the two incumbents have different strategies may exist. We don't provide a comprehensive equilibrium characterization. Rather, Proposition 1 is meant to provide a theoretical motivation for our experiments where both demand reduction and preemptive bidding (and, as shown, anything in between) can occur in equilibrium. It is not a priori clear which outcome is more focal, so our experiments provide an empirical test of the drawing power of these two opposing bidding forces.

3.2 Discriminatory auction

The demand reduction equilibrium cannot be sustained in the discriminatory auction since bidders cannot alter the prices they pay for the licenses they win by bidding low on other licenses. In fact, the discriminatory auction has an equilibrium in which a bidder places the *same* bid for each of the three licenses she is competing for.[15] We solely focus on this type of equilibrium in our theoretical analysis. Consider therefore the preemptive equilibrium where all three bidders bid on all three items, and incumbents take into account their true values and the externality $x > 0$. It will prove useful to introduce the inverses $\Phi_E(b)$ and $\Phi_I(b)$ of the bidding functions $B_E(v)$ and $B_I(v)$ respectively. The differential equations the inverse bid functions have to satisfy can be derived from a marginal analysis similar to the one in the previous subsection. For the entrant we have

$$-3\big(1 - (1 - \Phi_I(b))^2\big) + 3(\Phi_E(b) - b)\big(1 - (1 - \Phi_I(b))^2\big)' = 0. \tag{11}$$

To understand this equation recall that, in equilibrium, the gain for an entrant of type $\Phi_E(b)$ of bidding $b + \epsilon$ instead of b should balance the cost. The cost of such a deviation is 3ϵ when the entrant is *not* the lowest bidder, which happens with probability $(1 - (1 - \Phi_I(b))^2)$. The potential gain $3(\Phi_E(b) - b)$ occurs when the deviation changes her from a loser to a winner, which happens when the lowest of the two incumbent values was somewhere between $\Phi_I(b)$ and

[15] Suppose two bidders each submit only a single bid that applies to all three licenses. We have to show that the third bidder's best response is to also submit a single bid. Let v denote the bidder's value and $b_1 \leq b_2 \leq b_3$ her bids. The optimal b_3 is determined by trading off the profit conditional on winning, $v - b_3$, against the probability of winning as determined by the distribution of the sixth-highest of the others' bids. Likewise, the optimal b_2 (b_1) is determined by trading off $v - b_2$ ($v - b_1$) against the winning probability as determined by the distribution of the fifth-highest (fourth-highest) of others' bids, since one (two) of the own bids are higher. But if the other two bidders submit only a single bid then the distributions of the sixth, fifth, and fourth highest of others' bids are identical. Hence, $b_1 = b_2 = b_3$. Lebrun and Tremblay (2003) prove that for the case of two bidders (and no externalities), the equilibrium in which bidders submit only a single bid is the unique equilibrium of the discriminatory auction.

$\Phi_I(b+\epsilon)$: the probability of this event is $\epsilon(1-(1-\Phi_I(b))^2)'$. Similarly, for the incumbent bidders we have

$$-3\big(1-(1-\Phi_I(b))(1-\Phi_E(b))\big)+3(\Phi_I(b)-b)\big(1-(1-\Phi_I(b))(1-\Phi_E(b))\big)' \\ +x\Phi_E'(b)(1-\Phi_I(b)) \;=\; 0. \tag{12}$$

The two terms in the top line have the same interpretation as in equation (11). The extra term in the bottom line occurs when a losing incumbent, by raising her bid slightly, beats the entrant's bid, which has the extra benefit that the negative externality is avoided. This happens when the entrant's value was between $\Phi_E(b)$ and $\Phi_E(b+\epsilon)$ and the other incumbent's value was above $\Phi_I(b)$: the probability of this event is $\epsilon\Phi_E'(b)(1-\Phi_I(b))$.

For the case with no externality, $x=0$, the first-order differential equations (11) and (12) can be solved to yield $\Phi_I(b)=\Phi_E(b)=\Phi(b)$ where

$$\Phi(b) \;=\; \frac{1}{4}\Big(3+3b-\sqrt{9-30b+9b^2}\Big), \tag{13}$$

defined for $0\le b\le \frac{1}{3}$ with $\Phi(0)=0$ and $\Phi(\frac{1}{3})=1$. Since the differential equations (11) and (12) are necessary conditions and their solutions are unique, the inverse bid functions constitute the unique equilibrium (in which each bidder submits three identical bids). It is straightforward to invert (13) to yield the symmetric bidding function as shown by the thick solid line in Figure 5.[16]

Proposition 2. *With no externality, the unique symmetric equilibrium of the discriminatory auction in which each bidder submits three identical bids is given by*

$$B_I(v) \;=\; B_E(v) \;=\; \frac{v}{3}\Big(\frac{3-2v}{2-v}\Big). \tag{14}$$

In the presence of an externality, $x>0$, no analytic solutions to the above differential equations (11) and (12) exist. They can, however, be solved using numerical techniques. For the two values of x used in the experiment, $x=\frac{1}{2}$ and $x=1$, the bid functions for the entrant and the incumbents are shown as grey curves in Figures 6 and 7. Notice that incumbents' bids exceed those of an entrant with the same value. Also, low-value incumbents bid above their true values even though in a discriminatory auction they will have to pay their own bid when they win.

We are interested to what extent the ascending format of the previous subsection is more (or less) prone to preemptive bidding than the discriminatory auction studied here. One natural measure is the probability that the entrant wins a license in either format. The theoretical

[16] The equilibrium bid functions of Proposition 2 can also be derived more directly. Note that the payoff of a bidder who has value v but bids as if her value is w is given by $\pi^e(w|v)=(v-B(w))(1-(1-w)^2)$. Optimizing with respect to w and equating the result to zero at $w=v$ yields a first-order differential equation that is solved by (14).

prediction depends on which equilibrium is selected in the ascending auction. If the demand reduction equilibrium of the ascending auction is selected, the entrant always enters independent of the level of externality. As a result, the probability of entry is larger in the ascending auction than in the discriminatory auction in this case.

In a preemptive equilibrium where bidders place the same bids on all three licenses, entry occurs when the entrants' bid is not the lowest:

$$P_{entry} = \int_{\Phi_E(B_I(0))}^{1} (1 - (1 - \Phi_I(B_E(v_E)))^2) dv_E, \tag{15}$$

where $B_I(\cdot)$ and $B_E(\cdot)$ are the optimal bidding functions in the respective auction formats, and $\Phi_I(\cdot)$ and $\Phi_E(\cdot)$ are their inverses. Using the numerical solutions in Figures 1 and 5-7 it is straightforward to determine the entrant's entry probability for the different scenarios. In the preemptive equilibrium of the ascending auction they are 66.7%, 57.3%, and 45.4% when $x = 0$, $x = 50$, and $x = 100$ respectively. In the discriminatory auction they are 66.7%, 61.0%, and 55.3% when $x = 0$, $x = 50$, and $x = 100$ respectively. When bidding is coordinated on the preemptive equilibrium, the ascending auction is more prone to preemptive behavior by the incumbents.

Part of the intuition behind this result is that in the discriminatory auction incumbents face a strategic risk if they try to keep out newcomers, which they do not have in the ascending auction. In the discriminatory auction it may namely happen that an incumbent attaches high values to the licenses and bids high, while the fellow incumbent attaches low values to the licenses and bids low. As a consequence, the newcomer enters the market and a negative external effect materializes while at the same time the competitive incumbent pays a lot for the licenses that it obtains. Clearly, the incumbents' equilibrium bids take this risk into account (and incumbents bid less than in the absence of this risk). In the ascending auction, an incumbent bidder only bids above the licenses' values if the fellow incumbent is still active in the auction. So this strategic risk does not exist in the ascending auction.

Finally, we briefly discuss what would happen if the assumption that bids cannot exceed three units is dropped (without formal analysis). It is interesting to consider the possibility that a bidder can buy all 6 units, although we do not know of any practical cases where an individual bidder was allowed to capture the whole market. When the negative externality becomes sufficiently high, the demand reduction equilibrium will cease to exist with unrestricted bidders, because an incumbent will want to deviate from sharing the market equally and try to work out the entrant on its own.[17] Likewise, our conjecture is that unrestricted incumbents

[17] The demand reduction equilibrium may also vanish in some cases where the negative externality depends on the number of units bought by the entrant. If the difference in negative externality when the entrant acquires 2 licenses instead of 1 license is sufficiently large, an incumbent may want to deviate from equally sharing the licenses and try to obtain 3 units so that possibly only 1 unit remains for the entrant (when the other incumbent bids sufficiently high on the remaining 2 units).

will compete fiercer both in the preemptive equilibrium of the ascending auction and in the equilibrium of the discriminatory auction, because there is a chance that they can work out the entrant by themselves. As a result, the entrant will be forced to bid higher as well. Therefore, a seller who is only interested in maximizing revenue may be well advised to drop the restriction on the number of licenses that bidders can acquire. Since it allows for the possibility that a monopolist buys all the available licenses, it may lead to a serious cost to efficiency in the aftermarket, though.

4 Results

We present our findings in two parts. We start with an overview of the aggregate results and compare the performance of the two auctions in terms of revenue, efficiency and opportunities for entry. Under the ascending auction, a continuum of equilibria exist (cf. Subsection 3.1). In Subsection 4.1, we take the two extreme equilibria as natural benchmarks and refer to these as the Nash demand reduction equilibrium ($p^* = 0$) and the Nash preemptive equilibrium ($p^* = 1$), respectively. In Subsection 4.2, we subsequently discuss the main patterns in the individual bidding data and we address the matter of equilibrium selection in ascending auctions. There, we also relate our findings to the Nash 'cheap preemptive' equilibria with $0 < p^* < 1$ that exist in the ascending auction.

Most of our results are roughly the same for the 15 periods of part 1 where we used low stakes and the single period of part 2 where we used high stakes. To present our findings in a compact manner, we have chosen to pool the results of parts 1 and 2 and to report separate results only in those cases where they differ significantly.

When comparing different auctions, we make use of two testing procedures. The first is a prudent non-parametric procedure where we use independent averages per group as data points. Thus, the reported Mann-Whitney tests all make use of 8 data points per treatment. The second is a parametric testing procedure where we estimate the treatment effect on the basis of all data with the help of a regression with a random effect term for the groups. Here, we only include a dummy for the treatment variable and determine whether it is significant. The first procedure has the advantage that no assumptions are made about the distributions of the relevant variables. By taking the averages per group we obtain independent data as required for the test, but we lose statistical information about the variance within a group. The second procedure does not suffer from this disadvantage, at a cost of making assumptions about the distribution.

4.1 Revenue, efficiency and entry

Table 2 shows that, for all levels of the externality x, the discriminatory auction raises more revenue than the ascending auction. The table lists the observed average revenues together with the predicted revenues.[18] For the ascending auctions we show the predicted revenues based on the preemptive equilibrium ($p^* = 1$) and those based on the demand reduction equilibrium ($p^* = 0$).[19] As explained in the previous section, the latter equilibrium does not exist in the discriminatory auction. With negative external effects the ascending auctions raise about 50% of the revenue collected in the discriminatory auctions. Without external effects the ascending auction performs even worse. The differences between the two auctions are highly significant. In the discriminatory auctions, the actual revenues trace the predicted revenues very closely for disc0 and disc50 and reasonably well for disc100. In the ascending auctions, the average revenues fall short of the revenues predicted on the basis of the preemptive equilibrium. In these auctions, the demand reduction equilibrium with zero revenue turns out to be a strong force pulling the revenues downward.[20]

Figures 2 through 4 show histograms of the revenues for the cases $x = 0$, $x = 50$ and $x = 100$, respectively. Without external effects the frequency distribution of the ascending auction has a pronounced mode at zero revenue. Even when negative external effects are introduced the mode of the distribution stays at zero, although somewhat less pronounced. So demand reduction seems to be the strongest force in the ascending auctions, even when one of the bidders produces a substantial negative externality for the others.

[Figures 2 through 4 about here]

Sellers will typically be interested in the robustness of an auction and dislike formats that produce unpredictable outcomes. Note that the discriminatory auctions also beat the ascending

[18] Parts 1 and 2 of the experiment resulted in statistically similar revenues for 5 out of 6 treatments; when we consider the realized revenues as a fraction of the available Nash revenues at the preemptive equilibrium, the only significant difference is obtained for the treatment disc100. Here the relative revenue provides the appropriate measure for comparison between parts 1 and 2, because we kept values constant across treatments but not across parts. As it appears the randomly drawn values of part 2 are accidentally more favorable for raising revenue. In treatment disc100, average observed revenue equals 190.5 in part 1 and 284.8 in part 2, while the predicted Nash preemptive revenues equal 217.6 and 242.4, respectively. The ratios of these observed and predicted revenues differ significantly (Mann-Whitney rank test ($m = n = 8$, $p = 0.02$). The test results of the other 5 treatments are far from significant, however (all $p > 0.28$).

[19] In all cases the predictions listed in Table 2 are based on the *actual* private value draws used in the experiments. These predictions may slightly differ from the ones based on the $U[0, 100]$ distribution (cf. Section 3). E.g., when $x = 0$ the preemptive equilibria of the two auction formats are revenue equivalent in the general model and are predicted to yield the seller 150. Yet for the particular private values that we use the preemptive equilibrium in the ascending auction yields a slightly higher revenue (162.8 on average) than the equilibrium in the discriminatory auction (148.9).

[20] The levels of demand reduction in parts 1 and 2 are of the same magnitude. For instance, 6 of the 8 groups in asc0 successfully reduced demand in part 2. This suggests that bidders reduce their demand for the 'right' non-cooperative reasons, and that it is not due to a repeated game effect or low stakes.

Table 2: Revenues

		Ascending		Discriminatory		MW-test	Discr. dummy (Random Effects)		
$x = 0$	Actual	39.6	*71.4*	151.9	*54.3*	$p = 0.00$	112.3	*14.4*	$p = 0.00$
	Nash preempt	162.8	*117.4*	148.9	*36.1*				
	Nash dem red	0.0	*0.0*						
$x = 50$	Actual	93.0	*116.0*	182.3	*63.5*	$p = 0.00$	89.3	*19.2*	$p = 0.00$
	Nash preempt	197.1	*119.4*	183.3	*35.7*				
	Nash dem red	0.0	*0.0*						
$x = 100$	Actual	102.0	*140.9*	196.4	*89.3*	$p = 0.00$	94.3	*24.1*	$p = 0.00$
	Nash preempt	234.0	*128.3*	219.2	*34.8*				
	Nash dem red	0.0	*0.0*						
Mann-Whitney	x=0 vs x=50	$p = 0.04$		$p = 0.02$					
	x=50 vs x=100	$p = 0.83$		$p = 0.21$					
	x=0 vs x=100	$p = 0.02$		$p = 0.02$					

Notes: Standard deviations in italics. The Mann-Whitney ranksum tests reported in the column 'MW-test' compare the realized revenues for the ascending and discriminatory auctions using the 8 average observations per independent group as data. The final three rows report the similar Mann-Whitney test results for comparisons across different values of the externality x, for each auction format separately. The final column reports the significance of the discriminatory auction dummy variable in a regression with a random effect term for the groups.

auctions in this respect. Table 2 shows that, although the discriminatory auctions result in a higher variance of revenues than theory predicts, they do better than the ascending auctions. This result is confirmed graphically in Figures 2 through 4.

In both types of auctions the presence of a bidder who imposes negative externalities on others is good news for the seller who is interested in maximizing revenue. In the ascending auctions, the seller collects significantly more revenue when there is a moderate external effect of $x = 50$ than when there is no external effect. An increase of the negative external effect to $x = 100$ further enhances the revenue for the seller but not significantly so. The introduction of a bidder with negative effects for the others has quantitatively smaller effects in the discriminatory auctions. The test results are similar though: the difference in revenue between disc50 and disc0 is significant, while the difference between disc100 and disc50 is not. We summarize the above findings on revenue in the following result:

Result 1. (i) For every level of the external effect, x, the discriminatory auction raises significantly more revenue than the ascending auction. (ii) In both auction formats the presence of negative externalities (of either $x = 50$ or $x = 100$) increases the seller's revenue compared to the situation where externalities are absent ($x = 0$).

Ascending auctions are often promoted on efficiency grounds, i.e. they "put the licenses in the hands of the firms that value them the most." Although the argument is basically sound, there are two countervailing forces in the present situation. Consider the case where government sells licenses to use gas stations along highways. Here, colluding incumbents may coordinate to keep a price-fighting entrant out. Although the ascending auction may put the licenses in the hands of the incumbents who value them the most, this may very well harm consumer surplus and social efficiency (for this argument, see also Ewerhart and Moldovanu 2001). The other possibility why an ascending auction may harm efficiency occurs when firms decide to split the market as predicted by the demand reduction equilibrium. This equilibrium puts some licenses in the hands of firms with inferior private value components. So in the end it is an empirical question which of the auction formats should be chosen to pursue efficiency.

We first report the results for an efficiency measure that is valid for industries where the negative externality imposed on incumbents does not represent a social harm. Consider the example where a price-fighting entrant tries to penetrate a market of colluding incumbents. Here the price-fighter will produce a negative externality for the incumbents, but not for society.[21] For this type of example the 'traditional efficiency measure' seems most appropriate. This measure is calculated as the ratio of the sum of the realized private (or use) values and the

[21] To the contrary, society as a whole may actually become strictly better off when competition is intensified.

Table 3: Efficiency in % based on use values only

		Ascending		Discriminatory		MW-test	Discr. dummy (Random effects)		
$x = 0$	Actual	85.6	*10.4*	95.5	*6.7*	$p = 0.00$	9.9	*1.9*	$p = 0.00$
	Nash preempt	100.0	*0.0*	100.0	*0.0*				
	Nash dem red	81.0	*8.9*						
$x = 50$	Actual	88.6	*11.6*	95.9	*6.2*	$p = 0.00$	7.3	*1.9*	$p = 0.00$
	Nash preempt	99.7	*1.4*	99.8	*1.2*				
	Nash dem red	81.0	*8.9*						
$x = 100$	Actual	89.6	*10.2*	93.1	*10.2*	$p = 0.09$	3.5	*1.9*	$p = 0.06$
	Nash preempt	97.5	*5.8*	99.2	*3.5*				
	Nash dem red	81.0	*8.9*						
Mann-	x=0 vs x=50	$p = 0.21$		$p = 0.83$					
Whitney	x=50 vs x=100	$p = 0.92$		$p = 0.10$					
	x=0 vs x=100	$p = 0.09$		$p = 0.12$					

Notes: Standard deviations in italics. The Mann-Whitney ranksum tests reported in the column 'MW-test' compare the realized revenues for the ascending and discriminatory auctions using the 8 average observations per independent group as data. The final three rows report the similar Mann-Whitney test results for comparisons across different values of the externality x, for each auction format separately. The final column reports the significance of the discriminatory auction dummy variable in a regression with a random effect term for the groups.

maximally available sum of private values. To be precise, it is calculated as:

$$Eff_{\text{use value}} = \frac{\sum_{j=1}^{n} v_{\text{win, j}}}{3v_{\max} + 3v_{\text{mid}}} * 100\%$$

where n goods are sold, $v_{\text{win, j}}$ refers to the value of the winner of good j, $v_{\max}$ to the maximum private value in the group and v_{mid} to the middle private value of the group. Table 3 presents the results for this efficiency measure. Notice that the discriminatory auctions produce higher efficiency levels than the ascending auctions. The differences in efficiency levels are noteworthy and significant for the treatments without externalities ($x = 0$) and the ones with mild externalities ($x = 50$). For the auctions with strong externalities ($x = 100$) the effect is small and insignificant at the conventional level.

Without externalities the efficiency level in the ascending auctions is closer to the level predicted by the demand reduction equilibrium than the level predicted by the preemptive equilibrium. When externalities are introduced, actual efficiency moves slowly into the direction of the level predicted in the preemptive equilibrium. The realized efficiency level is about halfway between the two predicted levels when the negative externality is strong ($x = 100$). However, the increases in efficiency levels as the level of the external effect rises are not significant. A similar result applies for the discriminatory auctions; observed efficiency levels are independent of the level of the external effect.

There may also be situations where the negative externality represents a social harm, e.g. when a polluting firm acquires a license. In such cases it makes sense to incorporate the externality in the efficiency measure. A straightforward way to do this is to calculate the realized efficiency level as the ratio of the realized surplus and the theoretically available surplus. Here the realized surplus equals the sum of the realized private values minus the sum of the realized negative externalities. The theoretically available surplus is determined by the allocation that maximizes the sum of the private values diminished by the corresponding negative external effects. This leads to:

$$Eff_{\text{external effect}} = \frac{-2x \cdot I_{\{E \text{ enters}\}} + \sum_{j=1}^{n} v_{\text{win, j}}}{\max\{3v_{\text{A}} + 3v_{\text{B}}, 3v_{\max\{\text{A,B}\}} + 3v_{\text{E}} - 2x\}} * 100\%$$

where v_{A}, v_{B}, v_{E} refer to the values of incumbents A and B and entrant E respectively, $v_{\max\{\text{A,B}\}}$ to the maximum value of A and B and $I_{\{E \text{ enters}\}}$ an indicator function equal to one iff the entrants enters (and 0 otherwise). Table 4 shows the results for this efficiency measure. Again the discriminatory auction significantly outperforms the ascending auctions for the case of mild externalities, but not for the case of strong externalities.

In the ascending auctions, the efficiency levels are roughly halfway the level predicted by the demand reduction equilibrium and the preemptive equilibrium. Because these predicted levels decrease with the level of the external effect, so do the actual efficiency levels. In the discriminatory auctions the efficiency levels decrease significantly with the level of the external effect as well. Result 2 summarizes our findings concerning efficiency.

Result 2. (i) The discriminatory auction yields higher efficiency levels than the ascending auction for every level of the external effect, x. (ii) In both auction formats, the presence of negative externalities decreases efficiency (only) when the externality represents a social harm.

Policy makers often want to know how particular auction formats affect the chances for possible entrants. It has been argued that auctions with a discriminatory element offer better chances to entrants than ascending auctions since the latter offer incumbents the possibility to trail entrants and outbid them with the smallest possible margin. Discriminatory auctions contain an element of surprise, as incumbents face a difficult task when they trade off the probability

Table 4: Efficiency in % including external effects

		Ascending		Discriminatory		MW-test	Discr. dummy (Random Effects)		
$x = 0$	Actual	85.6	*10.4*	95.5	*6.7*	$p = 0.00$	9.9	*1.9*	$p = 0.00$
	Nash preempt	100.0	*0.0*	100.0	*0.0*				
	Nash dem red	81.0	*8.9*						
$x = 50$	Actual	73.5	*24.1*	82.8	*22.6*	$p = 0.03$	9.3	*3.3*	$p = 0.01$
	Nash preempt	94.7	*14.3*	94.4	*14.4*				
	Nash dem red	58.1	*24.5*						
$x = 100$	Actual	45.5	*68.5*	50.5	*59.9*	$p = 0.67$	5.0	*10.9*	$p = 0.65$
	Nash preempt	83.3	*52.6*	77.4	*53.8*				
	Nash dem red	20.6	*62.5*						
Mann-Whitney	x=0 vs x=50	$p = 0.01$		$p = 0.00$					
	x=50 vs x=100	$p = 0.01$		$p = 0.00$					
	x=0 vs x=100	$p = 0.00$		$p = 0.00$					

Notes: Standard deviations in italics. The Mann-Whitney ranksum tests reported in the column 'MW-test' compare the realized revenues for the ascending and discriminatory auctions using the 8 average observations per independent group as data. The final three rows report the similar Mann-Whitney test results for comparisons across different values of the externality x, for each auction format separately. The final column reports the significance of the discriminatory auction dummy variable in a regression with a random effect term for the groups.

of winning against the profit margin in case they win. There is, however, another argument in the opposite direction. In ascending auctions there exists a demand reducing equilibrium even when the entrant imposes negative external effects on the incumbents. In such an equilibrium, the newcomer enters independent of her private value for the licenses. Thus, ascending auctions may stimulate entry, although perhaps for the wrong reasons.

Table 5 reports the frequencies of market entry together with the number of goods the entrant obtains conditional on entry. This table does not include the treatments where external effects are absent, because when $x = 0$ bidders have symmetric roles. Comparing the two auction formats, there is no difference in the relative frequency with which entry occurs. This holds both with a mild and a strong externality. Notice that in the ascending auctions, entry levels are between the level predicted by the preemptive equilibrium and that predicted by the demand reduction equilibrium (100%).[22] In the discriminatory auctions the newcomer enters more often than predicted. In both types of auctions entry levels do not vary with the level of the external effect.

Table 5 also shows that, conditional on entry, the entrant wins slightly fewer licenses in the ascending auctions than in the discriminatory auctions. This observation is in line with the frequent play of the demand reduction equilibrium in the ascending auctions. The differences in number of licenses bought fail to reach significant levels however. The number of licenses the entrant gets (conditional on entry) is also independent of the extent of the external effect. Result 3 summarizes the findings on entry.

Result 3. Both the relative frequency of entry and the number of licenses the entrant buys conditional on entry are independent of the auction format and the level of the negative externality.

Overall, the aggregate results reveal that the discriminatory auction is preferred – or better, not outperformed – in terms of revenue, efficiency, and entry. A plausible explanation for this is that the demand reduction equilibrium has considerable drawing power in the ascending auction. The aggregate results for this auction typically fall in between the theoretical predictions of the demand reduction equilibrium and those of the preemptive equilibrium. In the discriminatory auctions the aggregate results are fairly well in line with the theoretical predictions. The main difference is that entry occurs more frequently than predicted (cf. Table 5). This in turn results in efficiency levels that are somewhat lower than predicted (cf. Table 4). A potential explanation for the surprisingly high frequencies of entry is that bidders do not submit the flat bidding schedules as predicted by the unique symmetric equilibrium. This allows for the

[22] Again note that the predictions appearing in Table 5 are based on the actual private value draws. For the theoretical $U[0, 100]$ distribution entry probabilities in the preemptive equilibria of the ascending auction equal 57.3% and 45.4% for $x = 50$ and $x = 100$, respectively (cf. Subsection 3.2). In the discriminatory auction these numbers equal 61.0% and 55.3%.

Table 5: Entry in %

		Ascending		Discriminatory		MW-test	Dummy Discr. (Random effects)		
Entry									
$x = 50$	Actual	83.6	*37.2*	81.3	*39.2*	$p = 0.52$	−2.3	*5.4*	$p = 0.67$
	Nash preempt	60.2	*49.2*	61.7	*48.8*				
	Nash dem red	100.0	*0.0*						
$x = 100$	Actual	82.8	*37.9*	85.2	*35.7*	$p = 0.48$	2.3	*8.9*	$p = 0.79$
	Nash preempt	46.1	*50.0*	57.8	*49.6*				
	Nash dem red	100.0	*0.0*						
MW-test	x=50 vs x=100	$p = 0.83$		$p = 0.34$					
# goods entrant (given entry)									
$x = 50$	Actual	2.14	*0.71*	2.27	*0.86*	$p = 0.34$	0.13	*0.13*	$p = 0.31$
	Nash preempt	3.0	*0.0*	3.0	*0.0*				
	Nash dem red	2.0	*0.0*						
$x = 100$	Actual	2.05	*0.77*	2.34	*0.82*	$p = 0.14$	0.29	*0.18*	$p = 0.11$
	Nash preempt	3.0	*0.0*	3.0	*0.0*				
	Nash dem red	2.0	*0.0*						
MW-test	x=50 vs x=100	$p = 0.56$		$p = 0.53$					

Notes: Standard deviations in italics. The Mann-Whitney ranksum tests reported in the column 'MW-test' compare the realized revenues for the ascending and discriminatory auctions using the 8 average observations per independent group as data. The final row of each panel reports the similar Mann-Whitney test results for comparisons across different values of the externality x, for each auction format separately. The final column reports the significance of the discriminatory auction dummy variable in a regression with a random effect term for the groups.

possibility that the entrant obtains 1 or 2 licenses, possibilities that will not materialize in the symmetric Nash equilibrium. In the next section we will come back to this aspect of the bidding process.

4.2 Individual bidding and equilibrium selection

The individual bidding in periods 9-16 resembles the bidding in periods 1-8 to a large extent. Therefore, we chose not to report on the time dimension of the data, except for the cases where it does matter.

Recall from section 3.2 that in the discriminatory auction, all three bidders bid on all three licenses in the unique symmetric equilibrium. In the absence of an externality, the optimal bid functions are the same for the entrant and the incumbents:

$$B(v) = \frac{v}{3}\Big(\frac{300-2v}{200-v}\Big), \tag{16}$$

where $0 \leq v \leq 100$ denotes the per-license private value of the bidder.[23] Figure 5 displays the average observed bids in treatment disc0 together with the Nash prediction reflected in (16). The Nash bids trace the average of the second-highest bids remarkably well. The absolute distance between the average of a bidder's three submitted bids and the corresponding Nash prediction is less than or equal to 3, 5 and 10 in respectively 39.8%, 59.9% and 86.4% of the cases. So a large proportion of the average bids are close to the Nash predictions and there are no systematic deviations in upward or downward direction.

[Figure 5 about here]

One aspect of observed bidding behavior that is not compatible with our theoretical predictions is that subjects tend to submit different bids for identical units. The same finding shows up in the discriminatory auction treatment of Engelmann and Grimm (2009). In fact, Figure 5 shows that the three bidding functions fan out for higher private values. A similar pattern of fanning out is present in the discriminatory treatments with external effects. In only 17.7% of the cases without external effects did the subjects submit exactly the same three bids. This number increases a little to 19.3% in the treatment with $x = 50$ and to 22.1% in the treatment with $x = 100$. A hedging motive may be responsible for bidders' tendency to submit different bids for identical licenses: with the high bid a bidder plays safe and makes it less likely that she ends up without any profit. With the low bid the bidder then tries to "hit the jackpot."

Figures 6 and 7 show the bidding patterns for the discriminatory auctions with $x = 50$ and $x = 100$, respectively. In each of these figures we separated the bids of the incumbents and the

[23] Recall that in section 3 the units were scaled down by a factor of 1/100.

entrants. We plotted the averages of subjects three submitted bids and the Nash predictions. The most striking departure from the theoretical predictions is that low-value incumbents bid too low. Theory predicts that when $x = 50$ all incumbents with $v \leq 13$ should bid above their value. In case $x = 100$ this applies to all types satisfying $v \leq 26$. For example, when $v = 0$, Nash incumbents should bid 50/6 when $x = 50$ and 100/6 when $x = 100$. In contrast to these predictions, low-value incumbents are unwilling to bid above their values. Perhaps they wish to avoid the worst-possible scenario in which they buy some licenses at prices above their values while still having to bear the negative externality caused by entry. Such an explanation may be compatible with the opposite deviation observed for high private values. Here incumbents bid below value but higher than the Nash prediction. Quite possible they do this to enhance the likelihood that the entrant is kept out.

Our theoretical analysis of the discriminatory auction with external effects assumes that incumbents employ the same strategy. To investigate the possibility of incumbent asymmetry, we estimated for each of the 8 groups of disc50 and disc100 the following OLS regression:

$$b_{i,t} = \beta_0 + \beta_1 v_{i,t} + \beta_2 v_{i,t}^2 + k\beta_3 + k\beta_4 v_{i,t} + k\beta_5 v_{i,t}^2 + \varepsilon_{i,t}, \tag{17}$$

where $b_{i,t}$ represents the average bid of incumbent i ($i = 1, 2$) in period t ($t = 1...16$), k represents a dummy that equals 0 when $i = 1$ and 1 when $i = 2$ and $\varepsilon_{i,t}$ refers to the error term that is independent across incumbents and periods. For each group, we used a Wald test to test whether the restriction $\beta_3 = \beta_4 = \beta_5 = 0$ is rejected at the 5% level. When $x = 50$ the restriction is rejected in 4 of the 8 groups and when $x = 100$ in 3 of the 8 groups. In total, incumbent symmetry is not rejected in a small majority of the groups.

The bidding behavior of the entrant is relatively closer to Nash. This is reflected by the relative frequency of bids close to the Nash predictions. In the treatment with weak external effects, 48.0% (82.7%) of the entrants' average bids are at most 5 (10) points away from the Nash predictions, while only 35.3% (72.2%) of the incumbents' bids are within a range of 5 (10) points from the Nash bids. In the treatment with strong externalities, the pattern is the same but the deviations are more pronounced. Here, 27.0% (60.7%) of the entrants average bids differ at most 5 (10) points from the Nash predictions, while only 15.9% (32.7%) of the incumbents' bids are within a range of 5 (10) points of the Nash bids.

[Figures 6 and 7 about here]

Theoretically, the introduction of an external effect in the discriminatory auctions should enhance the bids of both incumbents and entrants across the whole range of private values. The effect should be more pronounced for incumbents than for entrants. However, we do not observe any effect when bidders draw private values below 50. Pooled across all cases where bidders receive values below 50, they bid on average 11.7, 13.4 and 13.0 in the respective treatments with $x = 0$, $x = 50$ and $x = 100$. Neither the incumbents' nor the entrants' bids vary with the external effect for low private values. In contrast, when bidders draw private values above 50, their bids incorporate the external effect. Compared with the average bid of 29.3 observed in the absence of external effects, incumbents' bids increase to 36.7 while entrants' bids remain at 30.8 when $x = 50$. When the external effect is further enhanced to $x = 100$, incumbents' bids increase a little further to 38.7, while now the entrants' bids jump to 39.6. Thus the results suggest that the incumbents neglect the entrant when they have low private values, possibly because they think that they cannot prevent entry of the newcomer anyway. When they have high private values they are confident that their bids can make a difference and they bid more competitively than they do without external effects.

We summarize our main findings on bidding behavior in the discriminatory auctions in the following result:

Result 4. Individual bidding behavior in the discriminatory auctions deviates from our equilibrium predictions (cf. Subsection 3.2) in two important ways: (i) subjects tend to submit different bids for identical units and (ii) in the presence of negative externalities low-value incumbents bid too low.

Result 4 provides an explanation for our earlier observation that in the discriminatory auctions, entry occurs more often than predicted. First, because incumbents and entrants do not submit flat bidding schedules, actual bidding allows for the possibility that the entrant obtains 1 or 2 licenses. This happens in 37.5% (37.2%) of the cases when $x = 50$ ($x = 100$). Second, incumbents with low private values bid too low, also contributing to the higher frequencies of entry.

We next turn to the ascending auctions. These auctions present bidders with a coordination problem: do they split the market at low prices, thereby winning a moderate number of goods at high profit margins, or do they decide to bid competitively in an attempt to drive out one of their opponents? If each of the three subjects in a group starts bidding on two goods only, then the clock does not even start rising and each of the bidders buys two goods at a price of zero. In the experiments, bidders often reduce their demand in exactly this way. We also observe many cases that are very close to this 'ideal version' of demand reduction. For instance, there are cases where two of the three bidders start bidding on two goods while the third starts bidding on three goods. The clock starts rising and at a very low price the third bidder stops the clock by reducing her demand from three to two goods. Table 6 lists the perfect cases of strategic demand reduction in the row labeled DR1, together with the close-to-perfect cases in the rows DR2 and DR3.

Demand reduction is supported in equilibrium by an assumption about what happens off the equilibrium path. The theoretical analysis assumes that bidders should bid competitively once they find out that one of them greedily asks for 3 instead of 2 licenses. It turns out that this assumption agrees quite well with how subjects behaved in the experiment. When we focus on the ascending auctions where one bidder deviated from splitting the available licenses equally (i.e., on cases where one of the bidders initially demanded 3 licenses while the other 2 bidders initially demanded 2 licenses each), the bidders who reduced their demand in vain competed vigorously in the remainder of the auction by winning 2 units or by at least bidding up to their value in 86.9% of the cases. When we restrict the attention to periods 9-16, the relative frequency of competitive responses to cheating on demand reduction further increases to 92.3%.

The second panel of Table 6 labeled 'Preemption + Competition' depicts how often subjects bid competitively or preemptively in a serious attempt to get rid of a competitor. For the auctions without external effects, competitive bidding means that the realized price will at least be as high as the minimum private value in the group minus one. The three rows CO1, CO2 and CO3 list these cases; these rows differ in the actual price that results. In the treatments with external effects, preemptive bidding requires that each incumbent starts bidding on three goods, otherwise the external effect cannot be prevented. An ideal example of preemptive bidding occurs if the two incumbents successfully drive out the entrant by bidding above value and thereby prevent the negative effect. These cases are listed in the row PR. There are also cases where the incumbents successfully worked out the entrant but did not have to bid above value to do so. Such cases are consistent with preemptive bidding as well as with competitive bidding and are listed in the row (PR+CO)1. Even in the preemptive equilibrium entrants will sometimes enter the market if they have a sufficiently better private value than each of the incumbents. So the class of preemption+competition contains a subclass where the newcomer enters the market, despite the fact that each incumbent remained in the auction for three goods until the clock reached at least her private value (see row (PR+CO)2). Among these cases we have also included the 'close-to-perfect' cases where the entrant made an unsuccessful attempt to seduce the incumbents to collude by reducing her demand at a low price.

The percentages in Table 6 reveal that both demand reduction and competitive/preemptive bidding are observed in all regimes. Without external effects demand reduction is by far the most frequently observed outcome. With external effects the relative frequency of demand reduction drops dramatically, but still remains the most observed outcome. Preemptive bidding becomes more likely when the negative external effect inflicted by the entrant increases, but even when $x = 100$ only 20.4% of the outcomes are characterized as competitive/preemptive bidding (while 30.3% of the outcomes correspond to demand reduction). To get a clean estimate of preemptive bidding, the % of competitive outcomes observed in the ascending auction without

Table 6: Equilibrium selection in ascending auctions in %

		asc0		asc50		asc100	
		period 1-16	period 9-16	period 1-16	period 9-16	period 1-16	period 9-16
Demand	DR1	24.2	27.4	8.2	5.0	9.8	5.0
Reduction	DR2	30.6	32.3	6.6	11.7	14.8	21.7
	DR3	5.6	3.2	10.7	8.3	5.7	5.0
	Total	60.4	62.9	25.5	25.0	30.3	31.7
Preemption	CO1	0.8	1.6				
+ Competition	CO2	3.2	6.5				
	CO3	3.2	3.2				
	PR			0.8	1.7	1.6	1.7
	(PR+CO)1			11.5	13.3	13.1	15.0
	(PR+CO)2			2.5	1.7	5.7	6.7
	Total	7.2	11.3	14.8	16.7	20.4	23.3
Cheap Preemptive	PDR	8.1	9.7				
+Partial Demand	(CP+PDR)			22.1	20.0	18.9	20.0
Reduction	Total	8.1	9.7	22.1	20.0	18.9	20.0
Miscellaneous	Total	24.2	16.1	37.7	38.3	30.3	25.0

Notes: The categories are defined as follows. DR1: 'Clock does not start rising because sum of initial demands ≤ 6'; DR2: 'Sum of initial demands > 6, but the clock is already stopped at a price of 0'; DR3: '$0 <$ realized price ≤ 3 and minimum value of all bidders > 4'; CO1: 'asc0, price=minimum value all bidders - 1'; CO2: 'asc0, price=minimum value all bidders'; CO3: 'asc0, price $>$ minimum value all bidders'; PR: 'asc50 or asc100, each incumbent wins 3 goods, realized price $>$ minimum private value incumbents'; (PR+CO)1: 'asc50 or asc100, each incumbent wins 3 goods, realized price $\leq$ minimum private value incumbents'; (PR+CO)2: 'asc50 or asc100, each incumbent bids on 3 units until at least her private value, nevertheless entrant wins 1 or more goods'; PDR: 'ascending x=0, each bidders starts bidding on 3 goods, each bidder wins at least 1 good, 0.2*minimum value all bidders $\leq$ realized price $\leq$ 0.8*minimum value all bidders'; (CP+PDR): 'ascending $x > 0$, each incumbent starts bidding on 3 goods, entrant wins at least 1 good, 0.2*minimum value incumbents $\leq$ realized price $\leq$ 0.8*minimum value incumbents'.

external effect needs to be subtracted from the % of cases consistent with competitive and preemptive bidding in the ascending auctions with external effects. Thus, we find that only 7.6% of the cases when $x = 50$ and 13.2% of the cases when $x = 100$ are true examples of preemptive bidding.

The weak appeal of the preemptive bidding equilibrium might be related to the fact that this equilibrium potentially results in the worst-case scenario for an incumbent. This happens when she bids above her value on all three licenses while the other incumbent reduces demand from three to zero licenses. In that case she has to bear the negative external effect caused by the entrant and at the same time pay higher prices for the licenses than they are worth. Loss averse incumbents may only want to embark on the risky enterprise of preemptive/competitive bidding if they feel sufficiently confident that they will succeed in beating the entrant.[24] If this reasoning is sound, then one would expect that incumbents only opt for preemption/competition when they have a high private value. Figure 8 shows that this indeed appears to be a driving force behind equilibrium selection. The figure considers only the outcomes that received the labels 'preemption/competition' or 'demand reduction' in Table 6. For both regimes $x = 50$ and $x = 100$ the figure shows the percentage of preemptive/competitive outcomes as function of the minimum private value of the two incumbents. In both cases the demand reduction equilibrium prevails when the minimum private value is low while the preemptive/competitive outcome is dominant when this value is high. When the external effect is weak, incumbents pursue the preemptive/competitive outcome if both of them have a private value of at least 60.[25] In case $x = 100$ incumbents already opt for the preemptive/competitive path if both of them have private values higher than 40.

[Figure 8 about here]

There is an interesting pattern in the bidding of many of the experimental auction outcomes that do not belong to the class of pure demand reduction or the class of pure preemption. With negative external effects, the incumbents often start bidding on three goods each, like they are supposed to do in a preemptive bidding equilibrium. When it turns out that it is not possible to drive out the entrant at low prices, the incumbents often reduce their demand well before the clock has reached their private values. As a result, the entrant is able to enter the market at a price below the minimum private value of the incumbents. These outcomes have the flavor of the 'cheap preemptive' equilibria with $0 < p^* < 1$ derived in Subsection 3.1. Note, however,

[24]Ideally, one would like to identify a subject's loss-aversion in a different, unrelated task. We do not have such data, however.

[25]Notice that rather counterintuitively the curve for x=50 bends downward for very high minimum private values. However, this part of the figure is based on few data only.

that these outcomes are also consistent with what Engelmann and Grimm (2009) call ‘partial demand reduction’ equilibria. In a setting without negative external effects, they show that bidders may reduce demand at prices above 0 but below their values. We include them at the bottom of Table 6 in the row labeled (CP+PDR).[26] To assess the net % of cheap preemptive outcomes, we subtract the % of cases when $x = 0$ from the % of cases when $x = 50$ and $x = 100$. This way, we find that 14.0% of the cases in the regime with $x = 50$ and 10.8% of the cases in the regime with $x = 100$ represent true cheap preemptive attempts.

Notice that there is a whole range of cheap preemptive equilibria which allows for the possibility that the incumbents do not agree on the price level where they should reduce demand. In fact, in only 23.5% of the cases reported in (CP+PDR), the incumbents reduced demand at approximately the same price (measured as cases where the difference in prices where demand was reduced less than or equal to 2). Therefore, we prefer to refer to these cases as cheap preemptive outcomes instead of cheap preemptive equilibria. We observe an approximately equal number of cheap preemptive outcomes and preemptive cases. When faced with a stubborn entrant, many incumbents chicken out and settle for an outcome that still gives them a positive profit margin on the goods purchased.

Table 6 also reports the equilibrium selection results for periods 9-16 separately. The results for the second part of the experiment shine light on the questions whether the relative attractiveness of equilibria changes over time and whether subjects learn to play according to equilibrium over time. The relative frequencies of the equilibria in the second part of the experiment are by and large the same as the relative frequencies of the equilibria in the whole experiment. Thus, the relative attractiveness of the equilibria does not appear to change. In treatments $x = 0$ and $x = 100$, the relative frequency of outcomes in the Miscellaneous class decreases over time. This suggests that subjects learned to play better in accordance with equilibrium in the second part of the experiment. This learning effect was not observed when $x = 50$.

Our overall findings on equilibrium selection are summarized in Result 5.

Result 5. (i) In the ascending auction strategic demand reduction is observed more often than preemptive bidding, although the presence of negative externalities makes demand reduction less focal. (ii) Around $10 - 15\%$ of the observed outcomes can be classified as truly ‘cheap preemptive’; incumbents first try to get rid of the entrant at low prices, but then turn to demand reduction when this appears unsuccessful.

[26] The row labeled PDR contains the corresponding cases for the situation without external effects. Because in that regime the preemptive motive is absent, these cases are labeled as partial demand reduction outcomes.

5 Conclusion

When the ascending auction is employed to assign market licenses there often exist multiple equilibria that differ sharply in terms of revenue and efficiency. Which equilibrium is selected is an empirical issue that likely depends on market conditions before and after the auction, e.g. when incumbents compete with entrants for a limited set of licenses. At one extreme, there is the demand reduction equilibrium where all bidders collude and strategically reduce their demand at the lowest possible price. This way, each bidder cheaply obtains a (small) number of licenses. At the other extreme, there is a preemptive equilibrium where incumbents engage in predatory bidding to keep entrants out of the market. This way, incumbents avoid the negative externality that arises when entrants compete in the post-auction market. In addition, there are "cheap preemptive" equilibria that unify the two extremes. In such equilibria, bidders first behave preemptively but when the entrant does not concede early enough they switch to the demand reduction outcome. Like in the demand reduction equilibrium, all three bidders are needed for a successful reduction of demand.

Given the multitude of equilibria, the effectiveness of the ascending auction (in terms of revenue, efficiency, and entrants' chances) crucially depends on which of these equilibria is most likely selected. A practical and often used alternative is the sealed-bid discriminatory ('pay-your-bid') auction. This format has the advantage that it does not support demand reduction and, hence, collusion among all bidders is excluded. At the same time, preemptive bidding becomes more complicated as incumbents cannot track the behavior of other incumbents. Using controlled laboratory experiments we compare the performance of the discriminatory auction vis-a-vis the ascending auction.

In the experiments, demand reduction is always more common than the preemptive bidding outcome in the ascending auction, which generates less revenue and is less efficient than the discriminatory auction. Both auction formats induce similar high levels of entry; they are high in the ascending auction because of demand reduction and they are high in the discriminatory auction because bidders place different bids for the three items. With an increase in the negative external effect, the dominance of the demand reduction equilibrium diminishes.

The bidding data of the ascending auction reveal an intuitive empirical equilibrium selection device. Incumbents let their decision to pursue the demand reduction outcome depend on their private value. With low private values they figure they have no chance to drive the entrant out and they settle for demand reduction. With high private values they pursue preemption, conditional on the cooperation of the other incumbent. The threshold above which subjects opt for preemption decreases with the negative external effect.

The data reveal that the ascending auction fairly often results in outcomes that are consistent with "cheap preemption" . Incumbents first try to keep the entrant out of the market but when this appears unsuccessful, they revert to demand reduction. This result suggests that the outcome of the German UMTS auction is not as exceptional or irrational as it may have first appeared.

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Appendix A

Besides the on-screen instructions subjects also received a summary of these instructions on paper. Below a direct translation of this summary sheet is given for both the asc50 and disc50 treatments.

Summary of the instructions Today's experiment consists of two parts. At the beginning of part 1 you are assigned to a group of 3 participants. During the 15 rounds of part 1 the group composition remains unchanged. The three participants within a group are labeled type A, type B and type X. At the start of the experiment you will learn your type. You will keep the same type during the complete first part.

Products. Within each group there are in each round 6 identical products up for sale. For each group member the value of each product lies in between 0 and 100 points, and every integer number between 0 and 100 is equally likely. The value a particular group member has is independent of the values of the other group members. At the start of a round you will only learn your own value for each product. Your value of a product in one round does not depend on your value of a product in any other round.

Bidding and prices. After you have learnt your value, you indicate on how many products you would like to start bidding. You can start bidding on 0, 1, 2, or 3 products. We label this amount your "demanded quantity". If the sum of demanded quantities within a group is smaller than or equal to 6, then each group member is assigned his/her demanded quantity and pays a price of 0 points per product. The products that are possibly left over remain unsold.

In case the sum the demanded quantities exceeds 6, a "thermometer" starts rising from 0 points onwards. The thermometer indicates the price. At every price each group member has the opportunity to adapt the demanded quantity downwards. As soon as the sum of demanded quantities equals 6, the thermometer stops. The position of the thermometer determines the price that is paid for each product. All group members are assigned the number of products they demand at the time the thermometer comes to a stop.

From the moment the thermometer starts rising, you can decrease your demand quantity only such that the sum of the demand quantities remains larger than or equal to 6. In case a participant lowers his/her demanded quantity during a round, the other group members are informed immediately about this.

[In disc50: *Bidding and prices.* After you have learnt your value, you indicate for each of the products how much you would like to bid for it. You can make a bid on three products at most. A bid has to be in between 0 and 125 points. For each product you indicate how much you are willing to pay for it. You can decide yourself whether you make the same or different bids. You can also decide not to make a bid on one or more products.

After every group member has made his/her bids, the six products are assigned to the 6 highest bids. In case there are less than 6 bids in total, the products are assigned to all the bids that are made. The remaining products remain unsold. In case a product is assigned to you, your bid determines the price you pay for this product.]

Earnings. Your returns are equal the number of products that you buy multiplied by the difference between the value assigned to each of your products and the price you pay for each product:

$$\text{Your returns } = \text{number of products } * \text{ (your value} - \text{ the price)}$$

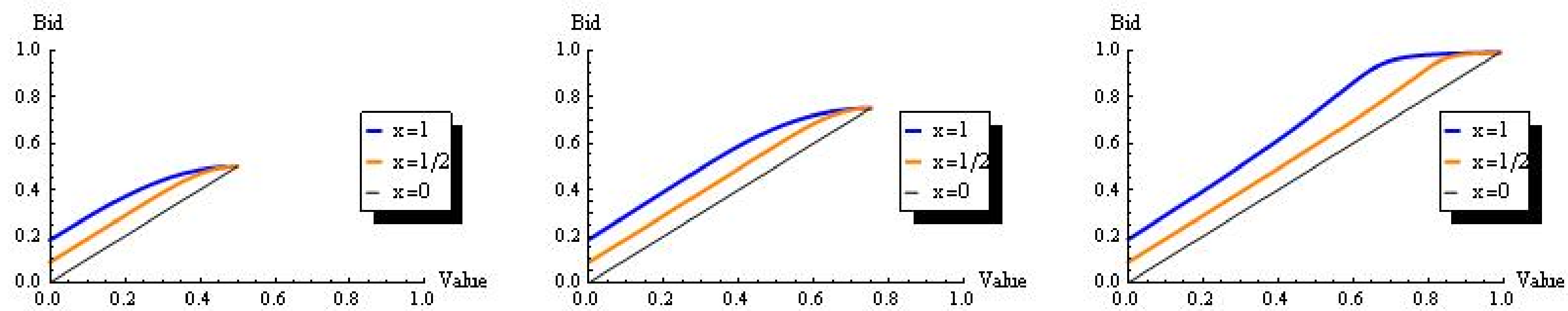
[In disc50: *Earnings.* For each product that you bought your returns are equal the difference between the value of the product and the price you pay for this product. Your overall returns equal the sum of the returns per product purchased. For example, if you buy 3 products, then your returns are equal to:

$$\begin{aligned} \text{Your returns} \quad = \quad & \text{(your value } - \text{ the price of } 1^{st} \text{ product you bought)} + \\ & \text{(your value } - \text{ the price of } 2^{nd} \text{ product you bought)} + \\ & \text{(your value } - \text{ the price of } 3^{rd} \text{ product you bought)} \qquad] \end{aligned}$$

If you are a participant with type X, then your earnings within a round equal your returns. In case you have either type A or type B, your earnings also depend on whether type X bought any products or not. If type X has bought one or more products, then the returns of both type A and type B are in that round reduced with 50 points. Only when type X buys no products at all there is no reduction on the returns of types A and B in that round.

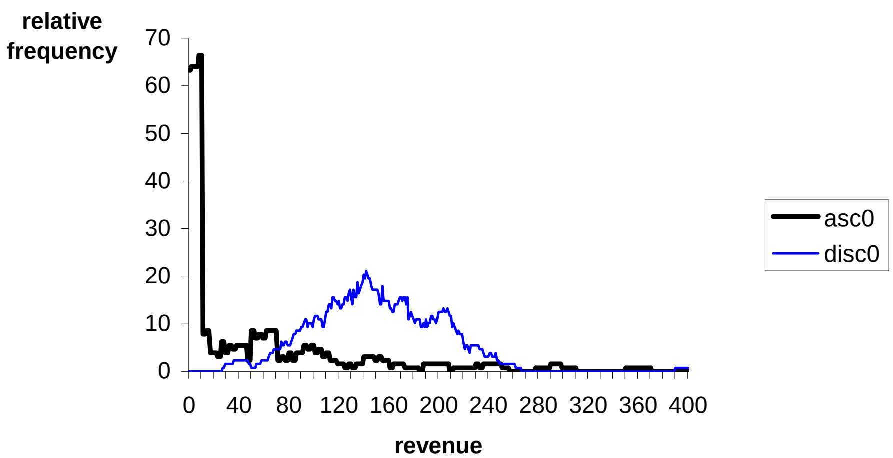
At the beginning of part 1 you receive a starting capital of 750 points. Your total number of points at the end of part 1 will be equal to the sum of this starting capital and your earnings in all 15 rounds. At the end of the experiment your points are exchanged into euros. Here it holds that 80 points correspond with 1 euro in money. Part 1 starts with a practice round. Your profits or losses during this practice round are not counted.

Figure 1: Incumbents' bidding behavior in the ascending auction



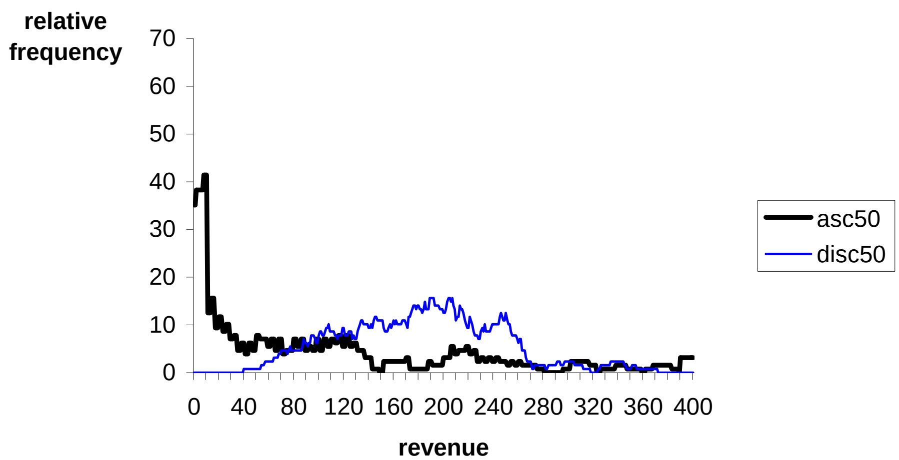
Notes: The figure displays the incumbents' bid functions in various cheap preemptive PBEs for the three different levels of *x* (scaled by 1/100) employed in the experiment. The panel on the l.h.s. displays the equilibrium for $p=0.5$, the middle panel for $p=0.75$ and the panel on the r.h.s. for $p=0.99$.

Figure 2: Revenue histograms when $x = 0$



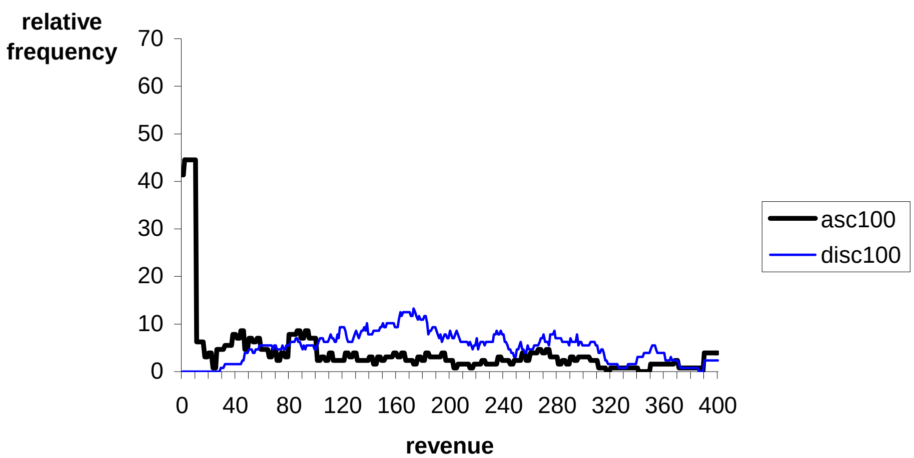
Notes: For every revenue level the % of outcomes that fall in the interval [revenue–10,revenue+10] is displayed.

Figure 3: Revenue histograms when $x = 50$



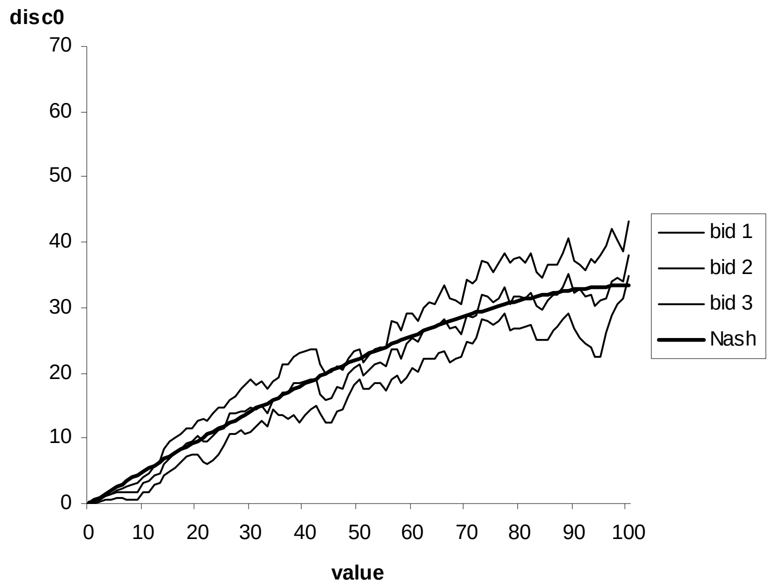
Notes: For every revenue level the *%* of outcomes that fall in the interval [revenue–10,revenue+10] is displayed.

Figure 4: Revenue histograms when x = 100



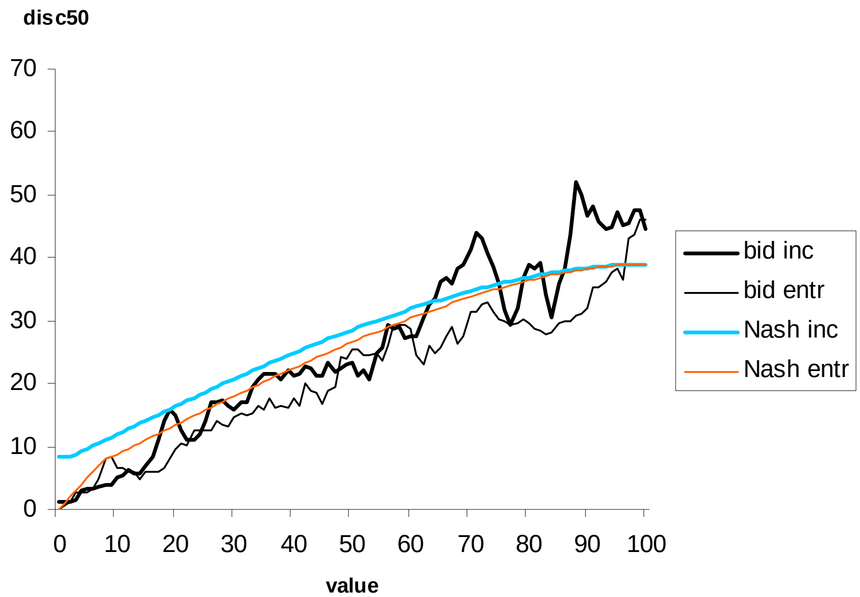
Notes: For every revenue level the % of outcomes that fall in the interval [revenue–10,revenue+10] is displayed.

Figure 5: Bidding behavior in the discriminatory auction with $x = 0$



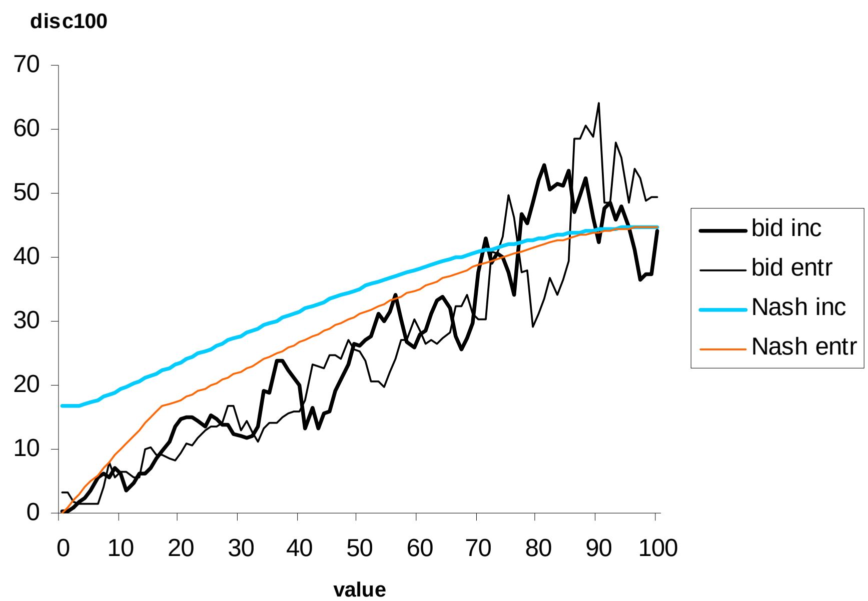
Notes: Bid 1 graphs the average highest bids submitted as function of the values. Likewise, bid 2 and bid 3 show the middle highest bids and the lowest bids respectively. For every value the average of bids in the interval [value–2, value+2] is reported.

Figure 6: Bidding behavior in the discriminatory auction with x = 50



Notes: Bid inc (bid entr) graphs the average of the three bids submitted by incumbents (entrants) as function of value. For every value the average of bids in the interval [value–2, value+2] is reported. Nash inc (Nash entr) shows the Nash bids of incumbents (entrants) of value.

Figure 7: Bidding behavior in the discriminatory auction with x = 100



Notes: Bid inc (bid entr) graphs the average of the three bids submitted by incumbents (entrants) as function of value. For every value the average of bids in the interval [value–2, value+2] is reported. Nash inc (Nash entr) shows the Nash bids of incumbents (entrants) of value.

Figure 8: Preemptive/competitive bidding versus demand reduction

Notes: For every minimum value of the incumbents the % of preemptive/competitive outcomes in the interval [minimum value incumbents–4, minimum value incumbents+4] is reported. The relative frequency of demand reduction outcomes equals 100% – the % of preemptive outcomes.