On the Micro-foundations of Money:
The Capitol Hill Baby-Sitting Co-op

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March 2002

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March 6, 2002

Abstract

We suggest a new micro-foundation of money in which markets are well-organized but consumers' preferences are stochastic. In this model, we solve for stationary equilibria and show that there is an optimum quantity of money. The rational solution of our model is compared with actual behavior in a laboratory experiment. It turns out that the experiment gives support to our theoretical results.

1   Introduction

Understanding the phenomenon of money is one of the most important challenges to economics. However, as Hellwig (1993) has argued convincingly, this

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challenge has not yet been mastered satisfactory by economic theory. In macroeconomics the relevance of money is not explained but *assumed*. In the standard microeconomic model of general equilibrium, money plays no role at all because markets are assumed to be very well organized and they are assumed to open once for all times. In sequential market models money is a potential store of value and can then be made compatible with the general equilibrium model, as has been demonstrated by Samuelson (1958) in the model of overlapping generations. However in this model the store of value can better be interpreted as a social insurance system rather than as money.

To improve upon this unsatisfactory situation, theorists looked deeper into the role of money by giving it a game theoretic foundation. Starting from the seminal papers of Kiyotaki and Wright (1989, 1991, 1993), an impressive literature developed in which markets are no longer considered to be well organized but traders meet randomly in pairs, see also Boldrin, Kiyotaki, and Wright (1993), Trejos and Wright (1995).

This paper suggests a micro-foundation of money that is somewhat orthogonal to the standard search model that aroused out of the papers by Kiyotaki and Wright. On the one hand, we assume that markets are well-organized in the sense that every potential supplier and every potential demander of a service can meet traders of the other side of the market in every period. On the other hand, we assume that the market participants have stochastic preferences. Hence some day a trader would prefer to be a supplier of a service and some other day she prefers to demand the service. The role of money is to allow the traders to transfer income between supply and demand days.

Our model of money is inspired by *trade circles*. Trade circles are of increasing importance in modern market economies and they provide good evidence for clinical studies on the role of money.[1] In a trade circle a set of participants can receive or deliver services at fixed prices. In return the supplier of a service receives some artificial money or coupon, which she can then spend to demand a service. Prices are fixed by some fairness consideration and credit is limited.

The trade circle that is perhaps most well-known to economic theorists is the Capitol Hill Baby-Sitting Co-op. This is because Krugman (1999)[2] illustrated the relevance of money to the everyday man/woman by means of this instructive example. In the Capitol Hill Baby-Sitting Co-op on average 150 couples tried to share baby-sitting fairly by introducing some coupon system. A coupon was the entitlement to receive one night of baby-sitting. Initially one coupon of baby-sitting was issued per couple. Supposing that coupons circulated, over time every couple would thus receive as many units of baby-sitting as it delivered. However, after a short while the system collapsed because there was insufficient demand of baby-sitting. Krugman (1999) attributes this breakdown to precautionary savings. The Co-op solved this problem simply by issuing more coupons. And having experienced that every couple was better off due to the increase in the number of coupons the Co-op issued more and more coupons so

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[1] See, for example, www.tauschring-archiv.de and http://www.talent.ch.

[2] See also Sweeney and Sweeney (1977) for a more detailed report on the Great Capitol Hill Baby-Sitting Co-op.

that the system broke down again. In the Capitol Hill Baby-Sitting Co-op fiat money has a positive value in exchange for services, it can lead to a Pareto-improvement over the situation without fiat money and there is an optimum quantity of money. That is why we got interested in a model of money that captures the main features of the Capitol Hill Baby-Sitting Co-op.

In the next section we give a sound theoretical foundation to the anecdote of the Capitol Hill Baby-Sitting Co-op. We first model an infinite game with stochastic preferences in which a service can be exchanged using a coupon system. Then we suggest a rational solution to the game which is based on individual optimization against market averages (playing the field). Markets are well organized in the sense that in every period in time there is a multilateral meeting of all suppliers and demanders. Money is essential because it allows to transfer purchasing power from a period with a low value of time to a period with a high value of time. Moreover, since credit is limited, in every period, there is a positive demand for money. We then show that there are always two types of equilibria. Non-monetary equilibria in which nobody accepts money and monetary equilibria in which money circulates. The latter Pareto-dominate the first. Moreover we show that in our model there is an optimum quantity of money. The role of money is to *coordinate* supply and demand. Money has this coordination role because every rational individual will hold money only up to some maximum. Hence, by issuing money demand can be stimulated.

Note that our model of the Capitol Hill Baby-Sitting Co-op has many features in common with the neo-Keynesian model as it was developed by Clower, Barro and Grossman, Benassy, Malinvaud and Dréze.[3] In both models prices are fixed and demand and supply is coordinated by quantity rationing. Whereas in the standard neo-Keynesian model money is justified by the notion of a temporary equilibrium the microeconomic foundation of money that our model provides is based on complete rationality under rational expectations.

Since our model is based on a rigorous idealization—the notion of stationary equilibria with rational expectations in a stochastic game—we found it important to contrast the predictions of our model with actual play of possibly bounded rational participants in a laboratory experiment. To this end we set up a market in which six participants per market could repeatedly buy or sell an abstract commodity. Buying was forbidden if the participant had run out of artificial money, called coupons, and a payoff could only be attained if the participant was able to buy. In this case the per period payoff was the time-value which was drawn every period randomly and independently across participants. While the probability distribution of the random process was common knowledge, the time values remained private information to the participants. By and large the experiment gives evidence that the completely rational solution concept we have analyzed also has descriptive merits. In all six markets, the behavior of the participants was exactly as our rational solution concept suggested. Even though each group forming a market consisted only of six participants, their behavior was conform with the best response to market averages which is well

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[3] For a comprehensive account of this theory see e.g. Malinvaud (1977) or Benassy (1982).

in line with our assumption that agents play the field. Moreover, the median maximal demand of money was exactly four units as the model predicted for those parameter values chosen in the experiment. We never observed the non-monetary equilibrium and money circulated fine in the experiment. It turned out that there is an optimum quantity of money, which was slightly higher in a setting with smaller expected time values, in which case there is a higher potential for a demand shortage.

2 The model

In this section we propose the formalization of the Capitol Hill Baby-Sitting Co-op which has been mentioned above. This formal representation will enable us to derive falsifiable conclusions from explicit assumptions on rationality and characteristics of consumers. The abstraction will also help to set up an experiment in which the implications of the model are tested in a laboratory. Our formal model of a monetary economy has to embrace three essential properties.

First, the object of trade is of a particular type. There is only one type of good or service (baby-sitting at night). The good cannot be stored, is not divisible and can only be supplied in one unit (i.e. one night) which is also the production time of the good. Each agent (i.e. the parents participating in the Co-op) can either consume or produce the good but cannot consume the good he produces. We assume that production of the good incurs no costs.

Second, the decision problem of the agent is subject to particular constraints. Since each agent can either supply or demand the good at any period in time, he needs to transfer income across time, if he wants to consume in some period. The only possibility to transfer income is via the acceptance of money in exchange for his good, because any form of private credit is ruled out by definition.[4] Since the price of the good is fixed to one (by the fairness criterion of the Co-op), trade takes place as an exchange of one unit of money against one unit of the good. These considerations highlight that each agent faces an intertemporal decision problem. In every period in time he needs to decide whether to participate in the market—which an agent only does if he accepts money—and whether to supply or demand the good. The actual decision will depend on his preferences and money holdings as well as on his perception of the value of money. We assume that preferences are stochastic and the realization which determines the potential instantaneous utility is observed at the beginning of every period in time.

Third, we have to formalize the market structure in which all potential demanders meet all potential suppliers of the good. We assume that there is one market in every period. At the beginning of the period (i.e. in the morning) all agents who demand or supply the good at the end of the period announce their position. Then the market coordinates demand and supply by some non-pricing mechanism, because the price of the good is fixed to one unit of money. Since

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[4] In a non-monetary barter economy agents would need access to a (complete) market of contingent commodities because the future states of the individual preferences are unknown.

the good comes in a non-divisible unit, we model rationing as follows. If both sides of the market do not match, the longer side is rationed by a mechanism that randomly selects as many agents from the longer side as there are on the shorter side. Any agent on the longer side is thus subject to a probability that he cannot carry out his planned transaction. The non-rationed agents are assigned randomly and the delivery of the good and the payments are made at the end of the period (i.e. at night). An agent has to show that he owns at least one unit of money if he demands the good.

In the following presentation we focus on the main ideas. A rigorous mathematical approach can be found in the appendix. Figure 1 shows the optimization problem of an agent which is denoted as Game O (the optimization game)[5].

Insert Figure 1 here

Figure 1: The optimization problem of an agent with rationing

The game consists of identical periods t $(t = 1, 2, 3, ...)$. The initial endowment of the agent is M units of money. In every period first a chance move determines the time value w: the probability of the high time value h is p_h and the probability of the low time value l is $p_l = 1 - p_h$. Then the agent selects one of the three alternatives S, B or I. Alternative S corresponds to "sell baby-sitting," alternative B can be interpreted as "buy baby-sitting" and alternative I is "stay idle." agents can only select alternative B if they have money (i.e. if the money holdings m in period t are greater than 0), which is the "cash in advance constraint." The choice of an agent is executed with success probability p_S if she chooses alternative S and with success probability p_B if she chooses alternative B. In the market the rationing determines the success probabilities of which one is always 1, because only one market side might be rationed. These probabilities depend on the choices of the other agents. For the first part of our analysis (theoretical and experimental) we take these probabilities as exogenously given. The quantity of money of the agent and her utility changes depend on her choice. The changes are given as the vector of the changes (money, utility) in the figure. Money holdings decrease (increase) if the agent is successful in buying (selling), and utility increases only if she is able to buy. In this case it increases by the time-value. At the end of the period the discounting with rate β takes place. The following restrictions are imposed on the parameters: $0 < l < h$, $0 \le p_S \le 1$, $0 \le p_b \le 1$, $0 < p_h < 1$, and $0 < \beta < 1$.

A policy $\psi(m, w, t) \in \{B, S, I\}$ of an agent determines the choice of an agent given the probabilities p_h, p_S, p_B and the discount rate β depending on her actual money holdings m, her actual time value w (high h or low l) and the period t. In the following part we characterize the stationary solution of this optimization problem. Therefore the policy does not depend on the period. The Bellman equations can be formulated depending on the state variable money holdings m

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[5]Although we model the market as a stochastic game we use the term agents instead of players in this paper

(see Appendix). $V(m)$ denotes the continuation value of the game if the money holdings are m, provided agents are risk neutral and all other parameters are fixed. The Bellman equations are:

$$V(0) = \beta \; \max \left\{ p_S \, V(1) + (1 - p_S) \, V(0), V(0) \right\}$$

(1)

$$V(m) = \beta \sum_{w=h,l} p_w \; \max \left\{ p_B \; [w/\beta + V(m-1)] + (1 - p_B) \, V(m), \right.$$

$$\left. p_S \, V(m+1) + (1 - p_S) \, V(m), V(m) \right\}$$

In every period the optimal stationary policy $\psi^\star$ of an agent is a solution to these equations. We do not derive the proofs of existence and uniqueness of the solution (which can be found in the Appendix), but here it is sufficient to give the intuition of the optimal policy $\psi^\star$ which is:

- Rule 1: $\psi^\star(0, w) = S$ for $w = h, l$;

- Rule 2: $\psi^\star(m, h) = B$, for all $0 < m$ and

- Rule 3: $\psi^\star(m, l) = S$, for all $0 < m < \overline{m}$; and $\psi^\star(m, l) = B$, for all $m \geq \overline{m}$, for some endogenously chosen $\overline{m}$.

Alternative I, staying idle, is never chosen because in the other alternatives utility or money might be gained. It is weakly dominated.

The intuition for rule 1 is: if an agent has no money it is optimal for her to choose alternative S (sell baby-sitting) and to try to get some money. Alternative B (buy baby-sitting) is not possible because of the cash-in-advance constraint. The best alternative for an agent is to obtain money.

The optimal choice of an agent who has the high time value h (and money) is alternative B (buy baby-sitting), because the maximal utility she can obtain for one unit of money is the high time value. Keeping the money will reduce the utility that she might obtain by at least the discounting rate. This motivates rule 2.

The third rule states that it is optimal for an agent not to hold more than $\overline{m}$ units of money. If she has more money, she chooses alternative B, even if her time value is low. An intuitive reason for this maximal quantity of money holdings is the following. If an agent has money holdings of $\overline{m}$, the decision taken in this period is irrelevant for the optimal continuation in the next $\overline{m}$ periods. An agent can use the additional unit of money that she might obtain this period in the $(\overline{m} + 1)$th period for the first time. Rule 3 results from the observation that an agent should go for the lower time value now if the discounting of the expected utility in the $(\overline{m} + 1)$th period leads to a lower expected utility than the lower time value l (times the success probability). If the inequality holds in the other direction an agent should choose the additional unit of money. That $\overline{m}$ is not infinite can be seen by a very crude boundary consideration: the maximal utility an agent can get in the $(\overline{m} + 1)$th period is

the high time value h. Discounting h for enough $(\overline{m})$ periods will lead to lower utility than the lower time value l (times the success probability) if $\overline{m}$ is high enough. Therefore, the lower time value is preferred to the additional unit of money. A numerical solution of the Bellman equation gives the result $\overline{m} = 4$ for the success probabilities being 1, the chances of a high and a low time value being equal, $p_h = p_l = 0.5$, and the continuation probability β being 95%.

3 The market equilibrium

We consider a market with a continuum identical agents as described above. Moreover, we will only investigate symmetric equilibria in which all agents use the same optimal policy. It is therefore sufficient to look at the behavior of a representative agent. Before we define the market equilibrium we look at the stationary distribution of money holdings that results from the agents' policies meeting in a market if the probability of the high time value and the success probabilities are given. Note that the policy of an agent solving the Bellman equations defines a Markov chain as shown in Figure 2.

Insert Figure 2 here

Figure 2: The Markov chain resulting from the optimal policy.

The state space is $\mathbb{N}$, the endogenous money holdings, of an agent. The probabilities of the time value and the success probabilities determine the transition probabilities. According to the rule for her optimal policy $\psi^\star$ the transition probabilities are zero if $m < 0$ or $m > \overline{m}$. For $0 \leq m \leq \overline{m}$ the probability for a transition from m to $m + 1$ is $(1 - p_h) \cdot p_S$ and for the transition from m to $m - 1$ the probability is $p_h \cdot p_B$. With probability $1 - p_S - p_h \cdot (p_B - p_S)$ the money holdings stay at m. For $m = 0$ the probability of the transition to $m = 1$ is p_S and with probability $1 - p_S$ the money holdings remain at 0. At $\overline{m}$ the probability to change to $m - 1$ is p_B and it is $1 - p_B$ to stay at $\overline{m}$. Since all probabilities are strictly positive, the Markov chain is irreducible and, thus, possesses a unique stationary distribution $\mu^\star$. A given stationary distribution μ determines the rationing, i.e. the success probabilities p_S and p_B by[6]

$$
p_S = \min\left\{1, \frac{\sum_{m \in \mathbb{N}} \mu(m) \sum_{w=h,l} p_w \, \mathbf{1}_{\psi(m,w)}(B)}{\sum_{m \in \mathbb{N}} \mu(m) \sum_{w=h,l} p_w \, \mathbf{1}_{\psi(m,w)}(S)}\right\}
$$

$$
(2)
$$

$$
p_B = \min\left\{1, \frac{\sum_{m \in \mathbb{N}} \mu(m) \sum_{w=h,l} p_w \, \mathbf{1}_{\psi(m,w)}(S)}{\sum_{m \in \mathbb{N}} \mu(m) \sum_{w=h,l} p_w \, \mathbf{1}_{\psi(m,w)}(B)}\right\}
$$

In this equation the probability that an agent wants to buy baby-sitting (choose alternative B) is determined and compared with the probability that

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[6]$\mathbf{1}_{\psi(m,w)}(S)$ is the indicator function which gives value 1 if $\mathbf{1}_{\psi(m,w)} = S$ and value zero otherwise.

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an agent wants to sell baby-sitting (alternative S). The probability of selling if the money holdings are m is the probability of getting the time value w if the choice given by the policy depending on m and w, $\psi(m, w)$, is selling, it is zero if the choice is buying. The total probability of selling is the sum over all these probabilities for fixed m weighted with the probability of having money holdings of m given by the stationary distribution. The probability of buying (alternative B) is determined in the same way. The success probability of the shorter market side is set to one and the success probability of the longer market side is the quotient of the probabilities which is smaller one.

To get some intuition on the rationing mechanism first look at the case $p_h = p_l = 0.5$. Provided the agents use the optimal policy $\psi^\star$ the only important probabilities are the probabilities at 0 and $\overline{m}$, because all other terms are identical for buying and selling. If the probability at zero is greater than at $\overline{m}$ more agents want to sell baby-sitting than agents want to buy it according to the optimal policy. Thus selling is rationed and vice versa.

One should realize that the success probabilities defined in (2) are equal to the prescribed ones (p_B and p_S). Let us consider the case $p_S = 1$ (the case $p_B = 1$ is analogous). By equation (2) given the policy $\psi^\star$, we get

$$p_B \left(\sum_{m=1}^{\overline{m}-1} \mu(m)\, p_l + \mu(\overline{m}) \right) = \sum_{m=1}^{\overline{m}-1} \mu(m)\, p_h + \mu(0)$$

This equation requires that in each period in time the number of agents whose money holdings decrease by one (due to actually buying the good) is equal to the number of agents whose money holdings increase by one (due to actually selling the good). This condition is satisfied by stationarity of μ.

In the market, given a policy ψ (which is optimal for fixed success probabilities p_S and p_B) the quantity of money M in that market also influences the stationary distribution of the Markov chain and the success probabilities p_S and p_B resulting from the stationary distribution μ. The intuition of this influence is that the higher the quantity of money the higher is the probability of high money holdings in the stationary distribution of the Markov chain. The higher the probability of high money holdings the more money agents have on average and the more agents want to buy baby-sitting compared to selling. The lower the initial money holdings the less agents want to buy. This effect determines the rationing. For the special case $p_h = p_l = 0.5$ and the agents using the optimal policy $\psi^\star$ the probability to stay at $\overline{m}$ compared to the probabilities to stay at zero increases if the quantity of money M increases which determines the rationing. Technically the "consistency" of the quantity of money and the success probabilities is implied by the condition that the quantity of money has to be equal to the average money holdings: $\sum_{m \in \mathbb{N}} m\, \mu(m) = M$.

All these factors that influence the market have to be "consistent" in an equilibrium, i.e. the policies have to result from the optimization of the agents given the other factors, the success probabilities given by the stationary distribution have to be the same as the ones taken for the determination of the policy and the quantity of money has to be equal to average money holdings of

an agent. These considerations define a stationary equilibrium.

A *stationary equilibrium* is a tuple $(\psi^\star, \mu^\star, p_S^\star, p_B^\star, M^\star)$, consisting of a stationary policy, a distribution over money holdings, success probabilities, and a quantity of money, such that

(i) given $p_S^\star$ and $p_B^\star$, $\psi^\star$ is an optimal policy for the Bellman equations (1);

(ii) given $\psi^\star$, $p_S^\star$ and $p_B^\star$, $\mu^\star$ is a stationary probability measure for the corresponding Markov chain on $\mathbb{N}$ (the money holdings);

(iii) given $\psi^\star$ and $\mu^\star$, the probabilities $p_S^\star$ and $p_B^\star$ satisfy the rationing given by (2);

(iv) the average money holdings are equal to the average money supply, i.e.
$\sum_{m \in \mathbb{N}} m\, \mu^\star(m) = M^\star$;

for given characteristics of the agent.

3.1 Monetary equilibria and the optimal quantity of money

Given the definition of a stationary equilibrium one can prove the existence of stationary equilibria. The existence of non monetary equilibria is easy to understand: For any characteristic of an agent and any amount of money a non-monetary equilibrium exists. In this equilibrium $p_B^\star$ is zero. In this market everybody wants to buy baby-sitting and nobody wants to sell. It is also obvious that no agent improves by selling baby-sitting, because with the additional unit of money received for selling baby-sitting she never can buy baby-sitting which is the only way to improve her utility.

It is more difficult to understand the existence of monetary equilibria for any characteristic of an agent and given success probabilities $p_B > 0$ and $p_S > 0$. The idea of this existence is analogous to the way we tried to subdivide the problem in the previous parts of this paper. Given the success probabilities an optimal policy is determined by solving the Bellman equations. Given this policy a stationary distribution on the money holdings is given which is "consistent" with the success probabilities taken for the determination of the optimal policy. The initial endowment with money is determined such that it is equal to the average money holdings. For details see the appendix A.

Another interpretation of this result is that agents assume certain success probabilities (for example: $p_S = p_B = 1$, i.e. no rationing will occur) and determine their policy by optimization. In a market the quantity of money and the policy determine a dynamic process which determine the success probabilities in the market. These success probabilities have to be identical to the ones assumed for the optimization. The quantity of money is the parameter that allows to adjust the success probabilities of the process to the success probabilities taken for the determination of the policy. The quantity of money is chosen such that the average money holdings under the assumption the policy works are equal to this quantity of money. This adjusts the success probabilities.

The general existence result for monetary equilibria when the quantity of money can be chosen by an institution outside the model of our monetary economy raises the question on the optimum quantity of money. Obviously, any non-monetary equilibrium is Pareto-dominated by every monetary equilibrium. However, this does not provide a satisfactory answer in the case of a benevolent social planner who maximizes the welfare in the economy. The most simple measure is that of the number of trades. The number of trades is maximal if no rationing occurs $(p_B^\star = p_S^\star = 1)$. If there is rationing, some agents cannot trade and thus obtain lower utility than in an equilibrium without rationing. While this criterion only measures individual but not social welfare it provides a simple benchmark for the social planner's policy.

We have the following result. There exists some quantity of money $M^\star$ and a corresponding monetary stationary equilibrium such that no rationing occurs $(p_B^\star = p_S^\star = 1)$. Every quantity of money which is different to $M^\star$ leads to a stationary equilibrium in which the number of trades is strictly less.

The existence of an equilibrium with $p_B^\star = p_S^\star = 1$ has been shown above. In the equilibrium without rationing and the quantity of money $M^\star$ the number of trades is maximal, because no rationing occurs. We have to compare the utility an agent obtains in this equilibrium with the utility she gets in any other equilibrium with different quantity of money M_1. In the second equilibrium rationing has to occur, because if $p_B^\star = p_S^\star = 1$ the policies of the agents would be the same, leading to the same Markov chain and the same quantity of money. If rationing occurs in an equilibrium the utility that a player obtains in this equilibrium is lower than in a non equilibrium policy combination without rationing in which the same policy is used, but the quantity of money M_2 is different. This non equilibrium policy combination is dominated by the equilibrium policy combination with $(p_B^\star = p_S^\star = 1)$ and the quantity of money $M^\star$, because the equilibrium policy is the solution to the Bellman equations without rationing. For details see the appendix A.

This shows that it is legitimate to call the quantity of money that is determined this way the *optimum quantity of money*.

4 The Experiment

Our goal in the experiment is to test the predictions of the model. Predictions fall within two categories: individual behavior in an artificial resp. actual market and market dynamics. We shortly summarize our hypotheses and the implementation of the model in an experimental setting before giving a detailed description of the experimental procedure and experimental results.

The first prediction is that individual behavior should render the optimal policy. That is, (1) no money holdings should imply choosing action S (sell); (2) if money holdings are non-zero observing a high time-value should result in playing B (buy) and (3) if money holdings are non-zero and below some certain quantity $\overline{m}$, observing a low time-value should imply S; if money holdings exceed $\overline{m}$, alternative B should be chosen. Especially if agents are risk neutral $\overline{m} = 4$.

The second hypothesis concerns our assumption that agents play the field and do not engage in strategic reasonings how to influence market averages. That is, the individual behavior described in the first hypothesis should be observed with or without actual markets. In particular a decrease in the probability of the high time-value should have a uniform impact regardless of the initial endowment of money.

The third hypothesis is derived from the result on the optimum quantity of money. In every market there is some quantity of money for which the average number of transactions is maximal. The optimal quantity depends on the fundamentals; in particular the less probable the high time-value, the lower the maximal money holdings. Especially if agents are risk neutral ($\overline{m} = 4$) and if $p_h = p_l = 0.5$ the optimal quantity of money $M^\star = \overline{m}/2 = 2$, because of the symmetry of the stationary distribution of the Markov chain.

The experiments are organized in two stages. First individual behavior is analyzed in games in which participants play against nature without markets. Subjects play these games repeatedly on computer terminals—referred to as the learning phase. After subjects have acquired experience in these games, a strategy game is carried out. In the strategy game subjects have to determine their individual strategies for each game. A form was supplied, but subjects were allowed to use a blank sheet of paper and did not need to fill in this form. Second we let subjects interact in a common market, facing the decisions of other human subjects. The procedure is the same as in the case discussed above. This approach is intended to reveal whether the notion of a stationary equilibrium in our model—which is based on complete rationality—is a realistic description of actual behavior in a situation with actual market-interaction.

In order to perform the experimental testing of our hypotheses in a laboratory, we have to operationalize our model. In the experiment the time-value corresponds to a potential monetary payoff. The discounting in the model is implemented as the break-off probability of the game. While the model postulates an infinite number of agents in the market, the number of subjects in the experiment has to be finite. A detailed description of the games follows.

Individual Behavior Game. Two games have been implemented to analyze the individual behavior in an artificial market. Each game consists of a maximum number of 100 periods. All periods are identical in both games. At the beginning of each period a random number generator determines the time-value of the subject. The probability of the two time-values is set to $p_h = p_l = .5$. After observing the time-value, agents have to choose one of the three alternatives S (selling), B (buying), or I (staying idle). Alternative B can only be chosen if money holdings are not zero. When the agent has made her choice, it is decided by a random draw whether the intention of the subject is put into execution.

In game N (no rationing), the success-probabilities are $p_B = p_S = 1$, i.e. no rationing takes place and each choice of agents is actually executed. In game R (rationing), the success-probabilities are $p_B = p_S = .8$, i.e. rationing takes place and each choice of the agents is only executed in 80% of all cases.

Depending on the agent's choice and on the outcome of the random draw determining whether this choice is put into action, the money holdings and the

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accumulated payoffs of the agent change. If S had been chosen then money holdings increase by one. If B had been chosen then money holdings decrease by one and the time-value is added to the payoff account. Nothing happens if choice I had been made. At the end of each period it is determined by a random draw whether the game ends. If the game ends, money holdings of agents become worthless and the accumulated payoffs turn into cash for the subject.

Market Game. The individual behavior in an actual market has also been analyzed in two games. Six participants interact in a market. The order of events is analogous to the game in an artificial market except that the rationing depends on the choices of the participants. In each period the longer side of the market is rationed according to an i.i.d. random draw such that the number of selected subjects matches the shorter side. All selected subjects on the longer side and all subjects on the shorter side actually trade. The time-value of each subject is also determined by a random draw that is i.i.d. across time and subjects.

In game $M.5$ the probability of the high time-value is set to $p_h = .5$; and in game $M.2$ we set $p_h = .2$. Each market was made up of six participants. With a total of 36 participants who were students at the university of Zurich, we have six independent observations of the market game. The rational for game R in which participants are rationed regardless of the side of the market they choose is founded in this implementation of the market game. Suppose money holdings do not matter and strategies of all subjects are buy/sell when observing a high/low time-value. Then for $p_h = .5$ the probability of actually making a trade in the current period from the perspective of a subject (regardless of the particular action chosen) is 68.75%. That is, due to the fact that a small number of subjects trade in a common market, the average number of high (resp. low) time-value actually realized differs from the expected number in most cases. However, this should not influence the result, because if the success probabilities for selling or buying are the same ($p_B = p_S$) the same behavior should be observed as if both were 1.

Payment of each participant is based on his/her individual performance in the experiment. The experiments were conducted in the computer laboratory of the Institute for Empirical Research in Economics at the University of Zurich. No communication was allowed between participants and computer terminals were separated from one another. Total time for the experiment was about four hours. Each game was played for about one hour with instructions (see Appendix) given in the first 15 minutes.

Throughout the experiment we let $\beta = 0.95$ (i.e. break–off probability is 5% in each round), $h = 10$, and $l = 5$. Each unit is worth 0.05 SFr. ($\approx \$0.03$). All parameter settings are common information. At the beginning of each game all participants were given an identical initial endowment of m ($m = 1, 3, 8$) units of money (i.e. helicopter-money), called coupons. The traded commodity was not termed. The average payoff of participants was about 100 SFr. ($\approx \$60$).

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4.1 Results

The strategy game is analyzed to compare the experimental results with the hypotheses we want to verify. We analyze the strategy game, because we are interested in the behavior of experienced agents who have passed through the "learning phase" with a repeated play of the respective game.

The main observation in the outcome of the strategy game concerns the structure of the strategies specified by the participants. The strategies of all participants consist of three phases: start phase, main phase, end phase. The start phase seems to be caused by the fact that 20 periods were played for sure. The end phase might be caused by the finite number (100) of periods. Since we are interested in the stationary behavior of subjects we analyze the main phase. All strategies are of the form described in our first hypothesis. Table 3 in the Appendix gives a full description of the result of the strategy game and lists the median of $\overline{m}$ across groups. The result is significant because 36 out of 36 observations are of this type.[7] Summarizing we can conclude that our first prediction is confirmed.

Let us next discuss the properties of the maximal number of coupons held by subjects. Table 3 shows that this quantity exhibits high variation across subjects within each game. The median $\overline{m}$ over all subjects is given by $\overline{m} = 4$ in the games N, R, and M.5 for all initial endowments $m = 1, 3, 8$ of coupons, except for game M.5 with $m = 1$ in which the median $\overline{m} = 3.5$. In game M.2 we find that $\overline{m} = 2.75$ for the initial endowment $m = 1$, and $\overline{m} = 3$ for the initial endowments $m = 3, 8$. Under the assumption that success probabilities are equal to 1, the theoretical model predicts (regardless of the initial money holdings) $\overline{m} = 4$ in games N and M.5, $\overline{m} = 2$ in game M.2 and $\overline{m} = 3$ in game R. These values are determined numerically, see Schenk-Hoppé (2001a). Summarizing we can state that the experimental results on the maximal amount of money holdings are in good agreement with the theoretical model.

We now turn to a comparison of different games to examine our second hypothesis that subjects play the field and do not engage in strategic reasonings how to influence market averages. Table 1 evaluates the six independent observations (one for each group) on the changes of the median between different games. Except for the clear pattern in the change in the mean from game M.5 to M.2, Table 1 does not provide any strong evidence about the direction of change of the median between different games.

Insert Table 1 here

The hypothesis that subjects play the field can be tested empirically by the change from game R (or N) to game M.5. Our prediction is that there is no change in the median of $\overline{m}$. It is clear from Table 1 that there is no strong empirical evidence against our hypothesis. Even on a significance level of 30%

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[7] The hypothesis that half of the population from which participants are drawn would pursue a different strategy is rejected in a one-sided binomial test on a significance level of 0.1%.

the hypothesis that $\overline{m}$ is constant is not rejected in a binomial test. However, the effect of a change in the initial endowment of coupons is not very strong. For instance in game M.5 the median $\overline{m}$ increases only slightly in merely 2 out of 6 groups when the initial endowment increases from $m = 1$ to $m = 8$. In game M.2 this effect is present in 3 groups. If subjects would fully take into account their own potential to effect market averages, an increase of at least about 5 coupons in the maximum money holdings should be observed. This is not the case here, see Table 3. The hypothesis that $\overline{m}$ increases more than one coupon is rejected in a one-sided binomial test on a significance level of 2%. Let us also compare the effect of a change in the probability p_h for different initial endowments. Table 1 shows that the direction of change of $\overline{m}$ from game M.5 to M.2 is significant. For each initial endowment $\overline{m}$ decreases in 6 out of 6 observations.[8] Thus changes in the fundamentals have the predicted effect and give a hint that subjects actually play the field.

Summarizing we may conclude that our second hypothesis is supported by the data.

We finally address the third hypothesis on the optimum quantity of money. We employ the strategies that subjects supplied in the strategy game in a numerical simulation of the markets M.5 and M.2. The simulations are implemented as MATLAB scripts and are available on the web (Schenk-Hoppé 2001a). It is worth to point out that maximal money holdings vary with the initial endowment in 17 (resp. 15) out of 36 case in game M.5 (resp. M.2). Thus one cannot expect to obtain a clear-cut result.

The strategy of every subject within a group are taken from Table 3. In order to obtain a most complete specification of the effect of different quantities of money, we linearly interpolate the maximal $\overline{m}$ of each subject over non-retrieved initial money holdings by using the available data for $m = 1, 3, 8$. For simplicity the results are rounded to integers.

Starting from a uniform initial distribution of money holdings m over subjects (thus m is the quantity of money), we simulate the market interaction for $10,000$ iterations (ignoring the break-off probability). From these data we calculate the average number of trades in a single period and the average time-values realized by the subjects in (Schenk-Hoppé 2001a). Table 2 details the numerical results.

Insert Table 2 here

One further finds that trade breaks down in game M.5 (game M.2) for all groups if the quantity of money $M \geq 8$ ($M \geq 5$).

For all groups and market games we observe an increase of transactions from an initial endowment of 1 to an optimal endowment m_O ($1 < m_O < 8$) and a decrease for higher endowments. The hypothesis that the number of transactions does not depend on the initial endowment is rejected on a 2%

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[8] The hypothesis that half of the population would hold more money in game M.2 than in game M.5 is rejected in a one-sided binomial-test on a significance level of 2%.

significance level for all markets. This strongly supports our third hypothesis. For a further test of our predictions we compare the optimal initial endowments m_O with the predicted optimal quantities.

The benchmarks for the average number of trades and realized time-values are as follows. The maximum number of trades in any one period is 3, i.e. both sides of the market are of the same length for all periods in time. The expected time-value of each subject is 7.5 in game M.5 resp. 6 in game M.2. Assuming that buyers have an expected time-value that matches the overall expected time-value, a proxy for the average time-value is 3.75 in game M.5 resp. 3 in game M.2.

Given these benchmarks we can state that the monetary economy determined by the strategy game works surprisingly well. The average number of trades reaches 2.15 in game M.5 and 2.26 in game M.2; and the average realized time-value tops at 3.37 in game M.5 and 2.66 in game M.2. The median of the quantities of money for which trades resp. payoffs are maximal is $M = 2$ in game M.5 whereas in game M.2 the median is 3 for trades and 2.5 for payoffs. In game M.5, 4 out of 6 observations are conform with $M = 2$ being the optimum quantity of money. The hypothesis that the optimum quantity of money is not in the interval $[2, 3]$ is rejected on a significance level of 5%. In group 1 the behavior of subject 3 stands out, because it has a very high $\overline{m}$. This behavior stimulates trade within the respective group even for high quantities of money. Beside the fact $\overline{m}$ exceeds the total amount of money in games M.5 and M.2 with $m = 1$, a reason for not being reluctant to hold so many coupons may be a dissenting individual perception of the break-off probability.

For the sake of comparison we simulate an artificial model in which $\overline{m} = 4$ (resp. $\overline{m} = 3$) for all agents in game M.5 (resp. game M.2). It turns out that average trades and payoffs are maximal for $M = 2$ in game M.5. The average number of trades is 2.13 and the average payoff is 3.35. In game M.2, the quantity of money $M = 2$ is also optimal. The average number of trades is 2.08 and the average payoff is 2.57.

We can state that performance of subjects—which exhibit a high degree of heterogeneity—is very good compared to this artificial market in which agents are homogenous.

Summarizing the experimental results we can conclude that there is experimental evidence for the correctness of our theoretical model which is based on complete rationality of agents. In the median the behavior of the participants in the experiment coincided with the optimal behavior of the representative agent as predicted by the model. The concept of a stationary (monetary) market equilibrium which assumes that agents "play the field" was also confirmed by the experiments. Even though each market consisted only of six participants, the median of their behavior was conform with the best response to market averages. We observed that money circulated fine in the different markets and the average number of trades and realized time-values were surprisingly high. In each market game an optimum quantity of money could be determined.

Insert Table 3 here

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5 Tables and Figures

Direction of change	N to R (initial m)			N to M.5 (initial m)			R to M.5 (initial m)			M.5 to M.2 (initial m)		
	1	3	8	1	3	8	1	3	8	1	3	8
increased	2	1	1	1	1	2	0	1	2	0	0	0
unchanged	3	5	4	2	2	1	3	2	1	0	0	0
decreased	1	0	1	3	3	3	3	3	3	6	6	6

Table 1: Evaluation of the strategy game: Number of groups in which the median of $\overline{m}$ increases/remains constant/decreases between different games.

Group		Game M.5 quantity of money						Game M.2 quantity of money			
		1	2	3	4	5	6	1	2	3	4
1	Trans.	1.85	2.05	2.09	2.08	1.81	0.50	0.85	1.19	1.37	2.25
	Value	3.03	3.31	3.29	3.18	2.58	0.62	1.30	1.74	1.93	2.64
2	Trans.	1.97	2.15	1.96	1.25	0.0	0.0	1.74	2.26	1.55	0.0
	Value	3.09	3.32	2.89	1.66	0.0	0.0	2.09	2.66	1.59	0.0
3	Trans.	1.89	2.14	1.99	0.91	0.0	0.0	1.13	1.77	2.22	0.0
	Value	3.09	3.35	2.94	1.16	0.0	0.0	1.57	2.27	2.34	0.0
4	Trans.	1.85	2.14	1.83	0.50	0.0	0.0	0.94	1.83	0.80	0.0
	Value	3.04	3.37	2.61	0.62	0.0	0.0	1.44	2.36	0.80	0.0
5	Trans.	2.10	2.14	1.97	0.93	0.0	0.0	1.02	1.16	1.99	0.80
	Value	3.20	3.25	2.87	1.20	0.0	0.0	1.51	1.68	2.46	0.81
6	Trans.	1.92	2.06	2.08	2.07	2.07	1.95	0.90	1.25	1.75	1.74
	Value	2.99	3.23	3.27	3.23	3.18	2.85	1.26	1.67	2.06	1.76

Table 2: Average number of transactions (Trans.) and average realized time-values (Value) per period in both market games for different quantities of money.

Grp.	Part.	Game N (initial m)			Game R (initial m)			Game M.5 (initial m)			Game M.2 (initial m)		
		1	3	8	1	3	8	1	3	8	1	3	8
1	1	6	4	4	6	4	4	6	4	6	5	3	5
	2	6	6	6	6	6	6	8	8	8	6	6	6
	3	12	12	12	12	12	12	12	12	12	12	12	12
	4	5	5	5	5	5	5	5	5	5	2	4	5
	5	3	3	3	3	3	3	3	3	3	2	2	2
	6	4	4	4	3	3	3	2	3	4	2	2	2
median		5.5	4.5	4.5	5.5	4.5	4.5	5.5	4.5	5.5	3.5	3.5	5
2	7	3	3	3	3	3	3	4	4	4	1	4	1
	8	3	3	3	4	3	4	1	3	9	1	3	9
	9	4	4	4	4	4	4	2	4	3	2	4	4
	10	5	5	5	4	5	5	5	5	5	3	3	3
	11	3	3	3	3	3	3	4	4	4	3	3	3
	12	10	10	10	10	10	10	4	6	6	2	3	3
median		3.5	3.5	3.5	4	3.5	4	4	4	4.5	2	3	3
3	13	3	4	5	6	7	2	3	6	3	3	7	5
	14	3	3	3	2	2	2	3	3	3	2	2	2
	15	6	6	6	6	6	6	3	3	3	2	2	2
	16	7	7	7	7	7	7	7	7	7	5	5	5
	17	7	7	7	7	7	7	3	3	3	3	3	3
	18	4	4	4	4	4	4	4	4	8	1	2	2
median		5	5	5.5	6	6.5	5	3	3.5	3	2.5	2.5	2.5
4	19	4	4	4	5	5	5	3	4	4	2	2	2
	20	6	8	10	9	6	10	5	4	8	3	3	4
	21	5	5	5	5	5	5	5	5	5	5	5	5
	22	5	5	5	4	4	4	4	4	4	3	3	3
	23	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	6.5
	24	4	4	4	4	4	4	4	4	4	3	3	3
median		4.5	4.5	4.5	4.5	4.5	4.5	4	4	4	3	3	3.5
5	25	5	7	12	5	7	12	3	6	10	2	4	9
	26	4	4	4	4	4	4	4	7	7	3	3	3
	27	1	1	1	1	1	1	1	3	3	3	2	2
	28	2	3	1	2	3	1	2	3	1	2	2	2
	29	3	3	3	3	3	3	3	3	3	2	2	2
	30	3	3	3	3	3	3	3	3	3	11	11	11
median		3	3	3	3	3	3	3	3	3	2.5	2.5	2.5
6	31	15	15	15	15	15	15	6	11	14	6	9	13
	32	5	5	5	5	5	5	3	3	3	1	1	1
	33	4	4	4	3	3	3	1	3	3	1	1	1
	34	3	3	3	2	2	2	2	2	2	2	1	1
	35	16	16	16	16	16	16	16	17	17	7	7	7
	36	5	6	8	4	6	8	5	4	6	5	5	8
median		5	5.5	6.5	4.5	5.5	6.5	4	3.5	4.5	3.5	3	4

Table 3: Evaluation of the strategy game: Maximal number of coupons $\overline{m}$ that subjects wanted to hold detailed for all games and initial endowments of money.

(Subject 23 is indifferent between 2 and 3 as maximal money holdings and randomizes with probability one half.)

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A Appendix: The formal approach

Let us describe the formal model. We assume that there is a continuum of infinitely-lived identical agents, denoted by the interval $[0, 1]$. We can thus speak of the representative agent, or simply "the agent." The assumption on the number of agents rules out any impact of strategic behavior of single individuals, i.e. each agent "plays the field." Time is discrete and divided into periods in which each agent can produce one unit of the good. Production incurs no cost for the agent. We assume that agents only derive utility from consumption of the good.

There is a fixed quantity of a tangible non-consumable good, called money. The amount of money is exogenously given. The price of the consumption good is fixed to one unit of money. We assume that agents have unknown histories. Thus private credit is ruled out and money is essential.

In each period in time, each agent has the following decision possibilities. He can offer to supply one unit of the good (S = sell), to demand a unit of the good (B = buy), or to stay idle (I). Each agent faces a cash-in-advance constraint; he cannot demand the good (buy) if his money holdings are zero. Whether a transaction actually takes place or not is depending on the ratio of buyers to sellers. As explained above, each agent on the shorter side is assigned a trading partner on the longer side by some random draw which is i.i.d. (independent and identically distributed) across time and agents. In the following, we call the probabilities to actually realize the sell/buy plan the success probabilities.

We assume that each agent has stochastic preferences that vary across time. The most simple approach is to postulate that each agent's preferences in each period are described by the realization of a two-valued i.i.d. random variable. Every agent privately observes the realization of the random draw before making a decision. The revealed value is the maximum (instantaneous) utility an agent can achieve in the respective period. The interpretation is straightforward. Two types of events can happen, say hardrock or classic concert; both of which the agent can only attend when he buys the good in the corresponding period. The two values of the random draw are associated to these events in the sense that they describe the personal ranking of the possible events expressed in terms of the derived utility from attending the event. While the individual realization differ in each period, the statistical properties of the realization are identical for all agents across time. Moreover, there will be a constant amount of agents with the high resp. low value in all periods. With this model of preferences we cannot expect to capture the phenomenon of precautionary savings, but we provide a very simple mechanism that stimulates trade which can only occur if some heterogeneity of agents prevails. The two values will represent potential pecuniary rewards in each round in the experiment.

The agent's objective is supposed to be the persuasion of a feasible plan that maximizes the expected present value of the total utility. For the sake of analytical tractability, we assume that agents are risk-neutral. One can interpret the discounting rate as the one-period survival probability of an agent who maximizes the no-discounting present value. A blend of both factors is also captured in this framework. Agents with finite lifetime of certain length would not accept money if they are rational.

We next derive the agent's optimization problem. Let us first define the exogenous variables. Let the time-value w_t, $t \geq 0$, be an independent and identically distributed process on the measurable space $(W, \mathcal{P}(W))$ with $W = \{l, h\}$, $h \geq l \geq 0$, where $\mathcal{P}(\cdot)$ denotes the power set. The probability distribution of w_t is denoted by $\mathbb{P}_w = (p_l, p_h)$, where $p_w = Prob\{w_t = w\}$, $w = l, h$. Further, to formalize the success probabilities, let z_t, $t \geq 0$, be a process on the measurable space $(Z, \mathcal{P}(Z))$ with $Z = \{0, 1\}^2$ such that $z_s(i)$ and $z_t(j)$ are independent for all $s, t \in \mathbb{N}$, $s \neq t$, and all $i, j = 1, 2$. Let the probability distribution of the first resp. second component of z_t be given by $(1 - p_B, p_B)$ resp. $(1 - p_S, p_S)$, where $p_B = Prob\{z_t(1) = 1\}$ and $p_S = Prob\{z_t(2) = 1\}$. p_S is the probability that an agent can actually sell if she decides to offer her good and, respectively, p_B is the probability that her demand will be satisfied.; afterwards called success probabilities. Accordingly, let $(W^t, \mathcal{W}^t)$ and $(Z^t, \mathcal{Z}^t)$ denote the measurable spaces associated to sequences of length t.

The state variable is a pair of money holdings m and time-value w. Money holdings is an endogenous variable. The state space of the system is therefore given by $(S, \mathcal{S}) := (\mathbb{N} \times W, \mathcal{P}(N) \otimes \mathcal{P}(W))$. The feasible actions can be formalized by a correspondence Γ :

$S \to A$, $A = \{B, S, I\}$. We have that $\Gamma(m, w) \equiv A$ for all $m \geq 1$, and – due to the cash-in-advance constraint – $\Gamma(0, w) \equiv \{S, I\}$. The graph of Γ is given by the measurable set $D = \{(m, w, a) \mid a \in \Gamma(m, w)\} \subset \mathbb{N} \times W \times A$. Note that the set of feasible actions is independent of the time-value.

Money holdings, $F : D \times Z \to \mathbb{N}$, is a flow variable whose evolution depends on the position an agent takes as well as on the outcome of the rationing procedure. We define it together with the return function $U : D \times Z \to \{l, h\}$ as follows,

$$\begin{array}{llll}
F(m, w, B, z) = m - 1, & \text{and} & U(m, w, B, z) = w, & \text{if } z(1) = 1; \\
F(m, w, S, z) = m + 1, & \text{and} & U(m, w, S, z) = 0, & \text{if } z(2) = 1; \\
F(m, w, a, z) = m, & \text{and} & U(m, w, a, z) = 0, & \text{otherwise.}
\end{array}$$

Hence, money holdings decrease (increase) if the agent is successful in buying (selling), and utility increases only if she is able to buy. In this case it increases by the time-value. A *plan* is a sequence of functions $\pi = \{\pi_t\}_0^\infty$, where $\pi_t : W^t \times Z^t \to A$ is $\mathcal{W}^t \otimes \mathcal{Z}^t$-measurable for all $t \geq 0$. Hence the action that is taken at t, $a_t \in \Gamma(m_t, w_t)$, depends on the history of the time-values, money holdings, and success probabilities.

A plan π is *feasible* from an initial state $s_0 = (m_0, w_0)$ if in addition it satisfies

- $\pi_0 \in \Gamma(s_0)$
- $\pi_t(w^t, z^t) \in \Gamma(m_t^\pi(w^{t-1}, z^t), w_t)$, for all $(w^t, z^t) \in W^t \times Z^t$, $t \geq 1$.

where the functions $m_t^\pi : W^{t-1} \times Z^t \to \mathbb{N}$ are defined recursively by

- $m_1^\pi(w_0, z_1) = F(m_0, w_0, \pi_0, z_1)$, for all $z_1 \in Z$
- $m_{t+1}^\pi(w^t, z^{t+1}) = F(m_t^\pi(w^{t-1}, z^t), w_t, \pi_t(w^t, z^t), z_{t+1})$, for all $(w^t, z^{t+1}) \in W^t \times Z^{t+1}$, $t \geq 1$.

The money holdings of the agent at the beginning of period t, denoted by m_t^π, are the key determinants of the feasibility of a plan π.

For each $s_0 \in S$, let $\Pi(s_0)$ be the set of feasible plans from s_0. For each $\pi \in \Pi(s_0)$, define $m^\pi = \{m_t^\pi\}_{t=1}^\infty$ as above. Further, let $m_0^\pi = m_0$.

We can now state the optimization problem of an representative agent with discount rate $\beta \in (0, 1)$. For any initial state s_0, the agent determines a plan that takes the supremum

$$v^\star(s_0) := \sup_{\pi \in \Pi(s_0)} \mathbb{E}\left(\sum_{t=0}^\infty \beta^t\, U(x_t^\pi, w_t, \pi_t, z_{t+1}) \right) \tag{3}$$

for any given initial state s_0.

The expected value of the supremum over the realization of w_0 is given by

$$V^\star(m) = p_h\, v^\star(m, h) + p_l\, v^\star(m, l) \tag{4}$$

because agents are risk–neutral by assumption. $V^\star : \mathbb{N} \to \mathbb{R}$ is a solution to the Bellman equation,

$$V(0) = \beta\, \max\left\{ p_S\, V(1) + (1 - p_S)\, V(0), V(0) \right\}$$

$$\tag{5}$$

$$V(m) = \beta \sum_{w = h, l} p_w\, \max\left\{ p_B\, [w/\beta + V(m - 1)] + (1 - p_B)\, V(m), \right.$$

$$\left. p_S\, V(m + 1) + (1 - p_S)\, V(m), V(m) \right\}$$

where the decision in the maximization operator is contingent on the realization of the stochastic preferences. This decision characterizes the optimal stationary policy $\psi^\star : \mathbb{N} \times W \to \{B, S, I\}$. Since the absolute value of the return function is bounded, the transversality condition holds and, thus, assuming stationarity of z_t, the sequence problem and the Bellman equation are equivalent. We have completely described the individual behavior of agents, given the data p_h, p_l, l, h, p_S, and p_B.

The individual decisions meet on the market. The market coordinates demand and supply by the rationing mechanism described above. Denote by μ a distribution of money, where $\mu(m)$

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is the Lebesgue-measure of agents with m units of money. Suppose all agents follow the same stationary policy ψ. Then, for given μ, the success probabilities p_S and p_B are given by

$$p_S = \min\left\{1, \frac{\sum_{m\in\mathbb{N}} \mu(m) \sum_{w=h,l} p_w \mathbf{1}_{\psi(m,w)}(B)}{\sum_{m\in\mathbb{N}} \mu(m) \sum_{w=h,l} p_w \mathbf{1}_{\psi(m,w)}(S)}\right\}$$

$$\tag{6}$$

$$p_B = \min\left\{1, \frac{\sum_{m\in\mathbb{N}} \mu(m) \sum_{w=h,l} p_w \mathbf{1}_{\psi(m,w)}(S)}{\sum_{m\in\mathbb{N}} \mu(m) \sum_{w=h,l} p_w \mathbf{1}_{\psi(m,w)}(B)}\right\}$$

If both numerator and denominator are zero, then both probabilities are set to zero. Hence, the success probabilities are proportional to the ratio of demand and supply.

For any fixed stationary policy ψ, and fixed success probabilities p_S and p_B, the trade on the market implies a dynamics on the set of money holdings over time. That is, each agent's money holdings evolves according to her policy and the rationing mechanism in each period in time. Assigning each agent to the state of her money holdings, we have defined a discrete Markov chain on $\mathbb{N}$, the set of money holdings.

Summarizing, a *stationary equilibrium* is a stationary policy ψ, a pair of stationary success probabilities p_S, p_B, and a stationary distribution of money μ such that: (i) the policy is optimal given the success probabilities, (ii) the distribution of money holdings μ is the limit distribution of the Markov chain given by the policy ψ and by the success probabilities p_S, p_B, (iii) the success probabilities are consistent with the ratios of demand and supply given ψ and μ and (iv) average money holdings are equal to average money supply.

We are interested in the existence of stationary equilibria and in the comparative statics of these equilibria with respect to the money supply. In the next section we first analyze the individual behavior, thereafter we give a formal definition of stationary equilibria that allows us to analyze their properties.

B   Analysis of the individual behavior

In this section we analyze the optimal individual decisions of agents which are described by the Bellman equation (5). Here we lay the foundation for the subsequent study of the equilibria of the monetary economy in Section C. The main result in this section is an explicit description of the agent's optimal policy. It turns out that an agent buys if he observes the high value and he is not cash constraint. This is because the high time-value is the best the agent can hope for. Hence immediately increasing the return function by this value is always better than acquiring one more unit of money or staying idle. The interpretation is straightforward. Moreover, agents only accumulate money up to a certain positive amount. For this amount of money, the opportunity costs of supplying the good are higher than the utility derived from consumption even at the low value. Hence, above this level of money holdings the agent always buys and below this level the agent sells if she has the low time-value. The results are achieved by a study of the properties of the value function (5).

Throughout the following we impose a restriction of the parameters by making the

Assumption B.1 $h > l > 0$, $p_h > 0$, $p_l > 0$, $p_B > 0$, and $p_S > 0$.

This assumption is imposed in all further results, unless otherwise noted.

Before deriving the qualitative properties of the value function, we show existence and uniqueness of the solution to the Bellman equation (5) together with some elementary properties.

Lemma B.1 *There exists a unique solution $V^\star$ to the Bellman equation (5). $V^\star$ is positive, increasing and bounded by $h/(1-\beta)$.*

Proof. Denote by X the set of bounded functions $V : \mathbb{N} \to \mathbb{R}$. Define the (non-empty) subset of X

$$Y = \{V : \mathbb{N} \to \mathbb{R} \mid 0 \le V(m) \le h/(1-\beta) \text{ for all } m \in \mathbb{N}\}$$

Y is complete with respect to the sup norm. Next, define an operator $T : X \to X$ via the Bellman equation (5) as follows. For each function $V \in X$, define the image TV by,

$$TV(0) \quad := \quad \beta \max \left\{ p_S \, V(1) + (1 - p_S) \, V(0), V(0) \right\}$$

$$TV(m) \quad := \quad \beta \sum_{w = h, l} p_w \, \max \Big\{ p_B \, [w/\beta + V(m-1)] + (1 - p_B) \, V(m),$$
$$p_S \, V(m+1) + (1 - p_S) \, V(m), V(m) \Big\}$$

Obviously, the solution to the Bellman equation is a fixed point of T.

We show that (i) T is a contraction on X; and (ii) T maps Y into itself. This implies that there exists a fixed point of T in Y which is unique in X. Any fixed point of T is obviously a solution to (5) and vice versa.

Let us prove property (i). We employ Blackwell's sufficient conditions for a contraction, see e.g. Stokey, Lucas, and Prescott (1989, Theorem 3.3), which states: Suppose the operator T is (1) monotone, i.e. if $V, W \in X$ with $V(n) \leq W(n)$ for all $n \in \mathbb{N}$ then $TV(n) \leq TW(n)$ for all $n \in \mathbb{N}$, and (2) discounting, i.e. there exists some constant $\gamma \in (0, 1)$ such that $T(V+c)(n) \leq TV(n) + \gamma c$ for all $V \in X$ and $c \geq 0$ (where the sum of $V \in X$ and a constant function is defined by $(V + c)(n) := V(n) + c$). Then T is a contraction on X. It is straightforward to check both conditions for the operator T defined above.

We check property (ii) next. First note that $V(m) \leq \sum_{t=0}^{\infty} \beta^t \, h = h/(1-\beta)$ for any policy. Since T is monotone and maps the set of positive functions into itself (because $l \geq 0$), it suffices to show that the image of the constant function $W(n) \equiv h/(1-\beta)$ satisfies $TW(n) \leq h/(1-\beta)$ for all $n \geq 0$. The definition of T yields that $TW(n) \leq p_B h + \beta h/(1 - \beta) \leq h/(1-\beta)$, for all $n \geq 0$. This yields property (ii).

Finally, note that the set of feasible plans is increasing in the initial endowment of money m_0 and, therefore, $V^\star(m+1) \geq V^\star(m)$ for all $m \geq 0$. This finishes the proof. $\qquad \square$

Remark B.1 *Lemma B.1 implies that playing I (staying idle) is always weakly dominated by S. We can thus exclude this choice in all further considerations.*

A first result on the optimal policy can be derived by studying the objective function of the agent. We show that it is optimal to choose action B, whenever $m \geq 1$ and the realization of the preferences is $w = h$.

Lemma B.2 *For all $m \geq 1$, $\psi^\star(m, h) = B$.*

Proof. First note that h is the maximal value the return function U can take in the agent's objective function (3). Since the success-probabilities p_B and p_S are independent of time, any action that postpones the action B when $m \geq 1$ and $w = h$, decreases total utility by at least the amount $p_B(1 - \beta)h$. Therefore a necessary condition for any optimal policy is playing B when $m \geq 1$ and $w = h$. $\qquad \square$

This result has the straightforward interpretation already mentioned above. Since utility is derived only from consumption and e.g. not from the possession of money, the highest possible value of the stochastic preferences has to "cashed." Lemma B.2 further enables us to gather more information on the properties of the value function. The main auxiliary result follows.

Lemma B.3 *$V^\star$ is strictly positive, strictly increasing, and strictly concave.*

Proof. Let $V^\star$ be the unique solution to (5) which exists and is unique by Lemma B.1.

We first show that $V^\star$ is strictly positive. For any $m \geq 1$, $V^\star(m) \geq p_B \mathbb{E}w + \beta V^\star(m-1) \geq p_B \mathbb{E}w > 0$. In particular this implies $V^\star(1) > 0$, which yields $V^\star(0) \geq \beta p_S V^\star(1) > 0$. Therefore, $V^\star$ is strictly positive.

Let us next show that $V^\star$ is strictly increasing. The set of feasible plans is increasing in the initial endowment of money m_0 and, therefore, $V^\star(m+1) \geq V^\star(m)$ for all $m \geq 0$. Moreover, the actual payoff for any elementary event, i.e. along any sample path of the exogenous stochastic processes, is increasing in the initial endowment of money, because the sequence of decisions made by an agent with endowment m can also be carried out if the endowment is

$m + 1$. To prove the assertion of the Lemma, it therefore suffices to show that for an event with strictly positive probability the total utility is strictly increasing in the initial endowment of money.

Suppose a run of m-times h occurs (an event that has strictly positive probability for each fixed $m \geq 1$). According to Lemma B.2 an agent with initial endowment $m + 1$ holds 1 unit of money, whereas an agent with endowment m is left with no money. The advantage of the agent with higher initial endowment is $V^\star(1) - V^\star(0)$ at the end of the run. As noted above, $V^\star(0) > 0$. This implies $V^\star(1) - V^\star(0) = \frac{1-\beta}{1-\beta+\beta p_S} V^\star(0) > 0$.

Finally, we show that $V^\star$ is strictly concave. Denote by $\psi^\star$ the corresponding optimal policy. Suppose $V^\star$ is not strictly concave, i.e. there exists an $\tilde{m} > 0$ such that $V^\star(\tilde{m} + 1) - V^\star(\tilde{m}) \geq V^\star(\tilde{m}) - V^\star(\tilde{m} - 1)$. Lemma B.2 ensures $\psi^\star(\tilde{m} + 1, h) = B$ and therefore, by the Bellman equation (5),

$$V^\star(\tilde{m} + 1) = \frac{\beta}{1 - \beta} \left[p_h \, p_B \left(\frac{h}{\beta} + V^\star(\tilde{m}) - V^\star(\tilde{m} + 1) \right) + ... \right]$$

where the term on the far right of this equation is given by $p_l \, p_B (l/\beta + V^\star(\tilde{m}) - V^\star(\tilde{m} + 1))$, if $\psi^\star(\tilde{m} + 1, l) = B$, and by $p_l \, p_S (V^\star(\tilde{m} + 2) - V^\star(\tilde{m} + 1))$, if $\psi^\star(\tilde{m} + 1, l) = S$.

By our assumption that $V^\star$ is not concave at $\tilde{m}$, $V^\star(\tilde{m}) - V^\star(\tilde{m}+1) \leq V^\star(\tilde{m}-1) - V^\star(\tilde{m})$. If $w = h$ is revealed and the agent would pursue from thereon the optimal policy corresponding to one unit of money less than she actually possesses, she would not be worse off because $V^\star(\tilde{m} + 1)$ is not decreased. In other words, the agent would not be worse off if she mentally disposes one unit of money in that case. However, this contradicts optimality of the policy, because disposing one unit of money leads to a strictly lower total utility because $V^\star$ is strictly increasing in the money holdings as has been proved above. □

The qualitative properties of the value function listed in Lemma B.3 enables us to give a complete description of the optimal policy. In particular, since the marginal utility derived from possessing one additional unit is diminishing and vanishes in the limit (because the value function is bounded), there is an endogenously determined upper bound of money that an agent is willing to hold, i.e. for all large enough money holdings m, the optimal policy is $\psi^\star(m, w) = B$ for $w = h, l$. This upper bound is at least one because if the agent does not accumulate any money, his total expected utility is zero when starting with no endowment of money. This, however, contradicts Lemma B.3 which ensures $V^\star(0) > 0$.

Theorem B.1 *There exists a maximal amount of money, $\overline{m}$, an agent wants to accumulate. It is given by*

$$\overline{m} := \min \left\{ m \geq 1 \mid \frac{p_B \, l}{\beta} > p_S \left[V^\star(m+1) - V^\star(m) \right] + p_B \left[V^\star(m) - V^\star(m-1) \right] \right\}$$

Further, the optimal policy is given by

- *$\psi^\star(0, w) = S$ for $w = h, l$;*
- *$\psi^\star(m, l) = S$, and $\psi^\star(m, h) = B$, for all $0 < m < \overline{m}$; and*
- *$\psi^\star(m, w) = B$, for $w = h, l$ and all $m \geq \overline{m}$.*

Proof. By Lemma B.2 we only have to show that $\psi^\star(m, l) = S$ for all $0 < m < \overline{m}$ and $\psi^\star(m, l) = B$ for all $\overline{m}$. According to the Bellman equation (5), these properties hold if and only if $p_B (l/\beta + V^\star(m-1)) + (1 - p_B) V^\star(m) \leq p_S V^\star(m+1) + (1 - p_S) V^\star(m)$ for all $0 < m < \overline{m}$, and $p_B (l/\beta + V^\star(m-1)) + (1 - p_B) V^\star(m) > p_S V^\star(m+1) + (1 - p_S) V^\star(m)$ for all $\overline{m}$. The conditions are equivalent to $p_B \, l/\beta \leq$ resp. $> p_B (V^\star(m) - V^\star(m-1)) + p_S (V^\star(m+1) - V^\star(m))$.

$V^\star(m)$ is increasing and bounded from above uniformly in m by Lemma B.1. Therefore, $\overline{V} := \lim_{m \to \infty} V^\star(m)$ exists and is bounded by $h/(1 - \beta)$. This implies that $\overline{m}$ exists and is finite ($p_B \, l/\beta > 0$).

Since $V^\star$ is concave, see Lemma B.3, $V^\star(m) - V^\star(m - 1)$ is decreasing in m. Therefore, the result is immediate from the definition of $\overline{m}$. □

Remark B.2 *If the condition in the defining equation of $\overline{m}$ holds with equality for $m = \overline{m} - 1$, the agent is indifferent between selling and buying at $\overline{m} - 1$. In this case it is also optimal to hold up to $\overline{m} - 1$ units of money, or to randomize between S and B when possessing $\overline{m} - 1$ units.*

Theorem B.1 completely characterizes the optimal policy of all agents. We have achieved this goal by proving certain properties of the value function, and thus did not need to solve the Bellman equation. However, the maximum money holdings $\overline{m}$ is not known explicitly but can be determined numerically for given parameter values. We will need this quantity when comparing the predictions of the theory with the experimental results. The numerical procedure employs the convergence procedure applied in the proof of Lemma B.1 (which is based on Blackwell's sufficient conditions). To make this approach tractable, we need an upper bound on $\overline{m}$; this allows to apply the procedure of Lemma B.1 on a finite set of money holdings. We proof the following auxiliary result.

Lemma B.4 *Suppose $p_S > 0$ and $l \geq 0$. Then, for all $m \geq 0$,*

$$V^\star(m+1) - V^\star(m) \leq \beta^m \frac{h}{1 - \beta + \beta p_S}$$

Proof. Since each optimal policy for the problem with initial endowment of $m + 1$ units of money can be pursued at least over a time-horizon of length m with a initial endowment of m units of money, one has that $V^\star(m+1) - V^\star(m) \leq \beta^m(V^\star(1) - V^\star(0))$. Using that $V^\star(1) - V(0)^\star = \frac{1-\beta}{1-\beta+\beta p_S}V(0)^\star$ (see proof of Lemma B.3) and $V(0)^\star \leq h/(1-\beta)$, we obtain the assertion of the Lemma. $\square$

Using that $V^\star$ is strictly concave (Lemma B.3), the estimate in Lemma B.4 yields an upper bound on $\overline{m}$ by inserting this estimate in the right-hand side of the defining equation of $\overline{m}$ (Theorem B.1). One finds that $\overline{m}$ is bounded by the smallest $m \in \mathbb{N}$ such that $p_B \, l \geq \beta^m(p_S \, \beta + p_B) \, h/(1 - \beta(1 - p_S))$.

C Analysis of stationary equilibria

The optimal individual decision of each agent—studied in the previous section—is based on perceived market conditions such as the probability of actually trading with a seller when being a buyer. The policy of every agent is confronted with the actual market conditions when all agents' buying and selling positions meet in the market in a period in time. The policies might turn out not to be compatible. We therefore introduce the notion of a stationary equilibrium of our monetary economy. In this equilibrium the perceived and the actual market conditions coincide, the quantity of money is constant, and the distribution of money holdings is stationary.

We consider an economy in which the individual behavior be the one defined in Theorem B.1. The individual decisions give rise – via the market – to a dynamics on the set of money holdings. This dynamics is appropriately described by a Markov chain on the space of money holdings $\mathbb{N}$. Note that the subset $\{0, ..., \overline{m}\}$ is invariant under the Markov chain. If the perceived success probabilities p_B and p_S are strictly positive, the induced Markov chain on $\{0, ..., \overline{m}\}$ is irreducible. This implies the existence of a unique stationary probability measure, say μ. The behavior of agents and the measure μ implies actual success probabilities, which may differ from the perceived ones. In particular, the quantity of money, which is defined by $M := \sum_{m \in \mathbb{N}} m \, \mu(m)$ (viz. the average-per-capita amount of money), may not be compatible with the perceived success probabilities in the sense. We next give a definition of an equilibrium in which the actual and the perceived probabilities coincide, and in which the quantity of money is stationary over time. On calling a tupel (β, l, h, p_l, p_h) the characteristics of the agent we summarize the description of the model by the definition of a stationary equilibrium.

Definition C.1 *A* stationary equilibrium *is a tuple $(\psi^\star, \mu^\star, p_S{}^\star, p_B{}^\star, M^\star)$, consisting of a stationary policy, a distribution over money holdings, success probabilities, and a quantity of money, such that*

(i) *given $p_S{}^\star$ and $p_B{}^\star$, $\psi^\star$ is an optimal policy for (5);*

(ii) *given $\psi^\star$, $p_S{}^\star$ and $p_B{}^\star$, $\mu^\star$ is a stationary probability measure for the corresponding Markov chain on $\mathbb{N}$;*

(iii) *given $\psi^\star$ and $\mu^\star$, the probabilities $p_S{}^\star$ and $p_B{}^\star$ satisfy (6);*

(iv) the average money holdings are equal to the average money supply, i.e. $\sum_{m \in \mathbb{N}} m\, \mu^{\star}(m) = M^{\star}$;

for given characteristics of the representative agent.

A stationary equilibrium is called monetary, if the value function solving (5) is strictly positive on $\mathbb{N}$.

We do not attempt here to discuss how such an equilibrium is achieved by the agents if the economy starts with a distribution of money that does not coincide with the stationary distribution but which induces the equilibrium quantity of money. This is possible (though analytical tractability is certainly challenging) in the framework of random dynamical systems, see e.g. Schenk-Hoppé (2001b). In a stationary equilibrium no agent has an incentive to follow a policy that is different to the one stated in Theorem B.1.

We first show that stationary equilibria exist.

Lemma C.1 *There exists a non-monetary stationary equilibrium for any characteristics of the representative agent and any amount of money; in this equilibrium $p_B^{\star} = 0$.*

Proof. Let $p_B = 0$ and $p_S = 1$. Then it is optimal to play buy if $m \geq 1$ and to stay idle if $m = 0$ for each realization of the preferences. If everyone wants to buy, then an agent is not better off when he sells, because he cannot buy afterwards. He is thus indifferent between selling, buying, or staying idle. The value function is identically zero. □

It is straightforward to see that non-monetary stationary equilibria are not unique in general. Let m be such that no agent wants to accumulate money up to $m - 1$. Then $\psi^{\star}(i, w) \equiv B$, for $i = m - 1, m, m + 1$, $p_S^{\star} = 1$, and $p_B^{\star} = 0$ is a (non-monetary) stationary equilibrium for $\mu^{\star}(m) = 1$ as well as for $\mu^{\star}(m - 1) = \mu^{\star}(m + 1) = 1/2$. Due to the existence of a non-monetary stationary equilibrium and the dependence of the agent's maximal money holdings on the success probabilities, one cannot expect that monetary stationary equilibria always exist. Suppose agents are very impatient (i.e. β close to zero) then there cannot exist a monetary equilibrium with a quantity of money larger than one. An appropriate choice of the quantity of money is therefore indispensable. However, Lemma B.3 ensures that any stationary equilibrium which satisfies Assumption B.1 is monetary. We have the following result on the existence of monetary stationary equilibria.

Theorem C.1 *For any characteristics of the representative agent and any prescribed success probabilities $p_B > 0$ and $p_S > 0$ (such that one of them is equal to one), there exists some quantity of money and a corresponding monetary stationary equilibrium such that $p_B^{\star} = p_B$, $p_S^{\star} = p_S$, and in which the optimal policy $\psi^{\star}$ is given by Theorem B.1.*

Proof. Denote by $V^{\star}$ the solution to the Bellman equation for the given characteristics of the representative agent and for the given success probabilities $p_B > 0$ and $p_S > 0$. Define the maximal amount of money holdings $\overline{m}$ according to Theorem B.1.

The policy $\psi^{\star}$, according to Theorem B.1, together with the probabilities p_h, p_l, p_B, and p_S defines a Markov chain on the set of money holdings $H := \{0, ..., \overline{m}\}$. Since all probabilities are strictly positive, the Markov chain is irreducible and, thus, possesses a unique stationary probability measure μ on H. In particular, the support of μ is H. Define the corresponding quantity of money $M^{\star} := \sum_{m=0}^{\overline{m}} m\, \mu(m)$.

It remains to check that $(\psi^{\star}, \mu, p_B, p_S, M^{\star})$ is a stationary equilibrium. First, according to Theorem B.1, the policy $\psi^{\star}$ is optimal, because both success probabilities are strictly positive. Second, μ is a stationary measure by construction, and condition (iv) of Definition C.1 is satisfied by the definition of $M^{\star}$.

Finally, we need to show that the success probabilities defined in (6) are equal to the prescribed ones p_B and p_S. Let us consider the case $p_S = 1$ (the case $p_B = 1$ is analogous). By equation (6), $p_B = p_B^{\star}$ for the policy $\psi^{\star}$, if and only if

$$p_B \left(\sum_{m=1}^{\overline{m}-1} \mu(m)\, p_l + \mu(\overline{m}) \right) = \sum_{m=1}^{\overline{m}-1} \mu(m)\, p_h + \mu(0)$$

This equation requires that in each period in time the number of agents whose money holdings decrease by one (due to actually buying the good) is equal to the number of agents whose money holdings increase by one (due to actually selling the good). This condition is satisfied by stationarity of μ. This equilibrium is monetary by Lemma B.3. □

Theorem C.1 is a general existence result for monetary equilibria when the quantity of money can be chosen by an institution outside the model of our monetary economy. This raises the question on the optimum quantity of money. Obviously, any non-monetary equilibrium is Pareto-dominated by every monetary equilibrium. However, this does not provide a satisfactory answer in the case of a benevolent social planner who maximizes the welfare in the economy. The most simple measure is that of the number of trades. If there is rationing, some agents cannot realize their plans and thus obtain lower utility than in an equilibrium without rationing. While this criterion only measures individual but not social welfare it provides a simple benchmark for the social planner's policy. We have the following result.

Corollary C.1 *For any characteristics of the representative agent there exists some quantity of money $M^\star$ and a corresponding monetary stationary equilibrium such that $p_B^\star = p_S^\star = 1$.*

Every quantity of money which is different to $M^\star$ leads to a monetary stationary equilibrium in which the number of trades is strictly less than with $M^\star$.

Proof. The number of trades is obviously maximal, if $p_B^\star = p_S^\star = 1$. According to Theorem C.1 such an equilibrium exists.

The number of trades is maximal if no rationing occurs. We have to compare the utility an agent obtains in the equilibrium without rationing ($p_B^\star = p_S^\star = 1$) and the quantity of money $M^\star$ with the utility in any other equilibrium (with the quantity of money M_1). In the second equilibrium rationing has to occur, because if ($p_B^\star = p_S^\star = 1$ the policies of the agents would be the same, leading to the same Markov chain and the same optimal quantity of money. If rationing occurs in an equilibrium the utility that a player obtains in this equilibrium is lower than in a non equilibrium policy combination without rationing in which the same policy is used, but the quantity of money M_2 is different. For the same policy a policy combination always gives at least the same utility to players in a market if nor rationing occurs. This non equilibrium policy combination is dominated by the equilibrium policy combination with ($p_B^\star = p_S^\star = 1$ and the quantity of money $M^\star$, because the equilibrium policy is the solution to the Bellman equations without rationing.

It is legitimate to call the quantity of money that is determined by Corollary C.1 the *optimum quantity of money.*

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