University of Zurich

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Department of Economics

Working Paper Series

ISSN 1664-7041 (print)
ISSN 1664-705X (online)

Working Paper No. 175

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October 2014

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Abstract: This paper analyzes two-stage rank-order tournaments. A principal decides (i) how to spread prize money across the two periods, (ii) how to weigh performance in the two periods when awarding the second-period prize, and (iii) whether to reveal performance after the first period. The information revelation policy depends exclusively on properties of the effort cost function. The principal always puts a positive weight on first-period performance in the second period. The size of the weight and the optimal prizes depend on properties of the observation error distribution; they should be chosen so as to strike a balance between the competitiveness of first- and second-period tournaments. In particular, the principal sets no first-period prize unless the observations in period one are considerably more precise than in period two.

JEL: D02, D44

Keywords: dynamic tournaments, repeated contests, information revelation, effort incentives

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1 Introduction

In many contexts, groups of economic agents supply efforts repeatedly, thereby giving rise to sequences of performance signals that principals can use to reward efforts. First, most organizations assess their employees' performance regularly. This performance information plays a crucial role for decisions on bonus payments, promotion and tenure. Second, in many arms-length relationships, buyers repeatedly procure goods and services from the same pool of suppliers. They can use past experience with these suppliers as a basis for the conditions of future interactions. Third, school and university teachers repeatedly observe the performance of students in their classes and can decide how to use this information for final grades.

Motivated by these real-world situations, we analyze the incentive effects of different approaches to rewarding repeated performance. Specifically, we ask the following questions:

1. How often should principals reward agents for good achievements? Should there be frequent small rewards or rare large rewards?

2. Which weight should principals give to recent performance relative to performance in the more distant past?

3. To which extent should the principal reveal the results of past performance measurement to the agents?

We answer these questions for dynamic tournaments. Tournaments are often used instead of contracts which condition explicitly and exclusively on each agent's own performance. In particular, organizations indeed provide incentives with promotion tournaments.[2]

Specifically, we consider a two-period tournament with two risk-neutral agents with identical and known abilities. To see the incentive effects of such tournaments most clearly, we abstract from the important issue of selecting

[2] A well-known argument for tournaments is that they are more credible because they are less prone to manipulation by the principal than contracts that depend explicitly on the details of performance: When performance is not verifiable, a principal may claim that performance was low to save on performance pay. Tournaments reduce this incentive, because the total payments to the agents are independent of performance.

the agent with the highest innate ability for a particular task. The principal chooses an incentive system, consisting of (i) the distribution of the prize money across the two periods, (ii) the weight of first-period performance in the second tournament and (iii) the information revelation policy.

After observing the policy, the agents choose effort levels in each period. The principal observes the performance of each agent, which is a noisy measure of effort. In period 1, she awards the prize (if any) to the agent with the higher performance. Under a full revelation policy, she communicates the performance of both agents in the first period. Under a no revelation policy, she neither communicates performance, nor who the winner was. In period 2, the agents choose efforts again. The principal then allocates the second-period prize to the agent for whom the weighted sum of first- and second-period performance is highest.[3]

In line with the existing literature, we consider the case that a principal regards efforts in different periods and by different agents as perfect substitutes and thus maximizes total effort. Contrary to most of the existing literature, we also analyze the optimal policy for a principal who regards efforts in different periods as imperfect substitutes and wants to balance them across periods.[4] We believe this is important, because excessively low efforts in some period may cause large harm, which cannot even be compensated by an extremely large effort in other periods.

Apart from allowing for imperfect intertemporal effort substitution, our approach differs from previous literature in three ways. First, we simultaneously consider information revelation, the prize distribution and performance weights as design tools of the principal. Second, we include the possibility that the distributions of the first- and second-period performance measures differ - e.g. in their precision - reflecting heterogeneity of tasks across periods. Third, we allow different cost functions across periods.

[3] In the no revelation case, the game is static. The model thus becomes a special case of a multi-battle contest where agents compete simultaneously in a multiplicity of dimensions (see, e.g., Clark and Konrad (2007) and Kovenock and Roberson (2010)). However, the dynamic cross-period effects which occur under our full revelation regime are totally absent in these papers.

[4] Specifically, she maximizes the product of first- and second-period efforts, or equivalently, the sum of the logarithms. Aoyagi (2010) also allows for more general objectives than maximizing total efforts.

Our contribution is threefold. First, we generalize existing results on information revelation. Previous analysis has shown for special cases that expected total efforts are lower with revelation than without when marginal effort costs are convex, and conversely for concave marginal effort costs (see Section 2). We show that this result holds for perfect and imperfect substitutes, and for arbitrary first-period prizes and performance weights.

Second, we clarify the relation between first-period prizes and first-period performance weights as incentives for first-period efforts. For both revelation policies and for perfect as well as imperfect substitutes, the optimal first-period prize is positive only if the distribution of the first-period observation error difference is very precise, that is, highly concentrated near zero. We then show that for quadratic cost functions and normally distributed observation errors, this condition is never satisfied. Even with more general distributional assumptions, the scope for using first-period prizes is limited: For imperfect substitutes and quadratic cost functions, the optimal first-period prize is never higher than the second-period prize.

Whereas the optimal first-period prize is typically zero, the optimal weight of first-period performance in the second-period tournament is strictly positive for both revelation policies, general cost functions and error distributions. The optimal weight is higher the lower the adverse effect of increasing the first-period weight on future competitive intensity is. For quadratic cost functions, normally distributed observation errors and perfect (imperfect) substitutes, the optimal weight is the ratio of the variances (standard deviations) of second-period and first-period observation error differences.

Third, we show that the potential gains from good design are quantitatively important. In the normal-quadratic example, the expected effort is at least 40% higher when a principal chooses prizes and weights optimally than when she distributes the prize money evenly across both periods without giving weight to first-period performance in the second period tournament.

The organization of the paper is as follows. Section 2 discusses related literature. In Section 3, we introduce the model. In Section 4, we analyze the behavior of agents for given policies. Section 5 characterizes the optimal policy. Section 6 interprets and sharpens our results in a normal-quadratic example. Section 7 concludes.

2 Relation to the Literature

In this paper, we focus on the optimal design of multi-period rank-order tournaments, in particular, on feedback policy, prize structure and weight of past performance.[5,6] The only paper we are aware of that simultaneously analyzes these three design dimensions is Gershkov and Perry (2009). However, their set-up differs substantially from ours. Most importantly, after period one, the principal merely knows whether there is a tie (arising with positive probability) or whether one of the agents has performed better (and, if so, which agent); there is no information on the size of the lead. In many contexts, such a coarseness of the information structure appears to be appropriate. However, in other contexts, the principal can collect and communicate information that provides the agents with a clear picture of how much their performance differs from the performance of others. This information will typically not be verifiable in a court, but for our purposes it is sufficient that the principal and the agents share a common understanding of the relation between promotion chances and the information communicated about the agents' relative positions.[7]

Also, Gershkov and Perry (2009) assume that the relation between winning probabilities and efforts is the same in both periods, while we allow for differences in the error structure. Finally, they only focus on maximization of total effort.

We mention in passing the substantial literature analyzing agent behavior in repeated tournaments without addressing optimal design. Several of these papers allow for effects of first-period play on the second period that are determined by technology rather than, as in our case, by the principal.[8] Moreover, some papers study two-period contests (rank-order, all-pay and

[5] Nitzan (1994) and Konrad (2009) provide surveys of the literature on tournaments.

[6] Another broadly related literature analyzes dynamic principal-agent relationships with moral hazard in a non-competitive setting. Lewis and Sappington (1997) examine how current incentives should optimally depend on past performance. Hansen (2013) and Chen and Chiu (2013) deal with the optimal revelation policy. For reasons of space, however, we will focus on studies that deal with repeated contests.

[7] With verifiable information, the principal could contract directly on efforts, and there would be no need to use tournaments.

[8] See Schmitt et al. (2004), Grossmann and Dietl (2009), Grossmann (2011) and Baik and Lee (2000).

Tullock, respectively) where the total effort in the two periods determines the winner of a final prize.[9,10]

2.1 Performance revelation

Several papers analyze the effect of interim performance revelation on efforts in dynamic tournaments. In a setting similar to ours, Aoyagi (2010) shows that expected effort is higher with information revelation than without if and only if marginal effort costs are concave.[11] Unlike in our paper, there is only one prize, and first-and second-period weights are the same. We endogenize these assumptions by providing conditions under which the principal optimally chooses the prizes and weights in this way. Moreover, we show that the optimal revelation policy has the same features when these assumptions do not hold.

Ederer (2010) introduces incomplete information about ability. The results are equivalent to those of Aoyagi (2010) if ability is non-complementary to effort.[12] If efforts and ability are complementary, it is possible that information revelation leads to higher expected efforts than no revelation even with quadratic effort costs.[13]

[9] See Hirata (2014) for all-pay auctions and Yildirim (2005) for Tullock contests. Casas-Arce and Martínez-Jerez (2009) consider a related rank-order tournament where all agents whose total performance is higher than a certain threshold win a prize.

[10] More broadly related, several papers analyze the agents' behavior in a sequence of contests where there is a prize for winning each contest, and an overall prize to the agent who is the first to win a certain number of contests. Examples are Konrad and Kovenock (2009) and Krumer (2013). Sela's (2011) model is similar, the difference being that there is no prize for winning a single contest.

[11] Aoyagi (2010) is quite general with respect to the objective of the principal, and he allows for partial revelation. Denter and Sisak (2013) show that effort may increase with revelation if marginal efforts are concave. They use their set-up to analyze the effect of polls on political campaign spending, allowing for an initial asymmetry before the beginning of the first period.

[12] Ederer and Fehr (2013) use a special case of this model with equal abilities.

[13] Other papers address the revelation policy in dynamic tournaments under very different assumptions. For example, Arbatskaya and Mialon (2012) analyze a lottery contest where first- and second-period efforts are complements in affecting the probability of winning. They find that revelation of first-period efforts decreases total efforts. Goltsman and Mukherjee (2011) consider a contest in which the agents either succeed or fail, and the prize is given to the agent who succeeded more often. The optimal policy reveals performance only if both agents fail. Zhang and Wang (2009) consider revelation policies

2.2 The weight of past performance

Several authors ask whether there should be a bias towards the first-period winner in the second period of a multi-period contest (Meyer 1992, Harbaugh and Ridlon 2011 and Ridlon and Shin 2013). Meyer (1992) considers a setting similar to our case with information revelation and a single prize, but with risk-averse agents. She shows that the cost-minimizing choice of an effort vector requires a bias towards the first-period winner.[14] Our analysis shows that the argument for giving a headstart also holds when the first-period prize is much higher than the second-period prize, when there is no information revelation, when intertemporal effort substitution is imperfect and when there is no information revelation. Finally, we provide results on the determinants of the size of the bias.[15]

2.3 Distribution of prize money

A small number of papers derives the optimal distribution of prize money across periods when there is an exogenously given technological link between the first and the second period, creating an asymmetry between the agents in the second period. The effects of such links are similar to those of a positive weight of past performance in the assignment of the second-period prize. Contrary to us, the authors focus on Tullock contests. For example, in Möller (2012), the prize money received in the first period does not yield direct utility to the agents, but reduces their effort costs in the second period. Under some circumstances, the optimal policy requires a positive prize both for the winner and for the loser in the first period.[16] In Clark et al. (2013), the winner in the first period may have lower effort costs in the second period.

in dynamic all-pay auctions with elimination.

[14] Ridlon and Shin (2013) show for a Tullock contest that an analogous result still holds for small asymmetries in the abilities of agents. However, if the asymmetry is high, favoring the first-period loser is optimal. In the dynamic all-pay auction of Harbaugh and Ridlon (2011), favoring the first-period loser is always optimal.

[15] Contrary to us, Meyer (1992) assumes that the size of the bias is fixed ex ante rather than a function of the performance difference in period 1.

[16] Since agents are initially symmetric, unequal prizes in the first period yield an asymmetry in the second period through their effect on second-period effort costs. This result is therefore similar to a positive weight on past performance in our setting.

The effort-maximizing prize structure is to give only a second-period prize. In Clark and Nilssen (2013), second-period effort costs fall with the first period effort. The authors provide conditions under which it is optimal to pay more than half of the total prize money in the second period.[17] Apart from the obvious difference in the structure of the contest, these papers do not analyze revelation policies, nor do they allow for imperfect substitutes.

Some papers derive the optimal distribution of prize money across stages in a two-period elimination tournament, where only the winners of the current period compete again in the next period. A seminal paper is Moldovanu and Sela (2006). Because elimination tournaments have a very different structure than our model, the results are difficult to compare to ours.

3 The Model

We consider a class of two-stage rank-order tournaments. Given a fixed budget $W > 0$, a principal chooses an *incentive system* $\mathcal{I}$, which is a tuple $(\eta, W_1, \rho) \in \mathbb{R} \times [0, W] \times \{0, 1\}$ to be explained below. For given $\mathcal{I}$, agents $i \in \{1, 2\}$ choose effort levels $e_{it} \geq 0$ $(t \in \{1, 2\})$.[18] The effort cost function $K_{it}(e_{it})$ has the following properties:

Assumption 1: *K_{it} is independent of i and differentiable three times. It satisfies $K'_{it} > 0$, $K''_{it} > 0$, $K_{it}(0) = K'_{it}(0) = 0$. $K'''_{it}(e_{it}) \geq 0$ or $K'''_{it}(e_{it}) \leq 0$ must hold globally.*

Thus, we can write $K_t \equiv K_{it}$. Note that we allow first- and second-period tasks to differ with respect to effort costs. This reflects the idea that the efforts in the two periods may be of very different types. Employees or suppliers may have to carry out different tasks in different periods; students learn different kinds of material in different phases of their education. Therefore, effort costs may differ across tasks.

The agents maximize expected utility and are risk-neutral. Their utility is additively separable in period-specific income and costs. At the end of each

[17] We have a similar result in the case of imperfect substitutes, but for very different reasons (see Proposition 7).

[18] In the following, the use of i and/or j as an index always implies $i, j \in \{1, 2\}$ and $i \neq j$.

period t, the principal observes performance, which is an imperfect measure $s_{it} = e_{it} + \varepsilon_{it}$ of effort. The error term ε_{it} is independently distributed across agents and periods. In each period, the error distribution is the same for agent 1 as for agent 2. However, the error distribution in period 1 may differ from the one in period 2. This captures the notion that tasks in different periods may also differ in terms of how easy it is to monitor effort.[19]

Based on the first-period performance, the principal awards the first-period prize W_1 to agent i if $s_{i1} > s_{j1}$. Agent i receives the second-period prize $W_2 = W - W_1$ if $s_{i2} + \eta s_{i1} > s_{j2} + \eta s_{j1}$.[20] The principal's choice of the first-period weight $\eta \in \mathbb{R}$ thus determines the influence of past performance on the chance of winning in the second period.

Under a *full-revelation policy* ($\rho = 1$), the principal communicates the measured performance of both players to the agents before they choose their second-period efforts. In practice, the principal will typically not communicate a concrete number. Instead, she may communicate whatever relevant information she has to the agents, thereby creating a common understanding about their relative performance.[21] Under a *no-revelation policy* ($\rho = 0$), the principal does not communicate the performance assessment. She does not even communicate who won the first-period prize and distributes both prizes at the end of period 2.

The following notation is helpful to describe the solution of the game.

Definition 1 *The* ***error difference*** *of player i in period t ($t = 1, 2$) is $\Delta\varepsilon_{it} = \varepsilon_{it} - \varepsilon_{jt}$, his* ***relative first-period performance*** *is $\Delta s_{i1} = s_{i1} - s_{j1} = \Delta e_{i1} + \Delta\varepsilon_{it}$, where $\Delta e_{it} = e_{it} - e_{jt}$.*

Clearly, $\Delta e_{it} = -\Delta e_{jt}$, $\Delta\varepsilon_{it} = -\Delta\varepsilon_{jt}$, $\Delta s_{i1} = -\Delta s_{j1}$.

[19] In a non-tournament setting, Ke et al. (2014) show that organizations optimally hire workers into easy-to-monitor jobs with low effort costs and then promote them into difficult-to-monitor jobs with high (marginal and absolute) effort costs. In our setting, this would correspond to $\sigma_1 < \sigma_2$ and $K_1(e) < K_2(e)$, $K_1'(e) < K_2'(e)$.

[20] In each period, in case of a tie, the principal assigns the prize to each agent with probability one half.

[21] As we will see, second-period efforts depend negatively on the absolute value of the performance difference in the first period. Hence, the principal has an incentive to always report equal performances. This problem becomes negligible if the principal leaves the communication to disinterested parties from within or outside the organization.

We make the following assumption on the error distributions:

Assumption 2 $\Delta\varepsilon_{it}$ *is distributed as* $F_t(s)$ *on* $\mathbb{R}$ *with a symmetric, single-peaked, strictly positive and continuously differentiable density* $f_t(s)$.

This implies $f_t(s) = f_t(-s)$, $f_t'(s) = -f_t'(-s)$ and $E(\Delta\varepsilon_{it}) = 0$.[22]

For some results, we assume that the cost functions are quadratic:

(C1) The cost function is $K_t(e_{it}) = \frac{k_t}{2}(e_{it})^2$ with $k_t > 0$.

We assume that, given a fixed prize budget, the principal's payoff is increasing in efforts, where the efforts of different agents within periods are perfect substitutes for the principal. We allow first- and second period efforts to be either perfect or imperfect substitutes. For *perfect substitutes*, the principal chooses the incentive system so as to maximize expected total efforts. For *imperfect substitutes*, she maximizes the expected product of first and second-period efforts. This corresponds to a complementarity between first- and second-period efforts, making it desirable to have similar efforts in both periods.

4 Behavior of the agents

We first analyze the equilibrium behavior of agents for given incentive system. The following simple result is mentioned without proof.

Lemma 1 *(i) The conditional probability that* $s_{i1} > s_{j1}$ *given* e_{i1} *and* e_{j1} *is* $F_1(e_{i1} - e_{j1})$.
(ii) The conditional probability that $s_{i2} + \eta s_{i1} > s_{j2} + \eta s_{j1}$ *given* e_{i2},e_{j2} *and* Δs_{i1} *is* $F_2(\eta\Delta s_{i1} + e_{i2} - e_{j2})$.

4.1 Full revelation

In period 2, a player's information set consists of all combinations of period 1 efforts and error differences that are consistent with the own first-period

[22] The assumptions on the distribution of the error differences are guaranteed to hold if the assumptions hold for the distributions of the observation errors.

effort e_{i1} and the observed relative performance Δs_{i1}.[23] We use the Perfect Bayesian Equilibrium (PBE) to deal with this imperfect information (Mas-Colell et al. 1995, p. 285). The task is simplified because there are no off-equilibrium events to consider, as f_1 is strictly positive on $\mathbb{R}$. Moreover, period 1 enters player i's payoffs only via Δs_{i1} and e_{i1}, so that the unobservable aspects of previous play (player j's effort choices) are irrelevant for the players' choices.

A pure strategy σ_i of player i consists of a first-period choice e_{i1} and a function E_{i2} mapping information sets $(e_{i1}, \Delta s_{i1})$ to actions e_{i2}. If player i chose e_{i1}, observes Δs_{i1} and assumes that player j plays the pure strategy $\sigma_j = (e_{j1}, E_{j2})$, he will assign probability one to the event that $\Delta\varepsilon_{i1} = \Delta s_{i1} - \Delta e_{i1}$. We will always assume that beliefs are formed in this way, without specifying them explicitly.

4.1.1 Second-period efforts

Using Lemma 1(ii), the expected second-period payoff of agent i, conditional on the relative first-period performance and second-period efforts, is

$$U_{i2}\left(e_{i2}, e_{j2}, \Delta s_{i1}\right) = F_2\left(\eta \Delta s_{i1} + \Delta e_{i2}\right) W_2 - K_2\left(e_{i2}\right). \qquad (1)$$

Thus, the first period influences the second-period payoff via the first-period relative performance Δs_{i1}. The corresponding first-order condition is

$$f_2(\eta \Delta s_{i1} + \Delta e_{i2}) W_2 = K_2'\left(e_{i2}\right). \qquad (2)$$

Though the game does not have any proper subgames because information sets in period 2 are not singletons, payoffs in period 2 are constant on information sets. We use this in the following definition.

Definition 2 *The second-period effort game induced by Δs_{i1} is the game with players $i = 1, 2$, strategy spaces $X_i = \mathbb{R}^+$ and payoffs given by (1) for $(e_{i2}, e_{j2}) \in X_i \times X_j$.*

[23] This statement holds no matter whether the principal publicly announces the absolute performance of each agent, or just the difference.

We obtain the following result:

Lemma 2 *Suppose $\rho = 1$ (full revelation) and $W_2 > 0$.*
(i) In any equilibrium of the second-period effort game, efforts are symmetric and satisfy

$$e^*_{i2}(\Delta s_{i1}) \equiv e^*_{i2}(\Delta s_{i1}; \eta, W_2, 1) = (K'_2)^{-1}[f_2(\eta \Delta s_{i1}) W_2] \tag{3}$$

(ii) If effort costs are sufficiently convex, (3) defines the unique Nash equilibrium of the second-period effort game.

Proof. See Appendix. ■

Lemma 2 has some simple comparative statics implications.

Corollary 1 *Suppose $\rho = 1$, $\eta \neq 0$ and $W_2 > 0$. Then e^*_{i2} is decreasing in $|\Delta s_{i1}|$ and $|\eta|$, and increasing in W_2.*

Proof. See Appendix. ■

The result on $|\Delta s_{i1}|$ implies that, if a *laggard* (an agent with $\Delta s_{i1} < 0$) increases own effort, or a *leader* (an agent with $\Delta s_{i1} > 0$) decreases efforts marginally in period 1, both players increase effort in period 2.[24] The other two results identify policy effects. In particular, increasing the absolute value of the first-period weight η reduces second-period efforts.

In the PBE, the symmetric equilibrium of the second-period effort game is played after each realization of Δs_{i1}. Thus, the expected second-period payoff, conditional on first-period performance, is

$$U^s_{i2}(\Delta s_{i1}) \equiv U_{i2}\left(e^*_{i2}(\Delta s_{i1}), e^*_{j2}(-\Delta s_{i1}), \Delta s_{i1}\right). \tag{4}$$

The expected second-period payoff, given first-period efforts, is

$$U^e_{i2}(e_{i1}, e_{j1}) \equiv E_{\Delta\varepsilon_{i1}} U^s_{i2}(\Delta e_{i1} + \Delta\varepsilon_{i1}). \tag{5}$$

[24] This result reflects the "well-known evaluation effect or lack-of-competition effect" (Ederer 2010, p. 742).

4.1.2 First-period efforts

Using Lemma 1(i), agent i's optimization problem in period 1 is

$$\max_{e_{i1}\geq 0} F_1\left(e_{i1}-e_{j1}\right)W_1+U_{i2}^e\left(e_{i1},e_{j1}\right)-K_1\left(e_{i1}\right).$$

The corresponding first-order conditions is

$$f_1\left(\Delta e_{i1}\right)W_1+\frac{\partial U_{i2}^e}{\partial e_{i1}}=K_1'\left(e_{i1}\right). \tag{6}$$

The following definition is crucial for the intuition.

Definition 3 *The* ***intensity of second-period competition*** *is given by*

$$C(\eta)=2\int_0^\infty f_2\left(\eta s\right)f_1\left(s\right)ds.$$

The logic of the definition is as follows. For each agent, $f_1(s)$ captures the density of the event that the relative first-period performance of this player is s when efforts are symmetric (as in equilibrium). Since both players choose identical equilibrium efforts in the second period, $f_2(\eta s)=f_2(-\eta s)$ captures the density of the event that a strike of luck of one agent in period 2 exactly compensates a strike of luck of the other agent of size s in period 1. Therefore, $C(\eta)$ measures the joint probability of the event that the second-period contest is a close run where a marginal effort increase of one agent will affect the outcome of the second-period contest and tip the balance in his favor: When $C(\eta)$ is high, an agent who has been lucky in the first period cannot be too sure about his winning prospects in the second period, and will therefore continue to put in some effort.

$C(\eta)$ is a function of the weight η with several simple properties. First,

$$C'(\eta)=2\int_0^\infty sf_2'\left(\eta s\right)f_1\left(s\right)ds<0 \text{ for } \eta>0: \tag{7}$$

An increase in the absolute value of the weight thus reduces the intensity of second-period competition. Moreover:

$$C(\eta) > 0 \quad (8)$$

$$C(0) = f_2(0) \quad (9)$$

$$C'(0) = 0 \quad (10)$$

$$C(\eta) = C(-\eta) \quad (11)$$

We sometimes invoke a regularity condition that simplifies the interpretation of our results:

(C2) For $\eta > 0$, $\eta C(\eta)$ is increasing in η.

This conditions holds, for instance, in Example E1 below. The following condition rules out corner solutions in period 1:

$$f_1(0) W_1 + \eta W_2 C(\eta) > 0. \quad (12)$$

(12) can only be binding for negative η.[25]

The following result uses (6) to derive equilibrium efforts:

Proposition 1 *Suppose $\rho = 1$ (full revelation).*
(i) In any symmetric interior PBE, first-period efforts must satisfy

$$e_1^*(\eta, W_1, W_2, 1) = (K_1')^{-1}[f_1(0) W_1 + \eta W_2 C(\eta)]. \quad (13)$$

(ii) Suppose the cost functions are sufficiently convex. If (12) holds, (3) and (13) describe the unique symmetric PBE strategies. If (12) is violated, $e_1^* = 0$ and (3) describe the unique symmetric PBE strategies.

Proof. See Appendix. ■

We defer the discussion of the second-order conditions to the appendix; there we will show that they require sufficiently convex cost functions.[26]

[25] We will show below that the principal will never choose negative values for η.
[26] The relevant condition is (32).

By Proposition 1, if (C2) holds, then a higher positive weight of past effort always induces higher first-period effort. The term in brackets on the right-hand side of (13) is the marginal benefit from increasing e_{i1}. The effect on the expected first-period payoff is $f_1(0)\,W_1$; the effect on the expected second-period payoff is $\eta W_2 C(\eta)$, which is positive if $\eta > 0$**.** This term reflects the direct effect of higher first-period effort on second-period winning chances. The term does *not* capture strategic effects on the future efforts of the other player. Such effects are relevant in the game, but they cancel out in the symmetric equilibrium.[27]

We now characterize second-period efforts. Symmetry of the equilibrium in Proposition 1 implies $\Delta s_{i1} = \Delta\varepsilon_{i1}$. Using (3) and taking the expectation over $\Delta\varepsilon_{i1}$, we obtain:

Corollary 2 *The expected efforts in period 2 in the full-revelation PBE described in Proposition 1 are*

$$E\left(e_2^*\left(\eta, W_2, 1\right)\right) = 2\int_0^\infty \left(K_2'\right)^{-1}\left[f_2\left(\eta s\right)W_2\right] f_1\left(s\right) ds \tag{14}$$

Proof. See Appendix. ■

Together with Assumption 2, Corollary 2 implies that second-period efforts decrease in $|\eta|$**.** Thus, first-period efforts must increase at least locally in $|\eta|$ near the optimal η. Therefore, by (13), (C2) must hold locally near the optimal η. Otherwise, by Proposition 1 the principal could increase efforts in both periods by reducing η, contradicting optimality of η.

[27]To see this, suppose $\eta > 0$; for $\eta < 0$, the argument is reversed. If, for any given first-period effort choice, a player knew he was ahead of the other player, he would have a strategic incentive to increase efforts to discourage player j from exerting effort in the future, whereas the converse would hold for a player who knows he is behind the opponent. Since the game is stochastic, players have to consider the expected strategic effects, which can be positive or negative, but cancel out when first-period efforts are identical.

4.2 No revelation

Under the no-revelation policy, agents simultaneously chose first- and second-period efforts according to

$$\max_{e_{i1}\geq 0, e_{i2}\geq 0} F_1\left(e_{i1}-e_{j1}\right)W_1 + \tag{15}$$

$$W_2 \int_{-\infty}^{\infty} F_2\left(\eta\left(e_{i1}-e_{j1}+s\right)+e_{i2}-e_{j2}\right) f_1\left(s\right) ds - K_1\left(e_{i1}\right) - K_2\left(e_{i2}\right).$$

The integral in (15) is the probability of winning the second-period prize, conditional on effort choices.[28] This leads to a simple characterization of the Nash equilibrium.

Proposition 2 *(i) Suppose* $\rho = 0$ *(no revelation). In any symmetric interior Nash equilibrium, efforts must satisfy:*

$$e_1^*\left(\eta, W_1, W_2, 0\right) = \left(K_1'\right)^{-1}\left[f_1\left(0\right)W_1 + \eta W_2 C(\eta)\right] > 0 \tag{16}$$

$$e_2^*\left(\eta, W_2, 0\right) = \left(K_2'\right)^{-1}\left[W_2 C(\eta)\right] > 0. \tag{17}$$

(ii) If the cost functions are sufficiently convex and (12) holds, (16) and (17) describe the unique symmetric Nash equilibrium of the game.[29]

Proof. See Appendix. ■

Both effort levels reflect standard cost-benefit considerations. The marginal benefit of first-period efforts depends on the increased winning probability in period 2 ($\eta C(\eta)$) as well as period 1 ($f_1(0)$).

By Propositions 1 and 2, first-period efforts in any symmetric equilibrium are non-stochastic and equal under both revelation policies; we thus write $e_1^*\left(\eta, W_1, W_2\right)$ for first-period equilibrium efforts.[30]

[28] This follows from Lemma 1(ii).

[29] In Appendix 8.1.6 we identify the meaning of "sufficient convexity". We also show that the second-order conditions hold locally for arbitrary convex cost function.

[30] The result reflects the fact that the marginal effect of first-period effort on expected second period payoffs is identical under both policies. Intuitively, a marginal increase of e_{i1} has positive effects on the second-period payoffs of player i if it suffices to tip the balance in the contest in period 2 in his favor. The probability that this happens, which is captured by $C(\eta)$ for both players, is independent of whether information on Δs_{i1} is revealed to players before they choose second-period efforts. In this argument, it is important to start from the respective equilibrium, with equal efforts in both periods.

5 Optimal policy

We now characterize the optimal policy of the principal.[31] To this end, we fix the total budget as W, so that $W_2 = W - W_1$. Since we focus on symmetric equilibria and efforts within periods are perfect substitutes, we can write the principal's objective in terms of the efforts of only one agent. As first-period efforts are non-stochastic, the principal's objective functions for perfect and imperfect substitutes, respectively, are:

$$V^P(\eta, W_1, \rho) \equiv e_1^*(\eta, W_1, W - W_1) + E(e_2^*(\eta, W - W_1, \rho)); \quad (18)$$

$$V^I(\eta, W_1, \rho) \equiv e_1^*(\eta, W_1, W - W_1) \cdot E(e_2^*(\eta, W - W_1, \rho)). \quad (19)$$

5.1 Optimal revelation policy

According to (18) and (19), the principal chooses the revelation policy that induces higher expected second-period efforts, no matter whether efforts are perfect or imperfect substitutes. Using Jensen's inequality, we can compare the expected second-period efforts in the equilibria characterized by Propositions 1 and 2:[32]

Proposition 3 *$\forall \eta \in \mathbb{R}, W_1 < W$:*
(i) If $K_2''' \geq 0$, then $e_2^*(\eta, W - W_1, 0) \geq E(e_2^*(\eta, W - W_1, 1))$.
(ii) If $K_2''' \leq 0$, then $e_2^*(\eta, W - W_1, 0) \leq E(e_2^*(\eta, W - W_1, 1))$.

Proof. See Appendix. ■

For quadratic costs, (i) and (ii) together imply that expected second-period efforts are equal under both revelation policies.[33] Proposition 3 ap-

[31] In the following discussion, we assume that, for given error distributions and effort cost functions, second-order conditions hold for all allowable choices of the policy variables. This is for instance true for the normal-quadratic example of Section 6.

[32] Intuitively, with revelation, the agents base their second-period decisions on the revealed asymmetry between players, whereas, without revelation, the expected asymmetry is decisive. Compare second-period decisions with and without revelation for given effort choices in the first period: For error realizations where the asymmetry is low (high) relative to expectations, efforts will be higher (lower) with revelation than without.

[33] Intuitively, the role of K_2''' is an immediate consequence of the fact that second-period efforts are the inverse of marginal costs for $\rho = 0$ and the expectation of the inverse of marginal costs for $\rho = 1$. Thus, concavity (convexity) of the inverse marginal costs is decisive for which regime yields higher efforts on expectation.

plies to all values of η and W_1 and, in particular, to those that maximize $e_2^* (\eta, W - W_1, 0)$ or $E (e_2^* (\eta, W - W_1, 1))$. Thus, even if the principal has chosen the optimal parameters for a given revelation policy, switching to the other revelation policy is beneficial if the corresponding condition on K_2''' holds. Hence, we have proven:

Corollary 3 *The optimal revelation policy is the same for perfect and imperfect substitutes, with $\rho = 0$ if $K_2''' > 0$ and $\rho = 1$ if $K_2''' < 0$. For $K_2''' = 0$, expected payoffs are independent of the revelation policy.*

The result extends Aoyagi (2010) who shows that, for one prize ($W_1 = 0$) and equal weights ($\eta = 1$), the effort cost function completely determines the optimal revelation policy.[34] Our result shows that this statement holds for arbitrary W_1 and η.

5.2 Optimal weight of past performance

The principal can give incentives for first-period efforts with W_1 or η. The next result shows that, no matter how high the first-period prize is, the principal should always assign a positive weight to past performance in the second-period contest. For perfect substitutes, we denote the optimal choice of η conditional on W_1 and ρ as $\eta^P(W_1, \rho)$ and the optimal choice of W_1 conditional on η as $W_1^P(\eta, \rho)$. For imperfect substitutes, we write $\eta^I(W_1, \rho)$ and $W_1^I(\eta, \rho)$.

Proposition 4 *$\eta^P(W_1, \rho) > 0$ and $\eta^I(W_1, \rho) > 0 \; \forall \; W_1 < W$ and $\rho = 0, 1$.*

Proof. See Appendix. ■

This result holds because, for $\eta = 0$, the marginal effect of η on first-period effort is positive and bounded away from zero (a first-order effect), whereas it is zero for second-period effort (a second-order effect). To understand the latter point, note that the adverse effect of increasing $\eta > 0$ on second-period efforts arises because the second-period contest becomes more asymmetric, that is, less competitive ($C' (\eta) < 0$). As $C' (0) = 0$, this adverse effect vanishes as η approaches 0.

[34] Ederer (2010) also treats this case in his discussion of non-complementary abilities.

Proposition 4 states that performance evaluation should always have some memory: Firms should consider not only the recent performance of employees and suppliers, but also the performance in the distant past. Similarly, students should not only be judged with respect to their recent performance. The open question is: How large should the "shadow of the past" be? To answer this question for perfect substitutes, the next result characterizes the weight of past performance for quadratic costs (C1). In this case, revelation and no revelation imply the same behavior. Thus, we write $\eta^P(W_1) \equiv \eta^P(W_1, 0) = \eta^P(W_1, 1)$, and similarly for $W_1^P(\eta)$. Furthermore, we write $\left(\eta^P, W_1^P\right) = \arg\max_{\eta, W_1} V^P(\eta, W_1)$.

Proposition 5 *Suppose (C1) holds. Then,* $\forall\, W_1 < W$, $\eta^P(W_1)$ *satisfies*

$$\left|\frac{C'(\eta)}{C(\eta)}\right| = \frac{1}{\frac{k_1}{k_2} + \eta} \tag{20}$$

Proof. See Appendix. ■

(20) captures the trade-off between strengthening first-period incentives and weakening second-period competition. Changes in the error distributions that increase the sensitivity $\left|\frac{C'(\eta)}{C(\eta)}\right|$ of second-period competition to the first-period weight η for all η reduce the optimal η.[35] Furthermore, the higher first-period marginal effort costs are compared to second-period marginal effort costs, the lower is the optimal η. Note that (20) and thus the optimal η is independent of the first-period prize W_1.[36]

5.3 Optimal first-period prize

We now supply several results on the optimal prize structure. We also use these results to obtain further insights on the optimal weights.

[35] We illustrate this in Figure 2 below.

[36] This is due to the fact that W_1 enters $\frac{\partial V^P(\eta, W_1)}{\partial \eta}$ linearly. The relevant expression is (43). Since $\frac{\partial^2 V^P(\eta, W_1)}{\partial \eta \partial W_1} < 0$, the increase in payoff by setting η optimally depends positively on $W - W_1$, the prize paid in the second period.

5.3.1 Perfect substitutes

For perfect substitutes, we confine ourselves to quadratic costs (C1) in the main text. The results are special cases of more general, but less transparent results that we state and prove in Appendix 8.4 (Propositions 9 and 10).

For quadratic costs, it is optimal to give only one positive prize. Depending on the observation error distributions, the prize should be based only on first-period performance ($W_1^P = W$) or on second-period performance as well ($W_1^P = 0$).

Corollary 4 *Suppose (C1) holds.*
(i) If $f_1(0) < \left(\frac{k_1}{k_2} + \eta\right) C(\eta)$*, then the optimal first-period prize conditional on* η *is* $W_1^P(\eta) = 0$*. If* $\exists\ \eta \in \mathbb{R}$ *s.t.* $f_1(0) < \left(\frac{k_1}{k_2} + \eta\right) C(\eta)$*, then the uncon-ditonally optimal first-period prize is* $W_1^P = 0$*.*
(ii) If $f_1(0) > \left(\frac{k_1}{k_2} + \eta\right) C(\eta)$*, then the optimal first-period prize conditional on* η *is* $W_1^P(\eta) = W$*. If* $f_1(0) > \left(\frac{k_1}{k_2} + \eta\right) C(\eta)\ \forall\ \eta \in \mathbb{R}$*, then the uncondi-tonally optimal first-period prize is* $W_1^P = W$*.*

Proof. See Appendix. ■

The intuition is straightforward. By (i), when second-period competition (as captured by $C(\eta)$) is intense enough relative to the precision of the first-period measurement (as captured by $f_1(0)$), then second-period efforts should be positive, which requires a second-period prize.[37] Otherwise (case (ii)), there should be no second-period prize. However, we will show in Section 6 that $W_1^P = 0$ always holds in a normal-quadratic example. Note that the condition under which there is no first-period prize is easier to satisfy when first-period marginal effort costs are high compared to second-period marginal effort costs.

Beyond quadratic effort costs In Appendix 8.4, we provide results on the optimal first-period prizes and weights for general cost functions. These results imply Proposition 4 for $K_t''' = 0$ as a special case. When $K_t''' \neq 0$, the effort choices with and without information revelation are no longer identical,

[37] Note that $f_1(0)$ is a purely local measure of precision, capturing the probability that identical efforts translate into identical performance measures.

so that the optimal policies do not coincide. Proposition 9 in the Appendix characterizes the optimal prize structure with information revelation for $K_t''' \leq 0$, in which case information revelation is superior to no revelation by Proposition 3. Conversely, Proposition 10 in the Appendix characterizes the optimal prize structure without information revelation for $K_t''' \geq 0$, where no information revelation is superior to revelation by Proposition 3. The interpretation of the general propositions is similar as for quadratic costs: If the first-period contest is too noisy, it is optimal not to give a first-period prize.

5.3.2 Imperfect Substitutes

For imperfect substitutes, we also obtain a general condition under which the optimal first-period prize for a given past weight is zero with performance revelation. The result applies if $K_t''' \leq 0$, so that revelation is optimal by Corollary 3.

Proposition 6 *Suppose $K_t''' \leq 0$ for $t = 1, 2$. For all $\eta > 0$, $W_1^I(\eta, 1) = 0$ if $f_1(0) < \eta C(\eta)$.*

Proof. See Appendix. ■

Thus, as with perfect substitutes, this (sufficient) condition is easier to satisfy if the first-period signal is not very precise ($f_1(0)$ is low) and second-period competition $C(\eta)$ is intense. To obtain stronger results, we now specialize to quadratic effort costs. As behavior is the same with and without revelation, we write $\eta^I(W_1) \equiv \eta^I(W_1, 0) = \eta^I(W_1, 1)$ and similarly for $W_1^I(\eta)$. Furthermore, we write $(\eta^I, W_1^I) = \arg\max_{\eta, W_1} V^I(\eta, W_1)$. We obtain:

Proposition 7 *If (C1) holds, the optimal first-period prize conditional on η is $W_1^I(\eta) \leq \frac{W}{2}\ \forall\ \eta$, so that the optimal unconditional first-period prize is $W_1^I \leq \frac{W}{2}$.*

Proof. See Appendix. ■

There is no counterpart of this result for perfect substitutes, where it can, in principle, be optimal to refrain from inducing second-period effort altogether. For imperfect substitutes, principals aim at a balanced effort distribution. Therefore, they need to make sure not to give excessive first-period prizes, because they are already providing indirect incentives for first-period effort through the weight η.

The following result specifies the optimal solution further:

Proposition 8 *Suppose (C1) holds.*
(i) $W_1^I(\eta) > 0$ *if and only if* $f_1(0) > 2\eta C(\eta)$. *In this case*

$$W_1^I(\eta) = W \frac{f_1(0) - 2\eta C(\eta)}{2f_1(0) - 2\eta C(\eta)} > 0.$$

(ii) The optimal $\left(W_1^I, \eta^I\right)$ *satisfies one of the following necessary properties:*

$$\begin{aligned} \text{(a) } W_1^I &= 0 \text{ and } \left|\frac{C'(\eta^I)}{C(\eta^I)}\right| = \frac{1}{2\eta^I}; \\ \text{(b) } W_1^I &= W \frac{f_1(0) - 2\eta^I C\left(\eta^I\right)}{2f_1(0) - 2\eta^I C(\eta^I)} > 0 \text{ and } \left|\frac{C'(\eta^I)}{C(\eta^I)}\right| = \frac{C\left(\eta^I\right)}{f_1(0)}. \end{aligned}$$

Proof. See Appendix. ■

Result (i) describes the optimal prize structure conditional on η. As with perfect substitutes, the optimal first-period prize is positive if first-period precision (captured by $f_1(0)$) is high and second-period competition $C(\eta)$ is low. Moreover, the result sharpens Proposition 6 by showing that, at least for quadratic costs, $f_1(0) < \eta C(\eta)$ is not necessary to guarantee that the conditionally optimal prize structure satisfies $W_1^I(\eta) = 0$. Finally, the result shows that, when (C1) and (C2) hold, first-period prizes and weights are substitutes: The optimal first-period prize is lower the higher the first-period weight η is.

Result (ii) describes the unconditionally optimal solution $\left(W_1^I, \eta_1^I\right)$ for quadratic costs. There are two possibilities, both depicted in Figure 1. According to (a), the first-period prize may be zero, in which case the optimal first-period weight is described by a simple condition that depends exclusively on $C(\eta)$ (see point A on the horizontal axis in Figure 1). As with perfect substitutes, the weight is lower if the adverse effect of η on future competition is higher. By (b), the first-period prize may be positive, in which case the optimal first-period weight is determined by a condition that

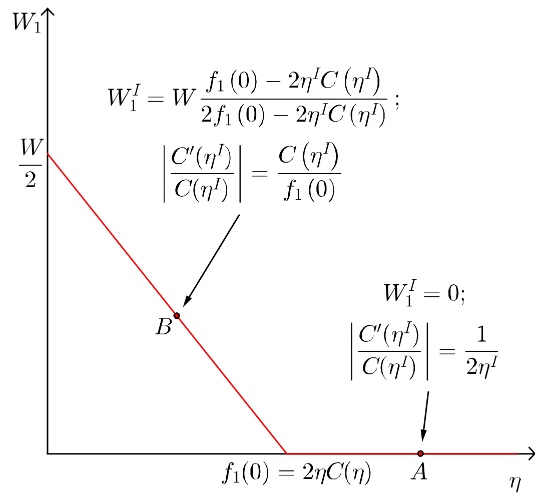

Figure 1: Necessary conditions for imperfect substitutes

depends on error distributions not only via $C(\eta)$, but also via $f_1(0)$ directly, as captured by $W_1^I(\eta)$ (see point B on the diagonal line in Figure 1).[38] The error distributions determine which of the two cases in Proposition 8 applies. For instance, with normal error distributions, the first-period prize is zero (see Corollary 6 below).

Note that in contrast to the perfect substitutes case (Propositions 5 and 4), the optimal weights and prizes are independent of the relation between first- and second-period effort costs and entirely determined by the properties of the observation error distributions.

5.4 Restrictions on allowable policies

In some circumstances, principals may not be free to choose arbitrary incentive systems. For instance, there may be a limit on the extent to which they may consider past performance in current evaluations. Then first-period prizes may act as substitutes for performance weights: For instance, with perfect substitutes, Proposition 4 shows that it is weakly easier to satisfy the condition for $W_1^P(\eta) = W$ and correspondingly weakly more difficult to satisfy the condition for $W_1^P(\eta) = 0$ when η is bounded above. Conversely,

[38]Note that $W_1^I(\eta)$ is typically not linear.

the prize structure may be restricted. For instance, a principal may have to spread the prize sum evenly. According to Proposition 7, $W_1 \leq \frac{W}{2}$ must hold for the optimal first-period prize with imperfect substitutes. In cases where an unconstrained principal would set $W_1 < \frac{W}{2}$, a principal who has to set $W_1 = \frac{W}{2}$ is giving excessive first-period incentives relative to the unconstrained case. To make up for this, she has to adjust the weight of first-period performance downwards.

6 A Normal-Quadratic Example

To obtain sharper results, we introduce a simple example.

Example E1: *The cost function is* $K_t(e_{it}) = \frac{k}{2}e_{it}^2$ *for* $t = 1, 2$. *The error difference* $\Delta\varepsilon_{it}$ *is normally distributed with variance* σ_t^2.[39]

Example E1 satisfies Assumptions 1 and 2.[40]

Corollary 5 *In E1, a PBE exists. The equilibrium efforts under revelation and no revelation are*

$$e_1^*(\eta, W_1, W_2) = \frac{1}{k\sqrt{2\pi}}\left(\frac{W_1}{\sigma_1} + \frac{\eta W_2}{\sqrt{\sigma_2^2 + \sigma_1^2\eta^2}}\right). \tag{21}$$

$$e_2^*(\eta, W_2, 0) = E\left(e_2^*(\eta, W_2, 1)\right) = \frac{W_2}{k\sqrt{2\pi}\sqrt{\sigma_2^2 + \sigma_1^2\eta^2}}. \tag{22}$$

Proof. See Appendix. ■

Comparative statics for second-period efforts are straightforward. Lower marginal effort costs, higher second-period prize, lower first-period weight and higher first- and second-period precision induce higher second-period efforts. Analogous results hold for period one. First-period efforts also increase if the second-period precision increases, given $\eta > 0$: The parameter change makes first-period effort more worthwhile, because the positive effect on winning the second-period prize increases. Finally, a redistribution of the prize sum from period 2 to period 1 increases first-period efforts, because the

[39] A normally distributed error difference follows, for example, from normally distributed observation errors.

[40] In the appendix, we also derive the second-order conditons ((66) and (68)).

positive effect of an increase in the first-period prize is always stronger than the negative effect of an identical decrease in the second-period prize.

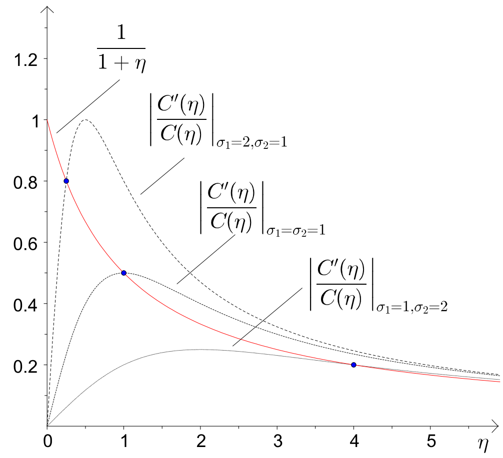

Figure 2: Necessary conditions for η with perfect substitutes

Figure 2 illustrates Proposition 5 for Example E1. It shows how the optimal weight η^P depends on the sensitivity $\left|\frac{C'(\eta)}{C(\eta)}\right|$ of second-period competition to η, which is low if the second-period performance measurement is relatively imprecise compared to the first-period measurement, implying a higher optimal weight of first-period performance.

Corollary 6 characterizes the optimal policy. The results endogenize the assumption that $W_1 = 0$ and $\eta = 1$ in Aoyagi (2010) for identically normally distributed error distributions ($\sigma_1 = \sigma_2$).[41]

Corollary 6 *In E1,*
(i) $\eta^P(W_1) = \frac{\sigma_2^2}{\sigma_1^2}\ \forall\ W_1 < W$. *Furthermore,* $W_1^P = 0$ *and* $\eta^P = \frac{\sigma_2^2}{\sigma_1^2}$.
(ii) Necessary conditions for the optimum are $\eta^I = \frac{\sigma_2}{\sigma_1}$ *and* $W_1^I = 0$.

Proof. See Appendix. ■

(i) shows that it is optimal to give only a second-period prize with perfect substitutes. Incentives for first-period efforts come exclusively from the weight of past performance, which is the ratio of the error variances in the

[41] Similarly, we provide a justification for Ederer's (2010) model with non-complementary abilities in which $W_1 = 0$ and $\eta = 0$.

two periods. (ii) yields a similar result for imperfect substitutes, with variances replaced by standard deviations. Note that $\eta^I > \eta^P$ if and only if $\frac{\sigma_2}{\sigma_1} < 1$: Greater precision of the second-period performance measure leads to higher second-period efforts compared to the first-period efforts. With imperfect substitutes, where an even effort flow is desired, a greater weight of the first period is used to mitigate the asymmetry.

The example demonstrates the importance of the right incentive system. To see this, suppose that initially the principal chooses $(\eta, W_1) = \left(0, \frac{W}{2}\right)$, so that there are two independent and identical tournaments. Next suppose that the principal introduces the optimal policy in two steps: First, she maintains the equal division of the prize sum across periods, but chooses the optimal weight $\eta^P\left(\frac{W}{2}\right)$, so that $(\eta, W_1) = (\frac{\sigma_2^2}{\sigma_1^2}, \frac{W}{2})$. Finally, she chooses prizes and weights optimally, so that $(\eta, W_1) = (\frac{\sigma_2^2}{\sigma_1^2}, 0)$. Simple calculations (available on request) show that if the principal sets only η optimally, the relative increase of her payoff, compared to the initial situation, is

$$E_\eta^P \equiv \frac{V^P\left(\frac{\sigma_2^2}{\sigma_1^2}, \frac{W}{2}\right) - V^P\left(0, \frac{W}{2}\right)}{V^P\left(0, \frac{W}{2}\right)} = \frac{\sqrt{\sigma_1^2 + \sigma_2^2} - \sigma_1}{(\sigma_1 + \sigma_2)}$$

By optimally adjusting the prize structure as well, she achieves an additional relative payoff increase of

$$E_{W_1}^P \equiv \frac{V^P\left(\frac{\sigma_2^2}{\sigma_1^2}, 0\right) - V^P\left(\frac{\sigma_2^2}{\sigma_1^2}, \frac{W}{2}\right)}{V^P\left(\frac{\sigma_2^2}{\sigma_1^2}, \frac{W}{2}\right)} = \frac{\sqrt{\sigma_1^2 + \sigma_2^2} - \sigma_2}{\sqrt{\sigma_1^2 + \sigma_2^2} + \sigma_2}.$$

The relative importance of these two effects depends on the precision of the performance measures. If second-period performance measurement is very precise ($\sigma_2 \approx 0$), whereas the first-period measure is not, then $E_\eta^P \approx 0$; If first-period performance measurement is very precise ($\sigma_1 \approx 0$), whereas the second-period measure is not, then $E_{W_1}^P \approx 0$. Thus, getting the performance weight right (rather than choosing $\eta = 0$) is only important when second-period performance measurement is imprecise; getting the second-period prize right (rather than splitting the price equally) only matters when first-period performance measurement is imprecise.

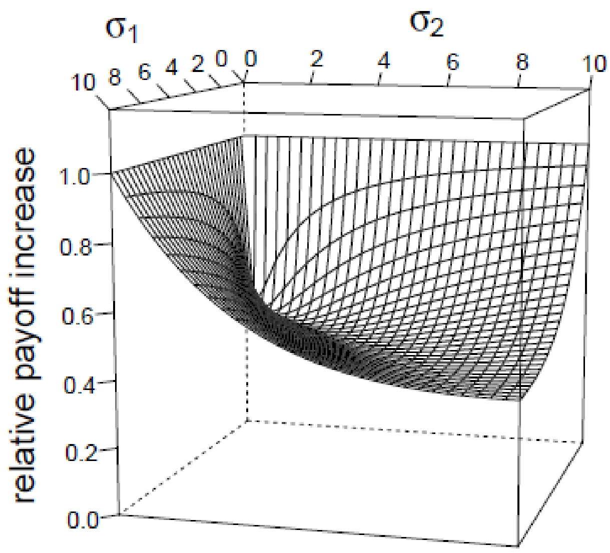

Figure 3: Relative payoff increase when setting W_1 and η optimally

Compared to the initial situation, the total relative payoff increase from setting both W_1 and η optimally is

$$E^P_{\eta,W_1} \equiv \frac{V^P\left(\frac{\sigma_2^2}{\sigma_1^2}, 0\right) - V^P\left(0, \frac{W}{2}\right)}{V^P\left(0, \frac{W}{2}\right)} = \frac{2\sqrt{\sigma_1^2+\sigma_2^2} - (\sigma_1+\sigma_2)}{(\sigma_1+\sigma_2)}$$

Figure 3 shows how the total relative payoff increase from choosing the optimal incentive system depends on the variances of the error distributions.

E^P_{η,W_1} attains its minimum for $\sigma_1 = \sigma_2$ at $\sqrt{2} - 1 \approx 41\%$. Thus, the percentage payoff increase from implementing the optimal policy is lowest if both performance measurements are equally precise. Even in this case, however, the benefits are substantial: The principal can achieve 41% higher payoff with a budget-neutral policy adjustment. Figure 3 further shows that if one of the performance measures is very precise ($\sigma_t \approx 0$), then $E^P_{\eta,W_1} \approx 1$. Hence, the more precise one of the performance measures, the more the principal can benefit from implementing the optimal policy.

7 Concluding Remarks

This paper analyzes intertemporal effort provision in two-stage tournaments. A principal with a fixed budget for prizes faces two risk-neutral agents. She observes noisy signals of effort in both periods. She aims at maximizing either total efforts (perfect substitutes) or the product of first- and second-period efforts (imperfect substitutes). She decides (i) how to spread prize money across the two periods, (ii) how to weigh performance in the two periods when awarding the second period prize, and (iii) whether to reveal performance after the first period.

We obtain several new insights. First, design matters. The potential losses from suboptimal incentive systems are substantial.[42] Second, several interesting results of existing research on revelation policy and performance weights are much more general than previously known, extending in particular to the important case that efforts in different periods are not perfect substitutes. Third, we provide new results on the determinants of optimal incentives. We show that the weight of past performance should depend negatively on the extent to which a higher weight of the past reduces competition. We also show how the spread of prizes across periods and the choice of weights depends on the relative precision of performance measures in the two periods. Finally, we show that, under quite general conditions, there should be no first-period prize.

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8 Appendix

8.1 Behavior of the Agents[43]

8.1.1 Proof of Lemma 2

(i) Equilibrium efforts must be positive because $f_2 > 0$ by Assumption 2 and $K_2'(0) = 0$ by Assumption 1. Since f_2 is symmetric by Assumption 2 and

$$-\left(\eta\Delta s_{11}+\Delta e_{12}\right) = \eta\Delta s_{21}+\Delta e_{22},$$

the left-hand side of the first-order condition (2) is equal for both agents. Hence $e_{i2}^*\left(\Delta s_{i1}\right) = e_{j2}^*(-\Delta s_{i1})$, so that the second-period efforts are the same for both agents. Thus, (2) becomes $f_2\left(\eta\Delta s_{i1}\right)W_2 = K_2'\left(e_{i2}\right)$. As $K_2'' > 0$ by Assumption 1, K_2' therefore is strictly increasing and thus invertible. Thus (3) must hold in any equilibrium.

(ii) The following inequality guarantees that the second-period payoffs (1) of player i are strictly concave in e_{i2}:

$$f_2'\left(\eta\Delta s_{i1}+\Delta e_{i2}\right)W_2 < K_2''\left(e_{i2}\right) \ \forall\ \Delta s_{i1} \in \mathbb{R},\, e_{i2}, e_{j2} \in \mathbb{R}^+. \tag{23}$$

(23) requires K_2 to be sufficiently convex.[44] If this condition holds globally, the first-order conditions (2) characterize a Nash equilibrium. Moreover, the equilibrium is unique, as (3) must necessarily hold in any equilibrium by Part (i) of the lemma.

8.1.2 Proof of Corollary 1

The inverse function theorem yields

$$\left[\left(K_2'\right)^{-1}\right]'\left(f_2\left(\eta\Delta s_{i1}\right)W_2\right) = \frac{1}{K_2''\left(\left(K_2'\right)^{-1}\left(f_2\left(\eta\Delta s_{i1}\right)W_2\right)\right)}.$$

[43] The proofs in this section generalize Aoyagi (2010) and Ederer (2010) (for non-complementary abilities) who assume $W_1 = 0$ and $\eta = 1$.

[44] By Assumption 2, $f_2'\left(\eta\Delta s_{i1}+\Delta e_{i2}\right) < 0$ if $\eta\Delta s_{i1} + \Delta e_{i2} > 0$, so that (23) always holds in this case. For the case that $\eta\Delta s_{i1} + \Delta e_{i2} < 0$, suppose f_2' is bounded above. Then (23) holds globally if K_2'' has a sufficiently high lower bound.

Thus (3) implies

$$\frac{\partial e_{i2}}{\partial \Delta s_{i1}} = \frac{\eta f_2'(\eta \Delta s_{i1}) W_2}{K_2''\left((K_2')^{-1}(f_2(\eta \Delta s_{i1}) W_2)\right)}; \tag{24}$$

$$\frac{\partial e_{i2}}{\partial \eta} = \frac{\Delta s_{i1} f_2'(\eta \Delta s_{i1}) W_2}{K_2''\left((K_2')^{-1}(f_2(\eta \Delta s_{i1}) W_2)\right)}; \tag{25}$$

$$\frac{\partial e_{i2}}{\partial W_2} = \frac{f_2(\eta \Delta s_{i1})}{K_2''\left((K_2')^{-1}(f_2(\eta \Delta s_{i1}) W_2)\right)}. \tag{26}$$

By Assumption 1, $K_2''> 0$. By Assumption 2, if $\Delta s_{i1}< (>)0 \wedge \eta \neq 0 \wedge W_2> 0$, then $\eta f_2'(\eta \Delta s_{i1}) > (<)0$ and thus $\frac{\partial e_{i2}}{\partial \Delta s_{i1}} > (<)0$. This implies that e_{i2} is decreasing in $|\Delta s_{i1}|$. As $\Delta s_{i1}= \Delta e_{i1}+\Delta \epsilon_{i1}$ we obtain the results for e_{i1} and e_{j1}. Similar arguments show that $\frac{\partial e_{i2}}{\partial \eta}> (<)0$ for $\eta < (>)0$ and thus e_{i2} decreasing in $|\eta|$. Since $f_2> 0$ by Assumption 2, we have $\frac{\partial e_{i2}}{\partial W_2}> 0$.

8.1.3 Proof of Proposition 1

(i) We first derive expressions for $\frac{\partial U_{i2}^e}{\partial e_{i1}}$ for symmetric first-period efforts. This allows us to state the FOC.

Lemma 3

$$\left.\frac{\partial U_{i2}^e}{\partial e_{i1}}\right|_{e_{i1}=e_{j1}} = \eta W_2 C(\eta) \tag{27}$$

Proof. Applying the envelope theorem to (4), we obtain

$$\frac{dU_{i2}^s(\Delta s_{i1})}{d\Delta s_{i1}} = \frac{\partial U_{i2}}{\partial e_{j2}} \frac{\partial e_{j2}^*(-\Delta s_{i1})}{\partial \Delta s_{i1}} + \frac{\partial U_{i2}}{\partial \Delta s_{i1}}. \tag{28}$$

Using (24) and the symmetry of the density (Assumption 2),

$$\frac{\partial e_{j2}^*(-\Delta s_{i1})}{\partial \Delta s_{i1}} = \frac{\partial e_{i2}(\Delta s_{i1})}{\partial \Delta s_{i1}} = \frac{\eta f_2'(\eta \Delta s_{i1}) W_2}{K_2''\left((K_2')^{-1}(f_2(\eta \Delta s_{i1}) W_2)\right)}.$$

(1) implies

$$\frac{\partial U_{i2}}{\partial e_{j2}} = -f_2(\eta \Delta s_{i1}+\Delta e_{i2}) W_2,$$

$$\frac{\partial U_{i2}}{\partial \Delta s_{i1}} = \eta f_2(\eta \Delta s_{i1}+\Delta e_{i2}) W_2.$$

Using these equations in (28) and inserting $\Delta e_{i2}= 0$, we obtain

$$\frac{dU_{i2}^{s}}{d\Delta s_{i1}}= -\frac{\eta f_{2}\left(\eta\Delta s_{i1}\right)f_{2}'\left(\eta\Delta s_{i1}\right)W_{2}^{2}}{K_{2}''\left(\left(K_{2}'\right)^{-1}\left(f_{2}\left(\eta\Delta s_{i1}\right)W_{2}\right)\right)}+\eta f_{2}\left(\eta\Delta s_{i1}\right)W_{2}.$$

Using this in (5), we obtain

$$\frac{\partial U_{i2}^{e}}{\partial e_{i1}}=$$

$$\int_{-\infty}^{\infty}\left[-\frac{\eta f_{2}\left(\eta\left(\Delta e_{i1}+s\right)\right)f_{2}'\left(\eta\left(\Delta e_{i1}+s\right)\right)W_{2}^{2}}{K_{2}''\left(\left(K_{2}'\right)^{-1}\left(f_{2}\left(\eta\left(\Delta e_{i1}+s\right)\right)W_{2}\right)\right)}+\eta f_{2}\left(\eta\left(\Delta e_{i1}+s\right)\right)W_{2}\right]f_{1}\left(s\right)ds$$

$$= \eta W_{2}\int_{-\infty}^{\infty}f_{2}\left(\eta\left(\Delta e_{i1}+s\right)\right)f_{1}\left(s\right)ds-$$

$$\eta W_{2}^{2}\int_{-\infty}^{\infty}\frac{f_{2}\left(\eta\left(\Delta e_{i1}+s\right)\right)f_{2}'\left(\eta\left(\Delta e_{i1}+s\right)\right)}{K_{2}''\left(\left(K_{2}'\right)^{-1}\left(f_{2}\left(\eta\left(\Delta e_{i1}+s\right)\right)W_{2}\right)\right)}f_{1}\left(s\right)ds.$$

Let

$$\begin{aligned} A &: = \int_{-\infty}^{\infty}f_{2}\left(\eta\left(\Delta e_{i1}+s\right)\right)f_{1}\left(s\right)ds, \\ B &: = \int_{-\infty}^{\infty}\frac{f_{2}\left(\eta\left(\Delta e_{i1}+s\right)\right)f_{2}'\left(\eta\left(\Delta e_{i1}+s\right)\right)}{K_{2}''\left(\left(K_{2}'\right)^{-1}\left(f_{2}\left(\eta\left(\Delta e_{i1}+s\right)\right)W_{2}\right)\right)}f_{1}\left(s\right)ds. \end{aligned}$$

With this notation,

$$\frac{\partial U_{i2}^{e}}{\partial e_{i1}}= \eta W_{2}A-\eta W_{2}^{2}B. \tag{29}$$

Substituting $s = t - \Delta e_{i1}$ and $ds = dt$ in A and decomposing the integral gives

$$A = \int_{-\infty}^{0}f_{2}\left(\eta t\right)f_{1}\left(t-\Delta e_{i1}\right)dt+\int_{0}^{\infty}f_{2}\left(\eta t\right)f_{1}\left(t-\Delta e_{i1}\right)dt.$$

Let $u = -t$. Symmetry of f_1 and f_2 by Assumption 2 implies $f_{2}\left(\eta t\right)= f_{2}\left(\eta u\right)$ and $f_{1}\left(t-\Delta e_{i1}\right)= f_{1}\left(u+\Delta e_{i1}\right)$. Hence,

$$\int_{-\infty}^{0}f_{2}\left(\eta t\right)f_{1}\left(t-\Delta e_{i1}\right)dt = \int_{0}^{\infty}f_{2}\left(\eta u\right)f_{1}\left(u+\Delta e_{i1}\right)du.$$

Thus,

$$\begin{aligned} A &= \int_0^\infty f_2(\eta u)\, f_1(u+\Delta e_{i1})\, du + \int_0^\infty f_2(\eta t)\, f_1(t-\Delta e_{i1})\, dt \\ &= \int_0^\infty f_2(\eta t)\left[f_1(t+\Delta e_{i1}) + f_1(t-\Delta e_{i1})\right] dt. \end{aligned}$$

Substituting $s = t - \Delta e_{i1}$ and $ds = dt$ in B and decomposing the integral, we obtain

$$B = \int_{-\infty}^0 \frac{f_2(\eta t)\, f_2'(\eta t)\, f_1(t-\Delta e_{i1})}{K_2''\left((K_2')^{-1}(f_2(\eta t)\, W_2)\right)} dt + \int_0^\infty \frac{f_2(\eta t)\, f_2'(\eta t)\, f_1(t-\Delta e_{i1})}{K_2''\left((K_2')^{-1}(f_2(\eta t)\, W_2)\right)} dt.$$

Again using $u = -t$ and appealing to symmetry, $f_2(\eta t) = f_2(\eta u)$, $f_2'(\eta t) = -f_2'(\eta u)$ and $f_1(t - \Delta e_{i1}) = f_1(u + \Delta e_{i1})$. Thus

$$\int_{-\infty}^0 \frac{f_2(\eta t)\, f_2'(\eta t)\, f_1(t-\Delta e_{i1})}{K_2''\left((K_2')^{-1}(f_2(\eta t)\, W_2)\right)} dt = \int_0^\infty \frac{f_2(\eta u)\, (-f_2'(\eta u))\, f_1(u+\Delta e_{i1})}{K_2''\left((K_2')^{-1}(f_2(\eta u)\, W_2)\right)} du.$$

Hence,

$$\begin{aligned} B &= \int_0^\infty \frac{-f_2(\eta u)\, f_2'(\eta u)\, f_1(u+\Delta e_{i1})}{K_2''\left((K_2')^{-1}(f_2(\eta u)\, W_2)\right)} du + \int_0^\infty \frac{f_2(\eta t)\, f_2'(\eta t)\, f_1(t-\Delta e_{i1})}{K_2''\left((K_2')^{-1}(f_2(\eta t)\, W_2)\right)} dt \\ &= \int_0^\infty \frac{f_2(\eta t)\, f_2'(\eta t)\left[-f_1(t+\Delta e_{i1}) + f_1(t-\Delta e_{i1})\right]}{K_2''\left((K_2')^{-1}(f_2(\eta t)\, W_2)\right)} dt. \end{aligned}$$

Substituting the expressions for A and B into (29) and using $s = t$, we obtain

$$\begin{aligned} \frac{\partial U_{i2}^e}{\partial e_{i1}} &= \eta W_2 \int_0^\infty f_2(\eta s)\left[f_1(s+\Delta e_{i1}) + f_1(s-\Delta e_{i1})\right] ds \qquad (30) \\ &+ \eta W_2^2 \int_0^\infty \frac{f_2(\eta s)\, f_2'(\eta s)\left[f_1(s+\Delta e_{i1}) - f_1(s-\Delta e_{i1})\right]}{K_2''\left((K_2')^{-1}(f_2(\eta s)\, W_2)\right)} ds. \end{aligned}$$

With $\Delta e_{i1} = 0$, we obtain (27). ■

Together, (6) and Lemma 3 imply

$$f_1(0)\, W_1 + \eta W_2 C(\eta) = K_1'(e_{i1}).$$

By Assumption 1, K_1' is invertible. We thus obtain (13) as a necessary condition for any symmetric interior PBE.

(ii) We know from Lemma 2(ii) that (2) implies sequential rationality in the second period. Moreover, from the discussion at the beginning of Section 4.1, beliefs are consistent.

As $K_1'(0) = 0$ by Assumption 1, efforts must be positive in any symmetric equilibrium if (12) holds. Thus, by Part (i), (13) is a necessary condition for an equilibrium. The second-order condition for player i is

$$f_1'\left(\Delta e_{i1}\right) W_1 + \frac{\partial^2 U_{i2}^e}{\partial e_{i1}^2} < K_1''\left(e_{i1}\right) \;\; \forall e_{i1}, e_{j1} \in \mathbb{R}^+. \tag{31}$$

Inserting (30) in (31) gives

$$\begin{aligned} & f_1'\left(\Delta e_{i1}\right) W_1 + \eta W_2 \int_0^\infty f_2\left(\eta s\right)\left[f_1'\left(s+\Delta e_{i1}\right) - f_1'\left(s-\Delta e_{i1}\right)\right] ds + \\ & \eta W_2^2 \int_0^\infty \frac{f_2\left(\eta s\right) f_2'\left(\eta s\right)\left[f_1'\left(s+\Delta e_{i1}\right) + f_1'\left(s-\Delta e_{i1}\right)\right]}{K_2''\left(\left(K_2'\right)^{-1}\left(f_2\left(\eta s\right) W_2\right)\right)} ds < K_1''\left(e_{i1}\right). \end{aligned} \tag{32}$$

The left-hand side of this inequality is decreasing in K_2'', while the right-hand side is increasing in K_1''. For given policy parameters and distributions, (31) therefore holds as long as $\min\left\{K_1''(0), K_2''(0)\right\}$, which is a lower bound for $K_1''\left(e_{i1}\right)$ and $K_2''\left(\left(K_2'\right)^{-1}\left(f_2\left(\eta s\right) W_2\right)\right)$, is sufficently large. In this case, the second-order condition can be guaranteed to hold whenever the slopes of f_1 and f_2 are bounded.

If these conditions hold globally, (13) thus describes an equilibrium, which is the unique symmetric equilibrium.

8.1.4 Proof of Corollary 2

Symmetry of the equilibrium implies $\Delta s_{i1} = \Delta\varepsilon_{i1}$. Hence, (3) implies

$$e_{i2}^*\left(\Delta s_{i1}, \eta, W_2, 1\right) = \left(K_2'\right)^{-1}\left(f_2\left(\eta\Delta\varepsilon_{i1}\right) W_2\right).$$

Taking the expectation over $\Delta\varepsilon_{i1}$, we obtain

$$E_{\Delta\varepsilon_{i1}}\left(e_{i2}^*\left(\Delta s_{i1}, \eta, W_2, 1\right)\right) = \int_{-\infty}^{\infty} \left(K_2'\right)^{-1}\left(f_2\left(\eta s\right) W_2\right) f_1\left(s\right) ds.$$

From the symmetry of the density by Assumption 2, we get (14).

8.1.5 Proof of Proposition 2

(i) From (15), the first-order conditions are

$$\begin{aligned} f_1(\Delta e_{i1}) W_1 + \eta W_2 \int_{-\infty}^{\infty} f_2(\eta(\Delta e_{i1}+s) + \Delta e_{i2}) f_1(s)\, ds &= K_1'(e_{i1}) \\ W_2 \int_{-\infty}^{\infty} f_2(\eta(\Delta e_{i1}+s) + \Delta e_{i2}) f_1(s)\, ds &= K_2'(e_{i2}) \end{aligned}$$

For the symmetric case $\Delta e_{i1} = \Delta e_{i2} = 0$, this simplifies to

$$\begin{aligned} f_1(0) W_1 + \eta W_2 C(\eta) &= K_1'(e_{i1}); \\ W_2 C(\eta) &= K_2'(e_{i2}). \end{aligned}$$

Inverting K_1' and K_2' yields (16) and (17).
(ii) If (12) holds, first-period equilibrium efforts are positive because $K_1'(0) = 0$ by Assumption 1. Equilibrium efforts in the second period are positive because $W_2 C(\eta) > 0$ by Assumption 2. By part (i), (16) and (17) are necessary equilibrium conditions.

Consider the following second-order conditions[45]

$$f_1'(\Delta e_{i1}) W_1 + \eta^2 W_2 \int_{-\infty}^{\infty} f_2'(\eta(\Delta e_{i1}+s) + \Delta e_{i2}) f_1(s)\, ds < K_1''(e_{i1}); \quad (33)$$

$$\begin{aligned} & K_1''(e_{i1}) W_2 \int_{-\infty}^{\infty} f_2'(\eta(\Delta e_{i1}+s) + \Delta e_{i2}) f_1(s)\, ds \\ & + K_2''(e_{i2}) \cdot \left[f_1'(\Delta e_{i1}) W_1 + \eta^2 W_2 \int_{-\infty}^{\infty} f_2'(\eta(\Delta e_{i1}+s) + \Delta e_{i2}) f_1(s)\, ds \right] \quad (34) \\ & - f_1'(\Delta e_{i1}) W_1 W_2 \int_{-\infty}^{\infty} f_2'(\eta(\Delta e_{i1}+s) + \Delta e_{i2}) f_1(s)\, ds < K_1''(e_{i1}) K_2''(e_{i2}). \end{aligned}$$

If these conditions hold globally, the expected payoff of player i is a strictly concave function of (e_{i1}, e_{i2}), so that (16) and (17) describe best responses, and thus characterize a Nash equilibrium. Furthermore, this is the unique symmetric equi-

[45] (33) is the condition that expected payoffs are strictly concave in (e_{i1}); (34) is the condition that the Hessian of the expected payoff function has strictly positive determinant.

librium.

8.1.6 Discussing Second-Order Conditions (No revelation)

Global Second-Order Conditions We first show that (33) and (34) hold for given policy parameters and distributions as long as K_t is sufficiently convex for $t = 1, 2$. For (33), this is obvious, as the right-hand side is increasing in $K_1{}''()$. To see that the statement is also true for (34), let

$$\begin{aligned} A &\equiv W_2 \int_{-\infty}^{\infty} f_2' \left(\eta \left(\Delta e_{i1}+s\right) + \Delta e_{i2}\right) f_1 \left(s\right) ds \\ B &\equiv f_1' \left(\Delta e_{i1}\right) W_1 + \eta^2 W_2 \int_{-\infty}^{\infty} f_2' \left(\eta \left(\Delta e_{i1}+s\right) + \Delta e_{i2}\right) f_1 \left(s\right) ds \\ C &\equiv -f_1' \left(\Delta e_{i1}\right) W_1 W_2 \int_{-\infty}^{\infty} f_2' \left(\eta \left(\Delta e_{i1}+s\right) + \Delta e_{i2}\right) f_1 \left(s\right) ds \end{aligned}$$

With this notation, (34) can be written as

$$K_1'' \left(e_{i1}\right) \cdot A + K_2'' \left(e_{i2}\right) \cdot B + C \leq K_1'' \left(e_{i1}\right) K_2'' \left(e_{i2}\right) \tag{35}$$

To prove that (34) holds for sufficiently convex cost functions, suppose it does not hold for some pair of cost function $\widetilde{K}_1$ and $\widetilde{K}_2$. Let $\widehat{K_t} \left(e\right) = \widetilde{K}_t \left(e\right) + \frac{\kappa}{2} e^2$. Then (35) for $\widehat{K}_1$ and $\widehat{K}_2$ is

$$\begin{gathered} \widetilde{K}_1'' \left(e_{i1}\right) \cdot A + \widetilde{K}_2'' \left(e_{i2}\right) \cdot B + C \leq \\ \widetilde{K}_1'' \left(e_{i1}\right) \widetilde{K}_2'' \left(e_{i2}\right) + \kappa \left(\widetilde{K}_1'' \left(e_{i1}\right) + \widetilde{K}_2'' \left(e_{i2}\right) - A - B\right) + \kappa^2 \end{gathered} \tag{36}$$

For all A and B, the right-hand side of this inequality can be made arbitrarily high by increasing κ, so that the inequality is satisfied and thus (34) holds.

Local Second-Order Conditions In the symmetric equilibrium, $\Delta e_{i1} = \Delta e_{i2} = 0$. Using this equation in (33) and (34), $f_1' \left(0\right) = 0$ and the symmetry of f_1

and f_2 (Assumption 2) gives

$$\eta^2 W_2 \int_{-\infty}^{\infty} f_2' (\eta s) f_1 (s) \, ds < K_1'' (e_{i1}) ; \tag{37}$$

$$\left(\frac{W_2}{K_2'' (e_{i2})} + \frac{\eta^2 W_2}{K_1'' (e_{i1})} \right) \int_{-\infty}^{\infty} f_2' (\eta s) f_1 (s) \, ds \leq 1. \tag{38}$$

By Assumption 2, $f_1 (s) = f_1 (-s)$ and $f_2' (\eta s) = -f_2' (-\eta s)$. This implies that $\int_{-\infty}^{\infty} f_2' (\eta s) f_1 (s) \, ds = 0$. Thus, the left-hand sides of (37) and (38) are all 0 and the inequalities hold automatically.

8.2 Revelation Policy: Proof of Proposition 3[46]

(14) and (17) imply

$$e_2^* (\eta, W - W_1, 0) - E (e_2^* (\eta, W - W_1, 1)) =$$
$$(K_2')^{-1} ((W - W_1) C (\eta)) - 2 \int_0^{\infty} (K_2')^{-1} (f_2 (\eta s) (W - W_1)) f_1 (s) \, ds.$$

Using Definition 3 and the symmetry of f_1 and f_2, the right-hand side can be written as

$$(K_2')^{-1} \left((W - W_1) \int_{-\infty}^{\infty} f_2 (\eta s) f_1 (s) \, ds \right)$$
$$- \int_{\infty}^{\infty} (K_2')^{-1} (f_2 (\eta s) (W - W_1)) f_1 (s) \, ds.$$

Substituting $g (s) \equiv (W - W_1) f_2 (\eta s)$, this becomes

$$(K_2')^{-1} \left(\int_{-\infty}^{\infty} g (s) f_1 (s) \, ds \right) - \int_{-\infty}^{\infty} (K_2')^{-1} (g (s)) f_1 (s) \, ds.$$

According to Jensen's inequality, this expression is weakly negative (weakly positive) if $(K_2')^{-1}$ is convex (concave), which is the case if and only if K_2' is concave (convex), that is, $K_2''' \leq 0$ ($K_2''' \geq 0$).

[46] The proof resembles Aoyagi (2010) and Ederer (2010) (case with non-complementary abilities), but allows for $W_1 > 0$ and $\eta \neq 1$.

8.3 Optimal Weights

8.3.1 Proof of Proposition 4

We start with several auxiliary results. Then, we show that $\eta < 0$ is never optimal. Finally, we show that it is always optimal to increase η from zero to some positive value.

Lemma 4 *Suppose* $\eta_1 > 0$ *and* $\eta_2 = -\eta_1$. *Then,*

(i)

$$e_1^* (\eta_1, W - W_1, 1) > e_1^* (\eta_2, W_1, W - W_1).$$

(ii)

$$E \left(e_2^* (\eta_1, W - W_1, 1) \right) = E \left(e_2^* (\eta_2, W - W_1, 1) \right).$$

(iii)

$$e_2^* (\eta_1, W - W_1, 0) = e_2^* (\eta_2, W - W_1, 0).$$

Proof. (i) From (13) and (16) and using (11), we have

$$\begin{aligned} e_1^* (\eta_1, W_1, W - W_1) - e_1^* (\eta_2, W_1, W - W_1) = \\ (K_1')^{-1} (f_1 (0) W_1 + \eta_1 (W - W_1) C(\eta_1)) - \\ (K_1')^{-1} (f_1 (0) W_1 - \eta_1 (W - W_1) C(\eta_1)). \end{aligned} \tag{39}$$

As $K_1'' > 0$, $(K_1')^{-1}$ is strictly increasing. Thus, (39) is strictly positive.

(ii) (14) and $f_2 (\eta_2 s) = f_2 (\eta_1 s)$ imply the result.

(iii) (17) and (11) imply the result. ■

Lemma 5 *Suppose* $W_1 < W$. *Then,*

(i)

$$\left. \frac{\partial e_1^* (\eta, W_1, W - W_1)}{\partial \eta} \right|_{\eta=0} > 0.$$

(ii)

$$\left. \frac{\partial E \left(e_2^* (\eta, W - W_1, 1) \right)}{\partial \eta} \right|_{\eta=0} = \left. \frac{\partial e_2^* (\eta, W - W_1, 0)}{\partial \eta} \right|_{\eta=0} = 0.$$

Proof. (i) From (13) and (16),

$$\frac{\partial e_1^*(\eta, W_1, W-W_1)}{\partial \eta} = \frac{(W-W_1)(C(\eta)+\eta C'(\eta))}{K_1''\left[(K_1')^{-1}(f_1(0)W_1+\eta(W-W_1)C(\eta))\right]}. \tag{40}$$

Hence,

$$\begin{aligned}\left.\frac{\partial e_1^*(\eta, W_1, W-W_1)}{\partial \eta}\right|_{\eta=0} &= \frac{(W-W_1)C(0)}{K_1''\left[(K_1')^{-1}(f_1(0)W_1)\right]} \\ &= \frac{(W-W_1)f_2(0)}{K_1''\left[(K_1')^{-1}f_1(0)W_1\right]}.\end{aligned}$$

where the second equality follows from (9). As $K_1'' > 0$ and $f_2(0) > 0$, $\left.\frac{\partial e_1^*(\eta, W_1, W-W_1)}{\partial \eta}\right|_{\eta=0} > 0$ provided $W_1 < W$.

(ii) From (14),

$$\frac{\partial E(e_2^*(\eta_1, W-W_1, 1))}{\partial \eta} = 2\int_0^\infty \frac{sf_2'(\eta s)(W-W_1)f_1(s)}{K_2''\left[(K_2')^{-1}(f_2(\eta s)(W-W_1))\right]}ds. \tag{41}$$

Hence,

$$\left.\frac{\partial E(e_2^*(\eta_1, W-W_1, 1))}{\partial \eta}\right|_{\eta=0} = 2\int_0^\infty \frac{sf_2'(0)(W-W_1)f_1(s)}{K_2''\left[(K_2')^{-1}(f_2(0)(W-W_1))\right]}ds = 0,$$

where the second equality follows from $f_2'(0) = 0$. Next, from (17),

$$\frac{\partial e_2^*(\eta, W-W_1, 0)}{\partial \eta} = \frac{(W-W_1)C'(\eta)}{K_2''\left[(K_2')^{-1}((W-W_1)C(\eta))\right]}.$$

Hence,

$$\left.\frac{\partial e_2^*(\eta, W-W_1, 0)}{\partial \eta}\right|_{\eta=0} = \frac{(W-W_1)C'(0)}{K_2''\left[(K_2')^{-1}((W-W_1)C(0))\right]} = 0,$$

where the second equality follows from (10). ■

To see that $\eta < 0$ is never optimal, note that Lemma (4) (i)-(iii) implies that for every $\eta < 0$, $-\eta > 0$ yields strictly higher first-period efforts and equally high second-period efforts. Thus, for any revelation policy and whether efforts are perfect or imperfect substitutes, the optimal η is non-negative.

To see that for $W_1 < W$ the optimal η is positive, note that by Lemma (5) (i) and (ii), increasing η marginally from zero increases first-period efforts, while there is no effect on second-period efforts. Hence, for any revelation policy and whether efforts are perfect or imperfect substitutes, the optimal η is positive provided $W_1 < W$.

8.3.2 Proof of Proposition 5

From (18),

$$\frac{\partial V^P(\eta, W_1, 1)}{\partial \eta} = \frac{\partial e_1^*(\eta, W_1, W - W_1)}{\partial \eta} + \frac{\partial E(e_2^*(\eta_1, W - W_1, 1))}{\partial \eta} \tag{42}$$

Using (C1) and (7) to simplify (40) and (41), (42) becomes

$$\frac{\partial V^P(\eta, W_1)}{\partial \eta} = \frac{(W - W_1)\left(C(\eta) + \eta C'(\eta)\right)}{k_1} + \frac{(W - W_1)\, C'(\eta)}{k_2} \tag{43}$$

Solving $\frac{\partial V^P(\eta, W_1)}{\partial \eta} = 0$ and rearranging gives the result.

8.4 Optimal Prize Structure for Perfect Substitutes

First, we provide results on the optimal prize structure for the case of general K_1 and K_2 that are not necessarily quadratic. As the revelation policy matters in this case, we first address the optimal prize structure for the full revelation case in Proposition 9. The result will rely on the Assumption that $K_t''' \leq 0$. This is not a serious restriction: Corollary 3 states that $K_2''' \leq 0$ is the case in which full revelation is optimal. Second, we consider the no revelation case in Proposition 10 for $K_t''' \geq 0$. Again, this is not a serious restriction because for $K_2''' \geq 0$ no revelation is optimal by Corollary 3. Third, we derive Corollary 4 for $K_t''' = 0$.

8.4.1 Full Revelation

Proposition 9 *Suppose* $K_t''' \leq 0$ *for* $t = 1, 2$*. For all* $\eta > 0$*,* $W_1^P(\eta, 1) = 0$ ($W_1^P(\eta, 1) = W$) *if and only if*

$$W f_1(0) < (>) K_1' \left[(K_1')^{-1} (\eta W C(\eta)) + 2 \int_0^\infty (K_2')^{-1} (f_2(\eta s) W) f_1(s) ds \right]. \tag{44}$$

Proof. Using (13) and (14) in (18) gives

$$\begin{aligned} V^P(\eta, W_1, 1) &= (K_1')^{-1} (f_1(0) W_1 + \eta (W - W_1) C(\eta)) \\ &\quad + 2 \int_0^\infty (K_2')^{-1} (f_2(\eta s)(W - W_1)) f_1(s) ds. \end{aligned} \tag{45}$$

This yields

$$\frac{\partial V^P(\eta, W_1, 1)}{\partial W_1} = \frac{f_1(0) - \eta C(\eta)}{K_1'' \left[(K_1')^{-1} (f_1(0) W_1 + \eta (W - W_1) C((\eta)) \right]} - 2 \int_0^\infty \frac{f_2(\eta s) f_1(s)}{K_2'' \left[(K_2')^{-1} (f_2(\eta s)(W - W_1)) \right]} ds,$$

and hence

$$\frac{\partial^2 V^P(\eta, W_1, 1)}{\partial W_1^2} = -\frac{K_1''' \left[(K_1')^{-1} (f_1(0) W_1 + \eta (W - W_1) C(\eta)) \right] (f_1(0) - \eta C(\eta))^2}{\left(K_1'' \left[(K_1')^{-1} (f_1(0) W_1 + \eta (W - W_1) C(\eta)) \right] \right)^3} - 2 \int_0^\infty \frac{(f_2(\eta s))^2 K_2''' \left[(K_2')^{-1} (f_2(\eta s)(W - W_1)) \right] f_1(s) ds}{\left(K_2'' \left[(K_2')^{-1} (f_2(\eta s)(W - W_1)) \right] \right)^3}.$$

Since $K_t''' > 0$, $K_t''' \leq 0$ implies $\frac{\partial^2 V^P(\eta, W_1, 1)}{(\partial W_1)^2} \geq 0$. Thus, there is no interior optimum. For W_1=0 and W_1=W, the principal's expected payoffs are

$$V^P(\eta, 0, 1) = (K_1')^{-1}(\eta W C(\eta)) + 2\int_0^\infty (K_2')^{-1}(f_2(\eta s) W) f_1(s) \, ds;$$
$$V^P(\eta, W, 1) = (K_1')^{-1}(f_1(0) W).$$

Therefore,

$$V^P(\eta, 0, 1) - V^P(\eta, W, 1) =$$
$$(K_1')^{-1}(\eta W C(\eta)) + 2\int_0^\infty (K_2')^{-1}(f_2(\eta s) W) f_1(s) \, ds - (K_1')^{-1}(f_1(0) W).$$

Hence, $V^P(\eta, 0, 1) - V^P(\eta, W, 1) > (<) 0$ if and only if (44) holds. ■

8.4.2 No Revelation

For the no revelation case, we again restrict the third derivative of the cost functions in such a way that the revelation policy is optimal by Corollary 3.

Proposition 10 *Suppose $K_t''' \geq 0$ for $t = 1, 2$. For all $\eta > 0$*
(i) $W_1^P(\eta, 0)$=0 if

$$\frac{f_1(0) - \eta C(\eta)}{K_1''\left[(K_1')^{-1}(\eta W C(\eta))\right]} - \frac{C(\eta)}{K_2''\left[(K_2')^{-1}(W C(\eta))\right]} < 0. \tag{46}$$

(ii) $W_1^P(\eta, 0)$=W if

$$\frac{f_1(0) - \eta C(\eta)}{K_1''\left[(K_1')^{-1}(f_1(0) W)\right]} - \frac{C(\eta)}{K_2''(0)} > 0. \tag{47}$$

(iii) If neither (46) nor (47) holds, $W_1^P \in [0, W]$.

Proof. Using (16) and (17) in (18) gives

$$V^P(\eta, W_1, 0) = \tag{48}$$
$$(K_1')^{-1}(f_1(0) W_1 + \eta (W - W_1) C(\eta)) + (K_2')^{-1}((W - W_1) C(\eta)).$$

This yields

$$\frac{\partial V^P(\eta, W_1, 0)}{\partial W_1} =$$

$$\frac{f_1(0) - \eta C(\eta)}{K_1''\left[(K_1')^{-1}(f_1(0) W_1 + \eta (W - W_1) C(\eta))\right]} - \frac{C(\eta)}{K_2''\left[(K_2')^{-1}((W - W_1) C(\eta))\right]}$$

and

$$\frac{\partial^2 V^P(\eta, W_1, 0)}{\partial W_1^2} =$$

$$-\frac{(f_1(0) - \eta C(\eta))^2 \cdot K_1'''\left[(K_1')^{-1}(f_1(0) W_1 + \eta (W - W_1) C(\eta))\right]}{\left(K_1''\left[(K_1')^{-1}(f_1(0) W_1 + \eta (W - W_1) C(\eta))\right]\right)^3}$$

$$-\frac{(C(\eta))^2 \cdot K_2'''\left[(K_2')^{-1}((W - W_1) C(\eta))\right]}{\left(K_2''\left[(K_2')^{-1}((W - W_1) C(\eta))\right]\right)^3}$$

Since $K_t'' > 0$, $K_t''' \geq 0$ implies $\frac{\partial^2 V^P(\eta, W_1, 0)}{(\partial W_1)^2} \leq 0$.
(i) Thus, the principal will set $W_1 = 0$ provided

$$\left.\frac{\partial V^P(\eta, W_1, 0)}{\partial W_1}\right|_{W_1=0} = \frac{f_1(0) - \eta C(\eta)}{K_1''\left[(K_1')^{-1}(\eta W C(\eta))\right]} - \frac{C(\eta)}{K_2''\left[(K_2')^{-1}(W C(\eta))\right]} < 0.$$

(ii) She will set $W_1 = W$ provided

$$\left.\frac{\partial V^P(\eta, W_1, 0)}{\partial W_1}\right|_{W_1=W} = \frac{f_1(0) - \eta C(\eta)}{K_1''\left[(K_1')^{-1}(f_1(0) W)\right]} - \frac{C(\eta)}{K_2''(0)} > 0.$$

■

8.4.3 Proof of Corollary 4

(i) With $K_t''' = 0$, Proposition 9 implies that $W_1^P(\eta)$ always is a boundary solution, with $W_1^P(\eta) = 0$ if $\frac{1}{k_1}(f_1(0) W) < \left(\frac{\eta}{k_1} + \frac{1}{k_2}\right) W C(\eta)$. This gives the first result. Note that the left-hand side of the last equation is total effort for $W_1 = W$, while the right-hand side is total effort for some η and $W_1 = 0$. Hence, if the inequality is satisfied for some η, then total effort for this η and $W_1 = 0$ is higher than for $W_1 = W$, which shows that $W_1 = W$ cannot be optimal. In this case, since there is no interior optimum by Proposition 9, $W_1 = 0$ is optimal.[47]

(ii) analogous.

8.5 Optimal Prize Structure for Imperfect Substitutes

8.5.1 Proof of Proposition 6

Using (13) and (14) in (19) yields

$$\begin{aligned} V^I(\eta_1, W_1, 1) &= (K_1')^{-1}(f_1(0)W_1 + \eta(W - W_1)C(\eta)) \cdot \\ &\quad 2\int_0^\infty (K_2')^{-1}(f_2(\eta s)(W - W_1))f_1(s)\,ds. \end{aligned} \tag{49}$$

Using (49), we have

$$\begin{gathered} \frac{\partial V^I(\eta, W_1, 1)}{\partial W_1} = \\ \frac{2(f_1(0) - \eta C(\eta))\int_0^\infty (K_2')^{-1}(f_2(\eta s)(W - W_1))f_1(s)\,ds}{K_1''\left[(K_1')^{-1}(f_1(0)W_1 + \eta(W - W_1)C(\eta))\right]} \\ -2(K_1')^{-1}(f_1(0)W_1 + \eta(W - W_1)C(\eta)) \cdot \\ \int_0^\infty \frac{f_2(\eta s)}{K_2''\left[(K_2')^{-1}(f_2(\eta s)(W - W_1))\right]} f_1(s)\,ds. \end{gathered} \tag{50}$$

[47] This can also be derived from Proposition 10.

Thus,

$$\frac{\partial^2 V^I\left(\eta, W_1, 1\right)}{\left(\partial W_1\right)^2}=$$

$$-\frac{2\left(f_1\left(0\right)-\eta C\left(\eta\right)\right)^2\cdot K_1'''\left[\left(K_1'\right)^{-1}\left(f_1\left(0\right)W_1+\eta\left(W-W_1\right)C\left(\eta\right)\right)\right]}{\left(K_1''\left[\left(K_1'\right)^{-1}\left(f_1\left(0\right)W_1+\eta\left(W-W_1\right)C\left(\eta\right)\right)\right]\right)^3}\cdot$$

$$\int_0^\infty\left(K_2'\right)^{-1}\left(f_2\left(\eta s\right)\left(W-W_1\right)\right)f_1\left(s\right)ds$$

$$-\frac{4\left(f_1\left(0\right)-\eta C\left(\eta\right)\right)}{K_1''\left[\left(K_1'\right)^{-1}\left(f_1\left(0\right)W_1+\eta\left(W-W_1\right)C\left(\eta\right)\right)\right]}\int_0^\infty\frac{f_2\left(\eta s\right)f_1\left(s\right)ds}{K_2''\left[\left(K_2'\right)^{-1}\left(f_2\left(\eta s\right)\left(W-W_1\right)\right)\right]}$$

$$-\left(K_1'\right)^{-1}\left(f_1\left(0\right)W_1+\eta\left(W-W_1\right)C\left(\eta\right)\right)\cdot 2\int_0^\infty\frac{\left(f_2\left(\eta s\right)\right)^2}{\left(K_2''\left[\left(K_2'\right)^{-1}\left(f_2\left(\eta s\right)\left(W-W_1\right)\right)\right]\right)^3}\cdot$$

$$K_2'''\left[\left(K_2'\right)^{-1}\left(f_2\left(\eta s\right)\left(W-W_1\right)\right)\right]f_1\left(s\right)ds.$$

Since $V^I\left(\eta, W, 1\right)=0$, $W_1=W$ is never optimal. Since $K_t''>0$, $K_t'''\leq 0$ implies $\frac{\partial^2 V^I(\eta,W_1,1)}{\partial W_1^2}>0$ if $f_1\left(0\right)-\eta C(\eta)<0$. For this case, there is no interior optimum and thus $W_1=0$.

8.5.2 Proof of Proposition 7

With (C1), (49) yields

$$V^I\left(\eta, W_1\right)=\frac{\left(W-W_1\right)C\left(\eta\right)}{k_1k_2}\left(f_1\left(0\right)W_1+\eta\left(W-W_1\right)C\left(\eta\right)\right). \tag{51}$$

Thus

$$\frac{\partial V^I\left(\eta, W_1\right)}{\partial W_1}=\frac{C\left(\eta\right)}{k_1k_2}\left[f_1\left(0\right)\left(W-2W_1\right)-2\eta C\left(\eta\right)\left(W-W_1\right)\right]. \tag{52}$$

Because $\eta>0$, $C\left(\eta\right)>0$ and $k_t>0$, $W_1>\frac{1}{2}W$ implies $\frac{\partial V^I(\eta,W_1)}{\partial W_1}<0$. Hence, $W_1<\frac{W}{2}\ \forall\ \eta$.

8.5.3 Proof of Proposition 8

(i) Clearly $W_1 > 0$ at the optimum if $\left.\frac{\partial V^I(\eta,W_1)}{\partial W_1}\right|_{W_1=0} > 0$, that is, using (52), if $f_1(0) > 2\eta C(\eta)$. To see that $W_1 = 0$ at the optimum if $\left.\frac{\partial V^I(\eta,W_1)}{\partial W_1}\right|_{W_1=0} < 0$ or, equivalently, $f_1(0) < 2\eta C(\eta)$, first note that (52) implies

$$\frac{\partial^2 V^I(\eta, W_1)}{\partial W_1^2} = \frac{C(\eta)}{k_1 k_2}\left[-2f_1(0) + 2\eta C(\eta)\right].$$

Thus $\frac{\partial V^I(\eta,W_1)}{\partial W_1}$ is monotone in W_1. Moreover, according to the proof of Proposition 7, $\frac{\partial V^I(\eta,W_1)}{\partial W_1} < 0 \ \forall W_1 > \frac{W}{2}$. The last two statements imply that, whenever $\left.\frac{\partial V^I(\eta,W_1)}{\partial W_1}\right|_{W_1=0} < 0$, then $\frac{\partial V^I(\eta,W_1)}{\partial W_1} < 0$ for all $W_1 \leq \frac{W}{2}$ and thus $W_1^I(\eta) = 0$. To see that $W_1 = 0$ at the optimum if $\left.\frac{\partial V^I(\eta,W_1)}{\partial W_1}\right|_{W_1=0} = 0$ or, equivalently, $f_1(0) = 2\eta C(\eta)$, note that $f_1(0) = 2\eta C(\eta)$ implies $\frac{\partial^2 V^I(\eta,W_1)}{\partial W_1^2} < 0$, so that $\frac{\partial V^I(\eta,W_1)}{\partial W_1} < 0$ $\forall W_1 > 0$ and thus $W_1^I(\eta) = 0$.

For $\left.\frac{\partial V^I(\eta,W_1)}{\partial W_1}\right|_{W_1=0} > 0$, the first-order condition $\frac{\partial V^I(\eta,W_1)}{\partial W_1} = 0$ yields $W_1^I(\eta) = W\frac{f_1(0) - 2\eta C(\eta)}{2f_1(0) - 2\eta C(\eta)} > 0$ for $f_1(0) > 2\eta C(\eta)$. Summing up, we obtain

$$W_1^I(\eta) = \begin{cases} W\frac{f_1(0)-2\eta C(\eta)}{2f_1(0)-2\eta C(\eta)} > 0, & f_1(0) > 2\eta C(\eta) \\ 0, & f_1(0) \leq 2\eta C(\eta) \end{cases} \tag{53}$$

(ii) (53) shows that W_1^I must correspond to one of the two cases mentioned in the proposition. To complete the proof, we derive the first-order condition for $\eta^I(W_1)$ for these two cases. From (51), we obtain

$$\frac{\partial V^I(\eta, W_1)}{\partial \eta} = \frac{(W - W_1)^2 C(\eta)}{k_1 k_2}\left(C(\eta) + \eta C'(\eta)\right) + \tag{54}$$
$$\frac{(W - W_1) C'(\eta)}{k_1 k_2}\left(f_1(0) W_1 + \eta (W - W_1) C(\eta)\right).$$

The first-order condition $\frac{\partial V^I(\eta,W_1)}{\partial \eta} = 0$ yields

$$W_1 = W\frac{(C(\eta))^2 + 2\eta C(\eta) C'(\eta)}{C(\eta)^2 + 2\eta C(\eta) C'(\eta) - f_1(0) C'(\eta)}. \tag{55}$$

According to (53), $W_1 = 0$ is a necessary condition for an optimum with $f_1(0) \leq 2\eta C(\eta)$. Inserting $W_1 = 0$ in (55) gives the first-order condition $\eta = -\frac{C(\eta)}{2C'(\eta)}$, which corresponds to Proposition 8(ii)(a). Analogously, $W_1 = W\frac{f_1(0)-2\eta C(\eta)}{2f_1(0)-2\eta C(\eta)}$ is a necessary condition for an optimum with $f_1(0) > 2\eta C(\eta)$. Inserting $W_1 = W\frac{f_1(0)-2\eta C(\eta)}{2f_1(0)-2\eta C(\eta)}$ in (55) and solving for $f_1(0)$ gives the first-order condition $f_1(0) = -\frac{(C(\eta))^2}{C'(\eta)}$, which corresponds to Proposition 8(ii)(b).

8.6 The Normal-Quadratic Example E1

8.6.1 Proof of Corollary 5

Part 1: Auxilliary Results

We will first provide several auxiliary results. Note that $\frac{2}{\sqrt{\pi}}\int_0^x \exp(-t^2)\,dt$ is the error function, for which

$$\frac{2}{\sqrt{\pi}}\int_0^\infty \exp\left(-t^2\right) dt = 1. \tag{56}$$

Next, for E1,

$$f_t(s) = \frac{1}{\sigma_t\sqrt{2\pi}}\exp\left(-\frac{s^2}{2\sigma_t^2}\right). \tag{57}$$

Hence,

$$\begin{aligned} f_t'(s) &= -\frac{1}{\sqrt{2\pi}}\frac{s}{\sigma_t^3}\exp\left(-\frac{s^2}{2\sigma_t^2}\right); \\ f_t''(s) &= \frac{1}{\sqrt{2\pi}}\frac{s^2-\sigma_t^2}{\sigma_t^5}\exp\left(-\frac{s^2}{2\sigma_t^2}\right); \\ f_t'''(s) &= \frac{1}{\sqrt{2\pi}}\frac{3\sigma_t^2 s - s^3}{\sigma_t^7}\exp\left(-\frac{s^2}{2\sigma_t^2}\right). \end{aligned} \tag{58}$$

As $s = -\sigma_t$ (the solution to $f_t''(s) = 0$ and $f_t'''(s) < 0$) maximizes $f_t'(x)$, we obtain $\forall x \in \mathbb{R}\ f_t'(x) \leq -\frac{1}{\sqrt{2\pi}}\frac{-\sigma_t}{\sigma_t^3}\exp\left(-\frac{\sigma_t^2}{2\sigma_t^2}\right)$ and thus

$$f_t'(x) \leq \frac{1}{\sigma_t^2\sqrt{2\pi\exp(1)}} \tag{59}$$

Furthermore, (57) implies

$$\int_0^\infty f_2(\eta s)\,ds = \frac{1}{\sigma_2\sqrt{2\pi}}\int_0^\infty \exp\left(-\left(\frac{s\eta}{\sqrt{2}\sigma_2}\right)^2\right)ds$$

Substituting $s = \frac{\sqrt{2}\sigma_2}{|\eta|}t$ and $ds = \frac{\sqrt{2}\sigma_2}{|\eta|}dt$ implies

$$\int_0^\infty f_2(\eta s)\,ds = \frac{1}{|\eta|\sqrt{\pi}}\int_0^\infty \exp\left(-t^2\right)dt.$$

With (56), we get

$$\int_0^\infty f_2(\eta s)\,ds = \frac{1}{2|\eta|}. \tag{60}$$

Next, (57) and (58) imply

$$\int_0^\infty f_2(\eta s)\,f_2'(\eta s)\,ds = -\frac{\eta}{2\pi\sigma_2^4}\int_0^\infty s\cdot\exp\left(-\frac{s^2\eta^2}{\sigma_2^2}\right)ds.$$

Substituting $s = \frac{\sqrt{t}\sigma_2}{|\eta|}$ and $ds = \frac{\sigma_2}{2\sqrt{t}|\eta|}dt$ and noting that $\int_0^\infty \exp(-t)\,dt = 1$, we obtain

$$\int_0^\infty f_2(\eta s)\,f_2'(\eta s)\,ds = -\frac{1}{4\pi\eta\sigma_2^2}. \tag{61}$$

Furthermore, (57) implies

$$C(\eta) = \frac{1}{\pi\sigma_1\sigma_2}\int_0^\infty \exp\left(-\left(s\frac{\sqrt{\sigma_1^2\eta^2+\sigma_2^2}}{\sqrt{2}\sigma_1\sigma_2}\right)^2\right)ds.$$

Substituting $s = \frac{\sqrt{2}\sigma_1\sigma_2}{\sqrt{\sigma_1^2\eta^2+\sigma_2^2}}t$ and $ds = \frac{\sqrt{2}\sigma_1\sigma_2}{\sqrt{\sigma_1^2\eta^2+\sigma_2^2}}dt$ yields

$$C(\eta) = \frac{\sqrt{2}}{\pi\sqrt{\sigma_1^2\eta^2+\sigma_2^2}}\int_0^\infty \exp\left(-t^2\right)dt$$

With (56), we get

$$C(\eta) = \frac{1}{\sqrt{2\pi}\sqrt{\sigma_1^2\eta^2+\sigma_2^2}}, \tag{62}$$

so that

$$C'(\eta) = -\frac{\eta\sigma_1^2}{\sqrt{2\pi}\left(\sigma_1^2\eta^2+\sigma_2^2\right)^{\frac{3}{2}}}. \tag{63}$$

Part 2: Second-Order Conditions

Next, we derive sufficient conditions for the second-order conditions to hold.[48] Using $K_t''(e_{it}) = k$, (23) simplifies to

$$f_2'(x)\, W_2 < k \ \forall x \in \mathbb{R} \tag{64}$$

From $W_2 \leq W$ and (59),

$$f_2'(x)\, W_2 \leq \frac{W}{\sigma_2^2\sqrt{2\pi\exp(1)}} \tag{65}$$

(64) and (65) imply that a sufficient condition for (23) to hold is

$$k > \frac{W}{\sigma_2^2\sqrt{2\pi\exp(1)}} \tag{66}$$

Similarly, (32) can be written as

$$\begin{aligned} & f_1'(x)\, W_1 + \eta W_2 \int_0^\infty f_2(\eta s)\left[f_1'(s+x) - f_1'(s-x)\right] ds \\ & + \frac{\eta W_2^2}{k}\int_0^\infty f_2(\eta s)\, f_2'(\eta s)\left[f_1'(s+x) + f_1'(s-x)\right] ds < k \end{aligned} \tag{67}$$

Using (59), we obtain $\forall x \in \mathbb{R}$

$$f_1'(x)\, W_1 \leq \frac{W_1}{\sigma_1^2\sqrt{2\pi\exp(1)}};$$

$$\eta W_2 \int_0^\infty f_2(\eta s)\left[f_1'(s+x) - f_1'(s-x)\right] ds \leq \frac{2W_2\left|\eta\int_0^\infty f_2(\eta s)\, ds\right|}{\sigma_1^2\sqrt{2\pi\exp(1)}};$$

$$\frac{\eta W_2^2}{k}\int_0^\infty f_2(\eta s)\, f_2'(\eta s)\left[f_1'(s+x) + f_1'(s-x)\right] ds \leq \frac{2W_2^2\left|\eta\int_0^\infty f_2(\eta s)\, f_2'(\eta s)\, ds\right|}{k\sigma_1^2\sqrt{2\pi\exp(1)}}.$$

[48] We only consider the second-order conditions for the full revelation case.

This yields an upper bound for the left-hand side of (67):

$$\frac{1}{\sigma_1^2\sqrt{2\pi\exp(1)}}\left[W_1+2W_2\left|\eta\int_0^\infty f_2(\eta s)\,ds\right|+\frac{2W_2^2}{k}\left|\eta\int_0^\infty f_2(\eta s)\,f_2'(\eta s)\,ds\right|\right].$$

With (60) and (61), this upper bound can be written as

$$\frac{W_1+W_2}{\sigma_1^2\sqrt{2\pi\exp(1)}}+\frac{W_2^2}{k\sigma_1^2\sigma_2^2(2\pi)^{\frac{3}{2}}\sqrt{\exp(1)}}\leq$$
$$\frac{W}{\sigma_1^2\sqrt{2\pi\exp(1)}}+\frac{W^2}{k\sigma_1^2\sigma_2^2(2\pi)^{\frac{3}{2}}\sqrt{\exp(1)}}.$$

A sufficient condition for (32) to hold is thus

$$k>\frac{W}{\sigma_1^2\sqrt{2\pi\exp(1)}}+\frac{W^2}{k\sigma_1^2\sigma_2^2(2\pi)^{\frac{3}{2}}\sqrt{\exp(1)}}. \quad (68)$$

Part 3: Characterizing the equilibrium

Proposition 1 thus characterizes the PBE. As $K_2'''=0$, Proposition 3 implies that efforts under both revelation policies are equal in expected value. Inserting $(K_t')^{-1}(x)=\frac{x}{k}$, (57) and (62) in (13) and (14) yields (21) and (22).

8.6.2 Proof of Corollary 6

(i) We first derive $\eta^P(W_1)$. With (62) and (63), we obtain

$$\frac{C'(\eta)}{C(\eta)}=-\frac{\eta\sigma_1^2}{\sigma_2^2+\sigma_1^2\eta^2}.$$

Proposition 5 (i) thus implies $\frac{\eta\cdot\sigma_1^2}{\sigma_2^2+\sigma_1^2\eta^2}=\frac{1}{1+\eta}$ as a necessary condition, which is uniquely (and positively) solved by $\eta=\frac{\sigma_2^2}{\sigma_1^2}>0$. Since the optimal η must be strictly positive by Proposition 4 and since the solution to the necessary condition is unique and positive, the necessary condition is sufficient and we have $\eta^P(W_1)=\frac{\sigma_2^2}{\sigma_1^2}\forall\ W_1<W$. Next, we show that $W_1^P=0$. By Corollary 4, $W_1^P=0$ if $\exists\eta$ such that $f_1(0)<(1+\eta)\,C(\eta)$. From (57) and (62), this condition is equivalent with $\frac{1}{\sigma_1\sqrt{2\pi}}<\frac{1+\eta}{\sqrt{2\pi}\sqrt{\sigma_1^2\eta^2+\sigma_2^2}}\Longleftrightarrow\eta>\frac{\frac{\sigma_2^2}{\sigma_1^2}-1}{2}$. In particular, this holds for

$\eta^P(W_1) = \frac{\sigma_2^2}{\sigma_1^2}$. Hence, $W_1^P = 0$ and $\eta^P = \eta^P\left(W_1^P\right) = \frac{\sigma_2^2}{\sigma_1^2}$.

(ii) (62) and (63) yield

$$C(\eta)^2 + C'(\eta)\, f_1(0) = \frac{\sqrt{\eta^2\sigma_1^2 + \sigma_2^2} - \eta\sigma_1}{2\pi\left(\eta^2\sigma_1^2 + \sigma_2^2\right)^{\frac{3}{2}}} > 0\ \forall\ \eta.$$

This is inconsistent with $\left|\frac{C'(\eta)}{C(\eta)}\right| = \frac{C(\eta)}{f_1(0)}$ as for $\eta > 0$, $\left|\frac{C'(\eta)}{C(\eta)}\right| = \frac{C(\eta)}{f_1(0)}$ is equivalent to $C(\eta)^2 + C'(\eta)\, f_1(0) = 0$**.** Therefore, according to Proposition 8(ii)(b), $W_1^I > 0$ cannot apply. Hence, Proposition (8)(ii) gives

$$\left|\frac{C'(\eta)}{C(\eta)}\right| = \frac{1}{2\eta}$$

as the necessary condition for η^I. Using (62) and (63), this can be written as

$$\frac{\eta\sigma_1^2}{\sigma_2^2 + \sigma_1^2\eta^2} = \frac{1}{2\eta},$$

which is solved by $\eta^I = \frac{\sigma_2}{\sigma_1}$.