Year: 2015

# Belief precision and effort incentives in promotion contests

Miklos-Thal, Jeanine ; Ullrich, Hannes

Abstract: The career concerns literature predicts that incentives for effort decline as beliefs about ability become more precise (Holmstrom, 1982/1999). In contrast, we show that effort can increase with belief precision when agents compete for promotions to better paid jobs that are assigned on the basis of perceived abilities. In this case, an intermediate level of precision provides the strongest incentive for effort, with effort increasing (decreasing) when beliefs are less (more) precise.

DOI: https://doi.org/10.1111/ecoj.12162



Originally published at:
Miklos-Thal, Jeanine; Ullrich, Hannes (2015). Belief precision and effort incentives in promotion contests. Economic Journal, 125(589):1952-1963.
DOI: https://doi.org/10.1111/ecoj.12162

# Belief Precision and Effort Incentives in Promotion Contests*

Jeanine Miklós-Thal[†] Hannes Ullrich[‡]

April 2, 2014

**Abstract**

The career concerns literature predicts that incentives for effort decline as beliefs about ability become more precise (Holmström 1982/1999). In contrast, we show that effort can increase with belief precision when agents compete for promotions to better paid jobs that are assigned on the basis of perceived abilities. In this case, an intermediate level of precision provides the strongest incentive for effort, with effort increasing (decreasing) when beliefs are less (more) precise.

**Keywords:** incentives, reputation, promotion contests, career concerns

*We are grateful to the editor Martin Cripps, two anonymous referees, Heski Bar-Isaac, Ryan McDevitt, Michael Raith, Veikko Thiele, Michael Waldman, and audiences at the 2010 "Tournaments, Contests and Relative Performance Evaluation" conference at NCSU and the 2011 IIOC for helpful comments.

[†]William E. Simon Graduate School of Business Administration, University of Rochester, NY - jeanine.miklos-thal@simon.rochester.edu.

[‡]Department of Business Administration, University of Zurich.

# 1 Introduction

Economic agents exert effort not just for immediate rewards but also to build reputations that yield future payoffs. This observation, first noted in Fama (1980), has given rise to a sizable literature studying such implicit incentives due to career concerns. In the seminal article of this literature, Holmström (1982/1999) showed that effort incentives decline as the beliefs about an agent's ability become more precise. Intuitively, additional performance observations have less impact on an agent's reputation when much is already known about him, which means that the expected marginal return to effort is lower when beliefs are more precise.[1]

We show that this key insight of the career concerns literature may no longer hold when effort incentives are driven by a worker's desire to build a reputation that improves his promotion prospects. In our model, a worker is promoted to a better paid job if and only if his posterior perceived ability exceeds a certain threshold; this threshold is ex ante uncertain if several agents compete for a limited number of slots, but fixed in situations without slot constraints. Such promotion rules are consistent with market-based promotion tournaments building on Waldman's (1984) theory of promotions as signals.[2] In these models, promotions are coupled with assignments to jobs with higher returns to ability, and the current employer learns more about a worker's ability than outside firms.[3] The signal about ability associated with a promotion thus raises potential employers' willingness to pay for promoted workers, which implies that promoted workers obtain a wage premium. In contrast, Holmström (1982/1999) assumes that all workers perform the same task (with constant returns to ability) and that all firms share the same information about workers, which implies that each worker's income is linearly increasing in the market's belief about his ability.

[1] This result has been confirmed in many other papers, including Dewatripont, Jewitt and Tirole (1999a, 1999b) and Casas-Arce (2009). A recent exception is Martinez (2009) who finds that the opposite may hold in a multi-period career concerns model with job assignments in which the strength of incentives depends on employment history and ability evolves over time in such a way that the precision of beliefs remains constant. Greater precision can strengthen effort incentives in such a model due to the fact that next-period learning has effects on current-period effort incentives. In contrast, we show that greater precision in beliefs can lead to stronger effort incentives in a simple model of promotion contests with only one period of learning.

[2] See also Gibbs (1995), Bernhardt (1995), Zabojnik and Bernhardt (2001), Ghosh and Waldman (2010), and Zabojnik (2012).

[3] Schönberg (2007) and Kahn (2013) provide empirical evidence of such asymmetric learning between firms.

We find that the precision of beliefs about abilities has two effects on effort incentives. First, it affects the probability that a worker's posterior reputation is close to the threshold for being promoted. Greater precision implies that the posterior reputation is likely to be close to the prior reputation, which strengthens (dampens) effort incentives if the prior reputation is close to (far from) the threshold. Second, it leads to the familiar "learning effect" already mentioned. Greater precision means that the agent's performance has less impact on the posterior belief about his ability, which dampens effort incentives. Putting together these two effects, we show that effort incentives are always maximized at an intermediate level of belief precision, and increasing (decreasing) at all lower (higher) levels of precision. This result holds both when two workers compete for a promotion awarded to the worker with the better posterior reputation and when the reputation threshold for being promoted is fixed. The result is also robust to adding a bias in favor of one of the workers.

The plan of this short paper is as follows. In Section 2, we set up and analyze a model of promotion contests based on perceived abilities, and establish our main result that the relation between effort incentives and the precision of beliefs about ability is non-monotonic. Next, we discuss differences with respect to rank-order tournaments in which the principal commits to promoting the agent with the better performance, even if the other agent has a superior reputation. Section 3 offers a brief conclusion.

# 2 Belief Precision in Promotion Contests

## 2.1 A Model of Promotion Contests

Consider a one-period game between a principal and two agents $j = 1, 2$. The agents' innate ability levels, $\eta_1$ and $\eta_2$, are unobservable to all parties. We assume that the prior distribution of beliefs about $\eta_j$ is normal with mean $m_j$ and precision (equal to the inverse of the variance) $h_j$. The prior distributions of $\eta_1$ and $\eta_2$ are independent. As in Holmström (1982/1999), all parties share the same prior beliefs.

At the beginning of the period, the agents simultaneously decide how much effort to exert. Agent $j$'s effort $a_j \in [0, \infty)$ is unobservable to the principal and agent $k \neq j$. The cost of effort is an increasing and strictly convex function $c(a_j)$ with $c(0) = c'(0) = 0$ and

$\lim_{a_j \to \infty} c'(a_j) = \infty$. Agent $j$'s performance is

$$y_j = \eta_j + a_j + \varepsilon_j,$$

where $\varepsilon_j$ is a stochastic noise term. We assume that $\varepsilon_1$ and $\varepsilon_2$ are independently and normally distributed with zero means and precision $h_\varepsilon$.

The principal has one slot at a higher job level available. After observing $y_1$ and $y_2$, the principal updates his beliefs and promotes agent $j \neq k \in \{1, 2\}$ if and only if

$$E\left[\eta_j \mid y_j\right] > E\left[\eta_k \mid y_k\right] + \Delta_k. \tag{1}$$

If $\Delta_k = 0$, the principal selects the agent with the highest perceived ability. If $\Delta_k > (<) 0$, the contest is biased in favor of (against) agent $k$. $\Delta_k = 0$ is a natural assumption in the context of promotions to higher job levels. Suppose that after the principal selects a worker, there is a second and final period in which the selected worker is assigned to a job level with a higher return to ability (Rosen 1982; Waldman 1984; Ghosh and Waldman 2010). Promoting the worker with the higher expected ability then maximizes the firm's expected second-period profits. Although biasing the contest can potentially improve effort incentives in the first period, it would require the principal to commit to an ex post sub-optimal decision, an unlikely scenario for promotions based on non-verifiable reputations.[4]

Agent $j$ maximizes

$$\Pi_j\left(a_j; a_j^e, a_k, a_k^e\right) = P_j\left(a_j; a_j^e, a_k, a_k^e\right) W - c\left(a_j\right),$$

where the probability that $j$ is promoted, $P_j\left(a_j; a_j^e, a_k, a_k^e\right)$, is a function of the agents' actual and anticipated ($a_1^e$ and $a_2^e$) effort levels, and $W > 0$ is the wage premium upon promotion.[5]

The equilibrium concept is (pure-strategy) perfect Bayesian equilibrium. Denote by

[4]See O'Keefe, Viscusi, and Zeckhauser (1984), Lazear and Rosen (1981, Section IV), and Meyer (1991, 1992) for detailed discussions of the possible benefits of biases in rank-order tournaments where the principal has commitment power. Other reasons why the principal may extend preferential treatment to some agents (i.e., $\Delta_k \neq 0$) are discrimination, affirmative action, or nepotism.

[5]In a full-fledged model of market-based promotion tournaments, the size of the wage premium $W$ would be determined endogenously and could thus depend on $h_1$ and $h_2$. We discuss how this would affect our results at the end of the analysis.

$(a_1^*, a_2^*)$ the equilibrium effort choices. In equilibrium, each agent's effort must be optimal given beliefs and the competing agent's effort choice, and all parties must rationally anticipate the equilibrium effort choices, i.e., $a_j^e = a_j^*$ for $j = 1, 2$.

If $a_j^e = a_j^*$, $j$'s posterior reputation given $y_j$ is

$$E\left[\eta_j \mid y_j\right] = \frac{h_j m_j + h_\varepsilon \left(y_j - a_j^*\right)}{h_j + h_\varepsilon} \tag{2}$$

Define the random variable

$$\zeta_j = \frac{h_k m_k + h_\varepsilon(\eta_k + \varepsilon_k)}{h_k + h_\varepsilon} + \Delta_k - \frac{h_j m_j + h_\varepsilon(\eta_j + \varepsilon_j)}{h_j + h_\varepsilon}.$$

Our independence and normality assumptions imply that the *prior* distribution of $\zeta_j$ is normal with mean

$$m_k + \Delta_k - m_j$$

and variance

$$\sigma^2 = \left(\frac{h_\varepsilon}{h_1 + h_\varepsilon}\right)^2 \left(\frac{1}{h_1} + \frac{1}{h_\varepsilon}\right) + \left(\frac{h_\varepsilon}{h_2 + h_\varepsilon}\right)^2 \left(\frac{1}{h_2} + \frac{1}{h_\varepsilon}\right) \tag{3}$$

We denote this distribution by $\varphi_j(\cdot)$ with c.d.f. $\Phi_j(\cdot)$. Note that $\varphi_1(-z) = \varphi_2(z)$ for all $z \in \mathbb{R}$, and denote by $\varphi(0) = \varphi_1(0) = \varphi_2(0)$ the prior density of $E[\eta_1 \mid y_1] + \Delta_1 = E[\eta_2 \mid y_2]$ given that $a_j = a_j^*$ for all $j$.

We now turn to $j$'s effort decision. Given $a_k = a_k^*$, $j$'s winning probability becomes

$$\begin{aligned} P_j(a_j; a_j^*, a_k^*, a_k^*) &= \Pr\left\{\frac{h_k m_k + h_\varepsilon(\eta_k + \varepsilon_k)}{h_k + h_\varepsilon} + \Delta_k < \frac{h_j m_j + h_\varepsilon\left(\eta_j + a_j + \varepsilon_j - a_j^*\right)}{h_j + h_\varepsilon}\right\} \\ &= \Phi_j\left(\frac{h_\varepsilon}{h_j + h_\varepsilon}(a_j - a_j^*)\right). \end{aligned}$$

The marginal impact of $a_j$ on $j$'s expected payoff given $a_k = a_k^*$ is hence

$$\frac{\partial \Pi_j(a_j; a_j^*, a_k^*, a_k^*)}{\partial a_j} = \varphi_j\left(\frac{h_\varepsilon}{h_j + h_\varepsilon}(a_j - a_j^*)\right) \frac{h_\varepsilon}{h_j + h_\varepsilon} W - c'(a_j),$$

and the first-order conditions for a pure-strategy equilibrium become

$$c'\left(a_j^*\right) = \varphi\left(0\right) \frac{h_\varepsilon}{h_j + h_\varepsilon} W \text{ for each } j = 1, 2. \tag{4}$$

Our assumptions on $c\left(\cdot\right)$ imply that each first-order condition has a unique and strictly positive solution.

However, $\Pi_j(a_j; a_j^*, a_k^*, a_k^*)$ can potentially reach its global maximum at a value $a_j > 0$ different from $a_j^*$ defined in (4), in which case the game has no pure-strategy equilibrium.[6] To rule out such cases, the ensuing analysis assumes that $\Pi_j$ is strictly concave in $a_j$ given $a_j^*$:

$$\varphi_j'\left(\frac{h_\varepsilon}{h_j + h_\varepsilon}(a_j - a_j^*)\right)\left(\frac{h_\varepsilon}{h_j + h_\varepsilon}\right)^2 W - c''\left(a_j\right) < 0 \text{ for all } a_j \geq 0. \tag{5}$$

If $m_k + \Delta_k - m_j \leq -\frac{h_\varepsilon}{h_j + h_\varepsilon} a_j^*$, (5) always holds; otherwise, (5) holds if and only if

$$W < \overline{W}_j \equiv \left(\frac{h_j + h_\varepsilon}{h_\varepsilon}\right)^2 \max_{a_j \in \left(0, a_j^* + \frac{h_j + h_\varepsilon}{h_\varepsilon}(m_k + \Delta_k - m_j)\right)} \left(\frac{c''\left(a_j\right)}{\varphi_j'\left(\frac{h_\varepsilon}{h_j + h_\varepsilon}(a_j - a_j^*)\right)}\right), \tag{6}$$

where $a_j^*$ is as defined in (4). Under the assumption that (5) holds for $j = 1, 2$, the game has a pure-strategy equilibrium with the effort levels $a_1^*, a_2^* > 0$ implied by (4).

## 2.2 Belief Precision and Effort Incentives

The career concerns literature predicts that greater precision in beliefs dampens effort incentives. We are interested in whether the same holds for promotion contests in which only relative perceived abilities matter, as described above. From the first-order conditions in (4), it is apparent that $h_1 < h_2$ implies $a_1^* > a_2^*$: the agent whose ability is better known exerts less effort. When beliefs about one or both agents' abilities become more precise, however, effort incentives can become stronger, as we show next. That is, consider two contests, $A$ and $B$, that are identical except that $h_1$ is higher in contest $B$; in this case, the equilibrium effort levels of both agents may be higher in contest $B$ than in contest $A$.

We first state the formal results and then discuss the underlying effects. The proof of

[6] $c'\left(0\right) = 0$ implies that $\frac{\partial \Pi_j}{\partial a_j} \mid_{a_j = 0} > 0$, hence $\Pi_j$ never reaches a maximum at $a_j = 0$.

Proposition 1 is relegated to the Appendix.

**Proposition 1** *Let $j \neq k \in \{1,2\}$.*

**(i)** *For any $(m_1, m_2, h_k, h_\varepsilon, \Delta_1)$, there exists a unique threshold $h_j^{self}(m_1, m_2, h_k, h_\varepsilon, \Delta_1) > 0$ such that*

$$\frac{da_j^*}{dh_j} > (<) 0 \text{ if and only if } h_j < (>) h_j^{self}.$$

*Moreover,* $\lim_{h_j \to 0} a_j^* = \lim_{h_j \to \infty} a_j^* = 0$, *and* $\lim_{h_j \to 0} \frac{da_j^*}{dh_j} = \infty$.

**(ii)** *If*

$$(m_1 + \Delta_1 - m_2)^2 \leq \left(\frac{h_\varepsilon}{h_j + h_\varepsilon}\right)^2 \left(\frac{1}{h_j} + \frac{1}{h_\varepsilon}\right), \tag{7}$$

*then*

$$\frac{da_j^*}{dh_k} > 0 \text{ for all } h_k.$$

*Otherwise, for any $(m_1, m_2, h_j, h_\varepsilon, \Delta_1)$, there exists a unique threshold $h_k^{comp}(m_1, m_2, h_j, h_\varepsilon, \Delta_1) > 0$ such that*

$$\frac{da_j^*}{dh_k} > (<) 0 \text{ if and only if } h_k < (>) h_k^{comp}.$$

*Moreover,* $\lim_{h_k \to 0} a_j^* = 0, \lim_{h_k \to \infty} a_j^* > 0$, *and* $\lim_{h_k \to 0} \frac{da_j^*}{dh_k} = \infty$.

The precisions of beliefs $h_1$ and $h_2$ affect the first-order conditions in (4) that determine equilibrium effort levels through two channels. First, $h_j$ affects the rate $\frac{h_\varepsilon}{h_j + h_\varepsilon}$ at which a marginal increase in $y_j$ improves $j$'s posterior reputation (see (2)). This is the standard learning effect, which ceteris paribus predicts that greater precision in the beliefs about $j$'s ability leads to lower effort by $j$.

Second, both $h_1$ and $h_2$ affect $\varphi(0)$, the prior density of $E[\eta_1 \mid y_1] + \Delta_1 = E[\eta_2 \mid y_2]$, through their impact on the variance $\sigma^2$ of $\varphi_1$ and $\varphi_2$ (see (3)). The first-order conditions in (4) imply that both agents' equilibrium effort levels are increasing in $\varphi(0)$. Intuitively, effort incentives are stronger if agents assign a higher probability to the race having a close outcome. The impact of an increase in either $h_1$ or $h_2$ on $\varphi(0)$ is ambiguous, however: given our normality assumptions, a marginal increase in the precision of beliefs about either agent's ability raises $\varphi(0)$ if and only if

$$(m_1 + \Delta_1 - m_2)^2 < \sigma^2. \tag{8}$$

Intuitively, greater precision in beliefs (about one's own or the rival's ability) strengthens effort incentives if the agents' starting positions are relatively symmetric, that is, if $m_1 + \Delta_1$ is close to $m_2$. If starting conditions are sufficiently asymmetric, however, then greater precision dampens effort incentives because it lowers the probability that the underdog can beat the frontrunner.

This second "closeness effect" explains why $h_k$ has an impact on $j$'s equilibrium effort. Although learning about $j$'s ability is independent of the beliefs about $k$'s ability, $h_k$ affects the probability that the contest outcome will be close, which, in turn, affects $j$'s effort incentives. More specifically, $\frac{da_j^*}{dh_k} > 0$ if and only if (8) holds. The variance $\sigma^2$ is strictly decreasing in $h_k$ with $\lim_{h_k \to 0} \sigma = \infty$. Two cases can occur: either $(m_1 + \Delta_1 - m_2)^2 > \lim_{h_k \to \infty} \sigma^2$, in which case there exists a unique $h_k^{\text{comp}}$ such that (8) holds if and only if $h_k < h_k^{\text{comp}}$, or $(m_1 + \Delta_1 - m_2)^2 < \lim_{h_k \to \infty} \sigma^2$, in which case (8) holds for all $h_k$. The latter inequality coincides with condition (7) in part (ii) of the proposition.[7]

The results in part (i) of the proposition stem from the combination of the learning effect and the closeness effect. Since $\varphi(0)$ is symmetric in $h_1$ and $h_2$, the closeness effect is again positive if and only if (8) holds; the learning effect is always negative.[8] If $h_j$ is high enough, then the negative learning effect always dominates: as the rate of belief updating about $j$'s ability converges to zero, $j$'s effort incentives completely vanish. When there is sufficient uncertainty about $j$'s ability ($h_j$ close to zero), on the other hand, the closeness effect is positive and dominates the negative learning effect. This is because the closeness effect goes to infinity as $h_j$ approaches zero, while the learning effect remains finite. Figure 1 illustrates Proposition 1(i) in an example.[9]

[7] The comparative statics results with respect to $h_\varepsilon$ are qualitatively similar. If $m_1 + \Delta_1 = m_2$, then $\frac{da_j}{dh_\varepsilon} > 0$ for all $h_\varepsilon$. If $(m_1 + \Delta_1 - m_2)^2 > \lim_{h_\varepsilon \to \infty} \sigma^2 = 0$, then there exists a threshold $\widehat{h}_\varepsilon > 0$ such that $\frac{da_j}{dh_\varepsilon} \gtreqless 0$ if and only if $h_\varepsilon \lesseqgtr \widehat{h}_\varepsilon$. See also Kwon (2013).

[8] Note that the symmetry of $\varphi(0)$ in $h_1$ and $h_2$ and the fact that the learning effect is always negative imply that $\frac{da_j^*}{dh_j} < \frac{da_j^*}{dh_k}$ whenever $h_1 = h_2$ and that $h_j^{\text{self}}(m_1, m_2, h, h_\varepsilon, \Delta_1) < h_k^{\text{comp}}(m_1, m_2, h, h_\varepsilon, \Delta_1)$ for any $h$ (in cases where $h_k^{\text{comp}}$ is defined).

[9] In the example depicted in Figure 1, strict concavity of the payoff functions implies the following upper

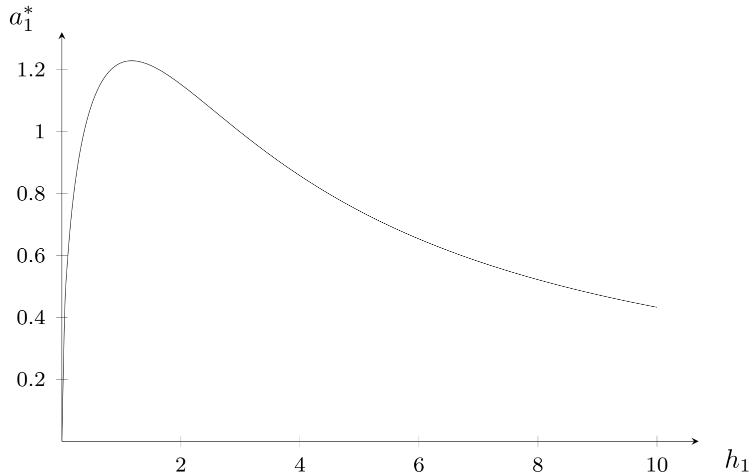


Figure 1: Agent 1's equilibrium effort if $c(a) = \frac{a^2}{2}$, $m_1 + \Delta_1 - m_2 = 0$, $h_2 = 2$, $h_\varepsilon = 1$, $W = 5$.

Comparative statics results with respect to the other model parameters are as expected. First, $\frac{da_j^*}{dW} > 0$: the higher the wage premium, the stronger are effort incentives. Second, $a_j^*$ is increasing in $m_j$ as long as $m_j < m_k + \Delta_k$ but decreasing in $m_j$ for $m_j > m_k + \Delta_k$. Intuitively, the marginal return to effort is higher in a more symmetric race, i.e., for $m_j$ closer to $m_k + \Delta_k$. Similarly, $a_j^*$ is increasing in $m_k + \Delta_k$ if and only if $m_j > m_k + \Delta_k$.

In the model discussed so far, relative reputations determine which agent obtains a wage premium. The resulting reward-to-reputation function is both non-linear (the agent obtains a wage premium if and only if his posterior reputation exceeds that of his rival) and ex ante uncertain (the threshold depends on the realization of the rival agent's performance). It is easy to see that the non-linearity in payoffs drives the result that belief precision has a non-monotonic effect on effort.[10] Suppose that $j$ is promoted if and only if

$$E\left[\eta_j \mid y_j\right] > \underline{\eta}, \tag{9}$$

bounds on $W$:

$$\overline{W}_1 = \frac{(1+h_1)\left(6+h_1+h_1^2\right)\sqrt{\frac{e^\pi}{2}}}{3h_1}, \overline{W}_2 = \frac{3\left(6+h_1+h_1^2\right)\sqrt{\frac{e^\pi}{2}}}{h_1(1+h_1)},$$

with $\min_{h_1} \overline{W}_1 \simeq 10.91$ and $\min_{h_1} \overline{W}_2 \simeq 6.20$. Hence, for $W = 5$, the game has a pure strategy equilibrium for all levels of $h_1$.

[10] See also Bar-Issac and Deb (2014) who, in independent work, allow for a general (deterministic) returns-to-reputation function in the original Holmström (1982/1999) model and show that effort incentives can be non-monotonic in the precision of beliefs.

where $\underline{\eta}$ is a fixed threshold. In Ghosh and Waldman (2010), for instance, firms have two different jobs and choose $\underline{\eta}$ such that assigning an agent to the job in which ability matters more (less) is optimal for the firm when $E[\eta_i \mid y_i] > (<) \underline{\eta}$. This promotion rule corresponds to situations in which there are no constraints on the number of promotions the firm can grant. Keeping all other assumptions unchanged, a model in which promotions are based on the rule in (9) is the limit case of our contest model for $h_k \to \infty$ and $m_k + \Delta_k = \underline{\eta}$ $(k \neq j)$. Since $j$'s equilibrium effort $a_j^*$ in the contest is continuous in $h_k$, the non-monotonicity result from Proposition 1(i) continues to hold:[11]

**Corollary 1** *Suppose* 1 *receives* $W > 0$ *if and only if* $E[\eta_1 \mid y_1] > \underline{\eta}$. *For any values of* $m_1$, $h_\varepsilon$, *and* $\underline{\eta}$, *there exists a threshold* $\widetilde{h}_1 > 0$ *such that* 1*'s equilibrium effort* $a_1^{**}$ *is strictly increasing (decreasing) in* $h_1$ *if and only if* $h_1 < (>) \widetilde{h}$.

As noted, the wage premia associated with promotions would need to be endogenous in a full-fledged model of market-based promotion tournaments (with or without slot constraints). While a full analysis of such a model is beyond the scope of this paper, a few comments on how this would affect our main result are in order. Any impact of $h_j$ on $W_j$ (agent $j$'s wage premium from promotion) would introduce an additional effect of $h_j$ on $a_j^*$. More specifically, if an increase in $h_j$ leads to an increase (decrease) in $W_j$, then $\frac{da_j^*}{dh_j}$ will be higher (lower) than in our model with fixed $W$. However, the qualitative finding that $a_j^*$ is first increasing and then decreasing in $h_j$ remains unchanged as long as the impact of $h_j$ on $W_j$ remains finite.

## 2.3 Discussion: Reputation- versus Performance-Based Promotions

Although most promotion decisions are based on assessments of ability, promotion contests are often modeled as rank-order tournaments à la Lazear and Rosen (1981) in which the

[11] One may wonder whether the uncertainty inherent in the rivalry between multiple agents weakens or strengthens $j$'s effort incentive for a given *expected* threshold, i.e., assuming $m_k + \triangle_k = \underline{\eta}$. The answer follows from the earlier analysis on the impact of uncertainty about the rival's ability summarized in Proposition 1(ii). If $\left(m_j - \underline{\eta}\right)^2 \leq \left(\frac{h_\varepsilon}{h_j + h_\varepsilon}\right)^2 \left(\frac{1}{h_j} + \frac{1}{h_\varepsilon}\right)$, then $\frac{da_1^*}{dh_2} > 0$, which implies that $a_1^{**} = \lim_{h_2 \to \infty} a_1^* > a_1^*$. In this case a reduction of uncertainty increases $j$'s effort incentive by raising the likelihood that the outcome will be close. If $\left(m_1 - \underline{\eta}\right)^2 > \left(\frac{h_\varepsilon}{h_1 + h_\varepsilon}\right)^2 \left(\frac{1}{h_1} + \frac{1}{h_\varepsilon}\right)$, Proposition 1(ii) implies that there exists a threshold $\widehat{h}_2 \in (0, h_2^{comp})$ such that $a_1^{**} < a_1^*$ if and only if $h_2 > \widehat{h}_2$.

principal commits to promote the agent with the better performance, even if another agent has a higher perceived ability.[12] Since the promotion decision is independent of the principal's beliefs about abilities, the previously discussed learning effect plays no role in a performance-based tournament. The precision of beliefs still affects effort incentives through the closeness effect, however, provided agents are uncertain about their abilities.

The effects of increases in $h_1$ or $h_2$ on effort incentives in rank-order tournaments are similar to those of a decrease in noise due to a higher $h_\varepsilon$, as analyzed by Hvide (2002) and Kräkel and Sliwka (2004) in tournaments with known abilities. If prior winning chances are sufficiently symmetric (asymmetric), greater precision increases (decreases) effort incentives. More formally, if the principal promotes agent $j$ if and only if

$$y_j > y_k + \Delta_k,$$

then it is easy to show that equilibrium effort levels in our setting are increasing in both $h_1$ and $h_2$ if and only if[13]

$$(m_1 + \Delta_1 - m_2)^2 < \frac{1}{h_1} + \frac{1}{h_2} + \frac{2}{h_\varepsilon}, \tag{10}$$

which is the counterpart to condition (8) in the reputation-based promotion contest. Turning to the impact of $h_j$ on $j$'s equilibrium effort, denoted by $a_j^{*T}$, this insight implies the following comparative statics results:

**(i)** If $(\Delta_1 + m_1 - m_2)^2 \leq \frac{1}{h_k} + \frac{2}{h_\varepsilon}$, then $\frac{da_j^{*T}}{dh_j} > 0$ for all $h_j$.

**(ii)** Otherwise, for any $(m_1, m_2, h_k, \Delta_k)$ there exists a threshold $h^T > 0$ such that $\frac{da_j^{*T}}{dh_j} > (<) 0$ if and only if $h_j < (>) h^T$.[14]

[12] Under perfect symmetry, a tournament leads to the same decision as a contest based on perceived abilities: if $m_1 = m_2$ and $h_1 = h_2$, then $E\left[\eta_j \mid y_j\right] > E\left[\eta_k \mid y_k\right]$ if and only if $y_j > y_k$. However, when agents prior reputations differ due to disparities in their past achievements, the employee with the highest posterior perceived ability need not be the one with the best recent performance. Adding a fixed bias to the decision rule does not restore equivalence, as the principal updates his beliefs about different agents' abilities at differing rates in a contest based on perceived abilities.

[13] The following discussion assumes that a pure-strategy equilibrium exists, which always holds provided $h_\varepsilon$ is not too large; see Lazear and Rosen (1981, p. 845, fn. 2).

[14] The qualitative relation between $a_j^{*T}$ and $h_j$ is similar in a model of performance-based promotions without slot constraints where $j$ is promoted if and only if $y_j > \overline{y}$. In that case, $a_j^{*T}$ is increasing in $h_j$ if and only if $(\overline{y} - m_j)^2 < \frac{1}{h_j} + \frac{1}{h_\varepsilon}$, which implies that either $a_j^{*T}$ is increasing in $h_j$ for all $h_j$ (if $m_j$ is close enough to $\overline{y}$), or the relation between $a_j^{*T}$ and $h_j$ is as described in (ii).

Comparing reputation- versus performance-based contests, the following key difference appears. In a reputation-based contest, the relation between $j$'s equilibrium effort and $h_j$ is always non-monotonic. For low $h_j$, $j$'s effort is increasing in $h_j$ because of the closeness effect; for high $h_j$, $j$'s effort is decreasing in $h_j$ because of the learning effect, even if the closeness effect is positive. In a performance-based contest, on the other hand, the absence of the learning effect implies that for sufficiently symmetric winning chances, $j$'s effort is always increasing in $h_j$. The relation is non-monotonic only if prior winning chances are sufficiently asymmetric, so that the closeness effect becomes negative at high levels of precision.

# 3 Conclusion

One of the insights from the career concerns literature that has become a received wisdom among economists is that having better information about a worker's ability dampens the worker's implicit effort incentives. Our paper sheds new light on the relation between precision of beliefs and effort incentives. Counter to the received wisdom, we show that when agents exert effort in order to build a reputation for high ability that improves their future promotion prospects, greater precision in beliefs can strengthen effort incentives.

Since employment relationships foster learning about ability (both for the employer and the employee), our analysis predicts a non-monotonic relation between tenure and implicit effort incentives due to future promotion prospects.[15] A testable implication of this is that, holding time to retirement constant, explicit incentives should be non-monotonic in tenure.[16] More specifically, if implicit and explicit incentives are substitutes, then explicit incentives are weakest for employees with intermediate tenure.[17] Our analysis thus suggests that future empirical research should explore the possibility of a non-monotonic relation between tenure

[15]To be more precise, this should hold as long as knowledge about the abilities of newly hired workers is bad enough and knowledge about workers with long enough tenure is good enough so that both the increasing and the decreasing sections of the relation are covered.

[16]In contrast, Gibbons and Murphy (1992), whose model features symmetric learning between firms as in Holmström (1982/1999), predict that explicit incentives are increasing in tenure.

[17]Most theories that combine explicit and implicit incentives predict that the two are substitutes, e.g., Gibbons and Murphy (1992). However, Dewatripont at al. (1997) show that implicit and explicit incentives can be complements, in which case our analysis would predict that explicit incentives are strongest for employees with intermediate tenure. In either case, our theory predicts a non-monotonic relation, which contrasts with existing work.

and explicit effort incentives.

# A Appendix

**Proof of Proposition 1:** Making use of the normality of $\varphi_j(\cdot)$, the first-order condition defining $a_j^*$ can be rewritten as

$$\frac{1}{\sqrt{2\pi}\sigma(h_j, h_k, h_\varepsilon)} \exp\left(-\frac{(m_k + \Delta_k - m_j)^2}{2\sigma^2(h_j, h_k, h_\varepsilon)}\right) \frac{h_\varepsilon}{h_j + h_\varepsilon} W_j = c'\left(a_j^*\right). \tag{11}$$

Applying the implicit function theorem to (11) and rearranging terms yields

$$\frac{da_j^*}{dh_j} = \underbrace{\frac{\exp\left(-\frac{(m_k+\Delta_k-m_j)^2}{2\sigma^2(h_j,h_k,h_\varepsilon)}\right)}{\sqrt{2\pi}\sigma(h_j, h_k, h_\varepsilon)} \frac{W_j \frac{h_\varepsilon}{h_j+h_\varepsilon}}{c''\left(a_j^*\right)}}_{>0} \times \left[-\frac{\frac{\partial\sigma(h_j,h_k,h_\varepsilon)}{\partial h_j}}{\sigma(h_j, h_k, h_\varepsilon)}\left(1 - \frac{(m_k + \Delta_k - m_j)^2}{\sigma^2(h_j, h_k, h_\varepsilon)}\right) - \frac{1}{h_j + h_\varepsilon}\right]. \tag{12}$$

We proceed by examining the limit values of $\frac{da_j^*}{dh_j}$. First, since $\lim_{h_j\to\infty} \frac{\partial P_j\left(a_j;a_j^*,a_k^*,a_k^*\right)}{\partial a_j} = 0$,

$$\lim_{h_j\to\infty} \frac{da_j^*}{dh_j} = 0.$$

Second, we show that $\lim_{h_j\to 0} \frac{da_j^*}{dh_j} = \infty$. Using $\lim_{h_j\to 0} \sigma = \infty$, the limit of (12) can be simplified as follows:

$$\lim_{h_j\to 0} \frac{da_j^*}{dh_j} = \frac{W_j}{\sqrt{2\pi}c''\left(a_j^*\right)} \lim_{h_j\to 0}\left(-\frac{\frac{\partial\sigma(h_j,h_k,h_\varepsilon)}{\partial h_j}}{\sigma^2(h_j, h_k, h_\varepsilon)}\right). \tag{13}$$

Using the expression for $\sigma^2$ in (3) and further simplifying leads to

$$\lim_{h_j\to 0}\left(-\frac{\frac{\partial\sigma(h_j,h_k,h_\varepsilon)}{\partial h_j}}{\sigma^2\left(h_j,h_k,h_\varepsilon\right)}\right)=\lim_{h_j\to 0}\frac{\frac{h_\varepsilon(2h_j+h_\varepsilon)}{2h_j^2(h_j+h_\varepsilon)^2}}{\left(\frac{h_\varepsilon[h_j(h_j+h_\varepsilon)+h_k(h_k+h_\varepsilon)]}{h_jh_k(h_j+h_\varepsilon)(h_k+h_\varepsilon)}\right)^{\frac{3}{2}}}$$
$$=\lim_{h_j\to 0}\frac{\frac{1}{2}\left(\frac{h_\varepsilon}{h_j(h_j+h_\varepsilon)}\right)^2}{\left(\frac{h_\varepsilon}{h_j(h_j+h_\varepsilon)}\right)^{\frac{3}{2}}}=\lim_{h_j\to 0}\frac{1}{2}\left(\frac{h_\varepsilon}{h_j\left(h_j+h_\varepsilon\right)}\right)^{\frac{1}{2}}=\infty. \qquad (14)$$

Since $c''>0$, (13) and (14) together imply that $\lim_{h_j\to 0}\frac{da_j^*}{dh_j}=\infty$.

Next we show that there is a *unique* $h_j^{self}$ such that $\frac{da_j^*}{dh_j}>0$ if and only if $h_j<h_j^{self}$. Since $c''>0$, $\frac{da_j^*}{dh_j}$ has the sign of the sum between square brackets in (12). Using the expression for $\sigma$ in (3) and simplifying, we find that

$$-\frac{\frac{\partial\sigma(h_j,h_k,h_\varepsilon)}{\partial h_j}}{\sigma\left(h_j,h_k,h_\varepsilon\right)}\left(1-\frac{\left(m_k+\Delta_k-m_j\right)^2}{\sigma^2\left(h_j,h_k,h_\varepsilon\right)}\right)-\frac{1}{h_j+h_\varepsilon}$$
$$=\frac{h_k\left(2h_j+h_\varepsilon\right)\left(h_k+h_\varepsilon\right)\left(1-\frac{(m_k+\Delta_k-m_j)^2}{\sigma^2(h_j,h_k,h_\varepsilon)}\right)-2h_j\left[h_j\left(h_j+h_\varepsilon\right)+h_k\left(h_k+h_\varepsilon\right)\right]}{2h_j\left(h_j+h_\varepsilon\right)\left[h_j\left(h_j+h_\varepsilon\right)+h_k\left(h_k+h_\varepsilon\right)\right]}. \qquad (15)$$

$\frac{da_j^*}{dh_j}$ has the same sign as the numerator in (15). The partial derivative of the numerator in (15) with respect to $h_j$ is

$$-2h_k\left(h_k+h_\varepsilon\right)\frac{\left(m_k+\Delta_k-m_j\right)^2}{\sigma^2\left(h_j,h_k,h_\varepsilon\right)}+h_k\left(2h_j+h_\varepsilon\right)\left(h_k+h_\varepsilon\right)\frac{\left(m_k+\Delta_k-m_j\right)^2}{\sigma^2\left(h_j,h_k,h_\varepsilon\right)}\underbrace{\frac{\partial\sigma^2}{\partial h_j}}_{<0}$$
$$-2\left[h_j\left(h_j+h_\varepsilon\right)+h_k\left(h_k+h_\varepsilon\right)\right]-2h_j\left(2h_j+h_\varepsilon\right),$$

which is negative. Together with the limit values $\lim_{h_j\to 0}\frac{da_j^*}{dh_j}=\infty$ and $\lim_{h_j\to\infty}\frac{da_j^*}{dh_j}=0$, this implies that there exists a unique $h_j^{self}>0$ such that $\frac{da_j^*}{dh_j}\gtreqless 0$ if and only if $h_j\lesseqgtr h_j^{self}$.

The limit values $\lim_{h_j\to 0}a_j^*$ and $\lim_{h_j\to\infty}a_j^*$ are implied by the first-order condition in (11). First, since $\lim_{h_j\to 0}\sigma=\infty$, the left-hand-side of (11) goes to 0 for $h_j\to 0$. Thus, $\lim_{h_j\to 0}a_j^*=0$. Second, since $\lim_{h_j\to\infty}\sigma$ is finite but $\lim_{h_j\to\infty}\frac{h_\varepsilon}{h_j+h_\varepsilon}=0$, the left-hand-side of (11) also goes to 0 for $h_j\to\infty$. Hence, $\lim_{h_j\to\infty}a_j^*=0$. This completes the proof of part

(i) of Proposition 1.

To prove part (ii), we apply the implicit function theorem to (11) to obtain

$$\frac{da_j^*}{dh_k} = \underbrace{\frac{\exp\left(-\frac{(m_k+\Delta_k-m_j)^2}{2\sigma^2(h_j,h_k,h_\varepsilon)}\right)}{\sqrt{2\pi}\sigma\left(h_j,h_k,h_\varepsilon\right)} \frac{W_j \frac{h_\varepsilon}{h_j+h_\varepsilon}}{c''\left(a_j^*\right)}}_{>0} \underbrace{\left(-\frac{\frac{\partial\sigma(h_j,h_k,h_\varepsilon)}{\partial h_k}}{\sigma\left(h_j,h_k,h_\varepsilon\right)}\right)}_{>0} \left(1-\frac{\left(m_k+\Delta_k-m_j\right)^2}{\sigma^2\left(h_j,h_k,h_\varepsilon\right)}\right). \quad (16)$$

From (16) it is easy to see that $\frac{da_j^*}{dh_k} > 0$ if and only if $\left(m_k+\Delta_k-m_j\right)^2 < \sigma^2$. Since $\lim_{h_k\to 0}\sigma^2 = \infty$ and $\sigma^2$ is strictly decreasing in $h_k$ (see (3)), this implies that $\frac{da_j^*}{dh_k} > 0$ for all $h_k$ if and only if $\left(m_k+\Delta_k-m_j\right)^2 \leq \lim_{h_k\to\infty}\sigma^2 = \left(\frac{h_\varepsilon}{h_j+h_\varepsilon}\right)^2\left(\frac{1}{h_j}+\frac{1}{h_\varepsilon}\right)$. Otherwise, there exists an $h_k^{comp} > 0$ such that $\frac{da_j^*}{dh_k} \gtreqless 0$ if and only if $h_k \lesseqgtr h_k^{comp}$.

Since $\sigma$ is symmetric in $h_j$ and $h_k$, (14) implies that also $\lim_{h_k\to 0}\left(-\frac{\partial\sigma(h_j,h_k,h_\varepsilon)}{\partial h_k}/\sigma^2\left(h_j,h_k,h_\varepsilon\right)\right) = \infty$. (16) therefore implies that $\lim_{h_k\to 0}\frac{da_j^*}{dh_k} = \infty$. Finally, since $\lim_{h_k\to 0}\sigma = \infty$, (11) implies that $\lim_{h_k\to 0} a_j^* = 0$, and since $\lim_{h_j\to\infty}\sigma$ is finite, (11) implies that $\lim_{h_k\to\infty} a_j^* > 0$ and finite. This completes the proof of part (ii) of Proposition 1.

**QED**

# References


[1] Bar-Isaac, Heski and Joyee Deb (2014), "What is a Good Reputation? Career Concerns with Heterogeneous Audiences," *International Journal of Industrial Organization*, forthcoming.

[2] Bernhardt, Dan (1995), "Strategic Competition and Compensation," *Review of Economic Studies*, 62(2): 315-339.

[3] Casas-Arce, Pablo (2009), "Career Tournaments," *Journal of Economics and Management Strategy*, 19(3): 667-698.

[4] Dewatripont, Mathias, Jewitt, Ian and Jean Tirole (1999a), "The Economics of Career Concerns, Part I: Comparing Information Structures," *Review of Economic Studies*, 66(1): 183-198.

[5] Dewatripont, Mathias, Jewitt, Ian and Jean Tirole (1999b), “The Economics of Career Concerns, Part II: Application to Missions and Accountability of Government Agencies,” *Review of Economic Studies*, 66(1): 199-217.

[6] Fama, Eugene F. (1980), “Agency Problems and the Theory of the Firm,” *Journal of Political Economy*, 88(2): 288-307.

[7] Ghosh, Suman and Michael Waldman (2010), “Standard promotion practices versus up-or-out contracts,” *RAND Journal of Economics*, 41(2): 301-325.

[8] Gibbs, Michael (1995), ”Incentive compensation in a corporate hierarchy,” *Journal of Accounting and Economics*, 19(2/3): 247-277.

[9] Holmström, Bengt (1982), ”Managerial Incentive Schemes: A Dynamic Perspective,” in: Essays in economics and management in honour of Lars Wahlbeck (Swenska Handelshogkolan, Helsinki, Finnland). Reprinted in: *Review of Economic Studies* (1999), 66: 169-182.

[10] Holmström, Bengt (1999), “Managerial Incentive Schemes: A Dynamic Perspective,” *Review of Economic Studies*, 66: 169-182.

[11] Hvide, H. K. (2002), “Tournament Rewards and Risk Taking,” *Journal of Labor Economics*, 20(4): 877-898.

[12] Kahn, Lisa B. (2013), “Asymmetric Information between Employers,” *American Economic Journal: Applied Economics*, 5(4): 165-205.

[13] Kräkel, M. and D. Sliwka (2004), “Risk Taking in Asymmetric Tournaments,” *German Economic Review*, 5(1): 103-116.

[14] Kwon, Illong (2013), “Risk-Taking in Subjective Promotion Tournaments,” *Applied Economics Letters*, 20(13): 1238-1243.

[15] Lazear, Edward P. and Sherwin Rosen (1981), “Rank-Order Tournaments as Optimum Labor Contracts,” *Journal of Political Economy*, 89(5): 841-864.

[16] Martinez, Leonardo (2009), "Reputation, Career Concerns, and Job Assignments," *B.E. Journal of Theoretical Economics (Contributions)*, 9(1), Article 15.

[17] Meyer, Margaret A. (1991), "Learning from Coarse Information: Biased Contests and Career Profiles," *Review of Economic Studies*, 58: 15-41.

[18] Meyer, Margaret A. (1992), "Biased Contests and Moral Hazard: Implications for Career Profiles," *Annales d'Economie et de Statistique*, 25/26: 165-187.

[19] O'Keefe, Mary, Viscusi, Kip and Richard Zeckhauser (1984), "Economic contests: Comparative reward schemes," *Journal of Labor Economics*, 2: 27-56.

[20] Rosen, Sherwin (1982), "Authority, Control, and the Distribution of Eearnings," *Bell Journal of Economics*, The RAND Corporation, 13(2): 311-323.

[21] Rosen, Sherwin (1986), "Prizes and Incentives in Elimination Tournaments," *American Economic Review*, 76(4): 701-715.

[22] Schönberg, Uta (2007), "Testing for Asymmetric Employer Learning," *Journal of Labor Economics*, 25(4): 651-692.

[23] Waldman, Michael (1984), "Job Assignments, Signalling, and Efficiency," *RAND Journal of Economics*, 15(2): 255-267.

[24] Zábojník, Ján and Dan Bernhardt (2001), "Corporate Tournaments, Human Capital Acquisition, and the Firm Size-Wage Relation," *Review of Economic Studies*, 68: 693-716.

[25] Zábojník, Ján (2012), "Promotion Tournaments in Market Equilibrium," *Economic Theory*, 51(1): 213-240.