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A unified approach to comparative statics puzzles in experiments

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Abstract: The paper shows that, in some important respects, the differences between the Nash equilibrium and competing concepts such as the quantal-response equilibrium are smaller than they appear. I start from the observation that, in many experiments, parameter shifts that leave the Nash equilibrium unchanged affect behavior. I explain the direction of change with a heuristic *structural approach*, relying on properties such as strategic complementarities and increasing differences. I justify the approach using existing comparative statics results for the Nash equilibrium and new comparative statics results for the quantal response equilibrium. Further, I show that the experimental observations can also be rationalized by a model of adjustment to change that does not rely on any equilibrium concept. Finally, I relate the structural approach to equilibrium selection concepts.

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1 Introduction

Laboratory experiments have cast doubt on the predictive value of the Nash equilibrium and its refinements. At least the joint hypothesis that monetary payoffs are maximized and the Nash equilibrium is played is often in conflict with the facts.[1] Nevertheless, as argued by Samuelson (2005), even when point predictions do not hold, comparative statics predictions may still be borne out in the lab.[2] However, in an insightful contribution, Goeree and Holt (2001), henceforth GH, report the results of ten pairs of experiments where the Nash equilibrium is the same in both cases, but nevertheless subjects behave differently. Thus, not only the point predictions are wrong, but even the comparative statics implication that behavior should not be affected by the parameter change fails to hold.

GH and various companion papers provide explanations for some of the observed deviations from Nash behavior in these experiments and in related work.[3] These explanations differ across experiments. In several cases, GH appeal to the quantal response equilibrium (QRE) of Mc Kelvey and Palfrey (1995) which does not presuppose that players choose best responses to the expected behavior of others, but allows for the possibility of errors. In other examples, they argue that social preferences can explain the paradoxes. Finally, in cases with multiple equilibria that are unchanged by the parameter change, they show that selection theories, based on risk dominance and potential maximization help to understand the observations.[4]

[1] For instance, subjects only rely on iterated elimination of dominated strategies to a limited extent (Beard and Beil 1994). Deviations from the Nash prediction also occur in games where social preferences matter, including public goods games (Ledyard 1995), ultimatum games (Güth et al. 1982) and trust games (Fehr et al. 1993).

[2] Samuelson himself points out the limitations of his statement, mentioning bargaining experiments of Ochs and Roth (1989) where the effects of the discount factor and the length of the game are inconsistent with standard predictions.

[3] Related papers include Anderson et al. (2001, 2002), Capra et al. (1999), Goeree and Holt (2005) and Goeree et al. (2003).

[4] Another promising approach to understanding the GH paradoxes was provided by

This paper presents a unified explanation of the treatment effects in several GH puzzles, *without making any attempt to provide point predictions*. I start from a simple observation that has gone unrecognized in the literature: 6 of the 10 pairs of experiments analyzed by GH share important structural properties. First, for suitable partial orders on strategy spaces they are games with strategic complementarities (GSC): Both players' best responses are weakly increasing in the actions of the other player. Second, an increasing difference condition (ID) holds: In one of the treatments (H), for each initial strategy profile, the incremental payoff from increasing the own action is weakly higher than in the other one (L). These two properties combined give a clear intuition why players are likely to choose higher actions in H than in L. First, because incremental payoffs are higher in H than in L, incentives to increase actions are higher in H for fixed behavior of the other player. Second, if players accordingly believe that the opponents will choose higher actions in game H, this reinforces the tendency to choose high actions by GSC. Based on these two structural properties of the game, it is therefore intuitive to predict that actions are weakly higher for H than for L, even though direct calculation of Nash equilibria predicts no change.

Crucially, the direction of change in the six GH puzzles satisfying strategic complementarities (SC) and ID is always predicted correctly in this fashion. In addition, a similar structure-based prediction in another GH example that is not a GSC is confirmed by the data.[5] In the remaining three cases, this heuristic *structural approach* does not yield the wrong predictions. It is not applicable, because the games are too complex to allow for comparative statics results that are based purely on the structural properties of the game.

The very fact that the structural approach is intuitive and provides correct predictions for 7 out of 10 GH examples (and many other similar experiments) might be regarded as a sufficient justification for its use. Nevertheless,

Eichberger and Kelsey (2007) who appeal to ambiguity aversion to explain the deviations from equilibrium behavior.

[5] This example (generalized matching pennies) is not a GSC, but is simple because the parameter only enters the payoffs of one player.

I offer several possible foundations. The first one is most closely related to existing literature. According to well-known *monotone comparative statics* results of Milgrom and Roberts (1990) and Vives (1990), the smallest and the largest Nash equilibrium of a parameterized GSC satisfying ID are weakly increasing in the parameter. Thus, by focusing on such general structural properties rather than on the specific payoff functions, one can obtain the *weak* comparative statics prediction identifying the direction of change *provided there is any change at all*. Whether the equilibrium changes, is not part of the prediction: The result for the GH examples that the equilibrium does not change is also consistent with such weak predictions. Thus, even though players do not play the Nash equilibrium in every single game, the (weak) comparative statics are predicted accurately by results that are based on the general structural properties of the Nash equilibrium rather than on the specific details. One reason why this is so may be that subjects are playing the Nash equilibrium of some game belonging to a wider class with the same structural properties. For instance, suppose actual payoffs result from a perturbation of monetary payoffs (for instance, because players have social preferences). The perturbation does not have to be small, as long as it does not destroy the basic structural properties. Then, the Nash equilibrium of the perturbed game still satisfies the weak comparative statics predicted by the structural approach. As long as one is exclusively concerned with comparative statics rather than point predictions, the source of the perturbation is irrelevant.

As another justification of the structural approach, I show that the QRE satisfies the same weak comparative statics as the Nash equilibrium: In GSC satisfying ID, if the parameter increases, the equilibrium weakly increases in the sense of first-order stochastic dominance. Thus, in spite of the well-known differences between the Nash equilibrium and the QRE, they provide similar comparative statics predictions in an important class of games.

Next, I show that a simple set of behavioral adjustment rules leads to the same comparative statics as the structural approach (and the logit equilibrium) and is thus also consistent with the experimental evidence. These adjustment dynamics share certain properties with well-known dynamics that are derived from standard Cournot best-response dynamics,[6] but they do not necessarily require that the adjustment process is justifiable via best responses.

Finally, I show that, in symmetric games with ID and multiple parameter-independent equilibria, the comparative statics predictions implied by equilibrium selection by risk dominance and potential maximization are consistent with the approach proposed here.

To sum up, the main message of the paper is that several different theoretical approaches can all rationalize existing comparative statics puzzles. This obviously makes it hard to discriminate between these theories. However, it is good news in the sense that, for an important class of games, we can quite confidently predict the direction of treatment effects, because the predictions can be based on a wide variety of different arguments.

In Section 2, I will sketch three of the GH examples. In Section 3, I will introduce the structural approach as a heuristic. Sections 4 and 5 relate the approach to the Nash equilibrium and the QRE, respectively. Section 6 derives the approach from adjustment dynamics. Section 7 discusses the relation to selection theories. Section 8 concludes.

2 Introductory examples

I shall first sketch three of the ten GH examples.

(i) In the *Kreps game*, players choose actions from $X_1 = \{0, 1\}$ and $X_2 = \{0, 1, 2, 3\}$, respectively. Table 1 gives payoffs, where $\theta \in \mathbb{R}^+$.

For all $\theta \in \Theta$, there are two pure Nash equilibria ($(0, 0)$ and $(1, 3)$). In addition, there is a mixed-strategy equilibrium where player 1 chooses $x_1 = 0$ with probability 30/31, and player 2 chooses $x_2 = 0$ with probability 1/21 and $x_2 = 1$ with probability 20/21. Thus, an increase of θ does not affect

[6] See, e.g., Milgrom and Roberts 1990, Echenique 2002.

	$x_2 = 0$	$x_2 = 1$	$x_2 = 2$	$x_2 = 3$
$x_1 = 0$	$200, 50$	$0, 45$	$10, 30$	$20, -250$
$x_1 = 1$	$0, -250$	$10, -100$	$30, 30$	$\theta + 50, \frac{6}{5}\theta + 40$

Table 1: Kreps Game

the equilibrium structure. However, GH report the following results. For $\theta = 0$, 32% of the subjects in the role of player 1 chose the high action 1; whereas 96% did so for $\theta = 300$. For $\theta = 0$, no subject in the role of player 2 chose $x_2 = 3$, but 84% did so for $\theta = 300$. Thus, the experimental evidence suggests that, as θ increases, more subjects choose high actions.

I am interested in this particular comparative statics observation of GH.[7] One could of course explain it with selection arguments, based for instance on payoff dominance. However, my goal is to find an explanation of treatment effects that also applies to games with unique parameter-independent Nash equilibria such as the following.

(ii) In the *Traveler's Dilemma*,[8] two players $i = 1, 2$ simultaneously choose integers $x_i \in \{180, ..., 300\}$. Each player is paid the minimum of the chosen numbers; in addition, the player with the lower number receives a transfer $R > 1$ from the player with the higher number. Therefore, defining $\theta = -R$,

$$\pi_i(x_i, x_j; \theta) = \min(x_i, x_j) + \theta \cdot sign(x_i - x_j). \qquad (1)$$

The dots on the lines in Figure 1 give the reaction functions for any $\theta \in \Theta = (-\infty, -1)$. Thus, for all θ the game has a unique Nash equilibrium $x_1 = x_2 = 180$.[9] GH considered $\theta = -5$ and $\theta = -180$.[10] For $\theta = -180$, 80% of the subjects chose actions between 180 (the minimum) and 185, whereas 80% choose actions between 295 and 300 (the maximum) for $\theta = -5$. Thus,

[7]GH emphasize that for $\theta = 0$ many subjects (68%) choose $x_2 = 2$, the only action that is neither part of a pure-strategy equilibrium nor of a mixed-strategy equilibrium.

[8]The game goes back to Basu (1994).

[9]This equilibrium is also the unique rationalizable strategy profile.

[10]Similar results have been obtained by Capra et al. (1999) for other parametrizations.

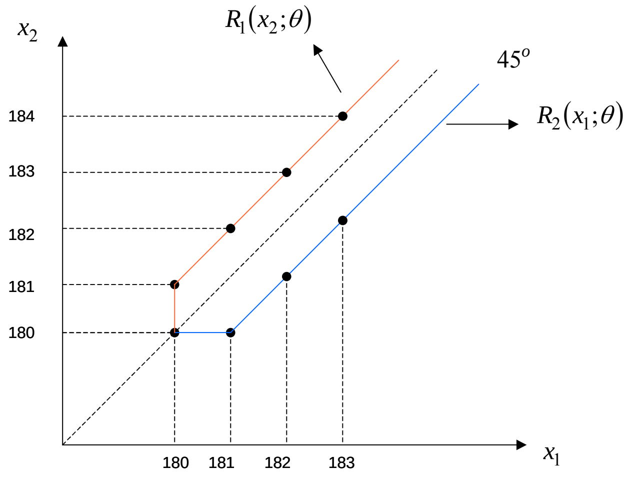

Figure 1: Traveler's Dilemma

as in the *Kreps Game*, even though the Nash equilibrium is independent of θ, a parameter increase induces higher actions.

(iii) In the *common-interest proposal game* (GH, Figure 3), two players move sequentially, according to the game tree in Figure 2.[11] Thus, the strategy spaces are $X_1 = X_2 = \{0, 1\}$. The parameter space is $\Theta = (0, 60)$. For all $\theta \in \Theta$, the unique subgame perfect equilibrium is $x_1 = x_2 = 0$. GH considered $\theta = 0$ and $\theta = 58$. For $\theta = 0$, 84% of the subjects in the role of player 1 and all the subjects in the role of player 2 chose the equilibrium actions $x_i = 0$. For $\theta = 58$, however, the corresponding figures are only 46% and 75% respectively. Hence, higher parameter values lead to higher actions.

Summing up, the following cases arise in the examples: (i) multiple pure-strategy equilibria, (ii) a unique pure-strategy equilibrium, or (iii) a unique subgame-perfect equilibrium. In all the examples, however, the set of pure-strategy equilibria is parameter-independent, but there are nevertheless clear treatment effects.

[11] I use the name "common-interest proposal game", because $(0, 0)$ is the optimal outcome for both players.

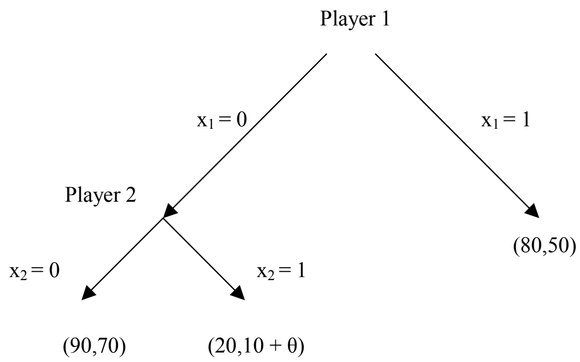

Figure 2: A Common-Interest Proposal Game

3 The structural approach

I will now introduce the heuristic *structural approach* to predict treatment effects even when the set of Nash equilibria is independent of treatments, as in the above examples. To repeat, the approach makes no attempt to explain why the observed play corresponds closely to the equilibrium in one case, but not in the other; it merely predicts the direction of change in behavior across treatments, not the relation to the equilibrium in any single experiment.

3.1 Defining the structural approach

In all the examples, there are players $i = 1, 2$, strategy spaces X_i and payoff functions $\pi_i(x_i, x_j, \theta)$, where $\theta \in \Theta$, a partially ordered set, such that:

1. X_i is independent of θ;

2. X_i is a finite set;[12]

[12] This assumption can be weakened considerably at the cost of greater technicalities. For the purposes of interpreting the experimental evidence, the set-up is sufficiently general.

3. X_i is equipped with a partial order $\geq$ that is independent of θ, with respect to which X_i forms a lattice.[13]

The following properties of the game are crucial.

Definition 1 *(i) π_i satisfies increasing differences in $(x_i;\theta)$,(ID), if*

$$\Delta_i\left(x_i^H, x_i^L; x_j;\theta\right) \equiv \pi_i\left(x_i^H, x_j;\theta\right) - \pi_i\left(x_i^L, x_j;\theta\right)$$

is weakly increasing in θ, that is, $\Delta_i\left(x_i^H, x_i^L; x_j;\theta^H\right) \geq \Delta_i\left(x_i^H, x_i^L; x_j;\theta^L\right)$ for all $x_i^H > x_i^L$, $\theta^H > \theta^L$, $i = 1, 2$, $j \neq i$.
(ii) π_i is supermodular (SUP) if $\Delta_i\left(x_i^H, x_i^L; x_j;\theta\right)$ is weakly increasing in x_j for all $x_i^H > x_i^L$, $i = 1, 2$, $j \neq i$.

By (i), an increase in θ has the direct effect of weakly increasing the incremental payoff for each player. Thus, for fixed behavior of the other player, increasing own actions becomes (weakly) more attractive, so that reaction functions are weakly increasing in θ.[14] By (ii), the payoff increase from increasing x_i is non-decreasing in x_j for $j \neq i$. Thus, the optimal response of player i is weakly increasing in x_j, that is, the game is a GSC. The positive direct effects of higher θ on x_i and the induced indirect effects on x_j are mutually reinforcing. Together, ID and SUP therefore suggest a (weakly) positive effect of θ on actions. I thus introduce the following central tool for predictions.

Definition 2 *For a GSC which satisfies ID with respect to $\theta \in \Theta$, the* ***structure-based prediction*** *of treatment effects is that the frequency distribution of observed play for θ^H weakly dominates the corresponding distribution for $\theta^L < \theta^H$ according to first-order stochastic dominance (FOSD).*[15]

[13] A lattice requires that the infimum and supremum of each pair of elements exists in X_i. In the following, the lattice structure will typically come from a complete order on a finite set.

[14] A formal version of this statement relies on Lemma 1 in the Appendix.

[15] In a finite game, this reduces to the requirement that, as θ increases, the fraction of players choosing an action up to and including any predetermined level of x_i weakly decreases.

3.2 Experimental evidence

The first justification for such structure-based predictions is that they are applicable in many examples, and that they are confirmed in this examples. As an illustration, take the Kreps game. Straightforward derivations show that this game satisfies SUP and ID with respect to the standard (total) orders on X_1, X_2 and θ.[16] The structure-based prediction is thus that for $\theta^H = 300$ players tend to choose higher actions than for $\theta^L = 0$. This is precisely the observed outcome. The overly strong independence prediction obtained by simple comparison of Nash equilibria for different parameter values is a boundary case of the structure-based prediction that the equilibrium is weakly increasing in θ. While the independence prediction does not survive empirical scrutiny because a change in θ changes the observed actions, the weaker comparative statics prediction is consistent with the facts.

This argument illustrates the central message of the paper: By ignoring details of the game and focusing instead on basic structural properties, one often obtains a weak prediction of treatment effects that is consistent with the evidence. The structural approach is a powerful tool for explaining comparative statics puzzles. For instance, the same logic can be applied to five other GH examples. Most immediately, the common-interest proposal game, the related *conflicting-interest proposal game*[17] and the *extended coordination game* also satisfy SUP and ID with respect to suitable parameters and partial orders.[18] In all three cases, like in the Kreps game, there are clear treatment effects, even though the equilibrium set is independent of θ.

Two other GH games, the traveler's dilemma and an auction game, are not supermodular, but nevertheless GSC. To illustrate, consider the traveler's

[16] As to ID, for both players, an increase in θ raises the benefit from choosing the highest action ($x_1 = 1$ and $x_2 = 3$) rather than any other one, whereas there is no relation between θ and the benefit for player 2 from increasing x_2 from 0 to 1 or 2, or from 1 to 2. As to SUP, for instance for player 1, the incremental payoffs increase from -200 to 10, 20 and finally $\theta + 30$ as player 2 increases his actions from 0 to 3.

[17] I use this term for the game described in Figure 4 of GH.

[18] Details of the arguments are available upon request.

dilemma. Because ID still holds,[19] a reduction in the transfer parameter R, or equivalently, an increase in θ, increases incremental payoffs. Hence, even though θ has no effect on the reaction function in the specific example, the game structure suggests that player i's reaction to x_j is weakly increasing in θ.[20] The traveler's dilemma corresponds to the boundary case where the reaction functions are unaffected by the parameter change even though ID holds. Ignoring all details of the game structure except ID and SC suggests that a parameter increase has the direct effect of increasing actions for both players, and that these effects are mutually reinforcing, so that actions should increase with θ, as required by the structure-based prediction.

Beyond the GH examples, many authors have investigated coordination games, which can be addressed similarly. As an example, consider an *effort coordination game*[21] with payoffs

$$\pi_i\left(x_i, x_j; \theta\right) = \min\left(x_i, x_j\right) + \theta \cdot x_i,$$

where $x_i \in \{0, 1, ..., M\}$ and $\theta = -c$ for some effort cost parameter $c \in (0, 1)$. For $c < 1$, the set of pure-strategy equilibria is the diagonal ($x_1 = x_2$). Thus, if one uses the set of pure-strategy equilibria to predict responses to parameter changes, increases in costs should have no effect on equilibrium effort. The comparative statics become more counter-intuitive if one allows for mixed-strategy equilibria. For instance, for $X_i = \{0, 1\}$, there is an equilibrium such that each player chooses $x_i = 1$ with probability c. Thus, as

[19] To see this, first note that, because of the term $\min\left(x_i, x_j\right)$ in the payoff function, there is an incentive to choose high actions. The term $\theta \cdot sign\left(x_i - x_j\right)$ acts as a counterbalance, but less so as θ approaches zero from below. Therefore, the incremental payoff from increasing x_i is non-decreasing in θ.

[20] Again, Lemma 1 in the Appendix provides the formal justification of this argument.

[21] Several authors have analyzed the effects of changing various parameters in other 2×2-coordination games satisfying (SUP) and (ID). For instance, in the experiments of Huettel and Lockhead (2000), Schmidt et al. (2003), and most of the experiments of Guyer and Rapoport (1972), the comparative-statics predictions correspond exactly to those obtained from the structural approach, and the arguments are similar as in the following discussion of effort coordination games. The propositions of this paper are not applicable for the "Benefit-to-other"-treatment of Guyer and Rapoport, because (ID) does not hold.

costs increase, agents put more weight on the high effort level, so that, paradoxically, effort *increases* with costs. Unsurprisingly, experimental results (van Huyck et al. 1990; Goeree and Holt 2005) show that for lower c more subjects choose higher effort. The structural approach resolves the tension between theoretical predictions and empirical observations. Effort coordination games are supermodular, because the net benefit from increasing effort is $1 - c > 0$ if the original effort level is smaller than the effort of the other player, and $-c < 0$ otherwise. Thus, π_i satisfies ID. Therefore, the structural prediction is that actions are weakly increasing in θ.

Another application concerns public goods experiments (e.g., Ledyard 1995), which display clear behavioral effects of the return on investment which again cannot be captured by comparison of Nash equilibria. The games satisfy ID, and SUP holds trivially because payoffs are additively separable.

The very fact that the predictions of the structural approach are consistent with the experimental evidence is a strong argument in its favor. In addition, the intuition for this observation is straightforward. Even subjects who, for whatever reason, do not display Nash behavior, are likely to understand the two basic structural properties: (i) High incremental payoffs make high actions attractive for given actions of the other player; (ii) incremental payoffs increase with the other player's action. If players understand these two properties, and if they believe that other players do so, too, then they should choose high actions for high parameter values.

Before turning to more precise justifications of the structural approach, I note that a slight modification of the idea can be used to show that a seventh GH example, the *generalized matching pennies* game, can be explained along similar lines, even though it is not a GSC (See Appendix 2 for details).[22]

[22]In this example, with an appropriate order on strategy spaces, a higher parameter increases the equilibrium action of one player, but leaves the action of the other player constant; while observed actions of both players are affected. Any order on the strategy space implies that the actions are SC for one player, but strategic substitutes (SS) for the other one. However, because the parameter only affects one of the two payoff functions, an intuitive structure-based prediction of treatment can be given even so, and this intuitive

Nash Prediction	Game	Observed Actions	Reason
Unique pure Nash equilibrium independent of θ	Traveler's dilemma (Capra et al. 1999, GH) Public goods games (Ledyard 1995)	Increasing in θ	SC + ID SUP + ID
Unique SPE independent of θ	Proposal games (GH Fig. 3 and 4)	Increasing in θ	SUP + ID
Unique mixed equilibrium: increasing in θ for player 2, constant for player 1	Matching pennies (Ochs 1995, GH)	Player 2: increasing Player 1: decreasing	SC/SS ID
Unique Bayesian Equilibrium independent of θ	Auction game (GH)	Increasing in θ	SC + ID
Multiple pure equilibria; mixed equilibrium is independent of θ	Kreps game (GH) Extended coordination game (GH)	Increasing in θ	SUP + ID
Multiple pure Nash equilibria where mixed equilibrium is decreasing in θ	Effort coordination (Van Huyck et al. 1990, Goeree and Holt 2005) Wolf's dilemma (Huettel-Lockhead 2000)	Increasing in θ	SUP + ID
Period-2 equilibrium independent of first-period play	Capacity game (Brandts et al. 2003)	Period-2 actions increasing in own period-1 action, decreasing in opponent's.	SUP + ID

Table 2: Summary of Results

As summarized in Table 2, the structural approach can thus explain the evidence in seven of the ten examples provided by GH. In the remaining cases, it does not provide a false prediction. It is simply not applicable because the games do not have suitable structural properties. Loosely speaking, the direct and indirect effects of parameter changes are not mutually reinforcing, so that general comparative statics results cannot be derived.

4 Nash equilibrium and structural approach

A first more formal justification of the structural approach relies on a familiar comparative statics results for the Nash equilibrium that was shown for supermodular games by Milgrom and Roberts 1990, but extends to GSC.[23]

Proposition 1 *Suppose ID and GSC hold. Then*
(i) A smallest and a largest pure-strategy Nash equilibrium exist.
(ii) For both equilibria, the actions of each player are weakly increasing in θ.

Statement (i) not only guarantees existence of a Nash equilibrium, but also makes sure that the smallest and largest equilibrium are well-defined.[24] Part (ii) provides a comparative statics prediction that is fully in line with the empirical evidence for the six GH-games that are GSC, namely that, if the equilibrium changes, it should move upwards. As a boundary case, Proposition 1 contains the prediction derived from direct calculation of the Nash equilibria in the GH-examples that the actions do not change with θ. However, consideration of the structural properties, "forgetting" the details of the payoff functions, suggests that, if actions change at all, they should increase with θ. One way to make this point more precise is to consider the following more specific argument.

prediction can be justified as in the GSC case.

[23] See Vives (1999, p. 35).

[24] The smallest equilibrium exists if and only if the profile consisting of the minimal equilibrium action for each player is itself an equilibrium; similarly for the largest equilibrium. Of course, for some or all values of θ, the smallest and largest equilibrium may coincide.

4.1 Nash equilibria of perturbed games

Suppose actual payoffs are perturbations of monetary payoffs, for instance, because players have social preferences. Specifically, suppose that instead of the monetary payoff functions π_i, players have objective functions

$$\widehat{\pi}_i(x_i, x_j; \theta) = \pi_i(x_i, x_j; \theta) + g_i(x_i, x_j; \theta), \tag{2}$$

such that π_i satisfies SC and ID and the perturbation g_i is such that $\widehat{\pi}_i$ still has this property. This could be true because g_i is small, but this is by no means the only possibility. For instance, if π_i and g_i both satisfy ID and SUP in (x_i, θ), so does $\widehat{\pi}_i$, no matter how large the perturbation is. Clearly, Proposition 1 can still be applied: Hence, even though the Nash equilibrium for $\widehat{\pi}_i$ may differ substantially from the equilibrium for π_i, the equilibrium is still weakly increasing in θ.

Of course, to obtain point predictions, the perturbation would have to be specified. As long as one is exclusively concerned with weak comparative statics, however, there is no need to do so. Any specification of perturbations such that actual payoffs $\widehat{\pi}_i$ and monetary payoffs π_i both have properties SC and ID justifies weak comparative statics conclusions for $\widehat{\pi}_i$ based on the structural properties of π_i. However, only suitable perturbations will guarantee that the equilibrium set for $\widehat{\pi}_i$ is increasing in θ, rather than merely non-decreasing. In situations where the equilibrium structure is independent of θ for π_i, but the observed behavior is increasing, the correct choice of $\widehat{\pi}_i$ should capture this possibility.

To illustrate the idea, consider the effort coordination game with $X_i = \{0, 1\}$. Let $k > 0$ and consider the effort coordination game with two effort levels and perturbation term $g_i(x_i, x_j; \theta) = k \max(x_j - x_i, 0)$. Thus, agents display spiteful behavior, gaining utility if the other player has exercised useless effort. The payoff matrix of the perturbed game is given in Table 3. Suppose $k < 1$. Then the perturbed game still satisfies ID and SUP, and Proposition 1 predicts that the smallest and largest equililibrium are both weakly increasing in θ. Closer inspection reveals that the equilibrium set depends on θ: For high costs ($\theta < -(1-k)$), the only equilibrium is $(0,0)$. For low costs ($\theta > -(1-k)$), there are multiple equilibria, including $(1,1)$ as well. Thus, with this specification of preferences, the largest equilibrium is *strictly* increasing in θ.

Summing up, for both π_i and $\widehat{\pi}_i = \pi_i + g_i$ the smallest and largest equilibrium are weakly increasing in θ. However, it is independent of θ for π_i, whereas it is strictly increasing in θ for $\widehat{\pi}_i$. Thus, if the true behavioral model is given by $\widehat{\pi}_i$, weak comparative-statics conclusions derived from the properties ID and SC of π_i are correct, but for the stronger conclusion that the equilibrium set changes with θ, it is necessary to have a perturbation of π_i (e.g., by g_i).

	$x_2 = 0$	$x_2 = 1$
$x_1 = 0$	$0, 0$	k, θ
$x_1 = 1$	θ, k	$1+\theta, 1+\theta$

Table 3: Payoffs in the Perturbed Effort Coordination Game Game

Of course, the interesting aspect of this section is not that there exist suitable perturbations for which the comparative-statics can be rationalized. In view of the substantial degrees of freedom, this should be expected.[25] It is much more interesting that the weak comparative statics carry over for wide classes of perturbations.

5 The quantal response equilibrium

The most popular approach to the GH puzzles is based on the *quantal response equilibrium* (QRE) introduced by Mc Kelvey and Palfrey (1995). This concept does not presuppose best responses; instead players can make errors. Specifically, consider a finite game with strategy spaces $X_i = \{0, 1, ..., n_i\}$; denote the probabilities with which player i chooses action x_i as $p^i_{x_i}$. Let

[25] See also the related discussion of the QRE below.

$\boldsymbol{\varepsilon}_i = (\varepsilon_{i1}, ..., \varepsilon_{in_i})$ be a vector of perturbations for player i, drawn from a joint density f_i. Then, by assumption, player i chooses $\nu \in X_i$ if and only if ν maximizes the sum of the expected payoff and the perturbation, that is,

$$\sum_{x_j=0}^{n_j} p^j_{x_j} \pi_i (\nu, x_j; \theta) + \varepsilon_{i\nu} \geq \sum_{x_j=0}^{n_j} p^j_{x_j} \pi_i (x_i, x_j; \theta) + \varepsilon_{ix_i} \; \forall x_i \neq \nu. \qquad (3)$$

Using this condition, one immediately arrives at the *stochastic best-response function* or *quantal response function* that assigns to each probability vector $\mathbf{p}^j$ for player j the probability vector $\mathbf{p}^i = \mathbf{p}^i(\mathbf{p}^j; \theta)$ of choices for player i defined by the requirement that $\boldsymbol{\varepsilon}_i$ satisfies (3). A QRE requires that each player's own error distribution is consistent with stochastic best response.

The next comparative statics result shows that the similarity in the predictions of the structural approach and the QRE is not a coincidence. As in Haile et al. (2007), I assume that a parameter shift leaves the error distribution unchanged; this *invariance assumption* is discussed by these authors.

Proposition 2 *Suppose a finite game with strategy spaces $X_i = \{0, 1, ..., n_i\}$ satisfies SUP and ID. Suppose for a fixed error distribution, a unique QRE $\mathbf{p}(\theta) = (\mathbf{p}^1(\theta), \mathbf{p}^2(\theta))$ exists for every θ. Then, an increase in θ shifts the equilibrium distribution $\mathbf{p}(\theta) = (\mathbf{p}^1(\theta), \mathbf{p}^2(\theta))$ weakly according to first-order stochastic dominance (FOSD).*

Thus, a parameter increase in a game satisfying (SUP) and (ID) implies that higher choices become more likely for the logit equilibrium. The intuition for the result is quite similar to the intuition for Proposition 1. As the parameter θ shifts upwards, increasing differences imply that, for fixed behavior of the opponent, players are more likely to respond with higher actions. Anticipating this, it becomes even more attractive for both players to increase their actions.

Paralleling the discussion in the previous section, the interesting aspect of Proposition 1 is not that that there exists some kind of QRE for which the comparative statics property holds, but that this holds quite generally (under the invariance assumption). Haile et al. (2008) have shown that, because of the degrees of freedom in specifying the error distributions, the QRE can explain any behavior in any given game. Nevertheless, as the authors themselves point out, the QRE may still put restrictions on possible comparative statics under the invariance assumption. However, Haile et al. (2008, Theorem 2) follow a completely different approach: They are interested in observing whether the behavior observed ex-post is consistent with agents putting greater weight on actions that, given the observed distribution of play, have benefited more from the parameter increase than others in terms of expected payoff. To the contrary, Proposition 1 gives comparative statics predictions purely on the basis of structural properties that are known ex ante.

6 Adjustment dynamics

The next contribution of the paper involves a more radical suggestion: To derive comparative statics results, it is not essential to apply an equilibrium concept. All that is required is an appropriate theory of change. This can best be explained by starting from standard adjustment theories, as presented for instance in Milgrom and Roberts (1990), Milgrom and Shannon (1994) and Echenique (2002). As the simplest case, take the framework of Section 3.1, and assume that best-responses are unique. Define Cournot best-response dynamics after a parameter increase from θ^L to θ^H as a sequence $\{x^k\} = \{x_1^k, x_2^k\}$; $k = 0, 1, ...$, as follows:

(ADJ1) $x^0 \in x(\theta^L)$, the old equilibrium set of the game

(ADJ2) For $k \geq 1$, $x^k = \phi(x^{k-1})$, that is, x^k results from best responses to x^{k-1}.

Then, for GSC, these adaptive dynamics have the following properties if the objective functions satisfy ID.

(ADJ3) $x^1 \geq x^0$ (because of ID)

(ADJ4) $x^{k+1} \geq x^k$ (because of GSC)

(ADJ5) $\lim_{k\to\infty} x^k \in \left[\inf x(\theta^H), \sup x(\theta^H)\right]$

Thus $\left\{x^k\right\}$ is monotone increasing, and hence $\lim_{k\to\infty} x^k \geq x^0$. The existing literature has shown that, for GSC, the limits of adjustment dynamics have similar properties, even when best-responses are non-unique and adjustment dynamics are more general. For instance, Milgrom and Roberts (1990) consider adjustment dynamics in supermodular games such that, for any date, there is a later date after which players select a strategy that is justifiable as a best response to behavior that is in the interval of past play. They show that all accumulation points are in the interval bounded by the smallest and largest new equilibrium. For similar "generalized adjustment dynamics",[26] Echenique (2002), shows that, if play starts below the smallest best response in a GSC (for instance, because of an upward parameter shift), then the set of limits is contained in the same interval.

To provide restrictions on the limit of such adjustment processes in terms of past play, as in (ADJ5), it is clearly necessary to impose conditions such as (ADJ1) and (ADJ2) or mild generalizations that relate adjustment behavior to best responses. For the weaker conclusion required here that the limit of the adjustment sequence exists and lies above the starting point, much weaker requirements are sufficient. This conclusion holds trivially for arbitrary sequences $x^k = \phi(x^{k-1})$ such that:

(ADJ6) The immediate response to a change of parameter is weakly positive ($x^1 \geq x^0$)

(ADJ7) ϕ is increasing.

These conditions can be justified without direct reference to (ADJ1) and (ADJ2) or even without assuming that choices are justifiable in terms of past

[26] Echenique makes the stronger requirement that play is *always* in the interval of best-responses to previous play (not only eventually).

play. Suppose that observed behavior in the treatment corresponding to θ^L is given by x^0, which can be below or above the old equilibrium (for instance, because players have perturbed objective functions). Next postulate that x^1 reflects the immediate adjustment to change, not taking into account possible adjustments of the other player. This adjustment may or may not be a best response to x^0. For instance, instead of resulting from a best response for monetary payoffs π_i, it could reflect best responses for $\widehat{\pi}_i$ (as in Section 4.1). If ID is satisfied, then the incremental value of increasing the action for fixed behavior of the other players increases. Even if the adjustment does not come from a best response, one should therefore expect that $x^1 \geq x^0$, as required by (ADJ6). Indeed, using Lemma 1, this conclusion will for instance be true if players best-respond to the previous choices with the perturbed payoffs $\widehat{\pi}_i$ rather than π_i. Thus, each player's immediate response to a parameter increase should be to increase the own action weakly. Similarly, (ADJ7) can be justified as reflecting indirect effects, even when the adjustment does not result from best responses: SUP guarantees that a player i who thinks that the other player has the immediate impulse to increase his actions following a parameter increase ($x_j^1 \geq x_j^0$) should realize that the marginal value of increasing an action increases even further. This would imply $x^2 = \phi(x^1) \geq \phi(x^0) = x^1$ which is (ADJ7) for $k = 1$.[27] Iterating the argument leads to a justification of (ADJ7) for arbitrary values of k.

To sum up, in a GSC satisfying ID, actions should increase after a parameter increase even if the adjustment dynamics does not explicitly follow from best-responses with monetary payoffs. As long as players increase their actions both as a direct response to a parameter increase and as an indirect response to higher actions of other players, which is natural under ID and SUP, actions will increase. Of course, convergence to a Nash equilibrium is not guaranteed without more specific assumptions. At any point of the sequence, behavior may be below or above the equilibrium set, depending on

[27] Again, this conclusion can be derived formally, by assuming that players best-respond with a perturbed payoff function as in Section 4.1.

whatever biases the players have.

7 Equilibrium Selection

The structural approach fits nicely with selection criteria such as risk dominance (Harsanyi and Selten 1988). Consider a *symmetric game* with $X_i = \{0,1\}$, that is, as in effort coordination games, payoff functions are such that $\pi_1(x', x''; \theta) = \pi_2(x', x''; \theta)$ for arbitrary $(x', x''; \theta) \in \{0,1\} \times \{0,1\} \times \Theta$. Suppose there are two pure-strategy equilibria $(0,0)$ and $(1,1)$. $(0,0)$ is *risk dominant* if both players prefer 0 if they expect the other player to choose 0 and 1 with probability 1/2 each. In effort games, risk dominance predicts that equilibria with higher effort levels are chosen as costs decrease (Goeree and Holt 2005). More generally, the comparative statics implied by the structural approach and by risk dominance coincide, as the following simple result shows.[28]

Proposition 3 *Consider a symmetric game with $X_i = \{0,1\}$, such that ID holds for the standard order on $\{0,1\}$. Suppose that the set of Nash equilibria is $\{(0,0), (1,1)\}$ for all $\theta \in \Theta$. Then, if $(1,1)$ is selected by risk dominance for θ^L; it is also selected for $\theta^H \geq \theta^L$.*

Proof. See Appendix. ■

An alternative approach to equilibrium selection that generalizes to games with more than two players and continuous actions is available for potential games (Monderer and Shapley 1996, Goeree and Holt 2005). Such games are characterized by the existence of a potential $V(x_1, x_2; \theta)$ with the defining property that $\pi_1(x_1'', x_2; \theta) - \pi_1(x_1', x_2; \theta) = V(x_1'', x_2; \theta) - V(x_1', x_2; \theta)$ for all x_1', $x_1'' \in X_1$, $x_2 \in X_2$, $\theta \in \Theta$, and analogously for π_2.[29] Potential-maximizing

[28] The discussion in Section 4.2 shows that, in symmetric games, (ID) suffices to generate monotone comparative statics.

[29] With continuously differentiable games, this boils down to the requirement that the partial derivatives of V with respect to each x_i coincide with those of $\pi_i(x_i, x_j; \theta)$.

strategy profiles are pure-strategy equilibria, but the converse is not necessarily true (Monderer and Shapley 1996): In games with multiple equilibria such as effort coordination games, there is typically a unique potential-maximizing profile which can be used for equilibrium selection. Monderer and Shapley (1996) have already argued that, in effort games, the observed effects of increasing costs can be explained using potential maximization, showing that, in the experiments of van Huyck et al. (1990), potential maximization selects the lowest equilibrium for high effort costs and the highest equilibrium for low effort costs. This is true more generally.[30]

Proposition 4 *In a symmetric game satisfying ID, suppose* $\left(x^H, x^H\right)$ *and* $\left(x^L, x^L\right)$ *are unique potential maximizers for* θ^H *and* θ^L *(*$\theta^H \geq \theta^L$*), respectively. Then* $x^L \leq x^H$*.*

Proof. See Appendix. ■

Summing up, the *structural approach* yields comparative statics predictions that are compatible with standard selection methods where they apply.

8 Conclusions

I have introduced a heuristic "structural" approach to predict treatment effects even when Nash equilibria are the same in the different treatments. The resulting comparative statics predictions are supported by the experimental observations in all cases that I am aware of, in particular, in the GH examples. I have shown that the structural approach is consistent with the predictions of the QRE and, where it applies, of equilibrium theories. Finally, I explain treatment effects by reference to an adjustment process that does not require any equilibrium concept.

[30] Relatedly, Echenique (2004) shows that finite two player ordinal potential games are GSC, so that when ID holds, the smallest and largest equilibrium must increase weakly with parameters. Proposition 4 sharpens the result by showing that the same is true for the potential maximizers.

As GH have already explained some of the experimental observations under consideration, it is legitimate to ask why another approach is needed. First, the structural approach brings together two literatures that rarely speak to each other, namely the experimental literature and the literature on monotone comparative statics in games with strategic complementarities. Hopefully, this exercise contains potential for further cross-fertilization.[31] Second, the structural approach is more basic than the alternative suggestions: Without imposing a particular story about what subjects do for any given parameter value, it shows that structural properties of the game are useful to explain treatment effects. Third, I provide a unified explanation of seven of the ten GH examples which, to my knowledge, no other single approach does. While the QRE, for instance, has several useful applications elsewhere, I am only aware of two types of GH paradoxes that are explained by the concept.[32] In addition, as I indicate in the paper, my predictions are also borne out in many examples that were not treated by GH. Fourth, the paper provides a suggestion that is interesting in its own right: To understand comparative statics, it is not absolutely essential to have a theory of point predictions – it suffices to have a theory of reaction to change. Finally, though this was not detailed here, the approach can be applied to problems that do not concern comparative statics directly. For instance, Brandts et al. (2006) consider a two-stage game of capacity choice where the structural approach correctly predicts the effects of (endogenous) capacity choices on second-period actions.[33]

[31] A vaguely related experimental contribution of Chen and Gazzale (2004) demonstrates that learning in certain games with strategic complementarities, namely supermodular games, works particularly well. However, the authors do not treat comparative statics.

[32] GH use the QRE to explain the traveler's dilemma and the matching pennies game; in addition, it is useful for effort cost games.

[33] Two players can make costly, but not fully binding capacity commitments C_i before they decide on investments I_i. Payoff functions $\pi_i(I_i, I_j; C_i, C_j)$ are supermodular in $(I_i, -I_j)$ and have increasing differences in $(I_i, I_j; C_i, -C_j)$. The subgame equilibria in stage 2 are independent of first-period choices. Interpreting C_i and C_j as exogenous parameters of the ensuing subgame, however, the structural approach correctly predicts

It would be interesting to extend the approach to other solution concepts. A natural candidate is the cognitive hierarchy model (e.g., Camerer et al. 2004), which assumes that players differ with respect to the extent of strategic thinking they carry out. Like the QRE, the model is consistent with comparative-statics observations in the games discussed here, for instance in coordination games. It appears quite conceivable that general comparative statics results are also available for cognitive hierarchy models in GSC.

In spite of the large number of conceivable applications, it is important to recognize the limitations of the approach. First, obviously, it does not provide point predictions. Second, there are examples where the direct and indirect effects of parameter changes are not mutually reinforcing, so that no comparative statics predictions are possible without relying on the concrete specification. Third, I am convinced that cleverly designed experiments can show that there are some GSC satisfying ID, for which the observed actions are not increasing in the parameter. The challenge for future experimental work is to discover under which circumstances such violations of the structural approach will occur.

9 Appendix

9.1 Appendix 1: Proofs

The following well-known monotone comparative statics (Topkis 1978) result will be helpful.

Lemma 1 *Let $f\left((x,\tau\right)$ be a real-valued function defined on $X \times T$, where X is a complete lattice and T is a partially ordered set. Suppose f satisfies increasing differences with respect to (x,τ). Then $g(\tau) \equiv \arg\max_{x\in X}\ f\left((x,\tau\right)$ is a weakly increasing correspondence.*[34]

that I_i should be non-decreasing in C_i and non-increasing in C_j for $j \neq i$ according to Proposition 1.

[34] $g(\tau)$ is weakly increasing if $\tau^L < \tau^H$ implies $\min g(\tau^L) \leq \min g(\tau^H)$ and $\max g(\tau^L)$

In the following applications, X will correspond to the strategy set of one player; τ will be the strategy set of the other player or the parameter θ.

9.1.1 Proof of Proposition 2

Proposition 2 is a simple corollary of the following result.

Lemma 2 *Suppose $\pi_i(x_i, x_j; \theta)$ satisfies SUP and ID. Then*
(i) For fixed choice probabilities of the opponent, $\mathbf{p}^j$, an increase in θ shifts the stochastic best response $\mathbf{p}^i(\mathbf{p}^j;\theta)$ according to FOSD.
(ii) The stochastic best response $\mathbf{p}^i(\mathbf{p}^j;\theta)$ is weakly increasing in $\mathbf{p}^j$.

Proof. (i) For $\nu \in \{0, 1, ..., n_i\}$, the probability that $x_i \leq \nu$ is chosen is

$$P_\nu(\theta) = prob\left(\max_{x_i\in\{0,1,...,\nu\}} \sum_{x_j=0}^{n_j} p^j_{x_j}\pi_i(x_i, x_j;\theta) + \varepsilon_{ix_i} \geq \sum_{x_j=0}^{n_j} p^j_{x_j}\pi_i(x'_i, x_j;\theta) + \varepsilon_{ix'_i}\right) \quad \forall x'_i > \nu. \quad (4)$$

By ID,

$$\sum_{x_j=0}^{n_j} p^j_{x_j}\pi_i(x_i, x_j;\theta) - \sum_{x_j=0}^{n_j} p^j_{x_j}\pi_i(x'_i, x_j;\theta)$$

is weakly decreasing in θ for all $x'_i > \nu$. Because $\varepsilon_{ix'_i} - \varepsilon_{ix_i}$ is independent of θ by the invariance property, $P_\nu(\theta)$ is therefore weakly decreasing in θ.

(ii) Suppose $r < s \in \{0, ..., x_j\}$. It suffices to show that replacing any $\mathbf{p}^j$ by $\mathbf{p}^{j\varepsilon} \equiv \left(\mathbf{p}^j_0, ..., \mathbf{p}^j_r - \varepsilon, ..., \mathbf{p}^j_s + \varepsilon, ..., \mathbf{p}^j_{n_j}\right)$ for $\varepsilon \in (0, q_r]$ leads to an FOSD-shift in $\mathbf{p}^i$. This will be true if, as ε increases, $P_\nu(\theta)$ is weakly larger for $\varepsilon = 0$ than for $\varepsilon > 0$ for all $\nu \in \{0, 1, ..., n_i\}$. This holds, because (SUP) implies

$\leq \max g(\tau^H)$, where the inequalities on X refer to some arbitrary partial order.

$$\sum_{x_j=0}^{n_j} p_{x_j}^{j\varepsilon}\pi_i\left(x_i,x_j;\theta\right) - \sum_{x_j=0}^{n_j} p_{x_j}^{j\varepsilon}\pi_i\left(x_i',x_j;\theta\right) -$$

$$\sum_{x_j=0}^{n_j} p_{x_j}^{j}\pi_i\left(x_i,x_j;\theta\right) + \sum_{x_j=0}^{n_j} p_{x_j}^{j}\pi_i\left(x_i',x_j;\theta\right) = \tag{5}$$

$$\varepsilon\left(\pi_i\left(x_i,s;\theta\right) - \pi_i\left(x_i',s;\theta\right) - \pi_i\left(x_i,r;\theta\right) + \pi_i\left(x_i',r;\theta\right)\right) \leq 0. \tag{6}$$

■

Now note that, with FOSD as a partial order, $\mathcal{P}_i$ the set of distributions on X_i is a complete lattice (Echenique 2003, Lemma 1); this structure carries over to $\mathcal{P} = \mathcal{P}_i \times \mathcal{P}_j$. Further, by Lemma 2, the stochastic best response correspondence shifts out as θ increases. Denote the interval of probability vectors in $\mathcal{P}$ that are greater or equal to some $\mathbf{p}$ as $U(\mathbf{p})$. Since the best-response correspondence for $\theta^H > \theta^L$ is weakly increasing by part (ii) of the lemma, it maps $U(\mathbf{p}(\theta^L))$ into itself. Its fixed point must therefore satisfy $\mathbf{p}(\theta^H) \geq \mathbf{p}(\theta^L)$.

9.1.2 Proof of Proposition 3

In this symmetric setting, $(1,1)$ is selected by risk-dominance for θ^L if and only if

$$\pi_i\left(1,1;\theta^L\right) - \pi_i\left(0,1;\theta^L\right) \geq \pi_i\left(0,0;\theta^L\right) - \pi_i\left(1,0;\theta^L\right). \tag{7}$$

Applying ID to both sides of (7) shows that analogous inequalities hold with θ^L replaced with θ^H, so that $(1,1)$ is selected by risk-dominance for θ^H.

9.1.3 Proof of Proposition 4

For $x^L, x^H \in X_1 = X_2$, $\theta \in \left\{\theta^L, \theta^H\right\}$, $V\left(x^L,x^L;\theta\right) - V\left(x^H,x^H;\theta\right) =$
$V\left(x^L,x^L;\theta\right) - V\left(x^H,x^L;\theta\right) + V\left(x^H,x^L;\theta\right) - V\left(x^H,x^H;\theta\right)$.

Thus, the definition of the potential function implies

$$V\left(x^L, x^L; \theta\right) - V\left(x^H, x^H; \theta\right) =$$
$$\pi_1\left(x^L, x^L; \theta\right) - \pi_1\left(x^H, x^L; \theta\right) + \pi_2\left(x^L, x^H; \theta\right) - \pi_2\left(x^H, x^H; \theta\right).$$

Using ID, therefore, if $x^L > x^H$ (contrary to the assertion of the proposition), for $\theta^H > \theta^L$

$$V\left(x^L, x^L; \theta^H\right) - V\left(x^H, x^H; \theta^H\right) \geq V\left(x^L, x^L; \theta^L\right) - V\left(x^H, x^H; \theta^L\right).$$

Further, because x^L is the unique maximizer of the potential function for θ^L,

$$V\left(x^L, x^L; \theta^L\right) - V\left(x^H, x^H; \theta^L\right) > 0.$$

Taking the last two inequalities together, x^H cannot maximize the potential for θ^H, which contradicts its definition.

9.2 Appendix 2: Generalized matching pennies

Even for asymmetric games that do not satisfy SC, the structural approach is sometimes useful. A case in point is *generalized matching pennies*, with $X_i = \{0, 1\}$ and $\Theta = \{44, 80, 320\}$ and payoffs as in Table 4.

	$x_2 = 0$	$x_2 = 1$
$x_1 = 0$	$\theta, 40$	$40, 80$
$x_1 = 1$	$40, 80$	$80, 40$

Table 4: Payoffs in the Generalized Matching Pennies Game

Identify a mixed strategy of player i, σ_i, with the probability of choosing action 1. For all $\theta \in \Theta = \{44, 80, 320\}$, the reaction correspondence for player 2 is given by the same dashed line $R_2(\sigma_1; \theta)$ in Figure 3, while it depends explicitly on θ for player 1. The unique mixed-strategy equilibrium is $\sigma_1^* = \frac{1}{2}$, $\sigma_2^* = 1 - \frac{40}{\theta}$. Thus, unlike in the earlier examples, only player 1's equilibrium action is independent of θ: Player 2's choice x_2 is increasing in θ,

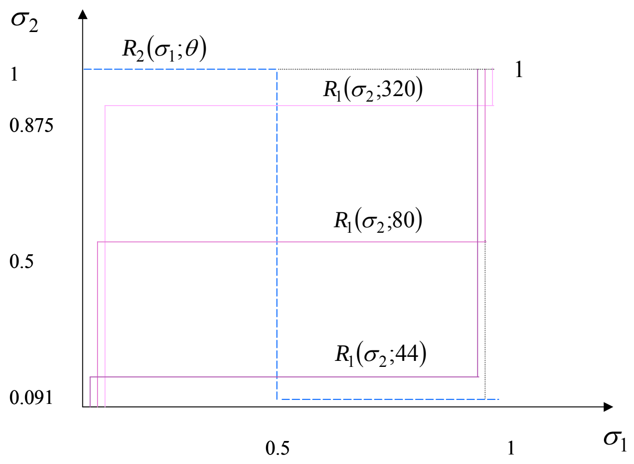

Figure 3: Generalized Matching Pennies

as the probability with which $x_2 = 1$ is played increases in θ. As θ increases from 44 to 80 and 320, the percentage of subjects in the role of player 1 choosing the high action decreases from 92% to 52% and then to 4%, whereas the corresponding values for player 2 increase from 20% to 52% and then to 84%. Thus, contrary to the prediction of the mixed-strategy equilibrium *both* players' actions change as θ does. Intuitively, as player 1's payoff function satisfies ID with respect to $(-x_1, \theta)$,[35] an increase in θ directly reduces his incremental benefits from higher actions.[36] Next, because $\pi_2(x_2, x_1; \theta)$ is supermodular in $(x_2, -x_1)$,[37] a reduction in x_1 from 1 to 0 has the indirect effect of increasing the incremental benefit for player 2 from increasing x_2 from 0 to 1.[38] These properties suggest that, when θ increases, player 1's action should decrease, whereas player 2's action should increase.

I first prove such a comparative statics result for games with four structural properties that hold for generalized matching pennies, but one addi-

[35] This means that $\Delta_1\left(x_1^H, x_1^L; x_2; \theta\right)$ is weakly decreasing in θ.

[36] When $x_2 = 0$, this benefit is $40 - \theta$; when $x_2 = 1$, this benefit is independent of θ.

[37] This means that $\Delta_2\left(x_2^H, x_2^L; x_1; \theta\right)$ is weakly decreasing in x_1.

[38] For $x_1 = 1$, this incremental benefit is -40, for $x_1 = 0$, it is 40.

tional requirement that obviously does *not* hold, namely existence of a unique pure-strategy Nash equilibrium.

Proposition 5 *Suppose both players have well-defined reaction functions; and there is a unique pure-strategy Nash equilibrium* $\mathbf{x}(\theta)$ *for each* θ*. Suppose further that the following properties hold:*
(SUP$_1$) $\pi_1(x_1, x_2; \theta)$ *is supermodular in* (x_1, x_2)*.*
(SUP$_2^-$) $\pi_2(x_2, x_1; \theta)$ *is supermodular in* $(x_2, -x_1)$*.*
(ID$_1^-$) $\pi_1(x_1, x_2; \theta)$ *satisfies increasing differences in* $(-x_1, \theta)$*.*
(IND$_2$) $\pi_2(x_2, x_1; \theta)$ *is independent of* θ*.*
Then $x_1(\theta)$ *is weakly decreasing in* θ*, and* $x_2(\theta)$ *is weakly increasing in* θ*.*

Proof. Let $\left(x_1^L, x_2^L\right)$ be the equilibrium for θ^L. By (SUP$_2^-$) and Lemma 1 the reaction function of player 2 is weakly decreasing in x_1, and by (IND$_2$), $R_2\left(x_1; \theta^L\right) = R_2\left(x_1; \theta^H\right)$. Thus, the equilibrium $\left(x_1^H, x_2^H\right)$ for θ^H lies on $R_2\left(x_1; \theta^L\right)$, implying

$$x_1^H \leq x_1^L \text{ and } x_2^H \geq x_2^L \text{ or } x_1^H \geq x_1^L \text{ and } x_2^H \leq x_2^L. \tag{8}$$

It therefore suffices to show that $x_1^H \geq x_1^L$ and $x_2^H \leq x_2^L$ cannot hold simultaneously except if both hold with equality. First, I show that $x_1^H > x_1^L$ and $x_2^H < x_2^L$ cannot hold simultaneously. Because $R_2\left(x_1^H; \theta^H\right) = R_2\left(x_1^H; \theta^L\right)$ is weakly decreasing by (SUP$_2^-$), clearly $x_1^H > x_1^L$ implies $x_2^H = R_2\left(x_1^H; \theta^H\right) \geq R_2\left(x_1^L; \theta^H\right) = R_2\left(x_1^L; \theta^L\right) = x_2^L$. Next, I exclude the possibility that $x_1^H > x_1^L$ and $x_2^H = x_2^L$. This would require that $R_2\left(x_1; \theta^H\right) = R_2\left(x_1^L; \theta^H\right)$ is horizontal to the right of $\left(x_1^L, x_2^L\right)$; the analogous statement would have to hold for $R_1\left(x_2; \theta^H\right)$. This would contradict uniqueness of the equilibrium. By similar reasoning I exclude the possibility that $x_2^H < x_2^L$ and $x_1^H = x_1^L$. Therefore (8) requires $x_1^H \leq x_1^L$ and $x_2^H \geq x_2^L$. ■

By (SUP$_1$) and (SUP$_2^-$), actions are strategic complements for player 1 and strategic substitutes for player 2. Figure 4 suggests why a clear comparative statics result can be obtained even so. For simplicity, the figure assumes continuous action spaces and presupposes that $R_1(\sigma_2; \theta)$ is *strictly*

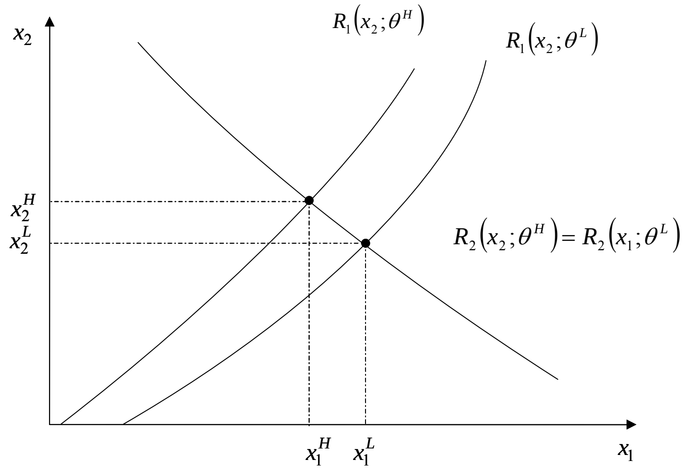

Figure 4: Understanding Generalized Matching Pennies

increasing, whereas $R_2(\sigma_1;\theta)$ is *strictly* decreasing; the proof of Proposition 5 extends the argument to reaction functions that are merely weakly increasing and weakly decreasing, respectively. Crucially, an increase in the parameter affects only the payoffs of one player, shifting his (increasing) reaction function inwards while leaving the other player's (decreasing) reaction function constant. Hence, the equilibrium must move to the left and upwards.

Even though generalized matching pennies only has a mixed-strategy equilibrium, the result applies. The mixed-strategy equilibrium can be shown to be the pure-strategy equilibrium of a game satisfying the assumptions of Proposition 5. This game has strategy space $\sum_i = [0,1]$, corresponding to the set of probability distributions on X_i; payoffs correspond to the expected payoffs of the original game.[39] Hence, comparative statics follow from basic structural properties.

[39] A related procedure was applied by Echenique (2003) who shows that the mixed extension of a GSC is still a GSC.

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