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Abstract Base-rate neglect is a robust experimental finding that individuals do not update their prior beliefs according to the Bayes' rule and, typically, underestimate their posterior probabilities. Another empirical finding is that individuals often do not acquire information even when there are no strategic considerations and the cost of new information is justifiable economically. This paper combines these two different fields of research. Specifically, it is demonstrated that base-rate neglect may lead to imperfect information acquisition. An application to the pricing of new financial assets as well as general implications for the socially optimal pricing of information are discussed.

JEL codes: C91, D83
Key words: Bayes' rule, base-rate neglect, decision making, information acquisition

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1 Introduction

Bayesian inference is a normative model for updating prior beliefs upon arrival of a new information (e.g. Baron, 2004). However, there is an extensive experimental evidence that individuals do not use the Bayes' rule, which became known as base-rate neglect or base-rate fallacy (e.g. Kahneman and Tversky, 1972, 1973; Bar-Hillel, 1980). Gigerenzer and Hoffrage (1995) find that the most frequently used non-Bayesian algorithm is a computation of the joint occurrence. Compared to the Bayesian inference, joint occurrence leads to the underestimation of the posterior probabilities.

This paper investigates the decisions of the non-Bayesian individuals when they acquire information. The imperfect information acquisition is often rationalized in the strategic setup (e.g. Hurkins and Vulkan, 2004) when an individual who acquires information reveals this information by his or her actions to the opponents and gets exploited by the latter. When information has no strategic value, the fact that individuals do not acquire full information is typically explained by information costs (e.g. Grossman and Stiglitz, 1980). However, Rötheli (2001) finds experimental evidence that individuals do not acquire information when the economic benefit from the new information exceeds its cost.

This paper demonstrates that, setting strategic and cost considerations aside, imperfect information acquisition can be explained by non-Bayesian updating, i.e. base-rate neglect. For example, consider a situation when the acquired information is used for updating the subjective beliefs about a bad state of the world (e.g. Rötheli, 2001). Individuals employing the non-Bayesian inference such as a joint occurrence underestimate the correct probability of the bad state given currently available information. Therefore, such individuals need to acquire less information than Bayesians to become sufficiently convinced that the bad state is not likely to occur. A simple model presented in this paper demonstrates that individuals employing joint occurrence as a proxy for the Bayes' rule may not acquire full information even if information is costless but arrives sequentially.

The remainder of this paper is organized as follows. Section 2 presents a formal model. Section 3 discusses applications of the results from Section 2. Section 4 concludes.

2 The Basic Model

Consider a project that delivers payoff $\underline{u} < 0$ when the state of the world is L and payoff $\overline{u} > 0$ otherwise. An individual decides whether to participate in the project or to abstain, in which case he or she receives a zero payoff for certain. Let $\eta(0) \in (0, 1)$ denote the prior subjective probability that the state of the world is L. Thus, ex ante, the expected payoff of the project is

$$\pi(0) := \eta(0) \cdot \underline{u} + (1 - \eta(0)) \cdot \overline{u}. \tag{1}$$

Before making the decision, the individual can sequentially acquire units of information at no cost. Only one unit of information is informative. If the state of the world is L, the acquired information is informative with probability $q \in (0, 1)$. If the state of the world is not L, the acquired information is always uninformative. Therefore, after receiving the informative unit of information, the individual knows with certainty that the state of the world is L.

Let $\eta(n)$ denote the subjective probability that the state of the world is L after n units of information are acquired, all being uninformative. In this case the expected payoff of the project is

$$\pi(n) := \eta(n) \cdot \underline{u} + (1 - \eta(n)) \cdot \overline{u}. \tag{2}$$

After n uninformative units of information, the next unit of information is informative with probability $q \cdot \eta(n)$. In this case, the individual gets a zero payoff. After n uninformative units of information, the next unit of information is uninformative with probability $1 - q \cdot \eta(n)$. In this case, the individual gets the expected payoff $\pi(n+1)$. The $n+1$-st unit of information is acquired if and only if

$$\pi(n) \leq (1 - q \cdot \eta(n)) \cdot \pi(n+1). \tag{3}$$

The last inequality can be rewritten as Condition 1 (see Appendix for mathematical derivation).

Condition 1 *After n uninformative units of information, further information is acquired if and only if*

$$\eta(n+1) + \frac{1}{q}(1 - \frac{\eta(n+1)}{\eta(n)}) \geq \frac{1}{1 - \underline{u}\,/\,\overline{u}}.$$

2.1 Bayesian Inference

Consider an individual updating his or her subjective probability $\eta(.)$ according to the Bayes' Rule (4).

$$\eta(n+1) = \frac{(1-q)^{n+1} \cdot \eta(n)}{(1-q)^{n+1} \cdot \eta(n) + 1 \cdot (1 - \eta(n))} \tag{4}$$

When equation (4) is plugged into Condition 1, simple algebra yields that Condition 1 is always satisfied (mathematical derivation is presented in the Appendix). Therefore, a Bayesian would always acquire all available information. Interestingly, the same result holds if the individual does not know q and uses adaptive learning to update sequentially the subjective beliefs about the state of the world and the information distribution (according to the Bayes' rule).

2.2 Non-Bayesian inference

Gigerenzer and Hoffrage (1995, p.694) find that the most frequently used non-Bayesian algorithm is *joint occurrence*. In our model, following this algorithm, an individual updates his or her subjective believes $\eta(.)$ according to equation (5).

$$\eta(n+1) = q \cdot \eta(n) \tag{5}$$

With every uninformative unit of information, the subjective probability $\eta(.)$ decreases by a constant factor, which equals to the probability of receiving an informative unit of information in the state L. Plugging equation (5) into Condition 1, we obtain inequality (6) as the condition for information acquisition after n uninformative signals.

$$\eta(n+1) + \frac{1-q}{q} \geq \frac{1}{1 - \underline{u} \,/\, \overline{u}}. \tag{6}$$

From equation (5) it follows that $\eta(n+1)$ converges to zero as n goes to infinity. Therefore, for large n, equation (6) can be rewritten as

$$q \leq \frac{\overline{u} - \underline{u}}{2\overline{u} - \underline{u}}. \tag{7}$$

Notice that $\frac{\overline{u}-\underline{u}}{2\overline{u}-\underline{u}} \in [0.5, 1]$, $\frac{\overline{u}-\underline{u}}{2\overline{u}-\underline{u}} = 0.5$ if $\underline{u} = 0$ and $\frac{\overline{u}-\underline{u}}{2\overline{u}-\underline{u}} = 1$ if $\overline{u} = 0$. Therefore, if $q < 0.5$, i.e. informative information comes with low probability, a non-Bayesian, similar to the Bayesian, will always acquire all available information. For $q \in [0.5, 1)$, a non-Bayesian may acquire incomplete information if the potential loss $\underline{u}$ from the project is small and informative information comes with high probability. Incomplete information acquisition does not occur if the potential gain $\overline{u}$ from the project is small and/or informative information comes with low probability. Intuitively, if the individual knows that participation in the project may be costly, i.e. $|\underline{u}\,|$ is large relative to $\overline{u}$, and if he or she knows that acquired information is unlikely to be informative, i.e. q is small, then he or she will seek full information.

Let us now determine the maximum number of informational units sought if information acquisition remains incomplete. Using recursive equation (5), the condition for further information acquisition (6) can be rewritten as

$$(q)^{n+1} \cdot \eta(0) + \frac{1-q}{q} \geq \frac{1}{1 - \underline{u}\,/\,\overline{u}}. \tag{8}$$

In case incomplete information acquisition is possible, let n^* be the first natural number for which inequality (8) does not hold. A non-Bayesian, then, chooses to acquire at most n^* units of information. The right hand side of inequality (8) is decreasing in $-\frac{\underline{u}}{\overline{u}}$. Therefore, n^* must be increasing in $-\frac{\underline{u}}{\overline{u}}$. In a population of heterogeneous individuals, more risk averse individuals fear the potential loss $|\underline{u}|$ relatively more than they value $\overline{u}$.[1] Thus, in a population of individuals with different risk attitudes, more risk averse individuals acquire more information.

It is interesting to note that, if information is idiosyncratic, even identical (non-Bayesian) individuals facing exactly the same decision problem may take different actions. At first sight, this appears to be intuitive as the individuals are facing a random process. In the present setting, however, all uncertainty can be resolved at no cost. This fact notwithstanding, if the state of the world is L, the individuals who receive by chance n^* uninformative units of information will stop the information acquisition and participate in the project. Other individuals with the same preferences who receive the informative unit of information will not participate in the project. Moreover,

[1]Increased loss aversion (e.g. Kahneman and Tversky, 1979, and Tversky and Kahneman, 1991), will have a similar effect.

the smaller n^* the larger the fraction of individuals choosing the project in the state L. Thus, the randomness of the process becomes prevalent although it could be resolved at no cost.

3 Applications

To highlight the consequences of our findings, we will discuss two applications - one economic and one political - in this section. First we will consider financial markets. Specifically, we will discuss the case of IPOs and show how our model may help to understand different patterns in the price movements for an asset when first traded at the stock market. Second, we will consider personal voting behaviour. We will argue that too many good news for a party before the election may be detrimental to their success. We will conclude this section with a reference to further potential applications. These will also emphasise the impact of the present approach on the socially optimal pricing of information.

3.1 IPOs

When a firm goes public, i.e. decides to sell firm shares (assets) at the stock market, we often observe that the new asset is oversubscribed and that first trading prices are far above the price at the end of the bookbuilding. Still, occasionally, only a short time after the emission, we observe that the asset price drops - sometimes even below the emission price.[2] In the following, we argue how our model may shed some light on this phenomenon.

Consider an asset A that has entered the bookbuilding phase. I.e. the assets are known to be sold at a price $p^* \in [p^-, p^+]$. For simplicity reasons,

[2] A perfect example for this is the IPO of Plasmaselect, DE0005471809.

we assume that $p^+ = p^-$ so that p^* is actually known to the investors at the beginning of the bookbuilding. The total number of assets is denoted by y. Bookbuilding closes at time T.

Until T, investors can acquire information about the fundamentals of the firm through different media. We will assume that this information is provided for free and that at T all information about the asset publicly known. With all information at hand, the asset is valued p_f by the market. The market value p_f can be either larger or smaller than p^*. To adapt this to our model, we can think of information as being either confirming or disconfirming for the hypothesis that $p_f > p^*$. Confirming and disconfirming information then corresponds to uninformative and informative information respectively. Information is released at random.

We assume that investors follow the media and order assets if their subjective belief that $p_f > p^*$ is weak. The number of assets ordered by the investors until time T is denoted by x. If more assets are ordered than sold, i.e. if $x > y$, we assume that there is an access demand at the first trading, t_1, and that the price follows the law of demand. We denote the price of the asset at t_1 by p_1. Thus, if $x > y$ then $p_1 > p^*$. The price at time t_2, some time after the emission, will be according to the fundamentals, i.e. the market will correctly incorporate all available information with time. The complete time scale is depicted in Figure 1.

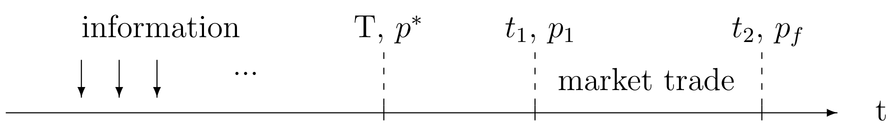

Figure 1: The sequence of events.

Consider the case where investors are non-Bayesians and where disconfirming information comes with low probability. According to our results from the previous section, there is a possibility of many investors believing that asset A is a desirable investment. Thus, with positive probability, $x > y$ even if the state of the world is L, in which case we will observe $p_f < p^* < p_1$.

Interestingly, if we consider the case of both Bayesians and non-Bayesians being present at the market, there may even be an incentive for the Bayesians to order the asset although the state of the world is L. Recall that Bayesians will always acquire full information before deciding on the investment. Yet, even if an investor is fully informed about the state of the world he may have observed that there was a long sequence of positive signals at the beginning. If he infers that most likely many of the non-Bayesians will order the asset, he may do so as well. The difference between the Bayesian and the non-Bayesians only is that the latter, if their demand was not satisfied due to oversubscription, will cause an access demand at the first trading at the stock market whereas the former will immediately sell the asset, knowing that the price is bound to fall.

3.2 Voting Behaviour

Another phenomenon our model may help to explain is related to elections. Often different measures of public sentiment some days prior to an election indicate that one party is almost certain to win. Yet, in some cases during the last days things become more contentious and finally, the eve of the election, the respective party may even be runners-up only - often accompanied by low election participation. The institutes responsible for opinion research then often go to some length explaining how and why this could have happened. In the following, we will try to highlight how our model may facilitate the arguing of these institutes.

For the sake of argument, we consider two parties, C and D. They are competing for the composition of a political body (or for the right to decide on a political position). We assume that all individuals would rather spend their day not voting if they knew, that their favorite party was going to win anyway. However, if they knew that things are less clear or that the outlook is bad, they would rather support their favorite party through their vote.

Now, each day the news will be full of information about the relative standing of the two parties. Although this information is assumed to be correct, it is random in that each day both of the two parties will engage in campaigning which may have both positive as well as negative effects on public sentiment. According to our model non-Bayesian supporters of, say, party C will follow the news and decide to go on vacation rather than to go to the elections if they are sufficiently confident that this will not be detrimental to their party. As soon as they have made their decision they will no longer follow the news.[3] Thus, if due to some unforseen event party D manages to (almost) catch up with party C on the last stretch, the missing voters may prove to be decisive - but in favour of party D.

3.3 Other Applications

Two other areas for which our results appear to be relevant are marketing and health economics.

For example, advertising a search good (see Nelson 1970 and 1974) in a way that conveys only little information will also be better than being more specific, especially if the product has close substitutes. If a consumer believes to have some positive information about one product but none about the others, this may suffice to decisively influence his buying decision. Assuming

[3]Another possibility is to say that the vacation entails a commitment.

that the target group for the product is very diverse, it may indeed be optimal to release only information that has almost no practical value, provided it is likely to be construed as positive by everyone.

Our model may also be relevant for decisions on information acquisition about ones own health status. In particular, presume that individuals disregard the impact of the frequency of their own doctoral visits on insurance premiums. In this context, our findings indicate that non-Bayesians are less likely to consult several doctors to reassure themselves about a certain diagnosis.[4] Thus, when contemplating, for example, the introduction of an additional cost per consultation which is privately payed by the patient, one should consider the impact on individual behaviour. Non-Bayesians, who are anyway prone to acquire incomplete information, will be affected more strongly than Bayesians. A cost that is socially optimal considering only Bayesian individuals will tend to be too prohibitive for non-Bayesians. Thus, smaller extra payments will be socially optimal if the relative proportion of non-Bayesians in the population is large.

4 Conclusion

This paper provides yet another explanation for imperfect information acquisition. When information has no strategic value and it is costless but acquired sequentially, an individual may not acquire full information if he or she does not update the subjective beliefs according to the Bayes' rule. Specifically, incomplete information acquisition occurs when the informative unit of information comes with high probability and the risky project bears a relatively small loss. Intuitively, a non-Bayesian underestimates the posterior probability of the bad state of the world to being true. Hence, such an individual can interpret a limited sequence of uninformative units of information

[4] Taking the opposite view, maybe hypochondriacs are simply overly eager Basyeians.

as a "convincing" evidence that the bad state of the world is false. In contrast, a Bayesian is never satisfied with a limited sequence of uninformative signals.

In many economic problems where incomplete information acquisition was traditionally explained by cost reasons, our results indicate that the role of the cost may be overestimated. Certainly the presence of costs negatively affects the amount of information gathered. However, if information acquisition is already incomplete when information is costless, it will be all the more so when the various costs of information are taken into account. Moreover, in the presence of costs, information acquisition may also be below the payoff maximising level. Therefore, our finding explains the experimental data reported in Rötheli (2001) where, in the presence of cost, less information is acquired than would be optimal considering expected payoffs only. Interestingly, subjects with an economic background seem to value information higher (and to process it more effectively) whereas non-economists show a high tendency to disregard the information or to construe it wrongly. One possible explanation might be that economists are more familiar with Bayes' rule. It would be interesting to learn more about the relative proportions of Bayesian and non-Bayesians in different social groups. In particular for the calculation of socially optimal costs of information, sound knowledge about the presence of non-Bayesians in the population appear to be crucial.

Appendix

Derivation of Condition 1 To derive Condition 1 note that equation (3) can be rewritten as

$$\eta(n)\underline{u} + (1 - q \cdot \eta(n))\overline{u} \leq (1 - q\eta(n)) \cdot [\eta(n+1)\underline{u} + (1 - \eta(n+1))\overline{u}].$$

Simple algebra yields

$$\underline{u}(\eta(n) - (1 - q\eta(n))\eta(n+1)) \leq \overline{u}((1 - \eta(n+1)) \cdot (1 - q\eta(n)) - 1 + \eta(n)))$$

$$\underline{u}(\eta(n) - (1 - q\eta(n))\eta(n+1)) \leq \overline{u}(-\eta(n+1) \cdot (1 - q\eta(n) - q\eta(n) + \eta(n)))$$

$$(\overline{u} - \underline{u}) \cdot (\eta(n) - (1 - q \cdot \eta(n)) \cdot \eta(n+1)) \geq \overline{u}q\eta(n)$$

$$\frac{(\eta(n) - (1 - q \cdot \eta(n)) \cdot \eta(n+1))}{q\eta(n)} \geq \frac{\overline{u}}{\overline{u} - \underline{u}}$$

$$\eta(n+1) + \frac{\eta(n) - \eta(n+1)}{q\eta(n)} \geq \frac{1}{1 - \underline{u} \,/\, \overline{u}}$$

$$\eta(n+1) + \frac{1}{q}(1 - \frac{\eta(n+1)}{\eta(n)}) \geq \frac{1}{1 - \underline{u} \,/\, \overline{u}}$$

This is exactly the condition 1. □

Proof that Bayesian inference leads to complete information acquisition The right hand side of Condition 1 is always smaller than 1, i.e.

$$\frac{1}{1 - \underline{u} \,/\, \overline{u}} < 1.$$

Let us show that the left hand side of Condition 1 is always larger than one when equation (4) holds, i.e. $\forall n \in \mathbb{N}$ it holds:

$$\eta(n+1) + \frac{1}{q}(1 - \frac{\eta(n+1)}{\eta(n)}) > 1.$$

This is equivalent to:

$$q\eta(n+1) + (\frac{\eta(n) - \eta(n+1)}{\eta(n)}) > q.$$

The next step is to express $\eta(n+1)$ in terms of $\eta(n)$. This gives:

$$q\eta(n)\frac{(1-q)^{n+1} \cdot \eta(n)}{(1-q)^{n+1}\eta(n) + 1 - \eta(n)} + \eta(n) - \frac{(1-q)^{n+1} \cdot \eta(n)}{(1-q)^{n+1}\eta(n) + 1 - \eta(n)} > q\eta(n).$$

Further equivalence transformations yield

$$(1-q)\eta(n) > (1 - q\eta(n))\frac{(1-q)^{n+1} \cdot \eta(n)}{(1-q)^{n+1} \cdot \eta(n) + 1 - \eta(n)}$$

$$1 > \frac{(1 - q\eta(n)) \cdot (1-q)^{n}}{(1-q)^{n+1} \cdot \eta(n) + 1 - \eta(n)}$$

which is satisfied for all $n \geq 1$. $\square$

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