Intellectual Property Rights in a Quality-Ladder Model with Persistent Leadership

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Abstract

This article analyzes the effects of intellectual property rights in a quality-ladder model in which incumbent firms preemptively innovate in order to keep their position of leadership. Unlike in models with leapfrogging, granting non-expiring forward protection reduces the rate of innovation and imposing a non-obviousness requirement reduces R&D spending. It is shown that full protection against imitation, granted independently of the size of the lead, maximizes the average innovation rate. (JEL L40, O31, O34)

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1 Introduction

This article studies the effects of different intellectual property right (IPR) policies in a quality-ladder model.[1] In contrast to most models of this type, where entrants undertake all R&D, so that there is leapfrogging, this article analyzes the case in which incumbent firms innovate preemptively in order to prevent being replaced by entrants, and in which there is persistence in leadership.

Empirical studies show that incumbent firms who remain the industry leaders for a sustained period of time carry out a considerable number of innovations. Malerba and Orsenigo (1999), Cefis and Orsenigo (2001) and Cefis (2003) find that persistent innovators account for a disproportionate share of all patent applications, and Peters (2009) documents large persistence in innovation behavior at the firm level.[2] Given that intellectual property rights (IPRs) are used to stimulate innovation, it is therefore important to understand how they should be designed in order to encourage cumulative innovation in a context where a large fraction of innovations are carried out by incumbent firms.

The following model setup is used in order to generate persistent leadership: There is free entry into R&D, and the R&D productivity of entrants and incumbents is the same. Contrary to most leapfrogging models, it is assumed that the R&D technology is characterized by decreasing returns at the industry level and that the incumbent moves first in the R&D game.[3] By increasing her own R&D effort, the incumbent can therefore decrease the profitability of R&D undertaken by entrants and preempt entry. It is assumed that an incumbent who has undertaken two successive innovations (is two steps ahead) can charge the unconstrained monopoly price and earns larger

[1] This setting seems appropriate as innovation continuously improves the quality or reduces the costs of existing goods in many industries, implying a process of creative destruction during which old innovations are displaced by new ones. For examples of cumulative innovation, see Scotchmer (2004), Chapter 5.

[2] Malerba and Orsenigo (1999) find that the percentage of patents granted between 1978 and 1991 to firms that had already innovated within their sector was 70% in Germany, 60% in France, 57% in the UK, 39% in Italy, 68% in the USA and 62% in Japan. Foster and Grim (2010) show that there was high persistence among the top 200 R&D performing firms in the US between 1976 and 2003. In a survey article, Bartelsman and Doms (2000) report that only around one-quarter of total factor productivity growth of US manufacturing establishments over a ten year period resulted from net entry and exit, while about half of it was due to within-plant growth. Foster, Haltiwanger and Krizan (2001) reach similar conclusions. Akcigit and Kerr (2010) find that large (incumbent?) firms undertake more incremental R&D directed at improving their existing product lines, while smaller firms focus more on drastic innovations that lead to the introduction of new products.

[3] Even without the latter assumption, a similar Walrasian equilibrium can be considered, as explained in footnote 24.

profits than an entrant who has to compete with the previous incumbent in the product market. Due to the Arrow replacement effect, the incumbent, however, has less stand-alone innovation incentives than an entrant. As entrants value entry (which brings a one step lead) less than the incumbent values not being replaced and keeping her (two step) lead, the incumbent preempts entry and carries out all R&D in equilibrium.[4]

The analysis builds on a simplified version of Denicolò (2001), who introduces the preemption mechanism of Gilbert and Newbery (1982) into a quality-ladder growth model, but does not study the role of intellectual property rights. In equilibrium, the amount of R&D that the incumbent undertakes in order to prevent being replaced depends positively on the value of an innovation for an entrant expecting to become the new leader upon entry. The prediction that incumbents invest more in R&D if entry pressure increases is supported by an empirical study of Czarnitzki, Etro and Kraft (2011), who use a dataset and a survey from the German manufacturing sector.

Within this setting of persistent leadership, the following IPR policies are analyzed: (i) forward protection (new innovations infringe the IPRs (patents) of previous innovators), (ii) a non-obviousness requirement (minimal inventive step), and (iii) protection against imitation (duration until rivals can copy a protected technology). The analysis generates three main results:

First, granting non-expiring forward protection reduces the innovation rate.[5] Under forward protection, entry is discouraged as incumbents can block entrants' innovations. Even if forward protection allows incumbents and entrants to collude in prices, entrants never obtain larger profits than without forward protection and without collusion. Therefore, granting forward protection of infinite duration unambiguously reduces entry pressure and the equilibrium rate of innovation. This result differs from that obtained by O'Donoghue and Zweimüller (2004), who study forward protection in the leapfrogging case. In their setting, patents for any two succeeding innovations are held by different firms so that equilibrium profits can be increased through collusion (which is not possible in case of persistent leadership where the incumbent already enjoys the maximal possible unconstrained monopoly profits). While forward protection also discourages innovation by leading to more backloaded profit flows in the leapfrogging context, its overall effect on growth can still be positive if the profit-increasing effect

[4]Entry can, however, occur if patents have expired, if there is collusion, or if innovations are drastic.

[5]In the case where consumers have unit demand, maximizing the innovation rate also maximizes welfare.

of collusion is larger than this backloading effect.

Extending on the first result, the case is analyzed where forward protection is only granted for a limited time span, while protection against imitation continues to be granted for an infinite duration. This means that courts become more lax over time in protecting IPR holders against follow-on innovations but always grant full protection against mere imitation. In this case, the rate of innovation falls when forward protection is granted, but increases once it has expired, as innovators anticipate being better protected against future entry. Given that forward protection allows the blockage of entry but does not facilitate collusion, the average innovation rate is maximal if innovators are granted forward protection of intermediate duration. This result has not been shown in the previous literature and also extends to the case of leapfrogging.[6] With persistent leadership, the average innovation rate can be increased if entering firms are granted forward protection of longer duration than the forward protection granted to incumbents.

The *second* main result is that imposing a non-obviousness (patentability) requirement reduces R&D spending. The reason for this is that such a requirement, given that it is binding for entrants, reduces entry pressure and therefore the amount of R&D the incumbent needs to undertake in order to preempt entry. If the requirement is only imposed on incumbents who have reached the maximal lead, it has no effect on their R&D spending, but can be useful in order to avoid inefficiently small inventive steps. These results differ from those obtained in the context of leapfrogging, where imposing a non-obviousness requirement can increase the innovation rate and welfare by avoiding an excessive rate of turnover, low markups, and inefficiently small inventive steps (see O'Donoghue (1998), O'Donoghue and Zweimüller (2004), and Hunt (2004)).

The *third* main result is that full protection against imitation for any size of the lead maximizes the average innovation rate. In order to allow for less than full protection, it is assumed that with a certain hazard rate, competitors can copy a given invention, enter the market, and compete away all profits. As in Acemoglu and Akcigit (2012), the general case of state-dependent IP protection is considered in which the rate at which protection against imitation expires can depend on the size of the lead of an IP holding firm. Unlike in Acemoglu and Akcigit (2012), however, the (average)

[6] If, however, protection against imitation expires at the same time as forward protection, granting finite protection cannot increase the average innovation rate compared to the case where there is no forward protection but infinite protection against imitation.

innovation rate is not maximal under state dependent IP protection, but rather in the case of full uniform protection, i.e. if protection never expires. If, however, IP protection is already sufficiently weaker for firms with a one step lead compared to those with a two step lead, further weakening it can, at a certain threshold value, increase the average innovation rate by inducing these firms to do more R&D than needed to preempt entry.

The analysis is extended to study various interesting additional issues:

If there is *trade secrecy* in the sense that entrants have to incur some fixed catch-up costs before they can use the state-of-the-art R&D technology, it is shown that incumbents can preempt entry without doing any R&D. In this case, the equilibrium innovation rate is zero in the case of full IP protection (against imitation). If, however, IP protection expires with a positive probability, so that incumbents lose their lead from time to time, this regularly creates a neck and neck situation with positive innovation incentives where firms try to become the next leader. Because of that, the average innovation rate is maximal for an intermediate probability of IP expiration. This result can also be obtained in the case where incumbents can make ex ante agreements with potential entrants in order to reduce R&D spending, or if they can employ researchers in areas unrelated to R&D.

In the case where *two R&D stages* must be completed in order to obtain an innovation, where the first of which consists of discovering an intermediate R&D input, the innovation rate is maximal if entrants are allowed to obtain IP protection on intermediate R&D inputs while incumbents are not.

The article is structured as follows: In *Section 2*, the related literature is reviewed, and in *Section 3*, the model is introduced and the equilibrium is derived. In *Section 4*, it is shown that permitting entrants and incumbents to collude in prices encourages innovation and that the equilibrium innovation rate is even higher if there is "extended lagging breadth", which grants the most recent innovator blocking power over the IPRs of previous innovators. *Section 5* studies the effects of protection against replacement. It is shown that the rate of innovation decreases if entrants have to pay fees upon entry in order to compensate the previous innovator (*Section 5.1*). Moreover, the effects of forward protection of infinite duration (*Section 5.2*) and finite duration (*Section 5.3*) are discussed. *Section 6* analyzes the effects of imposing a minimal inventive step and *Section 7* the case of state dependent protection against imitation. Several extensions are analyzed in *Section 8*. *Section 9* concludes. Proofs are collected in *Appendix 1.*

In the main part of the article, a partial equilibrium model is analyzed, and the simplifying assumption is made that increasing the lead beyond two steps does not lead to larger profit flows. In the supplementary *Appendix 2*, the more general case of an endogenous growth model (based on Barro and Sala-i-Martin (1995)) is analyzed in which also incumbents with a two step lead can increase profit flows by doing follow-on R&D. In this setting, the effects that IP policies have on the rate of growth are mainly the same as those that they have on the rate of innovation in the simplified model (see the summary in *Section 8.3*).

2 Related literature

The question of how antitrust policies should be designed in innovative industries where entrants expect to become the next incumbents is analyzed by Segal and Whinston (2007), who find that entrants should be well protected against incumbents in most cases in order to guarantee that profit flows for successful innovators do not become too backloaded. They therefore identify the same effect as O'Donoghue and Zweimüller (2004), who show that innovation increases if profit flows become more frontloaded. Chu (2009) studies a generalized version of the model of O'Donoghue and Zweimüller (2004) and quantitatively estimates the effect of blocking patents on R&D using US data. He finds that eliminating blocking patents would increase the steady-state R&D share of GDP by at least 10%. While Chu (2009) and the other articles do not analyze which policy can be used to obtain the maximal level of frontloading, the present article shows that it can be obtained through extended lagging breadth. O'Donoghue, Scotchmer and Thisse (1998) analyze the role of forward protection in a quality-ladder model with leapfrogging and compare the case of short and broad to long and narrow patents, assuming that forward protection allows firms to collude and that investment opportunities arrive at an exogenous rate.[7]

While most of the above-mentioned articles are concerned with the effects of forward protection, they (unlike this article) do not consider the case where forward protection expires (potentially earlier than protection against imitation) and where

[7]Bessen and Maskin (2009) and Llanes and Trento (forthcoming) analyze models of sequential innovation in which patents grant blocking power over follow-on innovations and in which licensing is inefficient due to asymmetric information. Assuming that firms can appropriate some surplus in the final goods markets even in the absence of IP protection, they show that innovation can be larger when there is no (forward) IP protection. In both articles, it is assumed that innovation does not lead to the replacement of the previous technology, so that issues like the Arrow replacement effect or the possibility of preemption that arise in a quality-ladder context are not considered.

the innovation rate fluctuates over time.

The main difference between the above-mentioned and the present article is that the former do not study the case in which incumbents undertake R&D.[8] While several articles have analyzed the conditions under which (some) persistence in leadership can arise in quality-ladder models[9], the role of IPR protection has only received little attention in settings in which incumbent firms also innovate.

Acemoglu and Akcigit (2012) analyze a model of step-by-step innovation in which there is a race between two firms in each sector and where the laggard first has to catch up through duplicative (but non-infringing) R&D before he can undertake frontier R&D (unless there is compulsory licensing of the leading edge technology). They argue that IP protection should be stronger for firms that have a larger technological lead over their rivals. In their model, IP protection affects innovation by affecting the incremental profits that the two incumbent firms obtain from moving one step ahead and it is assumed that parameters are such that the laggard never stops innovating and never drops out of the market. Reducing IP protection solely for firms with a smaller lead can therefore increase innovation incentives for laggards and the rate of innovation by increasing incremental profits. In contrast, there is free entry into (frontier) R&D in the model analyzed here, and reducing IP protection in a state-dependent way comes at the cost of reducing the value of entering and the amount of R&D incumbents need to undertake in order to preempt entry. This entry-discouraging effect is so strong that, contrary to Acemoglu and Akcigit (2012), who do not allow for entry, reducing IP protection in a state-dependent way always reduces the (average) innovation rate compared to the case of full uniform IP protection. These results are

[8]Segal and Whinston (2007) also study the case of innovation by leaders but do not analyze patent policies in this context.

[9]In most quality-ladder growth models (like Aghion and Howitt (1992)), the case of leapfrogging is analyzed, although incumbents are actually indifferent about their share in total R&D if the R&D sector is competitive and markets are Walrasian, so that there might as well be some persistence in leadership (see Cozzi (2007)). In many continuous-time patent race models (like Reinganum (1983 and 1985)) where marginal R&D costs are increasing at the firm-, but not at the industry level, preemption is not possible and incumbents invest less in R&D than challengers in the standard case with simultaneous moves and drastic innovations. In a similar setup with fixed costs of entering the R&D sector, Etro (2004) finds that in the case where there is free entry and industry leaders move first in the R&D game (are Stackelberg leaders), they do more R&D than entrants so that there is some persistence in leadership. Denicolò (2001), on which the current article builds, analyzes the case where preemption is possible due to decreasing R&D productivity at the industry level and finds that there is persistent leadership if innovations are non-drastic and incumbents move first. Fudenberg, Gilbert, Stiglitz and Tirole (1983) analyze the conditions under which preemption is possible and when there can be competition in patent races.

derived analytically, while Acemoglu and Akcigit (2012) obtain most of their results through numerical simulations.

Hopenhayn and Mitchell (2011) analyze a model with two innovators in which the duration of (forward) patent protection (preferential treatment) granted to a firm can depend on the whole innovation history. They find that the optimal level of protection is an increasing function of the past innovation success of a firm. Such backloading of rewards is optimal as it allows to encourage multiple innovations (early and late ones) at once. The authors show that the optimal policy can lead to a situation in which one firm is granted infinite protection over the whole quality ladder. Contrary to this article, the authors assume that inventment opportunities arrive at an exogenous rate, that preemption is not possible and that there is no Arrow replacement effect, so that (for a given strength of patent protection) innovation incentives do not depend on the size of a firm's lead.

Denicolò and Zanchettin (forthcoming) study a growth model with non-drastic innovations and constant returns to R&D in which incumbents and entrants simultaneously decide about the sizes of their R&D investments and in which there is no preemption. Unlike the present article, they assume that the R&D productivity of incumbents is larger than that of entrants so that incumbents might find it profitable to innovate, in spite of the Arrow replacement effect. They find that there can be stochastic leadership cycles, meaning that incumbents are not replaced immediately, but undertake some R&D and (on average) advance to a certain lead before they are replaced. The authors show that requiring successful outsiders to pay a licensing fee to the previous incumbent in this setting can increase the rate of growth, even if such forward protection does not facilitate collusion. Introducing state-dependent patent breadth is shown to affect the share of R&D the incumbent undertakes, but it has no effect on the rate of growth.[10]

Horowitz and Lai (1996) and Cadot and Lippman (1995) also find an inverted-U relation between patent length (capturing the strength of protection against imitation) and the (long run) rate of innovation in the case where there is a monopoly innovator. However, they simply assume that only one firm (the incumbent) is capable of doing R&D, while in this article a similar result is derived in a setting where entrants can

[10]Other articles in which (some) persistence in leadership results because incumbents are assumed to be more productive in doing R&D than entrants are Segerstrom and Zolnierek (1999) and Segerstrom (2007). Acemoglu and Cao (2010) assume that only incumbents are capable of undertaking incremental innovations, while only entrants can attain radical innovations.

also do R&D, but where incumbents can preempt entry without innovating themselves in equilibrium.

3 The model setup

There is a good, the quality of which can be increased (or the production costs of which can be reduced) step-by-step through innovation. There exist generations $i \in \{1, ..., k\}$ of the good. Time is continuous and (gross[11]) profit flows of a firm producing the newest generation k of the good depend on whether the firm has IP protection on it and on how far it is ahead of its rivals. If generation k is not protected by IPRs, everyone is allowed and able to produce it, and Bertrand competition pushes prices down to marginal cost. Profit flows are therefore zero in this neck-and-neck case where no firm has a lead over its rivals:[12] $\pi_0(k) = 0$. If a firm has IP protection on generation k, and another firm has IP protection on generation $k-1$ or if generation $k-1$ is in the public domain, there is limit pricing, meaning that firm k charges a price that is low enough to push firms $k-1$ out of the market. In this case of a one step lead, profit flows are given by $\pi_1(k)$. If the leading firm has IP protection on generations k and $k-1$ of the good, it is assumed to have enough market power to be able to charge the unconstrained monopoly price, meaning that a rival firm that can produce generation $k-2$ of the good does not find it profitable to enter and to undercut the monopoly price the leader charges. Therefore, leaders profit flows are the same if they lead by two or more steps (the latter being the case if the leader has IP protection on more than the two most recent versions of the good) and are given by $\pi_2(k) > \pi_1(k)$.

In order to simplify the analysis, it is assumed that profit flows only depend on the size of the lead, and not on k, meaning that they are independent of the absolute quality of the good:[13] $\pi_1(k) = \pi_1$, $\pi_2(k) = \pi_2$. The more general case of profit flows that increase in k is analyzed in *Appendix 2*. It is assumed that $\pi_1 \geq \frac{\pi_2}{2}$ (**Assumption**

[11] R&D expenditures are not yet subtracted from these profits.

[12] The subscript denotes the size of the lead.

[13] Such profit flows can be obtained as equilibrium outcomes in the following setup: consumers have unit demand, and the utility they derive from consuming the good (and which defines their maximal willingness to pay) is equal to its quality. Each innovation increases the quality of the good by the amount π_1, and marginal production costs are assumed to be zero so that profits of firms with a one step lead are equal to π_1. Due to limited IP (patent) breadth, a competitive fringe is allowed to supply copies of the newest good that are of inferior quality and/or are associated with higher marginal production costs. IP breadth is now assumed to be such that monopolists with a lead of two or more steps never charge a price in excess of π_2 in order to prevent entry of the competitive fringe (π_2 could also be interpreted as a cap on prices).

1), so that incremental profits that a firm obtains when it introduces a new generation $k+1$ of the good (weakly) decrease in the size of the lead that the firm has for generation k of the good. For entrants (who make zero profits before innovating), incremental profits are given by π_1, and are (weakly) larger than those of incumbents with a one step lead, which are given by $\pi_2 - \pi_1 \leq \pi_1$, and which are again larger than incremental profits of an incumbent who already has a two step lead, as those are equal to zero.[14]

Time is continuous, and the rate of interest is exogenous and given by r. All firms are risk neutral and not financially constrained.

An inventor of a new generation of the good gets IP protection on it, allowing her to exclude others from producing her generation of the good. In the first part of the analysis, IP protection is assumed to be of infinite duration (e.g. patents are infinitely lived).

3.1 R&D

Given k is the newest generation of the good, the next generation $k+1$ can be invented if R&D is undertaken.[15] In order to obtain the innovation with the instantaneous arrival rate ϕ, the total (industry) costs

$$C(\phi) = \begin{cases} c\phi^{1+\epsilon} \text{ if } \phi \leq \phi_m \\ \infty \text{ if } \phi > \phi_m \end{cases} \tag{1}$$

have to be incurred. All firms have access to this technology and if $\epsilon > 0$, marginal and average R&D costs increase in ϕ. If more than one firm does R&D, it is assumed that firm i obtains the innovation with arrival rate $\beta_i\phi$ if its share in total R&D costs is given by β_i. By increasing its own R&D effort, a firm therefore increases the R&D costs of all other firms (ceteris paribus). A reason for this might be that the supply curve for R&D inputs (like researchers) is upward sloping so that their price increases if demand for them increases. But even if R&D inputs are supplied elastically, it might be impossible to perfectly coordinate all R&D activities, meaning that the probability

[14]The feature that incremental profits decrease in the size of the lead arises in many contexts and more general settings and is the source of the "Arrow replacement effect" (Arrow (1962)) and the "escape competition effect" (Aghion et al. (2001)), which are widely discussed in the literature.

[15]It is not possible to skip a step in the ladder, as inventing the newest generation $k+1$ of the good requires access to the technology of generation k (or to the technologies of all previous generations $i \leq k$). An invention might, however, simply consist of adding a new component to an otherwise unchanged product (e.g. a new generation of software might simply consist of the previous generation with some added features).

of duplicative research increases in the total number of R&D inputs that are used.[16] While it is assumed here that a firm can only increase rivals R&D costs by undertaking R&D itself, the case where this is possible by simply buying/hiring R&D inputs and then using them for other purposes than R&D is analyzed in *Section 8.1.2*.[17] In order to simplify the analysis, it is assumed that the arrival rate cannot surpass the level ϕ_m for technological reasons.

3.2 Equilibrium

In the following, the stationary Markov perfect equilibrium is derived in which firms' strategies only depend on payoff-relevant variables.[18] In order to avoid a situation in which no firm ever finds it profitable to undertake R&D, it is assumed that $c < \frac{\pi_1}{r}$ (**Assumption 2**). Given an innovation has the value V_E for a newly entering firm[19], entry occurs until the average cost of innovating is equal to this value, i.e. until the following free entry condition is satisfied[20]:

$$V_E \leq c(\phi^*)^\epsilon \tag{2}$$

This condition pins down a lower bound for the equilibrium rate of innovation[21] ϕ^* as an increasing function of V_E. But the fact that this condition depends on the value of an innovation for an entrant does not imply that R&D is actually carried out

[16]In the case where N firms come up with the same innovation at the same instant of time, each would then obtain IP protection with probability $\frac{1}{N}$. In this case, the R&D effort exerted by one firm does not directly affect the price of inputs for other firms but increases their expected R&D costs per obtained IPR (patent) by lowering the probability of actually obtaining IP protection for the innovation.

[17]A case where a firm actually has to do R&D in order to increase rivals R&D costs is that where R&D costs increase due to duplication but not due to an upward sloping supply curve for R&D inputs. But also if R&D workers enjoy doing R&D and have to be paid more if they are forced to do other things, the cheapest way for a firm to increase a rivals R&D costs might be to do R&D itself.

[18]These variables include the size of the lead, profit flows, R&D costs and the rate of interest but not the generation k of the good, the identity of the market leader, or time. Nonstationary endogenous cycle equilibria that can occur in quality-ladder models are therefore not considered.

[19]This value is independent of whether an entrant replaces an incumbent with a one- or a two step lead, as (gross) profits in both cases are given by π_1.

[20]An entering firm that contributes to an overall increase in R&D effort does not take the fact that it increases the R&D costs of all other firms into account, and it therefore still finds entry profitable if the average costs are lower than V_E, even if the marginal costs are higher. As total R&D costs are given by $C(\phi) = c\phi^{1+\epsilon}$, average costs are given by $a(\phi) = c\phi^\epsilon$.

The qualitative results would, however, be the same if entry only occurred up to the point where V_E is equal to the marginal R&D costs $m(\phi) = (1+\epsilon)c\phi^\epsilon$.

[21]Equilibrium values are starred.

by entrants in equilibrium. Take the case of an incumbent firm with a two step lead. Without any threat of entry, this firm does not do any R&D, as it cannot increase profits above the current level π_2. If there is free entry, however, the incumbent knows that if she does not do any R&D, the instantaneous probability of replacement is given by the free entry condition as $\phi^* = \left(\frac{V_E}{c}\right)^{\frac{1}{\epsilon}}$. Assuming that incumbents move first (are Stackelberg leaders) and that entrants can adjust their R&D spending after observing the level of R&D the incumbent undertakes, this probability is independent of whether the incumbent does part of the R&D herself. This is because the average R&D costs that determine the profitability of entry only depend on total R&D spending, so that one unit of R&D undertaken by the incumbent crowds out one unit undertaken by entrants.[22] The incumbent can therefore reduce the instantaneous probability of replacement from ϕ^* to $(1-\beta)\phi^*$ if she bears the costs $\beta c(\phi^*)^{1+\epsilon}$. As successfully innovating entrants have to compete with the previous incumbents and only get (gross) profits π_1 upon entry (and maybe later on π_2 if they do a follow-on innovation), the value of an innovation V_E for an entrant is lower than an incumbents willingness to pay for keeping her two step lead. Because of this "efficiency effect" (which was first analyzed by Gilbert and Newbery (1982)), the incumbent finds it profitable to increase her R&D effort as long as $\beta < 1$, as the marginal value of reducing the probability of losing the two step lead exceeds the marginal costs, which in this case are equal to the average costs $c(\phi^*)^{\epsilon}$ and also equal to the value of an innovation for an entrant (V_E) due to the free entry condition.[23] Therefore, an incumbent with a two step lead finds it profitable to preempts entry by doing exactly as much R&D as needed to push average R&D costs up to (or slightly above) the value of an innovation for an entrant. In the case where the incumbent only has a one step lead she also does all the R&D in equilibrium, as she values obtaining the next innovation (which guarantees a two step lead) more than an entering firm does (which can only get a one step lead). We can therefore state:[24]

[22] Given the assumed R&D cost function, the free entry condition is the same, independent of the number of potential entrants. The analysis would therefore be the same even if there was only one single potential entrant.

[23] The value of incumbency is given by $V_I = \frac{\pi_2 - \beta c(\phi^*)^{1+\epsilon}}{r+(1-\beta)\phi^*}$. Inserting $\phi^* = \left(\frac{V_E}{c}\right)^{\frac{1}{\epsilon}}$ and deriving with respect to β, we obtain that $sign\frac{\partial V_I}{\partial \beta} > 0$ if $V_E\left(\left(\frac{V_E}{c}\right)^{\epsilon} + r\right) < \pi_2$. This condition is satisfied given that $V_E < V_I = \frac{\pi_2 - \beta c\left(\frac{V_E}{c}\right)^{\frac{1+\epsilon}{\epsilon}}}{r+(1-\beta)\left(\frac{V_E}{c}\right)^{\frac{1}{\epsilon}}}$.

[24] The equilibrium analyzed here can either be seen as one where the incumbent is a Stackelberg leader in the R&D game or as a Walrasian equilibrium where the total demand for R&D inputs is

Lemma 1 *Given the incumbent moves first in the R&D game, she undertakes all R&D in equilibrium so that there is persistent leadership*

Once the incumbent has reached a two step lead, the rate of innovation depends (through the free entry condition) positively on V_E, the value of an innovation for an entrant. To solve for the equilibrium, we therefore need to determine V_E given that an entering firm expects to expand its lead to two steps in the future and then to remain the leader, doing only as much R&D as needed to preempt entry. Denoting the value of being two (one) steps ahead by V_2 (V_1) and the R&D effort undertaken by an incumbent with a two step (one step) lead by ϕ_2 (ϕ_1), the following arbitrage conditions must be satisfied:

$rV_2 = \pi_2 - c\phi_2^{1+\epsilon}$ and

$rV_1 = \pi_1 - c\phi_1^{1+\epsilon} - \phi_1 V_1 + \phi_1 V_2$.

The right hand side in the first equation indicates the per period profits derived from selling the good minus the costs of conducting the level of R&D ϕ_2 that is needed to preempt entry. There are two additional terms in the second equation because the incumbent gains a two step lead and looses her one step lead in the case of an innovation (which occurs with arrival rate ϕ_1). As the value of an innovation for an entrant is exactly the value of obtaining a one step lead, we can solve the two equations for V_E:

$$V_E = V_1 = \frac{\pi_1 - c\phi_1^{1+\epsilon}}{r + \phi_1} + \frac{\phi_1}{r + \phi_1} \frac{\pi_2 - c\phi_2^{1+\epsilon}}{r} \tag{3}$$

The free entry condition therefore becomes:

$$\frac{\pi_1 - c(\phi_1^*)^{1+\epsilon}}{r + \phi_1^*} + \frac{\phi_1^*}{r + \phi_1^*} \frac{\pi_2 - c(\phi_2^*)^{1+\epsilon}}{r} = c(\phi_2^*)^{\epsilon} \leq c(\phi_1^*)^{\epsilon} \tag{4}$$

While this condition is always binding when the incumbent has a two step lead, it might be the case that an incumbent with a one step lead wants to get a two step lead more quickly, so that she does more R&D than required to increase average R&D costs to a level that preempts entry (meaning that $\phi_1^* > \phi_2^*$).

equal to the supply and in which the auctioneer allocates all R&D to the incumbent who is willing to pay at least as much for it as the entrants. As the entrants get zero profits in equilibrium, they are indifferent about the amount of R&D that they undertake and, if an incumbent would do less R&D than the equilibrium level, the Walrasian auctioneer would simply assign a larger amount of R&D to entrants in order to obtain the equilibrium (see Cozzi (2008) for a more detailed discussion).

A similar equilibrium might also be obtained as the outcome of an auction in which incumbents and entrants simultaneously bid for R&D inputs.

In order to simplify the analysis, the case where $\epsilon \to 0$ is considered in the following. In the case where $\epsilon = 0$, the equilibrium amount of R&D the incumbent or entrants undertake is undetermined if the free entry condition is satisfied with equality ($V_E = c$).[25] However, if ϵ is slightly positive, the preemption equilibrium results and the R&D rate is well determined for each point in time and equal to that that entrants expect to choose themselves in the case where they enter and become the next leaders. In the following, it is therefore assumed that in the case where $\epsilon = 0$, the equilibrium is selected that results as the limit if $\epsilon \to 0$. For $\epsilon = 0$, marginal and average R&D costs are equal to c, and the free entry condition is given by:

$$V_E = \frac{\pi_1 - c\phi_1^*}{r + \phi_1^*} + \frac{\phi_1^*}{r + \phi_1^*}\frac{\pi_2 - c\phi_2^*}{r} = c \tag{5}$$

We can now analyze how the equilibrium rate of innovation ϕ_2^* depends on the different parameters of the model.

Proposition 1 *The equilibrium innovation rate ϕ_2^* increases in π_1 and π_2 and decreases in r and c. An incumbent with a one step lead sets $\phi_1^* = \phi_2^*$*

Proof. See Appendix A ■

The intuition for these results is straightforward: once the incumbent has reached a two step lead, she does just as much R&D as needed to preempt entry. And this preemptive R&D level depends positively on entry pressure, i.e. on the incentives of entrants to undertake R&D in order to enter and to become the next incumbent. Entry pressure decreases in the R&D costs c and increases in the value of an innovation for an entrant which depends positively on profit flows π_1 and π_2 and negatively on the rate of interest r due to a discounting effect.

In the following sections, the effects that different IP policies have on the rate of innovation are analyzed.[26]

[25] While the value of an innovation for an entrant (V_E) depends on the amount of R&D ϕ_2^* that entrants expect to do in the future to preempt entry, it is independent of the level of current R&D.

[26] The analysis here focuses on the effects on the innovation rate without analyzing welfare implications. In the model setup described in footnote 13, however, it is easy to show that maximizing the innovation rate also maximizes welfare under Assumption 2 in the case where $\epsilon \to 0$ and where the rate of interest is equal to the rate of time preference.

4 Collusion and "extended lagging breadth"

If entrants are allowed to collude with previous incumbents in the product market or if one of the firms is allowed to sells its IPR (or an exclusive license) to the other, this permits them to avoid the phase of competition and to increase profits from π_1 to π_2. Given incumbents cannot commit to compete and are willing to collude once entry occurs, this increases the value of an innovation for entrants and therefore entry pressure and the rate of innovation and the more so, the larger the bargaining power of entrants is.[27]

In the case of collusion, the efficiency effect disappears, as entry no longer reduces joint profits. The incumbent now values not being replaced equally much as the entrants value entering, as she still gets a share of the surplus in case of entry. The incumbent is thus indifferent about her share in total R&D and there need not be persistent leadership anymore. The equilibrium rate of innovation that is determined by the free entry condition is, however, independent of whether the incumbent or the entrants carry out R&D.

From *Proposition 1* we know that the equilibrium innovation rate ϕ_2^* is maximal if entrants obtain the maximal market power (which has the value V_2) immediately upon entry, i.e. if $\pi_1 = \pi_2$. Collusion can, however, only lead to this outcome if entrants have all the bargaining power.[28] If entrants are weak bargainers, another possible way to increase innovation is to take all bargaining power away from incumbents by preventing them from competing in the product market once entry of a firm with an improved version of the good has occurred.[29] This can, for example, be achieved by

[27]Taking future entry pressure and the preemptive R&D level ϕ_2^* as given, the value $V_2 = \frac{\pi_2 - c\phi_2^*}{r}$ can be shared among the parties. Once entry has occurred and R&D costs are sunk, the entrant has to get at least the value $V_E = \frac{\pi_1 - c\phi_1}{r+\phi_1} + \frac{\phi_1}{r+\phi_1}\frac{\pi_2 - c\phi_2^*}{r}$ which he gets if there is no collusion (the outside option of the incumbent is zero). Suppose the entrant gets the fraction $\vartheta V_2 \geq V_E$ of the "profit pie" with ϑ denoting his bargaining power. Then, the free entry condition is given by $\vartheta \frac{\pi_2 - c\phi_2^*}{r} = c$, so that the equilibrium rate of innovation ϕ_2^* increases in ϑ and is larger than in the case without collusion if $\vartheta V_2 > V_E$.

The result that allowing collusion does not increase the innovation rate if entrants do not have any bargaining power does not hold in the leapfrogging case where equilibrium profits are always given by π_1 if there is no collusion. In this case, the value of an innovation and the innovation rate always increase under collusion as joint profits are increased and as entrants anticipate obtaining a larger share of the pie once the next entrant replaces them (see Segal and Whinston (2007)).

[28]That means if $\vartheta = 1$, using the notation of footnote 27.

[29]In the case where there are fixed (operating) costs associated with production, a replaced monopolist might exit the market completely and not constrain the price setting power of the entrant at all. In this case, entrants would directly get profit flows π_2.

making the IPR (patent) of the incumbent infringe on that of the entrant. Granting such "extended lagging breadth" therefore allows entrants to obtain the maximal[30] market power and profit flows π_2 immediately upon entry.[31] Then, there can again be either persistent leadership or leapfrogging (or a mixture of both), as incumbents and entrants have the same incentives to innovate in this case. The equilibrium rate of innovation can now be obtained from the free entry condition $V_2 = c$, implying that $\phi_2^* = \frac{\pi_2}{c} - r$.

5 Protection against replacement

5.1 Entry fees

As each innovation improves upon the last one, innovators use the knowledge accumulated by previous innovators as an R&D input. Let us now assume that there is a policy requiring a successful innovator to pay a fixed fee F upon entry into the product market in order to compensate previous innovators. Even in the case where the entire fee has to be given to the previous incumbent, there is still persistent leadership and preemption. The reason for this is that the incumbent firm values not being replaced more than the entrant values entering, so that there is no value of F for which, at the same time, the entrant is willing to undertake R&D in order to enter, and the incumbent is willing to permit entry by not doing any preemptive R&D. If entry nevertheless occurs (out of equilibrium), entrants therefore expect to become the next leaders and to do all the follow-on R&D themselves and to never receive any licensing fees from others. The value of an innovation for an entrant is therefore given by:

[30] In the case where a lead of more than two steps is required in order to become the unconstrained monopolist, the entrant's market power can be increased (up to the maximum) if not only the IPR of the second-newest generation of the good infringes on the IPR of the entrant, but also on the IPRs of the third, fourth... newest goods.

[31] In the case where instead of using IPRs firms can also protect their innovations through trade secrecy, the problem arises that if one firm opts for trade secrecy, the following will do so as well for this reason: as an entrant who obtains IPR protection in this case cannot block the production of the previous generation of the good, profits upon entry are given by π_1 in both cases, while the risk of being replaced in the future is lower in the case of trade secrecy where the following entrant does not obtain the full market power immediately. However, starting from a situation in which all firms use IPRs and in which there is extended lagging breadth, entrants do not prefer using trade secrecy as, under free entry, the value of an innovation is in each case equal to c (under extended lagging breadth, profits upon entry are higher but the expected rate of replacement (entry pressure) is also higher so that V_E is the same as in the case of trade secrecy, where profits are initially lower but where future entry pressure is lower as well).

$$V_E = \frac{\pi_1 - c\phi_1}{r + \phi_1} + \frac{\phi_1}{r + \phi_1}\frac{\pi_2 - c\phi_2}{r} - F \tag{6}$$

The free entry condition is now $\frac{\pi_1 - c\phi_1^*}{r+\phi_1^*} + \frac{\phi_1^*}{r+\phi_1^*}\frac{\pi_2 - c\phi_2^*}{r} = c + F$ (where either $\phi_1^* = \phi_2^*$ or $\phi_1^* = \phi_m$) and an increase in F unambiguously reduces the equilibrium innovation rate ϕ_2^*. The intuition for this is simply that any kind of licensing fee or other fixed fee (like IP litigation costs) that has to be paid upon entry in the product market decreases the value of an innovation for an entrant and therefore entry pressure and the incumbents R&D effort required to preempt entry.[32]

If it is possible to subsidize entry (impose a negative F), this increases the rate of R&D ϕ_2^* although the subsidy never has to be paid in equilibrium (given it is not too large).[33]

5.2 Forward protection

It is now assumed that an entrants IPR (generation $k + 1$) infringes on the IPR of the previous generation (k) of the good (but not on generation $k - 1$), so that the incumbent has blocking power over the innovation of the next entrant. O'Donoghue and Zweimüller (2004) call such an arrangement "forward protection" and assume that entrants and incumbents are only allowed to collude in prices if there is forward protection. As in the previous sections, allowing collusion was innovation-enhancing, while increasing the incumbent's blocking or bargaining power decreased the rate of innovation, it is not a priori clear whether forward protection, which includes both elements, increases or decreases the rate of innovation (when compared to a case without forward protection and without collusion). Again, it is assumed that incumbents

[32]While the effect of an increase in F on ϕ_2^* is unambiguously negative, increasing F can, however, induce an incumbent with a one step-lead to race faster. In the working paper version of this article (in *Section 12*), the more general case in which IP protection regularly expires is considered and it is shown that $\phi_1^* = \phi_m$ holds in equilibrium if F is sufficiently large. While increasing F still decreases the average innovation rate in most cases, there is a parameter range in which a slight increase in F actually increases the average innovation rate if it induces firms with a one step lead to set $\phi_1^* = \phi_m$ instead of $\phi_1^* = \phi_2^* < \phi_m$.

Even if only entrants are capable of doing R&D so that there is leapfrogging, imposing licensing fees reduces innovation incentives and the rate of innovation by leading to more backloaded profit flows. A similar reasoning applies in a product variety setting, where (for a given level of appropriability in the final goods markets) incentives for continuous innovation are larger if IPRs do not grant blocking power over future innovations and if there is no need for licensing (see also Llanes and Trento (forthcoming)).

[33]Such a policy would be especially effective in the case where the R&D productivity of entrants is lower (c is larger) than that of incumbents.

cannot commit not to collude ex post.

Proposition 2 *Forward protection reduces the equilibrium innovation rate (ϕ_2^*). An equilibrium under forward protection exists if $\phi_m > \frac{\pi_2}{c} - \frac{r^2}{\phi_m} - 2r > 0$. There is persistent leadership and an incumbent with a one step lead does the maximal amount of R&D in equilibrium ($\phi_1^* = \phi_m$).*

Proof. See Appendix B ■

This is the first main result of the article. As incumbents can prevent entrants from producing the next generation of the good, entry does not reduce their profits and there is no need to collude with entrants and to give them a share of the pie.[34] Forward protection therefore deters entry by decreasing π_1 to zero and unambiguously reduces the equilibrium innovation rate.[35] This result differs from the ones obtained in models where there is leapfrogging, like O'Donoghue and Zweimüller (2004) and O'Donoghue, Scotchmer and Thisse (1998). In these models, firms never lead by more than one step so that (gross) profit flows are given by π_1 in the standard case. Introducing forward protection then allows firms to collude and to increase joint profit flows to π_2. If the incumbents' bargaining power is not too large, so that entrants get a large enough share of the increased profit pie immediately upon entry (i.e. if profit flows are not too backloaded), forward protection can therefore increase the value of an innovation and the equilibrium innovation rate.[36]

[34]This is different in the more general model which is discussed in *Appendix 2, Section 12*, where the entrant's innovation allows an increase in joint profits above the level that the incumbent can obtain alone. But while incumbents want to collude with entrants in order to increase profits in this case, entrants never obtain a share of profits that is larger than in the case without forward protection, so that forward protection again decreases growth.

[35]This result, however, changes if forward protection allows incumbents and entrants to coordinate their joint R&D spending. In this case, they can increase joint profits by reducing the amount of R&D the entrant undertakes from $\phi_1^* = \phi_m$ to $\phi_1 = \phi_2^*$. Forward protection then reduces the rate of innovation if the bargaining power of entrants is weak and increases it if entrants are strong bargainers and if ϕ_m is large (see the working paper version of this article, *Section 13*).

[36]It should be noted that simply permitting collusion in both the case of persistent leadership and of leapfrogging without granting forward protection unambiguously increases the value of an innovation for an entrant and the equilibrium innovation rate. The reason for this is that by granting blocking power to incumbents, forward protection greatly discourages entry by increasing the incumbents' bargaining power. Again, introducing extended lagging breadth, which makes private negotiations unnecessary and gives all the market power to entrants immediately upon entry, leads to even larger innovation incentives in both cases.

5.3 Forward protection of finite duration

It was assumed in the previous section that forward protection is granted for an infinite duration. If this is not the case and if courts can become more lax over time in protecting IPR holders against follow-on innovators, the results can change substantially. It is now assumed that forward protection allows incumbents who have IP protection on generation k of the good to block entry of any improved version $k+i$ (with $i \geq 1$, so that not only entry of generation $k+1$ can be blocked as in the previous section[37]), but that this blocking power expires with hazard rate η. Once the blocking power expires, entrants with an improved version no longer infringe on the the incumbents IPR, while the IPR still prohibits copying the incumbents good (generation k). In order to introduce the mechanism in the clearest way, it is initially assumed that extended lagging breadth is granted in the case where the incumbents blocking power has expired, so that a successful innovator directly gets the maximal profit flows π_2 upon entry into the product market. Contrary to the previous sections, forward protection therefore only serves as a device for blocking entry and does not facilitate collusion. If forward protection on generation k of the good has expired and the next generation $k+1$ is invented, the new inventor is again granted forward protection, with the instantaneous probability of expiration again given by η. There are therefore two states in which the economy can be: state F, where the holder of the IPR on the newest generation of the good can block any entry due to forward protection, and state N, where forward protection has expired and in which there is free entry into R&D. The value of the IPR covering the newest generation of the good is denoted by V_F in state F and by V_N in state N. In state N, the free entry condition[38] $V_F = c$ pins down the innovation rate ϕ_N, while no R&D takes place in state F, so that $\phi_F = 0$.[39] As extended lagging breadth makes incumbents indifferent with respect to their share in total R&D in state N, V_F is the same as in the case where there is leapfrogging, and can therefore be derived from the arbitrage conditions $rV_N = \pi_2 - \phi_N V_N$ and $rV_F = \pi_2 - \eta V_F + \eta V_N$, so that $V_F = \frac{\pi_2}{r+\eta} + \frac{\eta}{r+\eta}\frac{\pi_2}{r+\phi_N}$. The free entry condition $V_F = c$ can then be solved for the innovation rate in state N:

[37] Instead, it could be assumed that prohibitively high entry fees are imposed on all follow-on innovators.

[38] While R&D only takes place in state N, the value of an innovation for an entrant is given by V_F, as there is a switch to state F if an innovation occurs.

[39] Given that $\epsilon \to 0$, no entrant has incentives to do R&D in the phase of forward protection, as profits can only be made once forward protection has expired, so that the value of an innovation is given by $\widetilde{V}_F = \frac{\eta}{r+\eta} V_F < V_F = c$.

$$\phi_N^* = \frac{\eta\pi_2}{c\,(r+\eta)-\pi_2} - r \tag{7}$$

ϕ_N^* is positive if $\eta > \frac{\pi_2}{c} - r > 0$ and decreases in η as the value of an innovation V_F declines if forward protection expires faster due to an increase in η. While reducing η increases ϕ_N^*, however, it also increases the likelihood that the economy is in state F in which there is no innovation, so that it is interesting to analyze the effect of η on the average rate of innovation $\widehat{\phi}$.

Proposition 3 *Assuming that there is extended lagging breadth when forward protection has expired (in state N) and that $\eta > \frac{\pi_2}{c} - r > 0$, the average innovation rate is given by $\widehat{\phi} = \frac{\eta\phi_N^*}{\eta+\phi_N^*}$ (with ϕ_N^* taken from equation (7)) and is maximal for the interior expiration rate $\eta^* = \frac{\pi_2}{c} - r + \frac{\sqrt{\pi_2(\pi_2 - rc)}}{c}$.*
Proof. *See Appendix C* ■

The average innovation rate $\widehat{\phi}$ can therefore be increased beyond the level $\phi_2 = \frac{\pi_2}{c} - r$ that can be obtained with extended lagging breadth without forward protection (that means if $\eta \to \infty$) if successful innovators are granted blocking power over future innovations for a limited time span. The reason for this is that granting such forward protection increases the effective length of IP protection by reducing the probability of replacement by a follow-on innovator, and therefore encourages innovation in state N. If forward protection is granted for an infinite duration ($\eta = 0$), the average innovation rate is, however, zero as follow-on R&D can then be blocked forever (so that the economy is never in state N). Replacing π_2 with π_1, this result clearly also applies to a standard leapfrogging setting without extended lagging breadth.[40]

In the following, the assumption of extended lagging breadth in state N is relaxed. In the cases where in state N entry leads to a phase of competition with reduced profits π_1, or in which profit flows for successful innovators are more backloaded because entrants have to compensate previous incumbents to some extent, the value of an

[40] An important assumption in this section is that expiration of forward protection does not imply a complete expiration of IP protection. If, however, expiration of forward protection also implies a complete expiration of IP protection, meaning that competitors are allowed to copy the good so that profits fall to zero, the results change. In *Appendix D* it is shown that introducing such IP protection of limited duration cannot increase the average innovation rate beyond the level that is obtained with extended lagging breadth and infinitely lived IP protection if there is no forward protection. The reason for this is that making IP protection expire pushes profits in state N to zero, and this discourages innovation more than the introduction of forward protection of limited duration increases it.

innovation V_F for an entrant is reduced for any value of η, so that ϕ_N^*, the rate of innovation in state N, declines. This unambiguously reduces the average rate of innovation $\widehat{\phi}$, implying that , extended lagging breadth for any given expiration rate η maximizes $\widehat{\phi}$. In the case where entrants first get profit flows π_1, and only in the case of two successive innovations profit flows π_2, there is again persistent leadership, and in order to prevent being replaced, incumbents with a two step lead undertake the preemptive amount of R&D ϕ_{2N}^* if the economy is in state N. Let us now assume that the probability of expiration η_i can be made conditional on the number i of consecutive innovations that an innovator has attained. Then, the following proposition holds:

Proposition 4 *Given the incumbent has undertaken $i \geq K$ consecutive innovations. Then, starting from a situation with uniform expiration rates $\eta_i = \overline{\eta} > 0$, the average rate of innovation $\widehat{\phi}$ can be increased by marginally reducing expiration rates η_i for firms with $i < K$ consecutive innovations.*
Proof. *See Appendix E* ■

If the expiration rates η_i are only marginally reduced for entrants or firms who have recently entered, this increases the value of an innovation for an entrant, but not enough to offset the efficiency effect, so that the incumbent still finds preempting entry worthwhile. Due to increased entry pressure, the preemptive amount of R&D ϕ_{2N}^* that the incumbent undertakes in state N increases so that the average innovation rate $\widehat{\phi} = \frac{\overline{\eta}\phi_N^*}{\overline{\eta}+\phi_N^*}$ increases if the rate $\overline{\eta}$, with which forward protection for the incumbent expires, is unchanged.

6 Minimal inventive step

So far it has been assumed that firms cannot influence the size of the quality improvement that an innovation brings, and that incremental profits as a function of the lead are therefore exogenously given. Moreover, the case has been considered in which a two step lead ensures that the quality difference between the leaders product and that of the closest follower is so large that the leader can set the unconstrained monopoly price and can obtain the maximal possible profits. This section looks at the case where R&D firms are capable of targeting different innovation sizes μ at different costs. It is assumed that there is still an upper bound $\overline{\pi}$ on profits for firms whose lead in terms of quality is larger than a certain threshold. However, it is now possible to reach this lead by either undertaking many little steps (small μ) or a few large steps (large μ).

As the step size measures the quality difference between the new and the previous generation of the good, marginal profits increase in μ, so that for example profits in the case of a one step lead are given by $\pi_1(\mu_1)$ with $\frac{\partial \pi_1(\mu_1)}{\partial \mu_1} > 0$ if the first step is of size μ_1.

In this setting, there is an additional instrument that IP policy can use: a non-obviousness requirement that sets a lower bound on the inventive step below which an inventor cannot obtain IP protection. Let us assume that R&D costs $C_i(\phi_i, \mu_i)$ for a firm i that targets inventive step (innovation size) μ_i and wants to obtain the arrival rate ϕ_i is given by $C_i(\phi_i, \mu_i) = c\phi_i\lambda(\mu_i)(C_{tot})^{\epsilon}$ with $\frac{\partial \lambda(\mu_i)}{\partial \mu_i} > 0$ and $C_{tot} = C_i + C_{-i}$ indicating the overall (industry-wide) R&D spending. For a given amount of R&D spending C_i, targeting a larger inventive step μ_i therefore implies a lower hazard rate ϕ_i. Moreover, it is assumed that it is prohibitively costly to target a drastic innovation which would give an entrant the maximal lead with profits $\overline{\pi}$ in a single step. Given that $\epsilon > 0$, R&D costs for firm i increase in total R&D spending.[41] Due to this assumption, an incumbent may again preempt entry by undertaking a large enough amount of R&D in order to increase the entrants' R&D costs.

Expected profits of an entrant who targets the step size μ_1 and innovation rate ϕ in order to obtain a one step lead can be written as $E\Pi_E = \phi V_E - c\phi\lambda(\mu_1)(C_{tot})^{\epsilon}$, where V_E denotes the value of obtaining a one step lead. V_E is a function of μ_1 and of the whole path of inventive steps and innovation rates that the entrant expects to choose in order to advance his lead and to preempt entry in the future. The free entry condition that must be satisfied in equilibrium is given by $V_E = c\lambda(\mu_1)(C_{tot})^{\epsilon}$ and pins down the total R&D spending C_{tot} as a positive function of V_E. As the incumbent pays the same prices for R&D inputs as entrants and values not being replaced more than entrants value entry[42], she again finds it profitable to do all the R&D in equilibrium and to preempt entry. The crucial assumption due to which preemption is always feasible and profitable is therefore that only the incumbents total R&D spending matters for the entrants' R&D costs, but not the combination of step size and innovation rate that the incumbent chooses. An incumbent whose lead is large enough to allow her to charge the maximal price $\overline{\pi}$ therefore does not care about the inventive step of her innovations but only about the total R&D spending which is needed to make entry

[41] A reason for that might be that the price of R&D inputs increases if demand for them increases.

[42] The value of incumbency V_I is larger than the value of entering V_E for all sizes of the lead due to the efficiency effect.

unprofitable.[43] An entrant's incentives to innovate, however, depend on the sequence of inventive steps chosen to reach a lead that is large enough to obtain the profit flow $\overline{\pi}$. Without any non-obviousness requirement, entrants chose the sequence and sizes of inventive steps that maximize the present discounted value of their R&D activity. Therefore, any binding requirement that restricts the R&D decisions of entrants and of incumbents who have less than the maximal lead necessarily decreases the value V_E that entrants derive from undertaking R&D, and therefore reduces entry pressure. And if entry pressure is reduced, an incumbent with the maximal lead who does all R&D in equilibrium needs to spend less on R&D in order to discourage entry. However, imposing a non-obviousness requirement solely on incumbents with the maximal lead has no effect on their overall R&D spending and can moreover prevent them from targeting inefficiently small inventive steps.[44] We therefore obtain the second main result of the article:

Proposition 5 *Imposing a non-obviousness requirement that is binding for entrants and/or incumbents who have not reached the maximal lead reduces equilibrium R&D spending. Inefficiently small inventive steps can be avoided by imposing a non-obviousness requirement only on incumbents who have obtained the maximal lead. Such a policy does not affect equilibrium R&D spending.*

This result differs from the previous literature. O'Donoghue and Zweimüller (2004) and Hunt (2004) show that, in the case of leapfrogging, imposing a patentability (non-obviousness) requirement can increase the rate of innovation and welfare. O'Donoghue (1998) reaches the same conclusion in a model where even incumbent firms do some R&D in equilibrium, but in which preemption is not possible, so that there is no persistent leadership. The following mechanisms are at work in these articles: imposing a patentability requirement makes innovating harder and prolongs incumbency by reducing the frequency of innovation and therefore the risk of being replaced. This again

[43] This can be different if the incumbents R&D activity affects the entrants' R&D profitability by increasing the risk of duplication. If in this case, incumbents are forced to pursue considerably larger inventive steps than entrants, this makes it more costly for them to target a certain innovation rate and duplication risk, so that they might no longer find it profitable to preempt entry. Weinschenk (2009) studies a two-period model with two firms and finds that preempting entry is only profitable for the incumbent if innovation is associated with a high success probability.

[44] In the simple model analyzed here, incumbents are indifferent with respect to the inventive step chosen so that they might not select the socially optimal step size if there is no non-obviousness requirement. In a more general model where incumbents can increase their profit flows through R&D, Denicolò (2001) shows that incumbents tend to pursue inventive steps that are too small, so that a non-obviousness requirement is more likely to impose a binding restriction.

increases the value of an innovation and the incentives to do R&D. Moreover, as firms are constrained by competition from the previous generation of the good, markups and therefore profit flows are increased if firms pursue larger quality improvements. Finally, imposing a patentability requirement can prevent firms from pursuing inefficiently small inventive steps. Contrary to that, incumbents already enjoy the maximal market power in the case of persistent leadership and need to be pushed in order to innovate more. Relaxing the non-obviousness requirement for entrants but not for incumbents therefore increases entry pressure and the equilibrium innovation rate. As entrants do not innovate in equilibrium, inefficiently small inventive steps are, however, never realized, although entrants could obtain IP protection on them.

Given the policy maker (the patent office) can observe whether an innovation passes a certain inventive step[45], it might still be impossible to impose different non-obviousness requirements on incumbents depending on whether they have reached the maximal lead (are unconstrained monopolists). However, making the non-obviousness requirement conditional on whether an entrant or an incumbent realizes an innovation only requires that the policy maker (patent office) can observe whether the firm filing for IP protection already has IP protection for the previous generation of the good. By not imposing a non-obviousness requirement on entrants, entry pressure and the rate of innovation could therefore be increased.[46]

7 State-dependent intellectual property protection

This section analyzes the effects of IP expiration in the basic model in which IPRs protect against imitation, but in which there is no forward protection and in which the

[45]Hopenhayn, Llobet and Mitchell (2006) study the optimal patent system in a quality-ladder model where the patent office cannot observe the size of the inventive step and therefore cannot impose a patentability requirement. Under the assumption all innovations are realized by entrants, they find that the optimal R&D incentives can be obtained through a patent buyout scheme. This requires that an inventor pays a certain fee to the previous patent holder in order to be allowed to replace her, and that the size of the fee depends on the level of protection that the previous inventor has "bought" from the patent office. Such a scheme, however, is likely to reduce innovation in a setting where incumbents innovate preemptively, as it allows the incumbent to preempt entry by simply buying protection against replacement, instead of doing R&D.

[46]If incumbents do not have perfect information about the entrants' R&D costs or the extent of decreasing returns to R&D, it might be optimal for them to target a level of R&D which allows for entry with a positive probability. In order to avoid inefficiently small innovations from being targeted and carried out by entrants, imposing a non-obviousness requirement also on entrants might be useful in this case. However, as long as entrants do not do much R&D in equilibrium, the non-obviousness requirement for them should still be weaker than that imposed on incumbents in order to keep entry pressure high.

step size is constant and exogenous. Like Acemoglu and Akcigit (2012), the general case of "state-dependent intellectual property protection" is analyzed in which the probability of IP expiration can depend on whether a firm has a one step or a two (or more) step lead over its rivals.[47] The IPRs of a firm with a one step lead are assumed to expire with the instantaneous Poisson arrival rate γ_1 and those of a firm with a two step lead with the instantaneous arrival rate γ_2. In the case of IP expiration, the newest generation of the good falls in the public domain, allowing competitors to copy it freely and to fully catch up.[48]

Assuming that there is no collusion, the value of an innovation for an entrant, V_E, is independent of whether the IPR of the previous generation of the good has expired and also independent of the size of the previous incumbents lead. In the case of persistent leadership, V_E (that means the value of being one step ahead) and the value V_2 of being two (or more) steps ahead can be derived from the following arbitrage conditions (again looking at the case where $\epsilon \to 0$):

$rV_2 = \pi_2 - c\phi_2 - \gamma_2 V_2$

$rV_E = \pi_1 - c\phi_1 - \gamma_1 V_E - \phi_1 V_E + \phi_1 V_2$

from which we get:

$$V_E = \frac{\pi_1 - c\phi_1}{r + \gamma_1 + \phi_1} + \frac{\phi_1}{r + \gamma_1 + \phi_1} \frac{\pi_2 - c\phi_2}{r + \gamma_2} \tag{8}$$

A firm with a one step lead sets its innovation rate ϕ_1 as a function of the IP expiration rates γ_1 and γ_2 which determine the relative profitability of a two step lead compared to a one step lead.

Lemma 2 *(i) If $\gamma_1 > \frac{\pi_1}{c} - r$ and if $\gamma_2 < \frac{\pi_2}{2c} - r$, a firm with a one step lead selects R&D effort $\phi_1^* = \phi_m$ given that ϕ_m is sufficiently large.*
(ii) If $\gamma_1 \leq \frac{\pi_1}{c} - r$ and if $\gamma_2 \leq \gamma_1 + \frac{\pi_2 - \pi_1}{c}$, a firm with a one step lead selects the preemptive R&D level $\phi_1^* = \phi_2^* < \phi_m$
(iii) If $\gamma_1 \leq \frac{\pi_1}{c} - r$ and if $\gamma_2 > \gamma_1 + \frac{\pi_2 - \pi_1}{c}$, a firm with a one step lead does not do any R&D ($\phi_1^* = 0$), so that there is leapfrogging.

[47] Conditioning intellectual property policy (or antitrust policy that has similar effects) on the size of a firm's lead is only possible if the policy maker (e.g. the patent office or the regulator) has detailed information about market conditions and can observe this lead. This might therefore be considerably harder than to just condition IPR policy on whether an innovation is undertaken by an entrant or an incumbent.

[48] This specification implicitly assumes that IP protection on second-newest goods never expires. The case in which firms with a two step lead can lose this lead for a one step lead (because their IPR on the second-newest good expires) is not considered.

Proof. *see Appendix F* ■

Due to Assumption 1 ($\pi_2 \leq 2\pi_1$), the conditions in Regime (i) imply that $\gamma_1 > \gamma_2$ needs to hold, meaning that IP protection needs to expire sufficiently more quickly in the case of a one step lead than in the case of a two step lead in this regime. This makes a two step lead relatively more profitable, so that firms with a one step lead do the maximal amount of R&D ϕ_m in order to reach a two step lead as quickly as possible. In Regime (ii), firms with a one step lead value not being replaced and reaching a two step lead more than entrants value entry, but they do not want to do more R&D than necessary to preempt entry so that $\phi_1^* = \phi_2^*$. In Regime (iii), IPRs expire so much faster in the case of a two step lead than in the case of a one step lead that firms with a one step lead do not find reaching a two step lead worthwhile and stop innovating. In Regimes (i) and (ii), the innovation rate in the case where the IPR on the currently newest generation of the good has expired ("zero step lead") is given by $\phi_0^* = \phi_2^*$. While entrants might undertake R&D in this case, the innovation rate is the same as the preemptive level ϕ_2^* that incumbents select in order to prevent entry, as it is pinned down by the same free entry condition $V_E = c$. In Regime (ii), the innovation rate is therefore independent of the state in which the economy is, while it is higher in the case of a one step lead than in the case of a zero or a two step lead in Regime (i). In Regime (iii), entrants undertake all R&D, and the innovation rate ϕ^* is independent of the state of the economy as it, again, does not depend on whether IP protection on the currently newest generation of the good has expired or not. We can now analyze the effect of IP expiration on the average innovation (arrival) rate $\widehat{\phi}$ and derive the third main result of the article.

Proposition 6 *Given a change in γ_i does not lead to a switch between regimes, the average innovation rate $\widehat{\phi}$ decreases in γ_1 in all Regimes and decreases in γ_2 in Regimes (i) and (ii). If a marginal shift in parameters leads to a switch from Regime (ii) to Regime (i), there is a discontinuous increase in $\widehat{\phi}$, while there is no discontinuity in the case of a switch between Regimes (ii) and (iii). The (average) innovation rate $\widehat{\phi}$ is maximal if $\gamma_1 = \gamma_2 = 0$, that means under full uniform IP protection.*
Proof. *See Appendix G* ■

Reducing IP protection by increasing γ_i decreases the value of an innovation for an entrant and entry pressure in all regimes. This directly leads to a reduction in the equilibrium rate of innovation in Regimes (iii) and (ii), and also to a decrease in the innovation rates $\phi_2^* = \phi_0^*$ in the cases of a two and a zero step lead in Regime (i). In Regime (i), there is, however, also a "composition effect" which goes in the opposite direction: increasing γ_2 can now increase the probability that the economy is in the state in which there is a one step lead and in which the innovation rate is maximal.[49] However, this composition effect is not strong enough to overcompensate the negative effect that an increase in γ_2 has on $\phi_2^* = \phi_0^*$, so that an increase in γ_2 still leads to a reduction in the average arrival rate $\widehat{\phi}$.[50] The average innovation rate in Regime (i) is therefore maximal if $\gamma_2 = 0$, in which case the composition effect disappears as leaders never loose a two step lead. But given that $\gamma_2 = 0$, the (equilibrium) innovation rate $\widehat{\phi} = \phi_2^*$ decreases in γ_1, as increasing the rate with which IP protection for firms with a one step lead expires merely reduces the value of an innovation for an entrant and therefore the amount of R&D the incumbent needs to undertake in order to preempt entry. Consequently, the (average) innovation rate is maximal under full uniform IP protection and, unlike in Acemoglu and Akcigit (2012), cannot be increased by reducing IP protection in a state-dependent way (i.e. by increasing γ_1 more than γ_2). However, given that γ_1 and γ_2 are already positive and at a certain threshold value, a small increase in γ_1 can lead to an increase in the average innovation rate $\widehat{\phi}$ by inducing a switch from Regime (ii) to Regime (i). The reason for this is that such a decrease in IP protection for firms with a one step lead can induce them to switch their strategy from only undertaking the amount of R&D that is necessary to preempt entry to undertaking the maximal amount of R&D in order to reach a two step lead as quickly as possible. There is therefore a nonlinear relation between γ_1 and the average innovation rate so that the "incentive effect" identified by Acemoglu and Akcigit (2012) is at work at a specific threshold value of γ_1.

8 Extensions

8.1 Potential perils of strong protection against imitation

In the previous section, it was shown that under free entry, reducing the strength with which intellectual property rights protect against imitation (that means increasing γ_i) cannot increase the average innovation rate through a composition effect, i.e. by

[49] Increasing γ_1 always decreases this probability (see Appendix G).

[50] In a more general model where $\epsilon > 0$ and where there is no upper bound ϕ_m on the innovation rate, firms with a one step lead would moreover reduce their R&D effort ϕ_1^* if obtaining a two step lead becomes less profitable (due to an increase in γ_2), so that the composition effect would be even weaker.

making a state in which R&D incentives are higher more likely. In the following subsections, cases are analyzed in which this is different and where reducing IP protection can increase the average innovation rate through such a composition effect (again, it is assumed that there is no forward protection and that the step size is constant and exogenous).

8.1.1 Trade secrecy

So far it has been assumed that there is free access to the R&D sector and that entrants have complete knowledge about the currently newest technology and can directly start doing frontier R&D. While, ideally, a firm is only granted IP (patent) protection when it discloses the functioning of its innovation, firms often succeed in keeping part of their knowledge secret so that competing firms first have to engage in some duplicative catch-up R&D before they can start conducting frontier R&D. In the following, it is therefore assumed that entrants first have to spend the fixed catch-up costs R before they can undertake frontier R&D. It is assumed that the incumbent can observe an entrant's catch-up and thus can adjust her R&D spending after observing entry into the R&D sector. IP protection expires with the constant hazard rate γ and, in the case of expiration, the newest available quality of the good falls in the public domain and is supplied at the marginal cost of zero.[51] In this case, the incumbent loses all profits, and it is assumed for simplicity that all knowledge about the currently newest version of the good falls in the public domain.[52]

The equilibrium can be derived through backward induction. Given the incumbent has a lead of one or two steps, and entry into the R&D sector has occurred, the analysis is the same as in the previous sections, and the incumbent does enough R&D to completely discourage R&D by the entrant, so that the entrant makes zero profits. Expecting this, no entrant ever finds it profitable to pay the catch-up costs R, so that there is no entry into the R&D sector if the currently newest generation of the good is protected by IPRs. This, however, implies that there is no entry pressure and that

[51] This specification is chosen for reasons of simplicity, and neglects the case where firms with a two step lead lose this lead for a one step lead because their IPR on the second-newest good expires. It is therefore implicitly assumed that IPRs on second-newest goods never expire.

[52] Alternatively, one could assume that , entrants can merely copy and produce the newest version of the good in the case of patent expiration, while the previous incumbent still keeps a lead in terms of knowledge and know-how that is necessary to do frontier R&D. Then, entrants would still need to pay the catch-up costs R before they can undertake frontier R&D, while previous incumbents could start right away. This change of assumptions would make the analysis more complicated but would not change the results qualitatively.

incumbents do not need to undertake any R&D in order to preempt entry. Incumbents thus stop innovating after having obtained a two step (or even a one step) lead. If IPRs have expired and if the currently newest version of the good is in the public domain, firms might, however, find it profitable to undertake R&D in order to become the next leader, given that IP protection does not expire too quickly. Denoting the innovation rate in the case of a one (two) step lead by ϕ_1 (ϕ_2) and that in the case where IPRs have expired by ϕ_0, the following proposition holds:

Proposition 7 *If* $0 \leq \gamma \leq \frac{\pi_2-\pi_1}{c} - r$ *(Case A),* $\phi_0^* = \phi_1^* = \phi_m$ *and* $\phi_2^* = 0$*. If* $\frac{\pi_2-\pi_1}{c} - r < \gamma \leq \frac{\pi_1}{c} - r$ *(Case B),* $\phi_0^* = \phi_m$ *and* $\phi_1^* = \phi_2^* = 0$ *and if* $\gamma > \frac{\pi_1}{c} - r$ *(Case C),* $\phi_0^* = \phi_1^* = \phi_2^* = 0$*. The average arrival rate* $\widehat{\phi}$ *("average innovation rate") is given by:*

$$\widehat{\phi} = \begin{cases} \frac{\gamma^3+2\gamma^2\phi_m}{(\phi_m+\gamma)^2} \text{ in Case A} \\ \frac{\phi_m\gamma}{\phi_m+\gamma} \text{ in Case B} \\ 0 \text{ in Case C} \end{cases}$$

Given that $\phi_m((\frac{2\pi_2-3\pi_1}{c}) - r) + (\frac{\pi_2-\pi_1}{c} - r)^2 > (<)0$*,* $\widehat{\phi}$ *is maximal if* $\gamma = \frac{\pi_2-\pi_1}{c} - r$ $(\gamma = \frac{\pi_1}{c} - r)$

Proof. *See Appendix H* ■

There is therefore an inverted-U relation between the average innovation rate and the strength of IP protection.[53] If IPRs are fully protected (the subcase of Case A where $\gamma = 0$), the average innovation rate is zero as there is no entry pressure and as monopolists rest on their laurels. Reducing the strength of IP protection then increases the probability of being in the state where no firm has a lead over its rivals and where innovation incentives are maximal, but also the probability of being in the state where the incumbent only has a one step lead and might (in Case A) find it profitable to do R&D in order to obtain a two step lead. If, however, IP protection is too weak (Case C), the average innovation rate is again zero, as appropriability is so low that firms do not find it profitable to do R&D, even if the currently newest version of the good

[53] It should be noted that this result holds for any size of R, as long as incumbents can observe catch-up and entry and can readjust their R&D effort ex post. However, they might not be capable of doing that, especially if catch-up costs (R) are low. If incumbents cannot observe catch-up, they actually have to undertake R&D in order to preempt entry, so that the analysis is again very similar to that in the previous sections. The fixed catch-up costs then only decrease entry pressure while stronger IP protection (a lower γ) again encourages innovation. The discontinuity that the effect of IP protection (γ) on the rate of innovation changes dramatically when even minor fixed costs of entering the R&D sector are introduced, therefore does not arise if incumbents cannot observe entry.

is in the public domain.[54]

8.1.2 Ex ante agreements and different forms of preemption

Proposition 7 holds more generally in any situation where an incumbent does not face any entry pressure[55] and it is in line with the findings of Horowitz and Lai (1996) and Cadot and Lippman (1995) who assume that only one incumbent firm is capable of doing R&D. But also in the case where a finite number of firms is capable of doing R&D and where they can make ex ante agreements about their joint R&D effort and about how to split profits, the analysis is the same. If these firms maximize joint profits, they do exactly as much R&D as the incumbent analyzed above. If it is difficult to prevent ex ante agreements and to force firms to compete in an innovation race, another possibility for increasing the average innovation rate is therefore to make IPRs expire with a positive probability.

It was assumed in the analysis above that incumbents can only discourage entrants' R&D by doing R&D themselves. There might, however, be cases in which incumbents can increase the R&D costs of potential entrants, or decrease their expected benefits of innovating, without doing R&D themselves. If there is an upward-sloping supply curve for R&D labour, incumbents might for example hire a certain amount of R&D labour in order to increase the wages that rivals need to pay, but employ the researchers in other areas than R&D. In the case where knowledge is to some extent tacit and where only researchers who were involved in past R&D are capable of doing frontier R&D, incumbents can simply offer them long term contracts in order to prevent them from doing research for entrants.[56] While preemption is not costless in these cases, the result that incumbents stop innovating (or only innovate a little) once they have obtained a sufficient lead is therefore the same. Because of that, the average innovation rate is again zero (or very small) if IPRs are fully protected and might increase if IP

[54] If instead of letting IP protection expire regularly (or reducing the length of IP protection) the breadth of IP protection was reduced with the effect of reducing the maximal profits to $\widetilde{\pi}_2$ (with $\pi_1 < \widetilde{\pi}_2 < \pi_2$), this would not increase the average innovation rate (see *Section 14* of the working paper version of this article, where also other extensions are discussed).

[55] Given the constant-return R&D technology with the upper bound ϕ_m, the innovation rate ϕ_0 in the case where the currently newest version of the good is supplied competitively is the same in the cases where one or several firms are capable of doing R&D.

[56] If it is costly for researchers to move from the incumbent to an entrant firm, this creates fixed costs of entering and facilitates preemption. And if incumbents are allowed to forbid their R&D workers to work for rivals in the future, preemption is even easier.

In the case where the R&D process requires different complementary inputs, it might even be possible to preempt entry by buying up a sufficient amount of a single R&D input which is in scarce supply.

protection is decreased.[57]

8.2 Intermediate R&D inputs

Let us now look at the case where two R&D stages have to be completed in order to improve the quality of the good by one step. In the first stage, an intermediate R&D input (which might be thought of as an idea) has to be invented and this input is used in the second stage to develop an improved version of the good. The two stages could as well be interpreted as a research and a development stage. The R&D technology at each stage is again stochastic and assumed to be of the same form as in the main model, allowing for preemption due to decreasing R&D productivity at the industry level. It is assumed that there is full IP protection against imitation in the final goods market ($\gamma = 0$). In the case where IP protection is granted on an intermediate R&D input, it allows to prevent other firms from using this input, and therefore from developing a better version of the good. Three cases are considered:

1) *No IP protection is granted on intermediate R&D inputs*

In this case, the innovation rate is zero as no firm has incentives to invent such an input.[58]

2) *Both entrants and incumbents can obtain IP protection on intermediate R&D inputs*

Innovation also comes to a halt in this case as incumbents always use the possibility of obtaining IP protection on the newest input in order to block entry and to eliminate the threat of being replaced by an entrant.[59]

3) *Only entrants can obtain IP protection on intermediate R&D inputs, but are not*

[57] The assumption that incumbents cannot increase their profit flows by increasing their lead beyond two steps is certainly unrealistic. However, even in a more general setting with increasing profit flows, an intermediate strength of IP protection maximizes the average innovation rate in the above-mentioned cases given that parameters lie within a certain range (see *Appendix 2, Section 14*).

[58] Once the input is invented, there is free entry into the second stage development race, so that expected profits for entrants are zero in this race. Expecting this, no entrant has incentives to spend money on inventing such an intermediate input in the first place. But neither does an incumbent who has already obtained a two step lead and cannot increase profits further by innovating.

[59] Given that an incumbent has obtained a two step lead and has also obtained an IPR on the R&D input which is needed to develop the next version of the good, she uses it to block follow-on R&D by entrants. If, instead, the entrant has IP protection on the newest version of the input, he finds it profitable to license it to the incumbent as he values doing follow-on R&D and entering the market with an improved version of the good less than the leader values blocking follow-on R&D. Even if entrants are not allowed to license to incumbents, the incumbent can prevent an entrant from obtaining IP protection on the intermediate R&D input by undertaking a sufficient amount of research effort in the race for the input.

allowed to license to incumbents

In this case, sustained innovation is possible, as the incumbent has incentives to preempt entrant R&D at each stage, without ever being able to block future entry completely. If the input is invented by the incumbent and freely accessible to entrants, the incumbent has incentives to preempt entry by exerting a large enough effort in the race for the second R&D stage. Expecting this, she finds inventing the intermediate input worthwhile (even if she does not obtain IP protection on it), as this prevents entrants from inventing and obtaining IP protection on it, which would allow them to replace her in the future. Once the second R&D stage is completed and the incumbent has developed the next version of the good, the whole process starts again.

8.3 A full-fledged general equilibrium growth model

In the previous sections, a partial equilibrium model was analyzed and the simplifying assumption was made that increasing the lead beyond two steps does not lead to larger profit flows. In the supplementary *Appendix 2* the more general case of an endogenous growth model (based on Barro and Sala-i-Martin (1995)) is analyzed in which also incumbents with a two step lead can increase profit flows by doing follow-on R&D. Assuming parameters to be such that each single innovation is non-drastic, while a two step lead allows the leader to charge the unconstrained monopoly price, the following results are derived:

As in the simple model, allowing collusion or granting extended lagging breadth encourages growth, and requiring entrants to pay licensing fees to incumbents, or granting forward protection reduces growth. It is shown in a welfare analysis that it is better to reduce innovation incentives in the case where equilibrium growth is excessive by introducing a price cap than by making profit flows more backloaded. The reason for this is that monopoly distortions are only reduced in the first case. Given the same non-obviousness requirement is imposed on entrants and incumbents, it is shown that growth can be increased if the non-obviousness requirement for entrants (and/or incumbents with less than the maximal lead) is slightly relaxed. This result is therefore weaker than that obtained in the simple model.

In the case where incumbents can preempt entry without innovating themselves, the average growth rate can again be maximal for an intermediate strength of IP protection, even in the case where innovations are drastic.

The effects that IP policies have on the rate of growth are therefore mainly the same as those they have on the rate of innovation in the simplified model

9 Conclusion

While most of the literature analyzing the role of intellectual property rights in models of cumulative innovation focuses on the case of leapfrogging, this article studies the case of persistent leadership. This is highly relevant from an empirical perspective, since most R&D is undertaken by firms that innovate on a permanent basis. Three main results come out of this analysis: *first*, non-expiring forward protection reduces the rate of innovation. *Second*, imposing a non-obviousness requirement reduces R&D spending. Both these results differ from the ones obtained in models with leapfrogging. The *third* main result is that full uniform protection against imitation, and not state-dependent IP protection, maximizes the average innovation rate. It is also shown that the average innovation rate can be increased if forward protection of finite duration is granted in combination with infinitely lived protection against imitation (a result that extends to the case of leapfrogging). Moreover, it is shown that if incumbents can preempt entry without innovating themselves, the average innovation rate is maximal if protection against imitation expires regularly. The article also analyzes how innovation can be encouraged if IP protection can be made conditional on whether an innovation is carried out by an entrant or an incumbent.

The model generates persistent leadership by assuming that innovations are non-drastic and that incumbents are capable of preempting entry, but - unlike other models - it does not rely on the assumption that incumbents are more productive in undertaking R&D than entrants. Within this setting, entry can occur if IP protection has expired, if entrants and incumbents can collude or sell their IPRs, or if there is extended lagging breadth. Therefore, the model is also consistent with cases in which there is less than perfect persistence. If, however, entry occurs because innovations are drastic, because entrants' R&D productivity is larger than that of incumbents, or because preemption is not possible (or not an optimal strategy for incumbents) due to technological or informational reasons, the leapfrogging model becomes more appropriate. Therefore, it might be beneficial to use different IPR policies depending on the degree of persistence of innovative activities in a given sector.[60]

[60] Cefis and Orsenigo (2001) and Cefis (2003) provide empirical evidence for the fact that the persistence of innovative activities depends on technological and industry characteristics.

10 References

Acemoglu, Daron and Akcigit, Ufuk (2012); "Intellectual Property Rights Policy, Competition and Innovation", Journal of the European Economic Association, 10 (1), pp. 1-42

Acemoglu, Daron and Dan Cao (2010); "Innovation by Entrants and Incumbents", working paper, September 12, 2010

Aghion, Philippe and Peter Howitt (1992); "A model of Growth through Creative Destruction", Econometrica 60 (2): 323- 351

Aghion, Philippe, C. Harris, P. Howitt and J. Vickers (2001); "Competition, Imitation and Growth with Step-by-Step Innovation", Review of Economic Studies, 68 (3): 467-492

Akcigit, Ufuk and William R. Kerr (2010); "Growth through Heterogeneous Innovations", PIER working Paper 10-035

Arrow, Kenneth (1962); "Economic Welfare and the Allocation of Resources for Invention", in National Bureau of Economic Research, *The Rate and Direction of Inventive Activity.* Princeton, NJ: Princeton University Press

Barro, R.J. and Sala- i- Martin, X. (1995); "Economic Growth", McGraw - Hill, New York

Bartelsman, Eric J. and Mark Doms (2000); "Understanding Productivity: Lessons from Longitudinal Microdata", Journal of Economic Literature vol. 38 (3), pp. 569-594

Bessen, James and Eric Maskin (2009); "Sequential Innovation, Patents, and Imitation", Rand Journal of Economics, vol. 40, issue 4, pp. 611 - 635

Cadot, O. and Lippman, S. (1995); "Barriers to Imitation and the Incentive to Innovate", INSEAD working paper

Cefis, Elena (2003); "Is there persistence in innovative activities?", International Journal of Industrial Organization, 21: 489-515

Cefis, Elena and Luigi Orsenigo (2001); "The persistence of innovative activities. A cross-countries and cross-sectors comparative analysis", Research Policy, 30: 1139-1158

Chu, Angus C. (2009); "Effects of Blocking Patents on R&D: A Quantitative DGE Analysis", Journal of Economic Growth, 2009, vol. 14, pp. 55-78.

Chu, Angus C., Cozzi, Guido and Silvia Galli (2012); "Does intellectual monopoly stimulate or stifle innovation?" Europen Economic Review

Cozzi, Guido (2007); "The Arrow Effect under Competitive R&D", The B.E. Journal of Macroeconomics, Volume 7, Issue 1

Czarnitzki, Dirk and Federico Etro, and Kornelius Kraft (2011); "Endogenous Market Structures and Innovation by Leaders: an Empirical Test", Ca Foscari University of Venice Working Paper No. 04/WP/2011

Denicolò, Vincenzo (2001); "Growth with non-drastic innovations and the persistence of leadership", European Economic Review, vol. 45(8), pp. 1399-1413

Denicolò, Vincenzo (2002); "Sequential innovation and the patent-antitrust conflict", Oxford Economic Papers, pp. 1-20

Denicolò, Vincenzo and Piercarlo Zanchettin (forthcoming); "Leadership Cycles in a Quality-Ladder Model of Endogenous Growth", The Economic Journal

Etro, F. (2004); "Innovation by Leaders", The Economic Journal, Vol. 114, 495 (April), pp. 281-310

Foster, Lucia and Cheryl Grim (2010); "Characterisics of the top R&D performing firms in the U.S.: evidence from the survey of industrial R&D", CES discussion paper 10-33

Foster, Lucia, John C. Haltiwanger and C. J. Krizan (2001); "Aggregate Productivity Growth: Lessons from Microeconomic Evidence", NBER Chapters, in: New Developments in Productivity Analysis, pp. 303-372

Fudenberg, Drew, Richard Gilbert, Joseph Stiglitz and Jean Tirole (1983); "Preemption, leapfrogging and competition in patent races", European Economic Review 22, 3-31

Gilbert, Richard J. & Newbery, David M. G., (1982); "Preemptive Patenting and the Persistence of Monopoly", American Economic Review, vol. 72(3), pages 514-26

Hopenhayn, Hugo & Gerard Llobet & Matthew Mitchell, (2006); "Rewarding Sequential Innovators: Prizes, Patents, and Buyouts", Journal of Political Economy, vol. 114(6), pages 1041-1068

Hopenhayn, Hugo & Matthew Mitchell (2011); "Optimal Patent Policy with Recurrent Innovators", working paper, University of Toronto

Horowitz, Andrew W. and Lai, Edwin L.- C. (1996); "Patent Length and the Rate of Innovation", International Economic Review vol. 37 (4), pages 785-801

Hunt, Robert M. (2004); "Patentability, Industry Structure, and Innovation", The Journal of Industrial Economics, Vol. 52, No. 3, pp. 401-425

Llanes, G. and S. Trento (forthcoming); "Patent Policy, Patent Pools, and the

Accumulation of Claims in Sequential Innovation"; Economic Theory

Malerba, F. and L. Orsenigo (1999); "Technological Entry, Exit and Survival: an Empirical Analysis of Patent Data", Research Policy, 28, 643 - 660

O'Donoghue, Ted (1998); "A patentability requirement for sequential innovation", RAND Journal of Economics Vol. 29, No. 4, pp. 654-679

O'Donoghue, Ted, Suzanne Scotchmer and Jacques François Thisse (1998); "Patent Breadth, Patent Life and the Pace of Technological Progress", Journal of Economics and Management Strategy, 7, 1-32.

O'Donoghue, Ted and Joseph Zweimüller (2004); "Patents in a Model of Endogenous Growth", Journal of Economic Growth, 9, 81-123.

Peters, Bettina (2009); "Persistence of innovation: stylised facts and panel data evidence", Journal of Technology Transfer, 34: 226-243

Reinganum, Jennifer (1983); "Uncertain Innovation and the Persistence of Monopoly", American Economic Review, Vol. 73, No. 4

Reinganum, Jennifer (1985); "Innovation and Industry Evolution", The Quarterly Journal of Economics, Vol. 100, No. 1

Scotchmer, Suzanne (2004); "Innovation and Incentives", MIT Press, Cambridge, MA

Segal, Ilya, and Michael D. Whinston (2007); "Antitrust in Innovative Industries", American Economic Review, 97(5): 1703–1730.

Segerstrom, Paul (2007); "Intel Economics", International Economic Review, 48: 247-280

Segerstrom, Paul and James Zolnierek (1999); "The R&D incentives of industry leaders", International Economic Review, 40: 745-766

Weinschenk, Philipp (2009); "Persistence of Monopoly and Research Specialization", MPI working paper

Appendix 1: Proofs

A

Proof. In order to determine the equilibrium value ϕ_2^* of ϕ_2, we need to derive the level of ϕ_1 chosen by an incumbent with a one step lead. This incumbent takes V_2, and therefore ϕ_2^*, as given and maximizes V_1 $(= V_E)$ with respect to ϕ_1. Due to the assumption of constant returns to R&D the analysis is quite simple: as $\frac{\partial V_E}{\partial \phi_1} = \frac{\pi_2 - \pi_1 - cr - c\phi_2^*}{(r+\phi_1)^2}$, we would get the corner solution $\phi_1(\phi_2^*) = \phi_m$ if $\frac{\partial V_E}{\partial \phi_1} > 0$, that means if $\phi_2^* < \frac{\pi_2 - \pi_1}{c} - r$ (**Condition 1**). If $\phi_1 = \phi_m$, we obtain $\phi_2^* = \frac{\pi_2}{c} + \frac{r}{\phi_m}\left(\frac{\pi_1}{c} - r\right) - 2r$ from the free entry condition (equation (5)), but this is incompatible with Condition 1, as $\left(\frac{\pi_1}{c} - r\right)\left(1 + \frac{r}{\phi_m}\right) < 0$ (**Condition 2**) is violated due to Assumption 2 $\left(\frac{\pi_1}{r} > c\right)$. Therefore, $\frac{\partial V_E}{\partial \phi_1} < 0$ holds in equilibrium and an incumbent with a one step lead sets $\phi_1^* = \phi_2^*$ and only conducts as much R&D as needed to preempt entry[61]. As $V_2 = \frac{\pi_2 - c\phi_2^*}{r}$ decreases in ϕ_2^* and $\frac{\partial V_1}{\partial \phi_1} < 0$ in equilibrium, V_E decreases in ϕ_2^*.[62] Using this result and implicitly differentiating the free entry condition, we find that ϕ_2^* increases in π_1 and π_2 and decreases in r and c. ■

B

Proof. Given the incumbent who produces generation k of the good already has a two step lead, she cannot increase the price and profits by colluding with an entrant, and simply uses her blocking power to prevent generation $k+1$ of the good from being produced. The entrant therefore gets profit flows $\pi_1 = 0$, but can increase them to the maximal profit flows π_2 in the future if he invents generation $k+2$ of the good, as the incumbent who has IP protection on generation k cannot block the production of generation $k+2$.[63] Assuming for the moment that entrants do follow-on R&D in order to become the next incumbents and then preempt entry to keep their two step lead, the value of an innovation for an entrant is given by $V_E = \frac{-c\phi_1}{r+\phi_1} + \frac{\phi_1}{r+\phi_1}\frac{\pi_2 - c\phi_2}{r}$, the same as in equation (5), but with π_1 replaced by 0. Using the arguments from

[61] Note that the amount of R&D needed to preempt entry is always given by ϕ_2^* as the value of an innovation for an entrant does not depend on the size of the lead of the current leader.

[62] The same also holds in the case where $\phi_1 = \phi_m$, which becomes relevant in later sections.

[63] Also in the case where the incumbent who has patented generation k only has a one step lead, the entrant with generation $k+1$ gets zero profits for the following reason: the incumbent can either block the entrant and collude with the firm that holds the patent on generation $k-1$, or collude with the entrant and push generation $k-1$ out of the market. By making firm $k-1$ and the entrant compete for offers, firm k can therefore extract the whole surplus π_2, leaving zero profits for the entrant.

the proof of *Proposition 1* (*Appendix A*) and replacing π_1 with 0 in Condition 2, we obtain that now $\phi_1^* = \phi_m$, as the relative profitability of obtaining a two step lead instead of a one step lead has increased. While entry does not reduce joint profits due to competition in the product market, it rises R&D above the level that is needed to preempt entry (as $\phi_1^* = \phi_m > \phi_2^*$), so that joint net profits decrease until the entrant has advanced to a two step lead. This creates a new sort of "efficiency effect" due to which the incumbent values preventing entry more than an entrant values entering. Therefore, incumbents have incentives to preempt entry completely and there is again persistent leadership. The equilibrium innovation rate ϕ_2^* can now be derived from the free entry condition $V_E = c$, and we obtain $\phi_m > \phi_2^* > 0$ if $\phi_m > \frac{\pi_2}{c} - \frac{r^2}{\phi_m} - 2r > 0$, which holds if ϕ_m is large and if $\pi_2 > 2cr$. As V_E is lower than in the case without forward protection (as π_1 is replaced by 0), the innovation rate is lower in the case with forward protection than in the case without. ■

C

Proof. As expiration of forward protection and innovation occur with the Poisson arrival rates η and ϕ_N^*, the expected time span during which the economy remains in state F (state N) before a switch to the other state occurs is given by $\frac{1}{\eta}$ ($\frac{1}{\phi_N^*}$), so that the proportion of time spent in state N is given by $\omega = \frac{\frac{1}{\phi_N^*}}{\frac{1}{\eta}+\frac{1}{\phi_N^*}}$. The average innovation rate is then given by $\widehat{\phi} = \omega\phi_N^* = \frac{\eta\phi_N^*}{\eta+\phi_N^*}$. Inserting $\phi_N^* = \frac{\eta\pi_2}{c(r+\eta)-\pi_2} - r$ and deriving $\widehat{\phi}$ with respect to η yields $sign\frac{\partial\widehat{\phi}}{\partial\eta} = sign\left[(\pi_2 - rc)(r^2 + 2\eta r) - \eta^2 rc\right]$. As the term in square brackets continuously decreases in η in the admissible range where $\eta > \frac{\pi_2}{c} - r$, we can derive the expiration rate $\eta^* = \frac{\pi_2}{c} - r + \frac{\sqrt{\pi_2(\pi_2 - rc)}}{c}$ for which the average innovation rate is maximal by solving the first order condition $\frac{\partial\widehat{\phi}}{\partial\eta} = 0$. ■

D

Proof. Given forward protection and protection against imitation expire with hazard rate η, the value of an innovation for an entrant is given by $V_F = \frac{\pi_2}{r+\eta}$. If $V_F \geq c$, the innovation rate in state N is therefore given by the corner solution $\phi_N^* = \phi_m$, so that the average innovation rate can be derived as $\widehat{\phi} = \frac{\eta\phi_m}{\eta+\phi_m}$. As $\widehat{\phi}$ increases in η, it is maximal if $\eta = \eta^* = \frac{\pi_2}{c} - r$, that means if the expiration rate is set at the maximal level that still induces R&D in state N (and for which $V_F = c$). As $\widehat{\phi}$ also increases in ϕ_m, the maximal value it can reach is therefore given by $\lim\limits_{\phi_m \to \infty} \widehat{\phi} = \eta^* = \frac{\pi_2}{c} - r$.

However, this is exactly the rate of innovation that can be obtained with extended lagging breadth if there is no forward protection and if IP protection (which prevents imitation) is granted for an infinite time. ■

E

Proof. Let us denote the value of an innovation in state $j \in \{F; N\}$ for a firm which has obtained i consecutive innovations by V_{ij}. Taking the fact that entrants expect to become the next incumbent and to do all follow-on R&D into account, the following arbitrage conditions need to hold:

$rV_{1F} = \pi_1 - c\phi^*_{1F} + \phi^*_{1F}V_{2F} - \eta_1 V_{1F} + \eta_1 V_{1N}$ with either $\phi^*_{1F} = 0$ or $\phi^*_{1F} = \phi_m$

$rV_{1N} = \pi_1 - c\phi^*_{1N} + \phi^*_{1N}V_{2F}$ with either $\phi^*_{1N} = \phi^*_{2N}$ (preemptive level) or $\phi^*_{1N} = \phi_m$

$rV_{2F} = \pi_2 - \eta_2 V_{2F} + \eta_2 V_{2N}$

$rV_{2N} = \pi_2 - c\phi^*_{2N} + \phi^*_{2N}V_{3F}$

$rV_{iF} = \pi_2 - \eta_i V_{iF} + \eta_i V_{iN}$ for $i < K$

$rV_{iN} = \pi_2 - c\phi^*_{2N} + \phi^*_{2N}V_{(i+1)F}$ for $i < K$

$r\overline{V}_F = \pi_2 - \overline{\eta}\overline{V}_F + \overline{\eta}\overline{V}_N$ for $i \geq K \geq 2$

$r\overline{V}_N = \pi_2 - c\phi^*_{2N} + \phi^*_{2N}\overline{V}_F$ for $i \geq K \geq 2$

where $\overline{V}_j$ denotes the value of incumbency for the current incumbent who has attained K or more consecutive innovations and for whom the rate of expiration of forward protection in state N is always given by $\overline{\eta}$. Combining the above equations, we can derive the value of an innovation for an entrant V_{1F} as a negative function of η_i[64]and of ϕ^*_{2N}. The preemptive amount ϕ^*_{2N} of R&D that the incumbent undertakes in equilibrium can be derived from the free entry condition $V_{1F} = c$, and therefore depends negatively on η_i. ■

F

Proof. Regime (i): A firm with a one step lead selects R&D effort $\phi_1 = \phi_m$ if $\frac{\partial V_E}{\partial \phi_1} > 0$,

which holds if $\phi^*_2 < \frac{\pi_2}{c} - r - \gamma_2 - \frac{\pi_1(r+\gamma_2)}{c(r+\gamma_1)}$ (**Condition 3**). Given $\phi_1 = \phi_m$, we can solve the free entry condition ($V_E = c$) for ϕ^*_2 to get: $\phi^*_2 = \frac{\pi_2}{c} - \frac{r+\gamma_2}{\phi_m c}(2c\phi_m + cr + c\gamma_1 - \pi_1)$. Plugging this expression into Condition 3 gives the condition $\phi_m(\pi_1 - cr - c\gamma_1) < -(r + \gamma_1)(\pi_1 - cr - c\gamma_1)$, which implies that $\pi_1 - cr - c\gamma_1 < 0$ (**Condition 4**) must hold in order to get $\phi_1 = \phi_m$. Moreover, the condition $\phi_m > \phi^*_2 > 0$ must be satisfied

[64]In case there is a parameter constellation for which $\phi_{1F} = \phi_{1N} = \phi_m > \phi^*_{2N}$, V_{1F} is, however, independent of η_1.

in order to have an equilibrium. $\phi_m > \phi_2^*$ is satisfied if ϕ_m is large enough, and for ϕ_m large, the condition $\phi_2^* > 0$ holds if $\frac{\pi_2}{c} - 2(r + \gamma_2) > 0$ (**Condition 5**). A firm with a one step lead therefore only selects R&D effort $\phi_1 = \phi_m$ if ϕ_m is sufficiently large, if $\gamma_1 > \frac{\pi_1}{c} - r$ (Condition 4) and if $\gamma_2 < \frac{\pi_2}{2c} - r$ (Condition 5).

Regimes (ii) and (iii): Given that $\gamma_1 \leq \frac{\pi_1}{c} - r$ (**Condition 6**), so that Condition 4 does not hold, a firm with a one step lead either undertakes the preemptive R&D effort $\phi_1 = \phi_2^*$ (Regime (ii)) or does not do any R&D ($\phi_1 = 0$, Regime (iii)), in which case there is leapfrogging. In Regime (ii), the value of an innovation for an entrant is given by $V_E^{ii} = \frac{\pi_1 - c\phi_2}{r+\gamma_1+\phi_2} + \frac{\phi_2}{r+\gamma_1+\phi_2}\frac{\pi_2 - c\phi_2}{r+\gamma_2}$ and in Regime (iii) by $V_E^{iii} = \frac{\pi_1}{r+\gamma_1+\phi_2}$, so that $V_E^{ii} > V_E^{iii}$ if $\phi_2 < \frac{\pi_2}{c} - r - \gamma_2$ (**Condition 7**). At the point of indifference where $V_E^{ii} = V_E^{iii}$, the free entry condition $V_E^{iii} = c$ determines the equilibrium innovation rate as $\phi^* = \frac{\pi_1}{c} - r - \gamma_1$ (**8**). Inserting (8) into Condition 7 we find that in order to be in Regime (ii) where a firm with a one step lead finds it optimal to do all the follow-on R&D and to set $\phi_1 = \phi_2^*$, the condition $\gamma_2 \leq \gamma_1 + \frac{\pi_2 - \pi_1}{c}$ must be satisfied. If $\gamma_2 > \gamma_1 + \frac{\pi_2 - \pi_1}{c}$, the economy is in Regime (iii) where the rate of innovation ϕ^* is given by (8). If Condition 6 holds, the equilibrium innovation rates ϕ^* in regime (iii) and ϕ_2^* in Regime (ii) are positive (which can be inferred from (8) and the fact that $V_E^{ii} > V_E^{iii}$ in Regime (ii)). ■

G

Proof. In Regime (i) where $\phi_1^* = \phi_m > \phi_2^* = \phi_0^*$, the arrival rate depends on the size of the lead, which itself changes stochastically over time. In order to calculate the average innovation rate (arrival rate) in this regime, we need to compute for which fraction of the time the economy is in which state (on average). To simplify the intuition, one can also think about a slightly modified version of the model in which there is a continuum of similar quality-good sectors of mass one, and compute in which fraction of the sectors the lead is equal to 0, 1 or 2 steps. Denoting the proportion of time or the fraction of sectors in which the lead is equal to k steps by σ_k, the following conditions need to be satisfied in Regime (i) in order to guarantee that the average entry into state k equals the average exit to other states:

$\sigma_o\phi_2^* = \gamma_1\sigma_1 + \gamma_2\sigma_2 \quad (k = 0)$

$\sigma_1(\phi_m + \gamma_1) = \sigma_0\phi_2^* \quad (k = 1)$

$\sigma_2\gamma_2 = \sigma_1\phi_m \quad (k = 2)$

The left hand sides stand for the exit from the corresponding states k and the right hand sides for the entry into these states. Taking as an example the case where $k = 0$, the average fraction of sectors leaving this state is given by the arrival rate of an innovation in this state ($\phi_0^* = \phi_2^*$) times the fraction of sectors where the state is given by a lead of zero (σ_0). Entry into this state occurs due to the expiration of IP protection in sectors with a lead of one or two steps and is given by the arrival rate of IP expiration in the case of a one step lead times the fraction of sectors with a one step lead ($\gamma_1\sigma_1$), plus the corresponding expression for $k = 2$ ($\gamma_2\sigma_2$). Using the condition $\sigma_0 + \sigma_1 + \sigma_2 = 1$ and these three equations, we can compute:

$$\sigma_0 = \frac{(\phi_m + \gamma_1)\,\gamma_2}{(\phi_m + \gamma_1)\,\gamma_2 + \phi_2^*\,(\gamma_2 + \phi_m)} \tag{9}$$

$$\sigma_1 = \frac{\phi_2^*\gamma_2}{(\phi_m + \gamma_1)\,\gamma_2 + \phi_2^*\,(\gamma_2 + \phi_m)} \tag{10}$$

$$\sigma_2 = \frac{\phi_2^*\phi_m\,(\phi_m + \gamma_1)\,\gamma_2}{(\phi_m + \gamma_1)\,\gamma_2\,((\phi_m + \gamma_1)\,\gamma_2 + \phi_2^*\,(\gamma_2 + \phi_m))} \tag{11}$$

The average arrival rate $\widehat{\phi}_i$ in Regime (i) is now given by the weighted sum of arrival rates in the different states, with the weights given by σ_k:

$$\widehat{\phi}_i = \phi_m\sigma_1 + \phi_2^*(\sigma_0 + \sigma_2) = \frac{\phi_2^*\,(\gamma_2\,(\phi_m + \gamma_1) + \phi_m(\phi_2^* + \gamma_2))}{(\phi_m + \gamma_1)\,\gamma_2 + \phi_2^*\,(\gamma_2 + \phi_m)} \tag{12}$$

The average arrival rates in regimes (ii) and (iii) are simply given by $\widehat{\phi}_{ii} = \phi_2^*$ and $\widehat{\phi}_{iii} = \phi^*$.

Now, the effects of IPR expiration can be analyzed in the different regimes:

Regime (iii): the innovation rate is given as $\phi^* = \frac{\pi_1}{c} - r - \gamma_1$ ((8) from *Appendix F*) and clearly decreases in γ_1.

In Regime (ii), ϕ_2^* also decreases in γ_i, which can be inferred from the free entry condition $V_E^{ii} = c$ as $V_E^{ii} = \frac{\pi_1 - c\phi_2}{r+\gamma_1+\phi_2} + \frac{\phi_2}{r+\gamma_1+\phi_2}\frac{\pi_2 - c\phi_2}{r+\gamma_2}$ decreases in γ_i and in ϕ_2.

For the parameter constellation $\gamma_2 = \gamma_1 + \frac{\pi_2 - \pi_1}{c}$ where the switch between Regimes (ii) and (iii) occurs, we have $V_E^{ii} = V_E^{iii}$, and as in both regimes the innovation rate is not state-dependent and determined by the free entry condition $V_E = c$, there is no discontinuous jump in the innovation rate at this point.

Regime (i): Solving the free entry condition $V_E = c$ for ϕ_2^*, we get: $\phi_2^* = \frac{\pi_2}{c} - \frac{r+\gamma_2}{\phi_m c}(2c\phi_m + cr + c\gamma_1 - \pi_1)$. From this, we obtain $\frac{\partial \phi_2^*}{\partial \gamma_1} < 0$ and $\frac{\partial \phi_2^*}{\partial \gamma_2} < 0$, given that

$cr+c\gamma_1-\pi_1>0$ (Condition 4 from *Appendix F*) holds. The average arrival rate is given by: $\widehat{\phi}=\phi_m\sigma_1+\phi_2^*(\sigma_0+\sigma_2)=\phi_m\sigma_1+\phi_2^*(1-\sigma_1)$. Using $\sigma_1=\frac{\phi_2^*\gamma_2}{(\phi_m+\gamma_1)\gamma_2+\phi_2^*(\gamma_2+\phi_m)}$, we obtain $sign\frac{\partial\sigma_1}{\partial\gamma_1}=sign\left\{\frac{\partial\phi_2^*}{\partial\gamma_1}\left(\phi_m+\gamma_1\right)\gamma_2-\phi_2^*\gamma_2\right\}<0$, so that we get $\frac{\partial\widehat{\phi}}{\partial\gamma_1}=\frac{\partial\sigma_1}{\partial\gamma_1}(\phi_m-\phi_2^*)+\frac{\partial\phi_2^*}{\partial\gamma_1}(1-\sigma_1)<0$.

Deriving the average arrival rate $\widehat{\phi}=\frac{\phi_2^*\left(\gamma_2(\phi_m+\gamma_1)+\phi_m(\phi_2^*+\gamma_2)\right)}{(\phi_m+\gamma_1)\gamma_2+\phi_2^*(\gamma_2+\phi_m)}$ with respect to γ_2 gives $sign\frac{\partial\widehat{\phi}}{\partial\gamma_2}=$
$sign\frac{\partial\phi_2^*}{\partial\gamma_2}\left[\left(\gamma_2\left(\phi_m+\gamma_1\right)+\phi_m(\phi_2^*+\gamma_2)\right)\gamma_2(\phi_m+\gamma_1)\ +\phi_2^*\phi_m\left(\gamma_2(\phi_m+\gamma_1)+\phi_2^*(\gamma_2+\phi_m)\right)\right]+$ $\phi_m(\phi_2^*)^2(\phi_m-\phi_2^*)<0$. This derivative is negative as $\frac{\partial\phi_2^*}{\partial\gamma_2}=-2-\frac{r+\gamma_1-\frac{\pi_1}{c}}{\phi_m}<-2$ under Condition 4. Therefore, setting $\gamma_2=0$ maximizes the average innovation rate. Given that $\gamma_2=0$, an incumbent who has obtained a two step lead never loses it, so that the average innovation rate is simply given by $\widehat{\phi}=\phi_2^*$, as $\sigma_0=\sigma_1=0$. But as ϕ_2^* decreases in γ_1 (because increasing γ_1 reduces the value of an innovation for an entrant V_E in all regimes and therefore the amount of R&D the incumbent needs to undertake in order to preempt entry), the average innovation rate is maximal if $\gamma_1=\gamma_2=0$.

At the switching point between Regimes (i) and (ii) (where Condition 4 holds with equality), there is no discontinuity in the value of an innovation for an entrant and $V_E^i=V_E^{ii}$ holds independently of whether entrants set $\phi_1=\phi_m$ or $\phi_1=\phi_2^*$. Therefore, the innovation rates ϕ_0^* and ϕ_2^* are also the same in both cases. However, the average innovation rate $\widehat{\phi}$ is higher if firms with a one step lead choose $\phi_1=\phi_m$, so that there is a discontinuous increase in $\widehat{\phi}$ when a slight shift in parameters leads so a switch from Regime (ii) to Regime (i). ■

H

Proof. An incumbent with a two step lead does not do any R&D ($\phi_2=0$) as this

would only create costs without increasing profit flows beyond the level π_2. The value of having a two step lead is therefore given by $V_2=\frac{\pi_2}{r+\gamma}$ and the value of having a one step lead by $V_1=\frac{\pi_1-c\phi_1}{r+\gamma+\phi_1}+\frac{\phi_1\frac{\pi_2}{r+\gamma}}{r+\gamma+\phi_1}$. An incumbent with a one step lead faces no entry pressure and sets its R&D level equal to $\phi_1^*=\phi_m$ if $\gamma\leq\frac{\pi_2-\pi_1}{c}-r$ (as then $\frac{\partial V_1}{\partial\phi_1}\geq0$) and does not do any R&D ($\phi_1^*=0$) if $\gamma>\frac{\pi_2-\pi_1}{c}-r$. In the case where IPRs have expired and where the currently newest good is in the public domain, the R&D incentives depend on V_1, the value of obtaining a one step lead. If $V_1>c$, the marginal benefits of doing R&D exceed the marginal costs, and at least one firm[65] finds it profitable to do R&D, so that the innovation rate is given by $\phi_0^* = \phi_m$. If $\gamma \leq \frac{\pi_2 - \pi_1}{c} - r$ (Case A), $V_1 = \frac{\pi_1 - c\phi_m}{r+\gamma+\phi_m} + \frac{\phi_m \frac{\pi_2}{r+\gamma}}{r+\gamma+\phi_m} > c$ holds due to the Arrow replacement effect ($\pi_2 < 2\pi_1$) and we have $\phi_0^* = \phi_1^* = \phi_m$. If $\frac{\pi_1}{c} - r \geq \gamma > \frac{\pi_2-\pi_1}{c} - r$ (Case B), we get $\phi_1^* = 0$ and $\phi_0^* = \phi_m$, so that incumbents with a one step lead do not do R&D, while firms undertake R&D if IPRs have expired. If $\gamma > \frac{\pi_1}{c} - r$ (Case C), IP protection is so weak that no firm does R&D even if the newest version of the good is in the public domain, so that $\phi_0^* = \phi_1^* = 0$. Denoting the probability to be in a state with a k-step lead by σ_k, the average arrival rate of an innovation is given by $\widehat{\phi} = \sigma_0\phi_0 + \sigma_1\phi_1 + \sigma_2\phi_2$. It can be derived from the equations below, in which the expected inflow into state k (left hand sides) is equal to the expected outflow (right hand sides):

$k = 0$: $\gamma(\sigma_1 + \sigma_2) = \sigma_0\phi_0^*$

$k = 1$: $\sigma_0\phi_0^* = \sigma_1\phi_1^* + \sigma_1\gamma$

$k = 2$: $\sigma_1\phi_1^* = \sigma_2\gamma = (1 - \sigma_0 - \sigma_1)\gamma$

Solving the system of equations for all σ_k, we obtain $\widehat{\phi}_a = \frac{\phi_m\left(\gamma^2+2\gamma\phi_m\right)}{(\phi_m+\gamma)^2}$ in Case A and $\widehat{\phi}_b = \frac{\phi_m\gamma}{\phi_m+\gamma}$ in Case B. In both cases, $\widehat{\phi}$ increases in γ, so that $\widehat{\phi}$ is maximal for the maximal level of γ that still lies within the admissible parameter range, that means for $\gamma_a = \frac{\pi_2-\pi_1}{c} - r > 0$ in Case A and for $\gamma_b = \frac{\pi_1}{c} - r > \gamma_a$ in Case B. $\widehat{\phi}_a(\gamma_a) > (<)\, \widehat{\phi}_b(\gamma_b)$ if $\phi_m(2\gamma_a - \gamma_b) + \gamma_a^2 > (<)\, 0$, that means if $\phi_m(\left(\frac{2\pi_2-3\pi_1}{c}\right) - r) + \left(\frac{\pi_2-\pi_1}{c} - r\right)^2 > (<) 0$. ■

[65] As marginal R&D costs go to infinity if $\phi = \phi_m$, average R&D costs rise in total R&D spending and there is entry up to the point where these average costs are equal to V_1.

Even though there is the upper bound on the total arrival rate ϕ_m, the free entry condition can be satisfied with equality if firms in the aggregate undertake more R&D than necessary to obtain ϕ_m, so that average costs increase above c. This can happen if the individual probability of obtaining an IPR (patent) depends on the ratio between individual and total R&D spending.

Appendix 2: Increasing profit flows

This section studies a more general setup in which even firms with a two step lead can increase their profits by conducting R&D. Like in Denicolò (2001), the analysis is based on a one-sector version of the growth model of Barro and Sala-i-Martin (1995, Chapter 7).

11 Model setup and equilibrium

The economy is populated by identical individuals of mass one who inelastically supply one unit of labour. Intertemporal preferences are given by $U(\tau) = \int_{t=\tau}^{\infty} c(t)\, e^{-\rho(t-\tau)} dt$ with $c(t)$ denoting consumption in period t. There is a final good y which can be consumed, used for research or used one-for-one to produce intermediate goods x_i, of which there exist generations $i \in \{1, ..., k\}$. The final good is produced using labour (which is in fixed supply) and intermediate goods according to the following production function: $y_k = X_k^{\alpha}$, with $X_k = \sum_{i=0}^{k} q^i x_i$ and $0 < \alpha < 1$. q^i indicates the quality of the intermediate good of generation i, and innovation allows the introduction of new intermediate goods, the quality of which is increased by the factor $q > 1$ compared to the previous generation.

The final good sector is assumed to be competitive, while intermediate goods can be protected by IPRs. Since different generations of intermediate goods are perfect substitutes, only the best quality (the newest generation) is used in equilibrium, so that the final good production function reduces to $y_k = q^{k\alpha} x_k^{\alpha}$. In equilibrium, the rate of interest r coincides with the rate of time preference ρ. Normalizing the price of the final good to one, the demand for the latest generation of the intermediate good as a function of its price p_k can be derived as

$$x_k = \alpha^{\frac{1}{1-\alpha}} q^{\frac{k\alpha}{1-\alpha}} p_k^{-\frac{1}{1-\alpha}} \tag{13}$$

In a stationary equilibrium, p_k is constant and from (13) and the production function it follows that the growth factor between two innovations in terms of the final good is given by $g \equiv \frac{y_{k+1}}{y_k} = q^{\frac{\alpha}{1-\alpha}}$.

Given the newest innovation (generation k of the intermediate good) is protected by an intellectual property right, its price is set in order to maximize profits $\pi_k = (p_k - 1)\, x_k$. If innovations are drastic or if the lead of the leading firm is so large (due to successive innovations or collusive agreements with the closest competitors) that no competitor can profitably underprice her, the (unconstrained) monopoly price, which is given by $p^M = \frac{1}{\alpha}$, is charged. Monopoly profits are then given as:

$$\pi_k^M = \alpha^{\frac{2}{1-\alpha}} \left(\frac{1}{\alpha} - 1 \right) q^{\frac{k\alpha}{1-\alpha}} \tag{14}$$

which can also be written as $\pi_k^M = \pi^M g^k$ with $\pi^M \equiv \alpha^{\frac{1+\alpha}{1-\alpha}} (1-\alpha)$.

Innovations are therefore drastic if $q \geq \frac{1}{\alpha}$. In the case where innovations are non- drastic ($q < \frac{1}{\alpha}$) and the last generation of the quality good is available to a competitor, there is limit pricing and the leader charges a price equal to its quality advantage $p^c = q < p^M$ in order to keep competitors out of the market. In this case profits are given by

$$\pi_k^C = \alpha^{\frac{1}{1-\alpha}} (q-1)\, q^{\frac{k\alpha}{1-\alpha}} q^{-\frac{1}{1-\alpha}} \tag{15}$$

which can also be written as $\pi_k^C = \pi^C g^k$ where $\pi^C \equiv \alpha^{\frac{1}{1-\alpha}} (q-1)\, q^{-\frac{1}{1-\alpha}}$.

In the following, it is assumed that $\alpha q^2 \geq 1 \geq \alpha q$ (**Assumption 3**), which implies that each single innovation is non-drastic, but that a two step lead allows the leader to charge the unconstrained monopoly price. π_k^C and π_k^M can therefore be interpreted as profits of firms with a one step and a two (or more) step lead.[66] Compared to the simple model used in the article, the main difference is therefore that these profits grow by the factor g when an innovation takes place.

R&D can be undertaken by using the final good as an input. The arrival rate of the $k+1$ th innovation is given by $\phi(k+1) = \min\left\{ \left(\frac{n}{cg^k} \right)^{\frac{1}{1+\epsilon}}, \phi_m \right\}$, where n denotes the total amount of the final good used in the R&D sector and $\epsilon \geq 0$. ϕ_m again sets an upper bound which the arrival rate cannot surpass due to technological reasons. The difference of this specification compared to that in the simple model is that R&D costs $n(\phi)$ increase from generation to generation. This assumption is needed to offset the effect of increasing profit flows and to obtain a balanced growth path with a constant innovation arrival rate. It is again assumed that all firms have access to the same R&D technology and that there is free entry into the R&D sector. In the following part of the analysis it is assumed that an innovator obtains IP protection of infinite duration that prevents others from producing her generation of the good, and that there is no

[66] If a price cap $q < \overline{p} < \frac{1}{\alpha}$ is imposed, profits of firms with a two- (or more-) step lead are decreased, but $\pi_k^C < \pi_k^M$ still holds.

forward protection and that voluntary deals like collusion are not permitted.

As profit flows grow with each innovation, even incumbents might gain from innovating in this setup, and it is not as clear as in the simple model whether entrants or incumbents have the larger stand-alone innovation incentives.

Lemma 3 *Given Assumption 3, incremental (stand-alone) profit flows of incumbents with a two step lead are lower than those for entrants and incumbents with a one step lead (Arrow replacement effect).*
Proof. *See Appendix X (at the end of this section)* ■

As R&D productivity is decreasing at the industry level ($\epsilon \geq 0$), incumbents can again preempt entry, and as they value not being replaced more than entrants value entry, they do all the R&D, so that there is again persistent leadership. Due to the Arrow replacement effect, incumbents with a two step lead only conduct as much R&D as necessary to prevent entry, so that the equilibrium innovation rate is again determined by a free entry condition and depends on the value of an innovation for an entrant expecting to become the new incumbent after entry.

For reasons of tractability, the analysis again focuses on the limit case where $\epsilon \to 0$. The expected value $V_2(k)$ of having a two step lead and supplying generation k of the intermediate good can now be derived from the following arbitrage condition:

$rV_2(k) = \pi^M g^k - \phi_2 c g^k - \phi_2 V_2(k) + \phi_2 V_2(k+1)$

The last terms on the right hand side indicate that, if an innovation occurs with the arrival rate ϕ_2, the firm stops supplying generation k and starts supplying generation $k+1$ of the good. Taking into account that $V_2(k+1) = gV_2(k)$ along a balanced growth path, this condition can be rewritten as

$$V_2(k) = g^k \frac{\pi^M - \phi_2 c}{r - \phi_2 (g-1)} \tag{16}$$

A balanced growth path, however, only exists if the denominator is positive, which is the case if $r > \phi_2^*(g-1)$ (**Condition 9**). The expected value $V_1(k)$ of getting a one step lead and supplying generation k of the intermediate good (which coincides with the value of an innovation for an entrant $V_E(k)$) can be derived from the arbitrage condition:

$rV_1(k) = \pi^C g^k - \phi_1 c g^k - \phi_1 V_1(k) + \phi_1 V_2(k+1)$

Inserting $V_2(k+1) = g^{k+1}\frac{\pi^M - \phi_2 c}{r-\phi_2(g-1)}$ from (16), we can solve for

$$V_1(k) = g^k \left(\frac{\pi^C - \phi_1 c}{r + \phi_1} + \frac{\phi_1}{r+\phi_1} \frac{g\left(\pi^M - \phi_2 c\right)}{r - \phi_2\left(g-1\right)} \right) \tag{17}$$

The terms ϕ_1 and ϕ_2 again denote the innovation arrival rates chosen by firms with a one and a two step lead, which turn out to be independent of k. As the value that the invention of the k th generation of the intermediate good has for an entrant is given by $V_1(k)$, and as the innovation costs in the case where the current version of the intermediate good has quality $k-1$ are given by $n = cg^{k-1}\phi$, the free entry condition is given by $V_1(k) = cg^{k-1}$, which can be written as:

$$\frac{\pi^C - \phi_1^* c}{r + \phi_1^*} + \frac{\phi_1^*}{r+\phi_1^*} \frac{g\left(\pi^M - \phi_2^* c\right)}{r - \phi_2^*\left(g-1\right)} = \frac{c}{g} \tag{18}$$

This condition is similar to that obtained in the simple model, with the difference that there is the growth factor g (an increase in g increases equilibrium growth) and that π_1 and π_2 are replaced by π^C and π^M. In order to determine the value of ϕ_1^*, we again need to determine how the value of an innovation for an entrant $V_1(k)$ depends on ϕ_1. Deriving (17), we find that $sign\frac{\partial V_1(k)}{\partial \phi_1} = sign\left\{\phi_2^*\pi^C(g-1) - \pi^C r + gr\pi^M - cr^2 - cr\phi_2^*\right\}$, which is independent of ϕ_1. Like in the simple model, we therefore have $\phi_1^* = \phi_m$ if $sign\frac{\partial V_1(k)}{\partial \phi_1} > 0$ and $\phi_1^* = \phi_2^*$ if $sign\frac{\partial V_1(k)}{\partial \phi_1} \leq 0$. The equilibrium rate of growth ϕ_2^* can then be determined from the free entry condition (18) once the appropriate value of ϕ_1^* is inserted.

12 The effects of IP policies

As there is persistent leadership and as incumbents only do as much R&D as needed to preempt entry, growth depends positively on entry pressure, i.e. on the value that an innovation has for an entrant. As in the simple model, the value of an innovation for an entrant increases if the profits of firms with a one step lead ($\pi^C g^k$) increase. Therefore, allowing **voluntary deals** that permit entrants to consolidate market power with previous incumbents and to avoid the phase of competition again increases growth.

Requiring entrants to pay **licensing fees** upon entry, in order to compensate previous incumbents, reduces the value of an innovation for an entrant and growth, like in the simple model, as entrants expect to become the next leaders and never to receive any licensing payments from others in the future.[67]

The analysis of **forward protection** in this setup is also very similar, but now collusion allows to increase joint profits by using the entrants' superior production technology. If an entrant has invented generation $k+1$ of the quality good, the incumbent can still obtain the previous profit flows $\pi^M g^k$ if she does not permit the entrant to produce, but if she negotiates with the entrant in order to obtain access to the new technology, joint profit flows can be increased to $\pi^M g^{k+1}$. If collusion does not allow coordinating joint R&D expenditures, forward protection again reduces growth. The reason for this is that the entrant can maximally (if he has all the bargaining power) get the additional profits $(g^{k+1} - g^k)\pi^M$ resulting from his innovation until he advances to a two step lead. But these additional profits are, due to the Arrow replacement effect, lower than the profit $\pi^C g^{k+1}$ which he would get without forward protection in the phase of competition.[68] Forward protection therefore decreases the value of an innovation for an entrant and equilibrium growth.[69]

As increasing π^C increases the value of an innovation for an entrant, entry pressure is maximal if entrants get the maximal possible profit flows $\pi^M g^k$ immediately upon entry. And this can again be obtained through **extended lagging breadth**. Therefore (not considering the possibility of forward protection of limited duration) growth is again maximized if there is extended lagging breadth.[70]

[67]Even if there was leapfrogging (because incumbents do not have ideas for R&D), it is not possible to increase innovation incentives by making licensing fees grow sufficiently over time. The reason for this is that licensing fees can maximally grow in line with profits (otherwise there would be a Ponzi game), but that the interest rate in equilibrium has to be at least as large as this expected rate of profit growth, so that discounted profits cannot increase if licensing fees are introduced.

[68]It is again assumed that collusion is not feasible or allowed if there is no forward protection.

[69]It is important to note that the result that innovation incentives are increased if a larger share of total profits is allocated to entrants at the expense of previous innovators depends on the assumption made in quality-ladder models that each innovation builds on a previous innovation. If there is, however, an initial product innovation, on which all following innovations build, the incentives to come up with this initial innovation clearly decrease if follow-on innovators can easily replace the initial innovator without any compensation. There is therefore a trade-off for patent policy in such a case, and encouraging initial R&D (by requiring entrants to compensate previous inventors with licensing fees, by granting forward protection or by forbidding ex post collusion) comes at the cost of reducing follow-on R&D (see Chu, Cozzi and Galli (2012) and also Denicolò (2002)).

[70]Llanes and Trento (forthcoming) analyze a product-variety setting in which new innovations build on the knowledge incorporated in all previously invented goods. They show that, for a given level of appropriability in the final goods markets, a transfer system which grants a subsidy to each newly entering innovator, and taxes the previously newest innovator, grants the optimal innovation incentives. Even in this different setting, making older innovations infringe on the newest one could be a way to implement such a transfer system. In order to reduce transaction costs associated with licensing and to grant the maximal possible profits to entrants, one could also more directly

In quality ladder models like the one studied here, an increase in the rate of growth might, however, not always be **welfare**-improving, as equilibrium growth might be excessive for certain parameter values[71]. However, this does not imply that a good way to reduce growth is to make profit flows more backloaded or to discourage entry by reducing π^C if the policy maker can instead impose a price cap equal to $\overline{p} < \frac{1}{\alpha}$ on incumbents.[72] Such a price cap decreases profits $\pi^M g^k$ of firms with a two step lead and therefore also the value of an innovation for an entrant and the rate of growth, but at the same time it reduces monopoly distortions stemming from IP protection, which is not the case if entry pressure is reduced by reducing π^C, or in another way which does not affect π^M. As there is persistent leadership, intermediate goods are only supplied by incumbents with a two (or more) step lead, so that monopoly distortions only depend on the price charged by firms with a two step lead and not on the hypothetical price that would arise in a phase of competition where a firm has a one step lead. Reducing the price and profits π^C of firms with a one step lead therefore only reduces entry pressure and growth without reducing monopoly distortions. We can therefore state:

Proposition 8 *In the case where innovation is excessive, it is better (in terms of social welfare) to reduce innovation incentives by limiting the maximal price $\overline{p}$ that firms with the maximal lead can charge than by reducing entry pressure in another way (like by reducing π_1, imposing entry fees etc.)*

Proof. See Appendix Y ■

"expropriate" previous innovators by allocating their patents (that generate profits by preventing imitation of a certain final good) to the most recent inventor. Such policies would, however, only work (contrary to the case of subsidies) if there is an initial stock of patents the profits of which can be allocated to the first innovator without the need to compensate the initial owners (the same holds true in the quality-ladder case).

[71] If an increase in growth is obtained at the cost of a decrease in current consumption arising from larger monopoly distortions, welfare might rise or fall. Denicolò (2001) gives numerical examples for both cases.

In the case where $\epsilon > 0$, there is a further effect (beside the "business stealing effect" that can arise in quality ladder models) through which overall innovation incentives might become excessive: if R&D productivity decreases at the industry level and there is free entry, firms do not take the fact that their R&D activity also increases the costs of other firms into account, so that more R&D is undertaken than in the case where entrants maximize their joint profits. Put differently, entry occurs until the value of an innovation is equal to the average and not the marginal costs of innovating, and this condition is satisfied for a larger overall amount of R&D. This effect is the same as that analyzed in the well-known problem of the "tragedy of the commons".

[72] This could also be done through patent policy, by allowing a competitive fringe to produce copied versions of the good that have higher production costs or a lower quality. Incumbents would then have to cut their price in order to prevent entry.

It is therefore optimal to make profit flows as frontloaded as possible and to set π^C equal to π^M like in the case of extended lagging breadth.[73]

13 Minimal inventive step

Let us now assume that the amount of the final good n_i that a firm i needs to use as a research input in order to improve the quality $\overline{q}$ of the current generation of the intermediate good by the factor μ_i (> 1) and to obtain the arrival rate ϕ_i, is given by $n_i = c\overline{q}^{\frac{\alpha}{1-\alpha}}\phi_i\lambda(\mu_i)(n_{tot})^\epsilon$, with $\frac{\partial\lambda(\mu)}{\partial\mu} > 0$ and with $n_{tot} = n_i + n_{-i}$ indicating the overall (industry-wide) amount of the final good which is used in R&D. The only difference between this specification and that in the simple model is that R&D costs are assumed to grow at the same rate as output (note that $y = K\overline{q}^{\frac{\alpha}{1-\alpha}}$ with K constant) in order to obtain a balanced growth path. Again, all that matters for preemption is therefore the total amount of resources used for R&D. As long as the incumbent values not being replaced more than entrants value entry, she therefore preempts entry, and if her stand-alone innovation incentives are lower than those of entrants, she just does as much R&D as needed to preempt entry. In this case, entry pressure and growth can again be increased if the non-obviousness requirement for entrants and incumbents with a one step lead is relaxed (if it was binding before). As *Lemma 3* proves the existence of the Arrow replacement effect if innovative steps are equal for entrants and incumbents (with the notation $\mu = q$), we know that there is preemption and that incumbents conduct all R&D in equilibrium if both entrants and incumbents (with a two step lead) face the same binding non-obviousness requirement. We can therefore state:

Proposition 9 *Suppose the same binding non-obviousness requirement is imposed on entrants and incumbents. Growth can then be increased if the non-obviousness requirement for entrants and/or incumbents with less than the maximal lead is slightly relaxed.*

It is, however, not so clear whether there is still persistent leadership if a considerably stronger non-obviousness requirement is imposed on incumbents than on entrants, as this might render preemption too costly (and even more so, if voluntary deals are

[73]The possibility of introducing forward protection of limited duration in order to achieve an even higher average rate of growth is not considered here. If it was introduced, full frontloading would, however, still be optimal in the case where forward protection has expired (the arguments being the same as in the case of the simple model).

possible after entry occurs). If incumbents tend to pursue smaller inventive steps than entrants, it is also possible that a (weak) non-obviousness requirement only binds for incumbents but not for entrants, so that relaxing it for entrants might not lead to increased entry pressure.

14 Perfect preemption

If a monopolist can preempt entry without innovating herself for the reasons discussed in *sections 8.1.1* and *8.1.2*, and if the entire stock of IPRs expires with hazard rate γ, the effects that IP protection has on growth are again similar to those it has on the innovation rate in the basic model:

Proposition 10 *Under the conditions* $\frac{\pi^M(g-1)}{r} < c$ *and* $\pi^c g > c(r+\gamma)$*, the equilibrium innovation rates* ϕ_i^* *in the case of an i step lead are given by* $\phi_0^* = \phi_1^* = \phi_m$ *and* $\phi_2^* = 0$ *if* $\pi^M g > \pi^c + c(r+\gamma)$*, while they are given by* $\phi_0^* = \phi_m$ *and* $\phi_1^* = \phi_2^* = 0$ *if* $\pi^M g < \pi^c + c(r+\gamma)$*. In both cases, average growth* $\widehat{\phi}$ *is maximal for an intermediate strength of IP protection* $0 < \gamma < \infty$*.*
Given that $\frac{\pi^M(g-1)}{r+\gamma} > c$ *and* $\phi_m < \frac{r+\gamma}{g-1}$*, we obtain* $\phi_0^* = \phi_1^* = \phi_2^* = \phi_m = \widehat{\phi}$
Proof. *See Appendix Z* ■

The only difference to the simple model is therefore that there is a parameter range ($\frac{\pi^M(g-1)}{r+\gamma} > c$) in which the firm keeps innovating even if it has surpassed a two step lead, and in which the innovation rate is always at its maximal level ϕ_m.

Let us now look at the **case where innovations are drastic** (i.e. where $q > \frac{1}{\alpha}$) and where there is only a single firm capable of doing R&D. If the IPR on the newest generation of the good is protected, this firm is a monopolist, charges the unconstrained monopoly price and makes profits $\pi^M g^k$. Such a monopolist does not find it profitable to do follow-on R&D if $\frac{\pi^M(g-1)}{r+\gamma} < c$ (see *Appendix Z*). If, however, $\frac{\pi^M}{r+\gamma} > c$, the firm wants to innovate, given that IP protection on the currently newest generation has expired. Therefore, average growth can again be maximal for an intermediate strength of IP protection. Given that $\frac{\pi^M(g-1)}{r} < c$, innovation only takes place if IPRs have expired, so that average growth is maximal if $\gamma = \frac{\pi^M}{c} - r$, as this strength still guarantees that firms find it profitable to do the maximal amount $\phi = \phi_m$ of R&D when IPRs have expired while it maximizes the probability that the economy is in this state.[74]

[74]While this article only analyzes the effects of protecting innovators from direct imitation (copying) by competitors, it does not address the more general question of the optimal breadth of IP protection

Appendix X

Proof. If an incumbent with a two step lead innovates, she obtains profit flows $\pi^M g^{k+1}$ instead of the previous profit flows that were given by $\pi^M g^k$. Incumbents with a one step lead also obtain profit flows $\pi^M g^{k+1}$ if they innovate, but these replace the lower one step lead profit flows that were given by $\pi^C g^k$. Therefore, incumbents with a one step lead have larger stand-alone innovation incentives than incumbents with a two step lead (as $\pi^M(g^{k+1} - g^k) < \pi^M g^{k+1} - \pi^C g^k$ due to the fact that $\pi^M > \pi^C$). Entrants gain $\pi^C g^{k+1}$ if they innovate (and do not lose any previous profits) and their stand-alone innovation incentives are larger than those of incumbents with a two step lead if $\pi^C g^{k+1} > \pi^M(g^{k+1} - g^k)$, that means if $\frac{\pi^M}{\pi^C} < \frac{g}{g-1}$ (**Condition 10**). It is now shown that this condition holds under Assumption 3 ($\alpha q^2 \geq 1 \geq \alpha q$), so that there is again the well-known Arrow replacement effect:

Replacing π^M, π^C and g by their values, Condition 10 implies that $\alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha) < \frac{q-1}{q\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}$ (i). This inequality is satisfied for the largest feasible value of q which is given by $q_{\max} = \frac{1}{\alpha}$ due to Assumption 3. If the right hand side (RHS) of (i) decreases in q, (i) therefore always holds. $\frac{\partial RHS}{\partial q} < 0$ holds if $1 - \alpha > q^{\frac{\alpha}{1-\alpha}} - \alpha q^{\frac{1}{1-\alpha}} \equiv R$ (ii). We have $\frac{\partial R}{\partial q} < 0$ so that (ii) always holds if it holds for the smallest feasible value of q which is given by $q_{\min} = \frac{1}{\sqrt{\alpha}}$ (due to Assumption 3). Inserting $q = q_{\min}$ into (ii) gives the inequality $\alpha + \alpha^{-\frac{\alpha}{2(1-\alpha)}} - \alpha^{\frac{1-2\alpha}{2(1-\alpha)}} < 1$ (iii). This condition (iii) is satisfied with equality if $\alpha = 1$. Using simulations it can be shown that (iii) is strictly satisfied if $0 < \alpha < 1$. Therefore, $\frac{\pi^M}{\pi^C} < \frac{g}{g-1}$ holds given that $\alpha q^2 \geq 1 \geq \alpha q$ and $0 < \alpha < 1$. ■

Appendix Y

Proof. Denoting the current generation of the quality good by k, consumption is given by $c(k) = y_k - x_k - cg^k\phi^*$, where the last term stands for the R&D spending in terms of the final good. Inserting $y_k = q^{k\alpha}x_k^\alpha$ and $x_k = \alpha^{\frac{1}{1-\alpha}}q^{\frac{k\alpha}{1-\alpha}}p^{-\frac{1}{1-\alpha}}$, we obtain $c(k) = g^k\left(\alpha^{\frac{\alpha}{1-\alpha}}p^{-\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}p^{-\frac{1}{1-\alpha}} - c\phi^*\right)$, which can be shown to decrease in p (given

if substitutable but differentiated goods can be invented. While granting IP protection on such goods tends to increase the incentives to invent them or to increase their quality, it creates direct competitors for the initial good and reduces the profits that inventors can earn if they improve the quality of this initial good. If two differentiated goods are very substitutable so that consumers do not benefit much from consuming both of them instead of only one, it is clearly more efficient if R&D is only undertaken to (continuously) increase the quality of one of the two goods and not of both goods at the same time. The costly invention of closely substitutable goods that do not have a higher quality than an existing good could be discouraged by making these inventions infringe on the IPR of the newest generation of the existing good.

that $p > 1$).

Along a balanced growth path[75], consumption, the input of intermediate goods and R&D investment all grow by the factor g when an innovation occurs, and expected intertemporal utility is given by $W(k) = g^k \frac{c(k)}{\rho - \phi^*(g-1)}$. The equilibrium rate of growth ϕ^* can be increased if either π^c or π^M, which is a positive function of p, are increased. If a given rate of growth can be obtained with different combinations of π^c and π^M, it is therefore always welfare-improving to choose the combination for which π^C is maximal and for which π^M and therefore p are as low as possible, as this minimizes the deadweight losses associated with IP policy and maximizes consumption and intertemporal utility, given the targeted rate of innovation. Entry pressure should therefore be kept as high as possible to push incumbents to innovate, and if the equilibrium rate of growth turns out to be excessive, it should be reduced by decreasing the market power of incumbents, but not by making entry more difficult. ■

Appendix Z

Proof. The value of a monopolist who produces generation k of the good, is two steps ahead, and innovates at the rate ϕ_2, can be determined from the arbitrage condition

$rV_2(k) = \pi^M g^k - \phi_2 c g^k - \phi_2 V_2(k) + \phi_2 V_2(k+1) - \gamma V_2(k)$

and is therefore given by

$$V_2(k) = \frac{\pi^M g^k - \phi_2 c g^k + \phi_2 V_2(k+1)}{r + \phi_2 + \gamma} \tag{19}$$

Taking $V_2(k+1)$ as given, the innovation rate ϕ_2^* which maximizes $V_2(k)$ is given by

$$\phi_2^* = \begin{cases} \phi_m \text{ if } V_2(k+1) > g^k \left(\frac{\pi^M}{r+\gamma} + c \right) \\ \in [0; \phi_m] \text{ if } V_2(k+1) = g^k \left(\frac{\pi^M}{r+\gamma} + c \right) \\ 0 \text{ if } V_2(k+1) < g^k \left(\frac{\pi^M}{r+\gamma} + c \right) \end{cases} \tag{20}$$

Assuming that the firm chooses innovation rate $\widetilde{\phi}_2$ after obtaining innovation $k+1$, we can use (19) and the fact that $V_2(k+1) = gV_2(k)$ along a balanced growth path,

[75] In this model, there are no transition dynamics and the economy jumps to a new steady state immediately, so that the welfare analysis boils down to a comparison of steady state utilities.

to derive

$$V_2(k+1) = \frac{\pi^M g^{k+1} - \widetilde{\phi}_2 c g^{k+1}}{r + \widetilde{\phi}_2(1-g) + \gamma} \tag{21}$$

where $\widetilde{\phi}_2 < \frac{r+\gamma}{g-1}$ needs to hold (**Condition 9a**)

Inserting (21) into (20), we obtain that for any $\widetilde{\phi}_2$,

$$\phi_2^* = \begin{cases} \phi_m \text{ if } \frac{\pi^M(g-1)}{r+\gamma} > c \\ \in [0;\phi_m] \text{ if } \frac{\pi^M(g-1)}{r+\gamma} = c \\ 0 \text{ if } \frac{\pi^M(g-1)}{r+\gamma} < c \end{cases} \tag{22}$$

so that (22) determines the innovation rate which a monopolist with a two step lead chooses for any quality level k in order to maximize expected profits. If $\frac{\pi^M(g-1)}{r+\gamma} > c$ and $\phi_m < \frac{r+\gamma}{g-1}$ (Condition 9a), the equilibrium innovation rate in the case of an i step lead is therefore always given by $\phi_i^* = \phi_m$, as incentives to innovate are even higher in the case of a zero or one step lead than in the case of a two step lead according to *Lemma 3.*

Let us now assume that $\frac{\pi^M(g-1)}{r} < c$ (**Condition 11**), so that a monopolist who can preempt entry without doing R&D herself actually finds it optimal not to do any R&D once she has reached a two step lead, even if IPRs are fully protected ($\gamma = 0$). The value of having a one step lead is then given by $V_1(k) = \frac{\pi^c g^k - \phi_1 c g^k + \phi_1 V_2(k+1)}{r+\phi_1+\gamma}$, with $V_2(k+1) = \frac{\pi^M g^{k+1}}{r+\gamma}$. A monopolist with a one step lead finds it profitable to do R&D and sets $\phi_1^* = \phi_m$ if $\frac{\partial V_1(k)}{\partial \phi_1} > 0$, which is the case if $\pi^M g > \pi^c + c\,(r+\gamma)$ (**Condition 12**). Given IPRs have expired and no firm has a lead over its rivals, firms always find it profitable to undertake R&D if $\pi^c g > c\,(r+\gamma)$ (**Condition 13**), as this condition ensures that R&D pays off even if a firm stops to innovate after having obtained a one step lead. Condition 11 is compatible with Condition 12 because $\pi^M(g-1) < cr < c(r+\gamma) < \pi^M g - \pi^c$ (as $\pi^M > \pi^c$) and it is also compatible with Condition 13 as $\pi^M(g-1) < cr < c(r+\gamma) < \pi^c g$ due to Condition 10 $\left(\frac{\pi^M}{\pi^c} < \frac{g}{g-1}\right)$. If parameters are such that Conditions 11, 12, and 13 hold, the monopolist therefore does not do any R&D when she has a two step lead, while the maximal amount of R&D is undertaken in the case of a one step lead, and also if IPRs have expired and no firm has a lead. If IPRs are fully protected ($\gamma = 0$), long run growth is therefore zero, as the monopolist stops innovating after having reached a two step lead. Reducing IP protection (increasing γ) up to the level that is still compatible with Conditions 12 and 13 therefore increases average growth as it increases the probability of being in a state where IPRs have expired or where there is a one step lead and in which innovation is maximal. Even if only Conditions 11 and 13 hold, so that the monopolist stops innovating after having reached a one step lead, the reasoning is the same and average growth is maximal for an intermediate strength of IP protection (the maximal γ that is compatible with Condition 13). ■