



## The diagnosis and management of dyscalculia

Kaufmann, Liane ; von Aster, Michael

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## CONTINUING MEDICAL EDUCATION

# The Diagnosis and Management of Dyscalculia

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## SUMMARY

**Background:** Dyscalculia is defined as difficulty acquiring basic arithmetic skills that is not explained by low intelligence or inadequate schooling. About 5% of children in primary schools are affected. Dyscalculia does not improve without treatment.

**Methods:** In this article, we selectively review publications on dyscalculia from multiple disciplines (medicine, psychology, neuroscience, education/special education).

**Results:** Many children and adolescents with dyscalculia have associated cognitive dysfunction (e.g., impairment of working memory and visuospatial skills), and 20% to 60% of those affected have comorbid disorders such as dyslexia or attention deficit disorder. The few interventional studies that have been published to date document the efficacy of pedagogic-therapeutic interventions directed toward specific problem areas. The treatment is tailored to the individual patient's cognitive functional profile and severity of manifestations. Psychotherapy and/or medication are sometimes necessary as well.

**Conclusion:** The early identification and treatment of dyscalculia are very important in view of its frequent association with mental disorders. Sufferers need a thorough, neuropsychologically oriented diagnostic evaluation that takes account of the complexity of dyscalculia and its multiple phenotypes and can thus provide a basis for the planning of effective treatment.

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**D**ifficulty in learning arithmetic, like difficulty in learning to read and write, is a common learning disorder in childhood. The prevalence of each of these disorders among primary-school pupils is about 5%, a relatively constant figure across the countries in which these disorders have been studied (1, e1). Dyscalculia is often associated with mental disorders (2, 3, e2). Many affected children acquire a negative attitude to counting and arithmetic, which, in turn, often develops into a specific mathematics anxiety or even a generalized school phobia (4). Unless specifically treated, dyscalculia persists into adulthood (5–7); it can lastingly impair personality development, schooling, and occupational training. Dyscalculia is also an economic issue, as adults with poor arithmetic skills suffer a major disadvantage on the job market. About 22% of young adults fall into this category (e3, e4). The mental disorders commonly associated with dyscalculia are expensive to treat (7). Thus, the early recognition and differential diagnosis of learning disorders are an important matter not just for child psychiatrists, who must often deal with the secondary conditions that arise from these disorders, but also for general practitioners and pediatricians, as the delayed acquisition of pre-scholastic skills in the nursery and kindergarten years may already be an early sign of a problem (3). In this article, we selectively review the literature on dyscalculia.

## Learning objectives

This article is intended to acquaint the reader with

- the typical development of numerical and arithmetical skills,
- the instruments used for the evaluation and differential diagnosis of dyscalculia, and
- current methods of treating dyscalculia.

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## Dyscalculia

The two learning disorders dyscalculia and dyslexia each have a prevalence of about 5% among primary-school pupils, a fairly constant figure internationally. Dyscalculia is often associated with mental disorders.

**BOX 1**

Various aspects of numerical-arithmetical functions and relevant skills in each functional area. This detailed categorization is mainly based on neuropsychological and neuroscientific classifications of calculating skills. The literature contains varying definitions and classifications of basic numerical and precursor functions. For the sake of simplicity, we here consider these two aspects together, because both are related to numerical-arithmetical knowledge that is acquired (mainly implicitly) before the child reaches school age.

Numerical-arithmetical functional areas (age/grade)	Skills and tasks
Basic numerical skills and precursor skills (preschool, nursery school, kindergarten)	Understanding of quantity (e.g., tasks requiring a comparison of quantities and numbers), rapid recognition of small quantities, mastery of counting skills, identification of Arabic numerals
Arithmetical factual knowledge (primary school)	Single-digit addition and multiplication, e.g., $3 + 2$ or $3 \times 2$ (see also Box 3)
Arithmetical procedures (from primary school onward)	Knowing the correct sequence of solution steps for multistep calculating tasks
Arithmetical reasoning; conceptual arithmetical knowledge (from primary school onward, highly dependent on teaching methods)	Knowledge of the involved quantities and partial quantities, knowledge of the similarities and differences between different types of operations, comprehension of arithmetical procedures

**The typical development of numerical and arithmetical skills**

Neuroscientific models let us view the development of numerical processing in the brain as a neuroplastic maturation process that leads, over the course of childhood and adolescence, to the establishment of a complex, specialized neural network (2, 8). This development begins with the very simple (basic) numerical skills that give even babies of a few months of age an elementary grasp of quantity and number. Infants can clearly distinguish one item from two items, and they are capable of a rough estimation of number for three or more items (e5). When language begins to appear, children become able to symbolize numbers linguistically with number-words: They learn to count out loud

**Numerical and arithmetical skills**

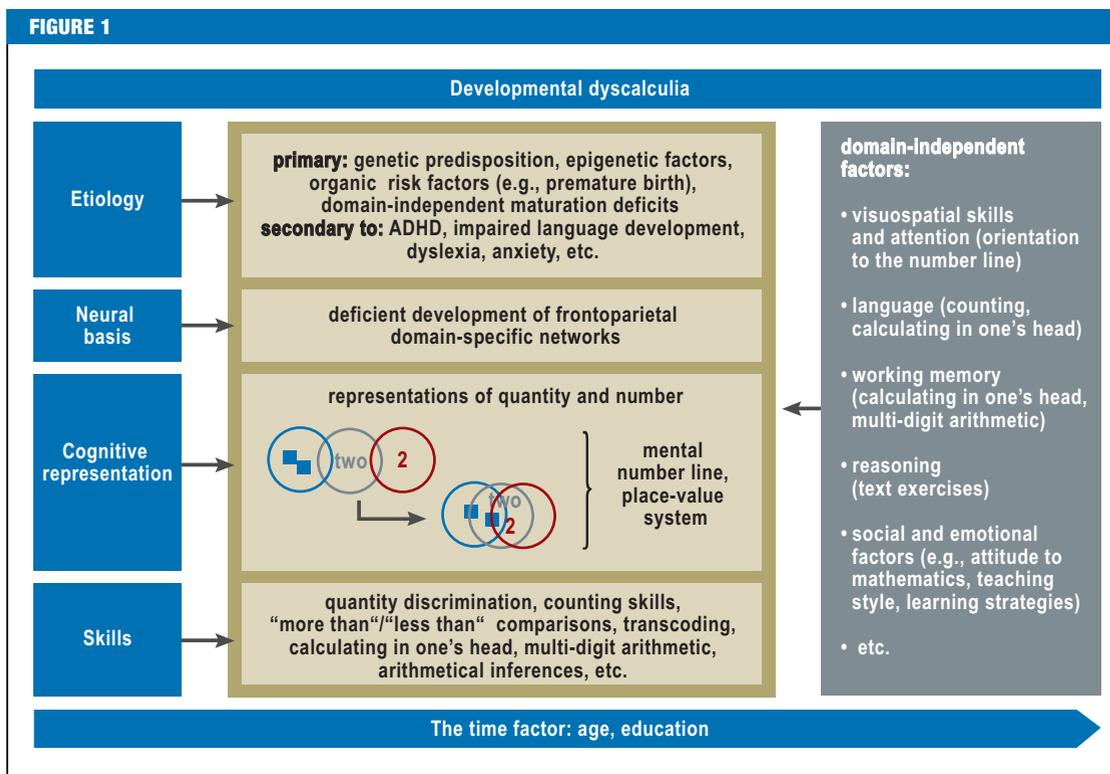
The development of numerical processing in the brain can be seen as a neuroplastic maturation process that leads, over the course of childhood and adolescence, to the establishment of a complex, specialized neural network.

and perform simple, verbal arithmetical manipulation of quantities and numbers. Next, a second way of symbolizing numbers is learned in the preschool and primary school years. This is the Arabic numeral system, with its own “grammar” that differs markedly from the representation of numbers in spoken language. The Arabic place-value system is a highly economical way of symbolizing numbers that simplifies numerical calculations. For example, the number 1 768 329 is written with a mere seven Arabic numerals, while its English expression, “one million seven hundred sixty-eight thousand three hundred twenty-nine,” is 62 letters long, not counting dashes and spaces. Finally, in parallel with the development of the spoken and written (Arabic-numeral) symbolization of numbers and the associated operative capacities, children develop a numerospatial conceptual ability (a “mental number line”) enabling them to operate with numerical symbols. The mental number line seems to be of fundamental importance for arithmetical thinking and for calculating in one’s head (e6). The early, basic numerical skills thus give an initial meaning to the processes of symbolization (number words, Arabic numerals), while the mental line extends the semantic range of the concept of number to a higher, more abstract level. An overview of the relevant aspects of numerical and arithmetical skills is provided in Box 1.

Functional imaging studies have shown that, with increasing practice and expertise, these components coalesce in a neural network in multiple brain areas that is activated when the performance of the corresponding tasks is required (8). Number-words are processed in the speech areas of the left perisylvian region, Arabic numerals in the occipital lobes. Basic numerical representations and the numerical-spatial representations that develop later on are subserved by parietal areas on both sides of the brain that become increasingly functionally specialized as the child grows older and acquires more education (e7, e8). The development and maturation of these domain-specific brain functions depends on the maturation of numerous domain-specific or multi-domain functions, including attention and working memory (mainly subserved by the frontal lobes), language, sensorimotor function (e.g., finger counting), and visuospatial ideation; they also depend on experience (e.g., practice and stimulation in everyday life and the type of teaching methods used). Clearly, at any time in a child’s development, many different factors could disturb or delay the maturation of

**The mental number line**

The mental number line seems to be of fundamental importance for arithmetical thinking and for calculating in one’s head. It extends the semantic range of the concept of number to a higher, more abstract level.



**Potentially causative factors** and different levels on which developmental dyscalculia can manifest itself

these neural networks, causing clinically evident manifestations of various kinds (Figure 1).

### Definition and etiology

#### Definition

Each of the two main classification systems for mental disorders that are currently in use, the ICD-10 (10) and the DSM-IV (11), classifies disorders of the acquisition of written language (ICD-10: F81.0 and F81.1), isolated dyscalculia (F81.2), and dyscalculia combined with dyslexia (F81.3) as independent categories of specific disorders of scholastic skills. Dyscalculia is defined as a serious impairment of the learning of basic numerical-arithmetical skills in a child whose intellectual capacity and schooling are otherwise adequate. It is supposed to be demonstrable by standardized psychometric testing that reveals poor calculating ability despite normal intelligence.

In the DSM-V, due to be published in 2013, the specific developmental disorders of scholastic skills will be

grouped together as dimensions within a single category, in each of which there can be a greater or lesser degree of impairment; there will no longer be a strict requirement for the psychometric demonstration of a discrepancy with otherwise normal intelligence. This change is being introduced to take better account of the heterogeneity of these disorders with respect to performance profiles and comorbidities, and thus to improve the clinical utility of the DSM diagnoses. In situations where proof of a discrepancy is required (as is still the case for medicolegal evaluations in Germany and Austria), we recommend the use of multi-component intelligence tests, consisting of several subtests, which can provide a detailed picture of the child's deficiencies—for example, the current (fourth) version of the Hamburg-Wechsler Intelligence Test for Children (e9).

The literature also reflects a distinction between "disorders" and "disabilities," with the latter term being more commonly used by education professionals. For example, a child is said to have a mathematical learning disability

### ICD-10 and DSM-IV

- Disorders of the acquisition of written language (ICD-10, F81.0 und F81.1)
- Isolated dyscalculia (ICD-10, F81.2)
- Dyscalculia combined with dyslexia (ICD-10 F81.3)

### Proof of discrepancy with normal intelligence

In situations where this is required, we recommend the use of multi-component intelligence tests, consisting of several subtests, which can provide a detailed picture of the child's deficiencies.

**TABLE**

**Comparison of two types of test for numerical-arithmetical skills**

	Curricular tests (school performance tests)	Neuropsychologically oriented tests
Concept	The test is oriented toward the mathematics curriculum in the child's grade. Its main purpose is to determine whether the child has met the learning objectives for his or her grade.	The test is designed to assess basic numerical and arithmetical skills and to take account of non-numerical functions. Its main purpose is to generate a performance profile for numerical-arithmetical skills and to determine the cause of difficulty calculating (i.e., to identify the underlying learning processes and mechanisms).
Manner of testing	Usually designed as power tests (i.e., only the correct answer is relevant); they are also usable as group or class tests (economy of use)	Attention is paid not only to correct answers, but also (at least for relevant components of the test) to processing speed and solving strategies, with the goal of a qualitative assessment of performance; thus, these tests can only be performed on an individual basis.
Diagnostic criterion (in many, but not all tests in the category in question)	PR<10	PR<10 combined with average intellectual performance (according to ICD-10 [10]); some tests not only include a strict cut off at PR<10 but also state the area in which additional help is likely to be needed (e.g., PR 10<25 in TEDI-MATH) (e21)
Examples	ERT for grades 1–4 (e.g., ERT 1+ (e22); ERT 2+ (e23); ERT 3+ (e24); ERT 4+ (e25); DEMAT for grades 1–4: DEMAT 1 (e26); DEMAT 2 (e27); DEMAT 3 (e28); DEMAT 4 (e29)	RZD 2–6 for grades 2–6 (e30); TEDI-MATH for pre-kindergarten to grade 3 (e21); ZAREKI-R for grades 1–4 (e31); ZAREKI-K for preschool children (e32)

PR, percentile rank. The following abbreviations stand for German-language tests: DEMAT, Deutsche Mathematiktests; ERT, Eggenberger Rechentest; RZD, Rechenfertigkeiten- und Zahlenverarbeitungs-Diagnostikum; TEDI-MATH, Test zur Erfassung numerisch-rechnerischer Fertigkeiten; ZAREKI-R, die neuropsychologische Testbatterie für Zahlenverarbeitung und Rechnen bei Kindern, revidierte Version; ZAREKI-K, die neuropsychologische Testbatterie für Zahlenverarbeitung und Rechnen bei Kindern, Kindergarten Version.

All of these tests except ZAREKI are published by Hogrefe ([www.testzentrale.de](http://www.testzentrale.de)).

The ZAREKI-R (e31) and the ZAREKI-K for 4- and 5-year-olds (e32) are available from Pearson Assessment ([www.pearsonassessment.de](http://www.pearsonassessment.de))

(MLD) when his or her performance on a test of mathematical skills falls below an arbitrarily chosen percentile in a study population, without reference to the child's general intelligence level (e10). Variations in defining criteria account for the wide variation among the study populations and testing procedures encountered in published studies, and this renders comparisons across studies difficult. The distinction between dyscalculia, considered as a disorder, and MLD is mainly important for research. In clinical treatment planning, the child's actual performance profile is of central importance, regardless of whether dyscalculia or MLD is said to be present.

**Etiology**

Dyscalculia has many contributory causes. For the ones that will be discussed below, no firm conclusions can yet

be drawn about the relations and interactions between them, as too little evidence is available. There is, however, a consensus in the current literature supporting the multifactorial origin of developmental dyscalculia and other learning disorders (2). Dyscalculia is also often present in children suffering from neurological diseases (e.g., epilepsy, premature birth, metabolic disorders) and genetic syndromes (e.g., fragile X syndrome, Williams-Beuren syndrome, velocardiofacial syndrome).

Developmental dyscalculia tends to run in families (e11), possibly because of a genetic predisposition (e12, e13).

There may be a primary genetic vulnerability to the impaired development of basic numerical functions or of linguistic, visuospatial, and executive ones. As we now know, the maturation of these functions can also be

**Mathematical learning disability**

A child is said to have a mathematical learning disability (MLD) when his or her performance on a test of mathematical skills falls below an arbitrarily chosen percentile in a study population, without reference to the child's general intelligence level.

**The etiology of dyscalculia**

Dyscalculia has many contributory causes. For the ones that will be discussed below, no firm conclusions can yet be drawn about the relations and interactions between them, as too little evidence is available.

affected by environmentally determined, epigenetically mediated influences, such as stress (Figure 1). This explains the marked association of dyscalculia with attention deficit hyperactivity disorder (ADHD) on the one hand, and with dyslexia on the other. Dyscalculia is not a single, uniform entity; rather, its subtypes can be classified systematically and in detail on the basis of their varying etiologies, underlying neural bases, cognitive representations, and skills levels (Figure 1).

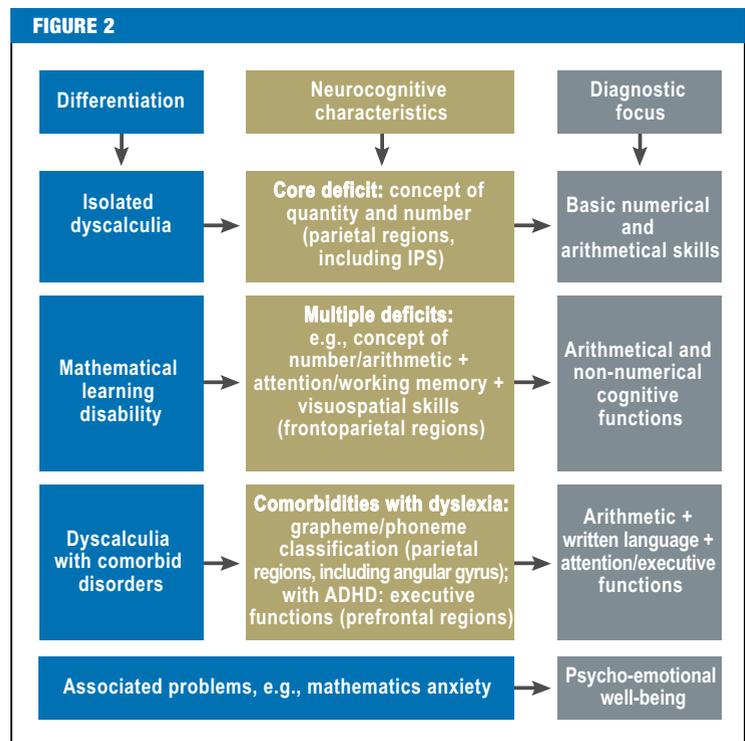
### Comorbidities and commonly associated accompanying phenomena

According to von Aster and Shalev (2), 20% to 60% of all persons with dyscalculia also have learning difficulties of other types, e.g., dyslexia (12) or ADHD (13, 14, e2). A combination of dyslexia and dyscalculia was found in 7.6% of 788 randomly sampled Dutch fourth- and fifth-graders (e14). Even among ninth-graders, 52% of the variance in calculating ability was accounted for by reading skills (e15). It even seems to be the case that deficient phonological skills (considered the most important precursor skill for written language) in preschool children are associated with poorer performance on formal calculating tasks in the early years of primary school (e16). A disorder of linguistic development in nursery-school and kindergarten age seems to be a risk factor for poor calculating ability (e17).

Only scant empirical data are available to date on the comorbidity of dyscalculia with attention deficit disorders. Rubinsten and colleagues (13) studied the effects of methylphenidate, a stimulant, on calculating ability in three groups of children with ADHD: The first group had no learning disorder, the second had a mathematical disability, and the third had dyscalculia. In all groups, the stimulant was found to improve those aspects of calculating ability that depend on working memory (e.g., “carrying” while calculating with multi-digit numbers), but it had no effect on basic numerical skills. The authors concluded that children who have both ADHD and either mathematical disability or dyscalculia need not only pharmacotherapy, but also specific learning therapy.

#### Accompanying manifestations

As is pointed out in a current review, mental illness is much more common among children with learning disorders than among age-matched children without learning disorders (30–50% versus 8–18%, [e2]). Children with dyscalculia often have severe emotional distress because they do poorly in school; this can lead, in turn,



**Differential-diagnostic considerations** for the identification and characterization of learning difficulties in the numerical-arithmetical area, taking special account of the distinction between dyscalculia and mathematical learning disability (MLD)

### BOX 2

#### Numerical-arithmetical skills that present particular difficulty to children with dyscalculia and mathematical learning disability<sup>1</sup>

- The acquisition, recall, and application of numerical-arithmetical knowledge, including numerospatial conceptualization and both factual and procedural arithmetical knowledge
- Wrong or inappropriate application of calculating strategies
- Difficulty generalizing learned content
- Little or no knowledge transfer, i.e., no automatic transfer of learned content to other tasks or problem areas without external help

<sup>1</sup>modified from (24)

### Comorbidities

**20% to 60% of all persons with dyscalculia also have learning difficulties of other types, e.g., dyslexia or attention deficit hyperactivity disorder.**

### An apparent link

**Deficient numerical precursor skills and deficient basic numerical skills seem to be associated with poorer performance in formal calculating tasks in the early years of primary school.**

**BOX 3**

Illustration of various solution pathways and strategies for single-digit addition and multiplication tasks (also called “arithmetical facts”)

Solution strategy	Task:	
	6 + 4	7 × 2
Direct recall	6 + 4 = 10	7 × 2 = 14
Count everything	1 + 1 + 1 + 1 + 1 + 1 = 6; 1 + 1 + 1 + 1 = 4; 6 + 4 = 10	-
Count from first summand up	6 + 1 = 7, 7 + 1 = 8, 8 + 1 = 9, 9 + 1 = 10	-
Breaking the problem down into components (focus on base 5)	5 + 4 = 9 9 + 1 = 10	(5 × 2) + (2 × 2)
Breaking the problem down (base 5) with movements	(6 - 1) + (4 + 1) = 5 + 5 = 10	-
Repeated addition	-	2 + 2 + 2 + 2 + 2 + 2 + 2
Other strategies	6 + 6 = 12 12 - 2 = 10	(10 × 2) - (3 × 2)

to a negative attitude about mathematical tasks or even to mathematics anxiety and school phobia (4). Mathematics anxiety tends to become chronic and to persistently impair skill development. Its effects can be seen on multiple levels—physiological (palpitations, diaphoresis), cognitive (feelings of helplessness, impaired working memory [e18]) and behavioral (avoidance). Severe mathematics anxiety also seems to impair basic numerical processing: The mental representation of number is less precise in persons with severe mathematics anxiety than in persons with mild or no mathematics anxiety (e19).

**The diagnostic assessment and differential diagnosis of dyscalculia**

Practical experience shows that there is a wide variety of performance profiles for numerical and calculating skills; this fact suggests that there are different subtypes

**Diagnostic instruments**

The diagnostic instruments used to evaluate dyscalculia are of two main types, the curricular and the neuropsychological.

of dyscalculia (15, 16, e20). Multiple cognitive factors have been discussed in the literature as potential contributory causes of dyscalculia (16, e10). Among these, a deficient understanding of quantity has been the most extensively studied by empirical means.

When dyscalculia is suspected, a detailed diagnostic evaluation is needed in order to take proper account of the complexity of this learning disorder and to produce an accurate picture of the affected child’s particular strengths and weaknesses in the area of numbers and calculations. The diagnostic instruments used for this purpose are of two main types, the curricular and the neuropsychological (Table). As the affected children often perform far below grade level on numerical and calculating tasks, the use of curricular tests alone may not yield a complete picture of the actual performance deficit; this can, in turn, lead to inappropriate interventions with little promise of efficacy, because the child’s performance is not really at the level for which the intervention was designed.

In children of kindergarten age, specific precursor skills can be identified (Box 1) that have been found to be reliable predictors of later calculating ability (e33, e34). Working memory has also been found to be such a predictor (17–19, e34, e35). Most standardized tests of calculating ability are designed for primary-school children and are not normed for any higher level than the sixth grade (e30). The BASIS-MATH 4–8 test, which was published in 2010, is the only German-language test that can be used up to the eighth grade (e36). It provides no normed data, but rather a single threshold value for the fourth through eighth grades, which can be interpreted as a rough indication of the mastery of basic mathematical concepts. BASIS-MATH has proved its usefulness in practice, as it yields not just a quantitative test score, but also information about the child’s calculating strategies, which can be of service in the planning of therapeutic interventions.

**Differential-diagnostic considerations**

Children with mathematical learning disability, with dyscalculia, and with combined dyscalculia and dyslexia have markedly different cognitive-neuropsychological performance profiles, as shown in Figure 2 (14, 20). In view of the multiplicity of the functional components participating in these disturbances and the wide range of potential mental and neuropsychiatric comorbidities, it is clear that the diagnostic evaluation must go beyond the strictly mathematical components to include a thorough

**Predictors**

In children of kindergarten age, specific precursor skills can be identified that have been found to be reliable predictors of later calculating ability.

personal, familial, and scholastic developmental history. The evaluation of the child's state of general cognitive development must also take account of non-numerical cognitive domains, social and emotional well-being, and findings from the fields of developmental neurology and, where appropriate, neurophysiology (EEG, neuroimaging) (2, 15, 21, 22) (*Box 4*).

### Methods for the treatment of dyscalculia

The treatment of children and adolescents with dyscalculia is a complex matter because of the heterogeneity of the disorder and the comorbid disorders often associated with it. For the best chance of a lasting therapeutic benefit, the treatment should be individually tailored to the findings of the diagnostic evaluation. It should be adapted to the patient's individual cognitive functional profile and may incorporate medications and psychotherapy as well, if these are warranted by severe accompanying psychopathological manifestations such as anxiety, depression, or ADHD (23, e37, e38).

Even though the disorder is heterogeneous, certain aspects of numerical calculating skills do seem to be especially common in, and characteristic of, patients with dyscalculia (*Box 2*).

The following skills are particularly important for the domain-specific aspects of treatment:

- basic numerical skills,
- the establishment and consolidation of numerospatial representations,
- the development of arithmetical reasoning,
- procedural knowledge, and
- the automatization of factual knowledge.

All of these skills fundamentally depend on successfully acquired layers of understanding, in constant interaction with the practicing of skills for their consolidation. For example, when a child acquires arithmetical facts, attention should be paid to the basic underlying arithmetical reasoning, as well as to frequent practice. As can be seen from the examples in *Box 3*, the various solving strategies differ both with respect to the complexity of the solution procedures involved and with respect to the underlying arithmetical reasoning. Counting up from the first summand to the answer is an immature solving strategy compared to breaking down the problem into its several components, but it is, at least, more mature and reflects a better comprehension of arithmetic than simply counting everything that is to be summed. The latter immature strategy demands more from working memory, which must store both interim

results, 6 and 4. Repeated practice fosters the automatic recall of learned content, thereby lessening the demand on working memory. This demand is especially high in multi-digit calculations with carrying, where interim results must be retained and manipulated (e.g., adding 87 and 45).

Most of the empirical studies on interventions to improve calculating ability have been carried out in English-speaking countries by special-education professionals. Their findings are, to some extent, inconsistent. A meta-analysis by Kroesbergen and van Luit (24) of 58 interventional studies among primary-school pupils revealed that

- most interventions dealt with basic numerical skills;
- interventions to promote basic numerical skills were more effective than interventions to promote precursor skills and/or problem-solving strategies; the sample size varied from 3 to 376, and the Cohen's *d* values (effect sizes) from  $-0.44$  to above 3 (24);
- shorter interventions were more effective than longer ones (the duration of interventions ranged from one week to one year across studies);
- interventions performed in person by teaching staff ("direct instruction") were more effective than computerized ones ("mediated instruction").

A further meta-analysis of studies of individualized intervention in the English-speaking countries revealed that individualized intervention is highly effective at improving calculating ability ( $d = 0.91$  [e40]), that the particular method of intervention that is used largely determines the degree of effectiveness, and that interventions conveying an understanding of strategy are much more effective than those in which the subject matter is passively communicated ("strategy instruction" versus "direct instruction").

The following methods of intervention were found to be especially effective:

- repeated practice;
- segmentation of subject matter;
- small, interactive groups;
- the use of cues in strategy-learning (e40).

A recently published meta-analysis by Ise and colleagues (25) evaluated eight studies from the German-speaking countries. The overall effect strength was 0.50, which is considered an intermediate value (neither strong nor weak). No difference in effectiveness was found between curricular and non-curricular treatment approaches, but all interventions tended to be more

### Diagnostic evaluation

The diagnostic evaluation must go beyond the strictly mathematical components to include a thorough personal, familial, and scholastic developmental history.

### The demand on working memory

The demand on working memory is especially high in multi-digit calculations with carrying, where interim results must be retained and manipulated.

effective the longer and the more intensively they were carried out. Most of the currently available German-language programs for promoting numerical calculating skills have not been empirically tested for efficacy, and many seem to lack an adequate theoretical basis.

The scientifically studied learning programs in the German-speaking countries include “Mengen, Zählen, Zahlen” (MZZ; “Quantities, Counting, and Numbers”), which was designed for preschool children (e41); the computerized and largely curricular intervention programs “Elfe und Mathis” (“Elfe and Mathis”) (e42); and the learning program “Calcularis” (e43), which is based on neuroscientific models of the development of number processing and arithmetic. The effect strength of these intervention programs has not been studied. MZZ is intended to promote the establishment of basic numerical and scholastic precursor skills (the concepts of quantity and number, counting order, and various types of notation). “Calcularis” begins with basic numerical skills and addresses the automatization of various number representations in increasingly large numerical domains, the understanding of arithmetical operations, and the establishment of factual arithmetical knowledge. “Calcularis” has an adaptive structure, i.e., it can be adapted to the individual child’s learning difficulties. In a study of a “Calcularis” prototype with functional neuroimaging (fMRI), the improvement in numerical-spatial and arithmetical skills was found to be associated with altered patterns of neural activation in the frontal and parietal lobes (e44).

**Overview: the role of the treating physician**

Dyscalculia, if untreated, persists into adulthood (5, 6, e3, e4). The affected persons often suffer from secondary, associated disturbances and are at a disadvantage on the job market (6, e2).

When dyscalculia is suspected, a detailed diagnostic evaluation should be performed. The primary task of specialists in child and adolescent psychiatry, and of the school health services, is to determine whether any comorbid (associated) disturbances are present. The differential evaluation of performance on numerical-arithmetical and non-numerical skills lies in the area of competence of school psychologists and neuropsychologists. Ideally, the performance profile that is generated by the diagnostic evaluation should serve as a point of departure for intervention planning.

The effective treatment of dyscalculia demands special expertise, which is most likely to be found

among graduates of specialized training and continuing-education programs that have been certified by recognized professional associations. In Germany, for example, the relevant organizations are the Bundesverband Legasthenie und Dyskalkulie (BVL, National Dyslexia and Dyscalculia Association) and the Fachverband Integrative Lerntherapie (FIL, Association for Integrative Learning Therapy). Recently, bachelor’s and master’s degree programs for specific training in learning therapy have been initiated at universities and professional training institutes.

Learning therapy can be carried out either in school, in conjunction with school, or outside school. In Germany, the cost of learning therapy is borne by the Youth Services Department after confirmation that the legally mandated condition of an impending mental impairment is fulfilled. As a rule, interventions can succeed only when they are ecologically valid, i.e., when they can take effect in the setting of the child’s everyday life.

A further role for the treating physician or psychologist may be to point out that an established legal framework exists for giving the affected persons special means to compensate for their learning difficulty in situations calling for high performance, including situations where their performance will be evaluated (tests). In Germany, Switzerland, and Austria, the availability of methods to acknowledge an individual’s learning difficulty within the school setting (e.g., by allowing more time to complete written examinations etc.) is far from uniform, and they may be entirely lacking. Whatever opportunities of this kind are available should be tried out in the individual case and made use of where appropriate.

In summary, the main role of the treating pediatrician or family physician centers on the early recognition of dyscalculia (and other learning disorders) and on counseling of the child’s parents and other carers about the further diagnostic and therapeutic measures that are indicated. Early recognition largely depends on information provided by the child’s parents or other carers. Depending on the age of the child, specific questions should be asked about his/her understanding of quantity, counting skills, and mathematical performance in school to date. The history should also include questions about any secondary disturbances that might be present, e.g., learning disorders in other areas and/or psychopathological manifestations, dislike of school, mathematics anxiety, and/or school phobia.

**Efficacy**

Most of the currently available German-language programs for promoting numerical calculating skills have not been empirically tested for efficacy.

**Payment for treatment**

In Germany, the cost of learning therapy is borne by the Youth Services Department after confirmation that the legally mandated condition of an impending mental impairment is fulfilled.

**BOX 4**

**Specific questions for history-taking in suspected dyscalculia**

These questions are a small component of the overall assessment of the child's health and mental and physical developmental state.

● **Etiology**

- Family history  
Are/were there any other family members (e.g., mother, father, siblings) who had difficulty learning to perform arithmetic?
- Primary vs. secondary  
Did the child have difficulty learning to perform arithmetic, or a particular aversion to arithmetic, from the very beginning, or did the problem only begin later, e.g., in the aftermath of an illness or other serious life event (including neurological and psychiatric disease and traumatic brain injury)

● **Isolated dyscalculia vs. general learning impairment**

- Does the child or adolescent attend a normal school?
- Does the child or adolescent have difficulty in other school subjects aside from mathematics? Are these difficulties so severe and/or present in so many subjects that the normal progression into the next grade might not be possible?

● **Precursor skills**

- Was the child or adolescent happy to deal with quantities and numbers as a preschool child? For example, did he or she perform normally with respect to number-rhymes, counting out loud, dividing groups of objects (e.g., sharing candies with friends), and playing board games?
- Could the child count to ten before starting school?
- Could the child recognize small numbers of objects (one, two, or three objects) at a glance before starting school?

● **Appropriate fostering and schooling**

- Did the child have adequate opportunity in the preschool and kindergarten years to incorporate quantitative concepts into play (e.g., to play board games)?
- Is there any reason to think that the child's instruction in mathematics was, or is, deficient? For example, do many other children in the same class also have difficulty calculating?

● **Associated problems**

- Language development  
Was language development delayed? (If so, then one should also expect difficulty with counting and other mainly linguistically

based aspects of arithmetic, such as the recall of arithmetical facts [e.g.,  $2 \times 3 = 6$ ] or the performance of textual exercises)

- Visuospatial skills  
Does the child or adolescent have difficulty drawing or copying geometrical figures (note: difficulties of this type can be hard to distinguish from impaired motor function while drawing) or difficulty with spatial and temporal orientation?
- Attention and working memory  
Does the child or adolescent have difficulty in everyday life when confronted with more than one task at once? Does he or she often forget appointments, homework, etc.?
- Other learning disorders  
Does the child or adolescent have, in addition to dyscalculia, difficulty acquiring written language (reading, spelling)?
- Accompanying social and emotional problems  
Does the child seem to suffer from mathematics anxiety, dislike of school, or school phobia?
- Psychosomatic complaints  
Does the child or adolescent ever complain of headache, abdominal pain, etc., when a mathematics test is scheduled or mathematics homework is due?
- Current school performance in mathematics  
What grade did the child most recently get in mathematics? What aspects of arithmetic does the child or adolescent currently do well, and what aspects or components present special difficulty? (Note: Here, one should ask not only about the elementary operations [which can be affected independently to differing degrees], but also about algebra, geometry, etc.)

● **Interventions and treatments to date**

- What, if anything, has been done till now to improve the child or adolescent's mathematical ability?
- Has any treatment been provided for other learning difficulties or behavioral problems?
- If so, what, and was it successful?

**Preconditions for effective treatment**

The treatment of dyscalculia is effective when it is individually tailored and adapted to the affected child or adolescent's performance profile.

**Highly effective treatment approaches**

Treatments addressing both the establishment and consolidation of arithmetical understanding and the automatization of the content to be learned have the highest chance of success. A particular challenge is the transfer of what is learned to its application in everyday life.

**Further Information on CME**

This article has been certified by the North Rhine Academy for Postgraduate and Continuing Medical Education.

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The solutions to the following questions will be published in issue 1–2/2013. The CME unit “Imported Viral Infections” (issue 41/2012) can be accessed until 23 November 2012. For issue 49/2012, we plan to offer the topic, “The Treatment of Hallux Valgus.”

Solutions to the CME questionnaire in issue 37/2012: Graf et al.: In-Flight Medical Emergencies. Answers: 1b, 2d, 3a, 4e, 5d, 6d, 7d, 8d, 9e, 10d

**Conflict of interest statement**

PD Dr. Kaufmann receives license fees for test procedures such as TEDI-MATH and BVN5-11, for the textbook “Dyskalkulie,” and for the edited text “Entwicklungsneuropsychologie.”

Prof. von Aster receives royalties from Pearson-Verlag for the ZAREKI-R/K test and from the publishing house Vandenhoeck & Ruprecht for the book “Rechenstörungen bei Kindern.” The Lilly company has paid him honoraria for the preparation of scientific continuing education events and has reimbursed his travel and accommodation costs.

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**REFERENCES**

1. Shalev RS, Gross-Tsur V: Developmental dyscalculia. *Ped Neurol* 2001; 24: 337–42.
2. von Aster MG, Shalev R: Number development and developmental dyscalculia. *Dev Med Child Neurol* 2007; 49: 868–73.
3. von Aster M,G, Schweiter M, Weinhold-Zulauf M: Rechenstörungen bei Kindern: Vorläufer, Prävalenzen und psychische Störungen. *Zeitschrift Entwicklungspsychol Pädagog Psychol* 2007; 39: 85–96.
4. Krinzing H, Kaufmann L: Rechenangst und Rechenleistung. *Sprache, Stimme, Gehör* 2006; 30: 160–4.
5. Gerber PJ: The impact of learning disabilities on adulthood: a review of the evidenced-based literature for research and practice in adult education. *J Learn Dis* 2012; 45: 31–46.
6. Kaufmann L, Pixner S, Goebel S: Finger usage and arithmetics in adults with math difficulties: Evidence from a case report. *Front Psychol* 2011; 2: 254.
7. Butterworth B, Varma S, Laurillard D: Dyscalculia: From brain to education. *Science* 2011; 332: 1049–53.
8. Kucian K, Kaufmann L: A developmental model of number representations. *Behav Brain Sci* 2009; 32: 340–1.
9. Landerl K, Kaufmann L: *Dyskalkulie*. München: Reinhardt Verlag 2008.
10. Dilling H, Mombour W, Schmidt MH: *Internationale Klassifikation psychischer Störungen. ICD-10 Kapitel V (F). Klinisch-diagnostische Leitlinien* (6., vollständig überarbeitete Auflage). Bern: Huber 2008.
11. Saß H, Wittchen H-U, Zaudig M: *Diagnostisches und Statistisches Manual Psychischer Störungen – Textrevision – DSM-IV-TR*. Göttingen: Hogrefe 2003.
12. Landerl K, Moll K: Comorbidity of learning disorders: Prevalence and familial transmission. *J Child Psychol Psychiatry* 2010; 51: 287–94.
13. Rubinsten O, Bedard AC, Tannock R: Methylphenidate improves general but not core numerical abilities in ADHD children with comorbid dyscalculia or mathematical difficulties. *J Open Psychol* 2008; 1: 11–7.
14. Rubinsten O: Co-occurrence of developmental disorders: The case of developmental dyscalculia. *Cognitive Development* 2009; 24: 362–70.
15. Kaufmann L, Nuerk H-C: Numerical development: Current issues and future perspectives. *Psychol Sci* 2005; 47: 142–70.
16. Wilson AJ, Dehaene S: Number sense and developmental dyscalculia. In: Coch D, Dawson G, Fischer K (eds.): *Human behavior, learning, and the developing brain: Atypical development*. New York: Guilford Press 2007; 212–38.
17. Gersten R, Jordan NC, Flojo JR: Early identification and intervention for students with mathematics difficulties. *J Learn Disabil* 2005; 38: 293–304.
18. Rosselli M, Matute E, Pinto N, Ardila A: Memory abilities in children with subtypes of dyscalculia. *Dev Neuropsychol* 2006; 30: 801–8.
19. Toll SW, Van der Ven SH, Kroesbergen EH, Van Luit JE: Executive functions as predictors of math learning disabilities. *J Learn Disabil* 2011; 44: 521–32.

**A prerequisite for efficacy**

As a rule, interventions can succeed only when they are ecologically valid, i.e., when they can take effect in the setting of the child’s everyday life.

20. Landerl K, Fussenegger B, Moll K, Willburger E: Dyslexia and dyscalculia: two learning disorders with different cognitive profiles. *J Exp Child Psychol* 2009; 103: 309–24.
21. Jordan NC, Levine SC: Socioeconomic variation, number competence, and mathematics learning difficulties in young children. *Dev Disabil Res Rev* 2009; 15: 60–8.
22. Rourke BP: Socioemotional disturbances of learning disabled children. *J Consult Clin Psychol* 1988; 56: 801–10.
23. Galonska S, Kaufmann L: Intervention bei entwicklungsbedingter Dyskalkulie. *Sprache Stimme Gehör* 2006; 30: 171–8.
24. Kroesbergen E, van Luit JEH: Mathematics Intervention for Children with Special Educational Needs. *Remedial and Special Education* 2003; 24: 97–114.
25. Ise E, Dolle K, Pixner S, Schulte-Körne G: Effektive Förderung rechenschwacher Kinder: Eine Metaanalyse. *Kindheit und Entwicklung* 2012; 21: 181–92.

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Please answer the following questions to participate in our certified Continuing Medical Education program. Only one answer is possible per question. Please select the answer that is most appropriate.

**Question 1**

**What is the prevalence of dyscalculia?**

- a) 1%
- b) 3%
- c) 5%
- d) 7%
- e) 9%

**Question 2**

**Which of the following seems to serve as a basis for arithmetical thinking and calculating in one's head?**

- a) the mental number line
- b) haptic finger dexterity
- c) visual conceptualization
- d) oral counting ability
- e) numerospatial visualizing ability

**Question 3**

**What particular cognitive aspect of dyslexia has been better documented than other aspects to date?**

- a) text comprehension and the ability to make abstractions
- b) writing speed
- c) spatial conceptualization
- d) the understanding of quantity
- e) algebraic knowledge

**Question 4**

**Which of the following conditions often accompany dyscalculia?**

- a) word-finding difficulty
- b) stuttering
- c) movement disorders
- d) mathematics anxiety and school phobia
- e) lack of drive

**Question 5**

**Which of the following underlies the diagnosis of dyscalculia, according to ICD-10 and DSM-IV?**

- a) subnormal performance on both a standardized calculating test and a standardized reading and writing test
- b) normal intelligence and subnormal performance on a standardized calculating test
- c) subnormal intelligence and subnormal performance on a standardized calculating test
- d) extensive history-taking and a structured psychiatric interview
- e) observation by parents and mathematics teachers

**Question 6**

**What types of intervention have been found particularly effective in the treatment of dyscalculia?**

- a) those aimed at establishing and consolidating arithmetical understanding along with repeated practice
- b) rote learning of the multiplication table
- c) extra help in a professional learning studio
- d) relaxation training
- e) cognitive behavioral therapy

**Question 7**

**What percentage of children with dyscalculia have a comorbid disorder, e.g., dyslexia or attention deficit hyperactivity disorder?**

- a) 10–30%
- b) 20–40%
- c) 30–50%
- d) 50–80%
- e) 20–60%

**Question 8**

**What is the earliest age at which specific precursor skills can serve as a reliable predictor of later calculating ability?**

- a) infancy (1 to 2 years)
- b) nursery-school and kindergarten age (3 to 5 years)
- c) primary-school age (6 to 9 years)
- d) prepuberty (10 to 12 years)
- e) puberty (13 to 14 years)

**Question 9**

**Which of the following is an empirically validated approach to the treatment of dyscalculia?**

- a) ergotherapy
- b) weekly supplemental lessons
- c) psychotherapy
- d) training with computer programs to improve calculating ability
- e) deficit-oriented, individually tailored learning therapy oriented toward the strengths and weaknesses of the affected child

**Question 10**

**Which of the following should receive special attention in the treatment of dyscalculia?**

- a) height development and BMI
- b) motor skills
- c) comorbidities
- d) social competence
- e) the child's ability to concentrate

## CONTINUING MEDICAL EDUCATION

# The Diagnosis and Management of Dyscalculia

Liane Kaufmann, Michael von Aster

## eReferences

- e1. Schulte-Körne G: The prevention, diagnosis, and treatment of dyslexia. *Dtsch Arztebl Int* 2010; 107(41): 718–27.
- e2. Vedi K, Bernard S: The mental health needs of children and adolescents with learning disabilities. *Curr Opin Psychiatry* 2012; 25: 353–8.
- e3. BSA/Basic Skills Agency (2001). *Adult Numeracy Core Curriculum*. London: Cambridge Training & Development.
- e4. NRDC/National Research and Development Centre for Adult Literacy and Numeracy (2008). *Research Briefing Numeracy*. London: NRDC.
- e5. Xu F: Numerosity discrimination in infants: evidence for two systems of representations. *Cognition* 2003; 89: B15–B25.
- e6. Schweiter M, Weinhold Zulauf M, von Aster MG: Die Entwicklung räumlicher Zahlenrepräsentationen und Rechenfertigkeiten bei Kindern. *Zeitschrift für Neuropsychologie* 2005; 16: 105–13.
- e7. Kaufmann L, Wood G, Rubinsten O, Henik A: Meta-analyses of developmental fMRI studies investigating typical and atypical trajectories of number processing and calculation. *Dev Neuropsychol* 2011; 36: 763–87.
- e8. Kucian K, von Aster MG, Loenneker T, Dietrich T, Martin E: Development of neural networks for exact and approximate calculation: a fMRI study. *Dev Neuropsychol* 2008; 33(4): 447–73.
- e9. Petermann F, Petermann U (Hrsg): *Hamburg-Wechsler-Intelligenztest für Kinder IV (HAWIK-IV)*. Bern: Verlag Hans Huber, 2008.
- e10. Geary D: Mathematical disabilities: cognitive, neuropsychological, and genetic components. *Psychol Bulletin* 1993; 114: 345–62.
- e11. Shalev RS, Manor O, Kerem B, Ayali M, Badichi N, Friedlander Y, et al.: Developmental dyscalculia is a family learning disability. *J Learn Disab* 2001; 34: 59–65.
- e12. Alarcon M, Defries J, Gillis Light J, Pennington B: A twin study of mathematics disability. *J Learn Disab* 1997; 30: 617–23.
- e13. Kovas Y, Plomin R: 2007. Learning abilities and disabilities: generalist genes, specialist environments. *Curr Directions in Psychol Sci* 2007; 16: 284–8.
- e14. Dirks E, Spyer G, van Lieshout EC, de Sonneville L: Prevalence of combined reading and arithmetic disabilities. *J Learn Disabil* 2008; 41: 460–71.
- e15. Korhonen JG, Linnanmäki K, Aunio P: Language and mathematical performance: a comparison of lower secondary school students with different levels of mathematical skills. *Scand J Educ Res* 2011 [DOI: 10.1080/00313831.2011.599423]
- e16. Jordan JA, Wiley J, Mulhern G: Phonological awareness and mathematical difficulty: a longitudinal perspective. *Br J Dev Psychol* 2010; 28: 89–107.
- e17. Manor O, Shalev RS, Joseph A, Gross-Tsur V: Arithmetic skills in kindergarten children with developmental language disorders. *Eur J Paediatr Neurol* 2001; 5: 71–77.
- e18. Hopko DR, Ashcraft MH, Gute J, Ruggiero KJ, Lewis C: Mathematics anxiety and working memory: Support for the existence of a deficient inhibition mechanism. *J Anx Dis* 1998; 12: 343–355.
- e19. Maloney EA, Ansari D, Fugelsang JA: The effect of mathematics anxiety on the processing of numerical magnitudes. *Q J Exp Psychol* 2011; 64: 10–16.
- e20. Von Aster MG: Developmental cognitive neuropsychology of number processing and calculation: varieties of developmental dyscalculia. *Eur Child Adolesc Psychiatry* 2000; 9 (Suppl 2): II41–57.
- e21. Kaufmann L, Nuerk H-C, Graf M, Krinzinger H, Delazer M, Willmes K: *TEDI-MATH: Test zur Erfassung numerisch-rechnerischer Fertigkeiten vom Kindergarten bis zur 3. Klasse*. Zürich: Verlag Hans Huber 2009.
- e22. Schaupp H, Holzer N, Lenart F: *ERT1+/Eggenberger Rechentest 1+*. Diagnostik für Dyskalkulie für das Ende der 1. Schulstufe bis Mitte der 2. Schulstufe. Bern: Verlag Hogrefe 2007.
- e23. Lenart H, Holzer N, Schaupp H: *ERT2+/Eggenberger Rechentest 2+*. Diagnostik für Dyskalkulie für das Ende der 2. Schulstufe bis Mitte der 3. Schulstufe. Bern: Verlag Hogrefe 2008.
- e24. Holzer H, Schaupp H, Lenart F: *ERT3+/Eggenberger Rechentest 3+*. Diagnostik für Dyskalkulie für das Ende der 3. Schulstufe bis Mitte der 4. Schulstufe. Bern: Verlag Hogrefe 2010.
- e25. Schaupp H, Holzer N, Lenart F: *ERT4+/Eggenberger Rechentest 4+*. Diagnostik für Dyskalkulie für das Ende der 4. Schulstufe bis Mitte der 5. Schulstufe. Bern: Verlag Hogrefe 2010.
- e26. Krajewski K, Küspert P, Schneider W, Visé M: *DEMAT 1: Deutscher Mathematiktest für erste Klassen*. Bern: Verlag Hogrefe 2002.
- e27. Krajewski K, Liehm S, Schneider W: *DEMAT 2: Deutscher Mathematiktest für zweite Klassen*. Bern: Verlag Hogrefe 2004.
- e28. Roick T, Gölick D, Hasselhorn M: *DEMAT 3: Deutscher Mathematiktest für dritte Klassen*. Bern: Verlag Hogrefe 2004.
- e29. Göllitz D, Roick T, Hasselhorn M: *DEMAT 4: Deutscher Mathematiktest für erste Klassen*. Bern: Verlag Hogrefe 2006.
- e30. Jacobs C, Petermann F: *Rechenfertigkeiten- und Zahlenverarbeitungs-Diagnostikum für die 2. Bis 6. Klassen*. Göttingen: Hogrefe 2005.
- e31. Von Aster MG, Weinhold Zulauf M, Horn R: *ZAREKI-R: Die neuropsychologische Testbatterie für Zahlenverarbeitung und Rechnen bei Kindern, revidierte Version*. Frankfurt: Harcourt 2006.
- e32. Von Aster MG, Bzufka M, Horn R: *ZAREKI-K: Die Neuropsychologische Testbatterie für Zahlenverarbeitung und Rechnen bei Kindern, Kindergarten Version*. Frankfurt: Pearson 2009.
- e33. Aunio P, Niemivirta M: Predicting children's mathematical performance in grade one by early numeracy. *J Learn Individ Diff* 2010; 20: 427–35.
- e34. Passolunghi MC, Vercelloni B, Schadee H: The precursors of mathematics learning: Working memory, phonological ability and numerical competence. *Cogn Dev* 2007; 22: 165–84.
- e35. Kaufmann L, Lochy A, Drexler A, Semenza C: Deficient arithmetic fact retrieval – storage or access problem? A case study. *Neuropsychologia* 2004; 42: 482–96.
- e36. Moser Opitz E, Reusser L, Moeri Müller M, Anliker B, Wittich C, Weesemann O, Ramseier E: *BASIS-MATH 4–8. Basisdiagnostik Mathematik für die Klassen 4–8*. Göttingen: Hogrefe 2010.

- e37. Dowker A. What works for children with math difficulties? Research report 2005; RR554; Department for Education and Skills, London: University of Oxford [ISBN 1 84478 261 1].
- e38. Wright RJ, Martland J, Stafford AK, Stanger G: Teaching Number. *Advancing Children's Skills and Strategies*, 2<sup>nd</sup> Ed. London: Paul Chapman Publishing Ltd. 2002.
- e39. Kaufmann L, Handl P, Thoeny B: Evaluation of a numeracy intervention program focusing on basic numerical knowledge and conceptual knowledge: a pilot study. *J Learn Disabil* 2003; 36: 564–73.
- e40. Swanson HL, Sachse-Lee C: A meta-analysis of single-subject-design intervention research for students with LD. *J Learn Disabil* 2000; 33: 114–36.
- e41. Krajewski K, Nieding G, Schneider W: „Mengen, zählen, Zahlen“. *Die Welt der Mathematik entdecken*. Berlin: Cornelsen 2007.
- e42. Lenhard A, Lenhard W, Klauer KJ: *Denkspiele mit Elfe und Mathis. Förderung des logischen Denkvermögens für das Vor- und Grundschulalter*. Göttingen: Hogrefe, 2011.
- e43. Von Aster MG, Käser T, Kucian K, Gross M: *Calcularis – Rechenschwäche mit dem Computer begegnen*. *Schweizerische Zeitschrift für Heilpädagogik* 2012; 18: 32–6.
- e44. Kucian K, Grond U, Rotzer S, Henzi B, Schönemann C, Plangger F, Gälli M, Martin M, von Aster MG: Mental number line training in children with developmental dyscalculia. *NeuroImage* 2011; 57: 782–95.