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ABSTRACT

The original intensity interferometers were instruments built in the 1950s and 1960s by Hanbury Brown and collaborators, achieving milliarcsec resolutions in visible light without optical-quality mirrors. They exploited a then-novel physical effect, nowadays known as HBT correlation after the experiments of Hanbury Brown and Twiss, and considered fundamental in quantum optics. Now a new generation of intensity interferometers is being designed, raising the possibility of measuring intensity correlations with three or more detectors. Quantum optics predicts two interesting features in many-detector HBT: (i) the signal contains spatial information about the source (such as the bispectrum or closure phase) not present in standard HBT and (ii) correlation increases combinatorially with the number of detectors. The signal-to-noise ratio (SNR) depends crucially on the number of photons – in practice always $\ll 1$ – detected per coherence time. A simple SNR formula is derived for thermal sources, indicating that three-detector HBT is feasible for bright stars. The many-detector enhancement of HBT would be much more difficult to measure, but seems plausible for bright masers.

Key words: instrumentation: interferometers.

1 INTRODUCTION

If two detectors are counting photons from an ordinary incoherent light source, there is a tendency for photons to arrive at both detectors together. This is known as photon bunching or HBT correlation, after the pioneering work of Hanbury Brown and Twiss, and is a consequence of the bosonic quantum statistics of photons. In a situation where coherent light would produce interference fringes, the HBT correlation with incoherent light varies according to those would-be but absent fringe patterns. This allows a type of interferometry with incoherent light and without optical-quality mirrors, known as intensity interferometry.

HBT correlation appears naturally in classical wave optics, and was actually first used to build a radio intensity interferometer, which resolved an extragalactic radio source for the first time (Hanbury Brown, Jennison & Das Gupta 1952). But when the same ideas were applied to visible light, and the effect was measured, first in the lab (Hanbury Brown & Twiss 1956) and then with starlight (Hanbury Brown & Twiss 1958), it became controversial, because it implied that different photons could interfere, contrary to conventional wisdom at the time. The controversies were eventually resolved with the development of a quantum-statistical theory for

incoherent light by Sudarshan (1963) and Glauber (1963) and the emergence of quantum optics. In quantum optics, ordinary light (or 'chaotic light') behaves like a random mixture of lasers, and the semi-classical picture of interfering waves, the squared amplitudes of which determine the emission rate of photo-electrons, turns out to be valid.[1] The general phenomenon of photon bunching or HBT correlation then spread to different areas of physics in different guises. For example, it is well known in the context of nuclear collisions (Baym 1998), and it may even be relevant to animal vision (Sim et al. 2012).

Meanwhile, Hanbury Brown and collaborators developed the Narrabri Stellar Intensity Interferometer (NSII) which measured stellar diameters down to milliarcsecs (Hanbury Brown 1968). But the photon detectors then available were only blue-sensitive, limiting the instrument to hot stars. So the NSII ran out of stars to observe in a few years, after which it was dismantled and almost forgotten. Decades later, with a new generation of detectors, intensity interferometry has become somewhat topical in astronomy again. A number of proposals and experiments have appeared in recent years (Ofir & Ribak 2006a,b; Borra 2008; Jain & Ralston 2008;

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[1] The semi-classical description is not valid in general. A nice counterexample is provided by electrons, which show HBT *anti*-correlation (Kiesel, Renz & Hasselbach 2002) from fermion statistics.

Foellmi 2009; Horch & Camarata 2012). Particularly ambitious are plans to adapt Cherenkov telescopes into a giant reincarnated NSII with resolution down to <0.03 mas (Dravins et al. 2012; Nuñez et al. 2012).

Future intensity interferometers are likely to have many detectors, not just two. Clearly, detectors can work in simultaneous pairs. The prospect of measuring N-fold photon coincidences has also been suggested (e.g. Ofir & Ribak 2006a). The basic theory is well established in the quantum optics literature and three-detector HBT has been measured in laboratory experiments (Sato et al. 1978; Zhou et al. 2010). But are the results interesting for an astronomical instrument? This paper examines the question.

2 STANDARD HBT

A nice introduction to HBT from a modern quantum-optics perspective appears in Glauber (2006). We quote some key results here.

The central concept is the field correlation functions. Let x_1 and x_2 denote the space–time locations of two detectors. The first-order correlation[2] is defined as the average:

$$G(x_1, x_2) \equiv \left\langle E^{(-)}(x_1) \, E^{(+)}(x_2) \right\rangle. \qquad (1)$$

Classically, $E^{(\pm)}$ denote the positive and negative frequency parts of the electric field at a detector. In quantum statistics, the fields become operators. We recognize $G(x_1, x_2)$ as the unnormalized correlation function or 'visibility' in radio interferometry, or the spatial Fourier transform of the source in the sky. Note that $G(x_2, x_1)$ is the complex conjugate (Hermitian conjugate in quantum statistics) of $G(x_1, x_2)$. Thus, $G(x_1, x_1)$, which is the count rate at x_1, is automatically real and non-negative. For a laser source, $G(x_1, x_2)$ would be independent of time, but for chaotic sources, the correlation falls away over a coherence time

$$\Delta\tau \approx 1/\Delta\nu. \qquad (2)$$

Now consider a form of second-order correlation:

$$G^{(2)}(x_1, x_2, x_2, x_1) \equiv \left\langle E^{(-)}(x_1) \, E^{(-)}(x_2) \, E^{(+)}(x_2) \, E^{(+)}(x_1) \right\rangle, \qquad (3)$$

which is the probability of coincident detection within $\Delta\tau$. Chaotic sources have the property (cf. Glauber's equation 37)

$$G^{(2)}(x_1, x_2, x_2, x_1) = G(x_1, x_1) \, G(x_2, x_2) + |G(x_1, x_2)|^2. \qquad (4)$$

The first term corresponds to the random coincidences that would also be expected for classical particles, but the second term describes the non-classical HBT correlations.

To estimate the signal-to-noise ratio (SNR), suppose we have two detectors with equal count rates r (photons per time unit), parametrized as

$$r \, \Delta\tau = G(x_1, x_1) = G(x_2, x_2), \qquad (5)$$

so that $G(x, x)$ provides the number of photons per coherence time. Let us now count photons over some time Δt. The time resolution Δt of the detector, typically $\Delta t \gg \Delta\tau$, is often called the reciprocal electrical bandwidth, because in the early experiments it was set by amplifier properties.

The product of photon counts (corresponding to random coincidences) will be close to $(r\Delta t)^2$, as given by the first term on the right of equation (4). The HBT signal given by the second term will make a small but non-zero contribution. Over a coherence time it will be $|G(x_1, x_2)|^2$. Over the much longer interval Δt, there are, so to speak, $\Delta t / \Delta\tau$ coherent slices, making the HBT signal $r^2 \Delta\tau \, \Delta t$. Using Poisson noise for the photon numbers and thus interpreting the square root of the total coincidence rate as the noise, we get a signal-to-noise ratio

$$\mathrm{SNR}(\Delta t) \sim r \, \Delta\tau \qquad (6)$$

for full correlation. This applies to a single counting time Δt. Over many counting times, the SNR adds in quadrature, so that we get[3]

$$\mathrm{SNR}(T) \sim r \, \Delta\tau \sqrt{\frac{T}{\Delta t}} \qquad (7)$$

when integrating over a duration T. When increasing the coherence time by using narrow-band filters, the photon rate r will decrease with $\Delta\nu \approx 1/\Delta\tau$ so that the 'spectral intensity' $r\Delta\tau$ stays constant. This has the remarkable effect that adding narrow-band filters does not reduce the achieved SNR. This makes it possible to increase the SNR further by using spectral detectors that can distinguish between different photon energies (within the limits of the uncertainty principle) and effectively record many narrow channels simultaneously. Or one can decrease the optical bandwidth so much that photon count rates are sufficiently low not to be affected by detector dead times.

In the NSII, the counting time Δt was ~10 ns. Current off-the-shelf instruments can achieve $\Delta t \simeq 50$ ps (not to mention better quantum efficiency and broader wavelength response). Clearly the time resolution is very good, but it is still orders of magnitude longer than the coherence time. Both these inequalities are important. The first inequality, or $\Delta t \gg \Delta\tau$, is what makes intensity interferometry interesting in the first place. In standard interferometry, optical paths have to be kept under control to better than λ, which is extremely demanding mechanically. For intensity interferometry, path differences do not matter as long as they are well below $c\Delta t$, corresponding to metres in the first experiments. This not only relaxes the mechanical tolerances, but it also makes the signal immune to atmospheric fluctuations. Compared to other techniques, intensity interferometry is possible with simpler technology. Put in another way, if we lack the precision to measure the first-order correlation $G(x_1, x_2)$ directly, we can still get information on it indirectly through the second-order correlation. The second inequality, or $\Delta t \ll T$, is what makes intensity interferometry usable with very low coincidence rates. The NSII could operate at $r \, \Delta\tau \sim 10^{-5}$, because it had 10^8 counting times per second and hence could build up $\mathrm{SNR} \sim 10^4 \, r \, \Delta\tau$ in a second. Current technology could deliver at least another order of magnitude better.

The above description, in terms of counting photons, is well suited to optical astronomy. In radio astronomy, on the other hand, a description in terms of waves and intensities is standard. For incoherent sources, the $E^{(\pm)}$ fields are considered as (complex) Gaussian random variables with autocorrelation functions determined by the characteristics of the receiving system. Each measurement naturally consists of a finite sum of random amplitudes, so that the fields and intensities vary with time even for sources of constant luminosity. This 'wave noise' or 'self-noise' defines the fluctuations whose correlations are measured as the HBT effect. The wave noise also adds

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[2] In a different terminology our 'first-order correlation' would be called 'two-point correlation', and our 'second-order correlation' is a 'four-point correlation'.

[3] It is understood that SNR must also include factors for throughput and detector efficiency (cf. equation 14 in Hanbury Brown 1968). But equation (7) is the essential SNR expression in standard HBT.

to the photon shot noise that is important in the optical domain of low photon rates, and this contribution actually dominates for high $r\Delta\tau$. We will return to this 'super-Poisson noise' (see e.g. Labeyrie, Lipson & Nisenson 2006) later.

Although the semi-classical picture is equivalent to the quantum optics picture on intensities versus photon counts (Sudarshan 1963), a conceptual difficulty arises when we consider the fields themselves. In standard radio interferometry (not HBT), the electric field is considered as a classical field which can be measured directly. But in quantum optics, the electric field is a non-Hermitian operator and hence not itself an observable. How to reconcile the radio astronomy and quantum-optics pictures? One possible resolution is given in Burke (1969). Here, we suggest another, which goes as follows. Let there be a source field $S^{(\pm)}$, and let us superpose it on a known local field $L^{(\pm)}$. The resulting field

$$E^{(\pm)}(x_1) = S^{(\pm)} + L^{(\pm)} \tag{8}$$

then gets its intensity measured:

$$G(x_1, x_1) = \left\langle S^{(-)}\,S^{(+)} \right\rangle + \left\langle L^{(-)}\,L^{(+)} \right\rangle + \left\langle S^{(-)}\,L^{(+)} \right\rangle + \left\langle L^{(-)}\,S^{(+)} \right\rangle. \tag{9}$$

Here, the first two terms on the right are just the intensities $|S|^2$ and $|L|^2$ of the two fields. The last two terms give a beating oscillation in the photon count rates, and it is from these oscillations that the phase $S^{(\pm)}$ is inferred. Thus, even if the source field cannot be measured directly, it can be inferred indirectly. To estimate the SNR, we note that the signal is in the last two terms of equation (9), and the noise will be mainly the noise in the second term. Thus,

$$\text{SNR}(G(x_1, x_1)) \simeq |S|\,|L| \,/\, \text{noise}\left(\left\langle L^{(-)}\,L^{(+)} \right\rangle \right). \tag{10}$$

If the local field is coherent, the denominator will follow Poisson noise and hence be $\propto |L|$, making the SNR $\propto |S|$, just as expected from the photon-counting picture. If the local field is not coherent, it will be subject to its own HBT fluctuations, or wave noise. We suggest the latter as a possible interpretation of the well-known 'receiver noise', which is the dominant noise in radio astronomy.

3 MORE THAN TWO DETECTORS

For chaotic sources, the higher order correlation functions are all given in terms of the first-order correlations. The result is presented in equation 10.27 of Glauber (1963), which, with a slight modification of notation, reads

$$G^{(n)}(x_1, \ldots, x_N, x_N, \ldots, x_1) = \sum_{\mathcal{P}} \prod_{k=1}^{N} G(x_k, \mathcal{P}x_k). \tag{11}$$

Here, $\mathcal{P}x_k$ denotes the kth element of a permutation of $\{x_1, \ldots, x_N\}$. The sum is over all permutations. In each term, the first argument always runs as $x_1, x_2, \ldots$, whereas the second argument runs as a permutation of that ordering. As we remarked earlier, in quantum optics, the G are correlations between the field operators, but the semi-classical approach of treating the fields as classical and then interpreting intensities as photon probabilities is valid for light (cf. Sudarshan 1963). Indeed, equation (11) also appears in the classical theory of random Gaussian variables, where it is known as Isserlis' theorem.

For three detectors, the formula (11) gives

$$G^{(3)}(x_1, x_2, x_3, x_3, x_2, x_1) = G(x_1, x_1)\,G(x_2, x_2)\,G(x_3, x_3)$$
$$+ G(x_1, x_3)\,G(x_2, x_2)\,G(x_3, x_1)$$

$$+ G(x_1, x_2)\,G(x_2, x_3)\,G(x_3, x_1)$$
$$+ G(x_1, x_1)\,G(x_2, x_3)\,G(x_3, x_2)$$
$$+ G(x_1, x_3)\,G(x_2, x_1)\,G(x_3, x_2)$$
$$+ G(x_1, x_2)\,G(x_2, x_1)\,G(x_3, x_3). \tag{12}$$

In each term here, the first argument always runs as x_1, x_2, x_3, whereas the second argument runs as a permutation of that ordering. One can rewrite this expression in another way, which is easier to interpret, by introducing the normalized correlations

$$g_{12} = \frac{G(x_1, x_2)G(x_2, x_1)}{G(x_1, x_1)G(x_2, x_2)},$$

$$g_{123} = \frac{G(x_1, x_2)G(x_2, x_3)G(x_3, x_1)}{G(x_1, x_1)G(x_2, x_2)G(x_3, x_3)}. \tag{13}$$

The three-point coincidence rate is then

$$1 + g_{12} + g_{13} + g_{23} + 2\Re\, g_{123} \tag{14}$$

times the chance coincidence rate. Here, the g_{ij} terms are the (normalized) power spectrum,[4] while the last term is the bispectrum, whose phase is well known in radio astronomy as the closure phase. Thus, whereas two-detector intensity interferometry only measures amplitudes but no phases, multidetector combinations are actually sensitive to certain combinations of phases and provide qualitatively new information.

If the detectors are close together, all the terms are equal ($g_{ij} = g_{123} = 1$), resulting in a six-fold enhancement over the chance coincidence rate. This has been measured in lab experiments (Zhou et al. 2010). At this point, one may get the idea, from the $N!$ terms in equation (11), that splitting up a single collecting area into N parts will increase the count rate. But in fact that will not happen. Splitting reduces the count rate in each detector by a factor of N; hence, the N-point coincidence rate would be $N!\,(r\,\Delta\tau/N)^N \propto \sqrt{N}\mathrm{e}^{-N}(r\,\Delta\tau)^N$, using Stirling's approximation for large N.

Let us now estimate the SNR. We have to be careful here, because the combinatorial formulas define the coincidences over $\Delta\tau$. We are interested in coincidences over Δt, and we have seen before in the case of two detectors that random and HBT coincidences scale differently. To take care of this, we have to understand that the intensities that are being correlated are integrated over a duration Δt consisting of many intervals of $\Delta\tau$, and correlations exist only for fields within the same short interval. The N-point HBT signal will therefore be

$$g_{1, \ldots, N} \times (r\,\Delta\tau)^N\,(\Delta t/\Delta\tau). \tag{15}$$

We should emphasize that equation (15) is not the number of coincidences, but the number that remains after subtracting off the chance coincidence rate and all the lower order HBT effects. Meanwhile, the chance coincidence rate is

$$(r\,\Delta t)^N. \tag{16}$$

Hence,

$$\text{SNR}(N, \Delta t) \sim g_{1, \ldots, N} \times (r\,\Delta\tau)^{N/2}\,(\Delta\tau/\mathrm{d}t)^{N/2-1}. \tag{17}$$

For $N = 2$ and $g_{12} \simeq 1$, we recover the simple expression (6) for standard HBT.

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[4] In terms of normalized visibilities γ_{jk}, we have $g_{12} = |\gamma_{12}|^2$ and $g_{123} = \gamma_{12}\gamma_{23}\gamma_{31}$.

4 SUPER-POISSON NOISE

In deriving the expression (17) for the SNR, we assumed that the chance coincidence rate that determines the noise follows Poisson statistics. This is a good assumption for low count rates $r\, dt \ll 1$. But for very high photon rates, a new source of noise enters: it is the well-known wave noise from radio astronomy, and as mentioned in the previous section, it can be considered as HBT correlation at a single space point over different times. Even in the limit of infinitely many photons, the received flux necessarily has to vary with time, because it consists of a finite number of samples from a stochastic process. These fluctuations that form the basis of the HBT effect in the first place eventually also limit its achievable SNR.

As we mentioned above, the N-detector coincidence rate includes all the lower order HBT signals as well. So, in principle, N-detector coincidences could be used to extract the two-point HBT signal. Is it advantageous to do so, compared to standard HBT? To answer this question, let us consider the number of N-detector coincidences over Δt

$$r^N \Delta t^N + r^N \Delta t^{N-1} \Delta \tau \sum_{j<k} g_{jk} + \dots. \tag{18}$$

The random coincidences are given by the first term, and two-point HBT gives the next term. In the same way as before, we may derive

$$\mathrm{SNR}(N \to 2, \Delta t) \sim \Gamma \times (r \Delta t)^{N/2} \frac{\Delta \tau}{\Delta t}, \tag{19}$$

where Γ denotes the sum in equation (18). Taking many counting times in quadrature and assuming full correlation ($g_{jk} = 1$), we have, analogous to equation (7),

$$\mathrm{SNR}(N \to 2, T) \sim \frac{N(N-1)}{2} (r \Delta t)^{N/2} \frac{\Delta \tau}{\Delta t} \sqrt{\frac{T}{\Delta t}}. \tag{20}$$

If we keep the total collecting area constant but split it into N detectors, so that the individual counting rate goes with r/N we have

$$\frac{\mathrm{SNR}(N \to 2, T)}{\mathrm{SNR}(2, T)} = (N-1) \left(\frac{r \Delta t}{N} \right)^{N/2-1}. \tag{21}$$

In the photon-counting regime, $r\Delta t/N < 1$, and hence splitting detectors does not help. The situation is different in the case of $r\Delta t/N > 1$, in which we may not be able to count individual photons, but can still measure intensities. The SNR would then apparently increase with N. Formally, the optimal number of detectors for $r\Delta t = 1/10/100$ is $N = 3/7/41$ with SNR benefits compared to two detectors of $1.15/14.6/(1.42 \times 10^9)$. These numbers at first look extremely promising. But there is a paradox: equation (21) has SNR increasing with Δt (that is, with coarser time resolution), when $r\Delta t$ is large enough. Clearly, this estimate based on photon shot noise cannot represent the whole truth, and resolving the paradox requires the inclusion of wave noise.

To see the effect of wave noise, let us return to the basic expression (4) of standard HBT, but now let x_1, x_2 be, not two detector locations, but the same detector at different times. Equation (4) then means the mean-square intensity at a single detector. Let us take this mean square over a time interval $\Delta t \gg \Delta \tau$. The first term on the right of equation (4) will contribute $(r\Delta t)^2$. The last term will contribute only if x_1, x_2 happen to be closer than a coherence time. There are $\Delta t/\Delta \tau$ time slices over which that happens, and over each of them, the last term contributes $(r\Delta \tau)^2$. The full contribution of the last term is hence $r^2 \Delta t \, \Delta \tau$. The expected photon count at a single detector is thus not constant, but has a variance of $r^2 \Delta t \, \Delta \tau$. Its square root is the single-detector wave noise (photon shot noise

comes on top of it) and is proportional to intensity, not square-root intensity. It is the single-detector wave noise, also described as super-Poisson noise, and dominates if $r\Delta \tau > 1$. For each sample of a Gaussian random process (like the field of chaotic light), wave noise limits SNR to order unity. Sampling the same field with larger collecting area does not circumvent this fundamental limit. Even for strong sources, the final SNR can thus never be higher than the square root of the number of samples. The only chance to increase the SNR is observing for longer or with more bandwidth. This is a well-known (although often neglected) effect in radio astronomy (see e.g. Radhakrishnan 1999).

Generalizing the N-point SNR formula (17) to high photon rates requires generalizing the above single-detector argument to N detectors. We leave a full calculation for a future paper, but the basic conclusion, that SNR over one counting time Δt cannot exceed unity, remains valid for multiple detectors, no matter how bright the source or how ridiculously large the light collectors.

5 FLUXES AND COUNT RATES

From the previous section, we see that the number of photons detectable in a coherence time, or $r\,\Delta \tau$, is central to the SNR.

In a passband around ν, a source with flux density F_ν gives

$$r\, \Delta \tau \simeq \frac{F_\nu}{h\nu} A, \tag{22}$$

where A is the collecting area. As mentioned before, the effect of $\Delta \nu$ cancels between r and $\Delta \tau$. Rewriting the expression, mixing wavelength and frequency, in the form

$$r\, \Delta \tau \simeq 0.05 \times \left(\frac{F_\nu}{\mathrm{Jy}} \right) \left(\frac{\lambda}{\mathrm{m}} \right) \left(\frac{A}{\mathrm{m}^2} \right) \tag{23}$$

makes it easy to compare sources.

Bright stars can have $F_\nu \sim 10^4$ Jy, but $\lambda \sim 10^{-6}$ m. Hence, $r\, \Delta \tau \ll 1$ per m^2. More detailed estimates are shown in Fig. 1. On the other hand, bright masers[5] can have $F_\nu \sim 100$ Jy at $\lambda \sim 1$ cm, so a large dish easily receives $r\, \Delta \tau > 1$. Now, intensity interferometry is not needed to resolve masers, because at radio wavelengths it is much easier to control the delays sufficiently accurately for standard interferometry. But high-order HBT using astrophysical masers would be a novel quantum-optics experiment.

Blackbody sources lead to a rather elegant estimate of $r\, \Delta \tau$. Consider a blackbody source at temperature T and angular area Ω on the sky. We detect photons from it using a collector of area A, in a passband $\Delta \lambda$. For convenience, we will work in terms of a logarithmic passband

$$\Delta(\ln \lambda) = \frac{\Delta \lambda}{\lambda}. \tag{24}$$

We also write

$$z \equiv \frac{hc}{\lambda k T}. \tag{25}$$

Recall the number density of a photon gas

$$\frac{4\pi}{\lambda^3} \frac{\Delta(\ln \lambda)}{e^z - 1}. \tag{26}$$

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[5] For natural masers to show HBT correlation, it is essential that they are not single-mode systems like artificial masers, but rather like chaotic superpositions of laboratory masers. This fact also makes them spatially incoherent so that the usual Fourier relations for the correlations hold.
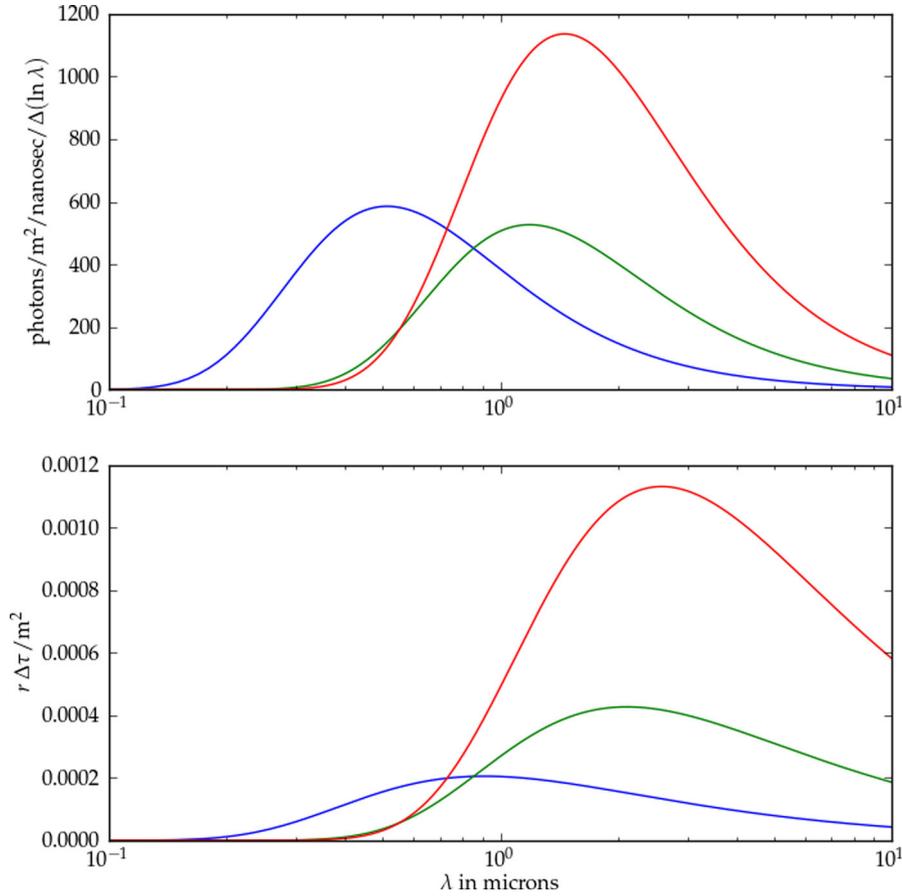
Figure 1. Photon flux (above) and photon flux per coherence time (below) for three blackbody sources, corresponding roughly to Sirius, Arcturus and Betelgeuse. The lower panel leads to simple estimates of SNR.

This is for one polarization state; since we are making rough estimates here, we will disregard the second polarization state. To get the photon flux, we multiply this by $c\Omega/(4\pi)$ or $\lambda\nu\Omega/(4\pi)$. The photon arrival rate in an area A is thus

$$r = \Omega A \, \frac{\nu}{\lambda^2} \, \frac{\Delta(\ln\lambda)}{e^z - 1} \, . \tag{27}$$

We can rewrite $\Delta\lambda$ in terms of the coherence time (2). The number of photons received in a coherence time is then

$$r \, \Delta\tau \sim \frac{\Omega A}{\lambda^2} \, \frac{1}{e^z - 1} \, . \tag{28}$$

We can also write $r \, \Delta\tau$ in another way. Consider the baseline needed to resolve the source. The square of the baseline is the area $A_{airy} \simeq \lambda^2/\Omega$ of an aperture whose Airy disc corresponds to the size of the source. This is a natural limit for the collecting area of detectors to just resolve the source and, e.g. measure its size, because larger detectors would require longer baselines and reduce the correlations g so much that the source would be 'resolved out'. With this definition, we have

$$r \, \Delta\tau \approx \frac{A}{A_{airy}} \, \frac{1}{e^z - 1} \, . \tag{29}$$

For very small z,

$$r \, \Delta\tau \approx \frac{A}{A_{airy}} \, \frac{kT}{hc}\lambda \, , \quad \lambda \gg \frac{hc}{kT} . \tag{30}$$

We see that for an HBT baseline adapted to the source size (that is, A/A_{airy} fixed), the SNR only depends on λT. Thus, by going far into

the Rayleigh–Jeans tail, $r\Delta\tau$ can in principle be made arbitrarily large (with the consequence of increasing wave noise), but in the brightest part of the spectrum $r\Delta\tau \ll 1$.

A simple physical interpretation for the expression (29) is obtained by noting that it is A/A_{airy} times the phase-space density of photons in blackbody radiation. As result of Liouville's theorem, this phase-space density is conserved when the radiation leaves the sources to travel towards the observer. When increasing the distance, the total flux gets diluted, but at the same time the apparent size of the source shrinks so that the momentum-space density increases, which compensates for the former effect. A_{airy} is a reciprocal measure for the apparent size of the source, and A/A_{airy} by definition takes care of both effects. With increasing distance, the light bucket has to grow to pick up more photons, but can also form a smaller field of view and be susceptible to a smaller part of momentum-space.

Fig. 1 shows the count rate and $r \, \Delta\tau$ for three blackbody sources. These have $T = 9940\,\mathrm{K}$ and angular diameter $\theta = 0.007$ arcsec, which approximates Sirius, $T = 4300\,\mathrm{K}$, $\theta = 0.02$ arcsec, similar to Arcturus, and $T = 3500\,\mathrm{K}$, and $\theta = 0.04$ arcsec, similar to Betelgeuse. Hanbury Brown & Twiss (1958) measuring Sirius had a collecting area of about 2 m^2, and quantum efficiency 15 per cent at 0.4 μm. From Fig. 1, we would predict SNR $\simeq 3 \times 10^{-5}$ per counting cycle, less absorption and other losses. The reported value is SNR $= 8.5$ in 345 min, using 5–45 MHz, which corresponds to SNR $\simeq 1 \times 10^{-5}$ per counting cycle.

Equation (29) is not limited to bright stars, however. As a more exotic example of an approximately thermal source, consider the

central accreting region of M87 (e.g. Doeleman et al. 2012). The innermost stable orbit lensed by the supermassive black hole has a diameter of the order of 50 μas. Resolving sources of this size at optical wavelengths requires baseline lengths of several kilometres. In order to achieve a similar $A/A_{\rm airy}$ as in the NSII, collecting areas of several hectares would be needed. Since optical-quality mirrors are not required, such light buckets could be built. Whether such an observation is possible probably depends more on how well the background light from the host galaxy can be excluded.

6 DISCUSSION

We may expand the title of this paper to two questions. First, is the quantum-optical effect of three-point or higher order HBT measurable for any astronomical source? If so, would it tell us new things about the source?

Measuring three-point HBT for bright stars appears feasible. Using equation (17) for $N = 2$ and $N = 3$ scaled to an integration time T,

$$\mathrm{SNR}(T, N = 2) \sim g_{12} \times (r\,\Delta\tau)\sqrt{\frac{T}{\Delta t}},$$

$$\mathrm{SNR}(T, N = 3) \sim g_{123} \times (r\,\Delta\tau)^{3/2}\frac{\sqrt{T\,\Delta\tau}}{\Delta t}. \qquad (31)$$

For visible light with a narrow-band filter, the coherence time $\Delta\tau \sim 10^{-12}$ s. Off-the-shelf photon counters can reach a time resolution of $\Delta t \sim 10^{-10}$ s. From Fig. 1, a square metre of collecting area gives $r\,\Delta\tau \sim 10^{-4}$ from a bright star. These numbers suggest SNR ~ 1 in an hour. For quicker results one would want to increase the collecting area – from (17) we see that $T \propto A^{-N}$ for a given SNR. As always in HBT, optical-path tolerances need only be $\ll c\Delta t$.

Three-point HBT would provide the three-point closure phase, or bispectrum, of the source on the sky. Is that worth having? Since two-point HBT gives only the power spectrum, and no phase information, having the bispectrum is likely to be an important advantage for image reconstruction of bright stars.

Going to four or more detectors, for bright stars or any other thermal sources, would be difficult. The geometric estimate (29) of the photon count per coherence time indicates that $r\,\Delta\tau \ll 1$ for any thermal source, except far in the Rayleigh–Jeans tail. But $r\,\Delta\tau$ appears at progressively higher powers in the SNR. For non-thermal sources, the situation may be very different. In particular, $r\,\Delta\tau > 1$ appears achievable for bright masers. Masers can be imaged using standard radio telescopes, so HBT may not provide any new

information on them. Nonetheless, it would be an interesting physics experiment to look for the combinatorial enhancement of the HBT effect for large N. The total number of photons would not increase, of course, they would just get more and more bunched.

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