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The Missing Link:
Unifying Risk Taking and Time Discounting

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Abstract

Almost all important decisions in people's lives entail risky and delayed consequences. Regardless of whether we make choices involving health, wealth, love or education, almost every choice involves costs and benefits that are uncertain and materialize over time. Because risk and delay often arise simultaneously, theories of decision making should be capable of explaining how behavior under risk and over time interacts. There is, in fact, a growing body of evidence indicating important interactions between behaviorally revealed risk tolerance and patience. Risk taking behavior is delay dependent, and time discounting is risk dependent. Here we show that the inherent uncertainty of future events conjointly with people's proneness to weight probabilities nonlinearly generates a unifying framework for explaining time-dependent risk taking, risk-dependent time discounting, preferences for late resolution of uncertainty, and several other puzzling interaction effects between risk and time.

JEL: D01, D81, D91
Keywords: Risk Taking, Time Discounting, Probability Weighting, Decreasing Impatience, Increasing Risk Tolerance, Preference for Late Resolution of Uncertainty, Preference for One-Shot Resolution of Uncertainty

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1 Introduction

Whatever we plan for the future, may it concern health, wealth, love or education, the consequences of our decisions are almost always uncertain and usually take time to materialize. Therefore, both our risk preferences and our time preferences are important drivers of our choices. In economics, risk preferences are conventionally assumed to be unaffected by the passage of time and time preferences to be unaffected by the presence of risk, even though some authors have pointed out parallels between the two domains (Prelec and Loewenstein, 1991; Quiggin and Horowitz, 1995). In contrast to this view, there is mounting evidence of complex interactions between behavior under risk and behavior over time that challenges the standard models of risk taking and time discounting. A new line of theoretical research, abandoning separability of risk and time, developed more realistic models that capture some aspects of the observed regularities (Halevy, 2008; Walther, 2010; Baucells and Heukamp, 2012). However, none of these approaches has identified all the interaction effects as manifestations of a single driving force, which is the objective of this paper.

Table 1 provides a summary of seven empirical facts, organized along five dimensions: First, revealed risk tolerance and revealed patience are delay dependent. Second, risk taking and time discounting behavior is also process dependent, i.e. it makes a difference whether a future prospect is evaluated in one shot or sequentially over the course of time. Third, the timing of uncertainty resolution affects risk taking behavior and, fourth, the presence of risk influences time discounting. Finally, people's evaluations of future risky payoffs depend on the sequence in which they are discounted for risk and for time.

Turning to the first phenomenon, delay dependence of risk taking behavior, Table 1 indicates that risk tolerance appears not to be a stable characteristic of people's behavior. Rather, it is higher for payoffs materializing in the future than for payoffs materializing in the present (Shelley, 1994; Ahlbrecht and Weber, 1997; Sagristano, Trope, and Liberman, 2002; Noussair and Wu, 2006; Coble and Lusk, 2010; Abdellaoui, Diecidue, and Öncüler, 2011). This finding may have far-reaching implications: If people are willing to tolerate high risks for events in the remote future, they may be reluctant to support policies combating global warming or to buy insurance for natural

Table 1: Seven Facts on Risk Taking and Time Discounting

Dimension	Fact	Revealed risk tolerance	Fact	Revealed patience
Delay dependence	#1	increases with delay	#2	increases with delay
Process dependence	#3	higher for one-shot valuation	#4	higher for one-shot valuation
Timing dependence	#5	higher for late uncertainty resolution		–
Risk dependence		–	#6	higher for risky payoffs
Sequence dependence	#7	depends on sequence of delay and risk discounting		–

The table describes seven facts regarding the effects of delay, process, timing, risk and sequence on risk taking and discounting behavior.

hazards. Greater risk tolerance for future payoffs goes against the grain of the most widely used models of risk taking behavior, such as expected utility theory and its prominent alternatives rank-dependent utility, cumulative prospect theory, and theories of disappointment aversion.

It is well known by now that delay dependence is also manifest in discounting behavior, which constitutes empirical fact #2. Contrary to the prediction of standard discounted utility theory, people behave more patiently with respect to more remote payoffs, which implies that people's discount rates are not constant but decline with the length of delay (Strotz, 1955; Benzion, Rapoport, and Yagil, 1989; Loewenstein and Thaler, 1989; Ainslie, 1991). This regularity has triggered a large literature on so-called hyperbolic preferences (e.g. Laibson (1997); Frederick, Loewenstein, and O'Donoghue (2002)). Hyperbolic discounting has been readily adopted by applied economics in many fields such as saving behavior, procrastination, addiction, and retirement decisions. Hyperbolic preference models provide, however, no explanation for interactions between time and risk.

Another regularity in the data concerns facts #3 and #4, the process dependence of risk taking and time discounting behavior. It generally makes a difference whether future prospects are judged in one shot or frequently over the course of time. In the domain of risk, people tend to invest less conservatively, i.e. they take on more risk, when they are informed about the outcomes of their decisions only at the end of the investment period rather than intermittently (Gneezy and Potters, 1997; Thaler, Tversky, Kahneman, and Schwartz, 1997; Bellemare, Krause, Kröger, and Zhang, 2005; Gneezy, Kapteyn, and Potters, 2003; Haigh and List, 2005). Stock market data is easily accessible and thus provides continuous feedback on portfolio performance. If people watch closely how uncertainty resolves, their risk tolerance may be much lower than when they have no access to this information. Hence, frequent portfolio evaluation may be an important factor driving the large equity premium, i.e. the return earned by stocks in excess of that earned by relatively risk-free government bonds, as it has been observed in the U.S. and in other industrialized countries (Mehra, 2006). The magnitude of the observed equity premium has been puzzling economists for more than 25 years because it is hard to reconcile with plausible levels of risk aversion when interpreted within the framework of expected utility theory. Some authors have attributed the aversion to frequent information and, consequently the large equity premium (Benartzi and Thaler, 1995; Barberis, Huang, and Santos, 2001), to loss aversion, an integral concept of prospect theory (Tversky and Kahneman, 1992), that has increasingly attracted economists' attention in the last decade (Gächter, Johnson, and Herrmann, 2007; Köszegi and Rabin, 2007; Abeler, Falk, Goette, and Huffman, 2011). While loss aversion may explain why people are more risk averse when assessing portfolio performance frequently it cannot account for the other findings on observed risk taking and discounting behavior.

In the domain of time discounting, a similar phenomenon of process dependence has been observed, listed as fact #4 in Table 1: The discounting shown over a particular delay is greater when the delay is divided into subintervals than when it is left undivided (Read, 2001; Read and Roelofsma, 2003; Ebert and Prelec, 2007; Epper, Fehr-Duda, and Bruhin, 2009; Dohmen, Falk, Huffman, and Sunde, 2012). For example, discounting over a one-year period will be greater when the year is divided into two subperiods of six months, and even more so when it is divided into, say, twelve monthly subperiods. This regularity has been labeled *subadditive discounting*. So evaluating a future payoff sequentially rather than in one shot typically decreases its value - decision makers exhibit less patience in this case, an effect equivalent to the process dependence of risk tolerance.

The third row in Table 1 refers to fact #5, the effect of the timing of uncertainty resolution on risk taking behavior. Principally, knowing the outcome of one's decision before the actual payment date should be beneficial because one can integrate this information into one's future plans. Therefore, according to the standard model of risky choice, information is valuable if it enables the decision maker to design better strategies. If she does not or cannot condition her actions on what she learns she should be indifferent toward the timing of uncertainty resolution, i.e. information about realized outcomes should be worthless to her (Grant, Kajii, and Polak, 1998). Hence, the value of information should be nonnegative. In contrast to this prediction, many people prefer uncertainty to resolve later rather than sooner (Chew and Ho, 1994; Ahlbrecht and Weber, 1996; Arai, 1997; Lovallo and Kahneman, 2000; Eliaz and Schotter, 2007; von Gaudecker, van Soest, and Wengström, 2011). For instance, some people with a family history of a genetic disorder may choose not to be informed whether they are affected by the disorder or not. Such an intrinsic preference for resolution timing cannot be accommodated by the standard theory of risk taking but is modeled by an additional preference parameter (Kreps and Porteus, 1978; Chew and Epstein, 1989; Grant, Kajii, and Polak, 2000).

Fact #6 pertains to a number of experimental studies that report systematic effects of risk on discounting behavior: Discount rates for certain future payoffs tend to be higher than discount rates for risky future payoffs (Stevenson, 1992; Ahlbrecht and Weber, 1997). Higher discount rates for certain payoffs not only run counter to intuition but also contradict the standard model of discounting according to which the same level of patience applies to risky and certain future prospects. Risk-dependent discounting is also evident in *diminishing immediacy*: People's preference for present certain outcomes over delayed ones, immediacy, weakens drastically when the outcomes become risky: Whereas many people prefer a smaller immediate reward to a larger delayed one, merely a minority continue to do so when both rewards are made probabilistic - they behave as if they discounted the risky reward less heavily than the original certain one (Keren and Roelofsma, 1995; Weber and Chapman, 2005).

Finally, the valuation of future prospects appears to be *sequence dependent*, labeled fact #7: It makes a difference whether a risky future payoff is first devalued for risk and then for delay or in the opposite order (Öncüler and Onay, 2009). When payoffs are discounted for risk first they are assigned a less favorable value than in the reverse case. Moreover, the delay-first valuation practically coincides with the value reported when both dimensions are accounted for in one single operation. This finding implies that risk tolerance revealed for future prospects systematically depends on the sequence in which discounting for risk and for time is performed.

The evidence discussed above demonstrates striking parallels between the susceptibility of observed risk tolerance and observed patience to the length of delay as well as the evaluation process: Deferring payoffs to the remote future apparently makes people both more risk tolerant and more patient, and one-shot evaluation has favorable effects on risk taking as well as on time discounting behavior. Since the effects appear not to be arbitrary aberrations from the predictions of the existing models, one might ask whether there is a common mechanism governing observed behavior that accounts for delay, process, timing, risk, and sequence dependence.

In the following we show that all these interaction effects between risk and time can indeed be rationalized within a single unifying framework that relies on two basic ideas. First, there is uncertainty attached to any future event as only immediate consequences can be totally certain. If future events are inherently uncertain, people's risk tolerance must play a role in the valuation of future prospects. Not surprisingly, risk has been identified as an important confound in the measurement of time preferences (Frederick, Loewenstein, and O'Donoghue, 2002). Therefore, the second pillar of our model pertains to the characteristics of risk preferences. Our model is based on one of the most widely replicated experimental regularities found in human and animal behavior, so-called *common-ratio violations*, which are inconsistent with classical theory (Allais, 1953; Hagen, 1972; Kahneman and Tversky, 1979; MacCrimmon and Larsson, 1979; Battalio, Kagel, and MacDonald, 1985; Kagel, MacDonald, and Battalio, 1990; Nebout and Dubois, 2012).

Researchers have developed a large number of alternative theories that are able to accommodate common-ratio violations.Proneness to common-ratio violations can be suitably captured by a probability weighting function that exhibits a specific characteristic, subproportionality. There is abundant evidence that risk taking behavior depends nonlinearly on the objective probabilities (for a recent review see Fehr-Duda and Epper (2012)). We show that subproportional probability weighting conjointly with inherent future uncertainty provides an integrative account of all the facts #1 to #7.

1.1 Related Literature

Our work utilizes other researchers' theoretical insights, which are generally limited to specific questions, and establish links that have remained unexplored so far. Furthermore, we show that these links provide a unifying account of a host of puzzling findings that have not been diagnosed as manifestations of the same underlying cause. In the following, we discuss in which way previous theoretical contributions are related to this paper.

Many authors have noted before that "[a]nything that is delayed is almost by definition uncertain" (Prelec and Loewenstein (1991), p.784) and, therefore, it is natural to conjecture that uncertainty shapes discounting behavior in one way or another. Several papers identified uncertainty as a potential cause of hyperbolic discounting (Sozou, 1998; Dasgupta and Maskin, 2005; Bommier, 2006). Most closely related to our research are the models introduced by Halevy (2008) and Walther (2010) that derive hyperbolic discounting from nonlinear probability weighting. Walther's approach is based on his model of affective utility (Walther, 2003) that endogenously generates an inverse S-shaped probability weighting function and, consequently, a short-run hyperbolic decline of the implied discount function. However, this discount function turns out to be U-shaped as discount rates start to increase again at some point in time. Halevy's discount function does not suffer from this deficiency because it is derived from subproportional probability weights,[1] a characteristic lacking in Walther's probability weighting function. Neither model addresses delay dependence of risk tolerance nor process, timing, and sequence dependence.

Inherent uncertainty does not play a role in Baucells and Heukamp (2012)'s axiomatic model for the limited domain of prospects with one non-zero outcome which captures interactions of risk tolerance and time discounting behavior by a psychological distance function. The psychological distance of an outcome is assumed to increase with increasing delay or decreasing

[1] Halevy (2008) based his analysis on probability weighting functions of increasing elasticity, which is equivalent to subproportionality.

probability. Similarly to ours, their model is inspired by the common-ratio effect in risk taking behavior but, contrary to our approach, it is a descriptive representation of behavior rather than a structural approach. In this setting, increasing risk tolerance and increasing patience are equivalent. Hence, hyperbolic discounting is built into the assumptions and not derived from probability weighting, in contrast to Halevy (2008) and Walther (2010).[2] Moreover, they do not deal with the issues of process, timing, and sequence dependence.

Process dependence is the focus of Palacios-Huerta (1999)'s contribution. He shows that, in the context of Gul (1991)'s model, a disappointment averse decision maker exhibits much larger risk aversion when she evaluates a multi-stage prospect sequentially rather than in one shot. Dillenberger (2010) provides an axiomatic underpinning for this result. He proves that, under the assumption of recursive valuation of multi-stage prospects, a weak preference for one-shot resolution of uncertainty is equivalent to risk preferences satisfying a novel axiom, negative certainty independence. This axiom weakens the standard independence axiom and allows for common-ratio violations but is silent on their actual occurrence. Dillenberger also provides an insightful discussion of the consequences of a preference for one-shot resolution of uncertainty on the value of information. Finally, our rank-dependent discussion of the relationship between subproportional probability weighting and one-shot resolution of uncertainty is based on the seminal work by Segal (1987a,b, 1990) who analyzes the evaluation of multi-stage prospects in the domain of rank-dependent utility. Since these papers deal with dynamic, but essentially atemporal, situations, they are not concerned with time discounting or the other facts listed in Table 1.

2 Key Assumptions

Our model builds on two basic ideas: First, there is uncertainty attached to any future event. This uncertainty inherent in the future, *inherent uncertainty* for short, may stem from different

[2] Baucells and Heukamp (2012) claim that their model can explain the experimental results in Abdellaoui, Diecidue, and Öncüler (2011) who find that the probability weighting function is more elevated for longer delays t. However, Baucells and Heukamp (2012)'s formulation of delay-dependent probability weights $w_t(p) = w(pe^{-r_x t})$ with probability p and discount rate r_x appears to imply the opposite (p. 836).

sources. At the personal level, it refers to a general feeling of "something may go wrong" due to unexpected contingencies, such as a contract party reneging on her promises or a check getting lost in the mail. Another important channel through which inherent uncertainty may manifest itself is the institutional environment. Environments where property rights are only weakly protected or institutions of contract enforcement are not reliable, as is the case in many developing countries, are characterized by high inherent uncertainty. This uncertainty turns allegedly guaranteed payoffs into risky ones and introduces an additional layer of uncertainty over and above the objective probability distributions of risky payoffs. Consequently, there are two distinct types of uncertainty, the objectively given *prospect uncertainty* and the subjective inherent uncertainty.

The second pillar of our model pertains to the characteristics of risk preferences. Abundant empirical evidence has demonstrated that risk taking behavior depends nonlinearly on the probabilities, which is inconsistent with expected utility theory. Consider the following famous example, introduced by Allais (1953): Imagine you were to choose between one million dollars for certain and five million dollars materializing with a probability of 98%. Most people choose the certain option of one million dollars. Now consider the choice between a 1%-chance of receiving one million dollars and a 0.98%-chance of receiving five million dollars. In this case, the majority opt for the five-million dollar alternative. Scaling down the probabilities of 100% and 98% by a common factor, in this example 0.01, induces many people to reverse their choices.[3] An intuitive explanation for common-ratio violations is the fear of disappointment: Losing a gamble over almost certain five million dollars is anticipated to be much more disappointing than losing a gamble over five million dollars that have only a tiny chance of materializing in the first place. Thus, if people fear disappointment their behavior appears to depend nonlinearly on the probabilities.

Not only emotions, such as disappointment and elation, were identified as potential drivers of probability distortions (Bell, 1985; Loomes and Sugden, 1986; Gul, 1991; Wu, 1999; Walther, 2003), but also perceptual and procedural factors. The fathers of prospect theory, Kahneman and Tversky, attributed probability dependence to the psychophysics of perception according to

[3]This example constitutes a special case of common-ratio violations, known as *certainty effect*, as the smaller outcome in the first decision situation, one million dollars, is to materialize with certainty.

which the sensitivity toward changes in probabilities diminishes with the distance to the natural reference points of certainty and impossibility (Tversky and Kahneman, 1992). Several other contributions focused on procedural aspects of choice (Rubinstein, 1988; Loomes, 2010). In these models, a prospect's value depends not only on the prospect's own characteristics but also on other prospects in the choice set. A recent contribution in this category is Bordalo, Gennaioli, and Shleifer (2012) who posit that probabilities are distorted in favor of payoffs that are perceived as particularly salient.

There is persuasive evidence of nonlinear probability weighting not only in the laboratory, but also in insurance, betting and financial markets (see the discussion of recent evidence in Fehr-Duda and Epper (2012)). Furthermore, brain activity during valuation of monetary gambles was discovered to be nonlinear in probabilities, providing a neurobiological foundation of observed behavior (Paulus and Frank, 2006; Berns, Capra, Chappelow, and Moore, 2008; Hsu, Krajbich, Zhao, and Camerer, 2009). In fact, probability distortions seem to be a ubiquitous feature of people's perception, action, and cognition (Zhang and Maloney, 2012).

In this paper, we rely on rank-dependent utility theory (RDU) (Quiggin, 1982), which captures probability distortions directly by a nonlinear probability weighting function. By convention, this function maps the weight attached to the probability of a prospect's best outcome. RDU has several attractive features. First, and most importantly, the common inverse S-shape of the probability weighting function generates overweighting of a prospect's extreme outcomes and underweighting of its intermediate outcomes, which nicely captures the notion that more extreme outcomes within a given prospect are more salient. Second, under appropriate assumptions, RDU respects transitivity, continuity, and first-order stochastic dominance, qualities that many economists are hesitant to dispense with. Finally, RDU displays first-order attitudes toward risk, i.e. preferences between prospects whose consequences are sufficiently close to one another do not necessarily tend to risk neutrality. In this sense, experimental evidence favors rank-dependent utility theory over many other non-expected-utility approaches that can accommodate the common-ratio effect but only permit second-order risk aversion (Sugden, 2004).

Experimental estimates of average probability weights typically yield inverse S-shaped probability weighting curves, underweighting large probabilities and overweighting small probabilities of the best outcome, which is also a common pattern in individual data (Gonzalez and Wu, 1999; Bruhin, Fehr-Duda, and Epper, 2010). Underweighting of large probabilities can be interpreted as manifestation of the disappointing potential of highly likely, yet uncertain, payoffs. Overweighting occurs when the realization of rather unlikely payoffs is expected to generate a feeling of elation. Aside from inverse-S shapes, convex weighting curves, globally underweighting probabilities, comprise another common category of shapes (van de Kuilen and Wakker, 2011). Since we are primarily concerned with common-ratio violations, we need to put more structure on the probability weighting function. Common-ratio violations are captured by a specific characteristic, subproportionality. In principle, subproportionality can be exhibited by both inverse-S shapes and convex shapes of probability weighting curves.

3 The Model

We consider the set of binary gain prospects $P = (x_1, p; x_2)$ with payoffs $x_1 > x_2 \geq 0$, probability p of the larger payoff x_1 and probability $1 - p$ of the smaller payoff x_2.[4] We assume that a decision maker's true atemporal risk preferences over such prospects, played out and paid out instantaneously, can be represented by a rank-dependent functional:

$$\begin{aligned} V(P) &= u(x_1)w(p) + u(x_2)(1 - w(p)) \\ &= [u(x_1) - u(x_2)]\, w(p) + u(x_2) \end{aligned} \tag{1}$$

where u measures the utility of monetary amounts x,[5] and w denotes the subjective probability weight attached to p, the probability of the better outcome x_1. As usual, both u and w are assumed to be monotonically increasing, w to be twice differentiable and to satisfy $w(0) = 0$ and $w(1) = 1$. A summary of the model variables is provided in Table 2. Technically, common-

[4]Our approach can be easily generalized to $n > 2$ outcomes provided that inherent uncertainty does not change the rank order of the prospects, i.e. if "something may go wrong" is encoded as an outcome no better than the prospects' minimum outcome.

[5]Wakker (1994, 2010) offer a theoretical argument in favor of our implicit assumption that instantaneous utility is transferable across the domains of risk and time. Abdellaoui, Attema, and Bleichrodt (2007); Abdellaoui, Barrios, and Wakker (2007) provide supportive evidence.

ratio violations are represented by *subproportionality* of the probability weighting function. Subproportionality decreases the decision maker's sensitivity to disappointment for scaled-down probabilities, i.e. risky outcomes are potentially more disappointing the higher is their ex-ante probability of materializing. In this sense, the loss of certainty hurts more than the scaling down of a probability less than one does.

Formally, subproportionality is defined as follows (Prelec, 1998): Subproportionality holds if $1 \geq p > q > 0$ and $0 < \lambda < 1$ imply the inequality

$$\frac{w(p)}{w(q)} > \frac{w(\lambda p)}{w(\lambda q)}. \tag{2}$$

Subproportionality implies the certainty effect, which constitutes the special case of $p = 1$. Therefore, $w(\lambda q) > w(\lambda)w(q)$ is satisfied for any λ, q such that $0 < \lambda, q < 1$. Many functional specifications proposed in the literature exhibit subproportionality over some probability range under appropriate parameter restrictions (see section 3.6 in Fehr-Duda and Epper (2012) for a review). Perhaps the most prominent representative of a globally subproportional function is Prelec (1998)'s flexible two-parameter specification, designed to capture common-ratio violations. Gul (1991)'s theory, for example, implies a strictly convex subproportional function.[6]

If the prospect is not played out and paid out in the present, but at some future time $t > 0$, two more factors become important. First, we follow the standard approach and model people's willingness to postpone gratification by a constant rate of time preference $\eta \geq 0$, yielding a discount weight of $\rho(t) = \exp(-\eta t)$. This assumption is not crucial for our results - neither a zero rate of time preference nor genuinely hyperbolic time preferences affect our conclusions. A prospect to be played out and paid out at $t > 0$ is discounted for time in the following standard way:

$$V_0(P) = ([u(x_1) - u(x_2)]\, w(p) + u(x_2))\, \rho(t) \tag{3}$$

Second, and most importantly, inherent future uncertainty changes the nature of the prospect.

[6] Incidentally, the salience-driven discontinuous probability distortions in Bordalo, Gennaioli, and Shleifer (2012) comprise a concave segment of a specific Rachlin, Raineri, and Cross (1991) variety and a convex segment of a Goldstein and Einhorn (1987) one, both of which are subproportional over their respective ranges.

Following Halevy (2008) and Walther (2010), let $0 < s \leq 1$ denote the constant subjective per-period probability of prospect survival, i.e. the probability that the decision maker will actually obtain the promised rewards by the end of the period. Then the probability that the allegedly guaranteed payment of x_2 materializes at the end of period t is perceived to be s^t, and the probability of the risky component $x_1 - x_2$ effectively amounts to ps^t. Therefore, the objective two-outcome prospect is subjectively perceived as a three-outcome prospect $\tilde{P} = (x_1, ps^t; x_2, 1 - ps^t; 0)$, where the zero outcome captures that "something may go wrong". With the passage of time, the probability of prospect survival gets progressively scaled down. Therefore, subproportional preferences are a natural framework for studying the effects of time on prospect valuation. At the present, the future prospect is evaluated according to

$$\begin{aligned} V_0\left(\tilde{P}\right) &= \left(\left[u(x_1) - u(x_2)\right] w(ps^t) + u(x_2) w(s^t)\right) \rho(t) \\ &= \left(\left[u(x_1) - u(x_2)\right] \frac{w(ps^t)}{w(s^t)} + u(x_2)\right) w(s^t) \rho(t) \end{aligned} \tag{4}$$

Now suppose that an observer assumes that there is no inherent uncertainty, i.e. that $s = 1$, while in fact $s < 1$. Consequently, she infers probability weights $\tilde{w}$ and discount weights $\tilde{\rho}$ from observed behavior on the presumption that the decision maker evaluates the objectively given prospect P. However, in the eye of the decision maker the prospect involves an additional layer of uncertainty. If the observer neglects $s < 1$, she estimates preference parameters according to Equation 3:

$$V_0(\tilde{P}) = \left(\left[u(x_1) - u(x_2)\right] \tilde{w}(p) + u(x_2)\right) \tilde{\rho}(t) \tag{5}$$

interpreting $\tilde{w}$ as true probability weights and $\tilde{\rho}$ as true discount weights, while in fact the weights are distorted by inherent uncertainty. Obviously, the measured weights are different from the true ones if $s < 1$. By comparing Equation 4 with Equation 5 we can see that the relationships between true and observed preference parameters are given by

$$\tilde{w}(p) = \frac{w(ps^t)}{w(s^t)} \tag{6}$$

$$\tilde{\rho}(t) = w(s^t)\rho(t) \tag{7}$$

These equations define the central relationships between observed and true underlying probability and discount weights. Concerning the discount weights, this representation is equivalent to Halevy (2008)'s who derives this relationship in the context of Yaari (1987)'s dual theory with a convex probability weighting function. Because $\tilde{w}(p) \neq w(p)$ and $\tilde{\rho}(t) \neq \rho(t)$ for subproportional preferences, inherent uncertainty drives a wedge between true underlying preferences and observed risk taking and discounting behavior. Thus, future uncertainty conjointly with proneness to Allais-type behavior provides the missing link between behavior under risk and over time.

Table 2: Model Variables

	Variable	Description	Characteristics
Prospects	x	monetary payoff	$x \geq 0$
	p	probability of x	$0 \leq p \leq 1$
	s	probability of prospect survival	$s \leq 1$
	$1-s$	inherent uncertainty	
	t	length of time delay	$t \geq 0$
Preferences	$u(x)$	utility function	$u(0) = 0, u' > 0$
	$w(p)$	true probability weight	$w(0) = 0, w(1) = 1, w' > 0$
	η	rate of pure time preference	$\eta \geq 0$, constant
	$\rho(t)$	discount weight	$\rho(t) = \exp(-\eta t)$
Behavior	$\tilde{w}(p)$	observed probability weight	$\tilde{w}(p) = w(ps^t)/w(s^t)$
	$\tilde{\rho}(t)$	observed discount weight	$\tilde{\rho}(t) = w(s^t)\rho(t)$
	$\tilde{\eta}(t)$	observed discount rate	$\tilde{\eta}(t) = -\tilde{\rho}'(t)/\tilde{\rho}(t)$

4 Model Predictions

In the following, we discuss the implications of our approach for the empirical phenomena listed in Table 1 and demonstrate that all of them can be explained by our framework. An important feature of future prospects concerns the timing of the resolution of uncertainty. We distinguish three different cases: First, the prospect is played out and paid out at the same time. This situation is represented by Propositions 1 and 2. Second, both prospect and inherent uncertainty are resolved gradually over the course of time. This case is covered by Proposition 3. Finally, prospect uncertainty is resolved before the payment date, the topic of Proposition 4. The proofs of the propositions are presented in Appendix B, as is a discussion of the necessity of subproportionality.

4.1 Fact #1: Delay Dependence of Observed Risk Taking Behavior

Turning to the simultaneous resolution of prospect and inherent uncertainty first, we see from Equation 6 that observed probability weights $\tilde{w}(p)$ deviate from true ones $w(p)$ in two respects: First, $w(s^t) < 1$ in the denominator boosts observed weights. Second, $w(ps^t)$ in the numerator distorts observed probability weights. The assumption of subproportional probability weights w generates unambiguous predictions for $\tilde{w}$:

PROPOSITION 1 (*Characteristics of observed probability weights*) Given subproportionality of w and $s < 1$:

1. The function $\tilde{w}$ is a proper probability weighting function, i.e. monotonically increasing in p with $\tilde{w}(0) = 0, \tilde{w}(1) = 1$.
2. $\tilde{w}$ is subproportional.
3. $\tilde{w}$ is more elevated than w. Elevation increases with time delay t and inherent uncertainty $1 - s$, at a decreasing rate.
4. $\tilde{w}$ is less elastic than w.
5. The increase in observed risk tolerance is more pronounced for more strongly subproportional risk preferences.

[Proof in Appendix B]

That $\tilde{w}$ is more elevated than w constitutes one of the central implications of our model. Probability weights are larger for delayed prospects and, hence, revealed risk tolerance appears to be higher than risk tolerance for present ones. The departure of $\frac{\tilde{w}(p)}{w(p)} = \frac{w(ps^t)}{w(p)w(s^t)} > 1$ from unity provides a measure of the wedge between observed and true probability weights and corresponds to the strength of the certainty effect inherent in the underlying risk preferences. The intuition behind this result is that, with the passage of time, payoffs become progressively less likely and, therefore, their disappointment potential diminishes commensurately. Since in our model utility from money u is not affected by future uncertainty, an increase in the elevation of the probability weighting curve gets directly translated into higher revealed risk tolerance. The presence of future uncertainty therefore makes people appear more risk tolerant for delayed prospects than for present ones. Moreover, $\tilde{w}$ is less strongly curved than w, implying a decreasing proneness to common-ratio violations. Therefore, the risk taking behavior of a typical subject in the lab, where uncertainty resolves almost immediately, is likely to overstate her risk aversion for real-world decisions as well as her proneness to Allais-type violations. Furthermore, the wedge between $\tilde{w}$ and w also increases with the degree of inherent uncertainty, implying, somewhat paradoxically, that observed risk tolerance increases with uncertainty.[7] It rises with the degree of subproportionality as well, implying *ceteris paribus* individual-specific sensitivities to delay.

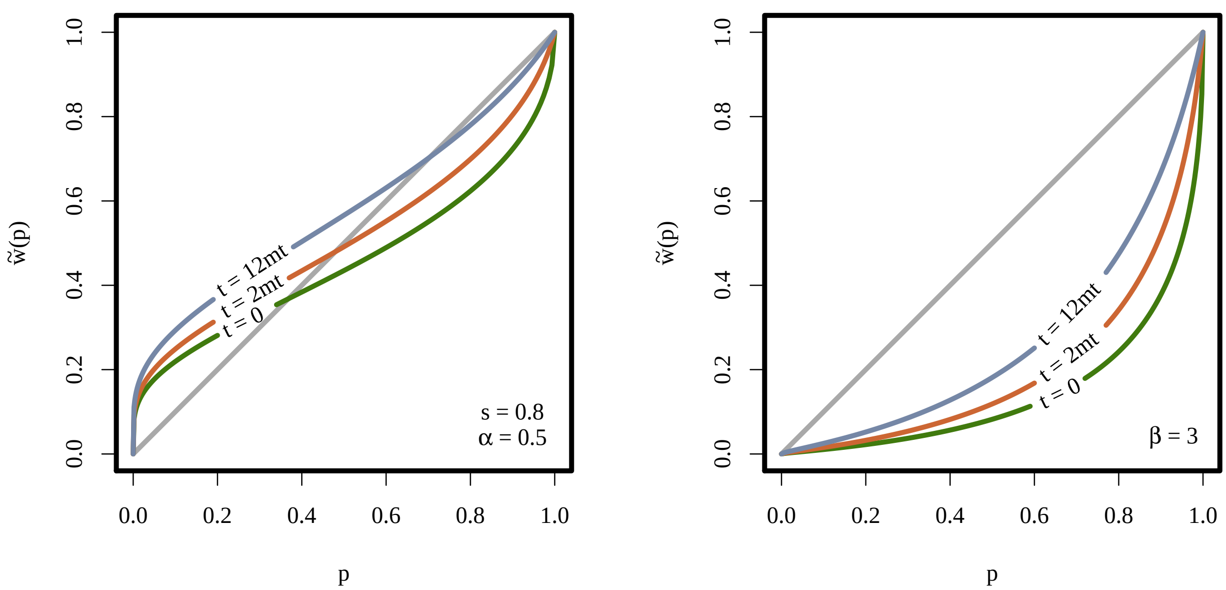
Illustration: Delay Dependence of Probability Weights
The delay dependence of observed probability weights is illustrated in Figure 1. Typically, decision makers exhibit an inverse S-shaped probability weighting function, characterized by underweighting of large probabilities and overweighting of small probabilities. The graph on the left side shows such a decision maker's true probability weights, labeled by $t = 0$, and the respective observed probability weights generated by increasing delay t, depicted by the curves for delays of two and twelve months, respectively. A different specimen of subproportional probability weights is displayed on the right hand side of Figure 1. Here, the probability weighting curve is globally convex, implying pronounced risk aversion.[8]

4.1.1 Evidence of Delay-Dependent Probability Weighting

Empirical evidence on the valuation of delayed prospects typically only provides results on summary measures of risk tolerance. The predominant finding in the literature is higher risk tolerance for delayed prospects than for present ones (Shelley, 1994; Ahlbrecht and Weber, 1997; Sagristano, Trope, and Liberman, 2002; Noussair and Wu, 2006; Coble and Lusk, 2010). Recently, Abdellaoui, Diecidue, and Öncüler (2011) conducted a carefully designed experiment eliciting probability weights for both present and delayed prospects, i.e. in our notation $w(p)$ and $\tilde{w}(p)$. Their results provide persuasive direct support for our approach. They find four distinctive characteristics of delay-dependent prospect valuation. First, the utility for money u does not react to time delay. Second, $\tilde{w}$ is significantly more elevated than w in the aggregate as well as for the

[7] This finding mirrors Quiggin (2003)'s result of atemporal risk tolerance increasing with background risk.
[8] Convex probability weighting implies risk aversion if utility u is, as usually assumed, (weakly) concave.

Figure 1: Effect of Delay on Observed Probability Weights $\tilde{w}$



The graphs show two typical specimens of atemporal risk preferences characterized by the probability weighting curves $w(p) = \tilde{w}(p)$ at $t = 0$: an inverse S-shaped curve on the left and a globally convex curve on the right. Moving the payoff date into the future by $t = 2$ and $t = 12$ months, respectively, shifts the probability weighting curves $\tilde{w}$ upwards. Furthermore, the probability weighting functions get progressively less strongly curved. For purposes of illustration, the curves are derived from Prelec's two-parameter probability weighting function $w(p) = \exp(-\beta(-\ln(p))^{\alpha})$ (Prelec, 1998), assuming degrees of subproportionality $\alpha = 0.5$ and convexity $\beta = 1$ (left hand side) and $\beta = 3$ (right hand side), respectively. Inherent uncertainty $1 - s$ is set at 0.2 per annum.

majority of the individuals. Third, an additional six-month delay affects elevation less strongly than the first six-month delay. Moreover, $\tilde{w}$ appears to be less strongly curved than w.[9]

4.2 Fact #2: Delay Dependence of Discounting Behavior

Repercussions on risk taking behavior are not the only effects of inherent uncertainty. The same mechanism also affects the valuation of allegedly guaranteed delayed payoffs as it drives a wedge between time preferences and observed discounting behavior. As shown in Equation 7, the observed discount weight for time equals $\tilde{\rho}(t) = w(s^t)\rho(t)$, which depends not only on the *pure* rate of time preference η, but also on the probability of prospect survival s as well as on the shape of the probability weighting function w. Clearly, if w is linear, $\tilde{\rho}$ declines exponentially irrespective of the magnitude of s. To see this, note that $\rho(t) = \exp(-\eta t)$ and $s^t = \exp(-(-\ln(s))t)$, implying a discount rate $\tilde{\eta} = \eta - \ln(s) > \eta$ for $0 < s < 1$. In this case, uncertainty *per se* increases the absolute level of revealed impatience, but cannot account for declining discount rates. Thus, an expected-utility maximizer will exhibit a constant discount rate that is higher than her underlying rate of pure time preference, but her behavior will not show any of the interaction effects addressed in this paper.

If, however, w is subproportional and $0 < s < 1$, the component $w(s^t)$ distorts the discount weight in a predictable way. Increasing patience is not a manifestation of underlying preferences but rather a consequence of inherent uncertainty changing the nature of future payoffs. At the level of observed behavior, increasing patience is the mirror image of increasing risk tolerance. In fact, the degree of proneness to common-ratio violations, the degree of subproportionality, can be interpreted as degree of time insensitivity. Intuitively, when the future is inherently uncertain promised rewards do not materialize with certainty and, therefore, they incorporate the potential of disappointment. Because more immediate payoffs are more likely to actually materialize than more remote payoffs, this potential is perceived to decline with the passage of time and becomes almost negligible for payoffs far out in the future. Technically, since shifting a payoff into the future amounts to scaling down its probability, a decision maker with subproportional preferences becomes progressively insensitive to a given timing difference. These insights are

[9] In their study on ambiguity, Abdellaoui, Baillon, Placido, and Wakker (2011) show estimates of a probability weighting curve derived from choices over prospects delayed by three months. This curve is also much more elevated than typical atemporal estimates are (see for example Bruhin, Fehr-Duda, and Epper (2010)).

formalized in the following proposition, prediction 3 of which is closely related to Theorem 1 in Halevy (2008). Note that our proposition holds for any subproportional function, even if it is inverse S-shaped.

PROPOSITION 2 (*Characteristics of observed discounting behavior*) Given subproportionality of w:[10]

1. $\tilde{\rho}(t)$ is a proper discount function for $0 < s \leq 1$, i.e. decreasing in t, converging to zero with $t \to \infty$, and $\tilde{\rho}(0) = 1$.
2. Observed discount rates $\tilde{\eta}(t)$ are higher than the rate of pure time preference η for $s < 1$.
3. Observed discount rates decline with the length of delay for $s < 1$.
4. Greater inherent uncertainty generates more strongly declining discount rates.
5. Comparatively more subproportional probability weighting generates comparatively more strongly declining discount rates.

[Proof in Appendix B]

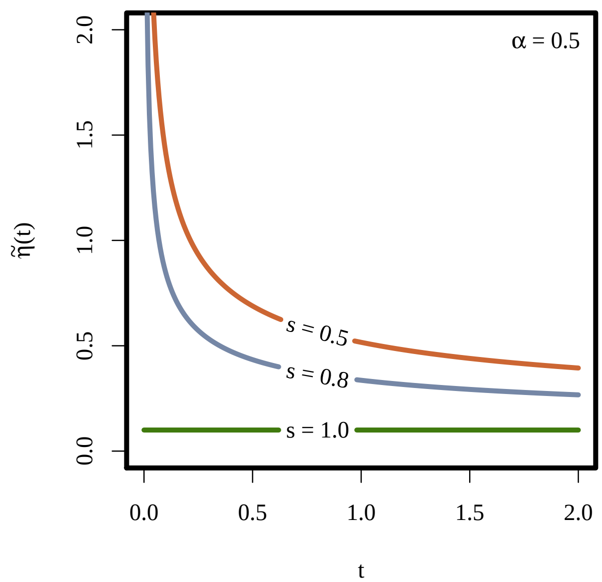
Illustration: Inherent Uncertainty and Discount Rates
The effects of inherent uncertainty on revealed discount rates are presented in Figure 2, which depicts a typical decision maker's observed *discount rates* $\tilde{\eta}$ as they react to varying levels of s. The horizontal line represents the case of no inherent uncertainty. In this case, the observed discount rate $\tilde{\eta}$ is constant and coincides with the true underlying rate of time preference η. When inherent uncertainty comes into play, however, discount rates decline in a hyperbolic fashion, and depart from constant discounting increasingly strongly with rising uncertainty, as shown by the curves for $s = 0.8$ and $s = 0.5$, respectively.

4.2.1 Evidence of Link between Subproportionality and Hyperbolicity

A large body of empirical evidence documents the prevalence of common-ratio violations as well as of non-exponential discounting, at least at the level of aggregate behavior (Kahneman and Tversky, 1979; Thaler, 1981; Benzion, Rapoport, and Yagil, 1989; Starmer and Sugden, 1989), which suggests that there may be a common cause driving behavior in both decision domains (Prelec and Loewenstein, 1991). However, there is vast heterogeneity in individuals' behaviors (Hey and Orme, 1994; Chesson and Viscusi, 2000; Bruhin, Fehr-Duda, and Epper, 2010) and the question arises whether common-ratio violations and non-constant discounting are actually exhibited by the same people. Our framework predicts not only the existence of a link between

[10]See Appendix B for a discussion of sufficient versus necessary conditions.

Figure 2: Effect of Inherent Uncertainty on Observed Discount Rates $\tilde{\eta}$



The graph shows discount rates as they move with the length of delay t for different levels of inherent uncertainty $1 - s$, where s denotes the probability of prospect survival. When there is no inherent uncertainty, $s = 1$, the observed discount rate is constant and equals the rate of pure time preference (line labeled by $s = 1.0$). The higher is the level of uncertainty, the lower s, the more pronounced the hyperbolic decline of discount rates over time is for decision makers with subproportional probability weights (curves labeled by $s = 0.5$ and $s = 0.8$). $\tilde{\eta}(t) := -\frac{\partial \tilde{\rho}}{\partial t} / \tilde{\rho}$. w is specified as Prelec's probability weighting function (in this example $\alpha = 0.5$ and $\beta = 1$).

subproportional risk preferences and hyperbolic discounting but also its strength: higher degrees of subproportionality are predicted to be associated with higher degrees of hyperbolicity.

Note that we predict merely a correlation between insensitivities to probabilities and delays, and not a one-to-one correspondence *à* la Baucells and Heukamp (2012), because it allows for many combinations of preferences: For example, an expected utility maximizer may have genuinely hyperbolic time preferences or a probability weighter may not perceive the future as inherently uncertain and, hence, will exhibit constant discounting. The model can, therefore, pick up a lot of individual heterogeneity, which is beyond the means of Baucells and Heukamp (2012)'s model. There is another advantage of our structural approach: It defines drivers of behavior that enable the researcher to understand real-world phenomena. Furthermore, these drivers can be experimentally manipulated and their effects tested.

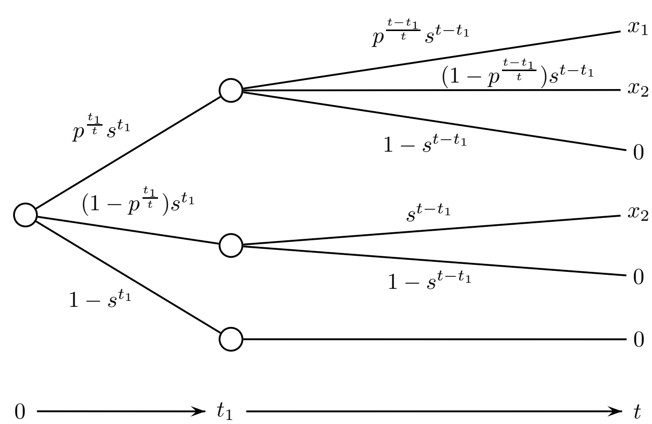
In a recent experimental study, Epper, Fehr-Duda, and Bruhin (2011) provide evidence that subjects' departures from linear probability weighting are indeed highly significantly correlated with the strength of decreasing discount rates. Moreover, in line with our framework, the curvature of the utility function seems not to be directly related to their decline. In fact, the only variable associated with decreasing discount rates turns out to be the degree of nonlinearity of probability weights, which explains a large percentage of the variation in the extent of the decline, whereas other individual characteristics, such as gender, age, experience with investment decisions and cognitive abilities have no significant impact on the degree of non-constant discounting.

4.3 Facts #3 and #4: Process Dependence of Observed Behavior

So far, we have considered the case of uncertainty resolving in one single stage, the domain over which atemporal risk preferences are defined. If uncertainty does not resolve in one shot but rather sequentially over the course of time, future prospects lose their single-stage quality and turn into multi-stage ones. In this case the question arises in which way multi-stage prospects are transformed into single-stage ones. In Appendix A we analyze in detail two different transformation methods, reduction by probability calculus and folding back, and their interactions with subproportional risk preferences. One of the issues concerns dynamic consistency. Dynamic consistency requires that choices made at, or plans formed at, different times conform with one another (Sugden, 2004). As Loomes and Sugden (1986) argue, any theory that accommodates the common-ratio effect must dispense either with dynamic consistency or with the compound probability axiom, i.e. reduction by the probability calculus. Therefore, if the decision maker cares only about the total probabilities of the final outcomes she will be dynamically inconsistent unless she precommits herself to stick to her original plans. Folding back, on the other hand, ensures dynamic consistency but, as Propositions 3 and 4 will show, has substantial consequences for revealed risk taking behavior. Therefore, as will become clear, we label adoption of folding back as myopic. In the following, we assume $\rho = 1$ for ease of exposition.

Suppose that uncertainty is partially resolved at some future time t_1 and fully resolved at the payment date t, as depicted in Figure 3. Proposition 3 shows that a decision maker with subproportional preferences prefers uncertainty to be resolved at the payment date t rather than at some earlier time. Note that this result does not hold generally under subproportionality in rank-dependent utility but only applies to the class of prospects studied here, i.e. prospects that are devalued by inherent uncertainty without effects on the rank order of the outcomes (see Dillenberger (2010) and the discussion in Appendix A).

Figure 3: Gradual Resolution of Uncertainty



PROPOSITION 3 (*Preference for one-shot resolution of uncertainty*) Given subproportionality of w, $s \leq 1$ and folding back:

1. A myopic decision maker prefers one-shot resolution of (total) uncertainty to gradual reso-

lution of uncertainty.

2. Her preference for one-shot resolution declines with (total) probability of the best outcome.
3. Her prospect valuation is lowest at midterm to maturity.

[Proof in Appendix B]

A special case is the valuation of allegedly certain future payoffs, which constitute simple prospects in our framework. A myopic decision maker, applying folding back, will exhibit a discount weight of $w(s^{t_1})w(s^{t-t_1}) < w(s^t)$, an incident of *subadditive discounting* (fact #4).

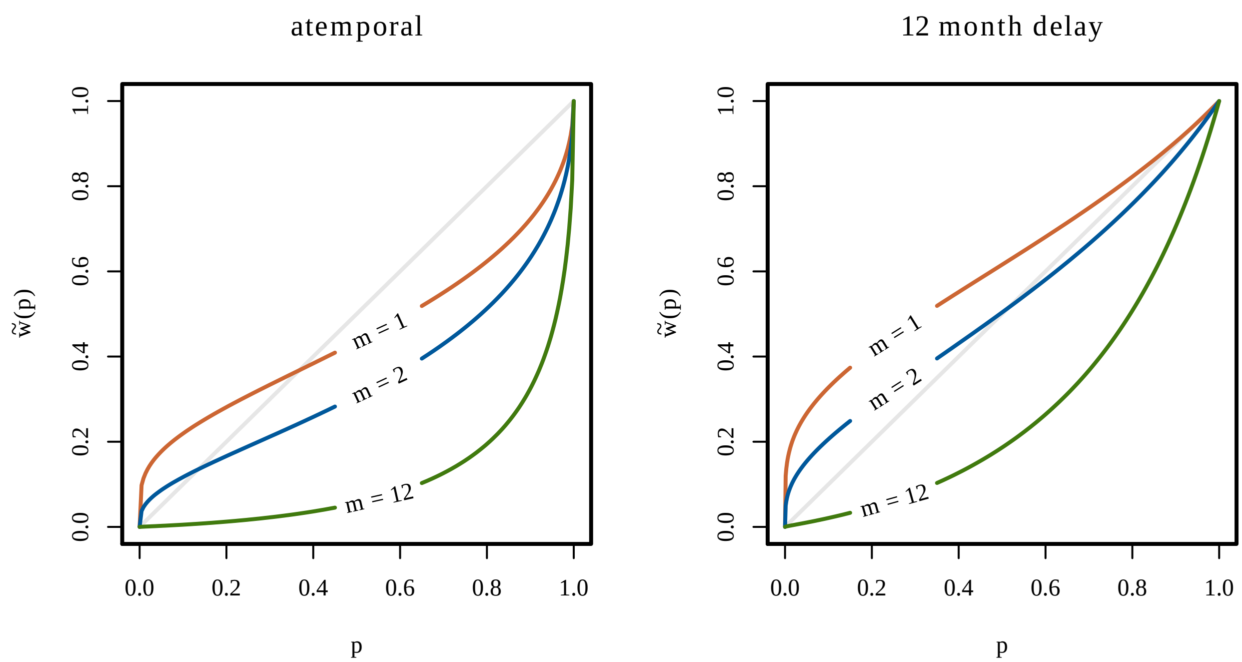
Preference for one-shot resolution of uncertainty is embodied in the characteristics of atemporal risk preferences and, therefore, all the insights of Segal (1990), who analyzes two-stage prospects in an atemporal setting, still apply. The effect of gradual resolution of uncertainty on subproportional probability weights is depicted in the left panel of Figure 4. The passage of time does not interact with this preference as long as there is no disassociation of prospect uncertainty from inherent uncertainty. However, revealed risk tolerance is additionally influenced by its delay dependence, as shown in the right panel of Figure 4. Consider a prospect with a long time horizon t. If its total uncertainty is resolved in one single stage, risk tolerance as well as the corresponding discount weight attains its maximum value. If uncertainty resolves gradually, both observed risk tolerance and the discount weight are smaller than in the one-shot case. The effect gets more pronounced the finer is the partition of delay t into subintervals. Therefore, anticipating to watch uncertainty resolve over time considerably dampens the effect of long time horizons on risk tolerance, because the decision maker is frequently exposed to the possibility of a disappointing outcome. The preference for one-shot resolution is strongest for improbable prospects and declines with rising probability. Moreover, our model predicts that partitions of the time interval of equal length will be valued particularly unfavorably. Partitions of equal length correspond to the least degenerate multi-stage prospect and can be interpreted as relatively most ambiguous situation, which is strongly disliked by people with subproportional preferences (Segal, 1987b).

Illustration: Process Dependence of Probability Weights
Figure 4 demonstrates the sensitivity of subproportional probability weights to the number of evaluation stages m, resulting from partitions of equal length (the most pronounced case). The more frequently feedback is provided, the more pronounced is the dampening effect on revealed risk tolerance, illustrated for the atemporal case in the left panel of Figure 4. The curve for

$m = 1$ represents a typical subproportional probability weighting function when outcomes are evaluated in one shot. If uncertainty resolves in two stages with equal probability rather than in one shot, the prospect is effectively evaluated with the probability weighting curve $m = 2$, which shows more pronounced underweighting. At $m = 12$, the curve looks extremely convex, implying strong risk aversion.

This insight is directly transferable to situations where the resolution of uncertainty involves the passage of real time. The combined effects of inherent future uncertainty and sequential evaluation are shown in the right panel of Figure 4. When the prospect is delayed by 12 months the curve for $m = 1$ is more elevated than the atemporal curve, a manifestation of delay-dependent risk tolerance. Compounding of weights semi-annually ($m = 2$) or monthly ($m = 12$) looks less dramatic than in the atemporal situation, but shows the same tendency toward convex probability weighting, i.e. an increase in risk aversion.

Figure 4: Process Dependence of Observed Probability Weights



The two panels demonstrate the effect of sequential evaluation on observed probability weights $\tilde{w}$ depending on the number of stages m. The left panel shows atemporal probability weighting functions for one-shot evaluation ($m = 1$) and multi-stage evaluations ($m = 2$ and $m = 12$). The right panel does the same for a 12-month delay under the assumption of an additional layer of uncertainty ($s = 0.8$). When resolution of uncertainty is delayed by 12 months, revealed risk tolerance for $m = 1$ is higher than in the atemporal case. Sequential evaluation ($m = 2, m = 12$), however, has the same qualitative, but less pronounced, effect as in the atemporal model.

4.3.1 Evidence on Gradual versus One-Shot Resolution

A number of prominent papers investigated the effects of feedback frequency and precommitment on people's risk taking behavior in investment games (Gneezy and Potters, 1997; Thaler, Tversky, Kahneman, and Schwartz, 1997; Gneezy, Kapteyn, and Potters, 2003; Bellemare, Krause, Kröger, and Zhang, 2005; Haigh and List, 2005) and generally find that investment behavior appears more risk averse when outcomes are evaluated more frequently.[11] This finding is often interpreted as a manifestation of *myopic loss aversion*, a term coined by Benartzi and Thaler (1995). In this context, myopia is defined as narrow framing of decision situations which focuses on short-term consequences rather than on long-term ones. Loss aversion, one of the key constituents of prospect theory, describes people's tendency to be more sensitive to losses than to gains.[12] According to this interpretation, if people evaluate their portfolios frequently, the probability of observing a loss is much greater than if they do so infrequently. In this sense, myopic loss aversion describes a mechanism similar to sequential probability weighting. Consequently, loss-averse investors shy away from risky assets as probability weighters do.

Several authors have challenged the loss aversion argument, however. Using conventional parameterizations of cumulative prospect theory, Blavatskyy and Pogrebna (2010) show that the effects of myopic loss aversion may be modified by probability weighting and conclude that myopic loss aversion alone cannot explain the observed patterns of behavior. Langer and Weber (2005) argue and support experimentally that, depending on the specific risk profile of the investment sequence, myopia may decrease or increase the attractiveness of a sequence. Regarding the equity premium, De Giorgi and Legg (2012) argue that probability weighting may raise the equity premium considerably above the level predicted by loss aversion alone. The upshot of these arguments is that probability weighting should not be ignored when studying investment behavior. In any case, all the studies on feedback frequency conducted so far have not controlled for subjects' inclinations toward probability distortions and have remained within the confines of atemporal risk preferences.

[11] In these experiments subjects evaluate *sequences* of identical two-outcome lotteries over several periods where the range of potential outcomes increases with the number of periods. Unlike Gul's disappointment aversion (Palacios-Huerta, 1999; Artsetin-Avidan and Dillenberger, 2011), our model does not deliver clear predictions for this class of prospects.

[12] Recently, Köszegi and Rabin (2009) extended the concept of loss aversion to changes in beliefs about present and future consumption. Their model also predicts decision makers to prefer information to be clumped together rather than apart.

For the domain of discounting, evidence of process dependence is presented in Read (2001) and Read and Roelofsma (2003). Interestingly, sequential discounting has another implication: People may exhibit hyperbolic discounting when the length of delay is increased, i.e. when uncertainty resolves in a single stage, but constant discounting when two events are shifted into the future by a common timing difference, which may induce folding back. Evidence for the simultaneous occurrence of constant and non-constant discounting is provided by Epper, Fehr-Duda, and Bruhin (2009) and Dohmen, Falk, Huffman, and Sunde (2012).

4.4 Fact #5: Timing Dependence of Risk Taking Behavior

The previous theoretical result rests on the assumption that prospect uncertainty is resolved simultaneously with inherent uncertainty. If the prospect is played out before payment takes place, prospect uncertainty is segregated from inherent uncertainty. As long as prospect uncertainty is unresolved both types of risks are effective, after resolution of prospect uncertainty only inherent uncertainty remains to be resolved, defining two distinct stages of uncertainty resolution. As Segal (1990) argues, folding back is particularly plausible when sufficiently long time passes between the stages or the stages are clearly distinct.

Figure 5 depicts three distinct cases: First, the prospect is played out immediately after prospect valuation. In this case, the decision will know the outcome after her decision and faces only inherent uncertainty. This situation corresponds to the left panel, labeled "immediate". The right panel shows the other extreme, labeled "late" when the prospect is played out and paid out at the same time t, the focus of Propositions 1 and 2. The middle panel is dedicated to the intermediate case when prospect uncertainty is resolved at some time t_1 in the future before the payment date.

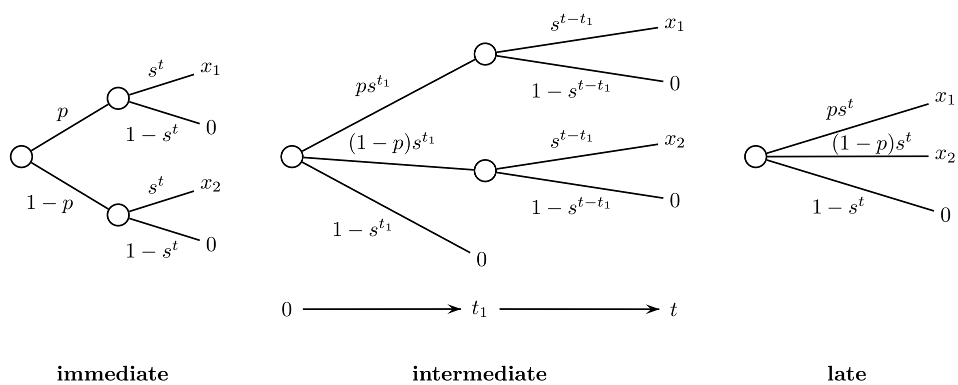
PROPOSITION 4 (*Preference for late resolution of prospect uncertainty*) Given subproportionality of w, $s < 1$ and folding back:

1. A myopic decision maker values prospects with prospect uncertainty resolving at the time of payment more highly than prospects with earlier resolution of prospect uncertainty.

2. The wedge between late and immediate resolution, $\frac{w(ps^t)}{w(p)w(s^t)}$, declines with probability p.

3. The wedge between late and immediate resolution increases with time horizon t and inherent uncertainty $1 - s$.

Figure 5: Resolution Timing of Prospect Uncertainty



The figure shows three different timings of the resolution of prospect uncertainty. Uncertainty gets resolved either immediately at $t = 0$ (left tree), in between the present and the time of payment (middle tree), or at the time of payment at $t > 0$ (right tree).

[Proof in Appendix B]

While it is always the case that late resolution at t is preferred to any earlier resolution time t_1, we cannot ascertain that intermediate resolution at $t_1 > 0$ is generally better than immediate resolution at $t_1 = 0$. Due to the ambiguity effect resulting from time partitions of equal length, discussed above, the discount weight $w(s^{t_1})w(s^{t-t_1})$ decreases with t_1 for $t_1 \in \left[0, \frac{t}{2}\right)$ and increases for $t_1 \in \left(\frac{t}{2}, t\right]$, while risk tolerance increases throughout. Therefore, total prospect value increases with t_1 as long as both factors increase, which is always the case for $t_1 > \frac{t}{2}$. Depending on the relative magnitudes of the effects before $\frac{t}{2}$, prospect value may decrease after $t_1 = 0$ for some time. Obviously, this depends on the prospect under consideration. Table 3 summarizes the effects of resolution timing on observed probability weights $\tilde{w}$ and discount weights $\tilde{\rho}$.

The value of a simple prospect (x, p) amounts to $u(x)w(ps^{t_1})w(s^{t-t_1})$. In this case, the movement of $w(ps^{t_1})w(s^{t-t_1})$ determines the preference for resolution timing. It is straightforward to show that the minimum of the utility weight $w(ps^{t_1})w(s^{t-t_1})$ is attained at $t_1^* = \frac{t}{2} - \frac{\ln(p)}{2\ln(s)}$, which lies below $\frac{t}{2}$. If $t_1^* > 0$, then immediate resolution may be preferred to some later times before $\frac{t}{2}$, otherwise prospect value increases monotonically in resolution time. The latter is the case for $p \leq s^t$. For a given prospect, this condition is more likely to be met for low inherent uncertainty and/or short time horizons. The greater the uncertainty or the longer the time horizon, the comparatively less desirable becomes intermediate resolution of uncertainty.

In our view, that atemporal risk preferences induce a preference for late resolution of prospect uncertainty constitutes the third important insight from our model, besides delay-dependent risk tolerance and hyperbolic discounting. If myopic decision makers perceive the future as inherently uncertain, this property follows endogenously from subproportionality and does not constitute an independent preference as in the theoretical literature on resolution timing (Kreps and Porteus, 1978; Chew and Epstein, 1989; Grant, Kajii, and Polak, 2000). These theories build on the assumption that the decision maker has an intrinsic preference for early or late resolution of uncertainty and examine the ramifications of this assumption for different models of atemporal risk preferences.

An intrinsic preference for late resolution of uncertainty can also be interpreted as an aversion to non-instrumental information. Information is non-instrumental when no further action can be taken that will change the decision maker's utility.[13] Grant, Kajii, and Polak (1998) present the following example of non-instrumental information:

"Consider, for example, the decision of whether to be tested for an incurable genetic disorder. A director of a genetic counseling program told the *New York Times* that there are basically two types of people. There are 'want-to-knowers' and there are 'avoiders'. There are some people who, even in the absence of being able to alter outcomes, find information of this sort beneficial. The more they know, the more their anxiety level goes down. But there are others who cope by avoiding, who would rather stay hopeful and optimistic and not have the unanswered question answered." (Grant, Kajii, and Polak (1998), p.234).

That there are different types of decision makers has not only been observed in the context of health-related information but also in the domain of financial prospects, as the following section shows.

4.4.1 Evidence of Preference for Late Resolution of Uncertainty

Several experimental studies have investigated people's intrinsic preferences for resolution timing, frequently based on hypothetical questions. The general finding is that there are varying percentages of people with preference for early resolution, preference for late resolution and

[13] There is a number of papers studying preference for instrumental information in non-expected utility models (see for instance Wakker (1988), Schlee (1990), Safra and Sulganik (1995). Li (2011) analyzes aversion to partial information in the context of an ambiguity averse preference model. See also the discussion of the value of information in Dillenberger (2010).

Table 3: Effects of Resolution Timing t_1 on Decision Weights

	Resolution Timing				
	immediate $t_1 = 0$		intermediate $0 < t_1 < t$		late $t_1 = t$
$\tilde{w}$	$w(p)$	$<$	$\frac{w(ps^{t_1})}{w(s^{t_1})}$	$<$	$\frac{w(ps^t)}{w(s^t)}$
$\tilde{\rho}$	$w(s^t)$	$>$	$w(s^{t_1})w(s^{t-t_1})$	$<$	$w(s^t)$

timing indifference (Chew and Ho, 1994; Ahlbrecht and Weber, 1996; Arai, 1997; Lovallo and Kahneman, 2000; Eliaz and Schotter, 2007; von Gaudecker, van Soest, and Wengström, 2011). However, in line with our predictions, preference for late resolution seems to be particularly pronounced for positively skewed prospects, i.e. for prospects with small probabilities of the best outcome, and increases with time delay. Epstein and Zin (1991) find preference for late resolution of uncertainty in market data on U.S. consumption and asset returns. As in the case for preference for one-shot resolution, none of the studies so far have controlled for nonlinear probability weighting.

4.5 Fact #6: Risk Dependence of Discounting Behavior

Researchers have been puzzled not only by delay-dependent risk tolerance and preferences with respect to resolution timing but also by other interactions between time and risk, encompassing risk-dependent discounting and diminishing immediacy. As we will show below, these findings can be naturally accommodated within our framework.

Several studies have found that decision makers appear to discount certain future outcomes more heavily than risky ones. Let V_0 denote the *present value* of the prospect $P = (x_1, p; x_2)$ delayed by t periods. Hence, for $\rho = 1$,

$$V_0 = \left([u(x_1) - u(x_2)] \frac{w(ps^t)}{w(s^t)} + u(x_2) \right) w(s^t) \tag{8}$$

Furthermore, let V_t denote the *future value* of P as of t:

$$V_t = [u(x_1) - u(x_2)]\, w(p) + u(x_2). \tag{9}$$

Discounting by $w(s^t)$ yields

$$V_t w(s^t) = ([u(x_1) - u(x_2)]\, w(p) + u(x_2))\, w(s^t). \tag{10}$$

According to standard discounting theory, the present value V_0 should be equal to the discounted value of V_t, namely $V_t w(s^t)$. However, because $w(p) < \frac{w(ps^t)}{w(s^t)}$, actually $V_t w(s^t) < V_0$. Therefore, it seems as if the certain value V_t is discounted more heavily than the (at t equally attractive) future prospect. The difference in the valuations is not caused by different rates of time preference for risky and certain payoffs, however, but by inherent uncertainty changing the nature of the future prospect when evaluated from the point of view of the present rather than from the point of view of the future. *Risk-dependent discounting* was found in several studies. Ahlbrecht and Weber (1997) replicated previous results of Stevenson (1992) only in matching tasks, involving elicitation of V_0 and V_t, but not in choice tasks. In their choice tasks, subjects were asked to choose between a prospect to be played at time t and a certain payment at t. Risk-dependent discounting was tested by varying t. The authors surmised that, as time passes, preference for the prospect over the certain payment should become more pronounced, which was not the case in their choice data, however. How can the absence of an effect in choice tasks be rationalized within our framework? When risky and certain prospects are evaluated concurrently only atemporal risk preferences play a role in subjects' elicited choices. Therefore, subjects' preference ordering over risky and certain payments should remain stable when varying t - this is exactly what Ahlbrecht and Weber found.

The same kind of risk dependence is at work when the revealed preference for a certain smaller present payoff over an allegedly certain larger later payoff decreases substantially when both payoffs are made (objectively) probabilistic. This finding was labeled *diminishing immediacy* (Keren and Roelofsma, 1995; Weber and Chapman, 2005) and motivated Halevy (2008)'s work. Because of the certainty effect, the additional layer of riskiness affects the later payoff much less than the present one because, due to inherent uncertainty, it is viewed as a risky prospect already from the outset.

4.6 Fact #7: Sequence Dependence of Measured Risk Tolerance

An equivalent analysis can be applied to the issue of sequence dependence of prospect valuation. In principle, there are three different methods of establishing a decision maker's value of a prospect $P = (x_1, p; x_2)$ delayed by t periods: the time-first sequence, the risk-first sequence, or the direct method. The time-first sequence encompasses, at the first stage, the elicitation of the present risky prospect which is considered to be equivalent to the future one and, at the second stage, the elicitation of the certainty equivalent of this present risky prospect. The risk-first sequence reverses the elicitation order and assesses the certainty equivalent as of time t first and its present value thereafter. The direct method, finally, elicits the present certainty equivalent of the delayed prospect without any intermediate steps.

When the decision maker is required to state the prospect's value when discounting solely for risk, she ignores the dimension of time and reports V_t, the value of which then gets discounted to $V_t w(s^t)$. Conversely, when discounting for time first, she states the present prospect which is equivalent to the delayed one, evaluated as $\left([u(x_1) - u(x_2)]\frac{w(ps^t)}{w(s^t)} + u(x_2)\right) w(s^t)$. Discounting for risk at the second stage results in its value V_0, which is equal to the present value elicited by the direct method.

Therefore, we predict that discounting for risk first results in a lower prospect valuation than discounting for time first. Moreover, discounting for time first is equivalent to prospect evaluation in one single operation. In their study on sequence dependence, Öncüler and Onay (2009) indeed found this pattern: While valuations resulting from the time-risk sequence and the direct method are not statistically distinguishable from each other, risk-time evaluations are significantly lower than the ones obtained from the other two methods (see also Ahlbrecht and Weber (1997)).

5 Discussion

Most economically important decisions, may they concern health, wealth, love or education involve a significant interval between the time that the relevant decision must be made and the time that all uncertainty is completely resolved. Therefore, our theoretical models of decision making should be able to handle these situations in a satisfactory way. Mounting evidence of significant interaction effects between time and risk call the descriptive validity of the standard models into question that view discounting for risk and discounting for time as independent operations.

Our approach provides not only a unifying explanation for seven puzzling facts uncovered by experimental research but also a novel view on perplexing real-world behaviors. For example, people buy warranties for household appliances at exorbitant prices but are reluctant to buy adequate health insurance unless forced to do so by law, even though many will agree that health is the most valuable good in one's life. Similarly, people seem overly risk averse when investing in the stock market but are not willing to buy highly subsidized insurance for natural disasters even though life and property are at stake. These examples suggest that risk tolerance, rather than being a manifestation of stable attitudes depends on the nature of the decision at hand. The puzzle of seemingly volatile preferences can be easily solved, however, if one accounts for the dimension of time along which real-world decisions typically differ. If people perceive the future as inherently uncertain our model of subproportional preferences predicts revealed risk tolerance to vary systematically with the timing and the process of uncertainty resolution. The longer the time horizon and the lower the frequency of feedback on uncertainty resolution the comparatively more risk tolerant decision makers will appear to be. Thus, the model provides a wide range of testable predictions that will generate new insights into people's economic behavior.

Since warranties for household appliances are typically rather short-term and products are used on a daily basis, consumers, anticipating their disappointment in the case of breakdown, will be easily persuaded to buy warranties. However, when deciding on health insurance they will be much more risk tolerant because health is anticipated to deteriorate very slowly and often does so imperceptibly for a long time. Similarly, it is hard to predict when natural disasters will actually occur and, therefore, potential floods and earthquakes are not on people's minds. Stock market investors' time horizons may also be long-term in principle but, contrary to the health and disaster insurance cases, information on portfolio performance is easily accessible and, due to its omnipresence in the news, hard to ignore. Frequent checking of newspapers and news tickers will substantially counteract the otherwise risk-tolerance increasing effect of long investment horizons. Delay- and process-dependent risk tolerance not only affects individuals' welfare but also society at large. People's reluctance to take out insurance for floods and earthquakes, for example, poses serious problems when disaster actually strikes. It is practically impossible for the public authorities to deny assistance once there are identified victims and their stories are publicized in the news (Viscusi, 2010). In the context of climate policy, it takes decades or even centuries until the stock of pollutants will be sufficiently reduced to see any gaugeable effect of society's abatement endeavors. If there is both great uncertainty about the effectiveness of abatement policies and lack of feedback, the risk tolerance of a large percentage of the population may be extremely high and, therefore, it is likely that they are opposed to supporting abatement measures.

The ultimate driver of our results is the certainty effect, i.e. people's tendency to give greater weight to certain outcomes than to uncertain outcomes. This effect not only produces higher risk tolerance for future prospects but essentially all the other interactions between time and risk found in the experimental data. But where does the certainty effect come from? Unfortunately, little is known empirically about the psychological mechanism producing common-ratio violations and the certainty effect. In the course of the paper, we have argued that disappointment aversion is a likely candidate for the source of probability-dependent risk preferences. However, it is still an open question whether emotional processes such as disappointment aversion, the psychophysics of perception, or simply some error of judgment is the driving force of behavior. In any case, when confronted with their allegedly irrational behavior in Allais-type situations many people insist on their original choices (MacCrimmon, 1968; Slovic and Tversky, 1974). Thus, the certainty effect does not arise from an error of judgment but seems to constitute a deeply rooted preference.

Appendix A: Dynamic Consistency, Myopia and Sophistication

From a normative point of view, there are two kinds of, logically distinct, problems with hyperbolic discounting. First, hyperbolic discounting violates stationarity, i.e. the requirement that the discount weight over a certain delay remains constant when the delay is shifted into the future. Second, there may be situations in which the decision maker is dynamically inconsistent, i.e. choices made or plans formed at different times do not coincide. In order to take a closer look at these claims and to pave the ground for Propositions 3 and 4, we use the technique of decision trees. Choice nodes are represented by squares and chance nodes are represented by circles. Without loss of generality, we will henceforth assume that the true discount weight ρ equals 1. In the decision trees in Figure 6, the upper branch represents an option U, the lower branch represents an option D. In Figure 6a, the option U entails an, allegedly certain, larger outcome x, to be paid in one period. The option D entails a smaller outcome y, payable immediately. Assume that the decision maker prefers the smaller immediate outcome y, i.e. $u(x)w(s) < u(y)$.

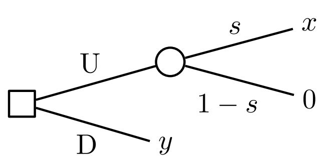
Now suppose that the decision maker faces the choice between x delayed by two periods and y delayed by one period, i.e. both outcomes are shifted into the future by one period. Option U in Figure 6b is associated with a value of $u(x)w(s^2)$ in this case, and option D is associated with $u(y)w(s)$. Due to subproportionality of probability weights w, $(w(s))^2 < w(s^2)$ and hence

$$\frac{u(x)w(s)}{u(y)} = \frac{u(x)w(s)w(s)}{u(y)w(s)} < \frac{u(x)w(s^2)}{u(y)w(s)}. \tag{11}$$

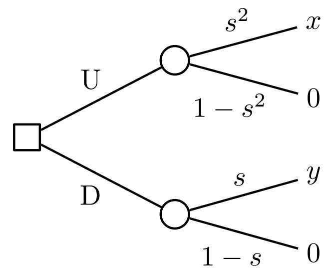
Therefore, the relative value of option U increases and may lead to a change of preference in favor of the larger later outcome x. This type of preference reversal has become known as *common difference effect*, a violation of stationarity, and constitutes one of the most robust empirical findings in intertemporal choice. In the framework of our model, the same mechanism that is responsible for common-ratio violations in risky choice produces violations of stationarity in intertemporal choice if inherent uncertainty comes into play. The parallelism between common-ratio violations in atemporal risky choice and violations of stationarity in intertemporal choice was noted by Prelec and Loewenstein (1991).

Let us assume that the common-ratio effect is sufficiently strong such that the decision maker choses option U in this decision situation, i.e. $u(x)w(s^2) > u(y)w(s)$. What happens if the

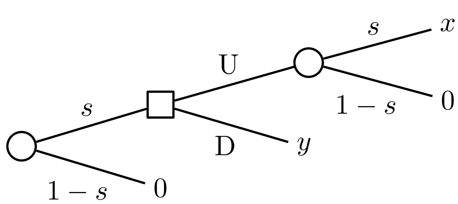
Figure 6: Static Choice, Dynamic Choice and Precommitment



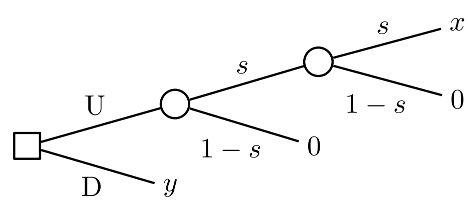
(a)



(b)



(c)



(d)

Tree (a) depicts the choice between an amount x to be paid next period if the prospect survives, and an amount y payable immediately, with $x > y$. The probability of prospect survival is denoted by s. In Tree (b) both options are deferred by one period. In Tree (c) the decision maker does not decide immediately over the deferred options, but at the end of the first period. Tree (d) represents the case of precommitment.

decision maker does not decide now but rather at the end of the first period? This decision situation is depicted in Figure 6c. From the point of view of the present, future uncertainty has to resolve favorably for the options to be still available at the end of the first period. Therefore, the decision maker effectively faces a genuinely dynamic two-stage problem.[14] Since preferences are defined over single-stage risks, multi-stage decision problems have to be transformed into single-stage ones by an appropriate mechanism. An obvious candidate is reduction by the calculus of probability. In this case, the probabilities of reaching the final outcomes are compounded and probability weighting is applied only to the resulting compounded probabilities. This procedure renders $u(x)w(s^2)$ for option U and $u(y)w(s)$ for option D and, therefore, a preference for U. At the end of the first period however, the options are valued as $u(x)w(s)$ and $u(y)$, respectively, leading to a change of plan in favor of D. Unless the decision maker foresees how she will behave in the future and precommits to maintain her original plan of choosing U, she will exhibit dynamically inconsistent behavior. Therefore, revealed behavior over time depends on several factors: the characteristics of atemporal risk preferences, the reduction method, and the use of precommitment. In a recent experiment Halevy (2011) finds that half of his subjects are time consistent, but only two thirds of them exhibit stationary choices. On the other hand, half of the inconsistent subjects display stationary preferences.

In a carefully designed experiment Starmer, Cubitt, and Sugden (1998) show that, in the context of atemporal dynamic risky choices, there is indeed a highly significant difference between behavior in situations with and without precommitment (see also Nebout and Dubois (2012)). In the situation without precommitment the majority of subjects, 71%, choose (in our notation) option D whereas in the (forced) precommitment case, which corresponds to the situation in Figure 6d, only a minority of 43% do so. Hence it seems to make a fundamental difference whether the choice has to be made now or later. This inconsistency constitutes a violation of the principle of timing independence, which requires that at each decision node the decision maker chooses the same path as in the corresponding tree where she precommits to a certain strategy.

Several authors made a case against reduction as an appropriate mechanism of transforming multi-stage prospects into single-stage ones. Segal (1990) argues that even if the decision maker accepts the basic laws of probability theory she may have a preference over the number of lot-

[14] A decision situation is dynamic if there is at least one chance node preceding a choice node (Machina, 1989).

teries she participates in, which invalidates reduction by probability calculus. Segal replaces the reduction axiom by a different axiom, compound independence,[15] which ensures the applicability of folding back as transformation mechanism. Folding back means that a two-stage prospect is evaluated recursively by replacing the second-stage prospect with its certainty equivalent and inserting the utility of the certainty equivalent into the single-stage valuation formula. If the decision maker puts herself into the shoes of her future self facing the decision, she will prefer D just as in the first decision problem and then discount the value of $u(y)$ to the present, which yields $u(y)w(s)$. The present value of option U amounts to $u(x)w(s)w(s)$ in this case. If this method is employed, the decision maker's behavior is consistent in the sense that her preferred option D today will also be the preferred option when she actually decides, i.e. she sticks to her original plan of action.

There is a severe problem with folding back, however. Given the decision maker's preferences in the previous decision situations, the following relationship holds:

$$u(x)w(s)w(s) < u(y)w(s) < u(x)w(s^2), \tag{12}$$

which implies that the decision maker would fare better (in terms of present utility) if she chose U instead of D at the end of the first period, i.e. if she precommitted herself to the plan yielding the compounded final prospect value. Therefore, folding back with subproportional preferences comes at a cost even though it is dynamically consistent. For this reason we will term sequential evaluation of multi-stage prospects by folding back as *myopic* and consistency with compounded final-stage evaluation as *sophisticated*.[16]

The decision situation with precommitment is depicted in Figure 6. Presumably, if the decision is made at the end of the first period rather than immediately, the multi-stage nature of the prospect becomes salient and folding back seems to be a natural, and prima facie perfectly

[15] Let $A = (Z_1, q_1; ...; X, q_i; ...; Z_m, q_m)$ be a two-stage prospect yielding m single-stage prospects Z_j with probabilities q_j, $j \in \{1, ..., i-1, i+1, ..., m\}$, and X with probability q_i, and let $B = (Z_1, q_1; ...; Y, q_i; ...; Z_m, q_m)$ yielding Z_j with probabilities q_j, $j \in \{1, ..., i-1, i+1, ..., m\}$, and Y with probability q_i. Compound independence holds if $A \succeq B \iff (X, 1) \succeq (Y, 1)$ (Segal, 1990).

[16] We do not want to imply that myopia, as defined here, is irrational, however. Loomes and Sugden (1986) argue that "...people seek consistently to maximize expected satisfaction, where that expectation includes the anticipation of possible disappointment and elation. We cannot see any reason for regarding such a maximand as irrational; nor do we think that any simple experience of satisfaction, whatever its source, can be designated either rational or irrational" (p.280).

rational, evaluation strategy. If the decision maker evaluates her options by folding back she will still choose D. However, if she integrates the probabilities of survival over the two periods into a single number, i.e. if she employs reduction by probability calculus, she ends up choosing option U. Irrespective of transformation strategy, precommitment serves an important purpose: It either ensures dynamic consistency (reduction) or maximum utility (folding back).

To get an impression of what kind of costs of myopic behavior may be involved consider the following illuminating example discussed by Palacios-Huerta (1999).

Example: The Costs of Sequential Evaluation
"On a given day in June 1994, in Los Angeles, the national soccer teams from Brazil and Italy played in the World Cup final. As most people in the world did, a well-known Brazilian professor of economics in the United States watched the game. After the regulation time the game was tied. After an extra thirty minutes the game remained tied. The soccer champion of the world for the next four years then had to be decided in a five-penalty-kick shoot-out. The professor then switched off his television set, as perhaps did many other people, especially Brazilians and Italians....Why did he do it?" (Palacios-Huerta (1999), p.250). Palacios-Huerta argues that taking the professor through the process of watching the penalty shoot-out increases the number of times that some disappointment may occur and, in this sense the process itself generates a loss of utility, the costs of emotional involvement.[17]

As recent theoretical developments show, nonlinear probability weighting can indeed be rationalized by anticipated emotions of elation and disappointment (Bell, 1982; Gul, 1991; Walther, 2003). For subproportional preferences, $(w(s))^m < w(s^m)$ is implied and, therefore, the difference between $w(s^m)$ and $(w(s))^m$ can be interpreted as an affect premium, the costs of evaluating an m-stage prospect sequentially rather than in one shot.[18] In the soccer example above, the professor avoids these costs by turning off the TV, i.e. by precommitment to be informed only of the final outcome of the shoot-out. In our terminology, he acts in a sophisticated way.[19] If precommitment is possible but costly, the affect premium provides a boundary for the costs of precommitment the decision maker is willing to incur.

In the following we apply this analysis to the valuation of two-outcome risky prospects. Let us abstract from the passage of real time and consider atemporal two-stage problems first. Assume that the prospect $(x_1, p; x_2)$ gets resolved in two stages $((x_1, r; x_2), q; (x_2, 1))$ such that $p = qr$.

[17]A similar reasoning is presented by Loomes and Sugden (1986).
[18]Dillenberger (2010) analyzes this premium in a general context.
[19]For another example of myopia versus sophistication, in the context of casino gambling, see Barberis (2012).

Applying folding back to the two-stage prospect $((x_1, r; x_2), q; (x_2, 1))$ renders a valuation of

$$[u(x_1) - u(x_2)]\, w(q)w(r) + u(x_2). \tag{13}$$

The value of its single-stage counterpart $(x_1, p; x_2) = (x_1, qr; x_2)$ amounts to

$$[u(x_1) - u(x_2)]\, w(qr) + u(x_2). \tag{14}$$

Subproportionality of w implies that $w(qr) > w(q)w(r)$, i.e. one-shot resolution of uncertainty is always preferred to gradual resolution. If gradual resolution is involved, the decision maker looks comparatively more risk averse than in the one-shot case. The difference between sequential and one-shot values, the affect premium, increases with the number of stages m.

The ratio $\frac{w(p)}{w(q)w(r)}$ provides a measure for the strength of the sequential evaluation effect, which exhibits a systematic relationship with respect to probability p (note that q is constant here):

$$\begin{aligned} \frac{\partial\left[\frac{w(p)}{w(p/q)}\right]}{\partial p} =& \frac{w(p)}{pw(p/q)}\left(\frac{w'(p)p}{w(p)} - \frac{w'(p/q)(p/q)}{w(p/q)}\right) \\ =& \frac{w(p)}{pw(p/q)}(\varepsilon_w(p) - \varepsilon_w(p/q)) \\ <& 0, \end{aligned} \tag{15}$$

as $p/q > p$ and the elasticity of w is increasing. Therefore, the wedge between one-shot evaluation and sequential evaluation is largest for highly unlikely prospects and decreases with p.

As is clear from Equation 13, it plays no role which stages probabilities q and r are attached to. In this sense, valuation by folding back is symmetrical. However, it makes a difference how total probability p is subdivided. The value of the two-stage prospect attains its minimum for $q = r = \sqrt{p}$, i.e. when the two stages are least degenerate (Segal, 1990). To see this let us examine the derivative of $w(q)w(r)$ w.r.t. r subject to the constraint that $p = qr$:

$$\begin{aligned} \frac{\partial\left[w(q)w(p/q)\right]}{\partial q} =& w'(q)w(p/q) + w(q)w'(p/q)(-p/q^2) \\ =& 0 \end{aligned} \tag{16}$$

$$
\begin{aligned}
&\Rightarrow \quad \frac{w'(q)}{w(q)}q = \frac{w'(r)}{w(r)}r \\
&\Rightarrow \quad q = r = \sqrt{p}
\end{aligned} \tag{17}
$$

because the elasticity of w is increasing. Therefore, the common-ratio effect observed in single-stage valuations carries over to two-stage prospects.[20] These insights have important implications for our problem of temporal prospect valuation when the resolution timing of the prospect does not coincide with the resolution timing of inherent uncertainty or when uncertainty resolves gradually, analyzed in Propositions 3 and 4.

A Note on Sequential Evaluation

In his Proposition 1, Dillenberger (2010) shows that, under recursivity, negative certainty independence (*NCI*) and a weak preference for one-shot resolution of uncertainty (*PORU*) are equivalent. The *NCI* axiom requires the following to hold: If a prospect $P = (x_1, r; x_2)$ is weakly preferred to a degenerate prospect $D = (y, 1)$ then mixing both with any other prospect does not result in the mixture of the degenerate prospect D being preferred to the mixture of P. This axiom is weaker than the standard independence axiom and does not put any restrictions on the reverse preference relation when a degenerate prospect is originally preferred to a nondegenerate one. The latter case characterizes the typical Allais common-ratio paradox. *NCI* allows for Allais-type preference reversals but does not imply them. Dillenberger's Proposition 3 demonstrates that *NCI* is generally incompatible with rank-dependent utility unless the probability weighting function is linear, i.e. unless *RDU* collapses to *EUT*. An intuitive explanation for Dillenberger's Proposition 3 is that under *RDU* prospect valuation is sensitive to the rank order of the outcomes and, therefore, mixtures with other prospects may affect the original rank order of outcomes in P (and D). How does Dillenberger's result relate to our claim that subproportional probability weights conjointly with recursivity imply a strong preference for one-shot resolution of uncertainty?

The crucial insight is that for the class of prospects studied in this paper changes in rank order do not occur and, hence, *NCI* is satisfied. To see this, assume that the prospect $(x_1, p; x_2)$, $x_1 > x_2 \geq 0$, gets resolved in two stages $((x_1, r; x_2), q; (x_2, 1))$ such that $p = qr$. In the atemporal case, when there is no additional inherent uncertainty, the two-stage prospect continues to be a strictly two-outcome one and the only relevant mixtures are those involving x_2. Suppose that $P = (x_1, r; x_2) \succeq (y, 1) = D$, with $x_1 > y > x_1$ and consider the following mixtures with $(x_2, 1 - \lambda)$ for some $\lambda \in (0, 1)$: $P' = (x_1, \lambda r; x_2)$ and $D' = (y, \lambda; x_2)$. The following relationships

[20] Segal (1987b) utilizes this result to explain the Ellsberg Paradox .

hold:

$$
\begin{aligned}
P \succeq D \Rightarrow V(P) &= [u(x_1) - u(x_2)]\, w(r) + u(x_2) \geq u(y) \\
V(D') &= u(y)w(\lambda) + u(x_2)(1 - w(\lambda)) \\
&\leq ([u(x_1) - u(x_2)]\, w(r) + u(x_2))\, w(\lambda) + u(x_2)(1 - w(\lambda)) \\
&= [u(x_2) - u(x_1)]\, w(r)w(\lambda) + u(x_2) \\
&< [u(x_2) - u(x_1)]\, w(\lambda r) + u(x_2) \\
&= V(P')
\end{aligned}
\tag{18}
$$

because $w(r)w(\lambda) < w(\lambda r)$ for any $\lambda \in (0,1)$ (and hence also for $\lambda = q$) due to subproportionality of w. Consequently, for mixtures with the smaller outcome x_2, NCI, and therefore also $PORU$, is *strongly* satisfied. If the mixing prospect may be any arbitrary prospect, in other words if surprises are possible in the course of uncertainty resolution, this result does not hold generally. The only surprise that is still admissible is the occurrence of an outcome worse than x_2, say z. Define $P'' = (x_1, \lambda r; x_2, \lambda(1-r); z)$ and $D'' = (y, \lambda; z)$.

$$
\begin{aligned}
V(D'') &= u(y)w(\lambda) + u(z)(1 - w(\lambda)) \\
&\leq ([u(x_1) - u(x_2)]\, w(r) + u(x_2))\, w(\lambda) + u(z)(1 - w(\lambda)) \\
&= [u(x_2) - u(x_1)]\, w(r)w(\lambda) + [u(x_2) - u(z)]\, w(\lambda) + u(z) \\
&< [u(x_2) - u(x_1)]\, w(\lambda r) + [u(x_2) - u(z)]\, w(\lambda) + u(z) \\
&= V(P'')
\end{aligned}
\tag{19}
$$

For $z = 0$, this case is exactly the situation studied in this paper when inherent uncertainty comes into play.

Appendix B: Proofs of Propositions

Proof of Proposition 1

.

1. Since $\tilde{w}(0) = \frac{w(0)}{w(s^t)} = 0$, $\tilde{w}(1) = \frac{w(s^t)}{w(s^t)} = 1$, and $\tilde{w}' = \frac{w'(ps^t)s^t}{w(s^t)} > 0$ hold, $\tilde{w}$ is a proper probability weighting function.

2. Subproportionality of $\tilde{w}$ follows directly from subproportionality of w as for $p > q$:

$$
\frac{\tilde{w}(\lambda p)}{\tilde{w}(\lambda q)} = \frac{w(\lambda s^t p)}{w(\lambda s^t q)} < \frac{w(s^t p)}{w(s^t q)} = \frac{\tilde{w}(p)}{\tilde{w}(q)}
\tag{20}
$$

3. Since w is subproportional,

$$\tilde{w}(p) = \frac{w(ps^t)}{w(s^t)} > \frac{w(ps)}{w(s)} > \frac{w(p)}{w(1)} = w(p) \tag{21}$$

holds for $s < 1$ and $t > 1$. Therefore, $\tilde{w}$ is more elevated than w. Obviously, elevation gets progressively higher with increasing t and an equivalent effect is produced by decreasing s. Since $\tilde{w}$ increases monotonically in t and $\tilde{w} \leq 1$ for any t, elevation increases at a decreasing rate.

4. For the elasticity of $\tilde{w}$, $\varepsilon_{\tilde{w}}(p)$, the following relationship holds:

$$\varepsilon_{\tilde{w}}(p) = \frac{\tilde{w}'(p)p}{\tilde{w}(p)} = \frac{w'(ps^t)ps^t}{w(ps^t)} = \varepsilon_w(ps^t) < \varepsilon_w(p), \tag{22}$$

as the elasticity ε_w is increasing in its argument iff w is subproportional (Segal, 1987a).

5. In order to show that a comparatively more subproportional probability weighting function entails a greater increase in observed risk tolerance we examine the relationship between the underlying atemporal probability weights w and observed ones $\tilde{w}$. Let w_1 and w_2 denote two probability weighting functions, with w_2 exhibiting greater subproportionality.

If $w_1(\lambda)w_1(p) = w_1(\lambda pq)$ holds for a probability $q < 1$, then $w_2(\lambda)w_2(p) < w_2(\lambda pq)$ follows as w_2 is more subproportional than w_1 (Prelec, 1998). Choose $r < 1$ such that $w_2(\lambda)w_2(p) = w_2(\lambda pqr)$. For $\lambda = s^t$, the following relationships hold:

$$\frac{\tilde{w}_1(p)}{w_1(p)} = \frac{w_1(\lambda p)}{w_1(\lambda)w_1(p)} = \frac{w_1(\lambda p)}{w_1(\lambda)w_1(p)} \frac{w_1(\lambda)w_1(p)}{w_1(\lambda pq)} = \frac{w_1(\lambda p)}{w_1(\lambda pq)}. \tag{23}$$

Applying the same logic to w_2 yields

$$\frac{\tilde{w}_2(p)}{w_2(p)} = \frac{w_2(\lambda p)}{w_2(\lambda)w_2(p)} = \frac{w_2(\lambda p)}{w_2(\lambda pqr)} > \frac{w_2(\lambda p)}{w_2(\lambda pq)}. \tag{24}$$

Therefore, the relative wedge $\frac{\tilde{w}_2(p)}{w_2(p)}$ caused by subproportionality is larger than the corresponding one for w_1. ■

Proof of Proposition 2

1. $\tilde{\rho}(0) = w(s^0)\rho^0 = 1$. Since $w' > 0$ holds, $\frac{\partial w(s^t)}{\partial t} < 0$ and, therefore, $\tilde{\rho}' < 0$. Finally, $\lim_{t\to\infty} \tilde{\rho}(t) = 0$ (in terms of discount rates: $\lim_{t\to\infty} \tilde{\eta}(t) = \eta$).

2. Discount rates are generally defined as the rates of decline of the respective discount functions, i.e. $\eta = -\frac{\rho'(t)}{\rho(t)}$ and $\tilde{\eta}(t) = -\frac{\tilde{\rho}'(t)}{\tilde{\rho}(t)}$. Therefore,

$$\begin{aligned}
\tilde{\eta}(t) &= -\frac{\tilde{\rho}'(t)}{\tilde{\rho}(t)} \\
&= -\frac{w'(s^t)s^t \ln(s)\exp(-\eta t) - w(s^t)\exp(-\eta t)\eta}{w(s^t)\exp(-\eta t)} \\
&= -\left(\frac{w'(s^t)s^t}{w(s^t)}\ln(s) - \eta\right) \\
&= -\ln(s)\varepsilon_w(s^t) + \eta \\
&> \eta
\end{aligned} \tag{25}$$

since $\ln(s) < 0, w > 0, w' > 0$. Note that $\frac{w'(s^t)}{w(s^t)}s^t$ corresponds to the elasticity of the probability weighting function w evaluated at s^t, $\varepsilon_w(s^t)$.

3. Since the elasticity of a subproportional function is increasing in its argument, the elasticity of $w(s^t)$ is decreasing in t. Thus,

$$\tilde{\eta}'(t) = -\ln(s)\frac{\partial \varepsilon_w(s^t)}{\partial t} < 0. \tag{26}$$

4. In order to derive the effect of increasing uncertainty, i.e. decreasing s, we examine the sensitivity of $\frac{\tilde{\rho}(t+1)}{\tilde{\rho}(t)\tilde{\rho}(1)} = \frac{w(s^{t+1})}{w(s)w(s^t)}$, which measures the departure from constant discounting between periods $t+1$ and t, with respect to changing s:

$$
\begin{aligned}
& \frac{\partial}{\partial s}\left[\frac{w(s^{t+1})}{w(s)w(s^t)}\right] \\
= \quad & \frac{1}{[w(s)w(s^t)]^2}\left[(1+t)s^t w(s)w(s^t)w'(s^{t+1}) - ts^{t-1}w(s)w(s^{t+1})w'(s^t) - w(s^t)w(s^{t+1})w'(s)\right] \\
= \quad & \frac{1}{s[w(s)w(s^t)]^2}\left[(1+t)s^{t+1} w(s)w(s^t)w'(s^{t+1}) - ts^{t}w(s)w(s^{t+1})w'(s^t) - sw(s^t)w(s^{t+1})w'(s)\right] \\
= \quad & \frac{w(s^{t+1})}{sw(s)w(s^t)}\left[\frac{(1+t)s^{t+1}w'(s^{t+1})}{w(s^{t+1})} - \frac{ts^t w'(s^t)}{w(s^t)} - \frac{sw'(s)}{w(s)}\right] \\
= \quad & \frac{w(s^{t+1})}{sw(s)w(s^t)}\left[(1+t)\varepsilon_w(s^{t+1}) - t\varepsilon_w(s^t) - \varepsilon_w(s)\right] \\
< \quad & 0.
\end{aligned}
$$

As $s^{t+1} < s^t < s$, $\varepsilon_w(s^{t+1}) < \varepsilon_w(s^t) < \varepsilon_w(s)$ and, hence, the sum of the elasticities in the final line of the derivation is negative. Therefore, increasing uncertainty, i.e. decreasing s, entails a greater departure from constant discounting and, consequently, a higher degree of hyperbolicity.

5. In order to examine the effect of the degree of subproportionality on hyperbolicity, the strength of decline, suppose that the probability weighting function w_2 is comparatively more subproportional than w_1, as defined in Prelec (1998), and that the following indifference relations hold for two decision makers 1 and 2 at periods 0 and 1:

$$
\begin{aligned}
u_1(y) = u_1(x)w_1(s)\rho \quad \text{for } 0 < y < x, \\
u_2(y') = u_2(x')w_2(s)\rho \quad \text{for } 0 < y' < x'.
\end{aligned}
\tag{27}
$$

Due to subproportionality, the following relation holds for decision maker 1 in period t:

$$
1 = \frac{u_1(x)w_1(s)\rho}{u_1(y)} < \frac{u_1(x)w_1(s^{t+1})\rho^{t+1}}{u_1(y)w_1(s^t)\rho^t}. \tag{28}
$$

Therefore, the subjective probability of prospect survival has to be reduced by compounding s over an additional time period Δt to re-establish indifference:

$$
u_1(y)w_1(s^t)\rho^t = u_1(x)w_1(s^{t+1+\Delta t})\rho^{t+1}. \tag{29}
$$

It follows from the definition of comparative subproportionality that this adjustment of the survival probability by Δt is not sufficient to re-establish indifference with respect to w_2, i.e.

$$u_2(y')w_2(s^t)\rho^t < u_2(x')w_2(s^{t+1+\Delta t})\rho^{t+1}. \blacksquare \tag{30}$$

Proof of Proposition 3

1. Consider the tree in Figure 3. Here, both prospect uncertainty and inherent uncertainty are assumed to resolve simultaneously in two stages, partially at t_1 and finally at t. Applying folding back, the resulting two-stage prospect is evaluated as

$$[u(x_1) - u(x_2)]\, w(p^{t_1/t}s^{t_1})w(p^{(t-t_1)/t}s^{t-t_1}) + u(x_2)w(s^{t_1})w(s^{t-t_1}). \tag{31}$$

   Subproportionality implies that $w(p^{t_1/t}s^{t_1})w(p^{(t-t_1)/t}s^{t-t_1}) < w(ps^t)$ and $w(s^{t_1})w(s^{t-t_1}) < w(s^t)$.

2. Follows directly from the derivation in Equation 15 in Appendix A.

3. Using the result of the derivation in Equation 16 in Appendix A, both utility weights attain their respective minima at $t_1 = \frac{t}{2}$ when partial probabilities are equal. $\blacksquare$

Proof of Proposition 4

1. Consider the graphs in Fig. 5. The tree on the right-hand side represents the subjective payoff probabilities if prospect uncertainty is resolved at the time of payment t. As discussed, this prospect is evaluated as $\left([u(x_1) - u(x_2)]\, \frac{w(ps^t)}{w(s^t)} + u(x_2)\right) w(s^t)$. If, however, prospect uncertainty is resolved immediately after prospect valuation the decision maker will know whether she is supposed to receive x_1 or x_2 at t. Therefore, after resolution of prospect uncertainty both possible outcomes are only affected by inherent uncertainty and get devalued by $w(s^t)$. This situation is shown on the left-hand side of Figure 5. Hence, the value of the prospect immediately before the prospect is played out amounts to

$$\begin{aligned} &[u(x_1) - u(x_2)]\, w(p)w(s^t) + u(x_2)w(s^t) \\ &= ([u(x_1) - u(x_2)]\, w(p) + u(x_2))\, w(s^t). \end{aligned} \tag{32}$$

As $w(ps^t) > w(p)w(s^t)$ is implied by subproportionality of w, prospects with resolution at the date of payment t are valued more highly than prospects with immediate resolution. In fact, in case of immediate resolution of prospect uncertainty, observed risk tolerance coincides with true risk tolerance and the present value of the prospect is only affected by (hyperbolic) discounting.

What happens if prospect uncertainty is not resolved immediately but rather at some later time t_1, $0 < t_1 < t$? After t_1, only inherent uncertainty remains to be resolved. In this case, the prospect's present value amounts to

$$\left([u(x_1) - u(x_2)] \frac{w(ps^{t_1})}{w(s^{t_1})} + u(x_2) \right) w(s^{t_1}) w(s^{t-t_1}). \tag{33}$$

Subproportionality implies $w(p) < \frac{w(ps^{t_1})}{w(s^{t_1})} < \frac{w(ps^t)}{w(s^t)}$ and, therefore, observed risk tolerance is highest for resolution at payoff time t. Moreover, the late-resolution discount weight $w(s^t) = w(s^{t_1}s^{t-t_1})$ is also greater than $w(s^{t_1})w(s^{t-t_1})$ for any earlier t_1, implying that late resolution is always preferred.

2. Examining the derivative of $\frac{w(ps^t)}{w(p)}$ with respect to p yields

$$\begin{aligned} \frac{\partial \left[\frac{w(ps^t)}{w(p)} \right]}{\partial p} &= \frac{w(ps^t)}{pw(p)} \left(\frac{w'(ps^t)ps^t}{w(ps^t)} - \frac{w'(p)p}{w(p)} \right) \\ &= \frac{w(ps^t)}{pw(p)} (\varepsilon_w(ps^t) - \varepsilon_w(p)) \\ &< 0, \end{aligned} \tag{34}$$

as $p > ps^t$ and the elasticity is increasing. Therefore, the wedge between late evaluation and immediate evaluation decreases with p.

3. The derivative of $\frac{w(ps^t)}{w(s^t)}$ with respect to t yields

$$\begin{aligned} \frac{\partial \left[\frac{w(ps^t)}{w(s^t)} \right]}{\partial t} &= \frac{\ln(s) w(ps^t)}{w(s^t)} \left(\frac{w'(ps^t)ps^t}{w(ps^t)} - \frac{w'(s^t)s^t}{w(s^t)} \right) \\ &= \frac{\ln(s) w(ps^t)}{w(s^t)} (\varepsilon_w(ps^t) - \varepsilon_w(s^t)) \\ &> 0, \end{aligned} \tag{35}$$

as $\ln(s) < 0$, $s^t > ps^t$ and the elasticity is increasing. Therefore, the wedge between late and immediate evaluation increases with time horizon t and, equivalently, with inherent uncertainty $1 - s$. ■

5.1 Miscellaneous Remarks

On the Necessity of Subproportionality

Clearly, subproportionality is sufficient to produce all the results of Propositions 1 and 2 (as well as of all the following propositions). But is subproportionality, aside from inherent uncertainty $1 - s > 0$, also necessary? For statements that refer to the present it is necessary that preferences exhibit the certainty effect, i.e. that $w(p)w(q) < w(pq)$ for any $p, q < 1$, which is implied by but does not imply subproportionality. Therefore, the result that risk tolerance is higher for future prospects than for present ones does not depend on subproportionality, only on the certainty effect. However, for statements pertaining to relationships between behaviors at different times in the future, for instance, that risk tolerance is increasing in t or that discount weights decline hyperbolically, subproportionality is necessary (for a proof with respect to observed discount weights see Saito (2011)). For example, preferences that are not generally subproportional but exhibit the certainty effect, such as the discontinuous weighting function $w(p) = \gamma p$ for $p < 1$ and $w(1) = 1$ defined for $0 < \gamma < 1$, will show an increase in risk tolerance relative to the present as well as quasi-hyperbolic discounting.

Special Cases: Simple and Degenerate Prospects

In our framework a simple prospect (x, p) with one non-zero outcome gets transformed into a prospect (x, ps^t) when it is played out and paid out at t. A degenerate prospect $(x, 1)$ delayed by t is perceived as (x, s^t). Subproportionality implies $\frac{w(1)}{w(s^t)} > \frac{w(p)}{w(ps^t)}$ for $t > 0$ and, therefore, allegedly certain prospects appear to get discounted more heavily than nondegenerate ones, and the effect is more pronounced for low-probability prospects. Note that observations on simple prospects alone do not allow to separate probability weights from discount weights. For this purpose, *nondegenerate two-outcome prospects are needed*.

Application: Constant-Sensitivity Discounting

Ebert and Prelec (2007) argue that time discounting is driven by two distinct forces, impatience and time sensitivity. The authors provide an axiomatic foundation of a constant-sensitivity discount function $\rho(t) = \exp(-(\theta t)^\alpha)$, where α measures time sensitivity and θ measures impatience.[21] θ marks the boundary between the near and far future: Times shorter than $1/\theta$ are in the near future, while times greater than $1/\theta$ are in the far future. So greater impatience leads to more immediate discounting. The time-sensitivity parameter α, on the other hand, decreases discounting for near-future outcomes and increases discounting for far-future ones. This function fits experimental data remarkably well.

In our framework, the discount function is defined as $\tilde{\rho}(t) = w(s^t)$ for $\rho = 1$. Inserting Prelec (1998)'s specification of the probability weighting function w yields $\tilde{\rho}(t) = \exp(-(\theta t)^\alpha)$, with impatience defined as $\theta = -\ln(s)$. The subproportionality parameter $\alpha < 1$, therefore, represents an index for time insensitivity: a 1% increase in delay implies an α% reduction in the

[21] A flexible specification is presented in Bleichrodt, Rohde, and Wakker (2009). See also Prelec (2004).

log-discounted-present-value of the reward. Hence, our model provides a natural link between subproportional probability weighting functions and constant-sensitivity discount functions.

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