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The Consumption-Income Ratio, Entrepreneurial Risk and the US Stock Market: Technical Appendix

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Derivation of cpy from the aggregate budget constraint

Letting lowercase letters denote logarithms, the aggregate budget constraint can be rewritten as

$$\log\left(1-\frac{\Pi_t}{\Psi_t}\right)=\theta_t-\psi_t. \tag{1}$$

The share of proprietary wealth in total wealth is $\Pi_t/\Psi_t=\exp(\pi_t-\psi_t)$, and I denote the long-run mean of Π_t/Ψ_t with γ. Hence, I can write $\gamma=\exp(\overline{\pi-\psi})$, where $\overline{\pi-\psi}$ is the logarithm of the long-run mean of Π_t/Ψ_t. I now expand the left-hand side of (1) around $\overline{\pi-\psi}$ to obtain

$$\log\left(1-\frac{\Pi_t}{\Psi_t}\right)\approx\kappa-\frac{\gamma}{1-\gamma}\left[\pi_t-\psi_t\right]$$

where $\kappa=\log(1-\gamma)-\gamma\left(\overline{\pi-\psi}\right)(1-\gamma)^{-1}$ is a constant. Plugging this back into (1) and rearranging yields

$$\psi_t=\gamma\pi_t+(1-\gamma)\theta_t-(1-\gamma)\kappa. \tag{2}$$

where $-(1-\gamma)\kappa$ is the linearization constant. This is equation (3) in the main article.

*This appendix is for publication as supplementary web-material only. The main article will appear in the *Journal of Money, Credit and Banking*.

Note that aggregate wealth Ψ_t is the present value of all dividends, $\Psi_t = C_t + \sum_{k=1}^{\infty} \left[\prod_{s=1}^{k} R_{c,t+s} \right]^{-1} C_{t+k}$ where $R_{c,t+s}$ is the gross return on total wealth. This expression can be written recursively as

$$\Psi_{t+1} = R_{c,t+1}(\Psi_t - C_t),$$

which allows the use of the approach adopted by Campbell and Mankiw (1989) for the log-linearization of the consumption–wealth ratio:

$$\frac{\Psi_{t+1}}{\Psi_t} = R_{c,t+1}(1 - \exp(c_t - \psi_t)).$$

Taking logs yields

$$\Delta\psi_{t+1} = r_{c,t+1} + \log(1 - \exp(c_t - \psi_t)).$$

The logarithmic term can now be expanded around the long-run consumption–wealth ratio $\exp(\overline{c - \psi})$ so that

$$\begin{aligned} \log(1 - \exp(c_t - \psi_t)) &= \log(1 - \exp(\overline{c - \psi})) + \frac{-\exp(\overline{c - \psi})}{(1 - \exp(\overline{c - \psi}))} \left[c_t - \psi_t - \overline{c - \psi} \right] \\ &= \kappa_C - \frac{\exp(\overline{c - \psi})}{(1 - \exp(\overline{c - \psi}))} \left[c_t - \psi_t \right] \end{aligned}$$

where

$$\kappa_c = \log(1 - \exp(\overline{c - \psi})) + \frac{\exp(\overline{c - \psi})}{(1 - \exp(\overline{c - \psi}))} \overline{c - \psi}.$$

Write $\Delta\psi_{t+1}$ tautologically as

$$\Delta\psi_{t+1} = \Delta c_{t+1} - (c_{t+1} - \psi_{t+1}) + (c_t - \psi_t)$$

to obtain

$$\kappa_c + r_{c,t+1} + \left[1 - \frac{1}{\rho}\right][c_t - \psi_t]$$
$$= \Delta c_{t+1} - (c_{t+1} - \psi_{t+1}) + (c_t - \psi_t)$$

where $\rho_c = 1 - \exp(\overline{c - \psi})$. Then rearrange to obtain

$$\kappa_c + \frac{1}{\rho}[c_t - \psi_t] = r_{c,t+1} - \Delta c_{t+1} - (c_{t+1} - \psi_{t+1}),$$

which can be solved forward with $\rho_c^k(c_{t+k} - \psi_{t+k}) \to 0$ to get

$$[c_t - \psi_t] = \frac{\rho_c}{1 - \rho_c}\kappa_c + \sum_{k=1}^{\infty} \rho_c^k [r_{c,t+k} - \Delta c_{t+k}].$$

If consumption and wealth are both integrated ($I(1)$) processes, then Δc will be stationary. Assuming that returns are also stationary, the right-hand side of this present-value relation reflects the discounted sum of expectations of stationary variables and will therefore be stationary. Hence, $c_t - \psi_t$ is stationary.

Applying the same log-linearization procedure to $p_t - \pi_t$, and $y_t - \theta_t$, I get

$$\psi_t = c_t + \mathbf{E}_t \sum_{k=1}^{\infty} \rho_c^k (\Delta c_{t+k} - r_{c,t+k}) \quad \text{(3a)}$$

$$\pi_t = p_t + \mathbf{E}_t \sum_{k=1}^{\infty} \rho_p^k (\Delta p_{t+k} - r_{p,t+k}) \quad \text{(3b)}$$

$$\theta_t = y_t + \mathbf{E}_t \sum_{k=1}^{\infty} \rho_y^k (\Delta y_{t+k} - r_{y,t+k}) \quad \text{(3c)}$$

where ρ_x is the mean reinvestment ratio of the respective wealth component; e.g., $\rho_c = 1 - \exp(\overline{c - \psi})$ and where I drop any linearization constants for brevity. Plugging into (2), one then obtains that $cpy_t \equiv c_t - \gamma p_t - (1 - \gamma) y_t$

is given by

$$
\begin{aligned}
cpy_t \quad = \quad & +\mathbf{E}\sum_{k=1}^{\infty}\rho_c^k\left(\Delta c_{t+k} - r_{c,t+k}\right) - \gamma\mathbf{E}_t\sum_{k=1}^{\infty}\rho_p^k\left(\Delta p_{t+k} - r_{p,t+k}\right) \\
& -(1-\gamma)\mathbf{E}_t\sum_{k=1}^{\infty}\rho_y^k\left(\Delta y_{t+k} - r_{y,t+k}\right) \\
& +constant,
\end{aligned}
$$

where I have used the notation $constant$ as a catch-all for linearization constants.

cpy as cointegrating relationship

To see formally that cpy must be a cointegrating relationship, rewrite this equation as

$$
\begin{aligned}
cpy = constant + \gamma E_t \sum_{k=1}^{\infty}\left(\rho_p^k \Delta p_{t+k} + \left(\rho_c^k - \rho_p^k\right) r_{p,t+k}\right) \qquad (4) \\
+ (1-\gamma)E_t \sum_{k=1}^{\infty}\left(\rho_y^k \Delta y_{t+k} + \left(\rho_c^k - \rho_y^k\right) r_{y,t+k}\right) - E_t \sum_{k=1}^{\infty}\rho_c^k \Delta c_{t+k}.and
\end{aligned}
$$

where I have decomposed the return on aggregate wealth, $r_{c,t+k}$, into a weighted average of the returns on proprietary (entrepreneurial) wealth and returns on other wealth.

$$
r_{c,t+k} \approx \gamma r_{p,t+k} + (1-\gamma) r_{y,t+k}. \qquad (5)
$$

From (4) it is apparent that cpy must be stationary: because c, p and y are all best characterized as individually $I(1)$, the present value of their changes must be stationary. If the returns on wealth are stationary, then their discounted sum must equally be stationary. This implies that cpy will be stationary. It therefore defines a cointegrating relationship that measures the temporary deviation of consumption, proprietary and other income from the common trends.

The deviation of the cointegrating relation from its long-run mean then predicts changes either in consumption or in one of the two components of income: away from the long-run trend, at least one of the three variables will have to adjust.

cpy as approximation of the entrepreneurial income ratio

Start from the consolidated present values of consumption of proprietors and workers $\Psi_t = \Psi_t^p + \Psi_t^w$. Rearranging and taking logarithms on both sides, we get an equation analogous to (1) above:

$$\log\left(1 - \frac{\Psi_t^p}{\Psi_t}\right) = \psi_t^w - \psi_t. \tag{6}$$

Maintain the assumption from the previous section that the share of proprietary wealth in total wealth is constant in the long run, so that $\gamma = \boldsymbol{E}(\Pi_t/\Psi_t)$ exists. It then follows from entrepreneurs' budget constraint that $\gamma = \boldsymbol{E}(\Psi_t^p/\Psi_t)$. Hence, log-linearizing (6) around γ we get

$$\psi_t = \gamma\psi_t^p + (1-\gamma)\psi_t^w + constant. \tag{7}$$

The stationarity of proprietors' and workers' respective consumption–wealth ratios allows us to obtain equations that are analogous to those obtained for the aggregate consumption–wealth ratio in (3a):

$$\begin{aligned}\psi_t^p &= c_t^p + \mathbf{E}_t \sum_{k=1}^{\infty} \rho_p^k \left(\Delta c_{t+k}^p - r_{p,t+k}\right) \\ \psi_t^w &= c_t^w + \mathbf{E}_t \sum_{k=1}^{\infty} \rho_y^k \left(\Delta c_{t+k}^w - r_{y,t+k}\right)\end{aligned}$$

where $r_{p,t}$ and $r_{y,t}$ are the internal rates of return on proprietary and non-proprietary wealth from above and constants have again be dropped for brevity. Substitute out for the ψ-terms in (7) and, ignoring constants, rearrange terms, again using $r_{c,t} = \gamma r_{p,t} + (1-\gamma) r_{y,t}$:

$$\begin{aligned}c_t = \; & \gamma c_t^p + (1-\gamma) c_t^w \\ & + \gamma \mathbf{E}_t \sum_{k=1}^{\infty} \left\{ \rho_p^k \Delta c_{t+k}^p + \left(\rho_c^k - \rho_p^k\right) r_{p,t+k} \right\} \\ & + (1-\gamma) \mathbf{E}_t \sum_{k=1}^{\infty} \left\{ \rho_y^k \Delta c_{t+k}^w + \left(\rho_c^k - \rho_y^k\right) r_{y,t+k} \right\} \\ & - \mathbf{E}_t \sum_{k=1}^{\infty} \rho_c^k \left(\Delta c_{t+k}\right).\end{aligned}$$

Hence, if aggregate consumption is not very predictable (as is the case in the data) and assuming that entrepreneurs' and workers' consumption growth are not too predictable either, the approximation error is

$$\begin{aligned} cpy-\left[\gamma\left(c_t^p-p_t\right)+(1-\gamma)\left(c_t^w-y_t\right)\right] &= \gamma \mathbf{E}_t \sum_{k=1}^{\infty}\left(\rho_c^k-\rho_p^k\right) r_{p,t+k} \\ &\quad +(1-\gamma)\mathbf{E}_t \sum_{k=1}^{\infty}\left(\rho_c^k-\rho_y^k\right) r_{y,t+k}. \end{aligned}$$

Note that the terms on the right-hand side of this equation also figure on the right-hand side of (4) and that the approximation error is independent of expected growth rates in p or y. Hence, cpy and $\gamma\left(c_t^p-p_t\right)+(1-\gamma)\left(c_t^w-y_t\right)$ contain the same information with respect to future changes of p and y. In the data, cpy mainly reflects expected changes in proprietary income whereas labor income and aggregate consumption are not very predictable. Hence, if c_t^p and c_t^W are assumed to be sufficiently close to random walks, then the temporary fluctuations in p identified by fluctuations in cpy largely reflect variation in c^p-p, the entrepreneurial consumption-income ratio.

Identifying permanent and transitory components

Specifically, Proietti (1997) proposes the following decomposition:

$$\begin{aligned} \mathbf{x}_t &= \mathbf{C}(\mathbf{1})\boldsymbol{\Gamma}(\mathbf{1})\mathbf{x}_t+\left[\mathbf{I}-\mathbf{C}(\mathbf{1})\boldsymbol{\Gamma}(\mathbf{1})\right]\mathbf{x}_t \\ &= \mathbf{x}_t^P+\mathbf{x}_t^T \end{aligned}$$

where $\mathbf{C}(\mathbf{1})$ is the long-run response of $\mathbf{x}_t$ to shocks; i.e., the loading associated with the random walk component in the Beveridge–Nelson–Stock–Watson decomposition of $\mathbf{x}_t$.

To identify permanent and transitory shocks directly, acknowledge that $\mathbf{C}(\mathbf{1})$ can be factored as $\mathbf{C}(\mathbf{1})=\mathbf{A}\alpha_{\perp}'$ so that

$$\pi_t = \alpha_{\perp}'\varepsilon_t$$

can be interpreted as the vector of permanent shocks, the innovations to the random walk component of $\mathbf{x}_t$. By construction, shocks that are transitory with respect to all components of the vector $\mathbf{x}_t$ must be orthogonal to π_t so that these shocks must be given by[1]

$$\tau_t = \alpha' \mathbf{\Omega}^{-1} \varepsilon_t.$$

Collecting permanent and transitory shocks into one vector θ_t,

$$\theta_t = \begin{bmatrix} \pi_t \\ \tau_t \end{bmatrix} = \begin{bmatrix} \alpha'_\perp \\ \alpha' \mathbf{\Omega}^{-1} \end{bmatrix} \varepsilon_t = \mathbf{P} \varepsilon_t.$$

From the estimated VECM, it is possible to obtain the Wold representation

$$\mathbf{\Delta x}_t = \mathbf{C}(\mathbf{L}) \varepsilon_t$$

so that with

$$\varepsilon_t = \mathbf{P}^{-1} \theta_t$$

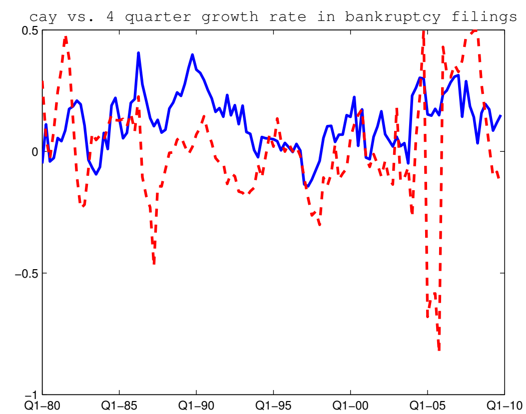
it is straightforward to identify the variance contribution of permanent and transitory shocks as well as impulse responses.[2]

[1] See Johansen (1991), Hoffmann (2001) and Gonzalo and Ng (2001)

[2] Note that the identification of the relative variance contributions of permanent and transitory shocks only requires knowledge of the (reduced-form) VECM parameters. The just-identification of the individual permanent and transitory shocks is not required. This will only be necessary once we are interested in conducting impulse response analysis. See e.g., Hoffmann (2001).

cpy and bankruptcy filings

FIGURE A.I:
The figure shows four-quarter growth rates in bankruptcy filings (dashed, red) and $cpy \times 10$ (blue, solid line). Data are obtained from the American Bankruptcy Institute at http://www.aib.org. Unfortunately, these are available only from 1980 onwards, so that a comparison with cpy in the early part of the sample is not possible. In addition, there seem to be changes in the definition of the AIB data that make it hard to interpret long time series of quarterly filings. Bankruptcy filings have generally trended downwards since 1980. Still there is a positive correlation with cpy at business cycle frequencies: bankruptcy filings are high when p is low, the correlation between the two lines in the figure is 0.23.



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