Institute for Empirical Research in Economics
University of Zurich

Working Paper Series
ISSN 1424-0459

Working Paper No. 272

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February 2006

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February 2006

Abstract

An individual makes random errors when evaluating the expected utility of a risky lottery. Errors are symmetrically distributed around zero as long as an individual does not make transparent mistakes such as choosing a risky lottery over its highest possible outcome for certain. This stochastic decision theory explains many well-known violations of expected utility theory such as the fourfold pattern of risk attitudes, the discrepancy between certainty equivalent and probability equivalent elicitation methods, the preference reversal phenomenon, the generalized common consequence effect (the Allais paradox), the common ratio effect and the violations of the betweenness.

Keywords: expected utility theory, stochastic utility, Fechner model, random error, risk

JEL Classification codes: C91, D81

[†] I am grateful to Wolfgang Kohler and the participants of the research seminar in IEW (Zurich, February 9, 2006) for helpful comments. John Hey generously provided the experimental data.

Stochastic choice under risk

I. Introduction

Experimental studies of repeated choice under risk (e.g. Knight, 1921) demonstrate that individual decision making is a stochastic rather than deterministic process. Camerer (1989), Starmer and Sugden (1989) and Wu (1994) provide extensive experimental evidence that there is some degree of randomness in the observed choices between risky lotteries. When individuals choose repeatedly between the same lotteries (with a possibility to declare indifference), they make identical decisions only in around 75% of all cases (e.g. Hey and Orme, 1994, p.1296). Apparently, decision making under risk is inherently stochastic (e.g. Ballinger and Wilcox, 1997). Moreover, Hey (2001) finds the variability of the subjects' responses to be generally higher than the difference in the predictive error of various deterministic decision theories.

While stochastic nature of choice under risk is persistently documented in experimental data, it remains largely ignored in the majority of decision theories (e.g. Loomes and Sugden, 1998). As a notable exception, Machina (1985) and Chew et al. (1991) develop a model of stochastic choice as a result of deliberate randomization by individuals with quasi-concave preferences (e.g. Starmer 2000). However, Hey and Carbone (1995) find that randomness in the observed choices between lotteries cannot be attributed to conscious randomization.

To reconcile predominantly deterministic decision theories with stochastic empirical data, a common approach is to embed a core decision theory into a stochastic choice model, for example, when estimating the parameters of a particular decision theory from experimental data. The models of stochastic choice employed for this purpose can be classified into three groups (e.g. Loomes and Sugden, 1998). The simplest stochastic choice model is proposed by Harless and Camerer (1994) who argue that individuals generally choose among lotteries according to some deterministic decision theory, but there is a constant probability that this deterministic choice pattern reverses (as a result of pure tremble). Carbone (1997) and Loomes et al. (2002)

find that this constant error model fails to explain the experimental data and it is essentially "inadequate as a *general* theory of stochastic choice".

Hey and Orme (1994) propose a stochastic choice model where a random error distorts the net advantage of one lottery over another (in terms of utility) according to some deterministic decision theory. Error term is an independently and identically distributed random variable with zero mean and constant variance.[1] Such random error or Fechner model (e.g. Fechner, 1860) can explain several choice anomalies including the common ratio effect (e.g. Loomes, 2005) and the violations of the betweenness (e.g. Blavatskyy, 2006). However, Loomes and Sugden (1998) find that random error model predicts too many violations of first order stochastic dominance. Camerer and Ho (1994) and Wu and Gonzalez (1996) argue that the probability of choosing one lottery over another is simply a logit function of the difference in their utilities (according to the deterministic underlying theory). Luce and Suppes (1965) prove a theorem that such (strong utility) model is in fact equivalent to random error model of Hey and Orme (1994).

Finally, Loomes and Sugden (1995) argue that individual preferences over lotteries are stochastic and can be represented by random utility model. Random utility model can explain a preference reversal phenomenon (e.g. Loomes, 2005) but it cannot rationalize rare violations of stochastic dominance that are observed in the data (e.g. Loomes and Sugden, 1998). Moreover, Sopher and Narramore (2000) find that variation in individual decisions is not systematic in a statistical sense, which strongly supports random error rather than random utility model.

Apparently, different models of stochastic choice that are used for generating stochastic choice pattern from a deterministic core decision theory are successful in explaining some choice anomalies but none of them is suitable for accommodating all known phenomena. To address this problem, I develop a new stochastic decision theory explaining many stylized empirical facts

[1] Hey (1995) and Buschena and Zilberman (2000) assume that error term is heteroscedastic *i.e.* the variance of errors is higher in certain decision problems, for example, when lotteries have many possible outcomes.

as a consequence of random mistakes that individuals make when evaluating a risky lottery. Thus, stochastic component is incorporated as a part of core decision theory, which makes explicit predictions about stochastic choice patterns. The latter are directly accessible for econometric testing on empirical data (without resorting to auxiliary stochastic choice models).

The main results of this paper can be summarized as follows. A new stochastic decision theory postulates that individuals behave as if maximizing the expected utility of risky lotteries but the exact calculation of the expected utility is distorted by random errors. Random errors are symmetrically distributed around zero as long as a lottery is valued at least as good as its lowest possible outcome and at most as good as its highest possible outcome (internality axiom). Errors are heteroskedastic (utility of degenerate lotteries or "sure things" is unaffected by errors) and correlated (errors are highly correlated when one lottery transparently dominates the other).

This stochastic decision theory explains many well-known violations of expected utility theory such as the fourfold pattern of risk attitudes, the discrepancy between certainty equivalent and probability equivalent elicitation methods, the preference reversal phenomenon, generalized common consequence effect (the Allais paradox), the common ratio effect and the violations of the betweenness. The reexamination of well-known experimental studies of repeated choice under risk reveals that the predictive power of new theory is comparable or even superior to that of other prominent descriptive decision theories such as the rank-dependent expected utility theory (e.g. Quiggin, 1981) and cumulative prospect theory (e.g. Tversky and Kahneman, 1992).

The remainder of this paper is organized as follows. The basic idea of new theory is demonstrated in Section II, which considers the simplest possible case of binary choice between one risky and one degenerate lottery. This basic theory is extended in Section III to deal with binary choice between two risky lotteries. The econometric estimation of new theory is presented in Section IV that reexamines three well-known experimental studies. Section V concludes.

II. Binary choice between a risky and a degenerate lottery

Let $L(x_1, p_1; \ldots x_n, p_n)$ denote a discrete lottery that delivers a monetary outcome $x_i \in \mathbf{R}$ with probability p_i, $i \in \{1,\ldots,n\}$. Let x_1 be the lowest possible outcome and let x_n be the highest possible outcome. An individual has deterministic preferences over lotteries that are represented by von Neumann-Morgenstern utility function $u : \mathbf{R} \to \mathbf{R}$. The latter is defined over changes in wealth rather than absolute wealth levels, as first proposed by Markowitz (1952) and advocated by Kahneman and Tversky (1979). The observed binary choices of an individual are, however, stochastic due to random errors that an individual makes when evaluating a risky lottery.

In particular, an individual chooses a lottery L over outcome x for certain if

(1) $$U(L) \geq u(x),$$

where the perceived expected utility of a lottery $U(L)$ is equal to the true expected utility of a lottery $\mu_L = \sum_{i=1}^{n} p_i u(x_i)$ according to individual preferences plus a random error ξ_L. An individual always chooses lottery L over outcome x for certain if condition (1) holds with strict inequality. Thus, an individual behaves as if maximizing the perceived expected utility (2).

(2) $$U(L) = \mu_L + \xi_L .$$

In general, an individual makes mistakes when evaluating a risky lottery, but he or she is unlikely to make transparent and obvious errors, which is captured by the following assumption.

Assumption 1 (internality axiom) An individual always chooses lottery L over outcome x for certain, if outcome x is smaller than x_1 and an individual always chooses outcome x for certain over lottery L, if outcome x is higher than x_n.

Assumption 1 implies that there are no errors in choice under certainty. In other words, if choosing between 20 dollars for sure and 10 dollars for sure, a rational individual always chooses the higher amount. Assumption 1 also restricts the distribution of random errors that distort the expected utility of risky lotteries. Let $F_L : \mathbf{R} \to [0,1]$ be the cumulative distribution function of error ξ_L occurring when an individual calculates the expected utility μ_L of a risky lottery L. Assumption 1 implies that $F_L(u(x) - \mu_L) = 0$ for all $x < x_1$ and $F_L(u(x) - \mu_L) = 1$ for all $x > x_n$.

Let CE_L be the certainty equivalent of a risky lottery L according to the deterministic preferences of an individual i.e. $u(CE_L) = \mu_L$. If there were no errors in the evaluation of lottery L, an individual would always choose lottery L over any outcome $x < CE_L$ for certain, would always choose any outcome $x > CE_L$ for certain over lottery L and would be exactly indifferent between choosing lottery L and amount CE_L for certain. When random errors affect choice, an individual may choose an outcome $x < CE_L$ for certain over lottery L and similarly, by mistake, he or she may choose lottery L over amount $x > CE_L$ for certain. Such mistakes can occur as long as they do not lead to the violation of Assumption 1.

Assumption 2 For any amount $\varepsilon > 0$ and a risky lottery L such that $CE_L \pm \varepsilon \in [x_1, x_n]$ the following events are equally likely to occur:

- lottery L is chosen over outcome $CE_L - \varepsilon$ for certain but not over outcome CE_L for certain
- lottery L is chosen over outcome CE_L for certain but not over outcome $CE_L + \varepsilon$ for certain.

The intuition behind Assumption 2 is the following. Consider an ε-neighborhood of the true certainty equivalent CE_L that does not contain outcomes, which are transparently inferior or superior to L due to Assumption 1. The perceived certainty equivalent[2] of lottery L is equally likely to be below or above CE_L in this ε-neighborhood. Thus, Assumption 2 effectively states that small random errors[3] are symmetrically distributed around zero on the outcome scale. In general, this may not hold for large errors. Assumption 1 prevents an individual from making

[2] That is an outcome whose utility is equal to the perceived expected utility (2) of a lottery.
[3] In this context random errors are considered small if $|\xi_L| \leq \min\{\mu_L - u(x_1), u(x_n) - \mu_L\}$.

transparent large errors but transparent negative errors $\xi_L \geq u(x_1) - \mu_L$ and transparent positive errors $\xi_L \geq u(x_n) - \mu_L$ are not necessarily symmetrically distributed. The remainder of this section shows how a simple stochastic theory (1)-(2) together with Assumptions 1-2 explains well-known empirical phenomena in binary choice between one risky and one degenerate lottery.

A. The fourfold pattern of risk attitudes

The fourfold pattern of risk attitudes refers to an empirical observation that individuals often exhibit risk aversion when dealing with probable gains or improbable losses. The same individuals often exhibit risk seeking when dealing with improbable gains or probable losses (e.g. Tversky and Kahneman, 1992). One of the implications of the fourfold pattern of risk attitudes is that individuals can simultaneously purchase insurance and public lottery tickets. The latter paradoxical observation was the first descriptive challenge for the expected utility theory (e.g. Friedman and Savage, 1948).

Consider a lottery L whose certainty equivalent CE_L according to undistorted individual preferences is closer to the highest possible outcome x_n than to the lowest possible outcome x_1 i.e. $x_n - CE_L < CE_L - x_1$. Tversky and Kahneman (1992) would refer to such lottery either as a probable gain or an improbable loss. Let $EV_L = \sum_{i=1}^{n} p_i x_i$ be the expected value of lottery L and M_L be an outcome such that an individual is equally likely: 1) to choose L over M_L for certain, 2) to choose M_L for certain over L. For lottery L, outcome M_L is smaller or equal to CE_L.[4]

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[4] Proof by contradiction. If $M_L > CE_L$ then probability that lottery L is chosen over outcome M_L for certain is

$$prob(\mu_L + \xi_L \geq u(M_L)) > prob(\mu_L + \xi_L \geq u(CE_L)) = prob(u(2CE_L - x_n) \leq \mu_L + \xi_L \leq u(CE_L)) \leq$$
$$\leq prob(u(x_1) \leq \mu_L + \xi_L \leq u(CE_L)) < prob(u(x_1) \leq \mu_L + \xi_L \leq u(M_L)) = prob(\mu_L + \xi_L \leq u(M_L)),$$

with the first equality due to Assumption 2, the second equality due to Assumption 1, and $2CE_L - x_n > x_1$ due to the property of L. Thus, the probability that lottery L is chosen over outcome M_L for certain is greater than the probability that outcome M_L for certain is chosen over lottery L, which contradicts to the definition of M_L.

If an individual has a concave utility function $u(.)$ then $EV_L \geq CE_L \geq M_L$. In binary choice between lottery L and its expected value EV_L for certain, an individual then is more likely (or equally likely) to choose EV_L for certain. This conclusion also holds for an individual with a convex utility function if $(CE_L \geq)\ EV_L \geq M_L$. Therefore, for probable gains or improbable losses individuals with a concave utility function and some individuals with a convex utility function exhibit a risk averse behavior at least as often as a risk seeking behavior.

Consider now a lottery L with a certainty equivalent CE_L closer to x_1 than to x_n so that $x_n - CE_L > CE_L - x_1$. Such lottery would be either an improbable gain or a probable loss in the terminology of Tversky and Kahneman (1992). For lottery L inequality $M_L \geq CE_L$ holds (proof is identical to the proof in footnote 4 with all inequalities reversed and x_1 and x_n interchanged). If an individual has a convex utility function then $M_L \geq CE_L \geq EV_L$. In binary choice between L and EV_L for certain, such individual is more (or equally) likely to choose L. This also holds for individuals with a concave utility function if $M_L \geq EV_L \quad (\geq CE_L)$. Thus, for improbable gains or probable losses individuals with a convex utility function and some individuals with a concave utility function exhibit a risk seeking behavior at least as often as a risk averse behavior.

To summarize, the fourfold pattern of risk attitudes has a very intuitive explanation. By construction, probable gains and improbable losses are lotteries whose certainty equivalent is close to the highest possible outcome. When evaluating such lotteries, an individual cannot make a large positive error, which would result in lottery being chosen over its highest outcome for certain. Thus, a lot of positive errors are immediately discarded as transparent mistakes but many negative errors are not recognized as obvious mistakes. Random errors are then likely to decrease the true expected utility of a lottery. This increases the chances of observing risk averse behavior for all individuals. For improbable gains or probable losses the situation is exactly reversed.

B. Discrepancy between certainty equivalent and probability equivalent elicitation methods

Certainty equivalent and probability equivalent elicitation methods are two classical procedures for inferring von Neumann-Morgenstern utility function of an individual from the observed binary choices. Unfortunately, two methods yield systematically different results (e.g. Wakker and Deneffe, 1996). Consider a simple lottery $L(x_1, 1/2; x_2, 1/2)$ and let c be a minimum outcome that an individual is willing to accept in exchange for L. Furthermore, let p be the highest probability such that an individual is willing to accept outcome c for certain in exchange for lottery $L'(x_1, 1-p; x_2, p)$. Obviously, any deterministic decision theory predicts that $p = 1/2$. Hershey and Schoemaker (1985) find instead that individuals, who initially reveal $c > (x_1 + x_2)/2$, often also declare $p > 1/2$ one week later (both for positive and negative outcomes).[5]

Outcome c is a minimum outcome which can be chosen over lottery L and it is simply a perceived certainty equivalent of L (see footnote 2). Thus, c is a random variable distributed in the interval $[x_1, x_2]$ with median M_L. Probability p is defined so that an individual chooses outcome c for certain over lottery $L'(x_1, 1-p; x_2, p)$ but not over lottery $L''(x_1, 1-p-\varepsilon; x_2, p+\varepsilon)$ where $\varepsilon > 0$ is arbitrary small. An individual reveals probability equivalent p with probability $F_{L'}(u(c)-\mu_{L'}) \cdot [1 - F_{L''}(u(c)-\mu_{L''})] \cong F_{L'}(u(c)-\mu_{L'}) \cdot [1 - F_{L'}(u(c)-\mu_{L'})]$. Probability equivalent that is most likely to be revealed is characterized by $F_{L'}(u(c)-\mu_{L'}) = 1/2$ or, equivalently, $M_{L'} = c$.

If an individual has a linear utility function, then the certainty equivalent CE_L of lottery L is simply $(x_1 + x_2)/2$. Assumption 2 then implies that $M_L = (x_1 + x_2)/2$ i.e. the perceived certainty equivalent c is symmetrically distributed in the interval $[x_1, x_2]$. If the realization of c is above $(x_1 + x_2)/2$, then an individual is most likely to reveal probability equivalent p such

[5] Figures 1a and 1c in Hershey and Schoemaker (1985) also demonstrate that individuals, who initially reveal $c < (x_1 + x_2)/2$, are slightly more likely to declare $p < 1/2$ one week later.

that $M_{L'} = c > (x_1 + x_2)/2$. This probability equivalent p must be greater than one half because if $p = 1/2$ then $M_{L'} = M_L = (x_1 + x_2)/2$. This conclusion also holds for an individual with a concave or a convex utility function if $M_L \leq (x_1 + x_2)/2$.

Intuitively, discrepancy between certainty equivalent and probability equivalent methods occurs due to error propagation. An individual makes random mistakes when evaluating a risky lottery so that the perceived certainty equivalent of the latter is equally likely to be below or above certain outcome M_L. For a risk neutral individual, outcome M_L is simply the midpoint of the interval $[x_1, x_2]$. Consider an individual who accidentally reveals too high perceived certainty equivalent $c >> M_L$ for a risky lottery and subsequently searches for a probability equivalent of this high outcome c. This individual is most likely to associate the sure outcome c with a lottery whose perceived certainty equivalent is equally probable to be below or above c. The probability of the highest outcome in such lottery is higher than one half. If it were exactly one half, the lottery would coincide with the original risky lottery whose certainty equivalent is equally likely to be below or above outcome M_L, but we know that $c >> M_L$.

Thus, conditional on high realization of a certainty equivalent, the probability equivalent elicited in the second stage is likely to be high as well. Similarly, conditional on low realization of a certainty equivalent, it is most likely that the probability equivalent elicited in the second stage is low as well. The argument also works when the sequence of tasks is reversed. Consider an individual who reveals a probability equivalent p for outcome $(x_1 + x_2)/2$ and then reports the certainty equivalent for lottery $L'(x_1, 1-p; x_2, p)$. He or she is most likely to deduce p from a lottery whose perceived certainty equivalent is equally likely to be below or above $(x_1 + x_2)/2$. If, by mistake, the revealed probability equivalent is too high, the perceived certainty equivalent of lottery L' is more likely to be above $(x_1 + x_2)/2$ (e.g. Hershey and Schoemaker, 1985).

C. The preference reversal phenomenon

The preference reversal phenomenon is a common violation of procedure invariance[6] that is often observed in the experimental studies. The phenomenon involves two lotteries of similar expected value. Lottery R yields a relatively high outcome with a low probability and it is often referred to as a dollar-bet. Lottery S yields a modest outcome with a probability close to one and it is typically called a probability-bet. Many individuals choose S over R in a direct binary choice, but at the same time they reveal that their lowest selling price is higher for R than for S. For instance, Tversky et al. (1990) find that 83% of subjects choose $S(\$0,0.03;\$4,0.97)$ over $R(\$0,0.69;\$16,0.31)$ and 71% of them price lottery R above lottery S.

Binary choice between lotteries R and S is a choice between two risky lotteries, which is the subject of the next section. However, lottery S delivers one outcome with probability close to one. Random errors do not distort the evaluation of degenerate lotteries as a consequence of Assumption 1. For simplicity, we assume that lottery S, which resembles a degenerate lottery, is also unaffected by random errors. Bostic et al. (1990) provide empirical support for such assumption. They find "only negligible differences" between a stated price for S and a certainty equivalent of S elicited from a series of binary choices between a risky and a degenerate lottery.

In a direct binary choice, an individual chooses probability-bet S over dollar-bet R if $\mu_S \geq U(R)$. Therefore, the likelihood that an individual chooses S over R is $F_R(\mu_S - \mu_R)$. An individual is more (or equally) likely to choose a probability-bet over a dollar-bet if $CE_S \geq M_R$.[7] The lowest selling price for lottery S is, naturally, just its certainty equivalent CE_S (e.g. Bostic et al., 1990). However, the theory of binary choice between a degenerate and a risky lottery that

[6] According to the principle of procedure invariance, equivalent methods of eliciting unobserved individual preference should yields identical results.

[7] If an individual makes no mistakes in the evaluation of a probability-bet, the perceived certainty equivalent of S is simply its certainty equivalent CE_S according to deterministic individual preferences.

we developed so far does not offer an immediate interpretation for the price of a risky lottery. One interpretation might be that the revealed price for a dollar-bet is an outcome that is perceived as equivalent in terms of utility to the *average* perceived expected utility of a risky lottery R.

By definition, dollar-bet yields a relatively high outcome with a low probability. Thus, for all individuals, who are not extremely risk-seeking, the certainty equivalent CE_R is located closer to the lowest possible outcome of lottery R. We established already in subsection A that for such lotteries the median M_R of the distribution of the perceived expected utility is greater or equal to CE_R. If the modal error that an individual makes when evaluating a risky lottery is zero, CE_R is the mode of the distribution of the perceived expected utility. A famous “mode, median, mean inequality” (e.g. Groeneveld and Meeden, 1977) then implies that the mean of the distribution of the perceived expected utility of R is greater or equal to the median M_R.

Therefore, it is possible to construct lotteries R and S such that the certainty equivalent CE_S is at least as high as the median M_R and not greater than the mean of the distribution of the perceived expected utility of R. In this case, an individual is more (or equally) likely to choose S over R in a direct binary choice while revealing a minimum selling price for R, which is greater or equal to the minimum selling price for S. Intuitively, such preference reversal phenomenon occurs because an individual makes larger errors (that are likely to cause an overvaluation of a dollar bet) in a pricing task than in a binary choice task.

Interestingly, one can also construct lotteries R' and S' such that the certainty equivalent $CE_{S'}$ is at least as high as the mean and not greater than the median $M_{R'}$ of the distribution of the perceived expected utility of R'.[8] In this case, stochastic decision theory predicts that one may observe a high incidence of reverse preference reversals (choice of R' over S' but a higher revealed price for S'). This implication can be further tested in the laboratory.

[8] S' delivers high outcome with probability close to one and R' yields even higher outcome with large probability.

III. Binary choice between two risky lotteries

A simple stochastic decision theory presented in section II can be easily extended to a more general case when an individual chooses between two risky lotteries. An individual chooses lottery L over lottery L' if the perceived expected utility $\mu_L + \xi_L$ of L is at least as high as the perceived expected utility $\mu_{L'} + \xi_{L'}$ of L'. Let $\xi_{L,L'} = \xi_L - \xi_{L'}$ be the difference in random errors that distort the evaluation of lotteries L and L'. An individual then chooses L over L' if

$$\mu_L + \xi_{L,L'} \geq \mu_{L'} \tag{3}$$

Lottery L is always chosen over lottery L' if condition (3) holds with strict inequality.

Inequality (3) defines the same choice rule as in the Fechner error model (e.g. Hey and Orme, 1994). However, the decision theory presented here differs from the Fechner model in the assumptions that are made about the distribution of an error term. In the Fechner model, an error term is symmetrically distributed. This is not necessarily the case for a random error $\xi_{L,L'}$.

Proposition 1 If Assumption 1 holds, then $prob(\xi_{L,L'} \leq u(x) - u(y) + \mu_{L'} - \mu_L) = 0$ for all $x < \underline{x}$, $y > \overline{y}$, and $prob(\xi_{L,L'} \leq u(x) - u(y) + \mu_{L'} - \mu_L) = 1$ for all $x > \overline{x}$, $y < \underline{y}$, where $\underline{x}$ and $\overline{x}$ ($\underline{y}$ and $\overline{y}$) are the lowest possible and the highest possible outcome of lottery L (lottery L').

All proofs are presented in the appendix

Intuitively, the difference in perceived expected utility of lotteries L and L' cannot be smaller than the difference between the utility of the lowest possible outcome of L and the utility of the highest possible outcome of L'. Similarly, an individual cannot perceive the difference in expected utilities of lotteries L and L' to be greater than the difference between the utility of the highest possible outcome of L and the utility of the lowest possible outcome of L'. Proposition 1 immediately implies that an individual always chooses L over L' if the highest possible outcome of L' is smaller than the lowest possible outcome of L and vice versa.

According to Proposition 1, any realization of a random error $\xi_{L,L'}$, which is lower than $u(\underline{x})-u(\bar{y})+\mu_{L'}-\mu_L$ or greater than $u(\bar{x})-u(\underline{y})+\mu_{L'}-\mu_L$, is immediately recognized and dismissed as a transparent mistake. The set of large negative errors $\{\xi_{L,L'} | \xi_{L,L'} < u(\underline{x})-u(\bar{y})+\mu_{L'}-\mu_L\}$ and the set of large positive errors $\{\xi_{L,L'} | \xi_{L,L'} > u(\bar{x})-u(\underline{y})+\mu_{L'}-\mu_L\}$ may not be symmetric around zero. Therefore, the distribution of error $\xi_{L,L'}$ is not necessarily symmetric. In section II we additionally assumed that small random errors are nevertheless symmetrically distributed around zero on the *outcome* scale (Assumption 2). In the context of binary choice between two risky lotteries it is more convenient to assume that small random errors $\xi_{L,L'}$ are symmetrically distributed around zero on the *utility* scale. Thus, Assumption 2 is replaced with Assumption 2a.

Assumption 2a $prob(-\varepsilon \le \xi_{L,L'} \le 0) = prob(0 \le \xi_{L,L'} \le \varepsilon)$ for any $\varepsilon > 0$ and any lotteries L and L' such that $\varepsilon \le u(\bar{x})-u(\underline{y})+\mu_{L'}-\mu_L$ and $-\varepsilon \ge u(\underline{x})-u(\bar{y})+\mu_{L'}-\mu$.

Assumption 1 implies that there is no error in choice between degenerate lotteries. If an individual makes no errors when choosing between 20 dollars for sure and 10 dollars for sure, it is plausible to assume that the chance of mistake is negligible when these outcomes are delivered with high probability (e.g. 0.999) but not for certain. In choice between such "almost sure things", the dispersion of random errors can be expected to become progressively narrower, the closer are risky lotteries to the degenerate lotteries. This intuition is embedded in the following assumption.

Assumption 3 $\lim_{k\to\infty} prob(|\xi_{L_k,L'_k}| \ge \varepsilon) = 0$ for any $\varepsilon > 0$ and any sequence of risky lotteries $\{L_k(x_1, p_1^k;...x_n, p_n^k)\}_{k=1}^{\infty}$ and $\{L'_k(x_1, q_1^k;...x_n, q_n^k)\}_{k=1}^{\infty}$ such that $p_i^k \xrightarrow{k\to\infty} 1$ and $q_j^k \xrightarrow{k\to\infty} 1$ for some $i, j \in \{1,...,n\}$.

The remainder of this section shows how choice rule (3) together with Assumptions 1, 2a and 3 explains well-known empirical phenomena in binary choice between two risky lotteries.

A. The generalized common consequence effect (the Allais paradox)

The common consequence effect involves four lotteries $S(x_2,1)$, $R(x_1, p-q; x_2, 1-p; x_3, q)$, $S'(x_1, 1-p; x_2, p)$ and $R'(x_1, 1-q; x_3, q)$ with probabilities $0 < q < p << 1$. These lotteries are constructed so that the difference in expected utility between S and R is the same as between S' and R' i.e. $\mu_S - \mu_R = \mu_{S'} - \mu_{R'}$. Expected utility theory predicts that if an individual chooses S over R then he or she should also choose S' over R' and vise versa (unless the individual is exactly indifferent between S and R and between S' and R'). However, empirical evidence shows that individuals often choose S over R and at the same time they choose R' over S' (e.g. Slovic and Tversky, 1974; MacCrimmon and Larsson, 1979). This choice pattern is known as the common consequence effect. The most famous example of this effect is the Allais paradox (e.g. Allais, 1953), which is a special case when $x_1 = \$0, x_2 = \$10^6, x_3 = \$5 \cdot 10^6, p = 0.11$ and $q = 0.1$.

Proposition 2 For any two lotteries L and L' that have equal expected utility ($\mu_L = \mu_{L'}$), an individual chooses L at least as often as L' if $u(\overline{x}) - u(\underline{y}) > u(\overline{y}) - u(\underline{x})$ and an individual chooses L' at least as often as L if $u(\overline{y}) - u(\underline{x}) > u(\overline{x}) - u(\underline{y})$, given Assumptions 1 and 2a.

According to Proposition 2, in binary choice between S and R that have equal expected utility, an individual chooses S more (or equally) often as R if $u(x_2) - u(x_1) > u(x_3) - u(x_2)$ or, equivalently, $u(x_2) > (u(x_3) + u(x_1))/2$. If lotteries S and R have equal expected utility, then S' and R' have also equal expected utility. Thus, according to Proposition 2, an individual chooses R' at least as often as S' if $u(x_3) - u(x_1) > u(x_2) - u(x_1)$, which is always satisfied. Thus, when $u(x_2) > (u(x_3) + u(x_1))/2$ and lotteries S and R have the same expected utility, an individual can choose S more often than R and at the same time he or she can choose R' more often than S'. This conclusion is immediately extendable to lotteries S and R that have similar (but not equal) expected utility, if we assume that errors $\xi_{R,S}$ can realize with strictly positive probability in any interval within the bounds of their admissible values, which are defined in Proposition 1.

The intuition behind common consequence effect is the following. A degenerate lottery S is not affected by random errors but the evaluation of a risky lottery R is affected by errors. The random errors are likely to decrease the perceived expected utility of R if its certainty equivalent is closer to its highest possible outcome x_3 (see section II).[9] Thus, an individual can choose S more often because random errors decrease the attractiveness of R. In a binary choice between S' and R', random errors are likely to overvalue the perceived expected utility of both lotteries. This effect is stronger for lottery R' because it has a wider range of possible outcomes.[10] Thus, an individual can choose R' more often than S' because random errors increase the perceived advantage of R' over S'.

According to the above explanation, the incidence of the common consequence effect diminishes if S becomes a risky lottery with the same range of possible outcomes as R, and S' has the same range of possible outcomes as R'. In this case, random errors are likely to decrease (increase) the perceived expected utility of S and R (S' and R'), and the strength of this effect is similar for both lotteries. Conlisk (1989) and Camerer (1992) find experimental evidence confirming this prediction: when four lotteries employed in the common consequence effect are all located inside the probability triangle (e.g. Marschak, 1950; Machina, 1982), the common consequence effect largely disappears.

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[9] If lotteries S and R have equal expected utilities, then the certainty equivalent of R is simply x_2. Thus, we end up with a condition that x_2 is closer to the highest possible outcome x_3 than to the lowest possible outcome x_1 of lottery R. If we use Assumption 2a instead of Assumption 2, we have a condition that $u(x_2)$ is closer to $u(x_3)$ than to $u(x_1)$, which is equivalent to $u(x_2) > (u(x_3) + u(x_1))/2$.

[10] Lotteries S' and R' are constructed so that the highest possible outcome of R' is greater than that of S' (both lotteries have the same lowest possible outcome x_1). If lotteries S' and R' have identical expected utility, random errors $\xi_{R',S'}$ are more likely to be positive rather than negative as a result of Proposition 1 and Assumption 2a.

B. The common ratio effect

The common ratio effect involves a binary choice between lottery $S(x_2,1)$ and lottery $R(x_1,1-\theta;x_3,\theta)$ and a binary choice between lottery $S'(x_1,1-r;x_2,r)$ and lottery $R'(x_1,1-\theta\cdot r;x_3,\theta\cdot r)$ with probabilities $0<r<<\theta<1$. Four lotteries are constructed so that $\mu_{S'}-\mu_{R'}=r(\mu_S-\mu_R)$. Therefore, expected utility theory predicts that if an individual chooses S over R, then he or she should also choose S' over R' (unless he or she is exactly indifferent in both choice situations). The common ratio effect refers to the empirical finding that individuals often choose S over R and at the same time they choose R' over S' (e.g. Bernasconi, 1994; Loomes and Sugden, 1998).

We already established that an individual may choose a degenerate lottery S more often than a risky lottery R if both lotteries are sufficiently similar in expected utility; random errors can realize in any interval within admissible bounds; and $u(x_2)>(u(x_3)+u(x_1))/2$. Intuitively, lottery S can be chosen more frequently, even if it has a lower utility, because random errors do not distort the utility of S but they are likely to decrease the perceived expected utility of R.

When probability $r\to 0$, then both lottery S' and lottery R' converge to a degenerate lottery that yields outcome x_1 for certain. Lottery R' has a higher expected utility than lottery S' if $(1-\theta)\cdot u(x_1)+\theta\cdot u(x_3)>u(x_2)$. In this case $\lim_{r\to 0} prob\left(\left|\xi_{S',R'}\right|\geq \mu_{R'}-\mu_{S'}\right)=0$ according to Assumption 3. Thus, it is possible to find probability r, which is close enough to zero, so that an individual chooses lottery S' with arbitrary small probability.

Intuitively, for small r, lottery R' may be chosen with very high probability because the distorting effect of random errors diminishes when lotteries S' and R' converge to a degenerate lottery. In such case, the chances are high that an individual simply chooses the lottery with a higher expected utility, which can be lottery R'. Thus, we can explain not only the common ratio effect but also a modal choice of R' over S' frequently found in the data (e.g. Loomes, 2005).

C. The violations of the betweenness

Consider two risky lotteries S and R, none of which dominates the other. A risky lottery $M = \alpha S + (1-\alpha)R$, $\alpha \in (0,1)$ is a probability mixture of S and R. When an individual chooses S over R and at the same time chooses M over S, he or she reveals quasi-concave preferences or preference for randomization (e.g. Starmer, 2000). When an individual chooses R over S and at the same time chooses S over M, the individual reveals quasi-convex preferences or aversion to randomization (e.g. Starmer, 2000). There exist lotteries S and R such that individuals reveal quasi-concave preferences more often than quasi-convex preferences and vice versa (e.g. Prelec, 1990; Gigliotti and Sopher, 1993). This finding is known as the violations of the betweenness.[11]

Since M is a probability mixture of S and R, its expected utility is $\mu_M = \alpha\mu_S + (1-\alpha)\mu_R$. According to the choice rule (3), the likelihood of observing quasi-concave preferences is simply $prob(\xi_{R,S} \le \mu_S - \mu_R) \cdot prob(\xi_{M,S} \ge (1-\alpha)(\mu_S - \mu_R))$ and the chances of observing quasi-convex preferences are $prob(\xi_{R,S} \ge \mu_S - \mu_R) \cdot prob(\xi_{M,S} \le (1-\alpha)(\mu_S - \mu_R))$. Thus, an individual reveals quasi-concave preferences more often if $prob(\xi_{R,S} \le \mu_S - \mu_R) \ge prob(\xi_{M,S} \le (1-\alpha)(\mu_S - \mu_R))$ and $prob(\xi_{M,S} \ge (1-\alpha)(\mu_S - \mu_R)) \ge prob(\xi_{R,S} \ge \mu_S - \mu_R)$. In the simplest case when errors $\xi_{R,S}$ and $\xi_{M,S}$ are identically distributed, these inequalities always hold when $\alpha < 1$ and $\mu_S > \mu_R$. Similarly, an individual reveals quasi-convex preferences more often if $\mu_S < \mu_R$.

Intuitively, when S has a higher expected utility than R, random errors are more likely to reverse the choice of S over M than the choice of S over R because M is located between S and R in terms of expected utility. For example, "intermediate" errors higher than $\mu_S - \mu_M$ and lower than $\mu_S - \mu_R$ would result in the choice of M over S, but they do not reverse the choice of S over R. Such random errors cause a higher incidence of quasi-concave preferences.

[11] The betweenness axiom is a weaker version of the independence axiom and it states that if an individual is indifferent between two lotteries, then a probability mixture of these lotteries is equally good (e.g. Dekel, 1986).

IV. Reexamination of experimental data

A. Parametric cumulative distribution function of random errors

Having demonstrated that the choice rule (3) together with Assumptions 1, 2a and 3 can theoretically explain many well-known violations of expected utility theory, the next obvious step is to estimate this stochastic decision theory on experimental data and compare its fit with other prominent decision theories. Direct non-parametric estimation of the distribution of an error $\xi_{L,L'}$ for a pair of lotteries L and L' appears to be problematic, because the experimenter cannot ask the subjects to choose between L and L' many times within a short period. Thus, for empirical estimation, we need to assume a parametric cumulative distribution function of random errors.

A natural candidate is that random errors are drawn from a normal distribution (e.g. Hey and Orme, 1994). However, normal distribution does not satisfy Assumptions 1 and 3. Moreover, Ballinger and Wilcox (1997) find that the normal distribution is "soundly rejected" by their data. Last but not least, normally distributed random errors may induce a high likelihood of violations of transparent stochastic dominance (e.g. Loomes and Sugden, 1998). To address these problems, the normal cumulative distribution function is used with the following modifications.

To satisfy Assumption 1, random errors are drawn from a *truncated* normal distribution. Specifically, lottery L is chosen over lottery L' with probability

$$(4) \qquad prob\left(\xi_{L,L'} \geq \mu_{L'} - \mu_L\right) = \frac{\Phi\left(u(\overline{x}) - u(\underline{y}) + \mu_{L'} - \mu_L\right) - \Phi\left(\mu_{L'} - \mu_L\right)}{\Phi\left(u(\overline{x}) - u(\underline{y}) + \mu_{L'} - \mu_L\right) - \Phi\left(u(\underline{x}) - u(\overline{y}) + \mu_{L'} - \mu_L\right)}$$

where $\underline{x}$ and $\overline{x}$ ($\underline{y}$ and $\overline{y}$) are the lowest and the highest possible outcome of lottery L (lottery L') and $\Phi(.)$ is cumulative distribution function of the normal distribution with zero mean and a standard deviation $\sigma_{L,L'}$. Obviously, equation (4) satisfies both Proposition 1 and Assumption 2a.

When an individual chooses between two risky lotteries and one lottery transparently dominates the other, the individual almost always chooses the dominant alternative. For instance, Carbone and Hey (1995) find only one violation of transparent dominance in 320 decisions (rate of violation 0.3%). Loomes and Sugden (1998) discover 12 violations of transparent dominance in 920 choice decisions (rate of violation 1.3%). Hey (2001) reports 24 violations of transparent dominance in 1590 choice decisions (rate of violation 1.5%). This empirical evidence suggests that the variance of errors $\sigma^2_{L,L'}$ is quite small when one lottery clearly dominates the other.

The notion of "transparent dominance" is not clearly defined in the literature. Intuitively, one lottery transparently dominates the other if it yields small outcomes with lower probability and large outcomes with higher probability. Apparently, transparent dominance is not equivalent to the first-order stochastic dominance. Tversky and Kahneman (1986) and Birnbaum (2004) provide experimental evidence that the first-order stochastic dominance can be frequently violated (with rates of violation up to 80%). Transparent dominance as defined in Definition 1 is equivalent to the first-order stochastic dominance if $n = 3$ but it is a stronger relation if $n \geq 4$.[12]

Definition 1 Lottery $L(x_1, p_1;...x_n, p_n)$ *transparently dominates* lottery $L'(x_1, q_1;...x_n, q_n)$ if there exist $i \in \{1,...,n\}$ such that $p_j \leq q_j$ for all $j \in \{1,...,i\}$ and $p_j \geq q_j$ for all $j \in \{i+1,...,n\}$.

If lottery L transparently dominates lottery L' or vice versa, a standard deviation $\sigma_{L,L'}$ is equal to a constant σ_D, which is likely to be (close to) zero for many individuals. When neither lottery transparently dominates the other, standard deviation $\sigma_{L,L'}$ is not necessarily small though it converges to zero if lotteries L and L' converge to degenerate lotteries (in order to satisfy Assumption 3). All these properties can be captured by the following formula:

(5) $$\sigma_{L,L'} = \begin{cases} \sigma_D, & L \text{ transparently dominates } L', \text{or vice versa} \\ \sigma \cdot \left(\prod_{i=1}^{n} (1 - p_i) + \prod_{i=1}^{n} (1 - q_i) \right), & \text{otherwise} \end{cases}$$

[12] Tversky and Kahneman (1986) and Birnbaum (2004) find frequent violations of the first-order stochastic dominance (when it is not transparent dominance) using lotteries with $n = 4$ and $n = 5$; whereas Carbone and Hey (1995), Loomes and Sugden (1998) and Hey (2001) find only few violations using lotteries with $n = 3$.

where σ and $\sigma_D << \sigma$ are individual specific parameters and p_i (q_i) is the probability of outcome x_i in lottery L (lottery L'), $i \in \{1,...,n\}$. Equations (4) and (5) complete the description of a parametric form of stochastic decision theory, which adds only two additional parameters (σ and σ_D) compared to the expected utility theory. Obviously, the expected utility theory is a special (limiting) case of the stochastic decision theory (4)-(5) when parameters σ and σ_D both converge to zero. In this case, probability (5) that an individual chooses lottery L over lottery L' converges to one if $\mu_L > \mu_{L'}$ and it converges to zero when $\mu_{L'} > \mu_L$.

B. Experimental data and estimation procedure

Stochastic decision theory (4)-(5) is estimated on experimental data of Hey and Orme (1994) and Loomes and Sugden (1998).[13] These datasets were already extensively reexamined for estimating various models of stochastic choice because they contain a sufficiently large number of observations per subject.[14] In the experiment of Hey and Orme (1994), 80 subjects are presented with 100 binary choice questions involving lotteries with outcomes £0, £10, £20 and £30. In every question, none of the lotteries transparently dominates the other alternative. At least three days after the initial experiment, the subjects repeat all 100 choice decisions again in a different randomized order. In the experiment of Loomes and Sugden (1998), 92 individuals face 45 binary choice questions involving lotteries with outcomes £0, £10 and either £20 or £30. Five questions are constructed so that one lottery transparently dominates the other lottery. Each of 45 questions is repeated two times within one experimental session.

The estimation of stochastic decision theory is conducted separately for every individual. Von Neumann-Morgenstern utility function $u(.)$ can be normalized for two arbitrary outcomes

[13] John Hey generously provided the data from Hey and Orme (1994). The data from Loomes and Sugden (1998) are reprinted in full detail in Loomes et al. (2002).
[14] Hey (1995), Hey and Carbone (1995), Carbone and Hey (2000), Buschena and Zilberman (2000) reexamined the data of Hey and Orme (1994). Loomes et al. (2002) reexamined the data of Loomes and Sugden (1998).

and normalization $u(£0)=0$ and $u(£10)=1$ is chosen for every subject in both datasets. Thus, every individual from Hey and Orme (1994) study is characterized by a triple of parameters $\{\sigma, u(£20), u(£30)\}$ and every subject from Loomes and Sugden (1998) study is characterized by a triple of parameters $\{\sigma, \sigma_D, u(£20)\}$ or $\{\sigma, \sigma_D, u(£30)\}$. These individual specific parameters are estimated by maximizing log-likelihood function[15]

$$(6) \qquad LL = \sum_{j=1}^{N} \begin{pmatrix} \ln prob\left(\xi_{L_j,L'_j} \geq \mu_{L'_j} - \mu_{L_J}\right) \cdot I_1(c_j) + \ln\left[1 - prob\left(\xi_{L_j,L'_j} \geq \mu_{L'_j} - \mu_{L_J}\right)\right] \cdot I_3(c_j) + \\ \frac{1}{2}\left(\ln prob\left(\xi_{L_j,L'_j} \geq \mu_{L'_j} - \mu_{L_J}\right) + \ln\left[1 - prob\left(\xi_{L_j,L'_j} \geq \mu_{L'_j} - \mu_{L_J}\right)\right]\right) \cdot I_2(c_j) \end{pmatrix}$$

where $c_j = 1$ if an individual chooses lottery L_j over lottery L'_j, $c_j = 2$ if an individual declares that he or she does not care which lottery L_j or L'_j to choose, and $c_j = 3$ if an individual chooses L'_j over L_j in question $j \in \{1,...,N\}$; $I_x(.)$ is an indicator function, and the number of questions N is 200 for Hey and Orme (1994) and 90 for Loomes and Sugden (1998) dataset. Probability $prob\left(\xi_{L_j,L'_j} \geq \mu_{L'_j} - \mu_{L_J}\right)$ is calculated according to formula (4) with standard deviation (5).

For every subject in both datasets, the fit of stochastic decision theory is compared with the fit of rank-dependent expected utility theory, or RDEU, (e.g. Quiggin, 1981) which coincides with cumulative prospect theory (e.g. Tversky and Kahneman, 1992), when lotteries involve only positive outcomes. Loomes et al. (2002) argue that RDEU is a good representative non-expected utility theory for contesting against stochastic models based on the expected utility theory. The estimates of RDEU parameters are obtained by maximizing log-likelihood function (6) given that $prob\left(\xi_{L_j,L'_j} \geq \mu_{L'_j} - \mu_{L_J}\right) = 1 - \Phi\left(\mu_{L'_j}^{RDEU} - \mu_{L_j}^{RDEU}\right)$, where $\Phi(.)$ is a cumulative distribution function of the normal distribution with zero mean and constant standard deviation s (estimated separately for every individual), and μ_L^{RDEU} is utility of lottery L according to RDEU (e.g. Quiggin, 1981).[16]

[15] Non-linear optimization was implemented in the *Matlab* 6.5 package (based on the Nelder-Mead simplex algorithm). The program files are available from the author on request.

[16] Normalization $u(£0)=0$ and $u(£10)=1$ is used for RDEU in both datasets.

C. Results

Table 1 shows mean, median and standard deviation of the estimated parameters of the stochastic decision theory and RDEU across all subjects in Hey and Orme (1994) dataset and Loomes and Sugden (1998) dataset.[17] Notably, the estimates of "noise" parameters σ and σ_D in stochastic decision theory and the standard deviation s in RDEU embedded into the Fechner error model are relatively low. Thus, although individuals make stochastic choices, there appears to be strong consistency in their decisions (e.g. Hey, 2001). Table 2 shows that an overwhelming majority of subjects have concave utility function but in Loomes and Sugden (1998) dataset nearly every second subject has an S-shaped probability weighting function, which contradicts to the theoretical foundations of RDEU (e.g. Tversky and Kahneman, 1992).

Stochastic decision theory presented in this paper and RDEU embedded in the Fechner error model are non-nested models that can be compared by means of Vuong's likelihood ratio test (e.g. Vuong, 1989).[18] Vuong's statistic z has a limiting standard normal distribution if two theories make equally good predictions. A significant positive value of z indicates that stochastic decision theory explains better the choice decisions of an individual and a significant negative value—that RDEU embedded in a simple Fechner error model makes a more accurate prediction.

Figure 1 and Figure 2 show the distribution of Vuong's likelihood ratio statistic across all subjects in Hey and Orme (1994) dataset and Loomes and Sugden (1998) dataset.[19] In Hey and Orme (1994) dataset RDEU has more parameters than the stochastic decision theory (because there are no questions with transparent dominance). Therefore, Vuong's likelihood ratio statistic

[17] In Loomes and Sugden (1998) dataset, half of the subjects faced lotteries with outcome £20 and half of the subjects faced lotteries with outcome £30. Therefore, aggregate statistics for parameters $u(£20)$ and $u(£30)$ in Loomes and Sugden (1998) dataset are calculated using only half of the sample.

[18] Loomes et al. (2002, p.128) describe the application of Vuong's non-nested likelihood ratio test to the selection between different stochastic choice models.

[19] Six subjects in Loomes and Sugden (1998) dataset turned out to be perfect expected utility maximizes. Since the expected utility theory is a special case of both stochastic decision theory ($\sigma, \sigma_D \to 0$) and RDEU ($\gamma = 1, s \to 0$), Vuong's likelihood ratio cannot be calculated in this case. Thus, these six subjects were excluded from Figure 2.

in Figure 1 is adjusted using Akaike and Schwarz information criteria. Figure 1 and Figure 2 show that for the majority of individuals the prediction of new stochastic decision theory is not significantly different from the prediction of RDEU embedded into Fechner model. Additionally, there appears to be more individuals (especially in Loomes and Sugden (1998) dataset) whose choice decisions are better predicted by stochastic decision theory rather than RDEU embedded into Fechner model. These results are consistent with conclusions of Buschena and Zilberman (2000) who reexamined Hey and Orme (1994) dataset. They find that the expected utility theory embedded into a stochastic choice model with heteroscedastic error performs at least as good as non-expected utility theories embedded into choice models with homoscedastic errors.

Parameters	**Hey and Orme (1994) dataset**			**Loomes and Sugden (1998) dataset**		
Stochastic decision theory	Mean	Median	Standard deviation	Mean	Median	Standard deviation
$u(£20)$	1.276	1.204	0.239	1.270	1.234	0.241
$u(£30)$	1.586	1.398	0.608	1.484	1.455	0.367
σ	0.081	0.038	0.230	0.074	0.053	0.101
σ_D				0.007	0.000	0.022
RDEU						
$u(£20)$	1.322	1.279	0.240	1.304	1.291	0.239
$u(£30)$	1.641	1.527	0.500	1.727	1.533	1.172
γ [20]	0.944	0.898	0.256	0.961	0.985	0.218
s	0.083	0.067	0.052	0.104	0.076	0.130

Table 1 Maximum likelihood estimates of parameters of stochastic decision theory and RDEU (given normalization $u(£0)=0$ and $u(£10)=1$)

Property	**Hey and Orme (1994)**		**Loomes and Sugden (1998)**	
	Stochastic decision theory	*RDEU*	*Stochastic decision theory*	*RDEU*
Convex utility function	2	1	1	2
S-shaped probability weighting function		19		44

Table 2 Number of subjects for whom the maximum likelihood estimates of parameters of stochastic decision theory and RDEU satisfy convexity/concavity properties

[20] Parameter γ is a power coefficient of the probability weighting function $w(p) = p^\gamma / \left(p^\gamma + (1-p)^\gamma\right)^{1/\gamma}$.

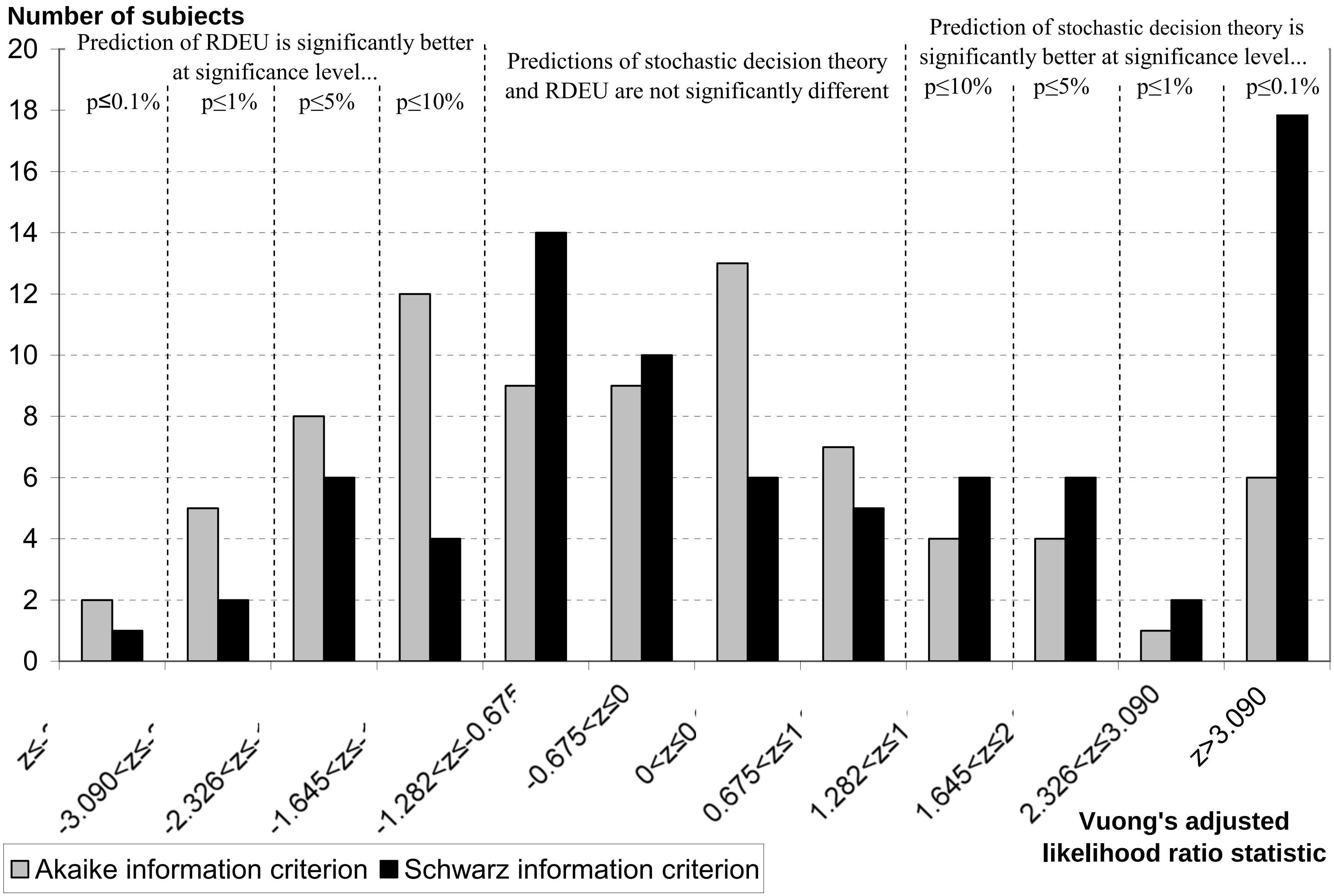

Figure 1 Comparison of stochastic decision theory vs. RDEU for subjects in Hey and Orme (1994) experiment

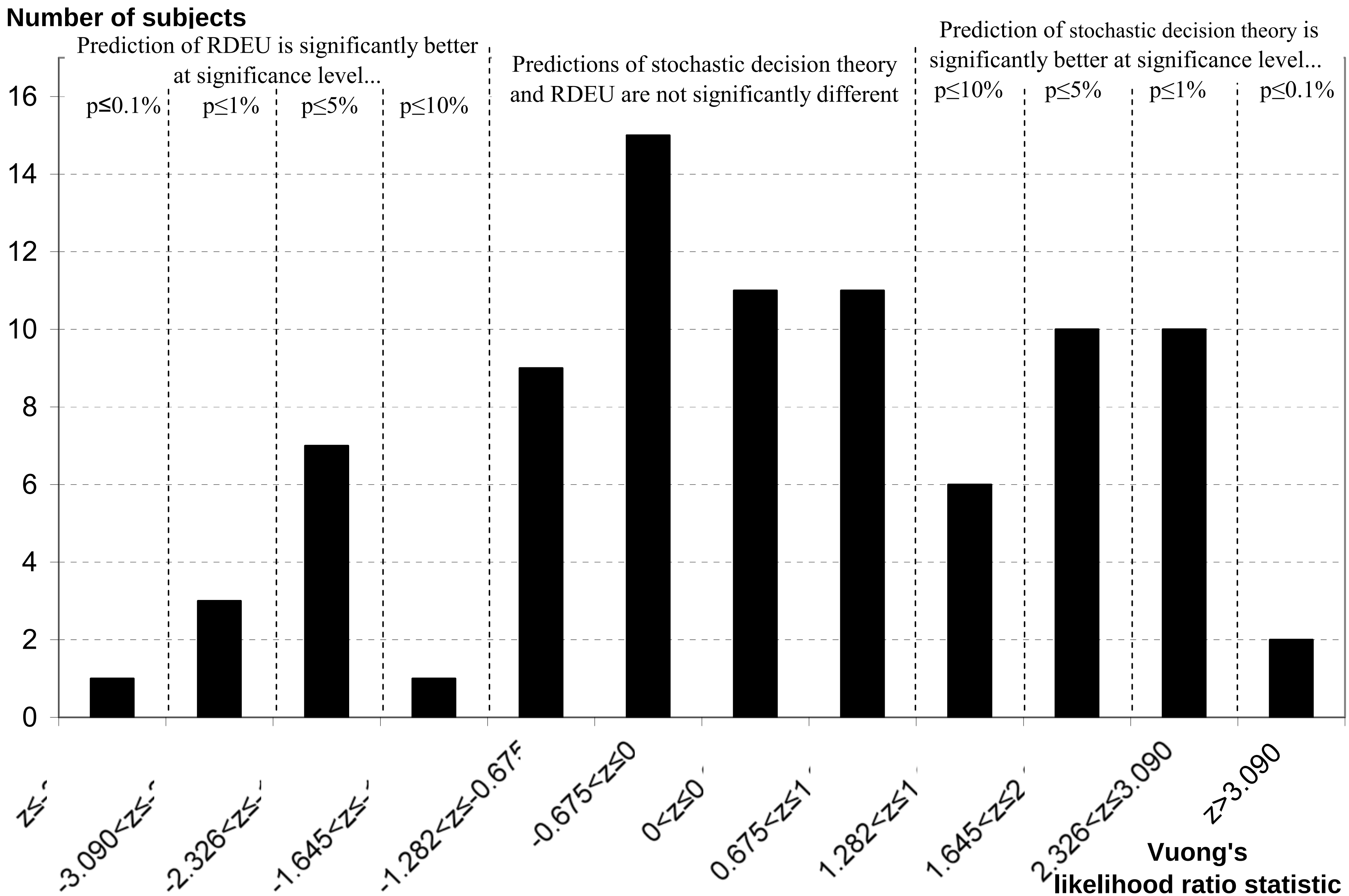

Figure 2 Comparison of stochastic decision theory vs. RDEU for subjects in Loomes and Sugden (1998) experiment

V. Conclusion

A vast experimental literature shows that individuals often make inconsistent decisions in a repeated choice under risk even when they are allowed to express indifference and the same binary choice problem is repeated within a short period of time. Variation in individual decisions appears to be nonsystematic, which supports the interpretation that stochastic choice under risk is a result of random mistakes rather than a reflection of stochastic preferences (random utility). In the existing models of stochastic choice, random errors are typically modeled as symmetrically distributed around zero so that an individual does not make systematic mistakes.

In this paper, a decision maker is modeled as an individual who maximizes his or her expected utility but makes random errors when evaluating a risky lottery. However, an individual does not make transparent and obvious mistakes. For example, an individual never chooses a risky lottery over its highest possible outcome for certain (or the lowest possible outcome for sure over a risky lottery). Thus, a minimum degree of rationality is imposed on the behavior of an individual who is allowed to make random errors as long as they do not lead to transparently irrational decisions. This restriction distinguishes the present paper from the numerous models of stochastic choice that were already proposed in the literature.

The main contribution of this paper is a demonstration that many well-known violations of expected utility theory (such as the fourfold pattern of risk attitudes, the common consequence effect, the violations of the betweenness etc.) can be the result of random errors. Several empirical phenomena analyzed in this paper were already explained by the existing error models. For example, Fechner error model can explain a more frequent choice of a riskier lottery in the common ratio effect but it cannot explain the switch to the modal choice of a riskier alternative. This switch is explained in this paper due to an assumption that the dispersion of random errors converges to zero when lotteries become similar to the degenerate lotteries.

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Appendix

Proof of Proposition 1

On the one hand, Assumption 1 implies $prob\left(\xi_L \le u(x) - \mu_L\right) = 0$, for all $x < \underline{x}$, and $prob\left(\xi_{L'} \le u(y) - \mu_{L'}\right) = 1$, for all $y > \overline{y}$. Since $\mu_{L'} + \xi_{L'} - u(y) \le 0$ for any realization of $\xi_{L'}$ and any $y > \overline{y}$ we can write $prob\left(\xi_{L,L'} \le u(x) - u(y) + \mu_{L'} - \mu_L\right) = prob\left(\xi_L \le u(x) - \mu_L + \mu_{L'} + \xi_{L'} - u(y)\right) \le$ $\le prob\left(\xi_L \le u(x) - \mu_L\right) = 0$. Thus, $prob\left(\xi_{L,L'} \le u(x) - u(y) + \mu_{L'} - \mu_L\right) = 0$ for all $x < \underline{x}$ and $y > \overline{y}$.

On the other hand, Assumption 1 implies that $prob\left(\xi_L \le u(x) - \mu_L\right) = 1$, for all $x > \overline{x}$, and $prob\left(\xi_{L'} \le u(y) - \mu_{L'}\right) = 0$ or $prob\left(\xi_{L'} > u(y) - \mu_{L'}\right) = 1$, for all $y < \underline{y}$. Since $\mu_L + \xi_L - u(x)$ is always smaller or equal to zero for any realization of ξ_L and any $x > \overline{x}$, it is possible to write $prob\left(\xi_{L,L'} \le u(x) - u(y) + \mu_{L'} - \mu_L\right) = prob\left(\xi_{L'} \ge u(y) - \mu_{L'} + \mu_L + \xi_L - u(x)\right) \ge prob\left(\xi_{L'} \ge u(y) - \mu_{L'}\right) \ge$ $\ge prob\left(\xi_{L'} > u(y) - \mu_{L'}\right) = 1$. $\Rightarrow$ $prob\left(\xi_{L,L'} \le u(x) - u(y) + \mu_{L'} - \mu_L\right) = 1$ for all $x > \overline{x}, y < \underline{y}$. *Q.E.D.*

Proof of Proposition 2

According to a choice rule (3), if $\mu_L = \mu_{L'}$, then lottery L is chosen over lottery L' with probability $prob\left(\xi_{L,L'} \ge 0\right)$ and L' is chosen over L with probability $prob\left(\xi_{L,L'} \le 0\right)$. In case when $u(\overline{x}) - u(\underline{y}) > u(\overline{y}) - u(\underline{x})$, we can write $prob\left(\xi_{L,L'} \ge 0\right) = prob\left(0 \le \xi_{L,L'} \le u(\overline{x}) - u(\underline{y})\right) =$ $= prob\left(0 \le \xi_{L,L'} \le u(\overline{y}) - u(\underline{x})\right) + prob\left(u(\overline{y}) - u(\underline{x}) < \xi_{L,L'} \le u(\overline{x}) - u(\underline{y})\right) \le prob\left(0 \le \xi_{L,L'} \le u(\overline{y}) - u(\underline{x})\right) =$ $= prob\left(u(\underline{x}) - u(\overline{y}) \le \xi_{L,L'} \le 0\right) = prob\left(\xi_{L,L'} \le 0\right)$, with the first and last equality due to Proposition 1. Thus, L is chosen at least as often as L'. Similarly, when $u(\overline{y}) - u(\underline{x}) > u(\overline{x}) - u(\underline{y})$, one can write $prob\left(\xi_{L,L'} \le 0\right) = prob\left(u(\underline{x}) - u(\overline{y}) \le \xi_{L,L'} \le 0\right) = prob\left(u(\underline{x}) - u(\overline{y}) \le \xi_{L,L'} < u(\underline{y}) - u(\overline{x})\right) +$ $+ prob\left(u(\underline{y}) - u(\overline{x}) \le \xi_{L,L'} \le 0\right) \le prob\left(u(\underline{y}) - u(\overline{x}) \le \xi_{L,L'} \le 0\right) = prob\left(0 \le \xi_{L,L'} \le u(\overline{x}) - u(\underline{y})\right) =$ $= prob\left(\xi_{L,L'} \ge 0\right)$. In this case lottery L' is chosen at least as often as lottery L. *Q.E.D.*