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Abstract

We consider the production of a generic system of non-strongly interacting particles
with a high total invariant mass M in hadron collisions. We examine the transverse-
momentum (q_T) distribution of the system in the small-q_T region ($q_T \ll M$), and we
present a study of the perturbative QCD contributions that are enhanced by powers
of large logarithmic terms of the type $\ln(M^2/q_T^2)$. These terms can be resummed
to all orders in QCD perturbation theory. The partonic production mechanism of
the final-state system can be controlled by quark–antiquark ($q\bar{q}$) annihilation and/or
by gluon fusion. The resummation formalism for the $q\bar{q}$ annihilation subprocess is
well established, and it is usually extrapolated to the gluon fusion subprocess. We
point out that this naïve extrapolation is not correct, and we present the all-order
resummation formula for the q_T distribution in gluon fusion processes. The gluon
fusion resummation formula has a richer structure than the resummation formula in
$q\bar{q}$ annihilation. The additional structure originates from collinear correlations that are
a specific feature of the evolution of the colliding hadrons into gluon partonic states.
In the q_T cross section at small values of q_T, these gluon collinear correlations produce
coherent spin correlations between the helicity states of the initial-state gluons and
definite azimuthal-angle correlations between the final-state particles of the observed
high-mass system.

1 Introduction

The properties of the transverse-momentum distributions of systems of high invariant mass that are produced at high-energy hadron colliders are important for QCD and electroweak studies and for physics studies beyond the Standard Model (SM).

We consider the inclusive hard-scattering process

$$h_1(p_1) + h_2(p_2) \rightarrow F(\{q_i\}) + X \ , \tag{1}$$

where the collision of the two hadrons h_1 and h_2 with momenta p_1 and p_2 produces the triggered final-state system F, accompanied by an arbitrary and undetected final state X. We denote by $\sqrt{s}$ the centre–of–mass energy of the colliding hadrons, which are treated as massless particles ($s = (p_1 + p_2)^2 = 2p_1p_2$). The observed final state F is a generic system of *one* or *more* particles with momenta q_i^μ ($i = 3, 4, 5, \dots$). The total momentum of F is denoted by q^μ ($q = \sum_i q_i$), and it can be expressed in terms of the total invariant mass M ($q^2 = M^2$), the transverse momentum $\mathbf{q_T}$ with respect to the direction of the colliding hadrons, and the rapidity y ($2y = \ln(p_2q/p_1q)$) in the centre–of–mass system of the collision. Throughout the paper, we limit ourselves to considering the case in which the system F is formed by non-strongly interacting particles, such as vector bosons ($\gamma, W, Z, \dots$), Drell–Yan (DY) lepton pairs, Higgs particles and so forth.

Provided the invariant mass M is large ($M \gg \Lambda_{QCD}$, Λ_{QCD} being the QCD scale), the production cross section and associated kinematical distributions of the process in Eq. (1), can be evaluated by using QCD perturbation theory. The cross section is expressed as a convolution of partonic cross sections $d\hat{\sigma}_{ab}$ ($a, b = q, \bar{q}, g$) with the parton densities of the colliding hadrons. The partonic cross section $d\hat{\sigma}_{ab}$ is computed as a power series expansion in the QCD coupling $\alpha_S(M^2)$ by considering the corresponding partonic subprocess $a + b \rightarrow F + \dots$, where the dots denote final-state partons. Since F is a system of colourless particles, the partonic subprocesses include quark-antiquark ($q\bar{q}$) annihilation,

$$q + \bar{q} \rightarrow F \ , \tag{2}$$

and gluon fusion,

$$g + g \rightarrow F \ . \tag{3}$$

In this paper we are interested in considering the process of Eq. (1) in kinematical configurations where the transverse momentum q_T of the system F is small (say, $q_T \ll M$). Unless the subprocesses in Eqs. (2) and (3) are both forbidden by selection rules related to the nature of F (e.g., $gg \rightarrow W^\pm$ is forbidden), these subprocesses produce the system F with $q_T = 0$. The system F acquires a non-vanishing transverse momentum through higher-order QCD radiative corrections to the subprocesses in Eqs. (2) and (3). Nonetheless, the *bulk* of the events is still produced in the small-q_T region.

The perturbative-QCD computation of the partonic cross sections $d\hat{\sigma}_{ab}$ in powers of $\alpha_S(M^2)$ shows that high-order coefficients contain logarithmic terms of the type $\ln^m(M^2/q_T^2)$. Although $\alpha_S(M^2)$ is small, these logarithmic terms can be large in the small-q_T region ($q_T \ll M$), thus spoiling the quantitative convergence of the expansion in powers of $\alpha_S(M^2)$ (at each fixed order in α_S, the partonic cross section eventually diverges to either $+\infty$ or $-\infty$ by considering the limit

$q_T \to 0$). To obtain reliable perturbative predictions in the small-q_T region, the logarithmically-enhanced terms have to be evaluated at sufficiently-high perturbative orders[†], and possibly *re-summed* to all orders in $\alpha_S(M^2)$.

The small-q_T logarithmic terms have their physical origin from multiple radiation of final-state partons that are soft and/or collinear to the colliding hadrons (partons). The method and the formalism to resum the logarithmically-enhanced terms at small q_T was developed in the eighties [1–11]. Subsequent, and important, theoretical progress in this field regards, for instance, the explicit computation of high-order resummation coefficients [12–15] (see additional comments in Sect. 2) and the understanding of their universality (process-independent) structure [12, 16].

In this paper we consider small-q_T resummation, and we deal with an issue that is also related to universality. The issue regards the relation between processes that are controlled by $q\bar{q}$ annihilation and by gluon fusion. Transverse-momentum resummation was originally worked out for the DY process [1–10], which is driven by $q\bar{q}$ annihilation. The resummation structure that emerges in the DY process was then *customarily* used (see, e.g., Refs. [17–26] and references therein) for many other processes of the class in Eq. (1). Such processes include, for instance, the production of the SM Higgs boson [17–21], of photon pairs [22], of vector boson pairs such as ZZ [23] and W^+W^- [24], and of slepton pairs [25, 26]. In particular, Higgs boson production is driven by gluon fusion, whereas diphoton and diboson production receives contributions from both the $q\bar{q}$ annihilation and gluon fusion subprocesses.

In the present contribution, we point out that there are some key differences between $q\bar{q}$ annihilation and gluon fusion. These differences have escaped detection until recent findings [13, 27, 28]. The physical origin of the differences is due to specific *collinear correlations* (see Sect. 3) that are a distinctive feature of the perturbative evolution of the colliding hadrons into gluon initial states. Analogous correlations are not produced by the perturbative evolution of spin unpolarized hadrons into quark or antiquark initial states.

As a consequence of these differences, transverse-momentum resummation in gluon fusion sub-processes has a 'richer' structure than in $q\bar{q}$ annihilation subprocesses. The small-q_T resummation formalism for the DY process [1–10] has to be modified and extended to deal with gluon fusion subprocesses. In particular, in gluon fusion subprocesses, gluon collinear correlations produce *spin* and *azimuthal* correlations that are logarithmically enhanced in the small-q_T region.

In the following we present and discuss our main general results on transverse-momentum resummation in gluon fusion processes. Then, we shall comment on Refs. [13, 27, 28]. Details about the derivation of our results, and the illustration of further related results, will appear elsewhere [29].

The outline of the paper is as follows. In Sect. 2, we briefly review the classical QCD results on transverse-momentum resummation. These results, which mostly derive from studies of the DY process, are presented using a general and process-independent notation that is useful for the subsequent presentation of q_T resummation in gluon fusion processes. In Sect. 3, we present our all-order resummation formula for generic transverse-momentum cross sections controlled by gluon fusion. We illustrate the structure of the resummation formula, and we discuss its origin from

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[†]The 'sufficiently-high' order depends on the specific q_T region of interest in each specific process; this order cannot be specified 'a priori'.

quantum-mechanical correlations (interference effects) produced by the collinear-parton radiation that accompanies the gluon fusion hard-scattering subprocess. We also explicitly consider the specific example of SM Higgs boson production. In Sect. 4, we reformulate q_T resummation in the helicity space of the colliding gluons. We show how gluon collinear correlations are related to helicity-flip phenomena in the hard-scattering subprocess. In Sect. 5, we specify the gluon fusion resummation formula for azimuthally-averaged transverse-momentum cross sections. We point out that the differences between the $q\bar{q}$ annihilation and gluon fusion channels persist even after having performed the integration over the azimuthal angle of the transverse-momentum vector. Section 6 is devoted to derive and discuss the general structure of the azimuthal-angle correlations embodied in the gluon fusion resummation formula. Few summarizing remarks are presented in Sect. 7.

2 Small-q_T resummation in impact parameter space

In this section we recall the 'classical' formalism [1–11, 16] of transverse-momentum resummation in impact parameter space. This illustration sets the stage for the presentation of our results on small-q_T resummation in gluon fusion processes (see Sects. 3–6).

We consider the process in Eq. (1), and we introduce the corresponding multidifferential cross section

$$\frac{d\sigma_F}{d^2\mathbf{q_T}\, dM^2\, dy\, d\mathbf{\Omega}} (p_1, p_2; \mathbf{q_T}, M, y, \mathbf{\Omega}) \ . \tag{4}$$

The differential cross section depends on the total momentum of the system F (i.e. on the variables $\mathbf{q_T}, M, y$) and, to be quite general, it can also depend on additional variables that specify the kinematics of the particles in the system F. In Eq. (4) these additional variables are generically denoted as $\mathbf{\Omega} = \{\Omega_A, \Omega_B, \dots\}$ (correspondingly, we define $d\mathbf{\Omega} \equiv d\Omega_A d\Omega_B \dots$). They can be, for instance, the rapidity y_i and the azimuthal angle $\phi(\mathbf{q_T}_i)$ of one of the particles (with momentum q_i) in the system F. In general, we only assume that the kinematical variables $\{\Omega_A, \Omega_B, \dots\}$ are independent of $\mathbf{q_T}, M$ and y.

Considering the $\mathbf{q_T}$ dependence of the multidifferential cross section in Eq. (4) within perturbative QCD, we introduce the following decomposition:

$$d\sigma_F = d\sigma_F^{(\mathrm{sing})} + d\sigma_F^{(\mathrm{reg})} \ . \tag{5}$$

Both terms on the right-hand side are obtained as convolutions of partonic cross sections and the scale-dependent parton distributions $f_{a/h}(x, \mu^2)$ ($a = q_f, \bar{q}_f, g$ is the parton label) of the colliding hadrons[‡]. The distinction between the two terms is purely theoretical. The partonic cross sections that enter the singular component (the first term on the right-hand side) contain all the contributions that are enhanced (or 'singular') at small q_T. These contributions are proportional to $\delta^{(2)}(\mathbf{q_T})$ or to large logarithms[§] of the type $\frac{1}{q_T^2}\ln^m(M^2/q_T^2)$. On the contrary, the partonic cross

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[‡]Throughout the paper we always use parton densities as defined in the $\overline{\mathrm{MS}}$ factorization scheme, and $\alpha_\mathrm{S}(q^2)$ is the QCD running coupling in the $\overline{\mathrm{MS}}$ renormalization scheme.

[§]To be precise, the logarithms are combined with corresponding 'contact' terms, which are proportional to $\delta^{(2)}(\mathbf{q_T})$. These combinations define regularized (integrable) 'plus distributions' $\left[\frac{1}{q_T^2}\ln^m(M^2/q_T^2)\right]_+$ with respect to $\mathbf{q_T}$.

sections of the second term on the right-hand side are regular (i.e. free of logarithmic terms) order-by-order in perturbation theory as $q_T \to 0$. To be precise, the integration of $d\sigma_F^{(\mathrm{reg})}/d^2\mathbf{q_T}$ over the range $0 \leq q_T \leq Q_0$ leads to a finite result that, at each fixed order in α_S, *vanishes* in the limit $Q_0 \to 0$.

The regular component $d\sigma_F^{(\mathrm{reg})}$ of the q_T cross section is definitely process dependent. In this paper we limit ourselves to considering the singular component, which has a universal (process-independent) structure.

To simplify the presentation of the all-order (resummed) structure of the singular component of the $\mathbf{q_T}$ differential cross section in Eq. (4), we introduce a shorthand (symbolical) notation in several places. For instance, the singular component of Eq. (4) is simply denoted by $[d\sigma_F]$; namely, we define

$$[d\sigma_F] \equiv \frac{d\sigma_F^{(\mathrm{sing})}}{d^2\mathbf{q_T}\, dM^2\, dy\, d\mathbf{\Omega}}\, (p_1, p_2; \mathbf{q_T}, M, y, \mathbf{\Omega})\ . \tag{6}$$

The transverse-momentum resummation formula can be written in the following factorized form [10, 16]:

$$[d\sigma_F] = \frac{M^2}{s} \sum_{c=q,\bar{q},g} \left[d\sigma_{c\bar{c},F}^{(0)}\right] \int \frac{d^2\mathbf{b}}{(2\pi)^2}\ e^{i\mathbf{b}\cdot\mathbf{q_T}}\ S_c(M,b)$$

$$\times \sum_{a_1,a_2} \int_{x_1}^1 \frac{dz_1}{z_1} \int_{x_2}^1 \frac{dz_2}{z_2}\ \left[H^F C_1 C_2\right]_{c\bar{c};a_1 a_2}\ f_{a_1/h_1}(x_1/z_1, b_0^2/b^2)\ f_{a_2/h_2}(x_2/z_2, b_0^2/b^2)\ , \tag{7}$$

where $b_0 = 2e^{-\gamma_E}$ $(\gamma_E = 0.5772\ldots$ is the Euler number) is a numerical coefficient, and the kinematical variables x_1 and x_2 are

$$x_1 = \frac{M}{\sqrt{s}}\, e^{+y}\ , \qquad x_2 = \frac{M}{\sqrt{s}}\, e^{-y}\ . \tag{8}$$

The function $S_c(M,b)$ and the functions symbolically denoted by $\left[d\sigma_F^{(0)}\right]$ and $\left[H^F C_1 C_2\right]$ are specified below.

The right-hand side of Eq. (7) involves the Fourier transformation with respect to the *impact parameter* $\mathbf{b}$ and two convolutions over the longitudinal-momentum fractions z_1 and z_2. The parton densities $f_{a_i/h_i}(x, \mu^2)$ of the colliding hadrons are evaluated at the scale $\mu = b_0/b$, which depends on the impact parameter.

We note that in the context of the study of the present paper, the resummation formula in Eq. (7), and the resummation formulae in Sects. 3–5, have a purely perturbative-QCD content[¶], analogously to customary fixed-order calculations of hard-scattering cross sections in hadron collisions. Using the Altarelli–Parisi evolution equations, the parton densities $f_{a/h}(x, b_0^2/b^2)$ can be expressed [16] in terms of the corresponding parton densities $f_{a/h}(x, \mu_F^2)$ at the evolution scale $\mu = \mu_F$, where μ_F is the customary *factorization scale* that enters fixed-order calculations. Having done that, all the remaining factors on the right-hand side of Eq. (7) are partonic contributions that can be expanded in powers of $\alpha_\mathrm{S}(M^2)$ at arbitrary perturbative orders.

[¶]Throughout the paper we do not consider the inclusion of any non-perturbative contributions, such as, for instance, those first introduced in Ref. [5].

The small-q_T region where $q_T \ll M$ corresponds in impact parameter space to the large-b region where $b \gg 1/M$. The perturbative expansion of the **b** space integrand in Eq. (7) produces large perturbative coefficients of the type $\ln^m(b^2 M^2)$: these coefficients lead to the small-q_T logarithmic terms $\left[\frac{1}{q_T^2} \ln^{m-1}(M^2/q_T^2)\right]_+$, through the evaluation of the Fourier transformation from **b** space to **q_T** space.

The factor $\left[d\sigma_{c\bar{c},F}^{(0)}\right]$ in Eq. (7) depends on the process (i.e. on the specific final state system F and its kinematics). This factor is the Born level cross section $d\hat{\sigma}^{(0)}$ (i.e. the cross section at its corresponding *lowest order* in α_S) of the partonic subprocesses $c + \bar{c} \to F$ in Eqs. (2) and (3). Making the symbolic notation explicit, we write:

$$\left[d\sigma_{c\bar{c},F}^{(0)}\right] = \frac{d\hat{\sigma}_{c\bar{c},F}^{(0)}}{M^2 \, d\mathbf{\Omega}} (x_1 p_1, x_2 p_2; \mathbf{\Omega}; \alpha_S(M^2)) \ , \tag{9}$$

where $x_1 p_1^\mu$ ($x_2 p_2^\mu$) is the momentum of the parton c ($\bar{c}$). In Eq. (7), we have included the contribution of both the $q\bar{q}$ annihilation[||] channel ($c = q, \bar{q}$) and the gluon fusion channel ($c = g$); one of these two contributing channels may be absent (i.e. $\left[d\sigma_{c\bar{c},F}^{(0)}\right] = 0$ in that channel), depending on the specific final state F.

The factor $S_c(M, b)$ in Eq. (7) is universal (process independent): it does not depend on the produced final-state system F and on its kinematics. It only depends on the partonic channel that produces the cross section $\left[d\sigma_{c\bar{c},F}^{(0)}\right]$. Thus, $S_c(M, b)$ is called quark ($c = q$ or $\bar{q}$) or gluon ($c = g$) Sudakov form factor in the cases of the $q\bar{q}$ annihilation or gluon fusion channel, respectively. The Sudakov form factor can be expressed in the following exponential form:

$$S_c(M, b) = \exp\left\{-\int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2} \left[A_c(\alpha_S(q^2)) \ln \frac{M^2}{q^2} + B_c(\alpha_S(q^2))\right]\right\} \ , \tag{10}$$

where the functions $A_c(\alpha_S)$ and $B_c(\alpha_S)$ are perturbative series in α_S:

$$A_c(\alpha_S) \ = \ \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n A_c^{(n)} \ , \tag{11}$$

$$B_c(\alpha_S) \ = \ \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n B_c^{(n)} \ . \tag{12}$$

The factor $\left[H^F C_1 C_2\right]$ in Eq. (7) has the following explicit form:

$$\left[H^F C_1 C_2\right]_{c\bar{c};a_1 a_2} = H_c^F(x_1 p_1, x_2 p_2; \mathbf{\Omega}; \alpha_S(M^2)) \ C_{c a_1}(z_1; \alpha_S(b_0^2/b^2)) \ C_{\bar{c} a_2}(z_2; \alpha_S(b_0^2/b^2)) \ , \tag{13}$$

where H_c^F and C_{ab} are both functions of α_S, and they can be perturbatively expanded as follows:

$$H_c^F(x_1 p_1, x_2 p_2; \mathbf{\Omega}; \alpha_S) \ = \ 1 + \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n H_c^{F \, (n)}(x_1 p_1, x_2 p_2; \mathbf{\Omega}) \ , \tag{14}$$

$$C_{ab}(z; \alpha_S) \ = \ \delta_{ab} \ \delta(1 - z) + \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n C_{ab}^{(n)}(z) \ . \tag{15}$$

[||]In the case of $q\bar{q}$ annihilation, the notation $c = q, \bar{q}$ is not completely precise, since the quark and the antiquark can have either equal or different flavour. The same comment applies to the factor $\left[H^F C_1 C_2\right]_{c\bar{c};a_1 a_2}$.

The function H_c^F is process dependent, whereas the perturbative functions C_{ab} are universal and depend only on the parton indices a and b (analogously to the dependence of the anomalous dimensions γ_{ab} that control the perturbative evolution of the parton densities, through the Altarelli–Parisi equations).

By inspection of the right-hand side of Eq. (13), we notice that the scale of α_S is not set to a unique value. We have $\alpha_S(M^2)$ in the case of the function H_c^F (as naturally expected for a process-dependent contribution), and $\alpha_S(b_0^2/b^2)$ in the case of the functions C_{ca_1} and $C_{\bar{c}a_2}$. The replacement $\alpha_S(M^2) \to \alpha_S(b_0^2/b^2)$ in Eq. (13) is feasible, provided it is properly compensated [16] by a corresponding factor to be inserted in Eq. (10): this procedure leads to a modification of the Sudakov form factor, which becomes a process-dependent quantity. Indeed, the hard-scattering function H_c^F was introduced in Ref. [16] to explicitly show (see also Ref. [12]) the universality (process independence) of both the Sudakov form factor and the coefficient function C_{ab}. In the version of Eq. (7) that was originally presented for the DY process (see Eq. (1.1) in Ref. [10]), the function $H_c^F = H_c^{DY}$ (note that H_c^{DY} depends only on α_S) is absorbed in the definition of the functions C_{ab} and of the function B_c of the form factor (these functions are thus 'those' of the DY process). As shown in Sect. 3, the process-dependent hard-scattering function H_c^F definitely plays a distinctive role in the case of gluon fusion subprocesses.

The present knowledge of the perturbative coefficients in Eqs. (11), (12), (14) and (15) is as follows (see also Sect. 2.3 of Ref. [20], where we used the same notation as in the present paper). The coefficients $A_c^{(1)}$, $B_c^{(1)}$ and $A_c^{(2)}$ are known since a long time for both the quark [6] and the gluon [11] form factors. The explicit expression of the coefficient $B^{(2)}$ for the DY process was first presented in Ref. [9]. The process-independent structure and the explicit form of the coefficients $B_c^{(2)}$ ($c = q, g$) was derived in Ref. [12]. The result of $A_c^{(3)}$ has been obtained very recently [15] by relating its value to the coefficient $B_c^{(2)}$ [9, 12] and to the coefficient of the soft part of the Altarelli–Parisi splitting functions at $\mathcal{O}(\alpha_S^3)$ [30]. The universal first-order coefficients $C_{qg}^{(1)}(z)$ and $C_{gq}^{(1)}(z)$ were first computed in Refs. [8, 10] and Ref. [31], respectively. The general result for the first-order coefficients $H_c^{F\,(1)}$ and $C_{ab}^{(1)}(z)$ was derived in Ref. [12], where the process dependence of $H_c^{F\,(1)}$ is explicitly related to the first-order virtual corrections of the partonic subprocesses $c + \bar{c} \to F$ in Eqs. (2) and (3). The coefficients of Eq. (13) at the second order in α_S have been computed for both SM Higgs boson production by gluon fusion** [13] and the DY process [14, 32].

We note that the resummation formula (7) involves perturbative functions of $\alpha_S(\mu^2)$, where the scale μ^2 is $\mu^2 = M^2$ (see Eqs. (9) and (13)), or $\mu^2 = q^2$ (see Eq. (10)), or $\mu^2 = b_0^2/b^2$ (see Eq. (13)). All these functions can be expressed in terms of $\alpha_S(\mu_R^2)$, $\ln(\mu^2/\mu_R^2)$ and the perturbative coefficients of the QCD β-function (μ_R is the *renormalization scale* that customarily appears in fixed-order calculations) by using the renormalization group equation for the perturbative evolution of the QCD running coupling $\alpha_S(\mu^2)$.

We add a relevant (though known) observation. Considering the dependence on the impact parameter **b**, all the factors in the integrand of the Fourier transformation on the right-hand side of the resummation formula (7) are functions of $\mathbf{b}^2$, with no dependence on the azimuthal angle $\phi(\mathbf{b})$ of **b** in the transverse plane of the collision. Therefore, in Eq. (7) we can straightforwardly

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**See, however, related comments in Sect. 3.

perform the integration over $\phi(\mathbf{b})$ and implement the replacement

$$\int \frac{d^2\mathbf{b}}{2\pi} \ e^{i\mathbf{b}\cdot\mathbf{q_T}} \ F(\mathbf{b}^2) = \int_0^{+\infty} db \ b \ J_0(bq_T) \ F(b^2) \ , \tag{16}$$

where $J_0(x)$ is the 0th-order Bessel function, and $F(\mathbf{b}^2)$ denotes a generic function of $\mathbf{b}^2$. This result implies a technical simplification of the resummation formula, since the *two*-dimensional Fourier transformation is replaced by the *one*-dimensional Bessel transformation. More importantly, this implies that the right-hand side of Eq. (7) depends only on $\mathbf{q_T}^2$, with no additional dependence on the azimuthal angle $\phi(\mathbf{q_T})$ of $\mathbf{q_T}$. Therefore, according to Eqs. (7) and (16), the singular part of the $\mathbf{q_T}$ differential cross section in Eq. (6) does not contain *any* azimuthal correlations with respect to $\mathbf{q_T}$. Equivalently, we can say that $d\sigma_F/d^2\mathbf{q_T}$ and $d\sigma_F/dq_T^2$ are simply proportional, and we can write:

$$\frac{d\sigma_F^{(\mathrm{sing})}}{d^2\mathbf{q_T} \ dM^2 \ dy \ d\mathbf{\Omega}} = \frac{1}{\pi} \ \frac{d\sigma_F^{(\mathrm{sing})}}{dq_T^2 \ dM^2 \ dy \ d\mathbf{\Omega}} \ . \tag{17}$$

Obviously, this does not mean that the multidifferential cross section $d\sigma_F/d^2\mathbf{q_T}$ in Eq. (4) has no azimuthal correlations. In general, azimuthal correlations are present in the regular part $d\sigma_F^{(\mathrm{reg})}$ (see Eq. (5)) of the cross section. In the small-q_T region ($q_T \ll M$), $d\sigma_F^{(\mathrm{reg})}$ is of $\mathcal{O}(q_T/M)$ (modulo powers of $\ln(M^2/q_T^2)$) with respect to $d\sigma_F^{(\mathrm{sing})}$ order-by-order in QCD perturbation theory. In the case of gluon fusion processes, these conclusions about azimuthal correlations at small q_T are no longer true in view of the results presented in the next section.

3 Transverse-momentum resummation in gluon fusion processes

The resummation formalism reviewed in Sect. 2 was originally developed and proven [1–10] for the DY process (and related observables, such as the energy–energy correlation function in e^+e^- annihilation). In the subsequent literature these results were extrapolated to various processes of the class in Eq. (1). In this section, we show that this 'naïve' extrapolation is not valid in the case of the (sub)processes that are controlled by gluon fusion.

To present our results, we start from the resummation formula in Eq. (7). It includes the contributions from both $q\bar{q}$ annihilation ($c = q, \bar{q}$) and gluon fusion ($c = g$). We thus separate these two types of contributions, and we write:

$$[d\sigma_F] = [d\sigma_F]^{(q\bar{q}-\mathrm{ann.})} + [d\sigma_F]^{(g-\mathrm{fus.})} \ . \tag{18}$$

Our new results refer to $[d\sigma_F]^{(g-\mathrm{fus.})}$, whereas we maintain the results of Sect. 2 for $[d\sigma_F]^{(q\bar{q}-\mathrm{ann.})}$. To be precise, also in the case of gluon fusion, we confirm the factorization structure on the right-hand side of Eq. (7). We explicitly report this structure:

$$[d\sigma_F]^{(g-\mathrm{fus.})} = \frac{M^2}{s} \left[d\sigma_{gg,F}^{(0)} \right] \int \frac{d^2\mathbf{b}}{(2\pi)^2} \ e^{i\mathbf{b}\cdot\mathbf{q_T}} \ S_g(M,b)$$

$$\times \sum_{a_1,a_2} \int_{x_1}^1 \frac{dz_1}{z_1} \int_{x_2}^1 \frac{dz_2}{z_2} \ \left[H^F C_1 C_2 \right]_{gg;a_1a_2} \ f_{a_1/h_1}(x_1/z_1, b_0^2/b^2) \ f_{a_2/h_2}(x_2/z_2, b_0^2/b^2) \ , \tag{19}$$

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where the lowest-order cross section $\left[d\sigma^{(0)}_{gg,F}\right]$ and the gluon form factor $S_g(M,b)$ are given in Eqs. (9) and (10), respectively. The new results regard the gluon fusion factor $\left[H^F C_1 C_2\right]_{gg;a_1 a_2}$.

The naïve expression on the right-hand side of Eq. (13) has to be replaced by the following result [29]:

$$
\begin{aligned}
\left[H^F C_1 C_2\right]_{gg;a_1 a_2} &= H^F_{\mu_1 \nu_1, \mu_2 \nu_2}(x_1 p_1, x_2 p_2; \mathbf{\Omega}; \alpha_S(M^2)) \\
&\times \ C^{\mu_1 \nu_1}_{g a_1}(z_1; p_1, p_2, \mathbf{b}; \alpha_S(b_0^2/b^2)) \ C^{\mu_2 \nu_2}_{g a_2}(z_2; p_1, p_2, \mathbf{b}; \alpha_S(b_0^2/b^2)) \ .
\end{aligned}
\tag{20}
$$

The first evident difference with respect to Eq. (13) is the presence of Lorentz tensors, rather than scalar functions. The Lorentz indices (symbolically) refer to the gluon fusion hard-scattering process

$$
g_{(\mu_1)}(x_1 p_1) + g_{(\mu_2)}(x_2 p_2) \to F \ ,
\tag{21}
$$

where μ_i $(i = 1, 2)$ is the Lorentz index carried by the external gluon leg with incoming momentum $x_i p_i$. The indices ν_1 and ν_2 refer to the external gluon legs of the process that is complex conjugate to that in Eq. (21).

The process-dependent factor H^F in Eq. (20) has the following perturbative expansion:

$$
\begin{aligned}
H^F_{\mu_1 \nu_1, \mu_2 \nu_2}(x_1 p_1, x_2 p_2; \mathbf{\Omega}; \alpha_S) &= H^{F\,(0)}_{\mu_1 \nu_1, \mu_2 \nu_2}(x_1 p_1, x_2 p_2; \mathbf{\Omega}) \\
&+ \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n H^{F\,(n)}_{\mu_1 \nu_1, \mu_2 \nu_2}(x_1 p_1, x_2 p_2; \mathbf{\Omega}) \ ,
\end{aligned}
\tag{22}
$$

with the lowest-order constraint:

$$
H^{F\,(0)}_{\mu_1 \nu_1, \mu_2 \nu_2} \ g^{\mu_1 \nu_1} \ g^{\mu_2 \nu_2} = 1 \ .
\tag{23}
$$

We also define the scalar function H^F_g as follows:

$$
H^F_g(x_1 p_1, x_2 p_2; \mathbf{\Omega}; \alpha_S) \equiv H^F_{\mu_1 \nu_1, \mu_2 \nu_2}(x_1 p_1, x_2 p_2; \mathbf{\Omega}; \alpha_S) \ g^{\mu_1 \nu_1} \ g^{\mu_2 \nu_2} \ .
\tag{24}
$$

A relevant property [29] of the Lorentz tensor $H^F_{\mu_1 \nu_1, \mu_2 \nu_2}$ is current conservation, namely:

$$
\begin{aligned}
p_1^{\mu_1} \ H^F_{\mu_1 \nu_1, \mu_2 \nu_2}(x_1 p_1, x_2 p_2; \mathbf{\Omega}; \alpha_S) &= p_1^{\nu_1} \ H^F_{\mu_1 \nu_1, \mu_2 \nu_2}(x_1 p_1, x_2 p_2; \mathbf{\Omega}; \alpha_S) = 0 \ , \\
p_2^{\mu_2} \ H^F_{\mu_1 \nu_1, \mu_2 \nu_2}(x_1 p_1, x_2 p_2; \mathbf{\Omega}; \alpha_S) &= p_2^{\nu_2} \ H^F_{\mu_1 \nu_1, \mu_2 \nu_2}(x_1 p_1, x_2 p_2; \mathbf{\Omega}; \alpha_S) = 0 \ .
\end{aligned}
\tag{25}
$$

The universal (process-independent) partonic tensor $C^{\mu_i \nu_i}$ in Eq. (20) exhibits an explicit dependence on the impact parameter $\mathbf{b}$, besides the implicit dependence on b^2 through the scale of α_S. The structure of the partonic tensor is:

$$
C^{\mu\nu}_{g a}(z; p_1, p_2, \mathbf{b}; \alpha_S) = d^{\mu\nu}(p_1, p_2) \ C_{g a}(z; \alpha_S) + D^{\mu\nu}(p_1, p_2; \mathbf{b}) \ G_{g a}(z; \alpha_S) \ ,
\tag{26}
$$

where

$$
d^{\mu\nu}(p_1, p_2) = -g^{\mu\nu} + \frac{p_1^\mu p_2^\nu + p_2^\mu p_1^\nu}{p_1 \cdot p_2} \ ,
\tag{27}
$$

$$
D^{\mu\nu}(p_1, p_2; \mathbf{b}) = d^{\mu\nu}(p_1, p_2) - 2 \frac{b^\mu b^\nu}{\mathbf{b^2}} \ ,
\tag{28}
$$

8

and $b^\mu = (0, \mathbf{b}, 0)$ is the two-dimensional impact parameter vector in the four-dimensional notation ($b^\mu b_\mu = -\mathbf{b^2}$). The gluonic coefficient function $C_{ga}(z; \alpha_S)$ has the same perturbative structure as in Eq. (15). The first-order coefficient $C_{ga}^{(1)}(z)$ of the function $C_{ga}(z; \alpha_S)$ in Eq. (26) and the first-order coefficient $H_g^{F\,(1)}$ of the function H_g^F in Eq. (24) have actually the same value [12] as obtained in the context of the 'naïve' expression in Eq. (13).

The partonic coefficient function $G_{ga}(z; \alpha_S)$ in Eq. (26) is a specific and distinctive feature of transverse-momentum resummation in gluon fusion processes[††]. Its perturbative expansion starts at order α_S, and we write:

$$G_{ga}(z; \alpha_S) = \frac{\alpha_S}{\pi}\, G_{ga}^{(1)}(z) \,+\, \sum_{n=2}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n G_{ga}^{(n)}(z) \ . \tag{29}$$

The first-order coefficients are [29]

$$G_{gg}^{(1)}(z) = C_A\, \frac{1-z}{z} \ , \tag{30}$$

$$G_{gq}^{(1)}(z) = G_{g\bar{q}}^{(1)}(z) = C_F\, \frac{1-z}{z} \ . \tag{31}$$

The second-order coefficients $C_{ga}^{(2)}(z)$ and $G_{ga}^{(2)}(z)$ are considered in Refs. [13, 29].

The tensors in Eqs. (27) and (28) fulfil the relations $g^{\mu\nu} d_{\mu\nu} = -2$ and $g^{\mu\nu} D_{\mu\nu} = 0$. Considering the centre–of–mass system of the collision and denoting by $\mu = 1, 2$ the Lorentz indices of the non-vanishing components of purely-transverse vectors (such as $\mathbf{q_T}$ and $\mathbf{b}$), we have $d_{\mu\nu} = D_{\mu\nu} = 0$ if $\mu = 0\,(\nu = 0)$ or $\mu = 3\,(\nu = 3)$. Therefore, the only non-vanishing components of $d_{\mu\nu}$ and $D_{\mu\nu}$ are those that correspond to Lorentz indices $j = 1, 2$ and $k = 1, 2$ of the transverse plane; we have

$$d^{jk}(p_1, p_2) = -\, g^{jk} \ , \quad D^{jk}(p_1, p_2; \mathbf{b}) = -\, g^{jk} - 2\, \frac{b^j\, b^k}{\mathbf{b^2}} \ . \tag{32}$$

The structure of Eqs. (20) and (26) has a definite physical origin: this structure is produced by *gluon collinear correlations* [29]. We refer to the correlations that occur in the universal (process-independent) partonic subprocess

$$a \ \rightarrow \ g + a_1 + a_2 + \dots \ , \tag{33}$$

where the initial-state colliding parton a ($a = q, \bar{q}, g$) 'evolves' in the colliding gluon g through *collinear* radiation of the final-state partons $a_1, a_2, \dots$.

To illustrate the role of gluon collinear correlations, we briefly sketch how they arise at the first non-trivial order in QCD perturbation theory. We consider the partonic hard-scattering process

$$a(p) + g(\bar{p}) \ \rightarrow \ F(q) + a(k) \ , \tag{34}$$

which leads to the first-order (real) radiative corrections to the gluon fusion process in Eq. (3), namely,

$$g(p) + g(\bar{p}) \ \rightarrow \ F(q) \ . \tag{35}$$

[††]Setting $G_{ga}(z; \alpha_S) = 0$ in Eq. (26), the gluon fusion factor of Eq. (20) coincides with the corresponding 'naïve' factor of Eq. (13).

Here the parton momenta are denoted by $p, \bar{p}$ and k. In the process (35), the final-state system F is produced with a vanishing transverse momentum. The final-state system F acquires a non-vanishing transverse momentum q_T^μ ($q_T^\mu q_{T\mu} = -\mathbf{q_T}^2$) through the radiative process in Eq. (34); here, the final-state parton a has transverse momentum k_T^μ, and momentum conservation implies $k_T^\mu = -q_T^\mu$. In the small-q_T region (formally, when $q_T \to 0$), the scattering amplitude of the process in Eq. (34) is singular, and the singular behaviour is controlled by a well-known universal factorization formula (see, e.g., Eq. (4.23) and related formulae in Sect. 4.3 of Ref. [33]). The factorization formula relates the scattering amplitude of the process in Eq. (34) with the scattering amplitude of the hard-scattering subprocess $g(zp) + g(\bar{p}) \to F$. The singular factor is produced by the transverse-momentum spectrum of the collinear splitting subprocess

$$a(p) \;\to\; g(zp) + a((1-z)p) \;\;, \tag{36}$$

where z is the longitudinal-momentum fraction that is transferred from the initial-state parton a to the initial-state gluon g. At the lowest perturbative order in the QCD coupling, the transverse momentum spectrum of the subprocess in Eq. (36) is proportional to

$$\frac{\alpha_S}{\pi} \frac{d^2 \mathbf{q_T}}{\mathbf{q_T}^2} \, dz \, \left[\hat{P}_{ga}^{(1)}(z, \mathbf{q_T}) \right]^{\mu\nu} \;\;, \tag{37}$$

where $k_T^\mu = -q_T^\mu$ is the transverse momentum of the the final-state parton a in Eq. (36), and the collinear splitting function $\left[\hat{P}_{ga}^{(1)}(z, \mathbf{q_T}) \right]^{\mu\nu}$ has the following explicit from:

$$\left[\hat{P}_{gg}^{(1)}(z, \mathbf{q_T}) \right]^{\mu\nu} = 2\,C_A \left[-g^{\mu\nu} \left(\frac{z}{1-z} + z(1-z) \right) + 2 \frac{q_T^\mu q_T^\nu}{\mathbf{q_T}^2} \frac{1-z}{z} \right] \;\;, \tag{38}$$

$$\left[\hat{P}_{gq}^{(1)}(z, \mathbf{q_T}) \right]^{\mu\nu} = \left[\hat{P}_{g\bar{q}}^{(1)}(z, \mathbf{q_T}) \right]^{\mu\nu} = 2\,C_F \left[-g^{\mu\nu} \frac{1}{2}\, z + 2 \frac{q_T^\mu q_T^\nu}{\mathbf{q_T}^2} \frac{1-z}{z} \right] \;\;. \tag{39}$$

In Eq. (37), μ and ν are the Lorentz indices of the gluon $g(zp)$ in the process (36) and in its complex conjugate process, respectively.

In the context of our present study, the most important feature of the gluonic splitting process in Eq. (36) is that it is intrinsically *polarized*. The corresponding collinear splitting function $\left[\hat{P}_{ga}^{(1)}(z, \mathbf{q_T}) \right]^{\mu\nu}$ has a non-trivial dependence on the Lorentz (and, thus, spin) indices of the gluon, and this dependence is controlled by the azimuthal angle of the small transverse momentum that is radiated in the splitting process. We remark that this polarization effect is present despite the fact that we are not considering polarized-scattering processes. Indeed, we have performed the *sum* over the spin polarizations of the final-state parton $a((1-z)p)$ *and* the *average* over the spin polarizations of the initial-state parton $a(p)$. The intrinsic gluon polarization effects that arise in the splitting process (36) (and, more generally, in the collinear splitting process of Eq. (33)) produce *correlations* between the initial-state gluon legs of the scattering amplitude of the factorized hard-scattering subprocess $g(zp) + g(\bar{p}) \to F$ and of the corresponding complex conjugate scattering amplitude.

Replacing the gluon g with a quark q (or an antiquark $\bar{q}$) in Eq. (33), we obtain the quark (antiquark) collinear evolution process

$$a \;\to\; q(\bar{q}) + a_1 + a_2 + \dots \;\;. \tag{40}$$

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In this case, if we sum over the spin polarizations of the final-state partons a_i and we average over the spin polarizations of the initial-state parton a, the collinear evolution of the quark (antiquark) turns out to be *unpolarized*, as a consequence of helicity conservation in QCD radiation from a massless quark (antiquark). This essential difference between the collinear evolution of quarks (antiquarks) and gluons is eventually the origin of the difference of transverse-momentum resummation between $q\bar{q}$ annihilation processes and gluon fusion processes.

Going back to Eqs. (37)–(39), we can rewrite the gluon splitting functions as follows:

$$\left[\hat{P}_{ga}^{(1)}(z,\mathbf{q_T})\right]^{\mu\nu} = -g^{\mu\nu}\,\hat{P}_{ga}^{(1)}(z) + \left(g^{\mu\nu} + 2\,\frac{q_T^\mu q_T^\nu}{\mathbf{q_T}^2}\right)2\,G_{ga}^{(1)}(z)\ ,\quad a = g, q, \bar{q}\ , \qquad (41)$$

where the functions $G_{ga}^{(1)}(z)$ are those in Eqs. (30) ($a = g$) and (31) ($a = q, \bar{q}$), and the functions $\hat{P}_{ga}^{(1)}(z)$ are

$$\hat{P}_{gg}^{(1)}(z) = 2\,C_A\left[\frac{z}{1-z} + \frac{1-z}{z} + z(1-z)\right]\ , \qquad (42)$$

$$\hat{P}_{gq}^{(1)}(z) = \hat{P}_{g\bar{q}}^{(1)}(z) = C_F\,\frac{1+(1-z)^2}{z}\ . \qquad (43)$$

We see that $\hat{P}_{ga}^{(1)}(z)$ are the (real part of) first-order Altarelli–Parisi probabilities that control the customary collinear evolution of the unpolarized gluon parton density $f_{g/h}(x,\mu^2)$. In Eq. (41), the unpolarized splitting functions $\hat{P}_{ga}^{(1)}(z)$ are multiplied by the tensor[‡‡] $-g^{\mu\nu}$ that does not produce any gluonic correlations; the collinear correlation effects are produced by the tensor that multiplies the functions $G_{ga}^{(1)}(z)$ in Eq. (41). Using Eq. (41) and performing the Fourier transformation of Eq. (37) from $\mathbf{q_T}$ space to $\mathbf{b}$ space, we reproduce the lowest-order structure of the resummation formulae in Eqs. (19) and (20) (more details are given in Ref. [29]). The term proportional to the unpolarized splitting function $\hat{P}_{ga}^{(1)}(z)$ produces the evolution of the parton density $f_{g/h}(x,\mu^2)$ up to the scale $\mu^2 = b_0^2/b^2$; this term also produces [12] a residual effect (included in the function $C_{ga}(z;\alpha_S)$ of Eq. (26)) that is related to the definition of the parton densities in the $\overline{\mathrm{MS}}$ factorization scheme. The term proportional to $G_{ga}^{(1)}(z)$ produces the first-order contribution to the function $G_{ga}(z;\alpha_S)$ in Eq. (26).

The factorized structure of Eq. (20) has no direct interpretation in terms of a probabilistic (classical or quasi-classical) partonic picture. In fact, the structure originates at the cross section level from an underlying quantum-mechanical *interference* between scattering amplitudes and their complex conjugates. In particular, the gluonic tensor $C_{ga}^{\mu\nu}$ in Eqs. (20) and (26) (unlike the quark coefficient C_{qa} in Eq. (13)) cannot be interpreted as the simple residual (and factorization-scheme dependent) effect of the customary evolution of the parton density $f_{a/h}(x,\mu^2)$ (which is a Lorentz scalar) from a non-perturbative scale up to the scale $\mu^2 = b_0^2/b^2$. Incidentally, we note that the first-order functions $G_{ga}^{(1)}(z)$ ($a = g, q, \bar{q}$) in Eqs. (30) and (31) do not depend on the factorization scheme of the parton densities.

The interference phenomenon that leads to Eq. (20) produces specific physical effects. The tensor $D_{\mu\nu}$ (see Eqs. (26) and (28)) explicitly depends on the direction of the impact parameter vector $\mathbf{b}$ in the transverse plane. This dependence produces enhanced (i.e. non-suppressed by

---

[‡‡]This tensor can be replaced by the tensor $d^{\mu\nu}(p,\bar{p})$ of Eq. (27), because of current conservation (gauge invariance) of the scattering amplitude of the factorized hard-scattering subprocess $g(zp) + g(\bar{p}) \to F$.

terms of $\mathcal{O}(q_T/M))$ *spin* and *azimuthal* correlations in the $\mathbf{q_T}$ differential cross section $d\sigma^F$ at small values of q_T. The spins of the gluons in the gluon fusion process of Eq. (21) are correlated through Eq. (20). The azimuthal angles of the particles of the final-state system F are correlated to the azimuthal angle of $\mathbf{q_T}$ through Eq. (20) and the Fourier transformation in Eq. (19).

Note that spin and azimuthal correlations do not necessarily show up their effect simultaneously. For instance, considering the differential cross section averaged over the azimuthal angle of $\mathbf{q_T}$, the azimuthal correlations cancel, whereas the spin correlations can survive. This observation also implies that, in gluon fusion processes, the naïve resummation factor of Eq. (13) is not correct even in the case of the azimuthally-averaged cross section $d\sigma_F/dq_T^2$ (see Sect. 5). To explicitly illustrate this observation in a very simple manner, below we consider a specific process: the production of the SM Higgs boson, H, by the gluon fusion mechanism. Since H is a scalar particle, its $\mathbf{q_T}$ cross section has no azimuthal correlations.

In the case of Higgs boson production, the final-state system F in Eq. (1) is simply H, with momentum q^μ. Within the SM, the corresponding gluon fusion production mechanism is mediated by a heavy-quark (mainly, top quark) loop. Our conclusions are unchanged if we consider a generic ggH effective coupling (such as, for instance, the SM effective coupling that is obtained in the large-M_t approximation, M_t being the mass of the top quark). Since H is a scalar particle of spin 0, the tensor structure of the corresponding hard factor $H_{\mu_1 \nu_1, \mu_2 \nu_2}^{F=H}$ in Eq. (20) is uniquely determined by Lorentz covariance, parity conservation and gauge invariance; we have

$$H_{\mu_1 \nu_1, \mu_2 \nu_2}^{F=H}(x_1 p_1, x_2 p_2; \alpha_{\rm S}(M^2)) = \frac{1}{2} \, H_g^{F=H}(\alpha_{\rm S}(M^2)) \left(g_{\mu_1 \mu_2} - \frac{p_{2\,\mu_1} p_{1\,\mu_2}}{p_1 \cdot p_2} \right) \left(g_{\nu_1 \nu_2} - \frac{p_{2\,\nu_1} p_{1\,\nu_2}}{p_1 \cdot p_2} \right) ,$$
(44)

where the scalar function $H_g^{F=H}$ (see Eq. (24)) only depends on $\alpha_{\rm S}$ (apart from the dependence on M_t/M or other parameters of the effective coupling ggH). Using Eqs. (26) and (44), the Higgs boson resummation factor of Eq. (20) is

$$\begin{aligned}
\left[H^{F=H} C_1 C_2 \right]_{gg;a_1 a_2} &= H_g^{F=H}(\alpha_{\rm S}(M^2)) \, \big[\, C_{g\,a_1}(z_1; \alpha_{\rm S}(b_0^2/b^2)) \, C_{g\,a_2}(z_2; \alpha_{\rm S}(b_0^2/b^2)) \\
&+ G_{g\,a_1}(z_1; \alpha_{\rm S}(b_0^2/b^2)) \, G_{g\,a_2}(z_2; \alpha_{\rm S}(b_0^2/b^2)) \, \big] \quad .
\end{aligned}$$
(45)

Note that the right-hand side does not depend on the direction of $\mathbf{b}$: this implies that the $\mathbf{q_T}$ distribution has no azimuthal correlations in the small-q_T region. This result is consistent with the fact that the $\mathbf{q_T}$ cross section for H production has no azimuthal correlations at any values of q_T.

We can compare the result in Eq. (45) with the structure in Eq. (13). Owing to the specific factorized dependence on z_1 and z_2 of the two terms in the square bracket of Eq. (45), the term proportional to $G(z_1)\,G(z_2)$ cannot be removed by a redefinition of the function $C(z; \alpha_{\rm S})$. This shows that the naïve gluon fusion expression in Eq. (13) is not valid even in the simple case of Higgs boson production. The structure of Eq. (45) obviously differs from the (correct) structure of Eq. (13) for $q\bar{q}$ annihilation processes. The additional term proportional to $G(z_1)\,G(z_2)$ is indeed produced by (gluon) spin correlations (see also Sect. 4), which have no analogue in $q\bar{q}$ annihilation processes. We finally observe that the term $G(z_1)\,G(z_2)$ in Eq. (45) starts at $\mathcal{O}(\alpha_{\rm S}^2)$ in QCD perturbation theory (see Eq. (29)). Therefore, its effect first appears in the computation of the next-to-next-to-leading order (NNLO) QCD radiative corrections to the Higgs boson q_T cross section.

We now comment on our paper in Ref. [13]. The paper deals with the class of processes in Eq. (1) and presents a practical formalism for the NNLO QCD calculation of the corresponding cross sections at the fully-differential level. The formalism exploits the subtraction method to cancel the unphysical infrared (IR) divergences that separately occur in the real and virtual radiative corrections. The explicit construction of the subtraction counterterms is based on the universal structure of transverse-momentum resummation formulae and on their expansion up to NNLO in QCD perturbation theory. These are the resummation formulae discussed in the present paper. Working on the research project of Ref. [13], we found the results presented in this section, and explicitly documented for the first time in this paper. These results are essential for the application of the subtraction method of Ref. [13] to gluon fusion processes. In these processes, the naïve expression in Eq. (13) does not reproduce the correct (and singular) perturbative behaviour of the q_T cross section in the limit $q_T \to 0$. Using Eq. (13), rather than the correct result in Eq. (20), leads to subtraction counterterms that would spoil the cancellation of the IR divergences. Although the theoretical results of the present paper were not explicitly illustrated in Ref. [13], they were actually taken into account in the NNLO computations presented therein. In particular, the explicit application to Higgs boson production (which was implemented in the Monte Carlo code HNNLO) considered in Ref. [13] is based on and implements the results (e.g. Eq. (45)) illustrated in this section.

We also add a brief comment on Ref. [28]. The authors of Ref. [28] study transverse-momentum cross sections at small q_T by introducing a factorization formalism that differs from the resummation formalism considered in Sect. 2 and in this section. In the case of SM Higgs boson production or, more generally, gluon fusion processes, a perturbative ingredient of the factorization formulae in Refs. [28, 34] is a collinear function $\mathcal{I}_{ga}^{\mu\nu}$ (the Lorentz indices μ and ν and the parton indices g and a refer to the notation used throughout our paper), which emerges from authors' analysis based on Soft Collinear Effective Theory. The function $\mathcal{I}_{ga}^{\mu\nu}$ is explicitly computed [28, 34] up to its first-order contribution in α_S. The result at $\mathcal{O}(\alpha_S)$ is expressed in terms of two form factors, $\mathcal{F}_1^{ga}$ and $\mathcal{F}_2^{ga}$, that multiply the tensors in Eqs. (27) and (28), respectively. Moreover, at order α_S, we note (see[†] Eqs. (35) and (37) in Ref. [34]) that the ratio $\mathcal{F}_2^{gg}/\mathcal{F}_2^{gq}$ is equal to C_A/C_F; the same value, C_A/C_F, is obtained by considering the ratio of our functions $G_{gg}^{(1)}(z)$ and $G_{gq}^{(1)}(z)$ in Eqs. (30) and (31). In Ref. [28], the authors also note that the form factor $\mathcal{F}_2^{ga}$ does not contribute to the Higgs boson q_T cross section at the next-to-leading order (NLO). These first-order features of the function $\mathcal{I}_{ga}^{\mu\nu}$ have clear analogies with the structure of our universal coefficient tensor $C_{ga}^{\mu\nu}$ in Eq. (26).

4   The gluon fusion resummation formula in helicity space

The factorization formula in Eq. (20) involves sums over the Lorentz indices of the gluons (i.e., of the gluon field A_μ). These sums can be replaced by corresponding sums over the spin polarization states of the gluon. In particular, it can be convenient to consider physical polarization states of definite helicity λ ($\lambda = \pm$). In this section we present the helicity space version of the factorization formula in Eq. (20).

[†]We assume that there is a typo in the overall sign of the argument of the hypergeometric function $_0F_1$ in Eq. (37) of Ref. [34].

Exploiting gauge invariance or, more precisely, current conservation (e.g. $p_{1\,\mu_1} C_{g\,a_1}^{\mu_1\,\nu_1}(z_1; \dots)$) and Eq. (25)), the right-hand side of Eq. (20) can be written in the following form:

$$
\begin{aligned}
\left[H^F C_1 C_2 \right]_{gg; a_1 a_2} &= \sum_{\lambda_1, h_1, \lambda_2, h_2} H^F_{(\lambda_1, h_1),(\lambda_2, h_2)}(x_1 p_1, x_2 p_2; \boldsymbol{\Omega}; \alpha_S(M^2)) \\
&\times\ C_{g\,a_1}^{(\lambda_1, h_1)}(z_1; p_1, p_2, \mathbf{b}; \alpha_S(b_0^2/b^2))\ C_{g\,a_2}^{(\lambda_2, h_2)}(z_2; p_1, p_2, \mathbf{b}; \alpha_S(b_0^2/b^2))\ , \quad (46)
\end{aligned}
$$

where λ_i and h_i are helicity space indices ($\lambda_i = \pm$, $h_i = \pm$). The relation between the 'helicity tensors' in Eq. (46) and the Lorentz tensors in Eq. (20) is:

$$
\begin{aligned}
H^F_{(\lambda_1, h_1),(\lambda_2, h_2)}(x_1 p_1, x_2 p_2; \boldsymbol{\Omega}; \alpha_S) &= H^F_{\mu_1\,\nu_1,\mu_2\,\nu_2}(x_1 p_1, x_2 p_2; \boldsymbol{\Omega}; \alpha_S) \\
&\times\ \varepsilon^{\mu_1}_{(\lambda_1)}(x_1 p_1) \left[\varepsilon^{\nu_1}_{(h_1)}(x_1 p_1) \right]^* \varepsilon^{\mu_2}_{(\lambda_2)}(x_2 p_2) \left[\varepsilon^{\nu_2}_{(h_2)}(x_2 p_2) \right]^* , \quad (47)
\end{aligned}
$$

$$
C_{g\,a}^{(\lambda_i, h_i)}(z_i; p_1, p_2, \mathbf{b}; \alpha_S) = \left[\varepsilon^{(\lambda_i)}_{\mu}(x_i p_i) \right]^* C_{g\,a}^{\mu\,\nu}(z_i; p_1, p_2, \mathbf{b}; \alpha_S)\, \varepsilon^{(h_i)}_{\nu}(x_i p_i)\ , \qquad i = 1, 2\ , \qquad (48)
$$

where $\varepsilon^{(\lambda)}_{\mu}(p)$ denotes the polarization vector of a gluon with on-shell momentum p ($p^2 = 0$) and helicity λ.

We recall that the gluon helicity vectors are not uniquely defined. Having chosen the two helicity vectors $\varepsilon^{(+)}_{\mu}(p)$ and $\varepsilon^{(-)}_{\mu}(p) = \left[\varepsilon^{(+)}_{\mu}(p) \right]^*$, there is still the freedom to change the helicity vector basis. For instance, the phase transformation

$$
\varepsilon^{(\lambda)}_{\mu}(p)\ \rightarrow\ \widetilde{\varepsilon}^{(\lambda)}_{\mu}(p) = e^{i\lambda \varphi_p}\, \varepsilon^{(\lambda)}_{\mu}(p)\ , \tag{49}
$$

defines helicity states $\widetilde{\varepsilon}^{(\lambda)}_{\mu}(p)$ that are physically equivalent to $\varepsilon^{(\lambda)}_{\mu}(p)$ (the phase φ_p on the right-hand side of Eq. (49) can also depend on the gluon momentum p). Note that Eq. (46) is invariant under the transformation in Eq. (49), whereas the helicity tensors $H^F_{(\lambda_1, h_1),(\lambda_2, h_2)}$ in Eq. (47) and $C_{g\,a}^{(\lambda, h)}$ in Eq. (48) are not separately invariant. In any actual computation of $H^F_{(\lambda_1, h_1),(\lambda_2, h_2)}$ and $C_{g\,a}^{(\lambda, h)}$, the definition of the helicity basis has to be clearly specified.

Some general and important properties of the universal coefficient function $C_{g\,a}^{(\lambda, h)}$ do not depend on the specific definition of the helicity basis. Owing to general relations between helicity vectors and using Eq. (26), we find that the helicity tensor $C_{g\,a}^{(\lambda, h)}$ in Eq. (48) has the following explicit structure:

$$
C_{g\,a}^{(\lambda_i, h_i)}(z_i; p_1, p_2, \mathbf{b}; \alpha_S) = C_{g\,a}(z_i; \alpha_S)\, \delta^{\lambda_i, h_i} + G_{g\,a}(z_i; \alpha_S)\, D^{(\lambda_i)}(p_i, \mathbf{b})\, \delta^{\lambda_i, -h_i}\ , \qquad i = 1, 2\ , \quad (50)
$$

where the scalar coefficient functions $C_{g\,a}(z; \alpha_S)$ and $G_{g\,a}(z; \alpha_S)$ are those of Eq. (26), $\delta^{\lambda, h}$ is the customary Kronecker symbol ($\delta^{+,+} = \delta^{-,-} = 1$, $\delta^{+,-} = \delta^{-,+} = 0$), and the helicity coefficients $D^{(\lambda)}(p_i, \mathbf{b})$,

$$
D^{(\lambda)}(p_i, \mathbf{b}) = -\frac{2}{\mathbf{b}^2} \left[\mathbf{b} \cdot \varepsilon^{(-\lambda)}(x_i p_i) \right]^2\ , \qquad i = 1, 2\ , \tag{51}
$$

are pure phase factors that have the following explicit form:

$$
\begin{aligned}
D^{(\lambda)}(p_1, \mathbf{b}) &= -e^{+2i\lambda\,[\phi(\mathbf{b}) - \varphi_1]}\ , \tag{52} \\
D^{(\lambda)}(p_2, \mathbf{b}) &= -e^{-2i\lambda\,[\phi(\mathbf{b}) + \varphi_2]}\ . \tag{53}
\end{aligned}
$$

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Note that the helicity coefficients $D^{(\lambda_i)}(p_i, \mathbf{b})$ have a distinctive dependence on $\phi(\mathbf{b})$ (which is the azimuthal angle of the impact parameter vector $\mathbf{b}$), whereas the phases φ_1 and φ_2 simply depend on the explicit definition (to be specified) of the helicity vectors $\varepsilon_\mu^{(\lambda)}(x_1 p_1)$ and $\varepsilon_\mu^{(\lambda)}(x_2 p_2)$ (see Eq. (49)). As already discussed, the dependence on φ_1 and φ_2 cancels in Eq. (46).

The helicity space representation (50) of the gluonic coefficient tensors explicitly shows the presence of two components: a helicity-conserving component and a helicity-flip component. This two-component structure originates from the gluon collinear correlations discussed in Sect. 3. The helicity-conserving component leads to the naïve (in the case of gluon fusion processes) factorization formula in Eq. (13). Inserting Eq. (50) in Eq. (46), the helicity-flip component, which is proportional to the coefficient function $G_{ga}(z; \alpha_S)$ (see Eqs. (29)–(31)), obviously produces non-trivial helicity (spin) correlations in the process-dependent factor H^F.

The helicity-flip coefficients in Eqs. (52) and (53) can be rewritten as follows

$$D^{(\lambda)}(p_1, \mathbf{b}) = -e^{+2i\lambda\,[\phi(\mathbf{b}\cdot\mathbf{q_T})-\varphi_1]}\, e^{+2i\lambda\phi(\mathbf{q_T})} = e^{+2i\lambda\phi(\mathbf{b}\cdot\mathbf{q_T})}\, D^{(\lambda)}(p_1, \mathbf{q_T}) \ , \qquad (54)$$

$$D^{(\lambda)}(p_2, \mathbf{b}) = -e^{-2i\lambda\,[\phi(\mathbf{b}\cdot\mathbf{q_T})+\varphi_2]}\, e^{-2i\lambda\phi(\mathbf{q_T})} = e^{-2i\lambda\phi(\mathbf{b}\cdot\mathbf{q_T})}\, D^{(\lambda)}(p_2, \mathbf{q_T}) \ , \qquad (55)$$

where $\phi(\mathbf{q_T})$ is the azimuthal angle of $\mathbf{q_T}$ (in the centre–of–mass frame of the collision), and $\phi(\mathbf{b}\cdot\mathbf{q_T}) \equiv \phi(\mathbf{b}) - \phi(\mathbf{q_T})$ is the relative angle between $\mathbf{q_T}$ and the impact parameter (i.e. $\mathbf{b}\cdot\mathbf{q_T} = b\,q_T \cos\phi(\mathbf{b}\cdot\mathbf{q_T})$). Using Eqs. (46) and (50), the helicity-flip phase factors in Eqs. (54) and (55) eventually enter the cross section formula in Eq. (19). Since $\phi(\mathbf{b}\cdot\mathbf{q_T})$ is simply the angular integration variable of the Fourier transformation in Eq. (19), the dependence on $\phi(\mathbf{q_T})$ of the $\mathbf{q_T}$ cross section $[d\sigma_F]$ is directly determined by the phase factors $\exp(\pm 2i\lambda\,\phi(\mathbf{q_T}))$ in Eqs. (54) and (55). Therefore, the helicity-flip component of the gluonic helicity tensor in Eq. (50) controls both the spin correlations and the azimuthal correlations of the $\mathbf{q_T}$ cross section in the small-q_T region.

In Sect. 3 we have discussed the case of SM Higgs boson production by gluon fusion. The corresponding helicity tensor $H^{F=H}_{(\lambda_1,h_1),(\lambda_2,h_2)}$ is obtained by using Eqs. (44) and (47). We have:

$$H^{F=H}_{(\lambda_1,h_1),(\lambda_2,h_2)}(x_1 p_1, x_2 p_2; \alpha_S(M^2)) = \frac{1}{2}\, H_g^{F=H}(\alpha_S(M^2))\, \delta^{\lambda_1,\lambda_2}\, \delta^{h_1,h_2}\, e^{i(\lambda_1-h_1)(\varphi_1+\varphi_2)} \ , \qquad (56)$$

where we have used the relation

$$\varepsilon_{\mu_1}^{(\lambda_1)}(x_1 p_1) \left(g^{\mu_1\mu_2} - \frac{p_2^{\mu_1} p_1^{\mu_2}}{p_1 \cdot p_2} \right) \varepsilon_{\mu_2}^{(\lambda_2)}(x_2 p_2) = -e^{i\lambda_1(\varphi_1+\varphi_2)}\, \delta^{\lambda_1,\lambda_2} \ . \qquad (57)$$

In Eq. (56), the constraint $\lambda_1 = \lambda_2$ (and $h_1 = h_2$) obviously originates from helicity conservation for the production of a boson with spin 0. Inserting Eqs. (50), (52), (53) and (56) in Eq. (46), we reobtain the result in Eq. (45). Terms proportional to $C(z_1)G(z_2)$ are absent from Eq. (45), since helicity conservation in the hard-process factor of Eq. (56) forbids contributions that are produced by a single helicity flip.

We comment on some of the results of Refs. [27, 35]. Studying diphoton production, the authors of Ref. [27] made the following important observation: the description of the small-q_T behaviour of the diphoton transverse-momentum cross section requires the introduction of new logarithmically-enhanced spin-flip contributions (which affect the azimuthal angle dependence of the produced diphoton system), through a mechanism that is unique to gluon scattering. This general observation was based [27] on the computation of the first-order QCD radiative corrections

to the lowest-order gluon fusion process $gg \to \gamma\gamma$. Expanding our resummed formulae (see, e.g., Eqs. (19), (46) and (50)) to the first order in α_S, we find agreement with the structure of the diphoton first-order results of Ref. [27]. To be precise, we 'almost' agree with those first-order results: our expression contains a term proportional to $\sum_{a=g,q,\bar{q}} G_{ga}^{(1)}(z) f_{a/h}(x/z)$, whereas the expression in Ref. [27] includes only the contribution $G_{gg}^{(1)}(z) f_{g/h}(x/z)$ (the function $P'_{g/g}(z)$ of Ref. [27] is related to $G_{gg}^{(1)}(z)$ in Eq. (30); namely, $P'_{g/g}(z) = 2\, G_{gg}^{(1)}(z)$). The authors of Ref. [27] also proposed an all-order generalization of their first-order results. The generalization (which is expressed in the Collins–Soper frame [36] of the diphoton pair) is achieved by introducing a new helicity-flip unintegrated parton density of the colliding hadrons. This unintegrated (transverse-momentum-dependent) parton density, or, more precisely, its Fourier transform to $\mathbf{b}$ space is denoted by $\delta^{\lambda,-h}\, \mathcal{P}'_{g/h}(x, \mathbf{b})$, where $\mathcal{P}'_{g/h}(x, \mathbf{b})$ does not depend on the gluon helicity state $\lambda = -h$. The general dependence of $\mathcal{P}'_{g/h}(x, \mathbf{b})$ on $\mathbf{b}$ and the possible relation between $\mathcal{P}'_{g/h}(x, \mathbf{b})$ and the customary parton densities are not specified in Ref. [27] (apart from the relation with $f_{g/h}$ that can be inferred from the first-order calculation therein). Our resummation formulae also contain one helicity-flip component for each of the colliding hadrons. This component is proportional to the following expression:

$$\delta^{\lambda,-h} \int_x^1 \frac{dz}{z} \sum_{a=g,q,\bar{q}} G_{ga}(z; \alpha_S(b_0^2/b^2))\, f_{a/h_i}(x/z, b_0^2/b^2)\, D^{(\lambda)}(p_i, \mathbf{b})\ , \qquad i = 1, 2\ , \qquad (58)$$

which is directly related to the parton densities $f_{a/h}$ through the longitudinal-momentum convolution with the perturbatively-calculable gluonic coefficient functions $G_{ga}(z; \alpha_S)$ ($a = g, q, \bar{q}$). Note also that the expression in Eq. (58) explicitly depends on the gluon helicity λ through the helicity coefficients $D^{(\lambda)}(p_i, \mathbf{b})$ ($i = 1, 2$). The helicity coefficients introduce a definite and non-trivial (though, simple) functional dependence on the azimuthal angle $\phi(\mathbf{b})$ (see Eqs. (52) and (53)) or, equivalently, on the relative azimuthal angle $\phi(\mathbf{b} \cdot \mathbf{q_T})$ (see Eqs. (54) and (55)). This functional dependence eventually produces definite *coherent* correlations (see, e.g., Eq. (45) and, more generally, Eqs. (66), (70) and (76) in Sects. 5 and 6) between the helicities, λ_1 and λ_2, of the two colliding gluons. Owing to this helicity dependence of the helicity-flip components in Eq. (58), by using our gluon fusion resummed formulae we are not able to reproduce the diphoton results of Ref. [27] (e.g., the structure of Eqs. (27), (30) and (38) in Ref. [27]): the differences show up starting from contributions of relative order α_S^2 with respect to the lowest-order process $gg \to \gamma\gamma$. Few additional comments about this point are included in the final part of Sect. 6.

5 Azimuthally-averaged cross sections in gluon fusion processes

Starting from the general $\mathbf{q_T}$ cross section in Eq. (4), we can consider its average over $\phi(\mathbf{q_T})$, at fixed values of the additional kinematical variables $\boldsymbol{\Omega}$ of the final-state system F. We thus define the following azimuthally-averaged cross section:

$$\left\langle \frac{d\sigma_F}{d^2\mathbf{q_T}\, dM^2\, dy\, d\boldsymbol{\Omega}} \right\rangle_\phi \equiv \int_0^{2\pi} \frac{d\phi(\mathbf{q_T})}{2\pi} \frac{d\sigma_F}{d^2\mathbf{q_T}\, dM^2\, dy\, d\boldsymbol{\Omega}}$$

$$= \frac{1}{\pi} \frac{d\sigma_F}{dq_T^2\, dM^2\, dy\, d\boldsymbol{\Omega}} (p_1, p_2; q_T^2, M, y, \boldsymbol{\Omega})\ . \qquad (59)$$

Following the shorthand notation of Eq. (6), the singular component (in the small-q_T region) of the cross section (59) is denoted as $[d\sigma_F]_\phi$. Then we consider the decomposition in Eq. (18). As recalled at the end of Sect. 2, in the case of $q\bar{q}$ annihilation subprocesses, small-q_T resummation does not produce any azimuthal correlations and, therefore, the corresponding $[d\sigma_F]^{(q\bar{q}-\text{ann.})}$ and $[d\sigma_F]_\phi^{(q\bar{q}-\text{ann.})}$ are obtained by the same resummation formula (see Eqs. (7) and (13) with $c = q, \bar{q}$). In the case of gluon fusion processes, the resummation formula for the azimuthally-averaged cross section is obtained from Eq. (19) by replacing

$$[d\sigma_F]^{(g-\text{fus.})} \rightarrow [d\sigma_F]_\phi^{(g-\text{fus.})} \quad , \tag{60}$$

and by performing the following replacement

$$\left[H^F C_1 C_2\right]_{gg;a_1a_2} \rightarrow \left[H^F C_1 C_2\right]_{gg;a_1a_2}^\phi \tag{61}$$

in the integrand on the right-hand side; the two-dimensional Fourier transformation con be replaced by the Bessel transformation as in Eq. (16). Therefore, the resummation formula for the azimuthally-averaged cross section is:

$$[d\sigma_F]_\phi^{(g-\text{fus.})} = \frac{M^2}{s} \left[d\sigma_{gg,F}^{(0)}\right] \int_0^{+\infty} \frac{db}{2\pi} \, b \, J_0(bq_T) \, S_g(M,b)$$
$$\times \sum_{a_1,a_2} \int_{x_1}^1 \frac{dz_1}{z_1} \int_{x_2}^1 \frac{dz_2}{z_2} \left[H^F C_1 C_2\right]_{gg;a_1a_2}^\phi \, f_{a_1/h_1}(x_1/z_1, b_0^2/b^2) \, f_{a_2/h_2}(x_2/z_2, b_0^2/b^2) \, , \tag{62}$$

where the integrand factor $\left[H^F C_1 C_2\right]_{gg;a_1a_2}^\phi$ has the following explicit expression:

$$\left[H^F C_1 C_2\right]_{gg;a_1a_2}^\phi = H_g^F(x_1p_1, x_2p_2; \boldsymbol{\Omega}; \alpha_S(M^2)) \, C_{ga_1}(z_1; \alpha_S(b_0^2/b^2)) \, C_{ga_2}(z_2; \alpha_S(b_0^2/b^2))$$
$$+ H_G^F(x_1p_1, x_2p_2; \boldsymbol{\Omega}; \alpha_S(M^2)) \, G_{ga_1}(z_1; \alpha_S(b_0^2/b^2)) \, G_{ga_2}(z_2; \alpha_S(b_0^2/b^2)) \, . \tag{63}$$

This result can easily be obtained by using Eqs. (46), (50), (54) and (55) and the following elementary integral:

$$\int_0^{2\pi} \frac{d\phi(\mathbf{q_T})}{2\pi} \, e^{\pm i n \phi(\mathbf{q_T})} = \delta^{n,0} \quad , \qquad n = 0, 1, 2, \ldots \quad . \tag{64}$$

The process-dependent factor H_g^F on the right-hand side of Eq. (63) is defined in Eq. (24). Its equivalent representation in helicity space is

$$H_g^F(x_1p_1, x_2p_2; \boldsymbol{\Omega}; \alpha_S) = \sum_{\lambda_1,\lambda_2} H_{(\lambda_1,\lambda_1),(\lambda_2,\lambda_2)}^F(x_1p_1, x_2p_2; \boldsymbol{\Omega}; \alpha_S) \quad . \tag{65}$$

The spin-correlated hard-scattering factor H_G^F has the following expression:

$$H_G^F(x_1p_1, x_2p_2; \boldsymbol{\Omega}; \alpha_S) = \sum_\lambda H_{(\lambda,-\lambda),(\lambda,-\lambda)}^F(x_1p_1, x_2p_2; \boldsymbol{\Omega}; \alpha_S) \, e^{-2i\lambda(\varphi_1+\varphi_2)} \tag{66}$$
$$= H_{\mu_1\nu_1,\mu_2\nu_2}^F(x_1p_1, x_2p_2; \boldsymbol{\Omega}; \alpha_S) \, d_{(4)}^{\mu_1\nu_1,\mu_2\nu_2}(p_1,p_2) \, , \tag{67}$$

where the 4th-rank tensor $d_{(4)}^{\mu_1\nu_1,\mu_2\nu_2}$ is

$$d_{(4)}^{\mu_1\nu_1,\mu_2\nu_2}(p_1,p_2) = \frac{1}{2} \left[d^{\mu_1\mu_2} d^{\nu_1\nu_2} + d^{\mu_1\nu_2} d^{\mu_2\nu_1} - d^{\mu_1\nu_1} d^{\mu_2\nu_2} \right], \tag{68}$$

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with $d^{\mu\nu} = d^{\mu\nu}(p_1, p_2)$ (see Eq. (27)). From the helicity space representation (66) of the process-dependent factor H_G^F, we see that both gluons (with momenta $x_1 p_1$ and $x_2 p_2$) undergo a helicity flip (i.e., $\lambda_1 = -h_1$ and $\lambda_2 = -h_2$); moreover, we note that the two helicity flips are *correlated*: they occur *coherently*, with the constraint $\lambda_1 = \lambda_2$.

As already anticipated in Sect. 3, in gluon fusion processes the naïve resummation factor of Eq. (13) is not correct even in the case of azimuthally-averaged cross sections. The correct resummation factor in given in Eq. (63). Comparing Eqs. (13) and (63), we note the presence of the additional gluonic coefficient function $G_{ga}(z; \alpha_S)$ and of the corresponding hard-scattering factor H_G^F (which, in general‡, differs from the spin-uncorrelated factor H_g^F).

6 Fourier vs. Bessel transformations and azimuthal correlations of the $\mathbf{q_T}$ cross section

Unlike the case of $q\bar{q}$ annihilation, the gluon fusion resummation factor $\left[H^F C_1 C_2 \right]$ in Eqs. (20) or (46) depends on the azimuthal angle $\phi(\mathbf{b})$ of the impact parameter vector $\mathbf{b}$. Therefore, in the resummation formula (19) we cannot simply use the relation (16) to perform the azimuthal integration involved in the Fourier transformation from $\mathbf{b}$ space to $\mathbf{q_T}$ space. Nonetheless, the dependence of $\left[H^F C_1 C_2 \right]_{gg;a_1 a_2}$ on $\phi(\mathbf{b})$ is fully specified at arbitrary perturbative orders. Moreover, this dependence is sufficiently simple to be handled in explicit form. As shown below, the azimuthal integration involved in the Fourier transformation can explicitly be performed. The two-dimensional Fourier transformation is thus replaced by one-dimensional Bessel transformations. The weight functions of these one-dimensional transformations are the 0th-order Bessel function $J_0(bq_T)$ and higher-order Bessel functions, such as, the 2nd-order and 4th-order functions $J_2(bq_T)$ and $J_4(bq_T)$.

To present the relation between Fourier and Bessel transformations, we first consider the gluon fusion resummation formula in helicity space. In this formulation, the $\phi(\mathbf{b})$ dependence of the integrand factor $\left[H^F C_1 C_2 \right]$ (see Eqs. (46) and (50)) is given by the helicity coefficients $D^{(\lambda)}(p_i, \mathbf{b})$ in Eqs. (52)–(55). We have to perform the Fourier transformation of contributions that are linear and quadratic with respect to these helicity coefficients. The explicit results of the corresponding integration over $\phi(\mathbf{b})$ are the following:

$$
\begin{aligned}
\int \frac{d^2\mathbf{b}}{2\pi} \, e^{i\mathbf{b}\cdot\mathbf{q_T}} \, D^{(\lambda)}(p_i, \mathbf{b}) \, F(\mathbf{b}^2) \; &= \; D^{(\lambda)}(p_i, \mathbf{q_T}) \int \frac{d^2\mathbf{b}}{2\pi} \, e^{i\mathbf{b}\cdot\mathbf{q_T}} \, e^{\pm 2i\lambda\,\phi(\mathbf{b}\cdot\mathbf{q_T})} \, F(\mathbf{b}^2) \\
&= \; -D^{(\lambda)}(p_i, \mathbf{q_T}) \int_0^{+\infty} db \, b \, J_2(bq_T) \, F(b^2) \;, \qquad (69)
\end{aligned}
$$

---

‡In the specific case of SM Higgs boson production, we have $H_G^{F=H} = H_g^{F=H}$ (see Eq. (45)).

$$\int \frac{d^2\mathbf{b}}{2\pi} \; e^{i\mathbf{b}\cdot\mathbf{q_T}} \; D^{(\lambda_1)}(p_1, \mathbf{b}) \; D^{(\lambda_2)}(p_2, \mathbf{b}) \; F(\mathbf{b}^2)$$

$$= D^{(\lambda_1)}(p_1, \mathbf{q_T}) \; D^{(\lambda_2)}(p_2, \mathbf{q_T}) \int \frac{d^2\mathbf{b}}{2\pi} \; e^{i\mathbf{b}\cdot\mathbf{q_T}} \; e^{+2i(\lambda_1-\lambda_2)\,\phi(\mathbf{b}\cdot\mathbf{q_T})} \; F(\mathbf{b}^2)$$

$$= D^{(\lambda_1)}(p_1, \mathbf{q_T}) \; D^{(\lambda_2)}(p_2, \mathbf{q_T}) \int_0^{+\infty} db\, b \left[\delta^{\lambda_1, \lambda_2} \, J_0(bq_T) + \delta^{\lambda_1, -\lambda_2} \, J_4(bq_T) \right] \; F(b^2)$$

$$= \delta^{\lambda_1, \lambda_2} \; e^{-2i\lambda_1(\varphi_1+\varphi_2)} \int_0^{+\infty} db\, b \; J_0(bq_T) \; F(b^2)$$

$$+ \; \delta^{\lambda_1, -\lambda_2} \; D^{(\lambda_1)}(p_1, \mathbf{q_T}) \; D^{(\lambda_2)}(p_2, \mathbf{q_T}) \int_0^{+\infty} db\, b \; J_4(bq_T) \; F(b^2) \; , \tag{70}$$

where $F(\mathbf{b}^2)$ denotes a generic function of $\mathbf{b}^2$. The results in Eqs. (69) and (70) are straightforwardly obtained by simply using the following integral representation of the Bessel function J_{2n}:

$$J_{2n}(x) = (-1)^n \int_0^{2\pi} \frac{d\phi}{2\pi} \; e^{+ix\cos\phi} \; e^{\pm i2n\phi} \; , \qquad n = 0, 1, 2, \dots \; , \tag{71}$$

where the variable x is a real number.

Considering the formulation in terms of Lorentz tensors (see Eqs. (20) and (26)), the dependence on $\phi(\mathbf{b})$ is produced by the tensor $D^{\mu\nu}(p_1, p_2; \mathbf{b})$ in Eq. (28). The results analogous to those in Eqs. (69) and (70) are:

$$\int \frac{d^2\mathbf{b}}{2\pi} \; e^{i\mathbf{b}\cdot\mathbf{q_T}} \; D^{\mu\nu}(p_1, p_2; \mathbf{b}) \; F(\mathbf{b}^2) = - \, D^{\mu\nu}(p_1, p_2; \mathbf{q_T}) \int_0^{+\infty} db\, b \; J_2(bq_T) \; F(b^2) \; , \tag{72}$$

$$\int \frac{d^2\mathbf{b}}{2\pi} \; e^{i\mathbf{b}\cdot\mathbf{q_T}} \; D^{\mu_1\nu_1}(p_1, p_2; \mathbf{b}) \; D^{\mu_2\nu_2}(p_1, p_2; \mathbf{b}) \; F(\mathbf{b}^2)$$

$$= d_{(4)}^{\mu_1\nu_1, \mu_2\nu_2}(p_1, p_2) \int_0^{+\infty} db\, b \; J_0(bq_T) \; F(b^2)$$

$$+ \; D_{(4)}^{\mu_1\nu_1, \mu_2\nu_2}(p_1, p_2; \mathbf{q_T}) \int_0^{+\infty} db\, b \; J_4(bq_T) \; F(b^2) \; , \tag{73}$$

where $d_{(4)}^{\mu_1\nu_1, \mu_2\nu_2}$ is given in Eq. (68), and the 4th-rank tensor $D_{(4)}^{\mu_1\nu_1, \mu_2\nu_2}$ is

$$D_{(4)}^{\mu_1\nu_1, \mu_2\nu_2}(p_1, p_2; \mathbf{q_T}) = D^{\mu_1\nu_1}(p_1, p_2; \mathbf{q_T}) \, D^{\mu_2\nu_2}(p_1, p_2; \mathbf{q_T}) - d_{(4)}^{\mu_1\nu_1, \mu_2\nu_2}(p_1, p_2) \; . \tag{74}$$

The derivation of Eqs. (72) and (73) is similar to the derivation of Eqs. (69) and (70), apart from few additional algebraic manipulations.

We note that the three Bessel functions J_0, J_2 and J_4 are not independent. Owing to general recursion relations between Bessel functions, we have:

$$J_4(x) = \frac{4\,(6 - x^2)}{x^2} \, J_2(x) - 3\, J_0(x) \; . \tag{75}$$

This relation can be used to express the results in Eqs. (69), (70), (72) and (73) in terms of two (rather than three) Bessel functions.

The representation in terms of Bessel transformations offers a technical simplification of the resummation formula, since the two-dimensional Fourier transformation is replaced by one-dimensional transformations. Moreover, in Eqs. (69) and (70) (or, equivalently, in Eqs. (72) and (73)) the dependence on the azimuthal angle $\phi(\mathbf{q_T})$ is fully and explicitly factorized with respect to the dependence on the magnitude, q_T, of the transverse momentum (the dependence on q_T is produced by the integration over b). Therefore, we can express the gluon fusion resummation formula in a form that manifestly exhibits the functional dependence on $\phi(\mathbf{q_T})$ of the $\mathbf{q_T}$ cross section, at small values of q_T.

To be explicit, we insert Eqs. (69) and (70) (or, Eqs. (72) and (73)) in the resummation formula (19), and we obtain:

$$
\begin{aligned}
[d\sigma_F]^{(g-\text{fus.})} &= [d\sigma_F]^{(g-\text{fus.})}_\phi \\
&+ [d\sigma_F]_{C_1 G_2} \left[H^F(\phi(\mathbf{q_T})) \right]_{C_1 G_2} + [d\sigma_F]_{G_1 C_2} \left[H^F(\phi(\mathbf{q_T})) \right]_{G_1 C_2} \\
&+ [d\sigma_F]_{GG} \left[H^F(\phi(\mathbf{q_T})) \right]_{GG} \quad .
\end{aligned}
\tag{76}
$$

The notation $\left[H^F(\phi(\mathbf{q_T})) \right]_I$ (rather than simply $\left[H^F \right]_I$) remarks that each of these factors (the subscript I is $I = C_1 G_2, G_1 C_2$ or GG) depends on $\mathbf{q_T}$ only through its azimuthal angle $\phi(\mathbf{q_T})$ (i.e., these factors do not depend on the magnitude of $\mathbf{q_T}$). All the other terms on the right-hand side of Eq. (76) depend on q_T, but they are independent of $\phi(\mathbf{q_T})$.

In Eq. (76), the gluon fusion cross section $[d\sigma_F]^{(g-\text{fus.})}$ is partitioned into several contributions. The first contribution is equal to $[d\sigma_F]^{(g-\text{fus.})}_\phi$, the azimuthally-averaged cross section. Since the cross section $[d\sigma_F]^{(g-\text{fus.})}$ is evaluated at fixed values of $\phi(\mathbf{q_T})$, our notation is imprecise. The notation actually means that the first contribution on the right-hand side of Eq. (76) is explicitly given by the expression presented on the right-hand side of Eq. (62); this expression coincides with the expression of the azimuthal average over $\phi(\mathbf{q_T})$ of $[d\sigma_F]^{(g-\text{fus.})}$. The other cross section contributions, $[d\sigma_F]_I$, in Eq. (76) have the following form:

$$
\begin{aligned}
[d\sigma_F]_I = \frac{M^2}{s} \left[d\sigma^{(0)}_{gg,F} \right] \int_0^{+\infty} \frac{db}{2\pi} \, b \, S_g(M,b) \\
\times \sum_{a_1,a_2} \int_{x_1}^1 \frac{dz_1}{z_1} \int_{x_2}^1 \frac{dz_2}{z_2} \, [JC_1C_2]^I_{gg;a_1a_2} \, f_{a_1/h_1}(x_1/z_1, b_0^2/b^2) \, f_{a_2/h_2}(x_2/z_2, b_0^2/b^2) \; ,
\end{aligned}
\tag{77}
$$

where the integrand factors denoted by $[JC_1C_2]^I$ are given by the following explicit expressions:

$$
[JC_1C_2]^{C_1 G_2}_{gg;a_1a_2} = J_2(bq_T) \, C_{g\,a_1}(z_1; \alpha_S(b_0^2/b^2)) \, G_{g\,a_2}(z_2; \alpha_S(b_0^2/b^2)) \; ,
\tag{78}
$$

$$
[JC_1C_2]^{G_1 C_2}_{gg;a_1a_2} = J_2(bq_T) \, G_{g\,a_1}(z_1; \alpha_S(b_0^2/b^2)) \, C_{g\,a_2}(z_2; \alpha_S(b_0^2/b^2)) \; ,
\tag{79}
$$

$$
[JC_1C_2]^{GG}_{gg;a_1a_2} = J_4(bq_T) \, G_{g\,a_1}(z_1; \alpha_S(b_0^2/b^2)) \, G_{g\,a_2}(z_2; \alpha_S(b_0^2/b^2)) \; .
\tag{80}
$$

The hard-scattering factors $\left[H^F(\phi(\mathbf{q_T}))\right]_I$ are process dependent; they are

$$
\begin{aligned}
\left[H^F(\phi(\mathbf{q_T}))\right]_{C_1 G_2} &= -H^F_{\mu_1\nu_1,\mu_2\nu_2}(x_1 p_1, x_2 p_2; \mathbf{\Omega}; \alpha_S(M^2))\, d^{\mu_1\nu_1}(p_1, p_2)\, D^{\mu_2\nu_2}(p_1, p_2; \mathbf{q_T}) \quad (81) \\
&= \sum_{\lambda_1,\lambda_2} H^F_{(\lambda_1,\lambda_1),(\lambda_2,-\lambda_2)}(x_1 p_1, x_2 p_2; \mathbf{\Omega}; \alpha_S(M^2))\; e^{-2i\lambda_2(\varphi_2+\phi(\mathbf{q_T}))} \\
&= \cos(2\phi(\mathbf{q_T})) \sum_{\lambda_1,\lambda_2} H^F_{(\lambda_1,\lambda_1),(\lambda_2,-\lambda_2)}(x_1 p_1, x_2 p_2; \mathbf{\Omega}; \alpha_S(M^2))\; e^{-2i\lambda_2\varphi_2} \quad (82) \\
&\quad - \sin(2\phi(\mathbf{q_T})) \sum_{\lambda_1,\lambda_2} i\,\lambda_2\, H^F_{(\lambda_1,\lambda_1),(\lambda_2,-\lambda_2)}(x_1 p_1, x_2 p_2; \mathbf{\Omega}; \alpha_S(M^2))\; e^{-2i\lambda_2\varphi_2}\,,
\end{aligned}
$$

$$
\begin{aligned}
\left[H^F(\phi(\mathbf{q_T}))\right]_{G_1 C_2} &= -H^F_{\mu_1\nu_1,\mu_2\nu_2}(x_1 p_1, x_2 p_2; \mathbf{\Omega}; \alpha_S(M^2))\, D^{\mu_1\nu_1}(p_1, p_2; \mathbf{q_T})\, d^{\mu_2\nu_2}(p_1, p_2) \quad (83) \\
&= \sum_{\lambda_1,\lambda_2} H^F_{(\lambda_1,-\lambda_1),(\lambda_2,\lambda_2)}(x_1 p_1, x_2 p_2; \mathbf{\Omega}; \alpha_S(M^2))\; e^{-2i\lambda_1(\varphi_1-\phi(\mathbf{q_T}))} \\
&= \cos(2\phi(\mathbf{q_T})) \sum_{\lambda_1,\lambda_2} H^F_{(\lambda_1,-\lambda_1),(\lambda_2,\lambda_2)}(x_1 p_1, x_2 p_2; \mathbf{\Omega}; \alpha_S(M^2))\; e^{-2i\lambda_1\varphi_1} \quad (84) \\
&\quad + \sin(2\phi(\mathbf{q_T})) \sum_{\lambda_1,\lambda_2} i\,\lambda_1\, H^F_{(\lambda_1,-\lambda_1),(\lambda_2,\lambda_2)}(x_1 p_1, x_2 p_2; \mathbf{\Omega}; \alpha_S(M^2))\; e^{-2i\lambda_1\varphi_1}\,,
\end{aligned}
$$

$$
\begin{aligned}
\left[H^F(\phi(\mathbf{q_T}))\right]_{GG} &= H^F_{\mu_1\nu_1,\mu_2\nu_2}(x_1 p_1, x_2 p_2; \mathbf{\Omega}; \alpha_S(M^2))\, D^{\mu_1\nu_1,\mu_2\nu_2}_{(4)}(p_1, p_2; \mathbf{q_T}) \quad (85) \\
&= \sum_{\lambda} H^F_{(\lambda,-\lambda),(-\lambda,\lambda)}(x_1 p_1, x_2 p_2; \mathbf{\Omega}; \alpha_S(M^2))\; e^{-2i\lambda(\varphi_1-\varphi_2-2\phi(\mathbf{q_T}))} \\
&= \cos(4\phi(\mathbf{q_T})) \sum_{\lambda} H^F_{(\lambda,-\lambda),(-\lambda,\lambda)}(x_1 p_1, x_2 p_2; \mathbf{\Omega}; \alpha_S(M^2))\; e^{-2i\lambda(\varphi_1-\varphi_2)} \quad (86) \\
&\quad + \sin(4\phi(\mathbf{q_T})) \sum_{\lambda} i\,\lambda\, H^F_{(\lambda,-\lambda),(-\lambda,\lambda)}(x_1 p_1, x_2 p_2; \mathbf{\Omega}; \alpha_S(M^2))\; e^{-2i\lambda(\varphi_1-\varphi_2)}\,.
\end{aligned}
$$

The factors $\left[H^F(\phi(\mathbf{q_T}))\right]_{C_1 G_2}$ and $\left[H^F(\phi(\mathbf{q_T}))\right]_{G_1 C_2}$ involve a single helicity flip (see Eqs. (82) and (84)). From the helicity space representation (86) of the factor $\left[H^F(\phi(\mathbf{q_T}))\right]_{GG}$, we see that both gluons (with momenta $x_1 p_1$ and $x_2 p_2$) undergo a helicity flip, and the two helicity flips are *correlated* by the constraint $\lambda_1 = -\lambda_2$. We recall that the double helicity flip with $\lambda_1 = \lambda_2$ leads to the factor H^F_G (see Eq. (66)) that enters the cross section contribution $[d\sigma_F]^{(g-\text{fus.})}_\phi$ (see Eq. (63)).

By direct inspection of Eqs. (76), (82), (84) and (86), we note that the $\phi(\mathbf{q_T})$ azimuthal dependence of the logarithmically-enhanced terms at small q_T is fully determined by the general structure of the transverse-momentum resummation formula. The $\mathbf{q_T}$ cross section $[d\sigma_F]^{(g-\text{fus.})}$ contains a contribution that is independent of $\phi(\mathbf{q_T})$ (namely, the term $[d\sigma_F]^{(g-\text{fus.})}_\phi$) plus a linear combination of the four trigonometric functions $\cos(2\phi(\mathbf{q_T}))$, $\sin(2\phi(\mathbf{q_T}))$, $\cos(4\phi(\mathbf{q_T}))$ and $\sin(4\phi(\mathbf{q_T}))$. No other functional dependence on $\phi(\mathbf{q_T})$ is allowed by the gluon fusion resummation formula.

A general remark about the azimuthal dependence is required. Since we are dealing with collisions of spin unpolarized hadrons, the corresponding cross sections are invariant under azimuthal rotations in the transverse plane. Therefore, the multidifferential cross section $[d\sigma_F]^{(g-\text{fus.})}$ cannot depend on the absolute value of $\phi(\mathbf{q_T})$. The azimuthal dependence of $[d\sigma_F]^{(g-\text{fus.})}$ can only appear

through final-state azimuthal *correlations*, namely, through functions of relative azimuthal angles $\Delta\phi_i$, such as, for instance, $\Delta\phi_i = \phi(\mathbf{q_T}) - \phi(\mathbf{q_{T}}_i)$. Here, $\phi(\mathbf{q_{T}}_i)$ denotes the azimuthal angle of one of the particles in the produced final-state system F (see Eq. (1)). Note that these azimuthal correlations are consistent with our previous conclusions about the functional dependence on $\phi(\mathbf{q_T})$; indeed, we have $\cos(2\Delta\phi_i) = \cos\big(2\phi(\mathbf{q_T})\big)\cos\big(2\phi(\mathbf{q_{T}}_i)\big) + \sin\big(2\phi(\mathbf{q_T})\big)\sin\big(2\phi(\mathbf{q_{T}}_i)\big)$, and analogous relations apply to $\sin(2\Delta\phi_i), \cos(4\Delta\phi_i)$ and $\sin(4\Delta\phi_i)$. According to the general notation that we have used in this paper, the $\phi(\mathbf{q_{T}}_i)$ dependence of the multidifferential cross section in Eq. (4) (or, Eq. (6)) is introduced through the final-state kinematical variables generically denoted by $\boldsymbol{\Omega} = \{\Omega_A, \Omega_B, \cdots\}$. In Eqs. (82), (84) and (86), the dependence on $\boldsymbol{\Omega}$ (e.g., on $\phi(\mathbf{q_{T}}_i)$) of $H^F(x_1p_1, x_2p_2; \boldsymbol{\Omega}; \alpha_S(M^2))$ combines with the explicit dependence on $\phi(\mathbf{q_T})$ to produce the final-state azimuthal correlations. The functional form of the azimuthal correlations is determined by the $\phi(\mathbf{q_T})$ dependence of the hard-scattering factors $\big[H^F(\phi(\mathbf{q_T}))\big]_I$, through perturbative coefficients (we recall that the Lorentz tensor $H^F_{\mu_1 \nu_1, \mu_2 \nu_2}$, and the equivalent helicity tensor $H^F_{(\lambda_1, h_1),(\lambda_2, h_2)}$, are power series functions of α_S) that are process dependent and observable dependent (i.e., they depend on the physical observable that is specified by the multidifferential cross section $d\sigma_F/d\boldsymbol{\Omega}$). In particular, if the multidifferential cross section is insensitive to the azimuthal angles $\phi(\mathbf{q_{T}}_i)$ (see, e.g., the simple case of inclusive production of the SM Higgs boson), the corresponding hard-scattering factors $\big[H^F(\phi(\mathbf{q_T}))\big]_I$ vanish order-by-order in perturbation theory.

The structure of the gluon fusion resummation formula (76) is much richer than the structure of the corresponding 'naïve' (i.e., extrapolated from $q\bar{q}$ annihilation) formula in Eq. (7). The additional structure is due to the helicity-flip contributions. These contributions are perturbatively driven by the gluonic coefficient function $G_{ga}(z; \alpha_S)$, and they lead to several hard-scattering factors: the factor H^F_G (see Eq. (66)), which contributes to the term $[d\sigma_F]^{(g-\text{fus.})}_\phi$, and the factors $\big[H^F(\phi(\mathbf{q_T}))\big]_I$. We briefly sketch the small-q_T singular behaviour produced by these various terms in QCD perturbation theory. To this purpose, we expand the integrand of the resummation formula in powers of $\alpha_S = \alpha_S(\mu_R^2)$ (with $\mu_R \sim M$), and we explicitly perform the Bessel transformations from b space to q_T space (technical details on this procedure, and explicit perturbative formulae can be found, for instance, in Ref. [20]).

The single helicity-flip terms first contribute at the relative order α_S. At this perturbative order, they lead to a partonic cross section contribution that is proportional to

$$\frac{\alpha_S}{\pi} \, \delta(1 - z_1) \, \delta_{g\,a_1} \, G^{(1)}_{g\,a_2}(z_2) \, \big[H^F(\phi(\mathbf{q_T}))\big]_{C_1 G_2} \left[\frac{1}{q_T^2}\right]_+ \, , \tag{87}$$

and to an analogous contribution that is obtained by the exchange $1 \leftrightarrow 2$ of the subscripts. Since at this order there is a leading logarithmic term of the type $\left[\frac{1}{q_T^2} \ln(M^2/q_T^2)\right]_+$ (this term is due to the 'naïve' contributions, with no helicity flips), the term in the expression (87) represents a next-to-leading logarithmic effect. The double helicity-flip term first contributes at the relative order α_S^2; it produces a partonic cross section contribution that is proportional to

$$\left(\frac{\alpha_S}{\pi}\right)^2 G^{(1)}_{g\,a_1}(z_1) \, G^{(1)}_{g\,a_2}(z_2) \left(2 \big[H^F(\phi(\mathbf{q_T}))\big]_{GG} \left[\frac{1}{q_T^2}\right]_+ + H^F_G \, \delta(q_T^2)\right) \, , \tag{88}$$

where the functions $G^{(1)}_{g\,a}(z)$ are given in Eqs. (30) and (31).

As we have discussed in Sect. 4, our gluon fusion resummation formula produces differences with respect to the diphoton results presented in Ref. [27]. These differences are evident starting from contributions at the relative order α_S^2. To explicitly point out the $\mathcal{O}(\alpha_S^2)$ differences, we can rewrite the structure of Eq. (38) in Ref. [27] by using our notation: this gives an expression that is proportional to

$$\left(\frac{\alpha_S}{\pi}\right)^2 G_{gg}^{(1)}(z_1) \, G_{gg}^{(1)}(z_2) \, \delta_{g\,a_1} \delta_{g\,a_2} \, 2 \left(\left[H^F(\phi(\mathbf{q_T}))\right]_{GG} + H_G^F\right) \left[\frac{1}{q_T^2}\right]_+ \,, \tag{89}$$

where the sum of hard-scattering factors in the round bracket originates from Eq. (30) of Ref. [27]. The q_T dependence of the expression (89) differs from that of our expression (88), and the difference is not removed by setting $G_{gq}^{(1)}(z) = G_{g\bar{q}}^{(1)}(z) = 0$ (the difference between the $\mathcal{O}(\alpha_S)$ expressions in Eq. (33) of Ref. [27] and in our Eq. (87) disappears by forcing $G_{gq}^{(1)}(z)$ and $G_{g\bar{q}}^{(1)}(z)$ to vanish).

At higher perturbative orders, the small-q_T singular behaviour in the expressions (87) and (88) is enhanced by powers of $\ln(M^2/q_T^2)$. The dominant logarithmic enhancement is produced by the gluon form factor $S_g(M, b)$, which appear in $[d\sigma_F]_\phi^{(g-\text{fus.})}$ (see Eq. (62)) and in each cross section contribution $[d\sigma_F]_I$ (see Eq. (77)). As is well known, at the relative order α_S^n, the customary (i.e., with no helicity-flip contributions) *leading* logarithmic terms have the following structure:

$$H_g^F \, \alpha_S^n \left(\left[\frac{1}{q_T^2} \ln^{2n-1}\left(\frac{M^2}{q_T^2}\right)\right]_+ + \dots\right), \qquad n \geq 1 \,. \tag{90}$$

At this order, the gluon fusion helicity-flip contributions produce the following logarithmic behaviour:

$$\left[H^F(\phi(\mathbf{q_T}))\right]_{C_1 G_2} \, \alpha_S^n \left(\left[\frac{1}{q_T^2} \ln^{2n-2}\left(\frac{M^2}{q_T^2}\right)\right]_+ + \dots\right), \qquad n \geq 2 \,, \tag{91}$$

$$\left[H^F(\phi(\mathbf{q_T}))\right]_{GG} \, \alpha_S^n \left(\left[\frac{1}{q_T^2} \ln^{2n-4}\left(\frac{M^2}{q_T^2}\right)\right]_+ + \dots\right), \qquad n \geq 3 \,, \tag{92}$$

$$H_G^F \, \alpha_S^n \left(\left[\frac{1}{q_T^2} \ln^{2n-5}\left(\frac{M^2}{q_T^2}\right)\right]_+ + \dots\right), \qquad n \geq 3 \,. \tag{93}$$

The dots in the round brackets of Eqs. (90)–(93) stand for subdominant terms in each corresponding expression. The comparison between the expressions (90) and (91) shows that the single helicity-flip terms produce *next-to-leading* logarithmic contributions at each perturbative order. The double helicity-flip terms lead to subdominant logarithmic contributions.

The singular q_T behaviour that is observed order-by-order in the QCD perturbative expansion is cured by q_T resummation. We recall that the resummed gluon form factor $S_g(M, b)$ (and, analogously, the quark form factor $S_q(M, b)$ in the $q\bar{q}$ annihilation channel) provides the integration over b in Eqs. (62) and (77) with a strong damping factor in the large-b region (roughly speaking, in the region where $b \gtrsim \mathcal{O}(1/M)$). This damping effect (the simple resummation of the leading double-logarithmic terms, $\alpha_S^n \ln^{2n}(b^2 M^2)$, in b space is sufficient to highlight the effect [2]) eventually leads to resummed perturbative predictions for the $\mathbf{q_T}$ cross section that are physically well-behaved in the small-q_T region. In particular, the qualitative behaviour of the resummed $\mathbf{q_T}$ cross section (76) at very low values of q_T can be examined by performing the limit $q_T \to 0$ of Eqs. (62) and (77). In this limit, we can write:

$$[d\sigma_F]_\phi^{(g-\text{fus.})} \sim [d\sigma_F]^{(q\bar{q}-\text{ann.})} \sim \text{const.} \,, \tag{94}$$

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$$[d\sigma_F]_{C_1 G_2} \sim [d\sigma_F]_{G_1 C_2} \sim q_T^2 \ , \tag{95}$$

$$[d\sigma_F]_{GG} \sim q_T^4 \ . \tag{96}$$

The constant behaviour in Eq. (94) is just a result of Ref. [2]. It follows [2] from a simple reasoning (and a minor modelling of the Sudakov form factor at very large values of b, $b \sim \mathcal{O}(1/\Lambda_{QCD})$). In few words, the reasoning amounts to the observation that the presence of the resummed Sudakov form factor justifies the use of the low-q_T approximation $J_0(bq_T) \sim 1$ to extract the behaviour of the resummation formula (62) at $q_T \sim 0$. The behaviour in Eqs. (95) and (96) follows from the same reasoning. We simply note that, in the case of the helicity-flip components $[d\sigma_F]_I$ of the $\mathbf{q_T}$ cross section, the resummation formula (77) involves higher-order Bessel functions. Thus, we have just used the low-q_T approximation $J_2(bq_T) \sim b^2 q_T^2$ for the single helicity-flip components (see Eqs. (78) and (79)) and the corresponding approximation $J_4(bq_T) \sim b^4 q_T^4$ for the double helicity-flip component (see Eqs. (80)).

7 Summary

Considering the hard-scattering production of high-mass systems in hadron–hadron collisions, in this paper we have examined the corresponding transverse-momentum cross sections at small values of $\mathbf{q_T}$. We have presented a study of the contributions that are logarithmically enhanced order-by-order in QCD perturbation theory. The enhanced contributions have the form of singular q_T-distributions of the type $\left[\frac{1}{q_T^2} \ln^n(M^2/q_T^2)\right]_+$. The all-order analysis and the perturbative resummation of these terms, in processes (such as the DY process) that are controlled by the $q\bar{q}$ annihilation channel, is a classical QCD result. We have pointed out that this result does not suffice for the treatment of processes (such as SM Higgs boson production) that are controlled by (or, simply, receive contributions from) the gluon fusion channel. The difference between the $q\bar{q}$ annihilation and gluon fusion channels originates from correlations that are intrinsically related to the collinear 'evolution' of the colliding hadrons into gluon partonic states.

We briefly summarize the main features of our general results on q_T resummation in gluon fusion processes. The gluon fusion resummation formula for generic $\mathbf{q_T}$ cross sections is presented in Eqs. (19), (20) and (26). The resummation formula controls all the singular (and logarithmically-enhanced) perturbative contributions to the $\mathbf{q_T}$ cross section in the small-q_T region. Gluon collinear correlations produce new (with respect to $q\bar{q}$ annihilation) structures from the perturbative evolution of the parton densities of the colliding hadrons. The additional structure (see Eq. (26)) enters q_T resummation through the factorization formula (20). The terms due to collinear correlations lead, in general, to next-to-leading logarithmic contributions to the $\mathbf{q_T}$ cross section (the leading logarithmic contributions still come from soft-radiation effects included in the customary Sudakov form factor). Gluon collinear correlations are directly related to helicity-flip phenomena in the gluon fusion hard-scattering subprocess (see Eq. (50)). These spin correlations originate from a quantum-mechanical interference: the flip occurs between the gluon helicity states in the scattering amplitude and in the complex-conjugate amplitude. The helicity-flip phenomenon due to gluon collinear correlations leads to definite correlations between the azimuthal angles of the particles in the high-mass system that is produced by the gluon fusion mechanism. These azimuthal-correlation effects accompany the dominant (singular) q_T behaviour of the perturbative cross section in the small-q_T region (azimuthal correlations with a similar q_T

behaviour are absent if the high-mass system is produced by $q\bar{q}$ annihilation). The functional form of the azimuthal correlations is fully specified, at arbitrary perturbative orders, by the gluon fusion resummation formula (see Eq. (76) and Eqs. (81)–(86)). The double helicity-flip component of the $\mathbf{q_T}$ cross section is characterized by a coherent interference between the spin-flipping gluons from the two colliding hadrons (see Eq. (70)). As a consequence of this interference, the double helicity-flip contribution produces two distinct terms with a different q_T behavior: a term with azimuthal correlations (the last term on the right-hand side of Eq. (76)) and a term, with no azimuthal correlations, that also contributes to azimuthally-averaged cross sections (see Eqs. (62) and (63)). Gluon collinear correlations thus imply that the differences in the structure of q_T resummation between the gluon fusion and $q\bar{q}$ annihilation channels persist even after having performed the integration over the azimuthal angle of the transverse-momentum vector.

An interesting and relevant issue regards the extension of q_T resummation to processes whose final-state system F contains strongly-interacting particles (partons) such as, for instance, high-$p_\perp$ hadrons and, more generally, jets. The general extension to this type of final-state systems (which have not been considered in this paper) is still lacking. It requires a proper treatment of soft radiation [37] in multiparton hard scattering, namely, the hard scattering of the two colliding partons *and* the final-state QCD partons in the system F. The main features of collinear radiation from the two colliding partons are not affected by the presence of the additional hard partons in the final state. Therefore the structure of the gluon collinear correlations that we have found and documented in this paper is relevant to any extensions of q_T resummation. In particular, the convolution of the parton densities with the perturbative gluonic tensor in Eqs. (26) or (50) is expected to appear in the q_T resummation formulae for the production of final-state systems that contain colour-charged partons. Note that these systems can be produced by $q\bar{q}$ annihilation (or, generally, $q\bar{q}$ and qq scattering), gluon fusion (or, generally, gg scattering) and $gq(\bar{q})$ scattering subprocesses on equal footing. Therefore, there is no escape from (almost) ubiquitous collinear correlations due to initial-state gluon hard scattering.

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