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Abstract: Loss aversion is traditionally defined in the context of lotteries over monetary payoffs. This paper extends the notion of loss aversion to a more general setup where outcomes (consequences) may not be measurable in monetary terms and people may have fuzzy preferences over lotteries, i.e. they may choose in a probabilistic manner. The implications of loss aversion are discussed for expected utility theory and rank-dependent utility theory as well as for popular models of probabilistic choice such as the constant error/tremble model and a strong utility model (that includes the Fechner model of random errors and Luce choice model as special cases).

Keywords: loss aversion, more loss averse than, nonmonetary outcomes, probabilistic choice, rank-dependent utility theory

JEL classification codes: D00, D80, D81

Loss Aversion

Loss aversion is one of the most important concepts in behavioral economics (Camerer, 2008). It is consistent with a wide range of empirical findings such as the endowment effect (Thaler, 1980; Kahneman et al., 1990), status quo bias (Samuelson and Zeckhauser, 1988), equity premium puzzle (Benartzi and Thaler, 1995), labor supply of cabdrivers (Camerer et al. 1997), disposition effects in condominium sales (Genesove and Mayer, 2001) and animal behavior (Chen et al. 2006) to name a few.

Loss aversion is traditionally defined in the context of lotteries over monetary payoffs (Kahneman and Tversky, 1979; Köbberling and Wakker, 2005; Schmidt and Zank, 2005). However, people often incur losses that may not be measurable in monetary terms (e.g. loss of a close friend or a relative, loss of faith, reputation or prestige, loss of a sports title, loss of animal species etc). This paper extends the notion of loss aversion to decision problems where outcomes (consequences) may not be measurable in monetary terms.

Numerous experimental studies demonstrate that people generally have fuzzy preferences over lotteries, i.e. they choose in a probabilistic manner (e.g. Camerer, 1989; Hey and Orme, 1994; Loomes and Sugden, 1998). Therefore, this paper also extends the notion of loss aversion to allow for the possibility of fuzzy preferences.

The paper is organized as follows. Section 1 defines comparative loss aversion in the context of an arbitrary outcome set. Section 2 considers the implications of this definition for expected utility theory (von Neumann and Morgenstern, 1944) and rank-dependent utility theory (Quiggin, 1981). Section 3 extends the notion of loss aversion to a more general setup where people have fuzzy preferences over lotteries. Section 4 discusses probabilistic loss aversion in the context of different models of probabilistic choice. Section 5 defines absolute loss aversion. Section 6 concludes.

1. Comparative Loss Aversion

Let X denote a finite set of outcomes (consequences) that contains at least two elements. We will treat X as an arbitrary abstract set, which is not necessarily a subset of Euclidean space $\mathbb{R}^n$. Let $X_- \subset X$ be a nonempty proper subset of X. The elements $x_- \in X_-$ are called losses and they can be, for example, "loss of \$100", "loss of a key chain", "loss of faith", "loss of virginity" etc. Let $X_+ \equiv X \setminus X_-$ denote the complement of X_-. The elements $x_+ \in X_+$ are called gains and they can be, for example, "gain of \$200", "gain in experience", "weight gain" etc.

A lottery L: $X \to [0,1]$ is a probability distribution on X, i.e. it delivers an outcome $x \in X$ with a probability $L(x) \in [0,1]$ and $\sum_{x \in X} L(x)=1$. The set of all lotteries is denoted by $\mathcal{L}$. Let $L_+ : X \to [0,1]$ denote a loss-free lottery that yields only gains with a positive probability i.e. $\sum_{x_+ \in X_+} L_+(x_+) = 1$ and $L_+(x_-)=0$ for any $x_- \in X_-$. Let $\mathcal{L}_+ \subset \mathcal{L}$ be the set of all such loss-free lotteries.

In this and the next section we consider a "traditional" decision maker who has a unique binary preference relation $\succsim$ on $\mathcal{L}$. As customary, we will use the sign $\succ$ to denote the asymmetric component of $\succsim$, and the sign $\sim$ to denote the symmetric component of $\succsim$. We will consider two individuals: an individual ♀ characterized by a preference relation $\succsim_{♀}$ and an individual ♂ characterized by a preference relation $\succsim_{♂}$.

Definition 1 An individual ♀ is strictly *more loss averse than* an individual ♂ if

a) $L_+ \succ_{♂} L$ implies $L_+ \succ_{♀} L$ for all $L_+ \in \mathcal{L}_+$ and all $L \in \mathcal{L}$;

b) $L_+ \sim_{♂} L$ implies $L_+ \succsim_{♀} L$ for all $L_+ \in \mathcal{L}_+$ and all $L \in \mathcal{L}$;

c) there exist $L_+ \in \mathcal{L}_+$ and $L \in \mathcal{L}$ such that $L_+ \sim_{♂} L$ and $L_+ \succ_{♀} L$.

According to Definition 1, a more loss averse individual strictly prefers a loss-free lottery over another lottery whenever a less loss averse individual does so as well. In addition, a more loss averse individual weakly prefers a loss-free lottery over another lottery whenever a less loss averse individual is exactly indifferent between the two. This definition of the more-loss-averse-than relation between individuals is quite general. Specifically, Definition 1 does not require that lottery outcomes are measurable in real numbers. It also does not require that individual preferences are represented by a specific decision theory (e.g. prospect theory). In particular, comparative loss aversion is defined in terms of observable preferences and not as a property of an unobservable function (e.g. a value function in prospect theory) that represents these preferences.

If an individual ♀ is more *loss* averse than an individual ♂, this does not imply that an individual ♀ is also more *risk* averse than an individual ♂. Specifically, it is possible that a less loss averse individual ♂ strictly prefers a sure loss of $x_{-}\in X_{-}$ over a lottery $L\in\mathcal{L}$ and at the same time a more loss averse individual ♀ strictly prefers L over a degenerate (risk-free) lottery that yields x_{-} for sure.[1] This implies that an individual ♀ is not always more risk averse than an individual ♂ (e.g. Blavatskyy, 2008a). Similarly, a more risk averse individual is not necessarily a more loss averse individual as well.

Proposition 1 If an individual ♀ is more loss averse than an individual ♂, or vice versa, then

a) $L_{+}\succ_{♀} M_{+}$ if and only if $L_{+}\succ_{♂} M_{+}$ for all $L_{+}, M_{+}\in\mathcal{L}_{+}$;

b) $L_{+}\sim_{♀} M_{+}$ if and only if $L_{+}\sim_{♂} M_{+}$ for all $L_{+}, M_{+}\in\mathcal{L}_{+}$.

Proof is presented in the Appendix.

[1] However, Definition 1 implies that if a less loss averse individual ♂ strictly prefers a sure gain of $x_{+}\in X_{+}$ over a lottery $L\in\mathcal{L}$ then a more loss averse individual ♀ does so as well and if an individual ♂ is exactly indifferent between the two then an individual ♀ weakly prefers the sure gain of x_{+} over L. In other words, an individual ♀ is more risk averse than an individual ♂ in the domain of gains.

Proposition 1 is an intuitive implication of Definition 1. We can unambiguously rank two individuals in terms of their loss preferences only if they have identical preferences over loss-free alternatives (gain lotteries). If the two individuals do not have the same preferences in choice without any losses, one of them may choose a specific loss-free lottery because it is her most preferred alternative and not because she is averse to losses. Thus, to have a meaningful concept of comparative loss aversion, we need to consider individuals with identical preferences over the set of loss-free lotteries.

2. Loss Aversion in Different Decision Theories

Let us first consider comparative loss aversion in the context of expected utility theory (von Neumann and Morgenstern, 1944). In expected utility theory there exists an utility function u:$X\to\mathbb{R}$ that is unique up to a positive linear transformation, such that

(1) $$L \succsim M \text{ if and only if } \sum_{x\in X} L(x)u(x) \geq \sum_{x\in X} M(x)u(x),$$

for any two lotteries $L, M \in \mathcal{L}$. According to formula (1), a lottery L is weakly preferred over a lottery M if and only if the expected utility of L is greater than or equal to the expected utility of M.

The following result follows immediately from Proposition 1.

Corollary 1 If an expected utility maximizer ♀ with utility function $u_{♀}$:$X\to\mathbb{R}$ is more loss averse than an expected utility maximizer ♂ with utility function $u_{♂}$:$X\to\mathbb{R}$, then there exist a>0 and $b\in\mathbb{R}$ such that $u_{♀}(x_+) = au_{♂}(x_+) + b$ for all $x_+\in X_+$.

Corollary 1 simply states that whenever two individuals can be ranked in terms of loss preferences, they must have the same utility function in the domain of gains, up to a positive linear transformation.

Proposition 2 An expected utility maximizer ♀ with utility function $u_♀: X \to \mathbb{R}$ is more loss averse than an expected utility maximizer ♂ with utility function $u_♂: X \to \mathbb{R}$ if and only if there exist $a>0$ and $b \in \mathbb{R}$ such that

a) $u_♀(x_+) = au_♂(x_+) + b$ for all $x_+ \in X_+$;

b) $u_♀(x_-) \leq au_♂(x_-) + b$ for all $x_- \in X_-$;

c) there exists a loss $x_- \in X_-$ such that $u_♀(x_-) < au_♂(x_-) + b$.

Proof is presented in the Appendix.

Proposition 2 effectively states that an individual ♀ is more loss averse than an individual ♂ if and only if we can normalize the utility function of an individual ♀ for two arbitrary gains so that ♀'s normalized utility function coincides with ♂'s utility function in the domain of gains and ♀'s normalized utility of any loss $x_- \in X_-$ is less than or equal to ♂'s utility of x_- (and it is strictly less for at least one loss $x_- \in X_-$).

Figure 1 illustrates Proposition 2 when X_+ is the set of positive real numbers $\mathbb{R}_+$ and X_- is the set of negative real numbers $\mathbb{R}_-$.

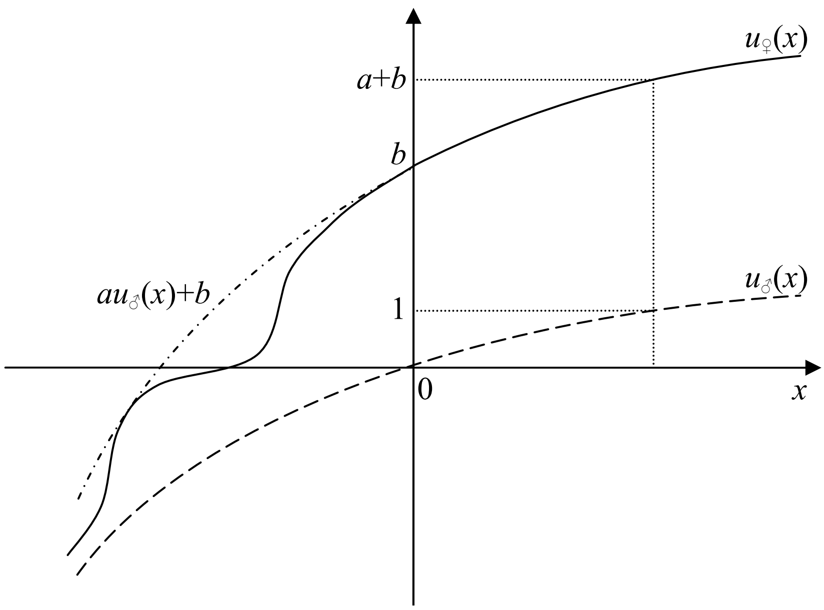

Figure 1 An expected utility maximizer ♀ with utility function $u_♀(x)$ is more loss averse than an expected utility maximizer ♂ with utility function $u_♂(x)$

Let us now apply the concept of comparative loss aversion in the context of rank-dependent utility theory (Quiggin, 1981). In rank-dependent utility theory there exists an utility function u:X→ℝ that is unique up to a positive linear transformation, and a unique strictly increasing probability weighting function w:[0,1]→[0,1] with $w(0)=0$ and $w(1)=1$, such that

$$L \succsim M \text{ if and only if } \sum_{x\in X} u(x)\left[w\left(\sum_{\substack{y\in X\\ u(y)\geq u(x)}} L(y)\right)-w\left(\sum_{\substack{y\in X\\ u(y)>u(x)}} L(y)\right)\right]\geq$$
(2)
$$\geq \sum_{x\in X} u(x)\left[w\left(\sum_{\substack{y\in X\\ u(y)\geq u(x)}} M(y)\right)-w\left(\sum_{\substack{y\in X\\ u(y)>u(x)}} M(y)\right)\right],$$

for any lotteries $L, M \in \mathcal{L}$. The following result follows immediately from Proposition 1.

Corollary 2 If a rank-dependent utility maximizer ♀ with an utility function $u_♀$:X→ℝ and a probability weighting function $w_♀$:[0,1]→[0,1] is more loss averse than a rank-dependent utility maximizer ♂ with an utility function $u_♂$:X→ℝ and a probability weighting function $w_♂$:[0,1]→[0,1], then $w_♀(p) = w_♂(p)$ for all p∈[0,1] and there exist a>0 and b∈ℝ such that $u_♀(x_+) = au_♂(x_+) + b$ for all $x_+ \in X_+$.

Recall that an unambiguous ranking of two individuals according to their loss attitudes is possible only if the two individuals share the same preferences over loss-free lotteries (Proposition 1). In the context of rank-dependent utility theory this implies the following. We can rank two rank-dependent utility maximizers according to their loss attitudes only if the two individuals have the same probability weighting function and the same utility function in the domain of gains, up to a positive linear transformation (Corollary 2). Note that Corollary 2 implies that the two rank-dependent utility maximizers have the same ranking of gains in terms of their desirability.

Proposition 3 A rank-dependent utility maximizer ♀ with an utility function $u_♀:X\to\mathbb{R}$ and a probability weighting function $w_♀:[0,1]\to[0,1]$ is more loss averse than a rank-dependent utility maximizer ♂ with an utility function $u_♂:X\to\mathbb{R}$ and a probability weighting function $w_♂:[0,1]\to[0,1]$ if and only if there exist $a>0$ and $b\in\mathbb{R}$ such that

a) $w_♀(p) = w_♂(p)$ for all $p\in[0,1]$;

b) $u_♀(x_+) = au_♂(x_+) + b$ for all $x_+\in X_+$;

c) $u_♀(x_-) \leq au_♂(x_-) + b$ for all $x_-\in X_-$;

d) there exists a loss $x_-\in X_-$ such that $u_♀(x_-) < au_♂(x_-) + b$.

Proof is presented in the Appendix.

Note that Proposition 3 does not require that the two rank-dependent utility maximizers have the same ranking of losses in terms of their desirability.

Proposition 3 characterizes the concept of loss aversion within a rank-dependent utility theory. In particular, Proposition 3 shows that loss aversion is entirely captured by the curvature of the utility function and it is not related to the shape of the probability weighting function. The restrictions on the curvature of the utility function, which are required for one individual to be strictly more loss averse than another individual, are exactly the same as in expected utility theory (cf. Proposition 2). Namely, the two individuals should have the same utility function in the domain of gains (up to a positive linear transformation) and a more loss averse individual should have an utility function that lies below the corresponding normalized utility function of a less loss averse individual in the domain of losses. Notably, these formal restrictions b)-d) in Proposition 3 are quite similar to the intuitive ideas of Kahneman and Tversky (1979) who pioneered the concept of loss aversion in behavioral economics.

3. Probabilistic Loss Aversion

Numerous experimental studies find that people do not always choose the same alternative when presented with exactly the same decision problem on two separate occasions within a short period of time (e.g. Camerer, 1989; Hey and Orme, 1994; Loomes and Sugden, 1998). In general, people often make contradictory choices if none of the lotteries transparently dominates other alternatives. In this section we will extend Definition 1 to a more general setup where people may choose in a probabilistic manner.

In the remainder of this paper we assume that the primitive of choice is a binary choice probability function $P:\mathcal{L}\times\mathcal{L}\rightarrow[0,1]$, which is also known as a fuzzy preference relation (e.g. Zimmerman et al., 1984). Notation $P(L,M)$ denotes probability that an individual chooses lottery $L \in \mathcal{L}$ over lottery $M \in \mathcal{L}$ in a direct binary choice. For any L, $M \in \mathcal{L}$, $L \neq M$, probability $P(L,M)$ is observable from the relative frequency with which an individual chooses L when asked to choose repeatedly between L and M. We consider two individuals: an individual ♀ and an individual ♂ characterized by binary choice probability functions $P_{♀}(.,.)$ and $P_{♂}(.,.)$ correspondingly.

Definition 2 An individual ♀ is probabilistically *more loss averse than* an individual ♂ if $P_{♀}(L_+,L)\geq P_{♂}(L_+,L)$ for all $L_+\in\mathcal{L}_+$ and all $L\in\mathcal{L}$ and there exist at least one loss-free lottery $L_+\in\mathcal{L}_+$ and one lottery $L\in\mathcal{L}$ such that $P_{♀}(L_+,L)>P_{♂}(L_+,L)$.

Definition 2 simply states that a more loss averse individual is always at least as likely to choose a loss-free lottery over any other lottery as a less loss averse individual. Definition 2 of the more-loss-averse-than relation between individuals is very general. In particular, lottery outcomes may not be measurable in real numbers. We also do not require that fuzzy preferences over lotteries are represented by a specific model of probabilistic choice. Thus, Definition 2 applies to very distinct models of probabilistic choice, e.g. when people have multiple preference relations on $\mathcal{L}$ (Loomes and Sugden, 1995) or when people have a unique preference relation on $\mathcal{L}$ but they make random errors (Hey and Orme, 1994; Blavatskyy, 2007). Last but not least, Definition 2 is more compact than Definition 1.

By replacing lottery $L\in\mathcal{L}$ in the first part of Definition 2 with a loss-free lottery $M_+\in\mathcal{L}_+$, we immediately arrive at the following result.

Corollary 3 If an individual ♀ is more loss averse than an individual ♂, or vice versa, then $P_♀(L_+, M_+)=P_♂(L_+, M_+)$ for all $L_+, M_+\in\mathcal{L}_+$.

According to Corollary 3, the ranking of individuals in terms of their loss attitudes is possible only if they choose in identical manner between loss-free lotteries. If this is not the case, heterogeneous loss attitudes are confounded with heterogonous tastes over loss-free lotteries and no clear comparison of individuals in terms of relative loss aversion can be made.

4. Loss Aversion in Different Models of Probabilistic Choice

One of the simplest models of probabilistic choice is the constant error/tremble model. Harless and Camerer (1994) argue that people have a unique preference relation on $\mathcal{L}$ but they do not always choose their preferred lottery. With a constant probability $\tau\in[0,0.5]$ a tremble occurs and people choose a less preferred alternative (for instance, due to a lapse of concentration). Specifically, in a constant error/tremble model there exists an utility function $u:X\rightarrow\mathbb{R}$ that is unique up to a linear transformation, such that

(3) $$P(L,M)=\begin{cases}\tau, & \sum_{x\in X} L(x)u(x)<\sum_{x\in X} M(x)u(x)\\ 0.5, & \sum_{x\in X} L(x)u(x)=\sum_{x\in X} M(x)u(x)\\ 1-\tau, & \sum_{x\in X} L(x)u(x)>\sum_{x\in X} M(x)u(x)\end{cases}$$

for any two lotteries L, $M \in \mathcal{L}$ and a probability $\tau \in [0,0.5]$. The following result follows directly from the proof of Proposition 2.

Corollary 4 An individual ♀ with utility function $u_{♀}:X \rightarrow \mathbb{R}$ and the probability of a tremble $\tau_{♀}$ is probabilistically more loss averse than an individual ♂ with utility function $u_{♂}:X \rightarrow \mathbb{R}$ and the probability of a tremble $\tau_{♂}$ if and only if $\tau_{♂} = \tau_{♀}$ and there exist $a>0$ and $b \in \mathbb{R}$ such that

a) $u_{♀}(x_+) = au_{♂}(x_+) + b$ for all $x_+ \in X_+$;

b) $u_{♀}(x_-) \leq au_{♂}(x_-) + b$ for all $x_- \in X_-$;

c) there exists a loss $x_- \in X_-$ such that $u_{♀}(x_-) < au_{♂}(x_-) + b$.

Let us now consider probabilistic loss aversion in the context of a strong utility model (e.g. Luce and Suppes, 1965). In this model there exists an utility function $u:X \rightarrow \mathbb{R}$ that is unique up to a positive linear transformation, and a strictly increasing function $\varphi:\mathbb{R} \rightarrow [0,1]$, which is unique up to a positive dimensional constant and satisfies $\varphi(v)+\varphi(-v)=1$ for all $v \in \mathbb{R}$, such that

(4) $$P(L,M) = \varphi \left(\sum_{x \in X} L(x)u(x) - \sum_{x \in X} M(x)u(x) \right)$$

for any two lotteries L, $M \in \mathcal{L}$.

Function $\varphi(.)$ captures the sensitivity of binary choice probabilities to differences in the expected utility of the two alternatives that an individual needs to choose from. If function $\varphi(.)$ is the cumulative distribution function of a normal distribution with zero mean and constant standard deviation, model (4) becomes the Fechner model of random errors (Fechner, 1860; Hey and Orme, 1994). If function $\varphi(.)$ is the distribution function of the logistic distribution, model (4) becomes Luce choice model (Luce, 1959). Blavatskyy (2008) provides axiomatic characterization of the choice rule (4).

Proposition 4 A strong utility maximizer ♀ characterized by a pair of functions ($u_♀$, $\varphi_♀$) is probabilistically more loss averse than a strong utility maximizer ♂ characterized by a pair of functions ($u_♂$, $\varphi_♂$) if there exist a>0 and $b\in\mathbb{R}$ such that

a) $u_♀(x_+) = au_♂(x_+) + b$ for all $x_+\in X_+$;

b) $u_♀(x_-) \leq au_♂(x_-) + b$ for all $x_-\in X_-$;

c) $\varphi_♀(av) = \varphi_♂(v)$ for all $v\in[-\delta, \delta]$, where $\delta = \max_{x_+\in X_+} u_♂(x_+) - \min_{x_+\in X_+} u_♂(x_+)$;

d) $\varphi_♀(av) \geq \varphi_♂(v)$ for all $v\in(\delta, \Delta]$, where $\Delta = \max_{x_+\in X_+} u_♂(x_+) - \min_{x_-\in X_-} u_♂(x_-)$;[2]

e) either there exists a loss $x_-\in X_-$ such that $u_♀(x_-) < au_♂(x_-) + b$ or there exists $v\in(\delta, \Delta]$ such that $\varphi_♀(av) > \varphi_♂(v)$ or both.

Proof is presented in the Appendix.

Proposition 4 shows that in a strong utility model loss aversion is related both to the curvature of the utility function $u(.)$ and the shape of the sensitivity function $\varphi(.)$. On the one hand, an individual ♀ can be more loss averse than an individual ♂ if they have the same utility function in the domain of gains (up to a positive linear transformation) but ♀'s utility function lies below ♂'s normalized utility function in the domain of losses. On the other hand, an individual ♀ can be more loss averse than an individual ♂ if they have the same sensitivity function in the neighborhood of zero (up to a positive dimensional constant) but individual ♀ is more sensitive to large differences in utility.

Interestingly, a strong utility model allows individual ranking in terms of relative *loss* aversion but not in terms of relative *risk* aversion. Wilcox (2008) and Blavatskyy (2008a) show that risk aversion cannot be defined within a strong utility model. Thus, there are models where loss aversion is well defined even though risk aversion is not.

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[2] Note that condition d) is equivalent to $\varphi_♀(av) \leq \varphi_♂(v)$ for all $v\in(-\Delta,-\delta]$ due to the skew-symmetric property of the sensitivity function $\varphi(.)$.

5. Absolute Loss Aversion

So far we considered only comparative loss aversion. To measure absolute loss aversion, we need to fix one binary choice probability function $P_{LN}: \mathcal{L}\times\mathcal{L} \rightarrow [0,1]$. An individual is called *loss neutral* if she has the binary choice probability function $P_{LN}(.,.)$. An individual is called *loss averse* if she is more loss averse (according to Definition 2) than the loss neutral individual. Similarly, an individual is called *loss seeking* or *loss loving* if the loss neutral individual is more loss averse than this individual.

Notice that the concept of absolute loss aversion depends on an ad hoc selection of a loss neutral binary choice probability function $P_{LN}(.,.)$.[3] This is similar to our temperature measurement that requires an arbitrary selection of zero temperature (e.g. the triple point of water in the Celsius scale or absolute zero in the Kelvin scale). Similarly, our time measurement also requires an ad hoc selection of an epochal date (e.g. the incarnation of Jesus in the Gregorian calendar, the creation of the world in the Hebrew calendar or the immigration of Muhammad in the Islamic calendar).

In a special case when lotteries have only monetary outcomes and people have deterministic preferences, Kahneman and Tversky (1979) arbitrary selected a loss neutral preference relation so that a loss neutral individual is exactly indifferent between accepting and rejecting a symmetric bet that yields a 50%-50% chance of either a loss of $-x$ or a gain of x, for all $x\in\mathbb{R}_+$. In other words, loss aversion is defined as aversion to symmetric 50%-50% lotteries. Several later studies also adopted this convention (e.g. Schmidt and Zank, 2005). However, it is not clear how this natural "normalization" can be extended to a more general case when outcomes are not measurable in real numbers.

[3] The definition of risk aversion also requires a priori "normalization" of risk neutral preferences (e.g. Epstein, 1999; Blavatskyy, 2008a). Similarly, in order to define uncertainty aversion we need an arbitrary definition of uncertainty neutrality (e.g. Epstein, 1999).

6. Conclusion

Loss aversion is a fundamental concept in behavioral economics. However, it is traditionally defined only in the context of lotteries over monetary payoffs. This paper extends the definition of loss aversion to a more general setup where outcomes are not necessarily measurable in real numbers and people do not necessarily have a unique preference relation over lotteries, i.e. they may choose in a probabilistic manner. Specifically, an individual ♀ is said to be probabilistically more loss averse than an individual ♂ if in any decision problem an individual ♀ chooses a loss-free lottery (that yields only gains with a positive probability) at least as frequently as does individual ♂.

This paper shows that the above definition of comparative loss aversion has very intuitive implications for well-known decision theories such as expected utility theory and rank-dependent utility theory as well as for popular models of probabilistic choice such as the constant error/tremble model, the Fechner model of random errors and Luce choice model. In particular, in these models loss aversion is related to the curvature of the utility function. If two individuals can be ranked in terms of their loss preferences, then they have the same utility function in the domain of gains (up to a positive linear transformation) but the utility function of a more loss averse individual lies below the normalized utility function of a less loss averse individual in the domain of losses.

In a strong utility model, loss aversion may be also driven by the curvature of the sensitivity function—a more loss averse individual may be more sensitive to large differences in expected utility of the two lotteries that are compared. Interestingly, comparative loss aversion is well-defined in a strong utility model, even though comparative risk aversion is not. This highlights an important point that stronger loss aversion does not necessarily imply stronger risk aversion, or vice versa.

References

Benartzi, S. and R. H. Thaler (1995) "Myopic Loss Aversion and the Equity Premium Puzzle" *The Quarterly Journal of Economics* **110**(1), 73-92

Blavatskyy, Pavlo R. (2007) “Stochastic Expected Utility Theory” *Journal of Risk and Uncertainty* **34**, 259-286

Blavatskyy, Pavlo R. (2008) “Stochastic Utility Theorem” *Journal of Mathematical Economics*, forthcoming

Blavatskyy, Pavlo R. (2008a) “Risk Aversion” IEW Working Paper 370

Camerer, Colin F. (1989) “An experimental test of several generalized utility theories.” *Journal of Risk and Uncertainty* **2**, 61-104

Camerer, Colin F. (2008) "Behavioral Economics", in T. Persson (Ed.) World Congress of the Econometric Society Proceeding, in press

Camerer, C., L. Babcock, G. Loewenstein and R. Thaler (1997) "Labor Supply of New York City Cabdrivers: One Day at a Time" *The Quarterly Journal of Economics* **112**(2), 407-441

Chen, Keith, Venkat Lakshminarayanan and Laurie Santos (2006) “How Basic are Behavioral Biases? Evidence from Capuchin-Monkey Trading Behavior” *Journal of Political Economy* **114**(3), 517-537

Epstein, L. (1999) “A Definition of Uncertainty Aversion” *Review of Economic Studies* **66**, 579-608

Fechner, G. (1860) “Elements of Psychophysics” NewYork: Holt, Rinehart and Winston

Genesove, D. and C. Mayer (2001) "Loss aversion and seller behavior: Evidence from the housing market" *Quarterly Journal of Economics* **116**(4), 1233-1260

Harless, D. and C. Camerer (1994) The predictive utility of generalized expected utility theories, *Econometrica* **62**, 1251-1289

Hey, J.D. and C. Orme (1994) Investigating generalisations of expected utility theory using experimental data, *Econometrica* **62**, 1291-1326

Kahneman, D., J. L. Knetsch and R. H. Thaler (1990) "Experimental Tests of the Endowment Effect and the Coase Theorem" *Journal of Political Economy* **98**(6), 1325-1348

Kahneman, D. and Tversky, A. (1979) “Prospect theory: an analysis of decision under risk” *Econometrica* **47**, 263-291

Köbberling V. and P. Wakker (2005) “An Index of Loss Aversion,” *Journal of Economic Theory* **122**, 263-291

Loomes, G. and Sugden, R. (1995) “Incorporating a stochastic element into decision theories” *European Economic Review* **39**, 641-648

Loomes, G. and Sugden, R. (1998) “Testing different stochastic specifications of risky choice” *Economica* **65**, 581-598

Luce, R. D. (1959) “Individual choice behavior” New York: Wiley

Luce, R. D. and Suppes, P. (1965) “Preference, utility, and subjective probability” in R. D. Luce, R. R. Bush & E. Galanter (eds.), *Handbook of mathematical psychology*, Vol. III, 249–410, Wiley, New York NY

Quiggin, John. (1981). “Risk perception and risk aversion among Australian farmers” *Australian Journal of Agricultural Recourse Economics* **25**, 160-169

Samuelson, W.F. and R.J. Zeckhauser (1988) “Status Quo Bias in Decision Making” *Journal of Risk and Uncertainty* **1**, 7-59

Schmidt, Ulrich and Horst Zank (2005) “What is Loss Aversion?“ *Journal of Risk and Uncertainty* **30**, 157-167

Thaler, Richard (1980) "Toward a Positive Theory of Consumer Choice" *Journal of Economic Behavior and Organization* **1**(1), 39-60

Tversky, Amos and Daniel Kahneman (1992) “Advances in Prospect Theory: Cumulative Representation of Uncertainty” *Journal of Risk and Uncertainty* **5**, 297-323

von Neumann, J. and Morgenstern, O. (1944) “Theory of Games and Economic Behavior, ” Princeton, Princeton University Press

Wilcox, N. (2008) “’Stochastically More Risk Averse’: A Contextual Theory of Stochastic Discrete Choice under Risk” *Journal of Econometrics* forthcoming

Zimmerman, H.J., Gaines, B.R. and Zadeh, L. A. (1984) “Fuzzy Sets and Decision Analysis”, North-Holland

Appendix

Proof of Proposition 1.

Consider an individual ♀ who is more loss averse than an individual ♂.

a) According to Definition 1, if $L_+ \succ_{♂} M_+$ then $L_+ \succ_{♀} M_+$ for all $L_+, M_+ \in \mathcal{L}_+$.

Let us now assume that there exist two lotteries $L_+, M_+ \in \mathcal{L}_+$ such that $L_+ \succ_{♀} M_+$ but $M_+ \succsim_{♂} L_+$. If $M_+ \succ_{♂} L_+$ then Definition 1 implies that $M_+ \succ_{♀} L_+$. However, this contradicts to our assumption that $L_+ \succ_{♀} M_+$. If $M_+ \sim_{♂} L_+$ then Definition 1 implies that $M_+ \succsim_{♀} L_+$. Again, this contradicts to our assumption that $L_+ \succ_{♀} M_+$. Therefore, it must be the case that $L_+ \succ_{♀} M_+$ if and only if $L_+ \succ_{♂} M_+$ for all $L_+, M_+ \in \mathcal{L}_+$.

b) According to Definition 1, if $L_+ \sim_{♂} M_+$ then $L_+ \succsim_{♀} M_+$ for all $L_+, M_+ \in \mathcal{L}_+$. Moreover, if $M_+ \sim_{♂} L_+$ then $M_+ \succsim_{♀} L_+$ for all $L_+, M_+ \in \mathcal{L}_+$. Hence, if $L_+ \sim_{♂} M_+$ then it must be the case that $L_+ \sim_{♀} M_+$ for all $L_+, M_+ \in \mathcal{L}_+$.

Let us now assume that there exist two lotteries $L_+, M_+ \in \mathcal{L}_+$ such that $L_+ \sim_{♀} M_+$ but $M_+ \nsim_{♂} L_+$. If $M_+ \succ_{♂} L_+$ then Definition 1 implies that $M_+ \succ_{♀} L_+$. However, this contradicts to our assumption that $L_+ \sim_{♀} M_+$. Similarly, if $L_+ \succ_{♂} M_+$ then $L_+ \succ_{♀} M_+$ due to Definition 1 and the analogous contradiction arises. Therefore, $L_+ \sim_{♀} M_+$ if and only if $L_+ \sim_{♂} M_+$ for all $L_+, M_+ \in \mathcal{L}_+$.

Similarly, we can prove that Proposition 1 holds when an individual ♂ is more loss averse than an individual ♀. *Q.E.D.*

Proof of Proposition 2.

We will first prove that if conditions a)-c) hold then an individual ♀ is more loss averse than an individual ♂. Consider two arbitrary lotteries $L_+ \in \mathcal{L}_+$ and $L \in \mathcal{L}$. We will first prove that $L_+ \succ_{♂} L$ implies $L_+ \succ_{♀} L$. Condition (1) implies that $L_+ \succ_{♂} L$ if and only if

(5) $$\sum_{x_+\in X_+} u_{♂}(x_+)L_+(x_+) > \sum_{x\in X} u_{♂}(x)L(x).$$

We can rearrange (5) into

(6) $$\sum_{x_+\in X_+} u_{♂}(x_+)\left[L_+(x_+)-L(x_+)\right] > \sum_{x_-\in X_-} u_{♂}(x_-)L(x_-).$$

Furthermore, we can multiply both sides of (6) on a positive constant $a>0$ and add $b\sum_{x_+\in X_+}\left[L_+(x_+)-L(x_+)\right] = b\sum_{x_-\in X_-} L(x_-)$ to both sides of (6), $b\in\mathbb{R}$. This results in

(7) $$\sum_{x_+\in X_+}\left[au_{♂}(x_+)+b\right]\left[L_+(x_+)-L(x_+)\right] > \sum_{x_-\in X_-}\left[au_{♂}(x_-)+b\right]L(x_-).$$

If part a) of Proposition 2 holds then there exist $a>0$ and $b\in\mathbb{R}$ such that $u_{♀}(x_+) = au_{♂}(x_+) + b$ for all $x_+\in X_+$. Hence, we can rewrite (7) as

(8) $$\sum_{x_+\in X_+} u_{♀}(x_+)\left[L_+(x_+)-L(x_+)\right] > \sum_{x_-\in X_-}\left[au_{♂}(x_-)+b\right]L(x_-).$$

If part b) of Proposition 2 holds then $u_{♀}(x_-) \le au_{♂}(x_-) + b$ for all $x_-\in X_-$. Thus, we can rewrite (8) as

(9) $$\sum_{x_+\in X_+} u_{♀}(x_+)\left[L_+(x_+)-L(x_+)\right] > \sum_{x_-\in X_-} u_{♀}(x_-)L(x_-).$$

Finally, we can rearrange (9) into

(10) $$\sum_{x_+\in X_+} u_{♀}(x_+)L_+(x_+) > \sum_{x\in X} u_{♀}(x)L(x),$$

which holds if and only if $L_+ \succ_{♀} L$ due to (1). Hence, $L_+ \succ_{♂} L$ implies $L_+ \succ_{♀} L$ for all $L_+\in\mathcal{L}_+$ and all $L\in\mathcal{L}$ if parts a) and b) of Proposition 2 hold.

To prove that $L_+ \sim_{♂} L$ implies $L_+ \succsim_{♀} L$ for all $L_+\in\mathcal{L}_+$ and all $L\in\mathcal{L}$ we just need to replace the sign “>” with the sign “=” in (5)-(8) and with the sign “≥” in (9)-(10).

Let us now prove that there exist $L_+\in\mathcal{L}_+$ and $L\in\mathcal{L}$ such that $L_+ \sim_{♂} L$ and $L_+ \succ_{♀} L$. If part c) of Proposition 2 holds then there exists $x_-\in X_-$ such that $u_{♀}(x_-) < au_{♂}(x_-) + b$. Let $y,z\in X_+$ be two gains such that $u_{♂}(x_-)<u_{♂}(y)<u_{♂}(z)$. Let L_+ be a lottery that yields y for sure

and let L be a lottery that yields x_- with probability p and z with probability 1-p. Obviously, there exists a probability p such that

(11) $$u_{♂}(y) = pu_{♂}(x_-) + (1\text{-}p)u_{♂}(z).$$

If (11) holds then condition (1) implies that $L_+ \sim_{♂} L$. However, if we multiply both sides of (11) on a>0 and add $b\in\mathbb{R}$ to both sides of (11) we obtain

(12) $$u_{♀}(y) = p(au_{♂}(x_-) + b) + (1\text{-}p)u_{♀}(z),$$

where we used the fact that $u_{♀}(x_+) = au_{♂}(x_+) + b$ for all $x_+\in X_+$ due to part a) of Proposition 2. Since $u_{♀}(x_-) < au_{♂}(x_-) + b$, then it must be the case that $u_{♀}(y) > pu_{♀}(x_-) + (1\text{-}p)u_{♀}(z)$. Hence, $L_+ \succ_{♀} L$ due to condition (1). In other words, we constructed two lotteries $L_+\in\mathcal{L}_+$ and $L\in\mathcal{L}$ such that $L_+ \sim_{♂} L$ but $L_+ \succ_{♀} L$.

To summarize, if parts a)-c) of Proposition 2 hold then conditions a)-c) of Definition 1 are satisfied i.e. an individual ♀ is more loss averse than an individual ♂. Let us now prove the necessity of parts a)-c) of Proposition 2. If an individual ♀ is more loss averse than an individual ♂ then part a) of Proposition 2 holds due to Corollary 1.

Suppose that an individual ♀ is more loss averse than an individual ♂ but there is a loss $x_-\in X_-$ such that $u_{♀}(x_-) > au_{♂}(x_-) + b$. In such case, for the two lotteries $L_+\in\mathcal{L}_+$ and $L\in\mathcal{L}$ that we constructed above we must have $L_+ \sim_{♂} L$ but $L \succ_{♀} L_+$. However, this contradicts to condition b) in Definition 1 i.e. in this case an individual ♀ is not more loss averse than an individual ♂. Thus, part b) of Proposition 2 must hold for any $x_-\in X_-$.

Finally, if part c) of Proposition 2 does not hold, i.e. $u_{♀}(x_-) = au_{♂}(x_-) + b$ for all $x_-\in X_-$, then $L_+ \sim_{♂} L$ implies $L_+ \sim_{♀} L$ for all $L_+\in\mathcal{L}_+$ and $L\in\mathcal{L}$ due to (1) and condition c) of Definition 1 cannot be satisfied. *Q.E.D.*

Proof of Proposition 3.

We will first prove the sufficiency of conditions a)-d) in Proposition 3. Consider two arbitrary lotteries $L_+ \in \mathcal{L}_+$ and $L \in \mathcal{L}$. We will first prove that part a) of Definition 1 must hold i.e. $L_+ \succ_{♂} L$ implies $L_+ \succ_{♀} L$.

Condition (2) implies that $L_+ \succ_{♂} L$ if and only if

$$\begin{aligned}(13)\quad &\sum_{x_+\in X_+} u_{♂}(x_+)\left[w_{♂}\left(\sum_{\substack{y_+\in X_+\\ u_{♂}(y_+)\ge u_{♂}(x_+)}} L_+(y_+)\right)-w_{♂}\left(\sum_{\substack{y_+\in X_+\\ u_{♂}(y_+)> u_{♂}(x_+)}} L_+(y_+)\right)\right]>\\ &>\sum_{x\in X} u_{♂}(x)\left[w_{♂}\left(\sum_{\substack{y\in X\\ u_{♂}(y)\ge u_{♂}(x)}} L(y)\right)-w_{♂}\left(\sum_{\substack{y\in X\\ u_{♂}(y)> u_{♂}(x)}} L(y)\right)\right].\end{aligned}$$

For any $a>0$ and $b\in\mathbb{R}$ we can rewrite condition (13) as follows

$$\begin{aligned}&\sum_{x_+\in X_+} \left[au_{♂}(x_+)+b\right]\left[w_{♂}\left(\sum_{\substack{y_+\in X_+\\ u_{♂}(y_+)\ge u_{♂}(x_+)}} L_+(y_+)\right)-w_{♂}\left(\sum_{\substack{y_+\in X_+\\ u_{♂}(y_+)> u_{♂}(x_+)}} L_+(y_+)\right)\right]>\\ (14)\quad &>\sum_{x_+\in X_+} \left[au_{♂}(x_+)+b\right]\left[w_{♂}\left(\sum_{\substack{y_+\in X_+\\ u_{♂}(y_+)\ge u_{♂}(x_+)}} L(y_+)\right)-w_{♂}\left(\sum_{\substack{y_+\in X_+\\ u_{♂}(y_+)> u_{♂}(x_+)}} L(y_+)\right)\right]+\\ &+\sum_{x_-\in X_-} \left[au_{♂}(x_-)+b\right]\left[w_{♂}\left(\sum_{\substack{y\in X\\ u_{♂}(y)\ge u_{♂}(x_-)}} L(y)\right)-w_{♂}\left(\sum_{\substack{y\in X\\ u_{♂}(y)> u_{♂}(x_-)}} L(y)\right)\right].\end{aligned}$$

If part a) of Proposition 3 holds, then the two individuals ♀ and ♂ have identical probability weighting functions. If part b) of Proposition 3 holds, there exist $a>0$ and $b\in\mathbb{R}$ such that $u_{♀}(x_+) = au_{♂}(x_+) + b$ for all $x_+\in X_+$ and we can rewrite (14) as follows

$$\begin{aligned}&\sum_{x_+\in X_+} u_{♀}(x_+)\left[w_{♀}\left(\sum_{\substack{y_+\in X_+\\ u_{♀}(y_+)\ge u_{♀}(x_+)}} L_+(y_+)\right)-w_{♀}\left(\sum_{\substack{y_+\in X_+\\ u_{♀}(y_+)> u_{♀}(x_+)}} L_+(y_+)\right)\right]>\\ (15)\quad &>\sum_{x_+\in X_+} u_{♀}(x_+)\left[w_{♀}\left(\sum_{\substack{y_+\in X_+\\ u_{♀}(y_+)\ge u_{♀}(x_+)}} L(y_+)\right)-w_{♀}\left(\sum_{\substack{y_+\in X_+\\ u_{♀}(y_+)> u_{♀}(x_+)}} L(y_+)\right)\right]+\\ &+\sum_{x_-\in X_-} \left[au_{♂}(x_-)+b\right]\left[w_{♀}\left(\sum_{\substack{y\in X\\ u_{♂}(y)\ge u_{♂}(x_-)}} L(y)\right)-w_{♀}\left(\sum_{\substack{y\in X\\ u_{♂}(y)> u_{♂}(x_-)}} L(y)\right)\right].\end{aligned}$$

Let $z_{-}\in X_{-}$ be the most desirable loss for an individual ♀ i.e. $u_{♀}(z_{-}) \geq u_{♀}(x_{-})$ for all $x_{-}\in X_{-}$. Let $Z_{-}\subset X_{-}$ be the set of all losses that an individual ♂ finds at least as good as z_{-} i.e. $u_{♂}(x_{-}) \geq u_{♂}(z_{-})$ for all $x_{-}\in Z_{-}$. If part c) of Proposition 3 holds then we can rewrite

$$
\begin{aligned}
&\sum_{x_{-}\in Z_{-}}\left[au_{♂}(x_{-})+b\right]\left[w_{♀}\left(\sum_{\substack{y\in X\\ u_{♂}(y)\geq u_{♂}(x_{-})}}L(y)\right)-w_{♀}\left(\sum_{\substack{y\in X\\ u_{♂}(y)>u_{♂}(x_{-})}}L(y)\right)\right]\geq\\
&\geq\sum_{x_{-}\in Z_{-}}\left[au_{♂}(z_{-})+b\right]\left[w_{♀}\left(\sum_{\substack{y\in X\\ u_{♂}(y)\geq u_{♂}(x_{-})}}L(y)\right)-w_{♀}\left(\sum_{\substack{y\in X\\ u_{♂}(y)>u_{♂}(x_{-})}}L(y)\right)\right]=\\
(16)\qquad &=\left[au_{♂}(z_{-})+b\right]\left[w_{♀}\left(\sum_{\substack{y\in X\\ u_{♂}(y)\geq u_{♂}(z_{-})}}L(y)\right)-w_{♀}\left(\sum_{\substack{y\in X\\ u_{♀}(y)>u_{♀}(z_{-})}}L(y)\right)\right]\geq\\
&\geq u_{♀}(z_{-})\left[w_{♀}\left(\sum_{\substack{y\in X\\ u_{♂}(y)\geq u_{♂}(z_{-})}}L(y)\right)-w_{♀}\left(\sum_{\substack{y\in X\\ u_{♀}(y)>u_{♀}(z_{-})}}L(y)\right)\right]\geq\\
&\geq\sum_{x_{-}\in Z_{-}}u_{♀}(x_{-})\left[w_{♀}\left(\sum_{\substack{y\in X\\ u_{♀}(y)\geq u_{♀}(x_{-})}}L(y)\right)-w_{♀}\left(\sum_{\substack{y\in X\\ u_{♀}(y)>u_{♀}(x_{-})}}L(y)\right)\right].
\end{aligned}
$$

We can repeat the above argument for a smaller set of losses $X_{-}\backslash Z_{-}$ and so forth. Since the set X_{-} is finite, we then arrive at the result

$$
\begin{aligned}
(17)\qquad &\sum_{x_{-}\in X_{-}}\left[au_{♂}(x_{-})+b\right]\left[w_{♀}\left(\sum_{\substack{y\in X\\ u_{♂}(y)\geq u_{♂}(x_{-})}}L(y)\right)-w_{♀}\left(\sum_{\substack{y\in X\\ u_{♂}(y)>u_{♂}(x_{-})}}L(y)\right)\right]\geq\\
&\geq\sum_{x_{-}\in X_{-}}u_{♀}(x_{-})\left[w_{♀}\left(\sum_{\substack{y\in X\\ u_{♀}(y)\geq u_{♀}(x_{-})}}L(y)\right)-w_{♀}\left(\sum_{\substack{y\in X\\ u_{♀}(y)>u_{♀}(x_{-})}}L(y)\right)\right].
\end{aligned}
$$

Using (17) we can rewrite (15) as follows

$$
\begin{aligned}
(18)\qquad &\sum_{x_{+}\in X_{+}}u_{♀}(x_{+})\left[w_{♀}\left(\sum_{\substack{y_{+}\in X_{+}\\ u_{♀}(y_{+})\geq u_{♀}(x_{+})}}L_{+}(y_{+})\right)-w_{♀}\left(\sum_{\substack{y_{+}\in X_{+}\\ u_{♀}(y_{+})>u_{♀}(x_{+})}}L_{+}(y_{+})\right)\right]>\\
&>\sum_{x\in X}u_{♀}(x)\left[w_{♀}\left(\sum_{\substack{y\in X\\ u_{♀}(y)\geq u_{♀}(x)}}L(y)\right)-w_{♀}\left(\sum_{\substack{y\in X\\ u_{♀}(y)>u_{♀}(x)}}L(y)\right)\right].
\end{aligned}
$$

If (18) holds then $L_{+}\succ_{♀}L$ due to (2). Hence, part a) of Definition 1 must hold.

To prove that part b) of Definition 1 must hold i.e. $L_{+}\sim_{♂}L$ implies $L_{+}\succsim_{♀}L$ for all $L_{+}\in\mathcal{L}_{+}$ and all $L\in\mathcal{L}$ we just need to replace the sign ">" with the sign "=" in (13)-(15) and with the sign "≥" in (18).

Finally, let us prove that part c) of Definition 1 must hold i.e. there exist $L_+ \in \mathcal{L}_+$ and $L \in \mathcal{L}$ such that $L_+ \sim_{♂} L$ and $L_+ \succ_{♀} L$. If part d) of Proposition 3 holds then there exists $x_- \in X_-$ such that $u_{♀}(x_-) < au_{♂}(x_-) + b$. Let $y,z \in X_+$ be two gains such that $u_{♂}(x_-) < u_{♂}(y) < u_{♂}(z)$. Let L_+ be a lottery that yields y for sure and let L be a lottery that yields x_- with probability 1-p and z with probability p. Since function $w_{♂}(p)$ is strictly increasing in p with $w_{♂}(0)=0$ and $w_{♂}(1)=1$, there exists a probability p such that

(19) $$u_{♂}(y) = (1- w_{♂}(p))u_{♂}(x_-) + w_{♂}(p)u_{♂}(z).$$

If (19) holds then $L_+ \sim_{♂} L$ due to (2). If parts a) and b) of Proposition 3 hold, we can rewrite (19) as follows

(20) $$u_{♀}(y) = (1- w_{♀}(p))(au_{♂}(x_-) + b) + w_{♀}(p)u_{♀}(z).$$

Since $u_{♀}(x_-) < au_{♂}(x_-) + b$, then (20) implies $u_{♀}(y) > (1- w_{♀}(p))u_{♀}(x_-) + w_{♀}(p)u_{♀}(z)$ i.e. $L_+ \succ_{♀} L$ due to (2). In other words, we constructed two lotteries $L_+ \in \mathcal{L}_+$ and $L \in \mathcal{L}$ such that $L_+ \sim_{♂} L$ but $L_+ \succ_{♀} L$.

Hence, if parts a)-d) of Proposition 3 hold then conditions a)-c) of Definition 1 are satisfied i.e. an individual ♀ is more loss averse than an individual ♂. Let us now prove the necessity of parts a)-d) of Proposition 3. If an individual ♀ is more loss averse than an individual ♂ then parts a) and b) of Proposition 3 hold due to Corollary 2.

Suppose that an individual ♀ is more loss averse than an individual ♂ but there is a loss $x_- \in X_-$ such that $u_{♀}(x_-) > au_{♂}(x_-) + b$. In such case, for the two lotteries $L_+ \in \mathcal{L}_+$ and $L \in \mathcal{L}$ that we constructed above we must have $L_+ \sim_{♂} L$ but $L \succ_{♀} L_+$. However, this contradicts to condition b) in Definition 1 i.e. in this case an individual ♀ is not more loss averse than an individual ♂. Thus, part c) of Proposition 3 must hold for any $x_- \in X_-$.

Finally, if part d) of Proposition 3 does not hold, i.e. $u_{♀}(x_-) = au_{♂}(x_-) + b$ for all $x_- \in X_-$, then $L_+ \sim_{♂} L$ implies $L_+ \sim_{♀} L$ for all $L_+ \in \mathcal{L}_+$ and $L \in \mathcal{L}$ due to (2) and condition c) of Definition 1 cannot be satisfied. *Q.E.D.*

Proof of Proposition 4.

Consider two arbitrary lotteries $L_+ \in \mathcal{L}_+$ and $L \in \mathcal{L}$. Let us prove that if conditions a)-d) of Proposition 4 are satisfied then $P_♀(L_+,L) \geq P_♂(L_+,L)$. Equation (4) implies that

(21) $$P_♀\left(L_+,L\right)=\varphi_♀\left(\sum_{x_+\in X_+} L_+\left(x_+\right)u_♀\left(x_+\right)-\sum_{x\in X} L\left(x\right)u_♀\left(x\right)\right).$$

If condition a) of Proposition 4 holds, we can rewrite equation (21) as follows

(22) $$P_♀\left(L_+,L\right)=\varphi_♀\left(\sum_{x_+\in X_+}\left[L_+\left(x_+\right)-L\left(x_+\right)\right]\left[au_♂\left(x_+\right)+b\right]-\sum_{x_-\in X_-} L\left(x_-\right)u_♀\left(x_-\right)\right).$$

If condition b) of Proposition 4 holds and given that function $\varphi_♀(.)$ is strictly increasing, we can rewrite equation (22) as follows

(23) $$P_♀\left(L_+,L\right)\geq\varphi_♀\left(\sum_{x_+\in X_+}\left[L_+\left(x_+\right)-L\left(x_+\right)\right]\left[au_♂\left(x_+\right)+b\right]-\sum_{x_-\in X_-} L\left(x_-\right)\left[au_♂\left(x_-\right)+b\right]\right).$$

Inequality (23) can be rearranged into

(24) $$P_♀\left(L_+,L\right)\geq\varphi_♀\left(a\left[\sum_{x_+\in X_+} L_+\left(x_+\right)u_♂\left(x_+\right)-\sum_{x\in X} L\left(x\right)u_♂\left(x\right)\right]\right).$$

If conditions c) and d) of Proposition 4 are satisfied then we can rewrite (24) as

(25) $$P_♀\left(L_+,L\right)\geq\varphi_♂\left(\sum_{x_+\in X_+} L_+\left(x_+\right)u_♂\left(x_+\right)-\sum_{x\in X} L\left(x\right)u_♂\left(x\right)\right).$$

The last inequality (25) simply states that $P_♀(L_+,L) \geq P_♂(L_+,L)$ due to equation (4).

Let us now prove that if conditions a)-e) of Proposition 4 are satisfied then there exist two lotteries $L_+ \in \mathcal{L}_+$ and $L \in \mathcal{L}$ such that $P_♀(L_+,L) > P_♂(L_+,L)$. According to condition e) of Proposition 4, at least one of the following conditions must hold: 1) there exists a loss $x_- \in X_-$ such that $u_♀(x_-) < au_♂(x_-) + b$; 2) there exists $v \in (\delta, \Delta]$ such that $\varphi_♀(av) > \varphi_♂(v)$.

If condition 1) holds, then for any lottery $L \in \mathcal{L}$ that yields such an outcome $x_- \in X_-$ with a positive probability inequalities (23)-(25) hold as strict inequalities and we have $P_♀(L_+,L) > P_♂(L_+,L)$. If condition 2) holds, then for any two lotteries $L_+ \in \mathcal{L}_+$ and $L \in \mathcal{L}$ such that $\sum_{x\in X} L_+(x)u_♂(x) - \sum_{x\in X} L(x)u_♂(x) = v$, inequality (25) holds as strict inequality and we have again $P_♀(L_+,L) > P_♂(L_+,L)$. *Q.E.D.*