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Author's addresses

Thomas Borek
E-mail: thomas.borek@math.ethz.ch

Stefan Buehler
E-mail: sbuehler@soi.unizh.ch

Armin Schmutzler
E-mail: arminsch@soi.unizh.ch

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Fax: +41-1-634 49 82
URL: www.soi.unizh.ch
E-mail: soilib@soi.unizh.ch

Mergers under Asymmetric Information—Is there a Lemons Problem?

Thomas Borek*
Stefan Buehler**
Armin Schmutzler***

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Abstract: We analyze a Bayesian merger game under two-sided asymmetric information about firm types. We show that the standard prediction of the lemons market model—if any, only low-type firms are traded—is likely to be misleading: Merger returns, i.e. the difference between pre- and post-merger profits, are not necessarily higher for low-type firms. This has two implications. First, under very general conditions, equilibria exist where mergers take place, and there is no presumption that there is inefficiently low trade. Second, in these equilibria it is typically not the case that only low-type firms enter an agreement.

Keywords: merger, asymmetric information, oligopoly, single crossing.

JEL: D43, D82, L13, L33.

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*Swiss Federal Institute of Technology Zurich, D-Math, Rämistr. 101, 8092 Zurich, Switzerland, thomas.borek@math.ethz.ch.

**University of Zurich and University of St. Gallen; Socioeconomic Institute, Hottingerstr. 10, 8032 Zurich, Switzerland, sbuehler@soi.unizh.ch

***University of Zurich and CEPR; Socioeconomic Institute, Hottingerstr. 10, 8032 Zurich, Switzerland, arminsch@soi.unizh.ch.

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1 Introduction

Since the nineteen sixties, there have been several extended periods of large scale merger activity. Nevertheless, some authors (e.g. Ravenscraft and Scherer 1987, 1989; Healy et al. 1992; Scherer 2002) have questioned the overall profitability of mergers. Also, numerous case studies suggest that at least one of the parties involved in a merger is likely to consider the deal a failure with the benefit of hindsight. For example, in the merger with the German Hypobank, the Hypovereinsbank discovered "the full horror of its partner's balance sheet" after two years (*The Economist*, July 29, 2000).

Thus, even though asymmetric information looms large in the market for firms, it seems hard to detect inefficiently low trade levels, let alone market breakdown, as suggested by the lemons market rationale emanating from Akerlof (1970). Is it therefore misleading to think of the merger market as a market for lemons?

In the following, we want to argue that the lemons market model is indeed not the ideal way to think of mergers, in spite of the considerable uncertainty surrounding them. Even though anecdotes on failed mergers and takeovers typically single out one of the partners as the lemon, the asymmetric information surrounding mergers is usually two-sided: Each of the firms knows more about its quality than the potential partner does. Both parties thus face the risk of joining a bad partner who adversely affects the profits of the merged entity. In the present paper, we therefore consider mergers under *two-sided asymmetric information* about firms' types.[1] High types are defined as having high stand-alone profits—that is, high profits in the absence of a merger—and as contributing to high merger profits if the transaction occurs.

[1] Hviid and Prendergast (1993) provide an analysis of merger games with one-sided asymmetric information. Assuming that the target firm has private information about its profitability, they show that an unsuccessful bid may increase the profitability of the target but reduce the profitability of the bidding firm (relative to the profitability before the merger offer) due to learning from rejection.

Even with two-sided asymmetric information, it may seem natural to expect that low types are more likely to become involved in a merger, as they expect low stand-alone profits and therefore have little to lose from giving up their identity. However, this is only one aspect of a firm's merger decision. To understand whether low types are really more willing to enter a merger agreement, it is necessary to consider the relation between a firm's type and its merger returns, that is, the difference between post-merger profits and stand-alone profits, where the latter can be interpreted as the firm's opportunity costs of the merger. As it turns out, it is not clear that low-type firms face higher merger returns than high-type firms.

To see this, consider a cash-financed transaction where the seller receives a fixed cash-payment if the merger takes place. Then, the seller's merger returns are the (type-independent) cash-payment minus the stand-alone profits. Thus, the seller's merger returns are decreasing in own type because stand-alone profits are increasing own type. For the buyer, however, the situation is less clear: His merger returns are the post-merger profits of the new firm minus stand-alone profits minus the cash payment. It is natural to assume that the post-merger profits are increasing in the buyer's type: A better buyer will, in general, increase the performance of the new firm. If this latter effect is strong enough, it will dominate over the stand-alone profit effect (which tends to drive good buyers out of merger deals).

Which of the two effects dominates depends critically on the technology of the merged firm. To illustrate this, we analyze the firms' merger returns in a linear Cournot framework.[2] First, consider the case where the buyer imposes its technology on the firm that has been taken over, so that the merged entity produces with the buyer's marginal costs (*buyer-dominated mergers*). In this case, the marginal effect of a higher type on stand-alone profits is always lower

[2] In spite of the well-known result that, in such a setting, two-firm mergers do not increase joint profits unless they lead to monopoly (Salant et al. 1983), Barros (1998) has shown that joint profits may rise due to rationalization effects when there is cost-heterogeneity between the firms. In principle, therefore, there is scope for efficiency-enhancing mergers.

than the marginal effect on post-merger profits, so that the buyer's merger returns are increasing in own type.[3] Second, consider the case where the new entity produces with the marginal costs of the more efficient firm, no matter whether this is the buyer or the seller (*rationalization mergers*). In this case, the buyer's merger returns are monotone increasing (decreasing) in own type if the buyer (the seller, respectively) is more efficient. Thus, for rationalization mergers, it is also not generally true that low-type firms have more to gain from a merger. Finally, consider the case where the seller has a strong impact on the new firm's technology (*seller-dominated mergers*), i.e. the new firm produces either with the seller's technology or with a biased average of the buyer's and seller's marginal costs. Only in this case will the buyer's merger returns be monotone decreasing in own type, so that standard adverse selection results translate directly to the merger setting.

With these findings in place, we proceed to the analysis of a more general reduced-form merger game in which two firms are matched whose types $z_i, i = 1, 2$, are drawn from distributions that are common knowledge.[4] After having observed their own type, both firms state whether they consent to a merger. If both firms consent, a merger takes place. If at least one firm declines, there is no merger. Following the merger game, an oligopoly game is played. If no merger occurs, both firms earn their stand-alone profits. If a merger occurs, the joint profit is shared according to some predetermined rule. In the simplest case, one firm buys the other one at a constant price p, as sketched in the Cournot example.[5] However, our results are also consistent with other ways of profit sharing.

Our main results are the following. First, if merger returns are monotone decreasing in own type for both firms, the standard results from the adverse

[3] The seller's merger returns are decreasing in own type, as in all other cases considered below.

[4] In specific applications, types may be interpreted as cost or demand parameters, with lower cost or higher demand corresponding to a better type.

[5] As will become clear in Section 4, much of our analysis goes through for more complex mechanisms.

selection literature carry over: In the *two-sided lemons equilibrium*, no trade is often the only equilibrium and, if trade takes place, only low-type firms consent to the merger. Second, if only the seller's returns are decreasing in own type, whereas the buyer's returns are increasing, the merger pattern looks dramatically different: Equilibria with trade exist quite generally, and in these *lemons and peaches equilibria*, low-type sellers merge with *high*-type buyers. Further, there will typically be non-degenerate type sets for which buyers regret the merger ex post, and non-degenerate type sets for which sellers regret the merger ex post. The last statement is true even if the seller receives a reimbursement that is independent of the competitor's type: As the buyer turns out to be worse than expected, sellers realize they could have made stand-alone profits that exceed the takeover price. Finally, independent of the properties of merger returns, there is always a *no-merger equilibrium* where firms merge with probability zero: If both firms believe that the other firm will not consent to the merger, irrespective of its type, it is a (weakly) best response not to consent, and beliefs are correct in equilibrium.

The remainder of the paper is organized as follows. In Section 2, we introduce the analytical framework. Section 3 analyzes the monotonicity properties of the merger return functions in the linear Cournot model with cash payment. Section 4 analyzes the Bayesian equilibria for various assumptions on merger return functions that are consistent with the examples from the Cournot case. Section 5 concludes.

2 Analytical Framework

We consider an oligopoly with an exogenous number of firms $n \geq 2$. Two of these firms, denoted as $i = 1, 2$, are matched to play a merger game. Each firm is characterized by a type $z_i \in \mathbb{R}$, which influences its profitability. There is asymmetric information about the value of z_i, i.e. firms know their own z_i, but not their competitor's $z_j, j \neq i$. The ex ante probability of z_i is described by a probability distribution F_i with density f_i and compact support $[\underline{z}_i, \overline{z}_i] \subset \mathbb{R}$. F_i is common knowledge. Note that we allow for ex ante heterogeneity between firms, i.e. firms' types z_i may be drawn from different distributions.[6]

Firms simultaneously announce whether they are willing to merge. The decision of firm i is summarized in a variable s_i such that $s_i = 1$ if it consents to an agreement and $s_i = 0$ if it rejects it. A merger occurs if and only if $s_i = 1$ for $i = 1, 2$. If no merger occurs, each firm earns its stand-alone oligopoly profit $\pi_i(z_i, z_j)$. If a merger occurs, the merged entity earns total profit $\pi^M(z_i, z_j)$. These functions are defined on some set $\mathcal{Z} = \mathcal{Z}_1 \times \mathcal{Z}_2$, where $[\underline{z}_i, \overline{z}_i] \subseteq \mathcal{Z}_i$. The properties of π_i and π^M reflect more primitive assumptions on the nature of product market interaction and the interpretation of the type variable. Throughout the paper, we shall require the following assumption on the firms' stand-alone profits to be satisfied.

Assumption 1 *π_i is non-decreasing in z_i; π^M is non-decreasing in z_1 and z_2.*

Thus, by definition, higher types are more efficient: The higher a firm's type, the higher its stand-alone profits and the higher the profit of the merged entity that it becomes a part of. The next assumption relates the other firm's type to own stand-alone profits.

Assumption 2 *$\partial\pi_i/\partial z_j \leq \partial\pi_i^M/\partial z_j$ for $i = 1, 2, j \neq i$.*[7]

This assumption is particularly appealing for horizontal mergers: There, stand-alone profits typically fall if the competitor becomes more efficient. Thus, the left-hand side of the inequality is negative. In addition, because

[6] This is of particular importance for vertical or conglomerate mergers where firms produce entirely different goods. Even the interpretation of the firms' types might differ: For vertical mergers, for instance, the types might correspond to the costs of input production for the upstream firm and marketing ability for the downstream firm.

[7] We are implicitly assuming that the profit functions are differentiable here for notational convenience only.

of Assumption 1, total post-merger profits increase if firm j becomes better. If the owners of both firms (weakly) benefit from these higher profits, then π_i^M is non-decreasing in z_j and the right-hand side of the inequality is non-negative.[8]

Denoting the individual post-merger profits of the formerly separate firms by $\pi_i^M(z_i, z_j), i, j = 1, 2, j \neq i$, we further require a balanced budget for the merger transaction.

Assumption 3 *The merging firms' individual profits add up to the joint profits of the merged firm, i.e.* $\pi_1^M(z_1, z_2) + \pi_2^M(z_2, z_1) = \pi^M(z_1, z_2)$.

We do not make any general assumptions on how exactly the functions π_i^M are determined, that is, how profits are split in case a of a merger. Nevertheless, even though much of our general analysis in Section 4 makes sense for alternative sharing rules, it will often be useful to think of a specific example where the owners of one firm, say firm 1, compensate the owners of the other firm, firm 2, by a cash payment $p > 0$ for the takeover, so that $\pi_1^M(z_1, z_2) = \pi^M(z_1, z_2) - p$ and $\pi_2^M(z_2, z_1) = p$. In fact, in Section 3, we will provide a linear Cournot example with cash compensation to illustrate the crucial relation between a firm's type and its merger returns.[9]

At first glance, the two-sidedness of asymmetric information no longer seems to be relevant with cash compensation: After all, only the buyer's post-merger payoff depends on the other player's type. However, the opportunity costs of a merger do depend on the other firm's type for both sellers and buyers: In a horizontal setting, the stand-alone profit is lower the better the other firm is. Therefore, merging with a high type is more attractive because the outside option is worse.

[8]In vertical relationships, however, both upstream and downstream firms usually have higher profits when the other party is of a high type. Therefore, the left-hand side might be positive and the relation does not necessarily hold.

[9]Later on in the paper, we will also refer to another specific example where firms commit to a particular split of joint profits ex ante, even if one firm turns out to be very inefficient ex post. This is essentially the way profits are shared if the owners of a merging firm are compensated by shares in the new firm.

3 The Merger Return Function

The last section suggested that merger decisions are closely related to merger returns, i.e. the difference between post- and pre-merger profits. We therefore introduce the following definition:

Definition 1 *(i) Firm i's* ***individual merger returns*** *are given by*

$$g_i(z_i, z_j) \equiv \pi_i^M(z_i, z_j) - \pi_i(z_i, z_j).$$

(ii) The merging firms' ***joint merger returns*** *are given by*

$$g(z_1, z_2) \equiv \pi^M(z_1, z_2) - \pi_1(z_1, z_2) - \pi_2(z_1, z_2).$$

These merger return functions reflect the nature of product market competition both before and after the merger, as well as the effect of the merger on the technology adopted by the new firm. Individual return functions also depend on the way in which profits are split. If Assumption 2 holds, individual merger returns for firm i are always non-decreasing in $z_j, j \neq i$. We shall now explore the monotonicity properties of g_i with respect to z_i. For definiteness, we shall think of merger transactions as being cash-financed in this section, which is not necessary for our general considerations in the remainder of the paper.

We shall analyze the properties of the merger return functions in a simple Cournot setting. Suppose that, initially, there are n firms with marginal costs $c_i, i = 1, ..., n$, and inverse demand is given by $P(Q) = a - Q$, where $Q = \sum_i^n q_i$ is aggregate output and $a > 0$. We consider a merger game between firm 1 and 2. Let the firms' types be defined as $z_i \equiv -c_i$, i.e. the negative of marginal costs. Suppose that z_1 and z_2 are uniformly and independently distributed with compact support $\mathcal{Z} = \mathcal{Z}_1 = \mathcal{Z}_2 = [\underline{z}, \overline{z}]$.

For simplicity, we assume that the marginal costs of the firms $j \neq 1, 2$ not involved in the merger are known to equal 1, so that $\sum_{j \neq 1,2} c_j = n - 2$. We focus on situations where all firms produce positive equilibrium outputs both before and after the merger. We denote the type of the merged entity by $z_M = -c_M$, where c_M is the marginal cost of the new firm. Finally, we shall assume that z_M is a function $z_M(z_i, z_j)$. Under these assumptions, firm i's stand-alone profits are given by

$$\pi_i(z_i, z_j) = \frac{(a + nz_i - z_j + (n-2))^2}{(n+1)^2},$$

whereas the new firm's post-merger profits are

$$\pi^M(z_i, z_j) = \frac{(a + (n-1)\, z_M(z_i, z_j) + (n-2))^2}{n^2}.$$

We use these formulae below to determine merger returns under different assumptions on $z_M(z_i, z_j)$, that is, on how the technology adopted by the merged firm depends on the pre-merger technologies. Figure 1 illustrates the model for $n = 3$. The triangle ABC delineates the admissible set of those (c_1, c_2) for which all firms produce positive outputs.

<Figure 1 around here>

We now proceed to show that merger returns are not necessarily decreasing in own type—so that simple analogies from Akerlof's (1970) lemons model are inadequate to characterize the potential equilibria of the merger game. The monotonicity properties of the merger return functions depend crucially on the merger technology $z_M(z_i, z_j)$ that we have not specified so far. The most common assumption in the merger literature is that the new firm works with the technology $z_M = \max(z_1, z_2)$ of the more efficient part, implying that the merger gives rise to rationalization effects. This makes sense if the technology of the superior firm can easily be implemented in the new firm and if, in addition, there are no synergies.[10]

[10] Even if one firm has higher marginal costs than the other one, it may be more efficient in some dimensions. Then, the new entity might benefit from adapting some aspects of the relatively bad firm's technology and the type might increase to some $z_M > \max(z_1, z_2)$.

There are, of course, very different ways to think of the effects of mergers on marginal costs. Suppose, for instance, that marginal costs depend mainly on the average productivity of a firm's employees. Then, the marginal costs of the merged firm should be some weighted average of the constituent parts, so that

$$z_M = \beta z_1 + (1-\beta) z_2, \quad \beta \in [0,1]. \tag{1}$$

Another possibility could arise when the productivity of a firm is mainly the result of tacit knowledge of its managers that is hard to communicate. In this case, it is not clear whether this knowledge will find its way into the new firm. For instance, when firm 1 buys firm 2 at some given price, there may be no incentive for the managers of firm 2 to inform the new firm's management about useful business knowledge they might have. The new firm might then have to use the buyer's technology, i.e. $z_M = z_1$. We refer to this as the case of a buyer-dominated merger. Obviously, this is a special case of (1), with $\beta = 1$. The other extreme in (1), $\beta = 0$, is harder to justify. There is no obvious reason why the seller's technology should be applied in the new firm, unless it is more efficient than the buyer's technology. But applying the seller's technology only when it is more efficient would correspond to a rationalization merger. Nevertheless, we shall deal with the seller-dominated merger below as a benchmark case, in addition to buyer-dominated mergers and rationalization mergers.

3.1 Case 1: Buyer-Dominated Mergers

First, consider the case where the new firm produces with the buyer's technology ($\beta = 1$), so that $z_M = z_1$ (buyer-dominated mergers). Then the buyer's merger returns are given by

$$\begin{aligned} g_1(z_1, z_2) &\equiv \pi^M(z_1, z_2) - \pi_1(z_1, z_2) - p \qquad (2) \\ &= \frac{(a + (n-1) z_1 + (n-2))^2}{n^2} - \frac{(a + n z_1 - z_2 + (n-2))^2}{(n+1)^2} - p, \end{aligned}$$

whereas the seller's merger returns are

$$g_2(z_1, z_2) \equiv p - \pi_2(z_2, z_1) \qquad (3)$$
$$= p - \frac{(a + nz_2 - z_1 + (n-2))^2}{(n+1)^2}.$$

Clearly, Assumption 2 holds, so that $\partial g_i / \partial z_j > 0, i, j = 1, 2, i \neq j$, because higher competitor types negatively affect own stand-alone profits but leave post-merger profits unaffected.[11] Also, for the seller, merger returns are decreasing in own type z_2: High-type sellers earn high stand-alone profits, whereas the takeover price is independent of types. For the buyer, however, merger returns are increasing in own type z_1: Not only do high-type buyers earn higher stand-alone profits, they also earn higher profits in the merged entity. Straightforward calculations show that the latter effect dominates over the former: Intuitively, the reduced intensity of competition after the merger (due to the elimination of one firm) means that the merged firm's output is higher than the buyer's stand-alone output. Thus, lower marginal costs have a stronger effect on profits after the merger. As a result, high-type buyers have more to gain from mergers than low-type buyers. By continuity, this result also holds for β sufficiently close to 1: The impact of the buyer on the post-merger technology is then sufficiently strong to guarantee that lower marginal costs have a stronger effect on post-merger profits than on stand-alone profits.

Before proceeding, we note that, for buyer-dominated mergers, the set of types for which the merger is efficient under certainty (i.e. where joint merger returns are positive), is bounded by ABE in Figure 1. Only if the buyer has a substantial efficiency advantage over the seller, can a buyer-dominated merger be efficient. For firms that are relatively homogeneous, the standard argument of Salant et al. (1983) shows that mergers are inefficient.

[11] The post-merger profit of the buyer, π^M, is a function of z_1 alone by definition, whereas the post-merger profit of the seller is simply p.

3.2 Case 2: Seller-Dominated Mergers

Next, consider the case where the new firm produces with the technology of the seller ($\beta = 0$), so that $z_M = z_2$ (seller-dominated mergers). As noted above, we do not want to argue that this is the most likely case. We nevertheless deal with it for two reasons. First, the analysis is useful as it illustrates that, in contrast to what one might expect, the requirements for merger returns to be decreasing in own type both for the seller and the buyer are rather strong. Second, the monotonicity properties of the pure seller-dominated merger carry over to somewhat less extreme cases of mergers where the new firm operates a biased mix of the technologies of its constituent parts (with $\beta > 0$ and sufficiently small).

As for buyer-dominated mergers, the seller's merger returns are given by (3) and thus remain decreasing in own type. The buyer's merger returns are now given by

$$g_1(z_1, z_2) = \frac{(a + (n-1) z_2 + (n-2))^2}{n^2} - \frac{(a + nz_1 - z_2 + (n-2))^2}{(n+1)^2} - p.$$

This function is decreasing in z_1, as the buyer's stand-alone profits are increasing in own type, whereas the post-merger profits are independent of the buyer's type by definition. That is, in this particular case, the merger returns are in fact decreasing in own type both for the buyer and the seller, as suggested by the lemons rationale.

For seller-dominated mergers, the set of types for which the merger is efficient under certainty is bounded by BCD in Figure 1. Only if the seller has a substantial efficiency advantage over the buyer, can a seller-dominated merger be efficient. Again, it follows from Salant et al. (1983) that seller-dominated mergers with relatively homogeneous firms are inefficient.

3.3 Case 3: Rationalization Mergers

Finally, consider the case where the new firm produces with the technology $z_M = \max(z_1, z_2)$ of the more efficient part (rationalization mergers). Merger returns for the seller are the same as above and thus given by (3). For the buyer, merger returns are given by

$$g_1(z_1, z_2) = \frac{(a + (n-1)\max(z_1, z_2) + (n-2))^2}{n^2} - \frac{(a + nz_1 - z_2 + (n-2))^2}{(n+1)^2} - p.$$

In this case, g_1 is clearly not monotone in z_1: If the buyer is more efficient than the seller ($z_1 \geq z_2$), the rationalization merger is similar to the buyer-dominated merger, where g_1 is monotone *increasing*. For $z_1 < z_2$, however, the rationalization merger is similar to the seller-dominated merger, where g_1 is monotone *decreasing*: A higher own type increases stand-alone profits, but leaves post-merger profits unaffected. That is, again the buyer's merger return function is not monotone decreasing in own type. Furthermore, joint merger returns can only be positive for sufficiently heterogeneous firms. However, as the buyer can now use the seller's technology if it is more efficient, mergers are efficient within both triangles ABE and BCD in Figure 1.

3.4 Summary

The examples discussed above serve to illustrate the point that it is anything but clear that low types have more to gain from mergers than high types: Even for the standard linear Cournot oligopoly model and the simplest assumption on the division of post-merger profits—cash compensation for the seller, post-merger profits of the new firm for the buyer—different types of mergers are likely to give rise to very different merger returns functions, as summarized in the following statement.

Observation 1 *In the linear Cournot model with cash compensation for the seller, merger returns for the seller are decreasing in own type. For the buyer, merger returns are*

(i) increasing in own type for buyer-dominated mergers;

(ii) decreasing in own type for seller-dominated mergers;

(iii) increasing in own type if the buyer is more efficient (and decreasing otherwise) for rationalization mergers.

Therefore, it is difficult to see how the conventional wisdom from the standard lemons market model carries over to the analysis of mergers. To better understand to what extent the lemons market rationale is useful for understanding mergers under two-sided asymmetric information, we next provide an analysis of the Bayesian equilibria of our reduced-form merger game under various assumptions on the merger return functions. Before doing so, we remark that Assumptions 1 to 3 hold in our linear Cournot example with cash compensation, so results that rely on these assumptions will be applicable.

4 Analyzing Merger Equilibria

In this section, we return to the merger game of Section 2 and characterize its Bayesian equilibria in general terms, depending on the properties of the merger return functions.

We use the following notation: For $i = 1, 2$, if firm i plays a strategy $s_i(z_i)$, we define $B_i \equiv B_i(s_i) \equiv \{z_i \,|s_i(z_i) = 1\}$, i.e., B_i denotes the set of types z_i for which firm i consents to a merger. Further, let

$$G_i(z_i; B_j, f_j) \equiv \int_{B_j} g_i(z_i, z_j) f_j(z_j)\, dz_j$$

denote the expected merger returns for firm i with type z_i when players j are distributed as f_j, and only players in B_j consent to a merger.

4.1 Two-Sided Lemons Equilibria

Though we believe the other cases are more relevant and more interesting, we start with conditions under which low types are more likely to merge in equilibrium, that is, there is a cut-off equilibrium where only low types consent to a merger. We call this a *two-sided lemons equilibrium*.[12]

We shall use the following terminology.

Definition 2 *The function $G_i : [\underline{z}_i, \overline{z}_i] \to \mathbb{R}$ satisfies* ***strong downward single crossing (SSC$^-$)*** *if, for all $z_i', z_i'' \in [\underline{z}_i, \overline{z}_i]$ such that $z_i' > z_i'', G_i(z_i') \geq 0$ implies $G_i(z_i'') \geq 0$ and $G_i(z_i') > 0$ implies $G_i(z_i'') > 0$.*

This definition is closely related to the familiar single-crossing property of incremental returns (Milgrom and Shannon 1994).[13] We first give a cut-off condition in terms of expected merger returns, and then consider more primitive conditions on actual merger returns.[14]

Lemma 1 *Suppose $G_i(z_i; B_j, f_j)$ satisfies SSC^- in z_i for all $B_j \subset \mathcal{Z}_j$, $i = 1, 2$, $j \neq i$ and all f_j. Then every Bayesian Equilibrium (s_1^*, s_2^*) in pure strategies with $\mathbb{P}[B_i(s_i^*)] > 0$ for $i = 1, 2$ satisfies the* ***cut-off-property****, that is, there are cut-off values $z_i^* \in \mathcal{Z}_i$ such that*

$$s_i^*(z_i) = \begin{cases} 1, & \text{if } z_i \leq z_i^*; \\ 0, & \text{if } z_i > z_i^*; \end{cases} \qquad i = 1, 2.$$

[12] Equilibria with a monotone relation between types and strategies are common in Bayesian games: Examples include first-price auctions where the type is the bidder's valuation and the strategy is the bid, double auctions where the types of buyers and sellers are valuations and costs, and the strategies are bids and asks (Chatterjee and Samuelson 1983), wars of attrition where the type is the valuation for the prize and the strategy is the quitting period, and games of public good provision where types correspond to the costs of providing a public good and actions correspond to the provision decision (see Fudenberg and Tirole (1991) for a discussion of these games). Athey (2001) analyzes more generally under which conditions such monotonicity results arise.

[13] Let $\Pi_i(s_i, z_i; B_j, f_j)$ define the expected payoff from strategy s_i for a firm with type z_i, facing a competitor characterized by B_j and f_j. Then $\Pi_i(s_i, z_i; B_j, f_j)$ satisfies the Milgrom-Shannon Single-Crossing Property in $(-s_i, z_i)$ if and only if G_i satisfies SSC^-.

[14] Using the equivalence between SSC^- and the Milgrom-Shannon condition, Lemma 1 is a special case of Theorem 1 in Athey (2001).

Proof. See Appendix. ■

The intuition for Lemma 1 is as follows: SSC^- states that, for any distribution of z_j, if some type z_i consents to a merger, so will any lower type $z_i' < z_i$, no matter what the distribution of z_j is. The result applies this property to the distribution of z_j corresponding to the equilibrium behavior of z_j.

Lemma 1 immediately implies the following result.

Proposition 1 (two-sided lemons) *If $g_i(z_i, z_j)$ is monotone decreasing in $z_i, i, j = 1, 2, i \neq j$, then every Bayesian Equilibrium satisfies the cut-off property.*

The intuition for this result is simple: If higher types have less to gain from a merger for arbitrary realizations of types, then clearly they must gain less in expectation. In the linear Cournot example, the monotonicity condition on merger returns in Proposition 1 corresponds to a seller-dominated merger (see section 3.2). Therefore, in this example, only low types (if any) consent to a merger.

As we demonstrated in Borek et al. (2003), Proposition 1 generalizes to a wide class of non-monotone functions. However, even these generalizations do not apply for some of the merger return functions discussed in Section 3 as we now show.

4.2 Lemons and Peaches Equilibria

We next consider the case where merger returns are increasing in own type for one firm, and decreasing in own type for the other, as suggested by the buyer-dominated merger in linear Cournot oligopoly (see Section 3.1).

Under these circumstances, Proposition 1 has the following straightforward implication:

Corollary 1 (lemons and peaches) *Suppose g_1 is monotone increasing in z_1, and g_2 is monotone decreasing in z_2. Then, in any Bayesian Equilibrium* *in pure strategies with* $\mathbb{P}[B_i(s_i^*)] > 0$ *for* $i = 1, 2$, *there exist* z_1^H *and* z_2^L *such that* $s_1(z_1) = 1$ *if and only if* $z_1 \geq z_1^H$ *and* $s_2(z_2) = 1$ *if and only if* $z_2 \leq z_2^L$.

Proof. Redefine the firms' types as $y_1 = -z_1$ and $y_2 = z_2$ and apply Proposition 1. ■

Corollary 1 reveals that the lemons rationale may be misleading in the context of mergers: As argued in Section 3, even though a high-type firm 1 foregoes higher stand-alone profits than a low-type firm 1 when entering a merger, its merger returns may nevertheless be higher, as it also performs better in the merged entity. In this case, only high types of firm 1 ("peaches") will consent to the merger in equilibrium. For firm 2, the lemons rationale remains correct: In equilibrium, only low types will consent to the merger. As a result, we obtain an equilibrium where low types of firm 2 merge with high types of firm 1. This is what we call a *lemons and peaches equilibrium.* As the linear Cournot model with buyer-dominated mergers satisfies the conditions of Corollary 1, we expect a lemons and peaches equilibrium to emerge under these circumstances.

In the Cournot example, we further saw that merger returns may actually be non-monotone in own type in the case of a rationalization merger: There, returns were decreasing for $z_1 < z_2$ and increasing in $z_1 \geq z_2$. We thus relax the monotonicity assumption of Corollary 1, requiring only that firm 1's returns are increasing in own type for $z_1 \geq z_2$, maintaining the assumption that firm 2's merger returns are decreasing in own type. In this setting, we obtain the following somewhat weaker result.

Proposition 2 *Suppose* g_1 *is monotone increasing in* z_1 *for* $z_1 \geq z_2$, *and* g_2 *is monotone decreasing in* z_2. *Then*

(i) there exists a $\widetilde{z}_2 \in \mathcal{Z}_2$ *such that* $s_2(z_2) = 1$ *if and only if* $z_2 \leq \widetilde{z}_2$.

(ii) if, for some $z_1 > \widetilde{z}_2, s_1(z_1) = 1$, *then* $s_1(z_1') = 1$ *for all* $z_1' > \widetilde{z}_2$.

Proof. (i) Follows immediately as g_2 is decreasing in z_2 by assumption. (ii) Suppose $z_1 > \widetilde{z}_2$. If $s_1(z_1) = 1$, we have $\int_{\underline{z}}^{\widetilde{z}_2} g_1(z_1, z_2) f_2(z_2) dz_2 \geq$

0. Since g_1 is increasing in z_1 for $z_1 \geq z_2$, and $z_1' > z_1 > \widetilde{z}_2$, we obtain $\int_{\underline{z}}^{\widetilde{z}_2} g_1(z_1', z_2) f_2(z_2) \, dz_2 \geq 0$. ■

For instance, Proposition 2 excludes equilibria such as those sketched in Figure 2, where bold lines correspond to sets of types consenting to a merger. Thus, there are no equilibria where $B_1 = \left[z_1^{\min}, z_1^{\max}\right]$ and

$$\widetilde{z}_2 \leq z_1^{\min} < z_1^{\max} < \overline{z} \quad \text{or} \quad z_1^{\min} \leq \widetilde{z}_2 < z_1^{\max} < \overline{z}.$$

In the case of rationalization mergers, our approach is thus limited in the sense that we can only exclude some types of equilibria.

<Figure 2 around here>

4.3 Necessary and Sufficient Conditions for Mergers

Our analysis in the above section strongly restricts the possible set of equilibria. However, it does not exclude the existence of equilibria where mergers do not come about in the first place. In this section, we therefore analyze more carefully whether trade actually occurs in equilibrium.[15]

The first result shows that, for arbitrary distributions of types, there is always a degenerate cut-off equilibrium where no type merges.

Proposition 3 (no-merger) *Each strategy pair* (s_1, s_2) *with*

$$\mathbb{P}\left[B_i(s_i)\right] \equiv \int_{B_i} f_i(z_i) \, dz_i = 0, \quad i = 1, 2,$$

is a Bayesian Equilibrium of the merger game.

Proof. See Appendix. ■

The result is very intuitive: If both firms believe that the other firm will not consent to a merger—no matter what its type is—it is a (weakly) best response not to consent, and beliefs are correct in equilibrium. Thus, there

[15] The analysis is closely related to the analysis of no trade equilibria in the standard adverse selection literature.

always is an equilibrium where firms merge with probability zero. Note, however, that the no-merger equilibrium is Pareto-dominated in terms of expected profits whenever a cut-off equilibrium exists where firms consent to a merger with strictly positive probability.

The next proposition deals with the case that merger returns are decreasing. It gives a sufficient condition under which there is no other than the no-merger equilibrium.

Proposition 4 *Suppose $\mathcal{Z}_i = \mathcal{Z}_j = \mathcal{Z}$. Further assume that, for $i, j = 1, 2, j \neq i$, $g_i(z_i, z_j)$ is monotone decreasing in z_i. If $g_i(z_i, z_j) \leq 0$ for $i = 1, 2$, $j \neq i$ and $z_i = z_j$, then there is no Bayesian Equilibrium where a positive measure of firms consents to a merger.*

Proof. See Appendix. ■

In brief, Proposition 4 states that if mergers between identical types are not beneficial under certainty for either firm and low types have stronger incentives to merge, then no trade is the only equilibrium.[16]

The result is useful in applications such as the linear Cournot model where, for homogeneous firms, joint merger returns are negative. Then, individual merger returns must also be negative for at least one firm for any budget-balancing split of profits. Suppose further that the transaction is share-financed rather than cash-financed as in the examples of Section 3. Specifically, assume that firms agree to predetermined shares $\alpha_i \in [0, 1], \alpha_1 + \alpha_2 = 1$, of the new firm's joint profits rather than cash payment, so that $\pi_i^M(z_i, z_j) = \alpha_i \pi^M(z_i, z_j)$. Clearly, if α_i is sufficiently close to $1/2$, both firms must have negative merger returns if they have identical types. Further, it is straightforward to show that if the firms' profit shares are not extremely asymmetric, merger returns are decreasing in own type for both

[16] The result generalizes to the case where $\mathcal{Z}_1 \neq \mathcal{Z}_2$ by demanding more generally that $g_i(z_i, z_j) \leq 0$ on the diagonal of $\mathcal{Z}$ rather than for $z_i = z_j$. Also, even this condition can be generalized further. However, though the present formulation of the proposition is not the most general, it is the easiest one to apply.

firms. Therefore, Proposition 4 applies and there is no equilibrium where firms merge with strictly positive probability.

So far, we have shown that, for merger returns that are decreasing in own types, no trade is the likely outcome. We have already seen, however, that this particular monotonicity requirement is surprisingly restrictive, even though it can be justified, for example, for seller-dominated mergers in the linear Cournot model. We therefore move to alternative assumptions on the merger return functions $g_i, i = 1, 2$. More specifically, we consider the case where g_1 is monotone increasing in z_1 and g_2 is monotone decreasing in z_2, as suggested by buyer-dominated mergers in the linear Cournot model, where in equilibrium high-type buyers acquire low-type sellers.

Let

$$G_1^L(z_1, z_2) \equiv \int_{\underline{z}_2}^{z_2} g_1(z_1, \widetilde{z}_2) f_2(\widetilde{z}_2) d\widetilde{z}_2$$

denote the expected returns of firm 1, anticipating that only low types of firm 2 (below z_2) will consent to a merger. Further, let

$$G_2^H(z_1, z_2) \equiv \int_{z_1}^{\overline{z}_1} g_2(\widetilde{z}_1, z_2) f_1(\widetilde{z}_1) d\widetilde{z}_1$$

denote the expected returns of firm 2, anticipating that only high types of firm 1 (above z_1) will consent to the merger. Clearly, an equilibrium with cut-off types (z_1^*, z_2^*) requires $G_1^L(z_1^*, z_2^*) = 0 = G_2^H(z_1^*, z_2^*)$. We use the notation

$$G_1(z_1) \equiv G_1^L(z_1, \overline{z}_2); \quad G_2(z_2) \equiv G_2^H(\underline{z}_1, z_2)$$

to describe the expected payoffs in the boundary case where player i expects all types $z_j \in \mathcal{Z}_j, j \neq i$, to consent. Then we obtain the following result.

Proposition 5 *Assume that g_1 and g_2 are continuous functions, with g_1 monotone increasing in z_1 and g_2 monotone decreasing in z_2.*

(i) Suppose

$$G_1(\overline{z}_1) < 0 \vee g_2(\overline{z}_1, \underline{z}_2) < 0, \tag{4}$$

or

$$\exists z_2 \in \mathcal{Z}_2 \text{ such that } \max\left(G_1^L(\overline{z}_1, z_2), g_2(\overline{z}_1, z_2)\right) < 0, \tag{5}$$

or

$$\exists z_1 \in \mathcal{Z}_1 \text{ such that } g_1(z_1, \underline{z}_2) > 0 > G_2^H(z_1, \underline{z}_2). \tag{6}$$

Then there is no equilibrium with mergers for a positive measure of types.

(ii) *Suppose neither of conditions (4)-(6) holds. Then there is an equilibrium with mergers for a positive measure of types.*

Proof. See Appendix. ■

The conditions under which the no-merger equilibrium is unique are fairly restrictive: Condition (4) requires that merger returns are negative even in the best possible scenario, so that a merger will never occur. Condition (5) requires the existence of a z_2 that is low enough that even the best type 1 does not want to merge if the cut-off value is $z_2^L = z_2$, but on the other hand high enough that z_2 himself does not want to merge with the best type of z_1. Finally, condition (6) requires that even the best type 1, say z_1', who is willing to merge with $\underline{z}_2$ is low enough that $\underline{z}_2$ does not want to merge if he expects type 1 players who consent to be from $[z_1', \overline{z}_1]$.

Figure 3 illustrates the result. The downward sloping line $G_1^L(z_1, z_2) = 0$ is the 'zero-returns curve' for player 1: If a player of type z_1 expects types z_2 and below to consent to a merger, the expected merger returns are zero if and only if (z_1, z_2) is on $G_1^L(z_1, z_2) = 0$. The line is downward sloping because $g_1(z_1, z_2)$ and therefore $G_1^L(z_1, z_2)$ are increasing in both z_1 and z_2, so that a higher type of player 1 is willing to merge with lower type 2 players. Similarly, consider $G_2^H(z_1, z_2) = 0$. If a player of type z_2 expects types z_1 and above to consent to a merger, the expected merger returns are zero if and only if (z_1, z_2) is on $G_2^H(z_1, z_2) = 0$. By analogous reasoning as before, as $g_2(z_1, z_2)$ and thus $G_2^H(z_1, z_2)$ are increasing in z_1 and decreasing in z_2, $G_2^H(z_1, z_2) = 0$ is upward sloping, so that a lower type of player 2 is willing to merge with lower type 1 players.

If condition (5) holds, $G_1^L(z_1, z_2) = 0$ intersects the right boundary of the type space above $G_2^L(z_1, z_2) = 0$. Intuitively, both types of players do not expect to gain enough from a merger. If condition (6) holds, $G_1^L(z_1, z_2) = 0$ intersects the lower boundary of the type space to the left of $G_2^L(z_1, z_2) = 0$. Intuitively, type 1 players are too keen on merging relative to type 2 players. Thus, if a type 1 player consents, this is not necessarily very positive information for type 2 players: Though the type 1 players that consent to a merger tend to be better than those that do not, they are still not good enough on average.

<Figure around 3 here>

The next result deals with the ex-post efficiency properties of the merger.

Proposition 6 *Suppose g_1 is monotone increasing in z_1 and g_2 is monotone decreasing in z_2. Then, whenever an interior cut-off equilibrium (z_1^*, z_2^*) exists, the following four cases coexist for non-degenerate sets of types.*

(i) Mergers with ***buyer regret (BR)****: In particular, low-type buyers regret merging with low-type sellers.*

(ii) Mergers with ***seller regret (SR)****: In particular, high-type sellers regret merging with low-type buyers.*

(iii) ***Pareto-improving mergers (PIM)****: In particular, both high-type buyers and high-type sellers benefit from the merger ex post.*

(iv) ***Inefficient Non-Mergers (INM)****: In particular, some efficient mergers of high-type sellers and high-type buyers do not occur.*

Proof. See Appendix ■

The intuition of Proposition 6 is straightforward. As cut-off types break even on average, they must regret a merger with the lowest type they expect to consent to a merger. Using the buyer/seller terminology introduced above, we therefore have that a buyer of type z_1^* regrets a merger with a seller of type $\underline{z}_2$. Similarly, a seller of type z_2^* regrets a merger with a buyer of type z_1^*. By continuity, there are regions like *BR* and *SR* in Figure 3 where buyers and sellers, respectively, regret the merger ex post (results (i) and (ii) in the proposition). Similarly, because cut-off types break even on average, they must have positive merger returns if they merge with the best type of the other player. In particular, a merger of a buyer with type $\overline{z}_1$ and a seller of z_2^* will benefit both firms. By continuity, there is a region like *PIM* where both high-type buyers and high-type sellers benefit ex post from the merger (result (iii) in the proposition). Finally, there is a region like *INM* where mergers do not occur even though they would be ex post efficient. This is because, in the cut-off equilibrium, some high-type sellers slightly above z_2^* do not consent to the merger for fear of selling out too cheaply.

5 The Standard Lemons Market Benchmark

Our analysis of the cut-off and no-trade equilibria in Section 4 is related to the standard literature on one-sided asymmetric information emanating from Akerlof (1970). This literature focuses on the circumstances under which an object of given quality—which is only known by the seller—will be traded at some price p, so that the seller's post-merger profits are independent of types. Also, stand-alone profits are independent of competitor types. Thus, translated to our setting the following conditions hold:

Condition 1 *The buyer's distribution function $F_1(z_1)$ is degenerate.*

Condition 2 *Both-firms stand-alone profits, π_i, are independent of $z_j, j \neq i$.*

Condition 3 *The seller's post-merger profit, π_2^M, is independent of z_2.*

These conditions make sense when a well-known firm diversifies into some other market by buying an unknown firm. We then interpret π^M as the sum of the buyer's stand-alone profit and the profit he obtains in the new activity. A cut-off result for this setting is a special case of our Proposition 1.

Corollary 2 *Suppose conditions 1-3 hold. Then every Bayesian equilibrium satisfies the cut-off property with respect to z_2.*

To understand the result, note that it suffices to consider the behavior of the seller, as the buyer's distribution function F_1 is degenerate by condition 1. Furthermore, since the seller's post-merger profit does not depend on own type by condition 3, his merger returns $g_2 = p - \pi_2(z_2)$ are monotone decreasing in own type; Proposition 1 thus immediately implies the result.

The next result gives a condition for no trade in the case of one-sided asymmetric information.

Proposition 7 *Suppose conditions 1-3 hold. Further assume that*

$$\int_{\underline{z}_2}^{z_2} \left(\pi^M\left(z_1, \widetilde{z}_2\right) - \pi_1\left(z_1\right)\right) f_2\left(\widetilde{z}_2\right) d\widetilde{z}_2 < \pi_2\left(z_1, z_2\right) \text{ for all } z_1, z_2. \tag{7}$$

Then there is no equilibrium where a positive measure of players merges.

Proof. By condition 2, π_2^M must be a fixed constant p. Thus player 2 consents to a merger if and only if $p \geq \pi_2\left(z_1, z_2\right)$. Now, denote the cut-off type of player 2 as z_2^*. By (7), we have

$$\int_{\underline{z}_2}^{z_2^*} \left(\pi^M\left(z_1, \widetilde{z}_2\right) - \pi_1\left(z_1\right)\right) f_2\left(\widetilde{z}_2\right) d\widetilde{z}_2 < \pi_2\left(z_1, z_2^*\right).$$

Thus, if $p \geq \pi_2\left(z_1, z_2^*\right)$, so that the cut-off type 2-player consents to merger, the expected merger returns for player 1 are negative for every potential cut-off type $z_2^* > \underline{z}_2$. ■

Condition (7) states that the buyer's expected merger returns, gross of the takeover price, are bounded above by the seller's stand-alone profits. If this condition holds, the buyer's merger returns are not sufficiently high to compensate the seller for foregone stand-alone profits, and the merger will thus not occur. This outcome is clearly inefficient whenever there exist some combinations of types for which the merger increases joint profits.

Even though conditions 1-3 are appropriate in some very specific settings, this paper demonstrated that they are misleading for a general theory of mergers under asymmetric information.

6 Conclusions

Mergers under asymmetric information differ from standard lemons problems in two ways. First, the informational asymmetry is generally two-sided. Second, high types usually earn more both before and after the transaction. Together, this implies that the prediction of the standard lemons model—if any, only low type firms will be willing to merge—describes merger behavior very inaccurately.

Using a simple linear Cournot model with cash payment, we illustrate that the buyer's merger returns are actually increasing (rather than decreasing) in own type when the buyer has a strong impact on the technology adopted by the merged firm. Only if the seller has a strong impact on the merged firm's technology will the buyer's merger returns be decreasing in own type. The merger pattern under two-sided asymmetric information crucially depends on the properties of the buyer's merger return function: For mergers with sufficient seller influence on the merged firm's technology, a two-sided lemons cut-off equilibrium with low-type sellers and low-type buyers emerges. However, for buyer-dominated mergers, a lemons and peaches equilibrium emerges where low-type sellers merge with high-type buyers.

The standard Akerlof (1970) lemons model further overemphasizes the no-trade problem in the context of mergers. Though it is still true that asymmetric information may prevent some efficient mergers, there is no general presumption that trade is inefficiently low. In fact, equilibria with trade arise quite generally for the case that only seller returns are decreasing in types.

Though trade is not necessarily inefficiently low in these equilibria, it does not necessarily arise if and only if it increases joint merger returns for the simple mechanisms we analyzed. This suggests a natural extension of the paper: It would be interesting to find out whether there are general incentive compatible and individually rational mechanisms with balanced budgets that avoid ex post regret on both sides of the market. The extension is non-trivial because it involves solving a mechanism design problem with interdependent valuations. However, our allocation problem has a relatively simple structure: The only task is to induce firms to merge if and only if the merger increases joint profits. Therefore, it does not seem hopeless to figure out mechanisms that solve the problems discussed in this paper.

Appendix

Proof of Lemma 1

Firm i's expected merger return, facing firm j with strategy s_j, is

$$\begin{aligned} G_i(z_i; B_j, f_j) &= \mathbb{P}[B_j]\,\mathbb{E}_{z_j}\left[\pi_i^M(z_i, z_j)\,|z_j \in B_j(s_j)\right] + \\ &\quad (1 - \mathbb{P}[B_j])\,\mathbb{E}_{z_j}[\pi_i(z_i, z_j)\,|z_j \notin B_j(s_j)] - \mathbb{E}_{z_j}[\pi_i(z_i, z_j)] \\ &= \int_{B_j} g_i(z_i, z_j)\, f_j(z_j)\, dz_j. \end{aligned}$$

If $G_i(z_i; B_j, f_j)$ is positive, firm i will consent to the merger, otherwise it will reject the merger. By assumption, $G_i(z_i; B_j, f_j)$ satisfies SSC^- in z_i. Thus, if a $z_i \in \mathcal{Z}_i$ exists such that $G_i(z_i; B_j, f_j) > 0$, then there exists a $z_i^\circ(s_j)$ such that $G_i(z_i; B_j, f_j) \geq 0$ if and only if $z_i \leq z_i^\circ(s_j)$. Now define

$$\tilde{R}_i(s_j) = \begin{cases} z_i^\circ(s_j), & \text{if } z_i^\circ(s_j) \leq \overline{z_i}; \\ \overline{z_i}, & \text{if } z_i^\circ(s_j) \geq \overline{z_i} \text{ or if } z_i^\circ(s_j) \text{ does not exist.} \end{cases}$$

Then firm i's optimal reaction is

$$R_i(z_i, s_j) = \begin{cases} 1, & \text{if } z_i \leq \tilde{R}_i(s_j); \\ 0, & \text{if } z_i > \tilde{R}_i(s_j). \end{cases}$$

In particular, for an equilibrium strategy s_j, the best reply has the required cut-off structure.

Proof of Proposition 3

Suppose firm i plays strategy $s_i(z_i)$ with $\mathbb{P}[B_i(s_i)] = 0$. Then the probability that a merger takes place is zero and therefore firm $j \neq i$ is indifferent between any strategies it can play; in particular, every strategy $s_j(z_j)$ with $\mathbb{P}[B_j(s_j)] = 0$ is a best response.

Proof of Proposition 4

Proof. Assume that, for all $i \in \{1,2\}$, there is no $\widehat{z}_i$ such that $g_i(\widehat{z}_i, \widehat{z}_i) \geq 0$. Suppose w.l.o.g. that there is a non-trivial cut-off equilibrium (z_1^*, z_2^*) with $z_1^* \geq z_2^*$. As $g_1(z_1^*, z_1^*) < 0$ and g_1 is non-decreasing in z_2, $g_1(z_1^*, z_2) < 0$ for all $z_2 \leq z_2^*$. Therefore, expected equilibrium profits for firm 1 are $\int_{\underline{z}_2}^{z_2^*} g_1(z_1^*, z_2) f_2(z_2)\, dz_2 < 0$, contradicting the condition that

$$\int_{\underline{z}_2}^{z_2^*} g_1(z_1^*, z_2) f_2(z_2)\, dz_2 = 0$$

for the cut-off values (z_1^*, z_2^*). ■

6.1 Proof of Proposition 5

Proof. (i) Suppose (4) holds. Then the merger returns are negative even in the best possible scenario either for firm 1 or firm 2, so that a merger will never occur.

Suppose now that (4) does not hold, but (5) instead. If $G_1^L(\overline{z}_1, z_2) < 0$ or $g_2(\overline{z}_1, z_2) = G_2^H(\overline{z}_1, z_2) < 0$ for all $z_2 \in \mathcal{Z}_2$, then clearly there is no equilibrium with mergers. Thus, suppose $G_1^L(\overline{z}_1, z_2) > 0$ for some $z_2 \in \mathcal{Z}_2$ and $G_2^H(\overline{z}_1, z_2) > 0$ for some $z_2 \in \mathcal{Z}_2$. By (5), $G_1^L(z_1, z_2) = 0$ intersects the right boundary of the type space in $(\overline{z}_1, z_2')$, whereas $G_2^H(z_1, z_2) = 0$ intersects the right boundary in $(\overline{z}_1, z_2'')$, with $z_2' > z_2''$ (see Figure 4, Panel (a)). Therefore, $G_2(z_2) = G_2^H(\overline{z}_1, z_2) < 0$ for all $z_2 > z_2''$. Thus, for type 2 players to consent to a merger if $z_1^H = \overline{z}_1$, we require $z_2^L \leq z_2''$. However, $G_1^L(z_1, z_2'') < G_1^L(z_1, z_1') = 0$, so that no type 1 player will consent to a merger if $z_2^L \leq z_2''$. As a result, a no-merger equilibrium comes about (see case a) in Table 1).

Now suppose (6) holds. $G_1^L(z_1, z_2) = 0$ then intersects the lower boundary of the type space in $(z_1', \underline{z}_2)$, whereas $G_2^H(z_1, z_2) = 0$ intersects with the lower boundary in $(z_1'', \underline{z}_2)$, with $z_1' < z_1''$ (see Figure 4, Panel (b)). Therefore, all type 1 players with $z_1 \geq z_1'$ consent to a merger even with the worst type 2 players. Thus, an equilibrium would require $z_1^H \leq z_1'$. As $z_1' < z_1''$, however, $G_2^H\left(z_1^H, \underline{z}_2\right) < 0$, $\forall z_1^H \leq z_1'$. That is, again a no-merger equilibrium comes about (see case a) in Table 1).

<Figure 4 around here>

(ii) If none of the conditions (4)-(6) holds, we obtain one of the following cases summarized in Table 1:

b) There is an interior intersection of $G_1^L(z_1, z_2) = 0$ and $G_2^H(z_1, z_2) = 0$ and therefore an interior equilibrium as in Figure 3.

c) $G_1^L(z_1, z_2) = 0$ intersects the upper boundary of the type space to the right of $G_2^H(z_1, z_2) = 0$ in $\left(z_1^b, \overline{z}_2\right)$ as in Figure 5, Panel (a). Then there is an equilibrium such that type 1 players above z_1^b and all type 2 players consent.

d) $G_1^L(z_1, z_2) = 0$ intersects the left boundary of the type space below $G_2^H(z_1, z_2) = 0$ as in Figure 5, Panel (b). Then there is an equilibrium where all type 1 players and type 2 players below z_2^s consent to a merger.

e) By $G_2^H(\underline{z}_1, \overline{z}_2) > 0$, all type 2 players consent to the merger. For the cut-off value $\overline{z}_2$ the expected merger returns are positive even for a type 1 player with the lowest possible type ($G_1^L(\underline{z}_1, \overline{z}_2) > 0$). There is thus an equilibrium where all types on both sides of the market consent to the merger.

f) By $G_2^H(\underline{z}_1, \overline{z}_2) > 0$, all type 2 players consent to the merger. $G_1^L(z_1, z_2) = 0$ intersects the upper boundary in $\left(z_1^b, \overline{z}_2\right)$ as in Figure 5, Panel (a). Then there is an equilibrium where all type 1 players above z_1^b and all type 2 players consent to a merger.

g) By $G_1^L(\underline{z}_1, \underline{z}_2) > 0$, all type 1 players consent. However, as $G_2^H(z_1, z_2) = 0$ intersects the lower bound of the type space in some $(z_1'', \underline{z}_2)$, with $z_1'' > \underline{z}_1$ (as in Figure 4, Panel (b)), $\exists z_1 \in \mathcal{Z}_1$ such that $g_1(z_1, \underline{z}_2) > 0 > G_2^H(z_1, \underline{z}_2)$. By (6), we therefore have a no-merger equilibrium.

h) By $G_1^L(\underline{z}_1, \underline{z}_2) > 0$, all type 1 players consent. $G_2^H(z_1, z_2) = 0$ intersects the left boundary in $(\underline{z}_1, z_2^s)$, with $z_2^s > \underline{z}_2$ (as in Figure 5, Panel (b)), and we thus have an equilibrium where all type 1 players and all type 2 players below z_2^s consent to a merger.

i) By $G_1^L(\underline{z}_1, \underline{z}_2) > 0$ and $G_2^H(\underline{z}_1, \overline{z}_2) > 0$, both type 1 and type 2 players face positive expected merger returns even in the worst possible scenario, so that in equilibrium, all types on both sides of the market consent to a merger.

Table 1: Summary of equilibria for $G_1^L(\overline{z}_1, \overline{z}_2) > 0$ and $G_2^H(\overline{z}_1, \underline{z}_2) > 0$

	$G_1^L(\underline{z}_1, \underline{z}_2) < 0$	$G_1^L(\underline{z}_1, \underline{z}_2) > 0$
$G_2^H(\underline{z}_1, \overline{z}_2) < 0$	a) no merger equilibrium b) interior equilibrium c) type 1 players above z_1^b and all type 2 players merge d) all type 1 players and type 2 players below z_2^s merge	g) $G_2^H(\underline{z}_1, \underline{z}_2) < 0$: no merger equilibrium h) $G_2^H(\underline{z}_1, \underline{z}_2) > 0$: all type 1 players and type 2 players below z_2^s merge
$G_2^H(\underline{z}_1, \overline{z}_2) > 0$	e) $G_1^L(\underline{z}_1, \overline{z}_2) > 0$: all types merge f) $G_1^L(\underline{z}_1, \overline{z}_2) < 0$: type 1 players above z_1^b and all type 2 players merge	i) all types merge

<Figure 5 around here>

■

6.2 Proof of Proposition 6

Proof. (i) Because $G_1^L(z_1^*, z_2^*) = 0$ and g_1 is increasing in z_1, we have $g_1\left(z_1^*, \underline{z_2}\right) < 0$. Thus, by continuity, $g_1(z_1, z_2) < 0$ for (z_1, z_2) that are sufficiently close to $(z_1^*, \underline{z_2})$.
(ii) is analogous: Seller regret occurs for (z_1, z_2) close to (z_1^*, z_2^*).
(iii), (iv) Because $G_1^L(z_1^*, z_2^*) = G_2^H(z_1^*, z_2^*) = 0$ and both g_1 and g_2 are increasing in z_1, we have $g_1(\overline{z}_1, z_2^*) > 0$ and $g_2(\overline{z}_1, z_2^*) > 0$. By continuity, there is thus an ε-neighborhood of $(\overline{z}_1, z_2^*)$ such that $g_1(z_1, z_2) > 0$ and $g_2(z_1, z_2) > 0$ for all (z_1, z_2) in this neighborhood. In this neighborhood, all the points with $z_2 < z_2^*$ satisfy (iii); the points above z_2^* satisfy (iv). ■

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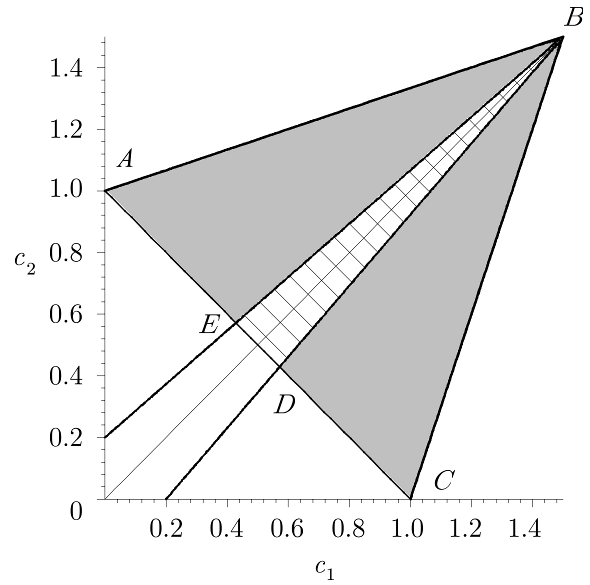

Figure 1: Admissible range of marginal costs ($a = 2, c_3 = 1$).

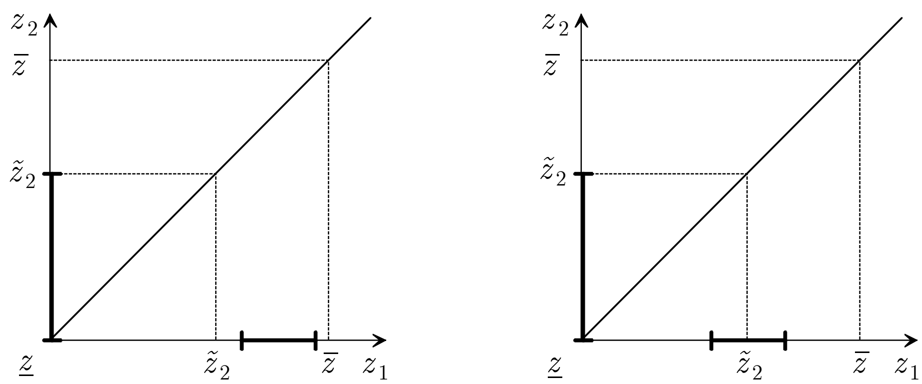

Figure 2: Equilibria excluded by Proposition 2

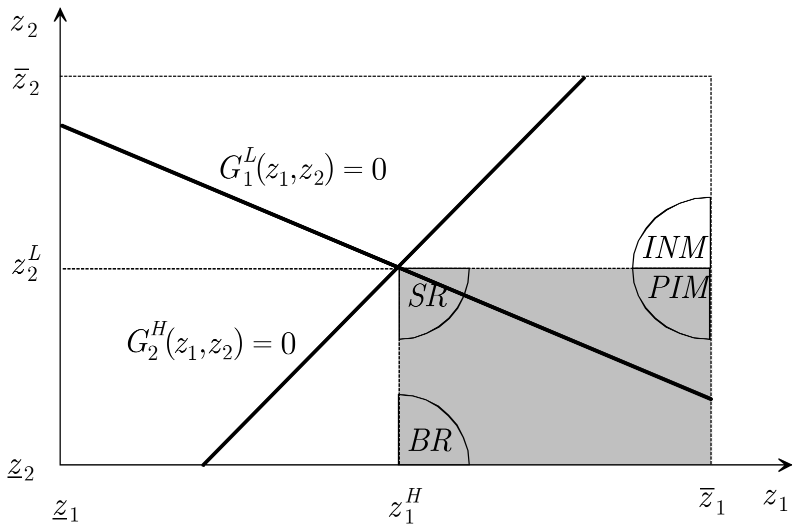

Figure 3: Characteristics of interior lemons and peaches equilibria

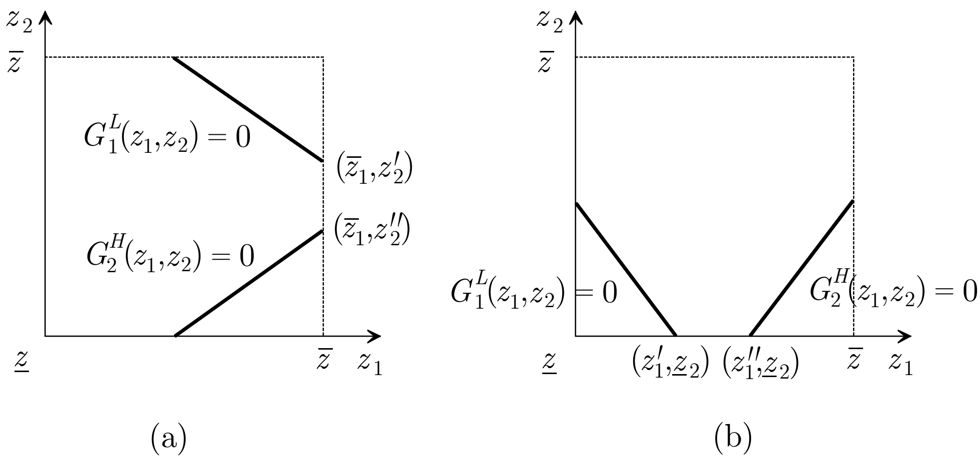

Figure 4: No-merger equilibria implied by (5) [Panel (a)] and (6) [Panel (b)]

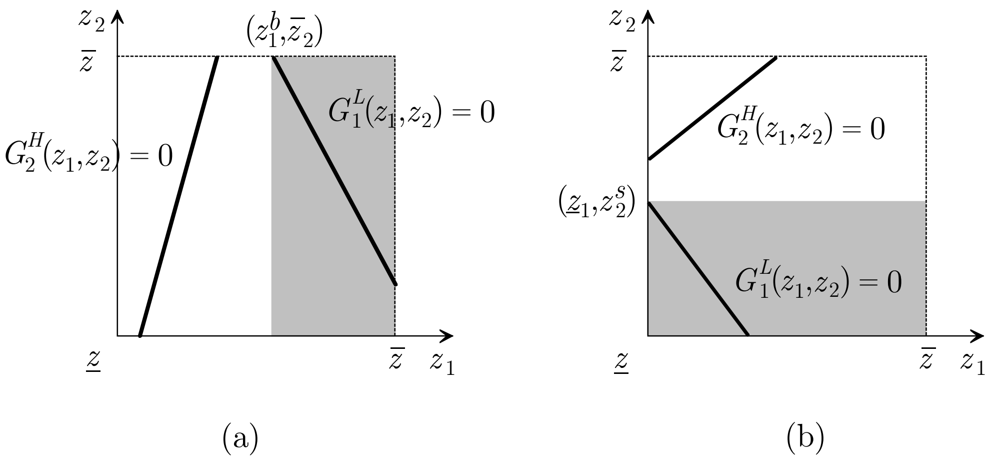

Figure 5: Lemons and peaches equilibria where all types on one side of the market consent

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